Effective Field Theory of Weakly Coupled Inflationary Models

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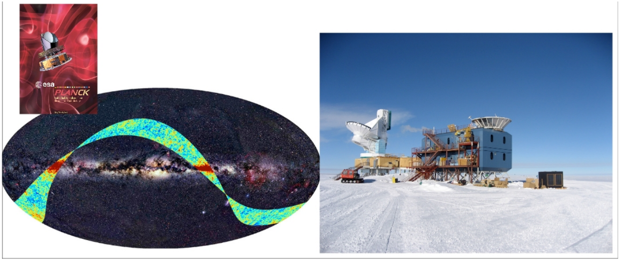
based on 1210.3020 with Rhiannon Gwyn, Gonzalo Palma and Mairi Sakellariadou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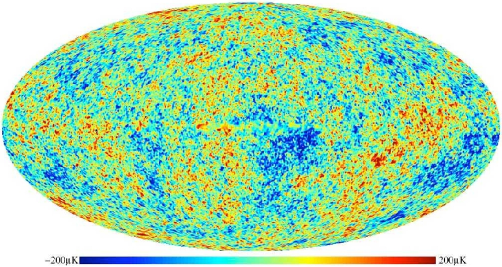
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Outline

When thinking about inflation we usually have a specific model ($\mathcal{L}_i$) in mind and then we study its predictions ($\mathcal{P}_i$).

$$\boxed{\mathcal{L}_i \Longleftrightarrow \mathcal{P}_i \Longleftrightarrow \text{Observations}}$$

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- General statement: Inflation = QFT on a time dependent gravitational background

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- General statement: Inflation = QFT on a time dependent gravitational background
- We want to study perturbations of a scalar field following a time-dependent solution

$$\phi(x,t) = \phi_0(t) + \delta\phi(x,t)$$

What about using the well known techniques of effective field theory ? Creminelli et al. '06, Cheung et al., Weinberg '08, Senatore/Zaldarriaga '09

How to construct the EFT for the fluctuations $\delta\phi$?

Use every possible operator that respects the symmetries of the theory !

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- Time translation invariance is spontaneously broken by the background! Under

$$t \mapsto \tilde{t} = t + \xi^0(x,t)$$
$$\Downarrow$$
$$\boxed{\delta\phi(t) \mapsto \delta\tilde{\phi}(\tilde{t}) = \delta\phi(t) + \dot{\phi}_0(t)\xi^0(t)}$$

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- Spatial diffs remain as a symmetry.

There are two useful gauges for the study of inflaton perturbations:

Mukhanov/Feldman/Brandenberger '92

The unitary gauge: $\boxed{g_{ij} = a^2 e^{\zeta} \delta_{ij} \;\text{ and }\; \phi = \phi_0(t)}$ where all the degrees of freedom are in the metric, and the

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- They are related by a Weyl rescaling.

The effective Lagrangian in unitary gauge

The effective Lagrangian expanded around an quasi dS background will be

$$\mathcal{L} = \frac{1}{2} M_{\rm Pl}^2 R - c(t) g^{00} - \Lambda(t) + \mathcal{L}^{(2)}(\delta g^{00}, \delta K_{\mu\nu}, \delta R_{\mu\nu\kappa\lambda}, \nabla_\mu ; t)$$

Requirement of homogeneous background $\rightarrow c(t), \Lambda(t)$
A subset of terms is

$$\boxed{\mathcal{L} \supset \sum_{n=2}^{\infty} \frac{1}{n!} M_n^4(t) (g^{00}+1)^n}$$

This Lagrangian when expanded out in the unitary gauge correctly reproduces the Lagrangian for the adiabatic curvature perturbation

Chen et al. '08

The effective Lagrangian in flat gauge

Weyl rescaling ! $\boxed{t \mapsto \tilde{t} = t - \pi(x,t)}$

$$\boxed{g^{00} = (1+\dot{\pi})^2 g'^{00} + 2(1+\dot{\pi})\partial_i \pi g'^{0i} + g'^{ij}\partial_i\pi\partial_j\pi}$$

and the effective Lagrangian reads (after lots of steps):

$$\mathcal{L}^{(2)} \supset -M_{\rm Pl}^2 \dot{H}\left[\dot{\pi}^2 - \frac{(\partial\pi)^2}{a^2}\right] + 2M_2^4\dot{\pi}^2 + \mathcal{M}_\pi^2\pi^2$$

$$\mathcal{L}^{(3)} \supset +2M_2^4\left[\dot{\pi}^3 - \dot{\pi}\frac{(\partial\pi)^2}{a^2}\right] - \frac{4}{3}M_3^4\dot{\pi}^3 + \cdots$$

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- $\mathcal{M}_\pi \sim \sqrt{\epsilon}H$: slow roll $\mapsto$ decoupling $\mapsto$ equivalence theorem

Cornwall/Levin/Tiktopoulos '74, Vayonakis '76

Reduction to known models

Setting $\boxed{M_n = 0}$ reproduces the standard single field inflation.

Setting $\boxed{M_n^4 = \dot{\phi}_0^{2n} \frac{\partial^n P}{\partial X^n}\Big|_{\phi=\phi_0}}$ reproduces a $P(X, \phi)$ model.

Keeping only extrinsic curvature terms $\mapsto$ Ghost Inflation

Arkani-Hamed/Creminelli/Mukohyama/Zaldarriaga '04

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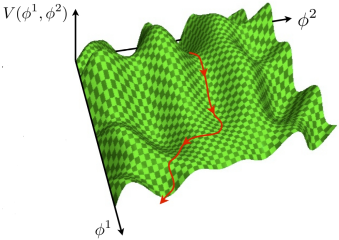
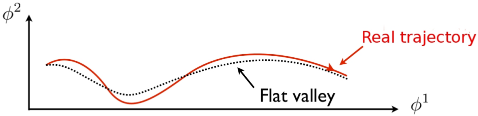
- $\boxed{\text{UV input} \longrightarrow \text{Predictability of the EFT}}$

Very massive fields: Truncate or integrate out ?

Massive fields fall off rapidly outside the horizon ...





Many stringy and SUGRA examples have moduli whose vevs depend on the inflaton !

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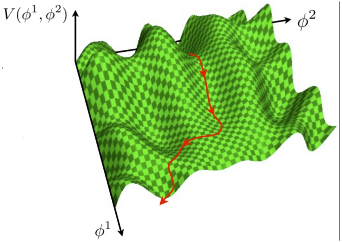

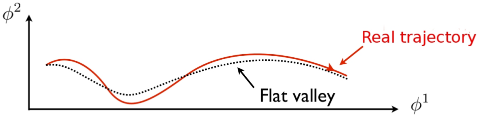

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- INTEGRATE OUT

Achucarro *et al.* '11

Heavy “imprints” in the EFT ?

Main idea:

self-interactions in the IR appear due to mediation of massive particle states in the UV

Baumann/Green '11

Gwyn/Palma/Sakellariadou/SS '12

In other words

$$\boxed{M_n \to M_n \frac{\mathcal{M}^2}{\mathcal{M}^2 - \Box}}$$

$$\Downarrow$$

$$\mathcal{L}^{(n)} \sim \left[(g^{00}+1) \frac{\mathcal{M}^2}{\mathcal{M}^2 - \Box} \right]^{n-1} (g^{00}+1)$$

EFT from integration of massive fields

$$\mathcal{L}_{\mathcal{F}} = \left\{ \dot{\mathcal{F}}^2 - (\nabla\mathcal{F})^2 - \mathcal{M}^2\mathcal{F}^2 - \beta\mathcal{F}^2(g^{00}+1) - \alpha\mathcal{F}(g^{00}+1) \right\}$$

By restricting ourselves to low energies we can integrate out $\mathcal{F}$.

EOM: $$\mathcal{F} = \frac{1}{\mathcal{M}^2-\nabla^2}\left[(g^{00}+1)\frac{\beta}{\mathcal{M}^2-\nabla^2}\right](g^{00}+1)$$

$$\frac{1}{c_s^2} = 1 + \frac{\beta}{\mathcal{M}^2}$$

EFT from integration of massive fields

In general the resulting effective Lagrangian reads:

$$\boxed{\mathcal{L} = -M_{\rm Pl}^2 a^3 \dot{H}\left[\dot{\pi}\left(1 + \frac{2M_2^4}{M_{\rm Pl}^2|\dot{H}|}\frac{\mathcal{M}^2}{\mathcal{M}^2-\tilde{\nabla}^2}\right)\dot{\pi} - (\tilde{\nabla}\pi)^2\right] + \mathcal{O}(\pi^3)}$$

Recall ~~Lorentz~~ so the system may find itself in a non-relativistic regime.

Low energy condition : $\boxed{\omega^2 \ll \mathcal{M}^2 + p^2 \quad \Longrightarrow \quad \Lambda_{\rm UV} = \mathcal{M}/c_{\rm s}}$

where $\frac{1}{c_{\rm s}^2} = 1 + \frac{2M_2^4}{M_{\rm Pl}^2|\dot{H}|}$ the speed of sound.

Dispersion relation

There is an important scale hidden in the dispersion relation!

$$\boxed{\omega^2(p) = c_s^2 p^2 + \frac{(1-c_s^2)}{\mathcal{M}^2 c_s^{-2}} p^4}$$

$p \ll \mathcal{M}$

$\Downarrow$

$$\boxed{\omega \simeq c_s p}$$

$p \gg \mathcal{M}$

$\Downarrow$

$$\boxed{\omega \simeq \frac{p^2}{\Lambda_{\rm UV}} + \frac{1}{2}\Lambda_{\rm new}}$$

$$\mathcal{M} c_s = \Lambda_{\rm new} \ll \Lambda_{\rm UV}$$

Light mode propagates in a medium $\rightarrow c_s \ll 1$.

Phonon excitations vs particle excitations

Non-locality and ghosts

Higher derivative theories: Ostrogradsky instability.
EFT is not such a case. Eliezer/Woodard '89, Sousa/Woodard '03

Pole structure: Biswas/Mazumdar/Siegel '06, Barnaby/Kamran '08

$$\boxed{D(p^2) \propto \frac{1}{\Gamma(p^2)}, \qquad \Gamma(p^2) = p^2 - \omega^2 - \frac{2\mathcal{M}^2\omega^2/c_\mathrm{s}^2}{\mathcal{M}^2+p^2-\omega^2}}$$

Poles: $\omega_+^2(p) \sim \Lambda_{\mathrm{UV}}^2 + \mathcal{O}(p^2)$,

$$\omega_-^2(p) = c_\mathrm{s}^2 p^2 + \frac{(1-c_\mathrm{s}^2)^2}{\mathcal{M}^2 c_\mathrm{s}^{-2}} p^4 + \mathcal{O}(p^6)$$

ω_+^2 has a negative residue! $\boxed{\text{no ghosts} \implies \omega \ll \Lambda_{\mathrm{UV}}}$

Weakly coupled inflation

Scattering of four scalars $\rightarrow$ loss of unitarity $\rightarrow$ strong coupling scale

$$\mathcal{L}_{\rm int} = \frac{(1-c_{\rm s}^2)}{16 M_{\rm Pl}^2 \epsilon H^2} (\nabla \pi_n)^2 \frac{\mathcal{M}^2 c_s^{-2}}{\mathcal{M}^2 - \nabla^2} (\nabla \pi_n)^2$$

$$\mathcal{A}(p_1, p_2 \rightarrow p_3, p_4) = 16\pi \left(\frac{\partial \omega}{\partial p} \frac{\omega^2}{p^2} \right) \sum_\ell (2\ell + 1) P_\ell(\cos\theta) a_\ell$$

optical theorem: $a_\ell + a_\ell^* \leq 1$

$$\boxed{\Lambda_{\rm s.c.} = (8\pi c_{\rm s}^2)^{2/5} \left[\frac{\Lambda_{\rm s.b.}}{\Lambda_{\rm UV}} \right]^{7/5} \Lambda_{\rm UV}}$$

Low derivative EFT: $\Lambda_{\rm s.c.} \sim c_{\rm s}^{5/4} (M_{\rm Pl}^2 |\dot{H}|)^{1/4} \rightarrow c_{\rm s} \gg 10^{-2}$

For horizon crossing in the new phys. regime $H > \Lambda_{\rm new}$,

$$p^2 \to \Lambda_{\rm UV} H, \qquad \partial_t \to H$$

Speed of sound hidden in the ration $\Lambda_{\rm UV}/H$

$$\boxed{\mathcal{P}_\zeta \simeq \frac{2.7}{100}\frac{H^2}{M_{\rm Pl}^2\epsilon}\sqrt{\frac{\Lambda_{\rm UV}}{H}}, \qquad r \simeq 7.6\epsilon\sqrt{\frac{H}{\Lambda_{\rm UV}}}, \qquad f_{\rm NL} \sim \frac{\Lambda_{\rm UV}}{H}}$$

$$\mathcal{P}_\zeta \simeq \frac{1.3}{100}\frac{H^2}{M_{\rm Pl}^2\epsilon c_{\rm s}}, \qquad r \simeq 16\epsilon c_{\rm s}, \qquad f_{\rm NL} \sim \frac{1}{c_{\rm s}^2}$$

No extra parameter! 3 measurements $\to$ 3 parameters

An example

Current bounds on $c_{\rm s}, \epsilon$ altered. e.g.
Let us say that $c_{\rm s} \sim 10^{-2}$

Natural assumption: $\Lambda_{\rm UV} \sim \Lambda_{\rm s.c.} \sim \Lambda_{\rm s.b.}$

If $P_\zeta \sim 10^{-9}$ (WMAP) $\rightarrow \Lambda_{\rm s.b.}/H = \Lambda_{\rm UV}/H = \sim 10^2 \rightarrow f_{NL} \sim 10^2$

$\Longrightarrow \Lambda^2_{\rm new} \sim 10^{-6} H^2 \rightarrow$ horizon crossing in the new phys regime.

$r < 10^{-2}$ (QUIET collaboration) $\rightarrow \epsilon < 10^{-2}$ (weaker than the current bound 10^{-4})

Conclusions

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- Including UV heavy modes corresponds to a derivative expansion of the standard EFT formalism
- Non-trivial change in the dispersion relation - “new physics” before the horizon
- Extends the validity regime of the theory up towards the UV (lift of strong coupling scale)
- Leads to broadened observational windows and novel interpretations for the cosmological observables

Future directions

- New Non-Gaussian signatures
  Scale-dependent self interaction $+$ change in the dispersion relation $=$ novel three point functions ?

Ashoorioon/Chialva/Danielsson, Chialva, Baumann/Green '11

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  Scale-dependent self interaction + change in the dispersion relation = novel three point functions ?

  Ashoorioon/Chialva/Danielsson, Chialva, Baumann/Green '11

- Extension to multi-field case, other types of Non-Gaussianity (e.g. local)?

Thank you !