cosmological consequences of noncommutative spectral geometry

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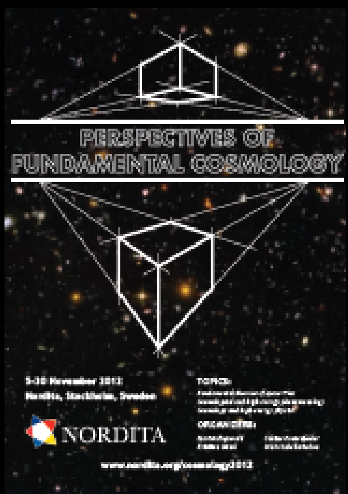

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outline

sakellariadou, 1205.5772 [hep-th]

- motivation
- noncommutative spectral geometry (NCSG)
- *criticisms*

  sakellariadou, stabile, vitiello, PRD 84 (2011) 045026

- phenomenological consequences
- cosmological consequences

  corrections to einstein's equations

  nelson, sakellariadou, PRD 81 (2010) 085038

  gravitational waves

  nelson, ochoa, sakellariadou, PRD 82 (2010) 085021

  nelson, ochoa, sakellariadou, PRL 105 (2010) 101602

  inflation

  nelson, sakellariadou, PLB 680 (2009) 263

  buck, fairbairn, sakellariadou, PRD 82 (2010) 043509

- conclusions

motivation

cosmology

early universe models can be tested with very accurate astrophysical data (CMB), while high energy experiments (LHC) test some of the theoretical pillars of these models

despite the golden era of cosmology, a number of questions:

- origin of DE / DM
- search for natural and well-motivated inflationary model (alternatives...)

...

are still awaiting for a definite answer main approaches:

- string theory
- LQC, SF, WdW, CDT, CS,...

- noncommutative spectral geometry (NCSG)

$$\mathcal{S}^{\rm E} = \int \Big(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^\star R^\star + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha} + \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \Big) \sqrt{g}\, d^4x \ ,$$

particle physics

at low energies:

$$S_{\text{Einstein-Hilbert}} + S_{\text{Standard Model}}$$

GR is governed by diffeomorphism invariance

(outer automorphism)

gauge symmetries are based on local gauge invariance

(inner automorphism)

the difference between these two symmetries may be responsible for difficulty in finding a unified theory of all interactions including gravity in addition:

- why the gauge group is $U(1) \times SU(2) \times SU(3)$?
- why the fermions occupy the particular representations they do ?
- why there are 3 families / why 16 fundamental fermions per each ?
- what is origin of Higgs mechanism and SSB of gauge symmetries?
- what is the Higgs mass and how are explained all fermionic masses?

...

to be answered by the ultimate unified theory of all interactions

noncommutative spectral geometry

connes (1994)

connes, marcolli (2008)

chamseddine, connes, marcolli (2007)

comment

"nothing" to do with $[\mathbf{x}^i, \mathbf{x}^j] = i\theta^{ij}$ used to implement fuzziness of space-time

anti-symmmetric real dxd matrix

however

euclidean version of moyal NCFT → spectral triples formulation of NCG

noncompact noncommutative spin manifold

compact noncommutative spin manifold

gayral, gracía-bondía, iochum, schücker, varilly (2004)

SM of electroweak and strong interactions:

a phenomenological model, which dictates geometry of space-time, so that the maxwell-dirac action functional produces the SM

⇓

geometric space defined by the product $\mathcal{M} \times \mathcal{F}$ of a continuum compact riemannian manifold $\mathcal{M}$ and a tiny discrete finite noncommutative space $\mathcal{F}$ composed of 2 points

geometry: tensor product of an internal geometry for the SM and a continuous geometry for space-time

NCSG approach is based on 3 ansatz:

1. at some energy level, ST is the product $\mathcal{M} \times \mathcal{F}$ of a continuous spin 4dim manifold $\mathcal{M}$ times a discrete noncommutative space $\mathcal{F}$

the noncommutative nature of $\mathcal{F}$ is given by a real spectral triple

$$\mathcal{F} = (\mathcal{A}_{\mathcal{F}}, \mathcal{H}_{\mathcal{F}}, D_{\mathcal{F}})$$

involutive algebra

represented as bounded operators on $\mathcal{H}_{\mathcal{F}}$ providing all information usually carried by a metric structure

self-adjoint operator with compact resolvent in $\mathcal{H}_{\mathcal{F}}$

given by the yukawa coupling matrix which encodes masses of fermions and kobayashi-maskawa mixing parameters

complex Hilbert space carrying a representation of the algebra

focus on $D_{\mathcal{F}}$ instead of $g_{\mu\nu}$

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$$\mathcal{F} = (\mathcal{A}_{\mathcal{F}}, \mathcal{H}_{\mathcal{F}}, D_{\mathcal{F}})$$

spectral geometry given by these product rules

$$\mathcal{A} = \underbrace{C^\infty(\mathcal{M}, C)} \otimes \mathcal{A}_{\mathcal{F}}$$

algebra of smooth complex valued functions on euclidean 4dim $\mathcal{M}$

$$D = \underbrace{D_{\mathcal{M}}} \otimes 1 + \gamma_5 \otimes \underbrace{D_{\mathcal{F}}}$$

Dirac operator on Riemannian spin manifold $\mathcal{M}$

self-adjoint fermion mass matrix

$$\partial\!\!\!/_{\mathcal{M}} = \sqrt{-1}\gamma^\mu \nabla^s_\mu$$

$$\mathcal{H} = \underbrace{L^2(\mathcal{M}, S)} \otimes \underbrace{\mathcal{H}_{\mathcal{F}}}$$

space of square integrable Dirac spinors over $\mathcal{M}$

finite dim space, which describes physical particle d.o.f. (helicity, chirality, flavour, charge, ...)

11. the finite dimensional algebra $\mathcal{A}_{\mathcal{F}}$ is (main input):

left-right symmetric algebra

$$\mathbb{C} \oplus \mathbb{H}_{\mathrm{L}} \oplus \mathbb{H}_{\mathrm{R}} \oplus M_3(\mathbb{C})$$

3x3 complex matrices

algebra of quaternions:

$$\mathbb{H} = \left\{ \begin{pmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{pmatrix} \; ; \; \alpha, \beta \in \mathbb{C} \right\}$$

$$\mathbb{H} \subset M_2(\mathbb{C})$$

II. the finite dimensional algebra $\mathcal{A}_{\mathcal{F}}$ is (main input):

left-right symmetric algebra

$$\mathbb{C} \oplus \mathbb{H}_{\mathrm{L}} \oplus \mathbb{H}_{\mathrm{R}} \oplus M_3(\mathbb{C})$$

3x3 complex matrices

however:

construct a model that accounts for massive neutrinos and neutrino oscillations

⇒ it cannot be a left-right symmetric model

- NCG imposes constraints on the involutive algebras of operators in Hilbert space
- avoid fermion doubling

11. the finite dimensional algebra $A_{\mathcal{F}}$ is (main input):

$$A_{\mathcal{F}} = M_a(\mathbb{H}) \oplus M_k(\mathbb{C})$$

$k = 2a$

algebra of quaternions

algebra of complex $k \times k$ matrices

$k = 4$ is the first value that produces the correct number of fermions in each generation; $k^2 = 16$ in each of 3 generations

chamseddine, connes (2007)

prediction

- the number of fermions is the square of an even integer
- the existence of 3 generations is a physical input

if more particles are found at LHC one <u>may</u> be able to accommodate them by choosing a higher value for k

- commutative geometries:

a real variable described by a real-valued function is given by the algebra of coordinates

- noncommutative geometries:

is represented as operators in a fixed hilbert space

since real coordinates are represented by self-adjoint operators, all information about space is encoded to the algebra $\mathcal{A}_{\mathcal{F}}$, which is related to the gauge group of local gauge transformations

III. Dirac operator connects the two pieces of product geometry nontrivially

idea behind spectral action:

while the topology is encoded by the algebra, all other information (e.g., metric) is encoded by the generalised covariant Dirac operator

$$\mathcal{D}_A = \mathcal{D}_{\mathcal{F}} + A + \epsilon' J A J^{-1}$$

$$A = A^\star \qquad A = \sum_j a_j [\mathcal{D}_{\mathcal{F}}, b_j] \;\; , \;\; a_j, b_j \in \mathcal{A}_{\mathcal{F}}$$

characterised completely by its spectrum

$$J^2 = \varepsilon \;\; , \;\; J\gamma = \varepsilon'' \gamma J$$

$$\varepsilon, \varepsilon', \varepsilon'' \in \{\pm 1\}$$

III. Dirac operator connects the two pieces of product geometry nontrivially

spectral action principal

the action functional depends only on the spectrum of the generalised Dirac operator and is of

it only accounts for the bosonic part of the model

$$Tr(f(D_A/\Lambda))$$

cut-off function

fixes the energy scale

a positive function that falls to zero at large values of its argument, so that

$$\int_0^\infty f(u)u\,du\ , \quad \int_0^\infty f(u)\,du$$

are finite

physical dim of a mass; no absolute scale on which they can be measured

III. Dirac operator connects the two pieces of product geometry nontrivially

<u>spectral</u> action principal

the action functional depends only on the <u>spectrum</u> of the generalised Dirac operator and is of the form:

$$Tr(f(D\!\!\!/_A/\Lambda)) + \frac{1}{2}\langle J\Psi, \mathcal{D}_A\Psi\rangle \ , \quad \Psi \in \mathcal{H}_{\mathcal{F}}{}^+$$

III. Dirac operator connects the two pieces of product geometry nontrivially

spectral action principal

the action functional depends only on the spectrum of the generalised Dirac operator and is of

$$Tr(f(D_A/\Lambda))$$

it only accounts for the bosonic part of the model

the action sums up eigenvalues of D_A which are smaller than Λ

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$$Tr(f(D\!\!\!/_A/\Lambda))$$

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the action sums up eigenvalues of $D\!\!\!/_A$ which are smaller than Λ

evaluate trace with heat kernel techniques, in terms of geometrical seeley-de witt coefficients:

$$\sum_{n=0}^{\infty} F_{4-n}\Lambda^{4-n}a_n \quad \text{where} \quad F(D\!\!\!/_A^2) = f(D\!\!\!/_A)$$

since f is a cut-off function, its taylor expansion at zero vanishes, so the asymptotic expansion of the trace reduces to:

$$\mathrm{Tr}\left(f\left(\frac{D_A}{\Lambda}\right)\right) \sim 2\Lambda^4 f_4 a_0 + 2\Lambda^2 f_2 a_2 + f_0 a_4$$

f plays a rôle through its momenta f_0, f_2, f_4

real parameters related to the coupling constants at unification, the gravitational constant, and the cosmological constant the full lagrangian of SM, minimally coupled to gravity in euclidean form,* is obtained as the asymptotic expansion (in inverse powers of Λ) of the spectral action for the product ST:

chamseddine, connes, marcolli (2007)

* the discussion of phenomenological aspects of the theory relies on a wick rotation to imaginary time

$$
\begin{aligned}
\mathcal{L}_{SM} = & -\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4} g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \\
& \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - \\
& W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - igs_w(\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu (W^+_\mu \partial_\nu W^-_\mu - \\
& W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \tfrac{1}{2} g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \tfrac{1}{2} g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + \\
& g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w (A_\mu Z^0_\nu (W^+_\mu W^-_\nu - \\
& W^+_\nu W^-_\mu) - 2 A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \tfrac{1}{2}\partial_\mu H \partial_\mu H - 2M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \tfrac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\
& \beta_h \left(\tfrac{2M^2}{g^2} + \tfrac{2M}{g} H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right) + \tfrac{2M^4}{g^2}\alpha_h - g\alpha_h M \left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) - \\
& \tfrac{1}{8} g^2 \alpha_h \left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2\right) - gM W^+_\mu W^-_\mu H - \\
& \tfrac{1}{2} g \tfrac{M}{c_w^2} Z^0_\mu Z^0_\mu H - \tfrac{1}{2} ig \left(W^+_\mu (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W^-_\mu (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)\right) + \\
& \tfrac{1}{2} g \left(W^+_\mu (H \partial_\mu \phi^- - \phi^- \partial_\mu H) + W^-_\mu (H \partial_\mu \phi^+ - \phi^+ \partial_\mu H)\right) + \tfrac{1}{2} g \tfrac{1}{c_w} (Z^0_\mu (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) + \\
& M \left(\tfrac{1}{c_w} Z^0_\mu \partial_\mu \phi^0 + W^+_\mu \partial_\mu \phi^- + W^-_\mu \partial_\mu \phi^+\right) - ig \tfrac{s_w^2}{c_w} M Z^0_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) + igs_w M A_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) - \\
& ig\tfrac{1-2 \ldots}{2c \ldots} \ldots \phi^+\phi^-) - \\
& \ldots \\
& \tfrac{1}{2} ig^2 \ldots \phi^+) -
\end{aligned}
$$

the NCSG spectral action offers an elegant geometric interpretation of the SM

$$
\begin{aligned}
& g^2 \tfrac{s_w}{c_w}(2c_w^2 - 1) Z^0_\mu A_\mu \phi^+\phi^- - g^2 s_w^2 A_\mu A_\mu \phi^+\phi^- + \tfrac{1}{2} ig_s \lambda^a_{ij} (\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu - \bar{e}^\lambda (\gamma\partial + m^\lambda_e) e^\lambda - \bar{\nu}^\lambda (\gamma\partial + \\
& m^\lambda_\nu)\nu^\lambda - \bar{u}^\lambda_j (\gamma\partial + m^\lambda_u) u^\lambda_j - \bar{d}^\lambda_j (\gamma\partial + m^\lambda_d) d^\lambda_j + igs_w A_\mu \left(-(\bar{e}^\lambda \gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j \gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j \gamma^\mu d^\lambda_j)\right) + \\
& \tfrac{ig}{4c_w} Z^0_\mu \{(\bar{\nu}^\lambda \gamma^\mu (1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{d}^\lambda_j \gamma^\mu (\tfrac{4}{3}s_w^2 - 1 - \gamma^5) d^\lambda_j) + (\bar{u}^\lambda_j \gamma^\mu (1 - \tfrac{8}{3}s_w^2 + \\
& \gamma^5) u^\lambda_j)\} + \tfrac{ig}{2\sqrt{2}} W^+_\mu \left((\bar{\nu}^\lambda \gamma^\mu (1+\gamma^5) U^{lep}{}_{\lambda\kappa} e^\kappa) + (\bar{u}^\lambda_j \gamma^\mu (1+\gamma^5) C_{\lambda\kappa} d^\kappa_j)\right) + \\
& \tfrac{ig}{2\sqrt{2}} W^-_\mu \left((\bar{e}^\kappa U^{lep\dagger}{}_{\kappa\lambda} \gamma^\mu (1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda} \gamma^\mu (1+\gamma^5) u^\lambda_j)\right) + \\
& \tfrac{ig}{2M\sqrt{2}} \phi^+ \left(-m^\kappa_e (\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa} (1-\gamma^5) e^\kappa) + m^\lambda_\nu (\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa} (1+\gamma^5) e^\kappa\right) + \\
& \tfrac{ig}{2M\sqrt{2}} \phi^- \left(m^\lambda_e (\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa} (1+\gamma^5)\nu^\kappa) - m^\kappa_\nu (\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa} (1-\gamma^5)\nu^\kappa\right) - \tfrac{g}{2}\tfrac{m^\lambda_\nu}{M} H (\bar{\nu}^\lambda \nu^\lambda) - \tfrac{g}{2}\tfrac{m^\lambda_e}{M} H (\bar{e}^\lambda e^\lambda) + \\
& \tfrac{ig}{2}\tfrac{m^\lambda_\nu}{M} \phi^0 (\bar{\nu}^\lambda \gamma^5 \nu^\lambda) - \tfrac{ig}{2}\tfrac{m^\lambda_e}{M} \phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda) - \tfrac{1}{4}\bar{\nu}_\lambda M^R_{\lambda\kappa} (1-\gamma_5)\hat{\nu}_\kappa - \tfrac{1}{4}\overline{\bar{\nu}_\lambda M^R_{\lambda\kappa} (1-\gamma_5)\hat{\nu}_\kappa} + \\
& \tfrac{ig}{2M\sqrt{2}} \phi^+ \left(-m^\kappa_d (\bar{u}^\lambda_j C_{\lambda\kappa} (1-\gamma^5) d^\kappa_j) + m^\lambda_u (\bar{u}^\lambda_j C_{\lambda\kappa} (1+\gamma^5) d^\kappa_j\right) + \\
& \tfrac{ig}{2M\sqrt{2}} \phi^- \left(m^\lambda_d (\bar{d}^\lambda_j C^\dagger_{\lambda\kappa} (1+\gamma^5) u^\kappa_j) - m^\kappa_u (\bar{d}^\lambda_j C^\dagger_{\lambda\kappa} (1-\gamma^5) u^\kappa_j\right) - \tfrac{g}{2}\tfrac{m^\lambda_u}{M} H (\bar{u}^\lambda_j u^\lambda_j) - \tfrac{g}{2}\tfrac{m^\lambda_d}{M} H (\bar{d}^\lambda_j d^\lambda_j) + \\
& \tfrac{ig}{2}\tfrac{m^\lambda_u}{M} \phi^0 (\bar{u}^\lambda_j \gamma^5 u^\lambda_j) - \tfrac{ig}{2}\tfrac{m^\lambda_d}{M} \phi^0 (\bar{d}^\lambda_j \gamma^5 d^\lambda_j)
\end{aligned}
$$

- full SM lagrangian
- majorana mass terms for right-handed neutrinos
- gravitational & cosmological terms coupled to matter

  - EH action with a cosmological term
  - topological term
  - conformal gravity term with the weyl curvature tensor
  - conformal coupling of higgs to gravity

the coefficients of the gravitational terms depend upon the yukawa parameters of the particle physics content

$$\mathscr{S}^{\mathrm{E}} = \int \Big(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^\star R^\star + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha}$$

$$+ \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \Big) \sqrt{g}\, d^4x \,,$$

$$\kappa_0^2 = \frac{12\pi^2}{96 f_2 \Lambda^2 - f_0 \mathfrak{c}} \,,$$

$$\alpha_0 = -\frac{3 f_0}{10\pi^2} \,,$$

$$\gamma_0 = \frac{1}{\pi^2} \left(48 f_4 \Lambda^4 - f_2 \Lambda^2 \mathfrak{c} + \frac{f_0}{4} \mathfrak{d} \right) ,$$

$$\tau_0 = \frac{11 f_0}{60\pi^2} \,,$$

$$\mu_0^2 = 2\Lambda^2 \frac{f_2}{f_0} - \frac{\mathfrak{e}}{\mathfrak{a}} \,,$$

$$\xi_0 = \frac{1}{12} \,,$$

$$\lambda_0 = \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2} \,;$$

$$\mathbf{H} = (\sqrt{a f_0}/\pi)\phi$$

$\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}$ describe possible choices of $\mathcal{D}_{\mathcal{F}}$

yukawa parameters and majorana terms for ν_{R}

$$\mathfrak{a} = \mathrm{Tr}\left(Y^\star_{(\uparrow 1)} Y_{(\uparrow 1)} + Y^\star_{(\downarrow 1)} Y_{(\downarrow 1)} + 3 \left(Y^\star_{(\uparrow 3)} Y_{(\uparrow 3)} + Y^\star_{(\downarrow 3)} Y_{(\downarrow 3)} \right) \right) ,$$

$$\mathfrak{b} = \mathrm{Tr}\left(\left(Y^\star_{(\uparrow 1)} Y_{(\uparrow 1)} \right)^2 + \left(Y^\star_{(\downarrow 1)} Y_{(\downarrow 1)} \right)^2 + 3 \left(Y^\star_{(\uparrow 3)} Y_{(\uparrow 3)} \right)^2 + 3 \left(Y^\star_{(\downarrow 3)} Y_{(\downarrow 3)} \right)^2 \right)$$

$$\mathfrak{c} = \mathrm{Tr}\left(Y_R^\star Y_R \right) ,$$

$$\mathfrak{d} = \mathrm{Tr}\left((Y_R^\star Y_R)^2 \right) ,$$

$$\mathfrak{e} = \mathrm{Tr}\left(Y_R^\star Y_R Y^\star_{(\uparrow 1)} Y_{(\uparrow 1)} \right) ,$$

Y matrics give fermion and lepton masses, as well as lepton mixing

criticisms

- simple almost commutative space

  extend to less trivial noncommutative geometries

- purely classical model

  it cannot be used within EU when QC cannot be neglected

- action functional obtained through perturbative approach in inverse powers of cut-off scale

  it ceases to be valid at lower energy scales (astrophysics)

- model developed in euclidean signature

  physical studies must be done in lorentzian signature

sakellariadou, stabile, vitiello, PRD 84 (2011) 045026

the doubling of the algebra is related to dissipation and the gauge field structure

⇒ the two-sheeted geometry is <u>the construction</u> that can lead to the gauge fields required to explain the SM

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the need to double the degrees of freedom is implicit even in the classical theory when considering the brownian motion

$$m\ddot{x}(t) + \gamma\dot{x}(t) = f(t)$$

this e.o.m. can be derived from a lagrangian in a canonical procedure, using a delta functional classical constraint representation as a functional integral the need to double the degrees of freedom is implicit even in the classical theory when considering the brownian motion

$$m\ddot{x}(t) + \gamma\dot{x}(t) = f(t)$$

x-system: open (dissipating) system

to set up a canonical formalism

constraint condition at classical level introduces new coordinate y

⇒ euler-lagrange eqs:

$$\frac{d}{dt}\frac{\partial L_f}{\partial \dot{y}} = \frac{\partial L_f}{\partial y} \; ; \qquad \frac{d}{dt}\frac{\partial L_f}{\partial \dot{x}} = \frac{\partial L_f}{\partial x}$$

$$L_f(\dot{x}, \dot{y}, x, y) = m\dot{x}\dot{y} + \frac{\gamma}{2}(x\dot{y} - y\dot{x}) + fy$$

⇒

$$m\ddot{x} + \gamma\dot{x} = f \; , \qquad m\ddot{y} - \gamma\dot{y} = 0$$

{x – y} is a closed system

canonical formalism for dissipative systems

the two-sheeted space of NCSG is related to the gauge structure

$$m\ddot{x} + \gamma\dot{x} + kx = 0$$

1dim damped h.o.

$$m\ddot{y} - \gamma\dot{y} + ky = 0$$

oscillator in doubled y-coord

canonical transformation:

$$x_1(t) = \frac{x(t)+y(t)}{\sqrt{2}} , \qquad x_2(t) = \frac{x(t)-y(t)}{\sqrt{2}}$$

vector potential

$$A_i = \frac{B}{2}\epsilon_{ij}x_j \quad (i,j=1,2)$$

$$B \equiv \frac{c}{e}\gamma \ , \ \epsilon_{ii} = 0 \ , \ \epsilon_{12} = -\epsilon_{21} = 1$$

$$L = \frac{m}{2}(\dot{x}_1^2 - \dot{x}_2^2) + \frac{e}{2}(\dot{x}_1 A_1 + \dot{x}_2 A_2) - e\Phi$$

$$\Phi \equiv (k/2/e)({x_1}^2 - {x_2}^2$$

it describes 2 particles with opposite charges $e_1 = -e_2 = e$ in the oscillator potential Φ and constant magnetic field $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$

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1dim damped h.o.

$$m\ddot{y} - \gamma\dot{y} + ky = 0$$

oscillator in doubled y-coord

canonical transformation:

$$x_1(t) = \frac{x(t) + y(t)}{\sqrt{2}} \ , \qquad x_2(t) = \frac{x(t) - y(t)}{\sqrt{2}}$$

vector potential

$$A_i = \frac{B}{2}\epsilon_{ij}x_j \quad (i, j = 1, 2)$$

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$$L = \frac{m}{2}(\dot{x}_1^2 - \dot{x}_2^2) + \frac{e}{2}(\dot{x}_1 A_1 + \dot{x}_2 A_2) - e\Phi$$

$$\Phi \equiv (k/2/e)({x_1}^2 - {x_2}^2$$

- doubled coordinate, e.g. x_2 acts as gauge field component A_1 to which x_1 coordinate is coupled
- energy dissipated by one system is gained by the other one
- gauge field as *bath/reservoir* in which the system is embedded

dissipation, implied by the algebra doubling, may lead to quantum features (loss of information within completely deterministic dynamics may lead to a quantum evolution)

⇛ the NCSG classical construction carries in the doubling of the algebra the seeds of quantisation

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in agreement with 't hooft's conjecture, loss of information (dissipation) in a regime of completely deterministic dynamics may be responsible of the system's QM evolution

$$m\ddot{x} + \gamma\dot{x} + kx = 0 \qquad m\ddot{y} - \gamma\dot{y} + ky = 0$$

$$H = \sum_{i=1}^{2} p_i \, f_i(q) \qquad H = H_{\mathrm{I}} - H_{\mathrm{II}}$$

impose constraint $H_{\mathrm{II}}|\psi\rangle = 0$

it defines physical states and guaranties that H is bounded from below

this constraint introduces information loss

quantisation as a consequence of dissipation (loss of information)

physical states are invariant under time reversal and periodical (τ)

in agreement with 't hooft's conjecture, loss of information (dissipation) in a regime of completely deterministic dynamics may be responsible of the system's QM evolution

$$m\ddot{x} + \gamma\dot{x} + kx = 0 \qquad m\ddot{y} - \gamma\dot{y} + ky = 0$$

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it defines physical states and guaranties that H is bounded from below

this constraint introduces information loss

dissipation term in H of a couple of classical damped-amplified oscillators manifests itself as a geometric phase

$$_{H}\langle\psi(\tau)|\psi(0)\rangle_{H} = e^{i\phi} = e^{i\alpha\pi}$$

in agreement with 't hooft's conjecture, loss of information (dissipation) in a regime of completely deterministic dynamics may be responsible of the system's QM evolution

$$m\ddot{x} + \gamma\dot{x} + kx = 0$$

$$m\ddot{y} - \gamma\dot{y} + ky = 0$$

$$H = \sum_{i=1}^{2} p_i \, f_i(q)$$

$$H = H_{\mathrm{I}} - H_{\mathrm{II}}$$

impose constraint $H_{\mathrm{II}}|\psi\rangle = 0$

it defines physical states and guaranties that H is bounded from below

this constraint introduces information loss

$$\Omega = \sqrt{\frac{1}{m}(k - \frac{\gamma^2}{4m})}$$

due to interaction with environment

$$\langle\psi_n(\tau)|H|\psi_n(\tau)\rangle = \hbar\Omega(n + \frac{\alpha}{2}) = \hbar\Omega n + E_0$$

dissipation term in H of classical damped-amplified oscillators manifests itself as geometric phase and leads to zero point energy

next steps

- include higher order corrections to the spectral action

test accuracy of approximated spectral action by first terms of its asymptotic expansion

$$S^3_a \times S^1_\beta$$

chamseddine, connes (2010)

- find noncommutative space whose limit is $\mathcal{M}_4 \times \mathcal{F}$

remark

spectral action is taken at unification scale;
it fixes the boundary conditions at unification scale

the model lives naturally at unification scale
⟹ the NCSG spectral action provides early universe models

extrapolations to lower energies: via (standard) renormalisation group analysis (*is this correct?*)

extensions to recent universe: considering nonperturbative effects in the spectral action

phenomenology

chamseddine, connes, marcolli (2007)

algebra $\mathcal{A}_{\mathcal{F}}$ of the discrete space $\mathcal{F}$: $M_2(\mathbb{H}) \oplus M_4(\mathbb{C})$

↓

$4^2 = 16$ fermions (the number of states on the Hilbert space) per family

- gauge bosons: inner fluctuations along continuous directions

- Higgs doublet: inner fluctuations along discrete directions

- mass of the Higgs doublet with –tive sign and a quartic term with a + sign ⟹ mechanism for SSB of EW symmetry

$$\mathcal{D} \to \mathcal{D} + A + \epsilon' J A J^{-1}$$

J: anti-linear isometry

$$A = A^\star = \sum_j a_j [\mathcal{D}, b_j] \quad , \quad a_j, b_j \in \mathcal{A}$$

- assuming f is approximated by cut-off function:

normalisation of kinetic terms:

$$\frac{g_3^2 f_0}{2\pi^2} = \frac{1}{4}$$

$$g_2^2 = g_3^2 = \frac{5}{3} g_1^2$$

coincide with those obtained in GUTs

$$\Rightarrow \quad \sin^2 \theta_W = \frac{3}{8}$$

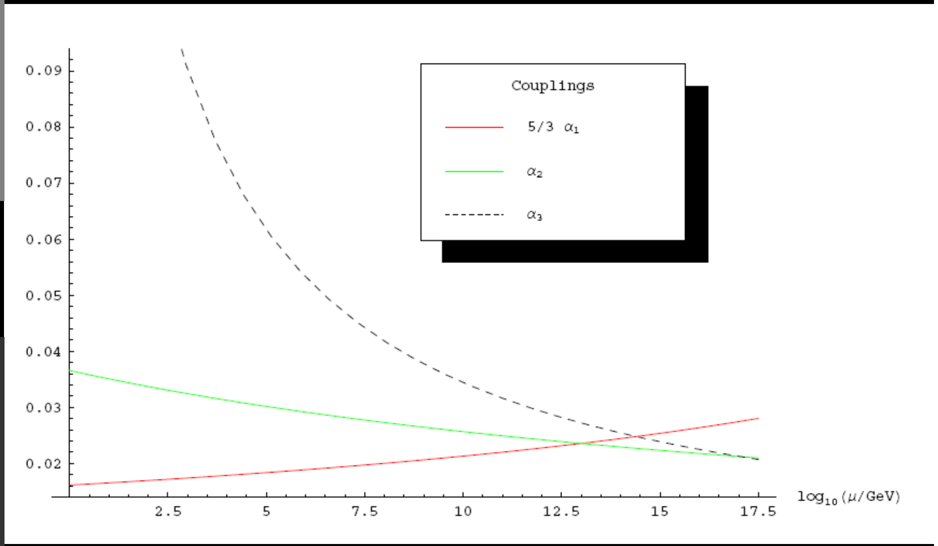
a value also obtained in SU(5) and SO(10)

chamseddine, connes, marcolli (2007)

- assuming big desert hypothesis, the running of the couplings $\alpha_i = g_i^2/(4\pi)$, $i = 1, 2, 3$ up to 1-loop corrections:

$$\beta_i = \frac{1}{(4\pi)^2} b_i g_i^3 \quad \text{with} \quad b = \left(\frac{41}{5}, -\frac{19}{6}, -7\right)$$

the graphs of the running of the three constants α_i do not meet exactly; they do not specify a unique unification energy



- big desert hypothesis approximately valid
- f can be approximated by the cut-off functions but there are small deviations

chamseddine, connes, marcolli (2007)

comment

if you leave g_1 unconstrained, you get the unification scale:

$$\Lambda = 1.1 \times 10^{17}\ \mathrm{GeV}$$

$$m_t \sim 170\ \mathrm{GeV}$$

stephan (2009)

- see-saw mechanism for m_ν with large $m_{\nu_{\text{right-handed}}}$

- constraint on yukawa couplings at unification scale:

$$\sum_\sigma (y_\nu^\sigma)^2 + (y_e^\sigma)^2 + 3(y_u^\sigma)^2 + 3(y_d^\sigma)^2 = 4g^2$$

- mass of top quark:

at unification scale $\Lambda \sim 1.1 \times 10^{17}\text{GeV}$, the $g \sim 0.517$
the RGE predicts $m_{\text{top}} \sim 179\ \text{GeV}$

chamseddine, connes, marcolli (2007)

- in zeroth order approximation: $m_{\text{higgs}} \sim 170\ \text{GeV}$

chamseddine, connes, marcolli (2007)

- sensitive to the value of unification scale
- sensitive to deviations of spectral function from cut-off function

the higgs mass will be determined by considering higher order corrections and incorporating them to the appropriate RGE remark

top quark mass consistent with experimental data, but predicted higgs mass is ruled out

m_{top} less sensitive to ambiguities of unification scale than m_{higgs}

$$S = S_{\text{bosonic}} + S_{\text{fermionic}}$$

determined by an infinite expansion assuming convergence of higher order terms

last developments

such low higgs mass may lead to an instability in V(H)
(quartic coupling of higgs becomes negative at high energy)

- -- big desert hypothesis ruled out (used here)
- -- invalidating positivity of coupling at unification (prediction of spectral action)

there is a real scalar singlet associated with the majorana mass of right-handed neutrino; this field is nontrivally mixed with higgs

responsible for breakdown of symmetry of discrete space:

$$\mathbb{H} \oplus \mathbb{H} \oplus M_4(\mathbb{C}) \longrightarrow \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$$

connes, chamseddine 1208.1030

comment

extension of SM with NCSG approach

model: minimal spectral triple which contains SM particles, new vector-like fermions and a new U(1) gauge subgroup

in addition, a new complex scalar field appears that couples to ν_R, the new fermions and the standard higgs

$m_{H_1} \sim 120$ GeV

$m_{H_2} \geq 170$ GeV

stephan (2009)

- number of fundamental fermions is 16
- algebra of the finite space is $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$
- correct representations of fermions w.r.t $SU(3) \times SU(2) \times U(1)$
- higgs doublet and SSB mechanism
- mass of top quark of around 179 GeV
- see-saw mechanism to give very light left-handed ν 's

chamseddine, connes, marcolli (2007)

problems

- 1-loop RG eqs. for running of gauge couplings and Newton constant do not meet exactly at one point; error within few percent
-- *higher order corrections will change running of all couplings*

- mass of higgs field in zeroth order approximation of spectral action is around 170 GeV
-- *depends on value of gauge couplings at unification scale, which is uncertain*

- no new particles besides those of the SM
-- *problematic if new physics is found at LHC*

- no explanation of the number of generations

- no constraints on values of the Yukawa couplings

chamseddine, connes, marcolli (2007)

cosmological consequences

corrections to einstein's equations

nelson, sakellariadou, PRD 81 (2010) 085038

bosonic action in euclidean signature:

$$\mathcal{S}^{\mathrm{E}} = \int \Big(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^\star R^\star + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha} + \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \Big) \sqrt{g}\, d^4x \;,$$

Yang-Mills

scalar minimal coupling

EH term

Weyl curvature term

cosmological term

scalar mass term

topological, thus nondynamical

$$R^\star R^\star = \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} \epsilon_{\alpha\beta\gamma\delta} \, R^{\alpha\beta}_{\mu\nu} R^{\gamma\delta}_{\rho\sigma}$$

$$\mathbf{H} = (\sqrt{a f_0}/\pi)\phi$$

coupling gravity with matter

scalar quartic potential

from bosonic action, consider the gravitational part including coupling between Higgs field and Ricci curvature

equations of motion

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R + \frac{1}{\beta^2}\delta_{cc}\left[2C^{\mu\lambda\nu\kappa}_{;\lambda;\kappa} + C^{\mu\lambda\nu\kappa}R_{\lambda\kappa}\right] = \kappa_0^2\delta_{cc}T^{\mu\nu}_{\text{matter}}$$

$$\beta^2 \equiv -\frac{1}{4\kappa_0^2\alpha_0}$$

$$\delta_{cc} \equiv [1 - 2\kappa_0^2\xi_0\mathbf{H}^2]^{-1}$$

$$\alpha_0 = \frac{-3f_0}{10\pi^2}$$

$$\delta_{cc} = 1$$

nelson, sakellariadou, PRD 81 (2010) 085038

neglect nonminimal coupling between geometry and higgs

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R + \frac{1}{\beta^2}\delta_{cc}\left[2C^{\mu\lambda\nu\kappa}_{;\lambda;\kappa} + C^{\mu\lambda\nu\kappa}R_{\lambda\kappa}\right] = \kappa_0^2\delta_{cc}T^{\mu\nu}_{\text{matter}}$$

$$\beta^2 \equiv -\frac{1}{4\kappa_0^2\alpha_0}$$

$$\delta_{cc} \equiv [1 - 2\kappa_0^2\xi_0\mathbf{H}^2]^{-1}$$

$$\delta_{cc} = 1$$

FLRW:
weyl tensor vanishes, so NCSG corrections to einstein eq. vanish

corrections to einstein's eqs. will be apparent at leading order, only in anisotropic models

bianchi V

$$g_{\mu\nu} = \mathrm{diag}\left[-1, \{a_1(t)\}^2 e^{-2nz}, \{a_2(t)\}^2 e^{-2nz}, \{a_3(t)\}^2\right]$$

integer

arbitrary functions

same order as standard EH term, but $\propto n^2$ so it vanishes for homogeneous types of bianchi V

$$A_i(t) = \ln a_i(t)$$

for slowly varying functions: small corrections

$$\kappa_0^2 T_{00} =$$
$$-\dot{A}_3\left(\dot{A}_1+\dot{A}_2\right) - n^2 e^{-2A_3}\left(\dot{A}_1\dot{A}_2 - 3\right)$$
$$+\frac{8\alpha_0\kappa_0^2 n^2}{3} e^{-2A_3}\left[5\left(\dot{A}_1\right)^2 + 5\left(\dot{A}_2\right)^2 - \left(\dot{A}_3\right)^2 - \dot{A}_1\dot{A}_2 - \dot{A}_2\dot{A}_3 - \dot{A}_3\dot{A}_1 - \ddot{A}_1 - \ddot{A}_2 - \ddot{A}_3 + 3\right]$$
$$-\frac{4\alpha_0\kappa_0^2}{3}\sum_i\Big\{\dot{A}_1\dot{A}_2\dot{A}_3\dot{A}_i + \dot{A}_i\dot{A}_{i+1}\left(\left(\dot{A}_i - \dot{A}_{i+1}\right)^2 - \dot{A}_i\dot{A}_{i+1}\right)$$
$$+\left(\ddot{A}_i + \left(\dot{A}_i\right)^2\right)\left[-\ddot{A}_i - \left(\dot{A}_i\right)^2 + \frac{1}{2}\left(\ddot{A}_{i+1} + \ddot{A}_{i+2}\right) + \frac{1}{2}\left(\left(\dot{A}_{i+1}\right)^2 + \left(\dot{A}_{i+2}\right)^2\right)\right]$$
$$+\left[\dddot{A}_i + 3\dot{A}_i\ddot{A}_i - \left(\ddot{A}_i + \left(\dot{A}_i\right)^2\right)\left(\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\right)\right] \times \left[2\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\right]\Big\}$$

nelson, sakellariadou, PRD 81 (2010) 085038

same order as
standard EH term,
but $\propto n^2$
so it vanishes for
homogeneous types
of bianchi V

$$A_i(t) = \ln a_i(t)$$

for slowly varying
functions:
small corrections

$$\kappa_0^2 T_{00} =$$

$$-\dot{A}_3\left(\dot{A}_1 + \dot{A}_2\right) - n^2 e^{-2A_3}\left(\dot{A}_1\dot{A}_2 - 3\right)$$

$$+\frac{8\alpha_0\kappa_0^2 n^2}{3} e^{-2A_3}\left[5\left(\dot{A}_1\right)^2 + 5\left(\dot{A}_2\right)^2 - \left(\dot{A}_3\right)^2\right.$$

$$\left.-\dot{A}_1\dot{A}_2 - \dot{A}_2\dot{A}_3 - \dot{A}_3\dot{A}_1 - \ddot{A}_1 - \ddot{A}_2 - \ddot{A}_3 + 3\right]$$

neglecting nonminimal coupling between geometry and higgs field, NCSG corrections to einstein's eqs. are present only in inhomogeneous and anisotropic space-times

$$+\frac{1}{2}\left(\left(\dot{A}_{i+1}\right)^2 + \left(\dot{A}_{i+2}\right)^2\right)\Bigg]$$

$$+\left[\dddot{A}_i + 3\dot{A}_i\ddot{A}_i - \left(\ddot{A}_i + \left(\dot{A}_i\right)^2\right)\left(\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\right)\right]$$

$$\times\left[2\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\right]\Bigg\}$$

nelson, sakellariadou, PRD 81 (2010) 085038

at energies approaching higgs scale, the nonminimal coupling of higgs field to curvature cannot be neglected

e.o.m. (neglecting conformal term, for simplicity):

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \kappa_0^2 \left[\frac{1}{1 - \kappa_0^2 |\mathbf{H}|^2 / 6} \right] T^{\mu\nu}_{\mathrm{matter}}$$

the effect of a nonzero higgs field is to create an effective gravitational constant

nelson, sakellariadou, PRD 81 (2010) 085038

alternatively, consider the effect on e.o.m. for the higgs field in some constant gravitational field

action for pure higgs field:

$$\mathcal{L}_{|\mathbf{H}|} = -\frac{R}{12}|\mathbf{H}|^2 + \frac{1}{2}|D^\alpha \mathbf{H}||D^\beta \mathbf{H}|g_{\alpha\beta} - \mu_0|\mathbf{H}|^2 + \lambda_0|\mathbf{H}|^4$$

for constant curvature, the self interaction of the higgs field is increased:

$$-\mu_0|\mathbf{H}|^2 \rightarrow -\left(\mu_0 + \frac{R}{12}\right)|\mathbf{H}|^2$$

for static geometries, the nominimal coupling of the higgs field to the curvature increases the higgs mass redefine higgs:

$$\tilde{\phi} = -\ln\left(|\mathbf{H}|/(2\sqrt{3})\right)$$

rewrite higgs lagrangian in terms of 4dim dilatonic gravity

$$\mathcal{L}_{|\mathbf{H}|} = -\frac{R}{12}|\mathbf{H}|^2 + \frac{1}{2}|D^\alpha \mathbf{H}||D^\beta \mathbf{H}|\, g_{\alpha\beta} - \mu_0|\mathbf{H}|^2 + \lambda_0|\mathbf{H}|^4$$

$$\mathcal{L}_{\tilde{\phi}} = e^{-2\tilde{\phi}}\left[-R + 6D^\alpha\tilde{\phi}D^\beta\tilde{\phi}g_{\alpha\beta} - 12\left(\mu_0 - 12\lambda_0 e^{-2\tilde{\phi}}\right)\right]$$

link with compactified string models

chameleon models

scalar field with nonminimal coupling to standard matter

NCG

scalar field (higgs) with nonzero coupling to bckg geometry

in a regime where e.o.m. are well approximated by einstein's eqs., the bckg geometry will be (approx.) given by standard matter

→ mass & dynamics of higgs field are explicitly dependent on local matter content

link with chameleon cosmology

gravitational waves in NCSG

nelson, ochoa, sakellariadou, RD 82 (2010) 085021

nelson, ochoa, sakellariadou, PRL 105 (2010) 101602 linear perturbations around minkowski background in synchronous gauge:

linear perturbations around minkowski background in synchronous gauge

$$g_{\mu\nu} = \mathrm{diag}\left(\{a(t)\}^2\left[-1, \left(\delta_{ij} + h_{ij}(x)\right)\right]\right)$$

$$a(t) = 1$$

$$\nabla_i h^{ij} = 0$$

linearised eqs. of motion from NCSG for such perturbations:

$$\left(\Box - \beta^2\right)\Box h^{\mu\nu} = \beta^2 \frac{16\pi G}{c^4} T^{\mu\nu}_{\mathrm{matter}}$$

with conservation eqs: $\frac{\partial}{\partial x^\mu} T^\mu_{\ \nu} = 0$

$$\beta^2 = -\frac{1}{32\pi G \alpha_0}$$

β^2 plays the rôle of a mass, so it must be positive $\Rightarrow \alpha_0 < 0$

$$\alpha_0 = \frac{-3f_0}{10\pi^2}$$

$$\frac{g_3^2 f_0}{2\pi^2} = \frac{1}{4}$$

$$g_3^2 = g_2^2 = \frac{5}{3} g_1^2$$

constraint on curvature squared terms (of different form but of the same order to the weyl term) from orbital precession of mercury

$$\beta > 3.2 \times 10^{-9} \mathrm{m}^{-1}$$

stelle (1978)

energy lost to gravitational radiation by orbiting binaries:

in the far field limit

$$-\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} \approx \frac{c^2}{20G}|\mathbf{r}|^2 \dot{h}_{ij}\dot{h}^{ij}$$

in terms of the quadrupole moment

strong deviations from GR at frequency scale

$$2\omega_c \equiv \beta c \sim (f_0 G)^{-1/2} c$$

set by the moments of the test function f

scale at which NCSG effects become dominant

binaries must have $\omega < \omega_c$

otherwise when $f_0 \longrightarrow 0$ GR cannot be reproduced

$$\omega < \omega_c \quad \text{i.e.} \quad \beta > 2\omega/c$$

PSR J0737-3039	$\beta > 7.55 \times 10^{-13}\ \mathrm{m}^{-1}$
PSR J1012-5307	$\beta > 7.94 \times 10^{-14}\ \mathrm{m}^{-1}$
PSR J1141-6545	$\beta > 3.90 \times 10^{-13}\ \mathrm{m}^{-1}$
PSR B1913+16	$\beta > 2.39 \times 10^{-13}\ \mathrm{m}^{-1}$
PSR B1534+12	$\beta > 1.83 \times 10^{-13}\ \mathrm{m}^{-1}$
PSR B2127+11C	$\beta > 2.30 \times 10^{-13}\ \mathrm{m}^{-1}$

future observations of rapidly orbiting binaries, relatively close to the earth, could improve this constraint by many orders of magnitude

amplitude of effects is proportional $(1 - 2\omega/c\beta)^{-1}$

inflation through the nonminimal coupling between the geometry and the higgs field

nelson, sakellariadou, PLB 680 (2009) 263

buck, fairbairn, sakellariadou, PRD 82 (2010) 043509 proposal: the scalar field of the SM, the higgs field, could play the rôle of the inflaton

but

within GR cosmology, to get the correct amplitude of density perturbations, the higgs mass would have to be 11 orders of magnitude **higher** than its particle physics value

re-examine the validity of this statement within NCSG

$$S^{\mathrm{L}}_{\mathrm{GH}} = \int \left[\frac{1 - 2\kappa_0^2 \xi_0 H^2}{2\kappa_0^2} R - \frac{1}{2}(\nabla H)^2 - V(H)\right] \sqrt{-g}\, d^4x$$

boundary conditions at unification scale Λ

$$V(H) = \lambda_0 H^4 - \mu_0^2 H^2$$

subject to radiative corrections as a function of energy

$$\kappa_0^2 = \frac{12\pi^2}{96 f_2 \Lambda^2 - f_0 \mathfrak{c}}$$

$$\xi_0 = \frac{1}{12}$$

$$\lambda_0 = \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2}$$

$$\mu_0 = 2\Lambda^2 \frac{f_2}{f_0}$$

$$f_0 = \pi^2/(2g^2)$$

a priori unconstrained

yukawa and majorana parameters subject to RGE

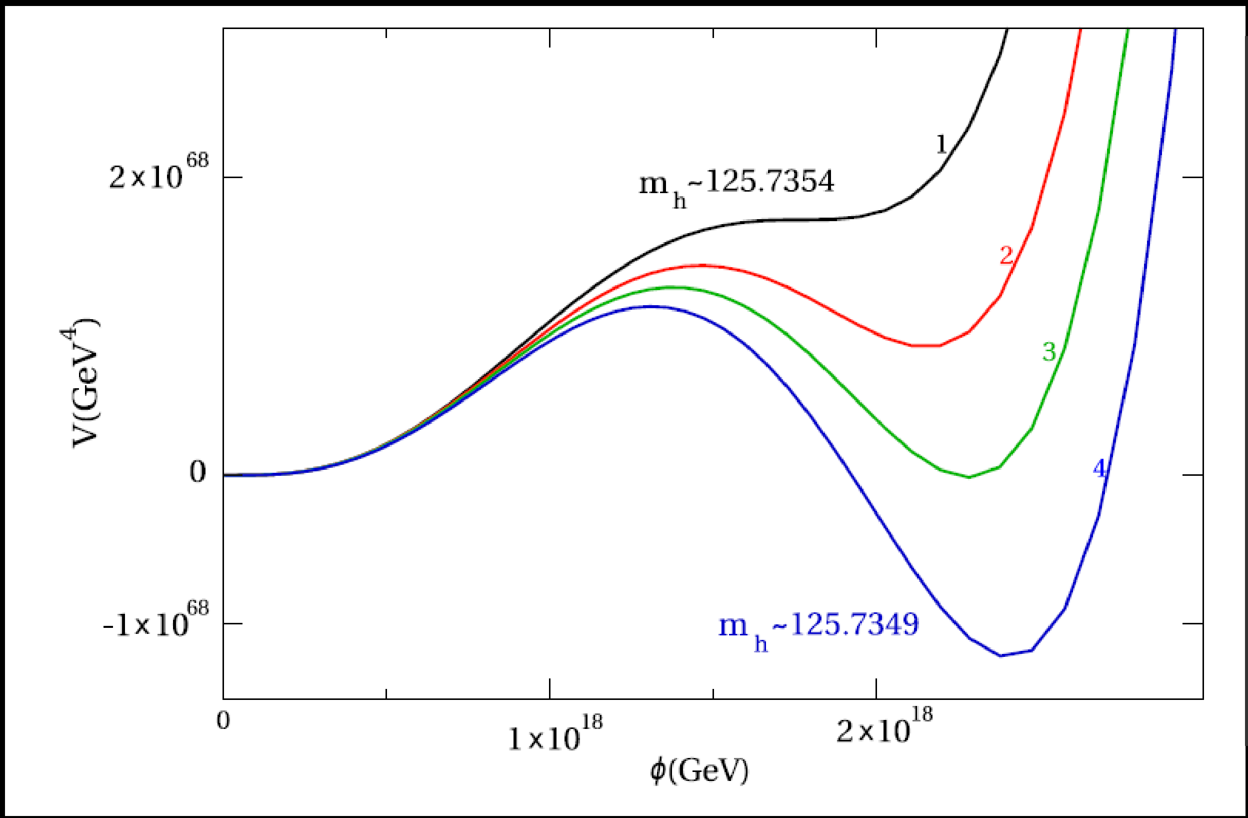
flat potential through 2-loop quantum corrections of SM

classical potential: $$V(H) = \lambda_0 H^4 - \mu_0^2 H^2$$

for very large values of the field $\mathbf{H}$, one needs to calculate the normalised value of the parameters λ_0 and μ_0

effective potential at high energies:

$$V(H) = \lambda(H)H^4$$



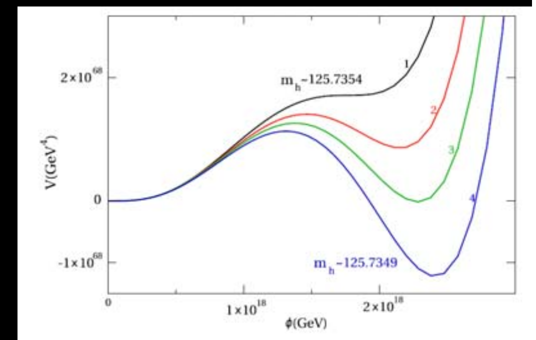
for each value of m_{top} there is a value of m_{higgs} where V_{eff} is on the verge of developing a metastable minimum at large values of **H** and V_{higgs} is locally flattened

approach

- calculate renormalisation of higgs selfcoupling for minimal coupling
- construct effective potential which fits the RG improved potential around flat region
- find modifications in that fit when conformal coupling is included

minimally coupled SM

analytic fit to the higgs potential in the region around the minimum:



$$
\begin{aligned}
V^{\text{eff}} &= \lambda_0^{\text{eff}}(H) H^4 \\
&= [a \ln^2(b\kappa_0 H) + c] H^4
\end{aligned}
$$

$$
a(m_{\rm t}) = 4.04704 \times 10^{-3} - 4.41909 \times 10^{-5} \left(\frac{m_{\rm t}}{\rm GeV}\right) + 1.24732 \times 10^{-7} \left(\frac{m_{\rm t}}{\rm GeV}\right)^2
$$

$$
b(m_{\rm t}) = \exp\left[-0.979261 \left(\frac{m_{\rm t}}{\rm GeV} - 172.051\right)\right]
$$

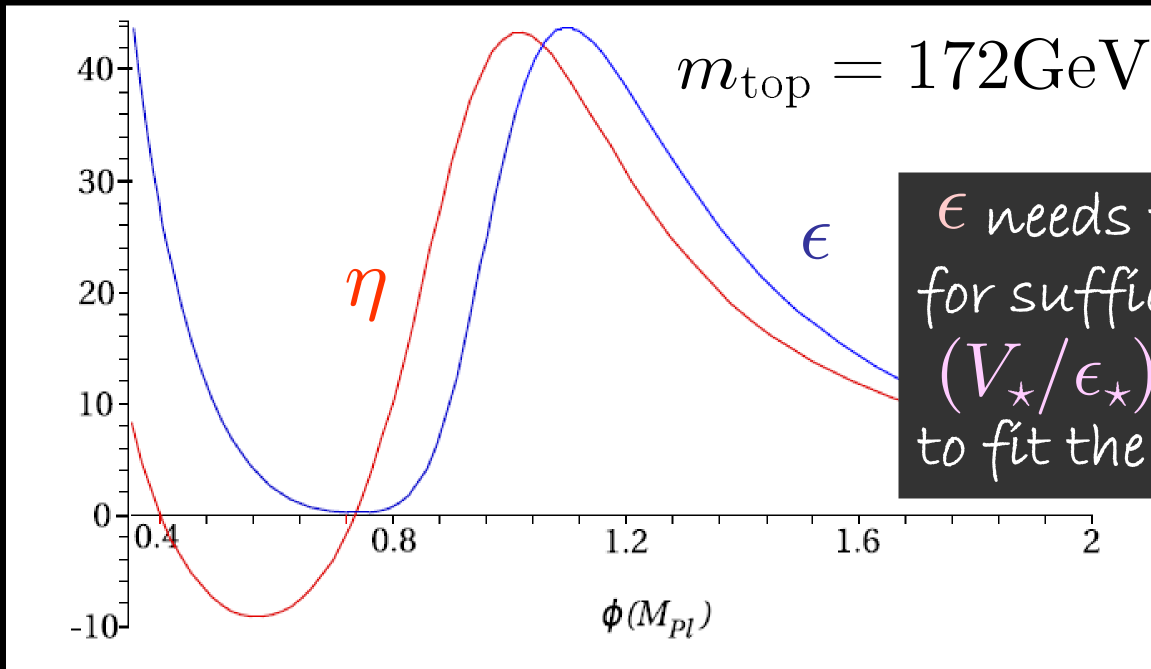
$c = c(m_{\rm t}, m_\phi)$ encodes the appearance of an extremum

an extremum occurs iff $c/a \leq 1/16$

find modifications in the fit when **conformal coupling is included**

for inflation to occur via the higgs field, the top quark mass fixes the higgs masss extremely accurately

buck, fairbairn, sakellariadou, PRD 82 (2010) 043509 the region where the potential becomes flat is narrow, so slow-roll must be very slow

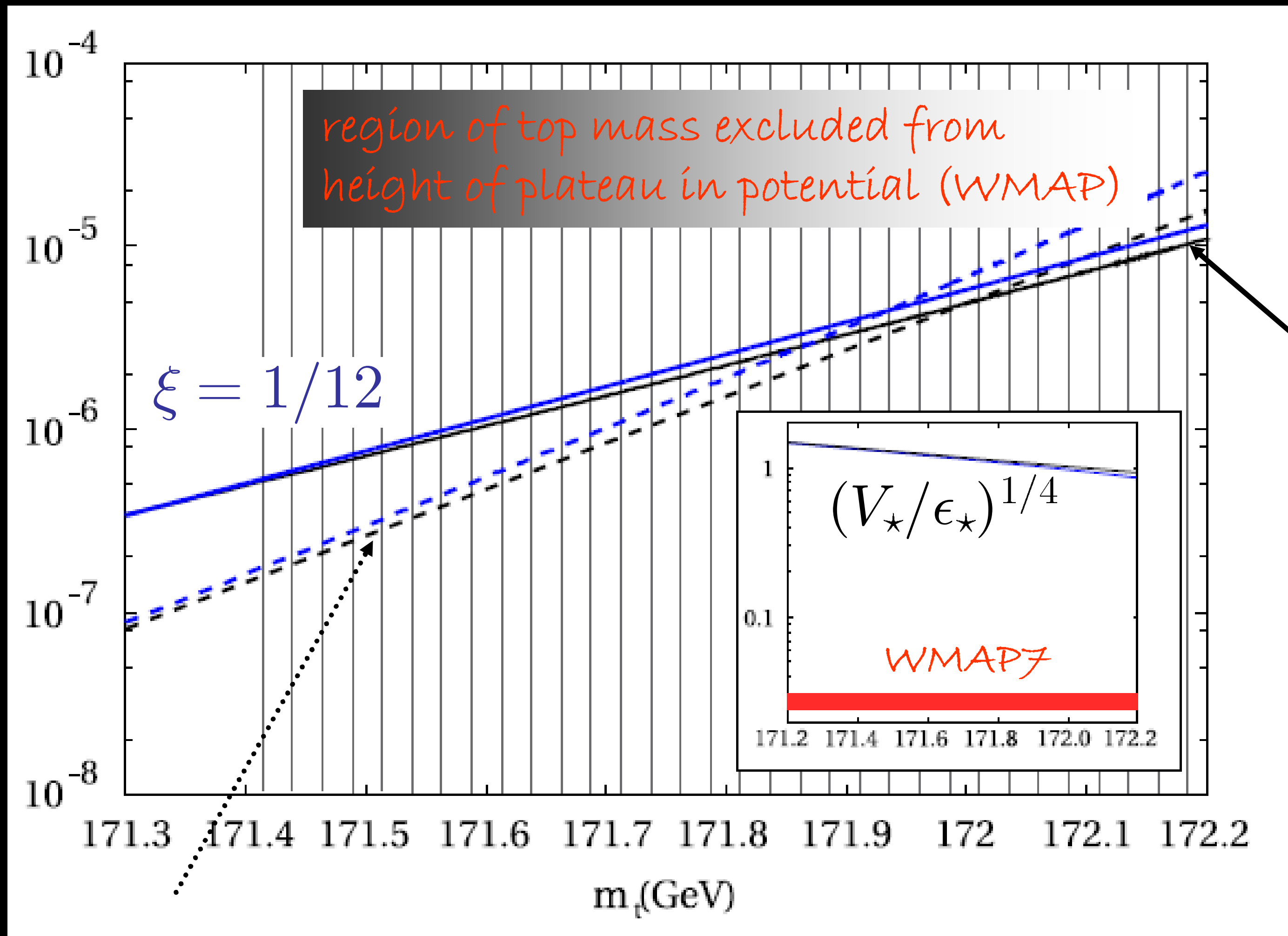


ϵ needs to be too small to allow for sufficient e-folds, and then $(V_\star/\epsilon_\star)^{1/4}$ becomes too large to fit the CMB constraint

$$\left(\frac{V_*}{\epsilon_*}\right)^{\frac{1}{4}} = (2.75 \pm 0.30) \times 10^{-2}\, m_{\rm Pl}$$

$$\epsilon_* \leq 1$$

$$N \sim \epsilon^{-1/2} \mathrm{d}\phi$$



potential in units of κ^{-4} for minimal coupling

maximum value of the first slow-roll parameter at horizon crossing for minimal coupling while the higgs field potential can lead to the slow-roll conditions being satisfied once the running of the self-coupling at two-loops is included, the constraints imposed from the CMB data make the predictions incompatible with the measured value of the top quark

could ξ be away from its conformal value?

there are no nonconformal values for the coupling ξ for which there is a renormalisation group flow towards the conformal value as one runs the SM parameters up in the energy scale

⇓

there are no quantum corrections to ξ, if it is exactly conformal at some energy scale

buchbinder, odintsov, lichtzier (1989)

youngsoo yoon, yongsung yoon (1997)

NCSG provides another (massless) scalar field σ which does not exhibit a coupling to the matter sector:

$$\mathscr{S} = \int \left[\frac{1}{2\kappa^2} R - \xi_H R H^2 - \xi_\sigma R \sigma^2 - \frac{1}{2} (\nabla H)^2 - \frac{1}{2} (\nabla \sigma)^2 - V(H, \sigma) \right] \sqrt{-g}\, d^4 x$$

$$V(H,\sigma) = \lambda_H H^4 - \mu_H^2 H^2 + \lambda_\sigma \sigma^4 + \lambda_{H\sigma} |H|^2 \sigma^2$$

σ cannot lead to successful slow-roll inflationary era

$$\xi_H = \frac{1}{12}\ , \qquad \xi_\sigma = \frac{1}{12}$$

$$\lambda_H = \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2}\ , \qquad \lambda_\sigma = \frac{\pi^2 \mathfrak{d}}{f_0 \mathfrak{c}^2}$$

$$\mu_H = 2\Lambda^2 \frac{f_2}{f_0}\ , \qquad \lambda_{H\sigma} = \frac{2\pi^2 \mathfrak{e}}{\mathfrak{a} \mathfrak{c} f_0}$$

can we accommodate an inflationary era without introducing (by hand) a scalar field?

the arbitrary mass scale in the spectral action for the Dirac operator can be made dynamical by introducing a dilaton field,

$$\mathcal{D}/\Lambda \rightarrow e^{-\Phi/2}\mathcal{D}e^{-\Phi/2}$$

$$\mathscr{S}_{\rm GDH} = \int \sqrt{G}\left[-\frac{1}{2\kappa_0^2}R + \frac{1}{2}\left(1+\frac{6}{\kappa_0^2 f^2}\right)G^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi \;+ G^{\mu\nu}D_\mu H'^* D_\nu H' - V_0\left(H'^* H'\right)\right]d^4x$$

f: dilaton decay constant

$$\Phi = (1/f)\tilde{\sigma}$$

dilaton (Φ) — scalar field ($\tilde{\sigma}$)

could this dilaton field play the rôle of the inflaton?

chamseddine and connes (2006)

conclusions

how can we construct a quantum theory of gravity coupled to matter ?

- purely gravitational theory without matter

or

- gravity-matter interaction is the most important aspect of dynamics

below planck scale: continuum fields and an effective action

NCSG:

the SM fields and gravity are packaged into geometry and matter on a certain kaluza-klein noncommutative space

alain connes' formulation of NCG:

mathematical/physical notions described in terms of spectral properties of operators

mapping

aim: differential geometry ⟷ algebraic terms

topology of space described in terms of algebras

NCSG depends crucially on choice of algebra $\mathcal{A}_{\mathcal{F}}$ represented on a Hilbert space $\mathcal{H}_{\mathcal{F}}$ and the Dirac operator $D_{\mathcal{F}}$

spectral triple $(\mathcal{A}_{\mathcal{F}}, \mathcal{H}_{\mathcal{F}}, D_{\mathcal{F}})$

information on ST geometry

- describes metric aspects of the model and the behaviour of matter fields represented by vectors on Hilbert space
- fluctuations of Dirac operator contain boson fields, including mediators of forces and Higgs field

physical picture of the discrete space

- left/right-handed fermions are placed on two different sheets
- Higgs fields: the gauge fields in the discrete dimensions
- inverse of separation between the two sheets: EW energy scale

picture similar to the randall-sundrum scenario

4dim brane embedded into 5dim manifold as 3dim brane placed at $x_5 = 0 \;,\; x_5 = \pi r_{\text{compactification}}$

meaning of the two-sheeted construction:

- the doubling of the algebra is related to dissipation and gauge field structure, required to explain the SM

- the classical construction of NCSG carries in the doubling of the algebra the seeds to quantisation ('t hooft's conjecture)

NCSG extends notion of commutative spaces, using data encoded in a spectral triple on a space composed by $\mathcal{M} \times \mathcal{F}$

- geometric explanation for SM phenomenology

- framework for early universe cosmology