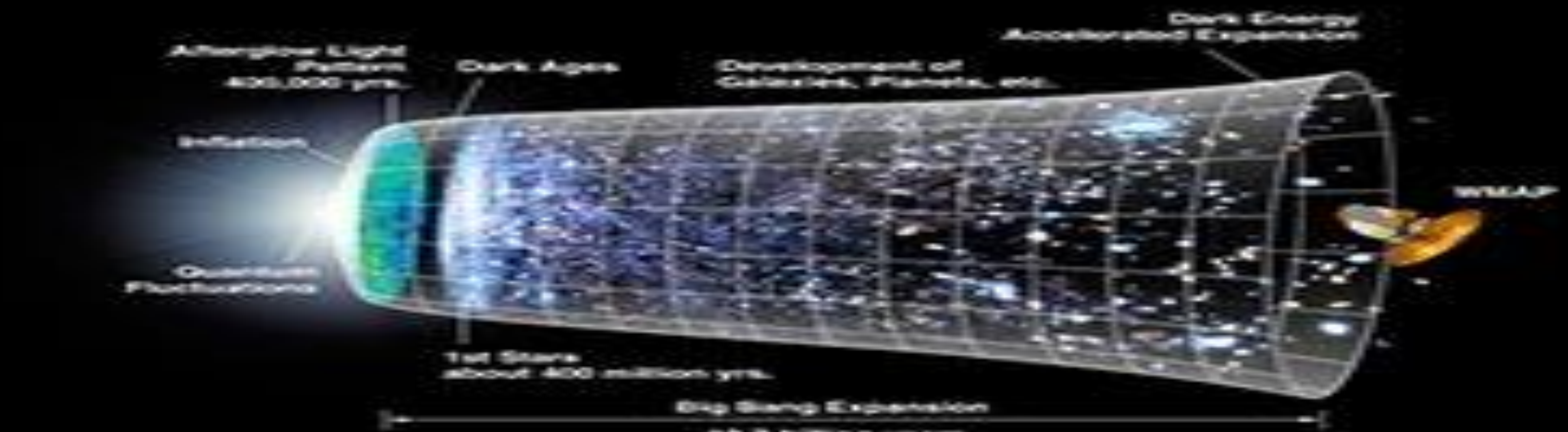
Gordian Knot of Eternal Inflation

Laura Mersini-Houghton **Stockholm 2012**

Inflation: Simple and in Agreement with Observations

- Big Bang Inflation: Universe Bangs into Accelerated Expansion
- Primordial Ripples Seed CMB and Structure
- Flatness, Homogeneity, Monopole Problems Solved.

Or are they?



Inflation: Such Extraordinary Unlikely Initial Conditions!

Quantum Gravity?

GR Breaks Down!

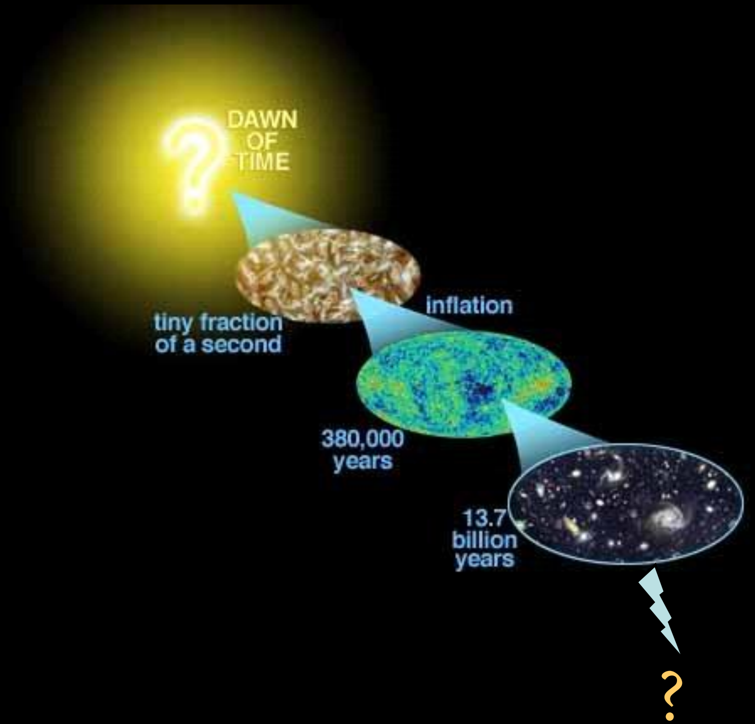
- **What selected these Initial Conditions?**

Chances: 1 /10 ^ {10 ^122} !

Equivalent to Demanding 2 Conditions:

i) Exquisite Homogeneity of Initial Patch,
ii) Slow Roll at High Energy

- **What Banged? Where did this come from?**



The Trouble with I.C.

Probability to start with Big Bang,

P = Exp [S], Entropy S = 1/Lambda.

Extraordinarily Unlikely Event I Anything Else More Likely !

Immediate Troubles:

{ “Too Special” ,   “Arrow of Time” ,   “Boltzman Brains” }

Eternal Inflation to the Rescue ?

Philosophy:

Once Inflation Starts, it never Stops.

No Need to Worry about Far Past I.C.

Reason:

Large Field Fluctuations Have a Nonzero Probability to Occur .

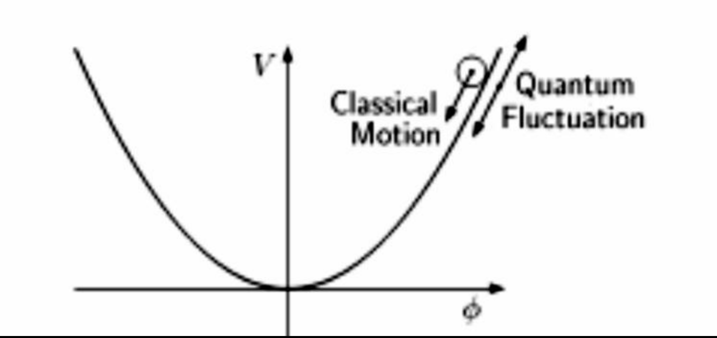
Sure, it's Miniscule but Multiply it With the Newly Produced Volume ?!

Get a Large Probability – More than 100%.

In Fact, It's Infinite !!!!!!

New Problem : The Measure(s) is Infinite !

Eternal Inflation: Basics



$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0\,.$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\left[\frac{\dot{\phi}^2}{2} + V(\phi)\right]\,.$$

Linde : 'Eternally Existing Selfreproducing Inflationary Universe', PLB175 '86.

- Long wavelength Perturbations Frozen. Coarse-Grain Subhorizon Modes. Get Diffusion Source: ' f (x,t)', i.e Langevin Eqn.

$$< f(x_1 t_1) f(x_2 t_2) >= \frac{H^3}{4\pi^2}\delta(t_1 - t_2)\frac{\sin(z)}{z}\,.$$

$$z = aH|x_1 - x_2|.$$

$$D = \frac{H^3}{8\pi^2}.$$

- Leads to Fokker Planck Equation for Probability Distribution Function of Field

$$3H\dot{\phi}(x,t) + V'(\phi) = f(x,t).$$

$$\begin{aligned}\frac{\partial P_\phi}{\partial t} &= \frac{1}{3H}\frac{\partial}{\partial \phi}\left[V'(\phi)P_\phi\right] + \frac{H^3}{8\pi^2}\frac{\partial^2 P_\phi}{\partial^2 \phi} \\ &= \frac{\partial}{\partial \phi}\left[\frac{1}{3H}P_\phi\frac{\partial V}{\partial \phi} + \frac{\partial}{\partial \phi}(DP_\phi)\right]\,.\end{aligned}$$

Diffusion of Large Flucuations :

Starobinsky Estimated Diffusion Term '86. Split the Field:

$$\phi(\mathbf{x},t) = \bar{\phi}(\mathbf{x},t) + \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}}\theta(k - \epsilon a(t)H)\left[a_{\mathbf{k}}\phi_{\mathbf{k}}(t)e^{-i\mathbf{kx}} + a^{\dagger}_{\mathbf{k}}\phi^{*}_{\mathbf{k}}(t)e^{i\mathbf{kx}}\right].$$

SuperH modes

Where:

SubH modes

$$f(\mathbf{x},t) = \epsilon a(t)H^2 \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}}\delta(k - \epsilon a(t)H)\left[a_{\mathbf{k}}\phi_{\mathbf{k}}(t)e^{-i\mathbf{kx}} + a^{\dagger}_{\mathbf{k}}\phi^{*}_{\mathbf{k}}(t)e^{i\mathbf{kx}}\right].$$

Diffusion dominates: $< \phi^2 > \cong D\, t.$

(e.g. Stationary Solutions:)

$$P_\phi \simeq e^{-\frac{(\phi-\phi_0)^2}{2D^2}}$$

Metric Perturbations Track Field Fluctuations: Related by Einstein Equations

Mukhanov et al., IJMPA '87. Use Newtonian gauge for metric:

$$ds^2 \simeq dt^2 - a(x,t)^2 dx^2$$ Perturb it: $$ds^2 \simeq (1 + 2\Phi(x,\tau))\, d\tau^2 - (1 - 2\Psi(x,\tau))\, dx^2$$

-By Einstein Eqns: $\delta\rho/\rho = -2\,\Psi$ and $\Psi = \Phi$. Thus metric PDF:

$$\Delta_\Phi^2 \simeq \langle \Phi^2 \rangle \simeq \frac{2^{1/2} V'^2}{3(3\pi V M_p^3)^{1/2}} \tau$$

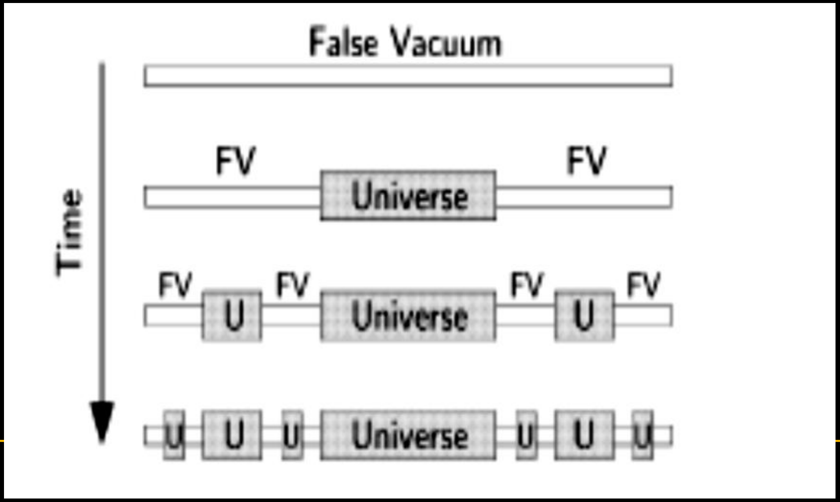
- Spacetime becomes a Fractal, d<3 : (Vilenkin et al. PLB199, '87),

where $$d = 3 - \frac{D\pi^2}{4\phi_0^2}$$, ϕ_0 is boundary of excursions.

Globally a Fractal Spacetime:

- Inflation produces Inhomogeneities. They all add up. On Superhorizon Scales: $\delta\rho/\rho \geq 0(1)$.
- Many Inflationary Pockets Leads to Highly Inhomogeneous Global Volume.
- Claim is EI is Generic: False Vacuum Decay or Chaotic Type

See e.g. Guth 2007, Vilenkin.



What is Wrong with This Picture?

1. **Counting Number of Fluctuation, Not Probability**
2. **Homogeneity Condition Broken** (Trodden et al. PRD61' 98)

- *Probability Occupied by Homog. Volume, Dim=d-3.*

$$P \simeq \int_{-\phi_0}^{\phi_0} P_\phi d\phi \approx \frac{4}{\pi} e^{-\frac{D\pi^2 t}{4\phi_0^2}}$$

- *Thus most Volume is Inhomogeneous (Trivially! it is a Fractal)*

$$V_3^{inh} \simeq e^{dHt} \qquad d = 3 - \frac{D\pi^2}{4\phi_0^2}$$

- *Clearly Condition to Get Inflating Domains Broken:*

$$[\, d - 3 \,] = \{\, m^2/3H^2,\ \lambda^{1/2}/12\pi,\ 4\pi/3\,\Gamma \,\}$$

Flaws in Eternal Inflation Argument :

1. Probability of Large Fluct. $\cong$ Concentration of Field Fluct. that Reach ϕ_* per unit 4-volume :

$$P_\phi = \frac{N(\phi_*)}{V_4}.$$

Thus- $P_\phi \times V_4 = N(\phi)$. Of Course it's ∞ !

Root of Measure Problem !!!

2. Homogeneity Requirement Not Included in the Measure Estimate.

But a Large Fluctuation Can Not Produce an Inflationary Domain if it Arises on an Inhomogeneous Domain. We Already Know Spacetime Is Incredibly Inhomogeneous and Fractal.

Finding a Smooth Homogeneous Domain on this Space Is Highly Unlikely

Our Proposal

- Include Homogeneity Requirement in Measure

$$P_i = P_{st} \times P_\phi$$

Since Field Excursion Random, h-Volumes Independent. Fractal Spacetime. Thus Probability of Field Randomly Finding Smooth Homog. Domains on this 4-Volume is:

- $$P_{st} = \frac{V^h}{V_{tot}} = \left(\frac{x}{R}\right)^{(3-d)}$$ ***e.g.*** $$P = P_\phi \times P_{st} \simeq B e^{-A(\phi^2-\phi^2)} (Hx)^{\frac{m^2}{3H^2}} e^{-\left(\frac{m^2}{3H^2}\right)Ht}$$

For False Vacuum: $P_\phi \cong \Gamma$ ***and*** $[3\text{-}d] = 4\pi/3\ \Gamma$

Conclusions

- Eternal Inflation is Not Eternal: It Ends within time t_f

$$t_f \simeq \frac{1}{(d-3)H}$$

- **New Measure Does Not Violate Unitarity**

- **Satisfy Einstein -Schomulsowsky Relation:**

$D/\mu = k\,T$ where $T \cong 1/H$, $\mu = D\,H$,

$$\mu \simeq \left(\frac{D}{d-3}\right)\frac{1}{t_f}.$$

There May Be Collisions

- Quite interesting Story: Oscillons