Towards Homogeneous Cosmologies from Group Field Theories

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What & Why

- Group Field Theories (GFTs) are a candidate to give a proper definition to the path integral for the gravitational field.
- The approach is based on combinatorial, discrete structures, containing geometric data but not based on the existence of a spacetime.
- Can we get effective description for simple situations like cosmology within a pregeometric scenario?

Plan

- GFT as models for quantum spacetime: a quick introduction
- GFT as second quantized theory for spin networks
- What do we know the continuum limit
- Coherent states for BEC
- Application of field coherent states to Bianchi cosmologies
- Effective equations from “fundamental dynamics”
- Discussion: Interpretation of the effective equations and outlook

BF theories

- A class of topological field theories seems particularly relevant.
- BF action & EOM

$$S_{BF} = \int d^4x \, B_{IJ} \wedge F^{IJ}(\omega)$$

$$\begin{cases} F^{IJ}(\omega) = 0 \qquad (\to \omega_\mu \sim 0, \text{locally}) \\ \\ D_\omega B^{IJ} = 0 \end{cases}$$

- As topological theories, they have only topological degrees of freedom
- Particularly good for quantization (see more later)

Plebanski's formulation of GR, or GR without a metric

- First order formalism, based on an SO(3,1) connection

$$S_{\text{Plebanski}} = \int d^4x \left(B_{IJ} \wedge F^{IJ}(\omega) + \underbrace{\Psi^{IJHK} B_{IJ} \wedge B_{HK}}_{\text{Simplicity Constraint}} \right)$$

- To the BF equation of motion, the Lagrange multiplier add the simplicity constraint

$$B^{IJ} = \pm \frac{1}{2} \epsilon^{IJ}{}_{KL} e^K \wedge e^L$$

- The theory reduces to a Palatini version of GR

$$S_{\text{Plebanski}}[B, \omega] \xrightarrow{\text{Simplicity Constraint}} S_{\text{Palatini}}[e, \omega]$$

Idea!

- Quantize BF and impose the simplicity constraints.
- Not obvious at all: work in progress
- Generate spinfoam/state sum models: partition function
- To have a finite partition function, regulate the theory putting it on a lattice
- Take a continuum limit (“lattice spacing goes to zero”)
- If the previous steps work, try to sum over triangulations.

Construction

$$Z = \int \mathcal{D}B\mathcal{D}\omega \exp\left(-\frac{i}{\hbar}\int_{\mathscr{M}} \delta_{ij} B^i \wedge F^j\right) \xrightarrow{\text{Formally}} \int \mathcal{D}\omega \prod_{x\in\mathscr{M}} \delta(F^i)$$

Discretize!

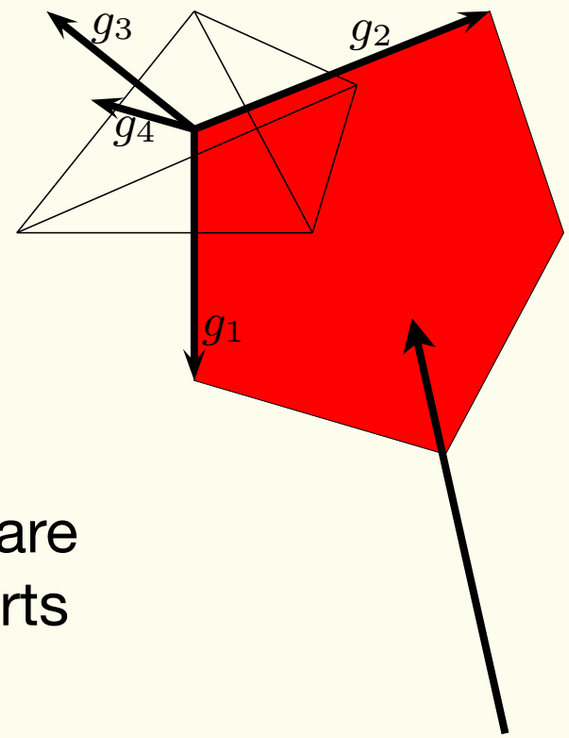
$$Z[\Delta^*] = \int \left(\prod_{e\in\Delta^*} dg_e\right) \prod_{f\in\Delta^*} \delta(g_f)$$

Triangulation dual to a simplicial complex

Essentially, the gravitational theory is reduced to a gauge theory (SU(2)) on a lattice (random)

Construction cont'd

Simplex of the simplicial complex



Edges of the dual triangulation are decorated with parallel transports

Face of the dual triangulation

<u>Restriction: dual triangulation with tetravalent vertices</u>

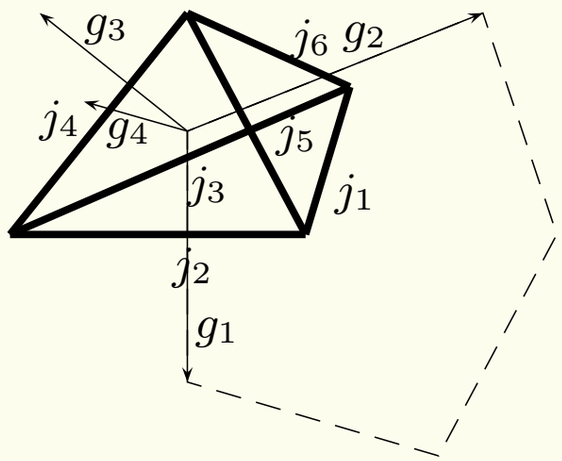
From each node there are four edges and six faces

Face amplitude:

$$\delta(g_1^{-1} g_{\text{ext}} g_2) = \sum_{j_1=0}^{\infty} d_{j_1} D^{j_1}_{m_1 n_1}(g_1^{-1}) D^{j_1}_{n_1 r_1}(g_{\text{ext}}) D^{j_1}_{r_1 m_1}(g_2)$$

Construction cont'd

Each dual face is associated to an edge, and to each edge of the tetrahedron we assign one spin (representation of the group)



The assignment of face amplitudes in the dual triangulation is translated into the assignment of a tetrahedron amplitude in the simplicial complex

Wigner 6j symbol $\left\{ \begin{array}{ccc} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{array} \right\}$

$$Z[\Delta^*] \sim Z_{\text{Ponzano–Regge}}[\Delta] = \sum_{\{j\}} \prod_{t \in \Delta} \left\{ \begin{array}{ccc} j_1^t & j_2^t & j_3^t \\ j_4^t & j_5^t & j_6^t \end{array} \right\}$$

Boulatov's model: the simplest Group Field Theory

[Boulatov 1992]

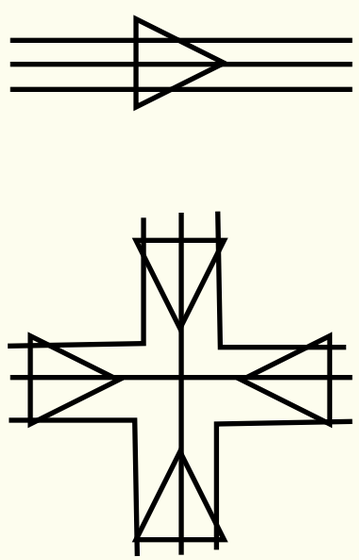
$$\phi : G \times G \times G \to \mathbb{C}; \qquad (g_1, g_2, g_3) \longmapsto \phi(g_1, g_2, g_3)$$

"Closure" → Edges of a triangle close (in Lie algebra)

$$\phi(g_1 h, g_2 h, g_3 h) = \phi(g_1, g_2, g_3) = \phi_{123}$$

$$S[\phi] = \int (dg)^3 \frac{1}{2} \phi_{123}\phi_{123} + \frac{\lambda}{4} \int (dg)^6 \phi_{123}\phi_{156}\phi_{453}\phi_{426}$$

Feynman Rules



$$Z(\lambda) = \int \mathcal{D}\phi \exp\left(-S[\phi]\right)$$

$$Z(\lambda) = \sum_{\Delta} \lambda^{n(\Delta)} Z_{\text{Ponzano–Regge}}(\Delta)$$

Coupling constant works as the exponential of a cosmological constant term

[Ooguri 1992] (4d model)

Higher dimensions

- A (single) field for d-dimensional models

$$\phi : \underbrace{G \times G \cdots \times G}_{d} \to \mathbb{C}$$

- The action:

$$S_{GFT}[\phi] = \int (dh)^d \phi_{a_1 \ldots a_d} \bar{\phi}_{a_1 \ldots a_d} + \frac{g}{d!} \int (dh)^K V(\{h\}) \phi_{a_1^1 \ldots a_d^1} \ldots \phi_{a_1^{d+1} \ldots a_d^{d+1}} + cc.$$

- The partition function for GFT

$$Z(g) = \int \mathcal{D}\phi \exp\left(-S_{GFT}[\phi]\right)$$

N.B.: this has to be defined through a resummation of the perturbative expansion (action not obviously bounded from below)

- And the partition function for QG

$$W(g) = \log Z(g)$$

Sum over connected graphs

GFTs: the general structure

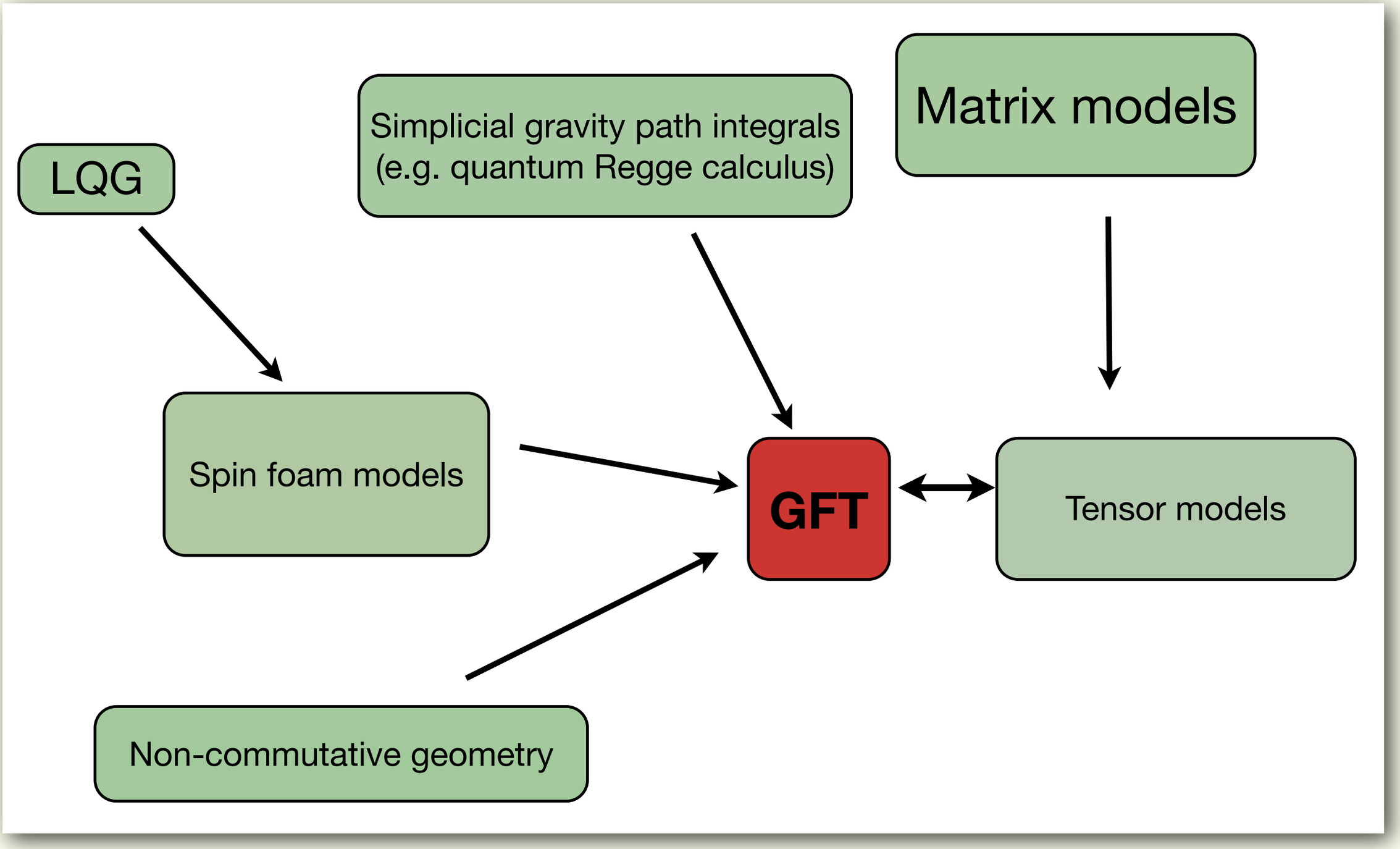
- GFTs are quantum/statistical field theories on a certain number of copies of group manifolds,
- not directly in space/spacetime,
- endowed with an action, nonlocal and with a variable amount of ingredients in its structure
- such that their partition function admits a Feynman expansion in colored graphs
- and each Feynman graph being a (simplicial) complex decorated with geometric data
- and whose amplitude is given by a spinfoam model for gravity

If GFTs work they promise to give spacetime as a sum over Feynman diagrams.

Reviews on GFT/Tensor models

- L. Freidel, arXiv: hep-th/0505016
- D. Oriti, arXiv: gr-qc/0512103
- D. Oriti, arXiv: gr-qc/0607032
- D. Oriti, arXiv: 0912.2441 [hep-th]
- V. Rivasseau, arXiv:1103.1900 [gr-qc]
- R. Gurau, J. Ryan, arXiv: 1109.4812 [hep-th]
- D. Oriti, arXiv: 1111.5606 [hep-th]
- V. Rivasseau, arXiv:1112.5104 [hep-th]

A map


LQG
Simplicial gravity path integrals
(e.g. quantum Regge calculus)
Matrix models
Spin foam models
GFT
Tensor models
Non-commutative geometry

Why do we need GFTs?

- Essentially, because it is a minimalistic proposal for the path integral approach to QG keeping a close contact with other approaches to QG, maintaining the basic features of GR: diffeo invariance, background independence.

- Might be able to deal with hardcore phenomena like big bang singularities: “no geometry” does not imply that the theory is unable to deal with the physical regime. True pregeometric models.

- They are field theories: we might be able to compute things.

- Analogy for the continuum limit: transition between Bose-Einstein condensate and normal dilute gas

- For most of the questions, one can work with GR and modifications as they were effective field theories, and be happy with that, but some important regimes are out of reach

What do we need?

- Extend to Lorentzian signature
- Impose consistently the constraints (see spinfoam)
- Make sense of the perturbative expansion (keep under control the simplicial complexes you are including in the sum)
- Try to find the continuum semiclassical limit
- Add matter
- Discuss the “transition amplitudes”, corresponding to insertion of boundaries, associated to certain correlation functions
- Try to get predictions & new insights for old problems

Comment

Here they go again with another bunch of promising but unmanageable and so far unphysical models...

What we want to do

- We want now to try to discuss some physics, at least in a primitive way
- Simplest case: Cosmology (less degrees of freedom)
- Two objectives: connect with other results in the area (LQC) and get new ideas on the way in which the models have to be developed in order to get to physical questions.

[Bojowald, Chinchilli et al 2012]
[Battisti, Marciano, Rovelli 2011]
[Bianchi, Rovelli, Vidotto 2010]

- Idea: model the state corresponding to the presence of macroscopic geometry/continuum limit as a condensed state like the ones used for BECs.

Reminder: kinematical states in LQG

Kinematical states constructed out of graphs, with L links and V vertices

The wavefunction will be a function of L variables (normally group elements), one for each link of the graph.

Furthermore, gauge invariance is imposed by means of projection (through integrations on each vertex of the graph)

$$\mathcal{L}^2\left((\mathrm{SU}(2))^L/(\mathrm{SU}(2))^V\right)$$

One can change perspective and look at spin networks as functions on the vertices convoluted together according to the connectivity of the graph

Reminder: Holonomies and fluxes

Triads and connections are stored into smeared quantities (fluxes & holonomies): parallel transports on the edge of the (embedded) graph and the flux as certain integral over plaquettes of the dual triangulation

$$E^i_{e,x_0} := tr\left(\frac{i\sigma^i}{2}\left[\int_{S_e} h(x_0 \to x) * E(x) h^{-1}(x_0 \to x)\right]\right)$$

$$A^a_i(x) \to h(\gamma) = \mathcal{P}\exp\left(-i\int_\gamma A^a T_a\right)$$

The physical states that we are seeking will store information about these discrete data specified on a given graph, and the fact that these data solve the constraints.

Reminder cont'd

Classical phase space coordinates lead to the holonomy-flux algebra

$$[\hat{E}^i_e, \hat{h}_{e'}] = i\hbar(8\pi G_N \gamma)\delta_{ee'} R^i \triangleright h_e$$

$$\left[\hat{E}^i_e, \hat{E}^j_{e'}\right] = i\hbar(8\pi G_N \gamma)\epsilon^{ij}{}_k \delta_{ee'} \hat{E}^k_e$$

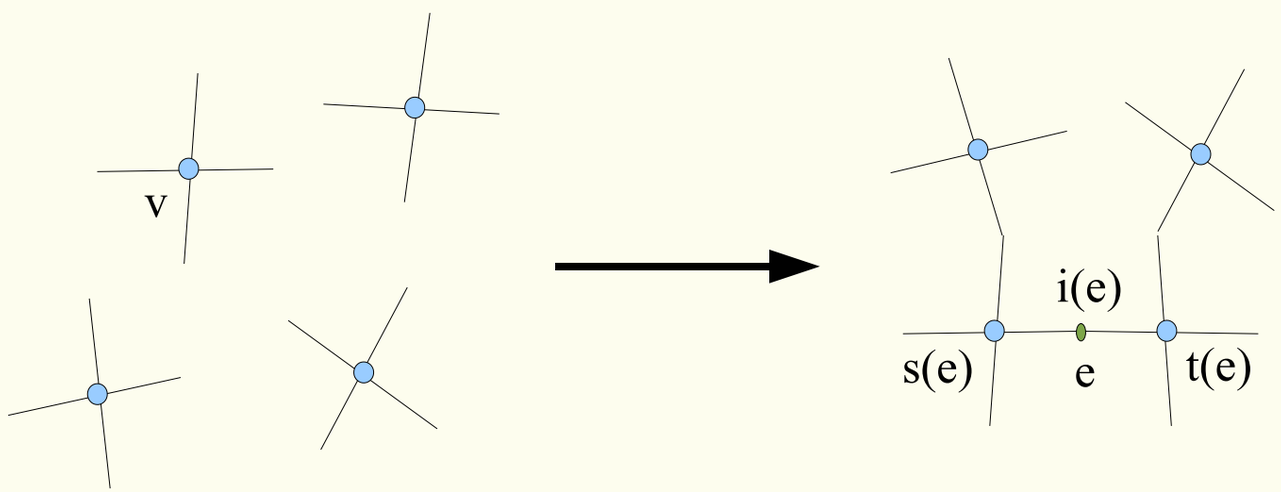
Note that h and fluxes are not a canonical basis as the familiar p&q's of the particle on a line.

Important: In the rest of the discussion I will redefine the fluxes by scaling them with an area, so that they are dimensionless.

GFT: operatorial version

[Livine, Oriti, Ryan 2011]

- You can think about GFT as a second quantized theory for spin networks
- GFT field creates/destroys spin networks vertices
- The spin network wavefunction can be seen as a peculiar n particle state, with gluings of the vertices



- The equation of motion for the field will implement the dynamics in terms of moves (creation/destruction of vertices and reconnection)

The continuum limit

- We have said that, in order to recover the continuum limit, a phase transition is required.
- We still have only partial results on the existence of such a transition, more work is needed (esp. to establish the properties)
- Can we get around this point, for the moment, and get a qualitative understanding of how we can get EOM for the continuum limit, using other methods?

A very condensed presentation of BECs

- Second quantized description of weakly interacting, dilute Bose gas

$$\hat{\Psi} = \frac{1}{\sqrt{V}} \sum_k \hat{a}_k e^{ik\cdot x} \qquad \hat{H}_0 = \int \hat{\Psi}^\dagger(x) \left(-\frac{\hbar}{2m}\nabla^2 - \mu + \frac{\kappa}{2}|\hat{\Psi}|^2 \right) \hat{\Psi}(x) d^3x$$

- Condensation: mean field approximation

$$\hat{\rho}_{\text{cond}} = ?$$

$$\hat{\Psi} \approx \psi \mathbb{I} + \hat{\chi} \qquad \langle \Omega | \hat{\Psi} | \Omega \rangle = \psi$$

$$\hat{\Psi} |0\rangle = 0 \qquad \hat{\Psi} |\Omega\rangle \neq 0$$

atomic Fock vacuum — ground state

- Equations of motion for the classical field, induced by the quantum equations of motion:

$$i\hbar \frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi - \mu\psi + \kappa|\psi|^2\psi$$

Condensate for GFT: a first sketch

- What if we consider coherent states, in GFT?

$$|\varphi\rangle = \exp\left(\int dh \varphi_{1\ldots d} \hat{\phi}^{\dagger}_{1\ldots d}\right)|0\rangle$$

- There is a mean field

$$\hat{\phi}_{1\ldots d}|\varphi\rangle = \varphi_{1\ldots d}|\varphi\rangle$$

- It's a many body state (not with fixed number of "particles")

$$\underbrace{\hat{\phi}_{1\ldots d}\cdots\hat{\phi}_{1'\ldots d'}}_{n}|\varphi\rangle = \underbrace{\varphi_{1\ldots d}\cdots\varphi_{1'\ldots d'}}_{n}|\varphi\rangle$$

- Idea: it might be used for cosmological models, since the answer to the question "what is the local structure of the wavefunction?" is always the same (more on this, later).

EOM

- We can apply the EOM for the field theory to these particular states to get effective EOM for the mean field

$$\hat{\phi} + \lambda \int dh \hat{\phi} \ldots \hat{\phi} = 0 \qquad \rightarrow \qquad \varphi + \lambda \int dh \varphi \ldots \varphi = 0$$

- Nonlinear version of WdW? If we interpret the mean field as a wavefunction, it seems so

- We can however interpret the mean field as a mere hydrodynamic quantity, giving us a distribution over the configuration space to be interpreted statistically, but not quantum mechanically.

- BEC: condensate wavefunction is not a wavefunction, but a field storing the hydrodynamic info about the condensate

Geometrical content

[Oriti, LS 2010]

- The mean field allows us to compute expectation values of geometric operators associated to the vertex

$$\varphi \qquad \rightarrow \qquad \langle h \rangle, \langle E^i \rangle, \ldots$$

- Ideally, the scale factor is one of such operators, so we need to translate the equations for the mean field into equations relating these expectation values (~Ehrenfest)

- Problem: the data stored into a vertex are not gauge invariant, we need a different sort of quantity

- However, just to convince you, when you work with coherent states in the field configurations, you get the desired result (BF)

$$\varphi \propto \int dh\, K^t(g_1 h G_a) K^t(g_2 h G_b) K^t(g_3 h G_c) \qquad \begin{cases} G_a = G_c \\ G_b \qquad \text{arbitrary} \end{cases}$$

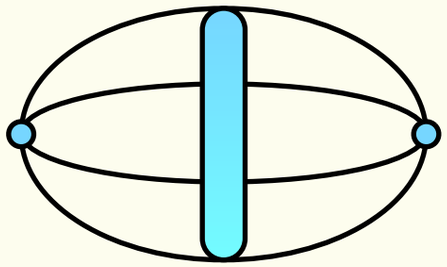
Next possibility: the dipole

- Instead of a mean field approximation, consider the analogue of the Bogoliubov approximation for the condensate, a sort of coherent state associated to a pair:

$$|\varphi\rangle = \exp\left(\int dh D_{1\ldots 2d}\hat{\phi}^{\dagger}_{1\ldots d}\hat{\phi}^{\dagger}_{d+1\ldots 2d}\right)|0\rangle$$

- D is now playing the role of the order parameter, and it is in terms of this that the theory will be rewritten

- Graph



- Notice that this state contains an arbitrary number of these dipoles, we are not using the interpretation of the dipole as a (dual) triangulation of sphere (i.e. we are not fixing a scale of a triangulation)

- Better gauge transformations: the dipole depends on the three residual independent vectors up to the adjoint action of SU(2).

Work in progress

- Translate the dipole into geometric info (check gauge invariance)

- Play the same game of the mean field, try to get reasonable equations

$$\int dh K_{123,h}(g)\varphi_0^*(h) + \lambda N \left(K_{453,426}(g) \int dh K_{123,h}(g)\varphi_0^*(h) + \text{permutations} \right) +$$

$$+ \lambda N^2 \int dh K_{453,h}(g)\varphi_0^*(h) \int dh' K_{426,h'}(g)\varphi_0^*(h) \int dh'' K_{123,h}(g)\varphi_0^*(h) = 0$$

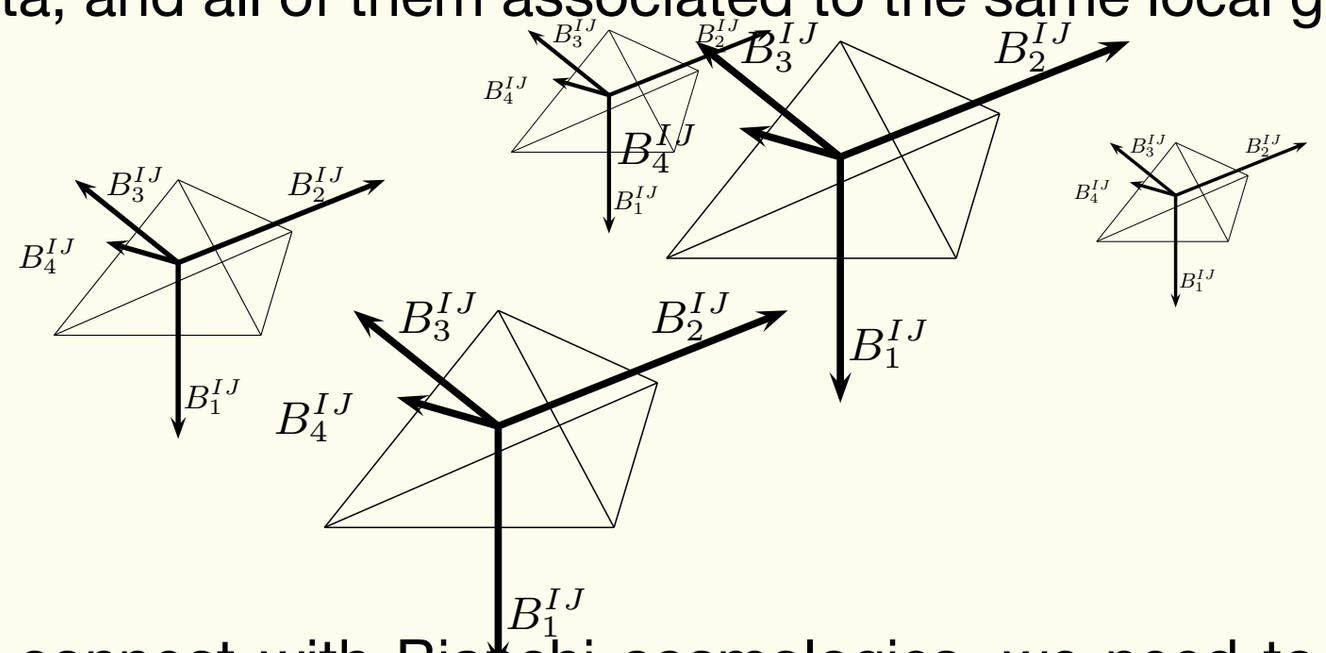
- Relate the equations of motion to some form of gravitational equations

- This requires a precise mapping between the arguments of the functions and the scale factor(s)

Towards homogeneous cosmologies/shortcut

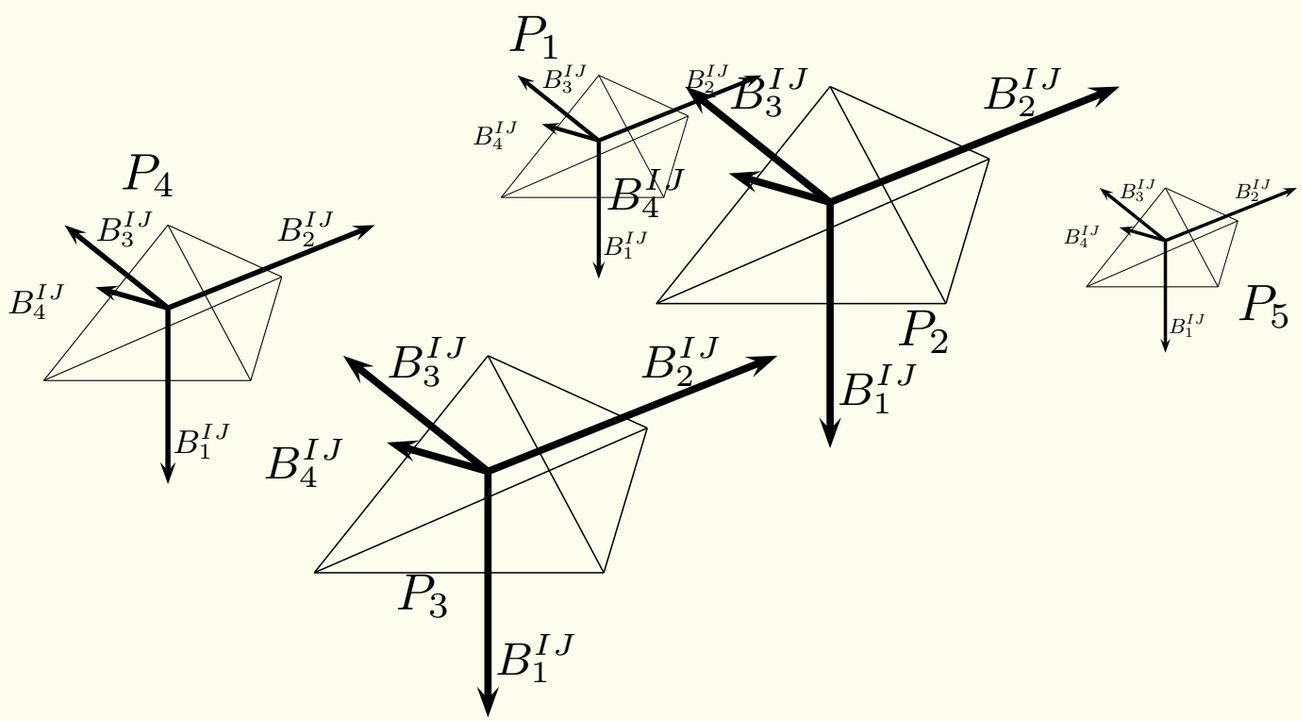
- To say anything about cosmology, we should mention how we plan to get a grasp of the geometric structure of a spatial slice (a group manifold)

- We have a bunch of tetrahedra, all of them decorated with the same data, and all of them associated to the same local geometry



- To connect with Bianchi cosmologies, we need to embed them into a spatial slice, aligning the tetrahedra with a basis of left invariant vector fields and translating then the data on the tetrahedra in terms of scale factors.

Some equations



$$B^{0I}(\triangle_i(P)) = \left(\epsilon^{0I}{}_{JK} e^I \wedge e^J\right)(\triangle_i(P))$$

$$g_{ij}(P) := \eta_{IJ} e_i^I e_j^J$$

"internal metric"

$$\downarrow$$

$$g_{ab} = ?$$

If we assume that the tetrahedra are embedded in a group manifold, in such a way that the edges have always the same orientation with respect to a basis of left invariant vector fields, then we can translate the tetrahedron metric into a physical metric

$$g_{ab}(P) = v_a^i(P) v_b^j(P) g_{ij}(P)$$

Thus, the condensate, giving a constant g_ij, is compatible (not implying) with Bianchi cosmologies

Towards homogeneous cosmologies/2

- Alternatively, triangulate a spatial slice, construct a correlation function for GFT corresponding to that structure, compute its value with the given ansatz and try to make sense of the equation of motion implemented in terms of Schwinger-Dyson equations.

$$\langle P_{N_b}(\phi)\rangle = \left\langle \int (dg)^{N_b} \underbrace{\phi_{abc}\cdots\phi_{amn}}_{N_b} \right\rangle$$

$$\downarrow$$

$$\left\langle \frac{\delta P_{N_b}(\phi)}{\delta\phi} \right\rangle - \left\langle \frac{\delta S[\phi]}{\delta\phi} P_{N_b}(\phi) \right\rangle = 0$$

Final comments

- GFT are an interesting class of models for QG
- They still need a lot of work to show that they do make sense as we hope (definition of the path integral, properties of the phase transitions, correct dynamics, matter fields, etc.)
- In the meantime, we can use insights from other areas of Physics and try to make educated guesses on what could be an effective description of macroscopic geometries in terms of microscopic “quanta of space”, at least in simple cases like homogeneous cosmologies.
- Many ambiguities (we are following BECs, in which we can have a physical motivation for choosing one state or another) and interpretational problems (nonlinear WdW?)
- If condensed states correspond to FRW/Bianchi, where are inhomogeneities? Excitations over the condensate?
- Framework is different from other proposals, since we are not necessarily working at fixed triangulation (we are still keeping all the possible triangulations, in a sense)

Conclusions

Stay tuned