Spinfoam and Cosmology

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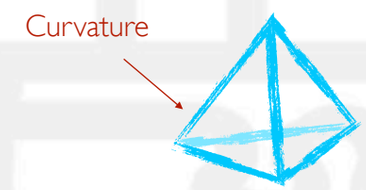
HISTORY OF THE MAIN IDEAS

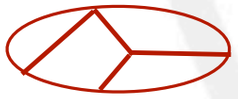
old quantum gravity

- 1957 $Z(q) = \int_{\partial g = q} Dg \;\; e^{iS_{EH}[g]}$ [Misner]
- 1961 Regge calculus → truncation of GR [Regge]
- 1967 W-DeW equation [Wheeler, DeWitt]
- 1971 Spin-geometry theorem → spin network [Penrose]



old LQG

- 1988 Complex variables for GR [Ashtekar]
- 1988 Loop solutions to WdW eq → LQG [Rovelli-Smolin]
- 1994 Spectral problem for geometrical operators → spin network
- 1996 Covariant dynamics → spinfoams [Reisenberger-Rovelli]



- 1999 LQC [Bojowald]

new results

- 2008 Covariant dynamics of LQG [Engle-Pereira-Livine-Rovelli, Freidel-Krasnov]
- 2010 Asymptotic of the new dynamics → recovery of Regge action [Conrady-Freidel, Barrett et al, Bianchi]
- 2011 Cosmological constant → finiteness of the transition amplitudes [Han, Fairbairn-Moesburger]

→

- 2010 Spinfoam Cosmology [Bianchi-Rovelli-FV]

IN THIS TALK

- (covariant) LQG

Input:

- **Dynamics of quanta of spacetime**
  - Group variables on graphs
- **Local product of interaction vertex**
  - Feynman rules
- **Lorentzian signature**

Output:

- **UV finite**
  - Physical cutoff at the Planck scale
- **IR finite**
  - Cosmological const. = q-deformation
- **GR recover in the semiclassical limit!**
- Easy to couple YM fields

- COSMOLOGY

- **Quantum cosmology based on the full quantum theory**
  - Possibility to explore the deep quantum regime in the early universe
  - Possibility to include quantum fluctuations naturally

(covariant) LQG

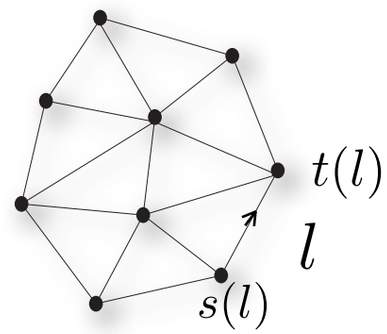
THE THEORY

- The theory is defined by:
  - $\mathcal{H}$ is a Hilbert space
  - $\mathcal{A}$ is an algebra of operators
  - $\mathcal{W} : \mathcal{H} \to \mathbb{C}$ is a map that defines the dynamics

$(\mathcal{H}, \mathcal{A}, \mathcal{W})$ defines a background independent quantum field theory whose classical limit is general relativity

HILBERT SPACE

- Abstract graphs: Γ={N,L}



- Group variables: $\begin{cases} h_l \in SU(2) \\ \vec{L}_l \in su(2) \end{cases}$

- $\tilde{\mathcal{H}} = \bigoplus_{\Gamma} \mathcal{H}_\Gamma$

- Graph Hilbert space: $\mathcal{H}_\Gamma = L_2[SU(2)^L / SU(2)^N]$ separable! [Fairbain, Rovelli, 2004]

  - Gauge transformations $\psi(U_l) \to \psi(V_{s(l)}\, h_l V_{t(l)}^{-1}), \qquad V_n \in SU(2)^N$

- $\mathcal{H} = \tilde{\mathcal{H}} / \sim$ where $\sim$ defined identifying states on subgraph.

- The space $\mathcal{H}_\Gamma$ admits a basis $|\Gamma, j_\ell, v_n\rangle$ labelled by a spin for each link and an intertwiner for each node. These states are called spinnetwork states. They solve the gauge constraint.

Nodes: **discrete quanta of volume** with quantum number v_n *(quanta of space)*

Links: **discrete quanta of area** with quantum number j_l

OPERATOR ALGEBRA

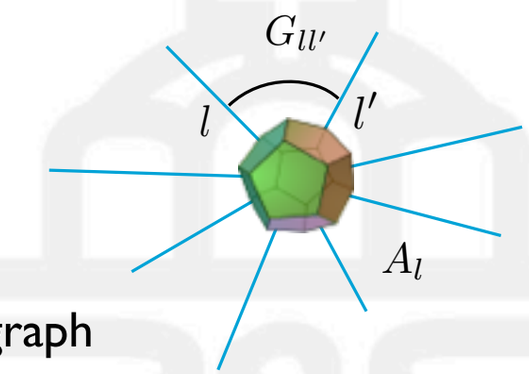
- h_l : "Holonomy of the Ashtekar-Barbero connection along the link"
- $\vec{L}_l = \{L_l^i\}, i = 1, 2, 3$ left-invariant vector field for each link l : $L^i \psi(h) \equiv \frac{d}{dt}\psi(he^{t\tau_i})\Big|_{t=0}$

  gravitational field operator (tetrad)

- Composite operators:
  - Area: $A_\Sigma = \sum_{l\in\Sigma} \sqrt{L_l^i L_l^i}.$
  - Volume: $V_R = \sum_{n\in R} V_n, \qquad V_n^2 = \frac{2}{9}\,|\epsilon_{ijk} L_l^i L_{l'}^j L_{l''}^k|.$
  - Angle: $L_l^i L_{l'}^i.$

- Area and volume (A_l, V_n) form a complete set of commuting observables and have discrete spectra.
- The spin network basis $|\Gamma, j_\ell, v_n\rangle$ diagonalizes the area and volume operators.

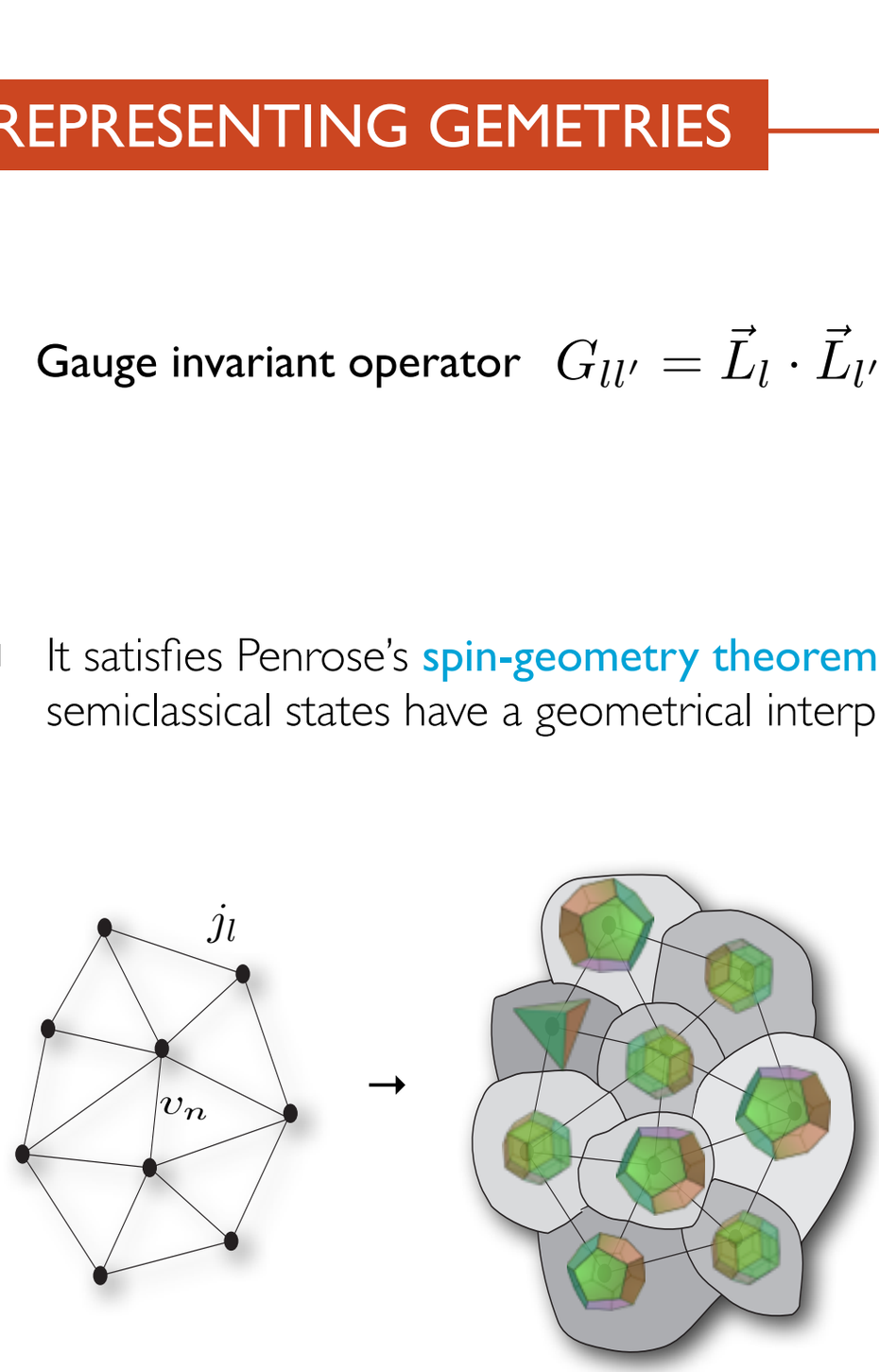
REPRESENTING GEMETRIES



- Gauge invariant operator $G_{ll'} = \vec{L}_l \cdot \vec{L}_{l'}$ with the constraint $\sum_{l \in n} G_{ll'} = 0$

Penrose metric operator on the graph

- It satisfies Penrose's spin-geometry theorem (1971), and Minkowski theorem (1897): semiclassical states have a geometrical interpretation as polyhedra.



Geometry is quantized:

- eigenvalues are discrete
- the operators do not commute
- a generic state is a quantum superposition

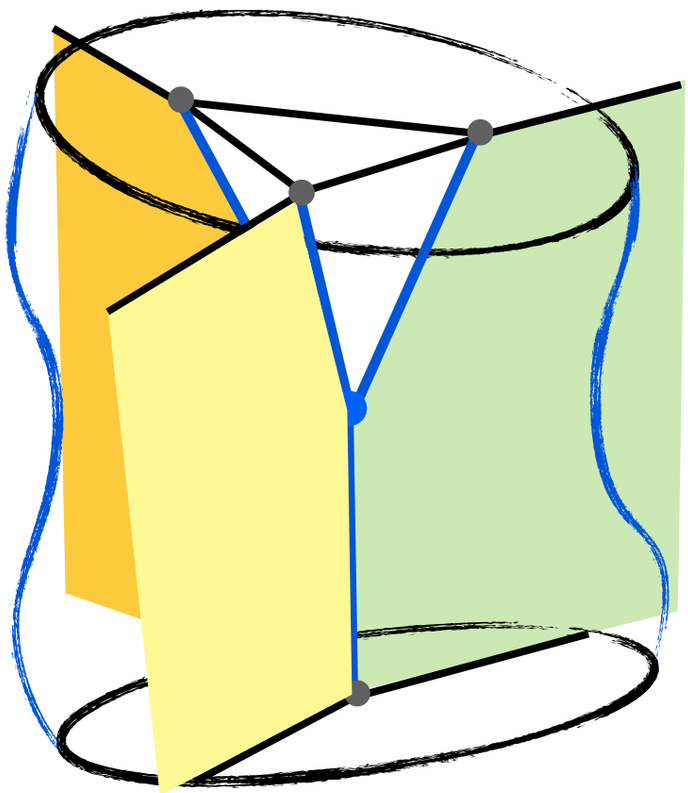
→ coherent states theory

Spinnetwork states represent classical geometries.

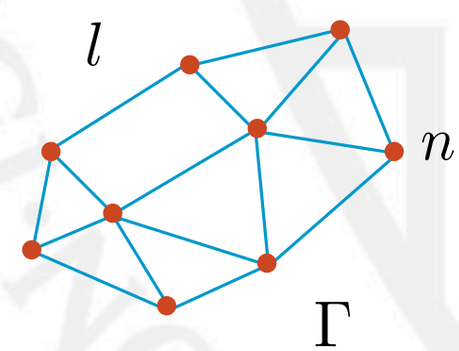
Quantum states of space, rather than states on space.

TRANSITION AMPLITUDE

$$W(q'_{ij}, q_{ij}) \sim \int_{\partial g = q', q} Dq \; e^{i\,S}$$



graph = discretization of space



- expansion: discretization of spacetime

$$\mathcal{C} = \{V, E, F\}, \quad E \subset V \times V, \quad F \subset P(V)$$

Two-complex = the 2-skeleton
of a triangulation (any dimension)

$$\Gamma = \{N, L\}, \quad L \subset N \times N$$

A REMINDER OF THE CLASSICAL THEORY

Tetrads $\quad g_{ab} \to e^i_a \qquad g_{ab} = e^i_a \; e^i_b \qquad e = e_a dx^a \in \mathbb{R}^{(1,3)}$

Spin connection $\quad \omega = \omega_a dx^a \in sl(2,\mathbb{C}) \qquad \omega(e): \qquad de + \omega \wedge e = 0$

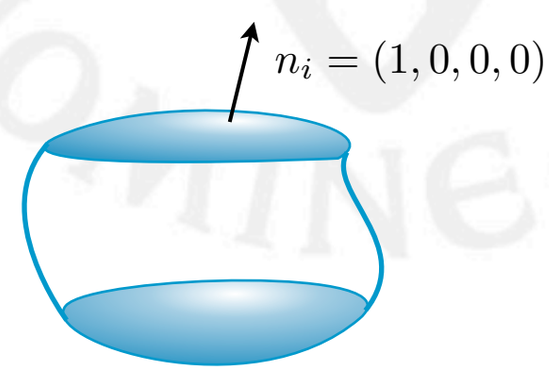
GR action
$$S[e,\omega] = \int e \wedge e \wedge F^*[\omega]$$

GR Holst action
$$S[e,\omega] = \int e \wedge e \wedge F^*[\omega] + \frac{1}{\gamma} \int e \wedge e \wedge F[\omega]$$

Canonical variables
$$\omega, \qquad B = (e \wedge e)^* + \frac{1}{\gamma}(e \wedge e)$$

On the boundary $\quad n_i = e^a_i n_a \qquad n_i e^i = 0 \qquad SL(2,\mathbb{C}) \to SU(2)$

$$B \to (K = nB, L = nB^*)$$



Linear simplicity constrain $\quad \vec{K} + \gamma \vec{L} = 0$

SL(2,C) UNITARY IRRIDUCIBLE REPRESENTATIONS

SU(2) unitary representations: $2j \in Z$ $\quad |j;m\rangle \in \mathcal{H}_j$

SL(2,C) unitary representations: $2k \in N, \quad \nu \in R$ $\quad |k,\nu;j,m\rangle \in \mathcal{H}_{k,\nu} = \bigoplus_{j=k,\infty} \mathcal{H}^j_{k,\nu},$

γ-simple representations: $\nu = \gamma(k+1)$

SU(2) → SL(2,C) map:

$$\begin{aligned} Y_\gamma : \quad & \mathcal{H}_j \quad \to \mathcal{H}_{j,\gamma j} \\ & |j;m\rangle \mapsto |(j,\gamma(j+1))\,;\ j,m\rangle \end{aligned}$$

Image of Y_γ : $\quad j = k \quad$ minimal weight subspace

Main property: $\quad \vec{K} + \gamma \vec{L} = 0 \quad$ weakly on the image of Y_γ

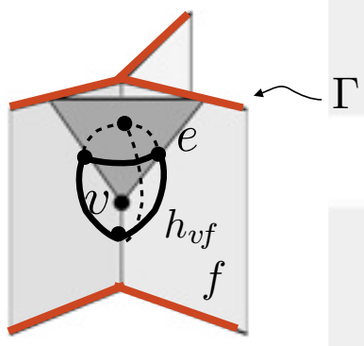
Boost generator $\quad$ Rotation generator

COVARIANT LQG DYNAMICS

- Transition amplitudes $$W_{\mathcal{C}}(h_l) = \int_{SU(2)} dh_{vf} \prod_f \delta(h_f) \prod_v A(h_{vf}) \qquad h_f = \prod_v h_{vf}$$

- Vertex amplitude $$A(h_f) = \sum_{j_f} \int_{SL(2,\mathbb{C})} dg_e \prod_f (2j_f + 1)\, Tr_j[h_f Y_\gamma^\dagger g_e g_{e'}^{-1} Y_\gamma]$$

- Simplicity map $$\begin{aligned} Y_\gamma \;:\; \mathcal{H}_j &\to \mathcal{H}_{j,\gamma j} \\ |j;m\rangle &\mapsto |j,\gamma(j+1);j,m\rangle \end{aligned}$$



2-complex $\mathcal{C}$
(vertices, edges, faces)

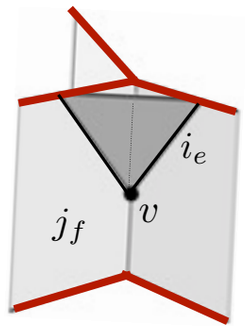
- Minimal area $a_o = 8\pi G\hbar\gamma\frac{\sqrt{3}}{2}$ → natural UV cut-off
- Cosmological constant $\Lambda > 0$ → natural IR cut-off

Han, Fairbairn, Moesburger, 2011
see also Bianchi, Rovelli 20111

Amplitude A^q: SL(2,C) → $\mathrm{SL(2,C)_q}$ network evaluation.

- A^q ~ SL(2,C) Chern-Simon expectation value of the boundary graph of the vertex
- A^q = Vassiliev-Kontsevich invariant of the boundary graph of the vertex graph.

LARGE DISTANCE LIMIT



Two-complex $\mathcal{C}$
(dual to a cellular decomposition)

$$Z = \sum_{j_f, i_e} \prod_f d_{j_f} \prod_v A(j_f, i_e)$$

Theorem :
[Barrett, Pereira, Hellmann, Gomes, Dowdall, Fairbairn 2010]

$$A(j_f, i_e) \underset{j \gg 1}{\sim} e^{iS_{\text{Regge}}} + e^{-iS_{\text{Regge}}}$$

[Freidel Conrady 2008, Bianchi, Satz 2006, Magliaro Perini, 2011]

$$W_{\mathcal{C}} \xrightarrow[j \gg 1]{} e^{iS_\Delta} \qquad Z_{\mathcal{C}} \xrightarrow[C \to \infty]{} \int Dg \; e^{iS[g]}$$

Theorem :
[Han 2012]

$$A^q(j_f, i_e) \underset{j \gg 1, q \sim 1}{\sim} e^{iS^\Lambda_{\text{Regge}}} + e^{-iS^\Lambda_{\text{Regge}}} \qquad q = e^{\Lambda \hbar G}$$

STRUCTURE OF THE THEORY

$$
\begin{tikzcd}
\text{Exact quantum gravity transition amplitudes } W(h_l) \arrow[r, "\hbar\to 0"] & \text{General relativity Hamilton function } S(q) \\
\boxed{\text{LQG transition amplitudes } W_{\mathcal{C}}(h_l)} \arrow[u, "\mathcal{C}\to\infty"] \arrow[r, "\hbar\to 0"] & \text{Regge Hamilton function } S_{\Delta}(l_{i_b}) \arrow[u, "\Delta\to 0"] \arrow[l, "\text{Classical limit}"']
\end{tikzcd}
$$

Continuous limit

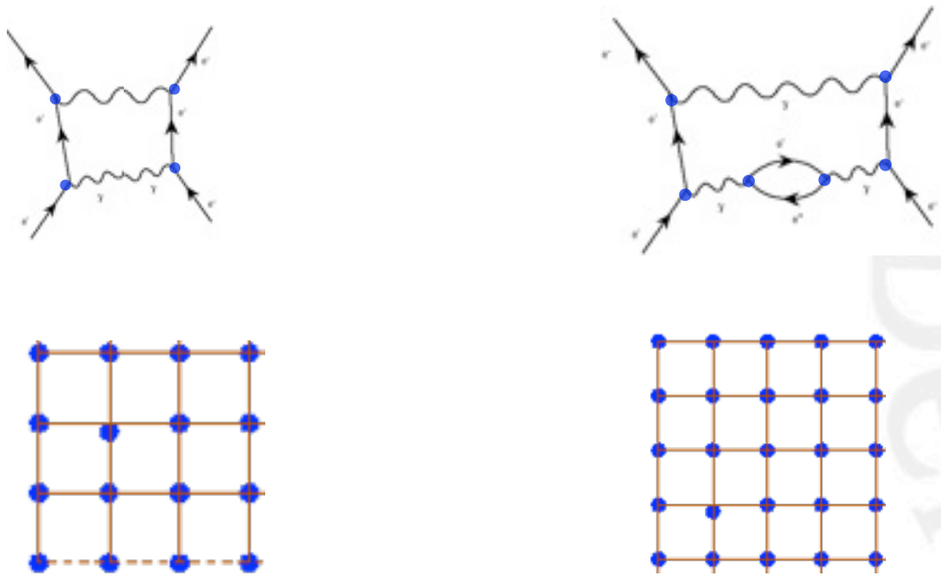
- No critical point
- No infinite renormalization
- Physical scale: Planck length

Regime of validity of the expansion:

$$L_{Planck} \ll L \ll \sqrt{\frac{1}{R}}$$

CONVERGENCE BETWEEN THE QED AND THE QCD PICTURES

- *All* physical QFT are constructed via a truncation of the *d.o.f.* (cfr: particles in QED, lattice in QCD)
- All physical calculation are performed within a truncation.
- The limit in which all *d.o.f.* is then recovered is pretty different in QED and QCD:



QUANTUM GRAVITY Diff invariance ! [Rovelli, Ditt-invariance, 2011]

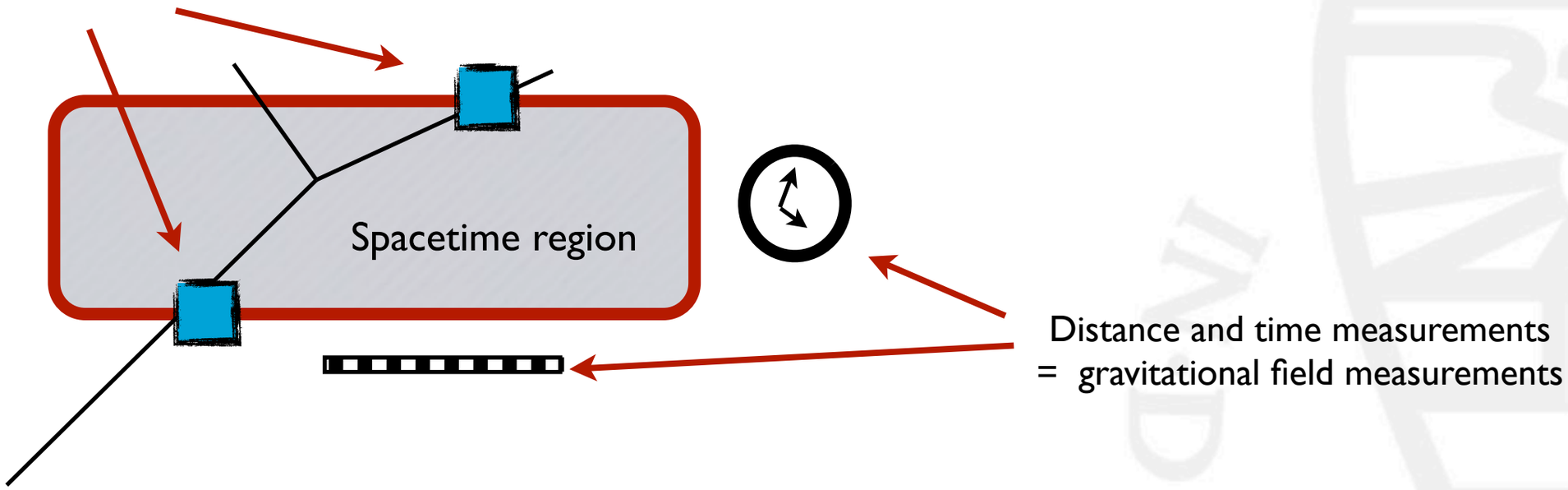
Lattice site = small region of space = excitations of the gravitational field = quanta of space = quanta of the field

applications

APPLICATIONS: GRAVITONS

Boundary values of the gravitational field = geometry of box surface = distance and time separation

Particle detectors = field measurements



Distance and time measurements are field measurements like the other ones: **boundary data** of the problem.

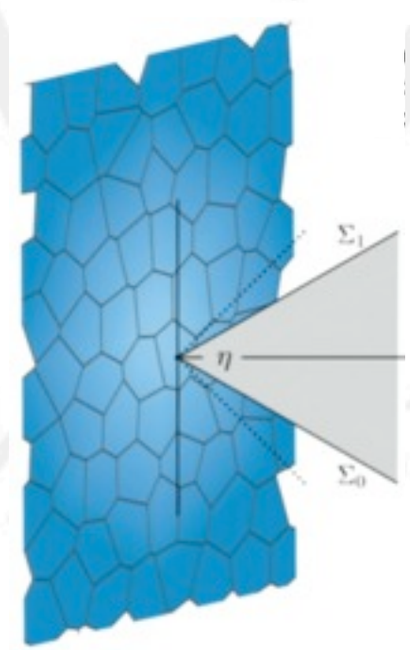
- Gravitational waves — The free graviton propagator is recovered [Alesci, Bianchi Magliaro Perini 2009 , Ding 2011]
- Scattering — The Regge n-point function is recovered in the large j limit [Zhang, Rovelli 2011]

APPLICATIONS: BLACK-HOLE ENTROPY

- Local near-horizon geometry is Rindler geometry, where the stationary killing field is boost
- Local equilibrium time evolution is the generator of boosts aK
- Local energy is $\mathrm{A}a$ $\quad E = \langle j|aK|j\rangle = \gamma j = \dfrac{Aa}{8\pi G}$ [Frodden, Gosh, Perez]
- State $|\Psi\rangle = \otimes_f |j\rangle$ is thermal at temperature $T = \dfrac{a\hbar}{2\pi}$ [Bianchi]
- Entropy $\quad dS = \dfrac{dE}{T} = \dfrac{dAa}{8\pi G}\dfrac{2\pi}{a\hbar} = \dfrac{dA}{4\hbar G}$ [Bianchi, 2012]

$$S = \frac{A}{4\hbar G}$$

recovered in the full non-pertubative theory



APPLICATIONS: SPINFOAM COSMOLOGY

canonical / covariant quantization

gravity → quantum gravity

$$R_{\mu\nu} - \tfrac{1}{2}\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

$$W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma\ \psi_v)(\mathbb{1})$$

symmetry reduction

cosmology → quantum cosmology

$$ds^2 = dt^2 - a^2(t)\, d^3\vec{x}$$

+ perturbations

Friedmann eq. for a(t)
eq. for perturbations

CANONICAL QUANTIZATION: *LQC*

(lectures by Bojowald and Calcagni)

- Hamiltonian quantum theory for $a(t)$ $\longrightarrow$ $\Psi(a) \in \mathcal{H}_{LQC}$

$$H\,\Psi = 0$$

- *quantum theory* $\longrightarrow$ *effective theory*

effective quantum Friedmann eq.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho\left(1 - \frac{\rho}{\rho_{cr}}\right)$$

- *Fluctuations on an effective quantum background*

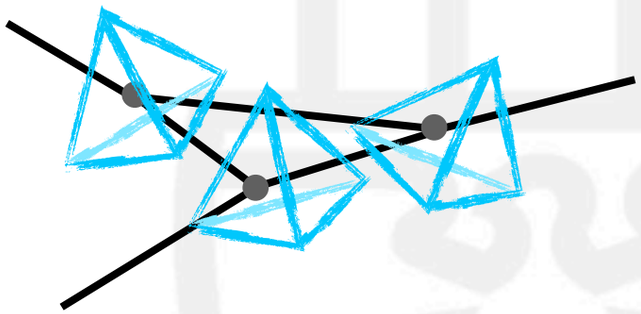
Mukhanov-Sasaki eq.
for scalar/tensor perturbations

$$v'' - \left(1 - 2\frac{\rho}{\rho_{cr}}\right)\nabla^2 v - \frac{z''}{z}v = 0$$

[Catilleau, Grain, Barrau, Linsefors, FV]

LQG KINEMATICS

$$W(q',q) = \langle q|\ e^{\frac{i}{\hbar}\,H(q',q)\,t}\ |q'\rangle$$



- ***States labelled by quantum numbers of a discrete geometry***
  - *spinnetwork states*

$$|\Gamma, j_\ell, v_n\rangle \in \tilde{\mathcal{H}} = \bigoplus_\Gamma \mathcal{H}_\Gamma$$

- boundary states defined on graphs in LQG
- these states capture a(t) + anisotropies + roughest inhomogeneities

concretely...

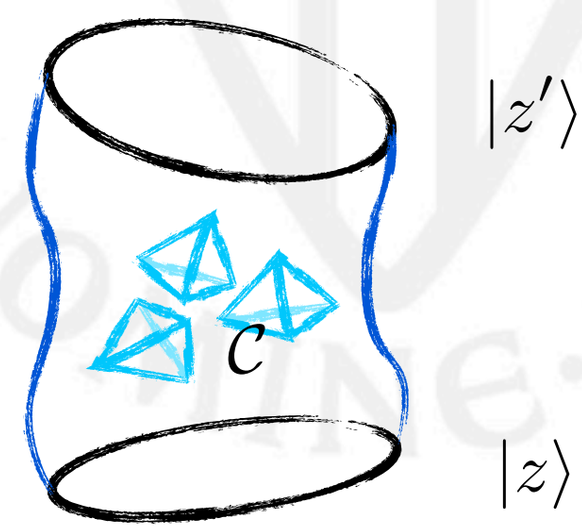
- the states on the graph are well defined in full LQG — Bianchi Magliaro Perini (Ashtekar...Thiemann...)
- transition amplitudes — Engle Pereira Rovelli Livine.. Freidel... Alesci,.. Lewandowski, Kaminski, Kisielowski
- classical limit — Barrett, Dowall, Fairbain, Gomes, Hellmann, Alesci...

- idea:
  - **coherent states** superposition of spinnetwork states, but peaked on a given geometry $\langle W_{\mathcal{C}} | z, z' \rangle$

$$|a, \dot{a}\rangle = \sum_{j_\ell, v_n} C_{j_\ell, v_n} \, |\Gamma, j_\ell, v_n\rangle$$

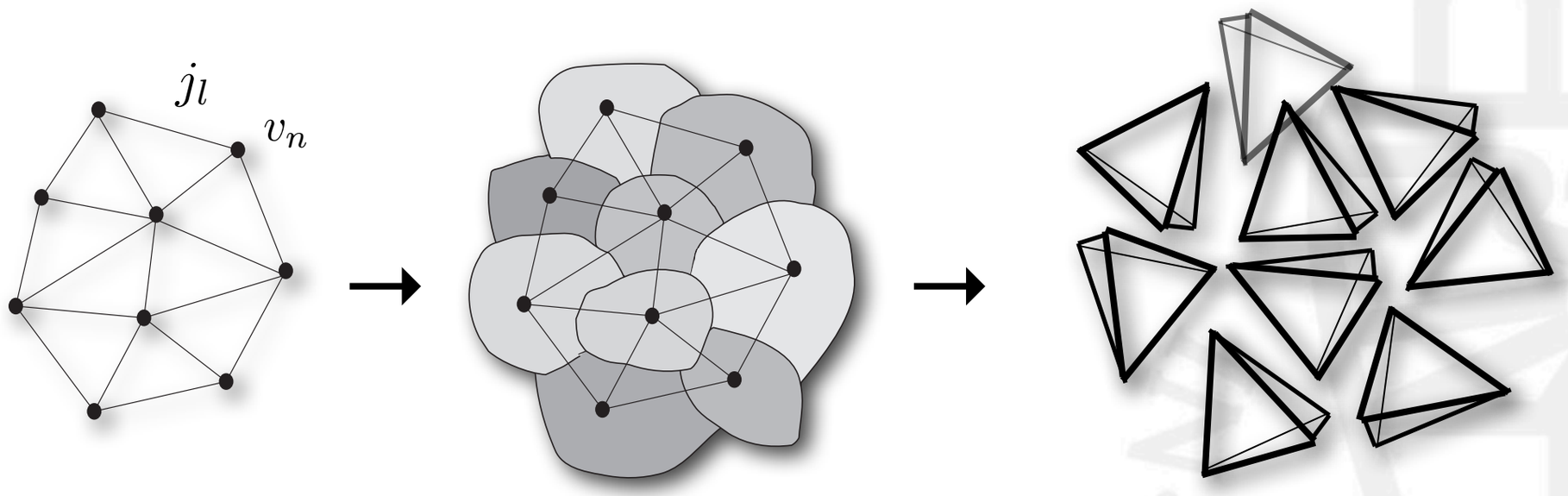
  - **RLRW**

$$|a, \dot{a}\rangle \longrightarrow |z\rangle \qquad z = \alpha \, \dot{a} + i\beta a$$



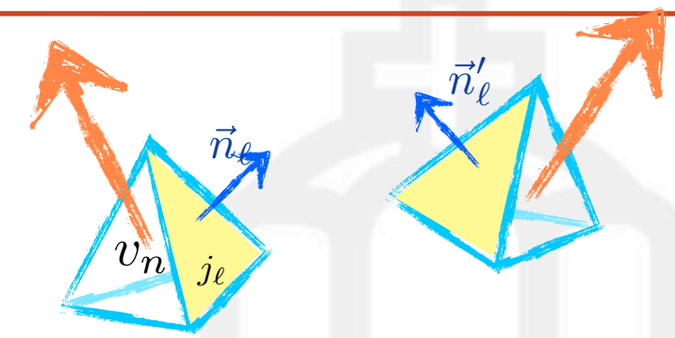
[Rovelli FV Bianchi Marcianò Magliaro Perini]

COHERENT STATES



- Spin network diagonalize metric and have quantum spread extrinsic geometry
- Coherent states: peaked in a given (discrete) **intrinsic and extrinsic** geometry [Thiemann, Speziale, Rovelli, Livine, Bianchi Magliaro Perini]
- Holomorphic representation: Basis of coherent states [Ashtekar Lewandowski Marolf Mourao, Bianchi Magliaro Perini]
- Triangulation interpretation: Regge or "twisted" [Dittrich, Freidel, Livine, Speziale, Wieland]

COHERENT STATES



- Spinnetwork states $|\Gamma, j_\ell, v_n\rangle$ ↔ Coherent states $|\Gamma, z_\ell, \vec{n}_\ell, \vec{n}'_\ell\rangle$

$$\psi_{H_\ell}(h_\ell) = \int_{SU(2)^N} dg_n \prod_{\ell=1}^{L} K_t(\, g_{s(\ell)}\, U_\ell\, g_{t(\ell)}^{-1}\; H_\ell^{-1}) \qquad H_\ell \in SL(2,\mathbb{C})$$

"group average"
to get gauge invariant states

The heat kernel K_t peaks each U_ℓ on H_ℓ

Bianchi, Magliaro, Perini

- **Geometrical interpretation** for the labels $(z_\ell, \vec{n}_\ell, \vec{n}'_\ell)$:

Freidel, Speziale

$\vec{n}_\ell, \vec{n}'_\ell$ are the 3d normals to the faces of the cellular decomposition;

- $Im(z_\ell)$ ↔ curvature at the faces and $Re(z_\ell)$ ↔ area of the face

$$Re(z_\ell) = \theta(\gamma K + \Gamma)$$

- **Hom&Iso coherent states** $|\Gamma, z\rangle$
  $\vec{n}_\ell, \vec{n}'_\ell$ fixed by requiring a regular cellular decomposition

Marcianò, Magliaro, Perini, Rovelli, FV

- in terms of the scale factor $Re(z) \sim \dot{a}$ and $\sqrt{Im(z)} \sim a$

1st-ORDER FARCTORIZATION

- classical dynamics

$$S_H = const \int \mathrm{d}t \, (a\dot{a}^2 + \frac{\Lambda}{3} a^3)\Big|_{\dot{a} = \pm\sqrt{\frac{\Lambda}{3}}a} = const \frac{2}{3}\sqrt{\frac{\Lambda}{3}}(a_f^3 - a_i^3)$$

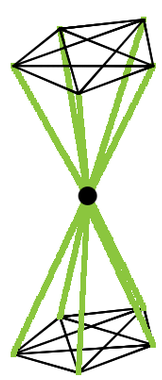
- quantum dynamics

$$W(a_f, a_i) = e^{\frac{i}{\hbar} S_H(a_f, a_i)} = W(a_f)\overline{W(a_i)}$$

- loop dynamics

$$\langle W | \psi_{H_{(z,z')}} \rangle = W(z, z') = W(z)\,\overline{W(z')}$$

order (1)



$$W_{\mathcal{C}_\infty}(z',z) = \int h_\ell \int h'_\ell \; \overline{\psi_{z'}(h'_\ell)} \, W_1(h'_\ell, h_\ell) \, \psi_z(h'_\ell)$$

$$W_1(h'_\ell, h_\ell) = \int_{SL(2,\mathbb{C})} \prod_{n=1}^{N-1} dG_n \prod_{\ell=1}^{L} P(h_\ell, G_\ell) P(h'_\ell, G'_\ell)$$

$$G_\ell = G_{n_s} G_{n_t}^{-1}$$

VERTEX AMPLITUDE

$$\langle (j,\gamma j); j', m' | \, Y \, | j, m \rangle = \delta_{p,\gamma j} \delta_{kj} \delta_{jj'} \delta_{mm'}$$

$$W_v(h_\ell) = \int_{SL(2,\mathbb{C})} \prod_{n=1}^{N-1} dG_n \prod_{\ell=1}^{L} P(h_\ell, G_\ell)$$

kernel of the simplicity map:

$$\begin{array}{llll} Y: & \mathcal{H}^{(j)} & \longrightarrow & \mathcal{H}^{(j,\gamma j)} \\ & |j,m\rangle & & |(j,\gamma j); j, m\rangle \end{array}$$

$$P(h_\ell, G_\ell) = \sum_{j_\ell} (2j_\ell + 1) \; D^{(j_\ell)}(h_\ell)^{m'}_{m} \; D^{(\gamma j_\ell, j_\ell)}(G_\ell)^{jm}_{jm'}$$

- **coherent states**

$$\psi_{H_\ell}(h_\ell) = \int_{SU(2)^N} dg_n \prod_{\ell=1}^{L} \sum_{j_\ell} (2j_\ell + 1) \; e^{-2t\hbar \, j_\ell (j_\ell+1)} \; \mathrm{Tr}\, [D^{(j_\ell)}(\, g_{s(\ell)} \, U_\ell \, g^{-1}_{t(\ell)} \; H_\ell^{-1})]$$

$$P_t(H_\ell, G_\ell) = \int dh_\ell \; K_t(h_\ell, H_\ell) \; P(h_\ell, G_\ell)$$

$$= \sum_{j_\ell} (2j_\ell + 1) \; e^{-2t\hbar j_\ell (j_\ell+1)} \; \mathrm{Tr}\, [D^{(j_\ell)}(H_\ell) Y^\dagger D^{(\gamma j_\ell, j_\ell)}(G_\ell) Y \;]$$

SADDLE POINT

- **Coherent states introduces** $D^{(j)}(H_\ell(z)) = D^{(j)}(R_{\vec{n}_s})\ D^{(j)}(e^{-iz\frac{\sigma_3}{2}})\ D^{(j)}(R^{-1}_{\vec{n}_n})$

$$D^{(j)}(e^{-iz\frac{\sigma_3}{2}}) = \sum_m e^{-izm}\ |m\rangle\langle m|$$

$Im(z)$ large: $D^{(j)}(e^{-iz\frac{\sigma_3}{2}}) \approx e^{izj}\ |j\rangle\langle j|$

$$D^{(j)}(H_\ell(z)) = D^{(j)}(R_{\vec{n}_s})\ e^{-izj}|j,+j\rangle\langle j,+j|D^{(j)}(R^{-1}_{\vec{n}_n}) = e^{-izj}|j,\vec{n}_\ell\rangle\langle j,\vec{n}_\ell|$$

- $\mathrm{Tr}\,[\,D^{(j_\ell)}(H_\ell)Y^\dagger D^{(\gamma j_\ell, j_\ell)}(G_\ell)Y\,]$ $\quad e^{-izj}\langle j,\vec{n}_\ell|\ Y^\dagger D^{(\gamma j_\ell, j_\ell)}(G_\ell)Y\ |j,\vec{n}_\ell\rangle$

$$\sum_{j_\ell}\prod_{\ell=1}^{L}\ (2j_\ell+1)\ e^{-2t\hbar j_\ell(j_\ell+1)}e^{-izj}\int_{SL(2,\mathbb{C})}\prod_{n=1}^{N-1}dG_n\prod_{\ell=1}^{L}\langle(\gamma j,j);j,\vec{n}_\ell|\ D^{(\gamma j_\ell,j_\ell)}(G_\ell)\,|(\gamma j,j);j,\vec{n}_\ell\rangle$$

- $j_\ell \to \alpha j_\ell$ and $\alpha \gg 1$ $\quad \Omega(j_\ell) \approx \dfrac{1}{\alpha^3\sqrt{\det\ Hess(j_\ell)}}\ e^{-\frac{1}{2}ij_\ell\,\theta}$

EVALUATION OF THE AMPLITUDE

$$W(z) = \sum_{j_\ell} \prod_{\ell=1}^{L} \frac{1}{\alpha^3 \sqrt{\det\ Hess(j_\ell)}} \, (2j_\ell + 1)\, e^{-2t\hbar j_\ell (j_\ell+1) - i z j_\ell} \; e^{-\frac{1}{2} i j_\ell \, \theta}$$

$$\theta\,(\gamma K + 1) - \theta = 0$$

- **Gaussian sum** peaked at j_o for all j_ℓ $\qquad j \sim j_o + \delta j$
- max (real part of the exponent) gives where the gaussian is peaked; $\qquad j_o \sim Im\, \tilde{z}/4t\hbar$
- imaginary part of the exponent $=2k\pi$ gives where the gaussian is not suppressed. $\qquad Re\, \tilde{z} = 0$

$$\dot{a} \sim 0$$

- We obtain Minkowski space!

$$W(z) = \left(\sqrt{\frac{\pi}{t}}\, e^{-\frac{\tilde{z}^2}{8t\hbar}}\; 2j_o \right)^L \frac{N_\Gamma}{j_o^3}$$

DE SITTER SPACE

[Bianchi FV Krajewski Rovelli]

$$Z_{\mathcal{C}} = \sum_{j_f, \mathrm{v}_e} \prod_f (2j+1) \prod_e e^{i\lambda\, \mathrm{v}_e} \prod_v A_v(j_f, \mathrm{v}_e)$$

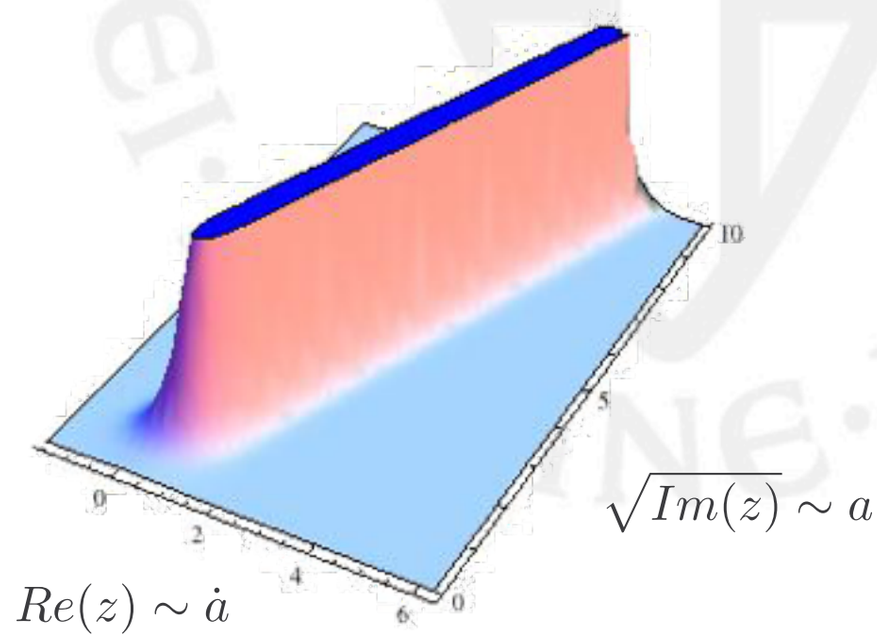
$$\mathrm{v}_e \sim \mathrm{v}_o\, j^{3/2}$$

$$W(z) = \sum_j (2j+1)\, \frac{N_\Gamma}{j^3}\, e^{-2t\hbar j(j+1) - izj - i\lambda \mathrm{v}_o j^{\frac{3}{2}}}$$

$$i\lambda \mathrm{v}_o j^{\frac{3}{2}} \sim i\lambda \mathrm{v}_o j_o^{\frac{3}{2}} + \frac{3}{2} i\lambda \mathrm{v}_o j_o^{\frac{1}{2}} \delta j$$

- the gaussian is peaked on $j_o = \dfrac{Im(z)}{4t\hbar}$
- the gaussian is not suppressed for $Re(z) + \lambda \mathrm{v}_o j^{\frac{1}{2}} = 0.$
- $\dfrac{Re(z)^2}{Im(z)} = \dfrac{\lambda^2 \mathrm{v}_o^2}{4t\hbar} \longrightarrow \left(\dfrac{\dot{a}}{a}\right)^2 = \dfrac{\Lambda}{3}$

$$\Lambda = const\, \lambda^2 G^2 \hbar^2$$

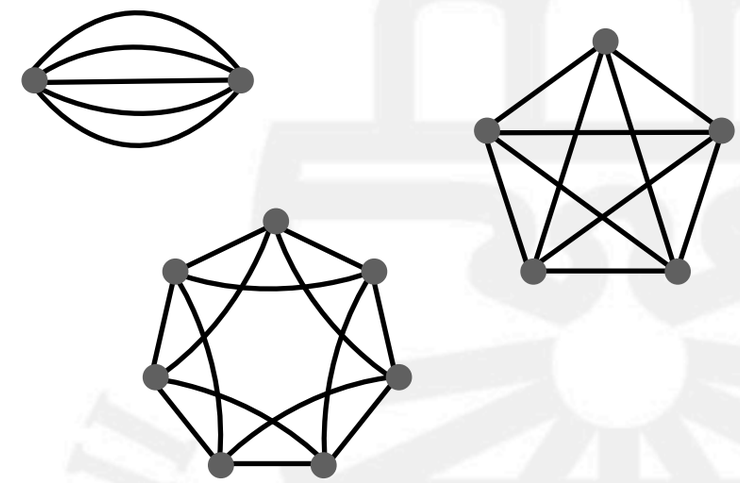


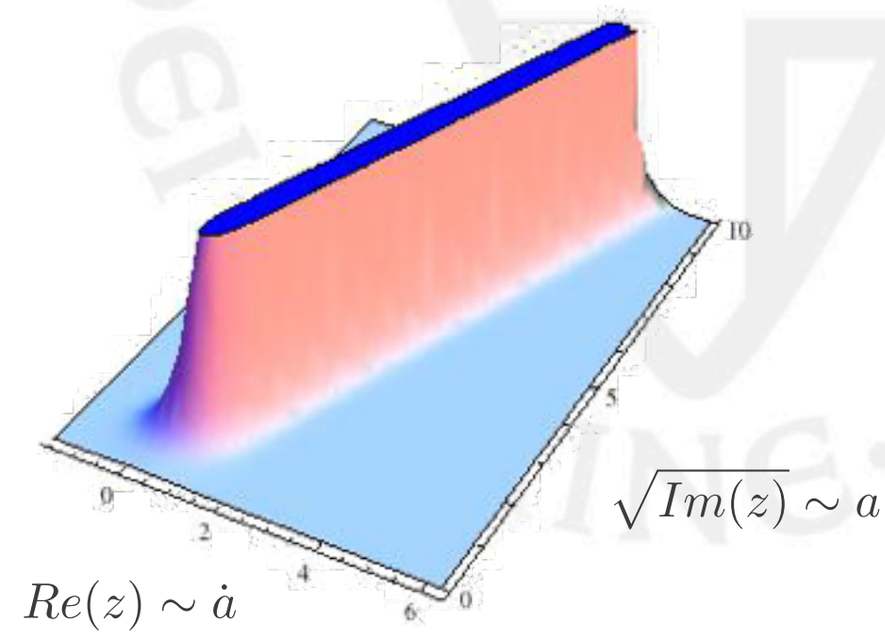
WHAT HAS BEEN DONE

- Regular graphs (same # link attached to each node)
- Asymptotic for large scale factor
- Analytic evaluation
- Numerical evaluation

} Friedmann equation

- Euclidean and Lorentzian
- Cosmological constant: $\Lambda = 0$ and $\Lambda > 0$





WHAT NEEDS TO BE DONE

- *Exploit different graphs*
- *Study the deep quantum regime*
- *Cosmological constant: quantum groups*
- *Coupling of matter*
- *Perturbations*

TO CONCLUDE

SUMMARY: LQG

- **The boundary states represent classical geometries.**
  Canonical LQG 1990, Penrose spin-geometry theorem 1971
- **Boundary geometry operators have discrete spectra.**
  Canonical LQG main results, 1990
- **The classical limit of the vertex amplitude converges (appropriately) to the Regge Hamilton function (with cosmological constant).**
  Barrett et al, Conrady-Freidel, Bianchi-Perini-Magliaro, Engle, Han..., 2009-2012
- **The amplitudes with positive cosmological constant are UV and IR finite.**
  Han, Fairbairn, Moesburger, 2011
- **Amplitudes are locally Lorentz covariant.**
  The short-scale discrete geometry does not break Lorentz invariance.
- **The theory has been extended to fermions and Yang Mills fields.**
  Bianchi, Han, Magliaro, Perini, Rovelli, Wieland 2010

SUMMARY: COSMOLOGY

- There are approximations in the quantum theory that yield cosmology.
- The theory recover general relativity in the semiclassical limit, also for non-trivial solutions.
- There is a simple way to add the cosmological constant to the dynamics of LQG.

- **Covariant LQG provides a framework for cosmology**
- A tool to explore the deep quantum regime
- Study of fluctuations from the full quantum theory

references: 1003.3483, 1101.4049, 1107.2633.