Effective relational dynamics of a non-integrable cosmological model

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Abstract

We apply the effective approach to evaluating semiclassical relational dynamics to the **closed Friedman–Robertson–Walker** cosmological model filled **with** minimally coupled **massive scalar field.**

This model is interesting for studying relational dynamics in more general setting because
(i) it features **non–trivial coupling of relational clock to evolving degrees of freedom**,
(ii) **no** temporally **global clock** variable exists,
(iii) it is **non–integrable** which is typical for **generic dynamical systems**.

Effective framework

- Effective framework = extracting **representation independent information**.
- Natural phase space structure defined by **Poisson bracket**

$$\{\langle\hat{\mathbf{A}}\rangle,\langle\hat{\mathbf{B}}\rangle\}=\frac{\mathbf{1}}{\mathbf{i}\hbar}\langle[\hat{\mathbf{A}},\hat{\mathbf{B}}]\rangle$$

+ 'classical variables' $\mathbf{q_i}=\langle\hat{\mathbf{q}}_\mathbf{i}\rangle, \mathbf{p_i}=\langle\hat{\mathbf{p}}_\mathbf{i}\rangle$
+ associated **quantum moments**

$$\mathbf{\Delta}(q_1^a p_1^b q_2^c p_2^d) = \langle(\hat{q}_1-\langle\hat{q}_1\rangle)^a(\hat{p_1}-\langle\hat{p_1}\rangle)^b (\hat{q}_2-\langle\hat{q}_2\rangle)^c(\hat{p_2}-\langle\hat{p_2}\rangle)^d\rangle$$

order of a given moment $n=(a+b+c+d)\geq 2$.

Method

- Perform effective **analogue of Dirac quantization** by taking $C_{pol}=\langle\widehat{pol\hat{C}}\rangle=0$ for all polynomials in basic variables.
- **Semiclassical truncation at order** $\hbar$ → get four gauge flows/constraints in terms of expectation values of variables and moments: spreads $(\Delta x_i)^2$ and covariances $\Delta(x_i x_j)$.
- Follow classical type constraint analysis.
- Partially **fix gauge freedom** – choose one of configuration variables as internal clock, e.g. q_i, set $(\Delta q_i)^2=0$ and covariances to vanish.
- Gauge-fixing and positivity conditions with interpreting remaining quantum flow as dynamics of the system w.r.t. clock q_i = ***Zeitgeist*** **associated with** $\mathbf{q}_i$.
- **Clock picks up imaginary contribution** so that constraints are satisfied and evolving variables remain real along its flow.
- Sometimes clock not globally valid because it is 'too slow' to resolve relational evolution at some point → **breakdown of semiclassical approximation**. Breakdown **is only relative to specific gauge choice** and q_j can serve as good clock near turning point of q_i.
- To completely evolve the system **switch between** different ***Zeitgeister*** near turning points of their clocks by transfering relational data **between the two gauge frameworks.**

Classical dynamics

Hamiltonian constraint for closed FRW

$$\mathbf{C_H}=\mathbf{p}_\phi^2-\mathbf{p}_\alpha^2-\mathbf{e}^{4\alpha}+\mathbf{m}^2\phi^2\mathbf{e}^{6\alpha}=\mathbf{0},$$

$\mathbf{m^2\phi^2e^{6\alpha}}$ provides **coupling between relational clock**, α or ϕ, **and evolving configuration variable**, ϕ or α, respectively.

Two **typical classical solutions** to closed FRW spacetime – both ϕ and α fail to be good clocks. **Generically no globally valid clock exists.**

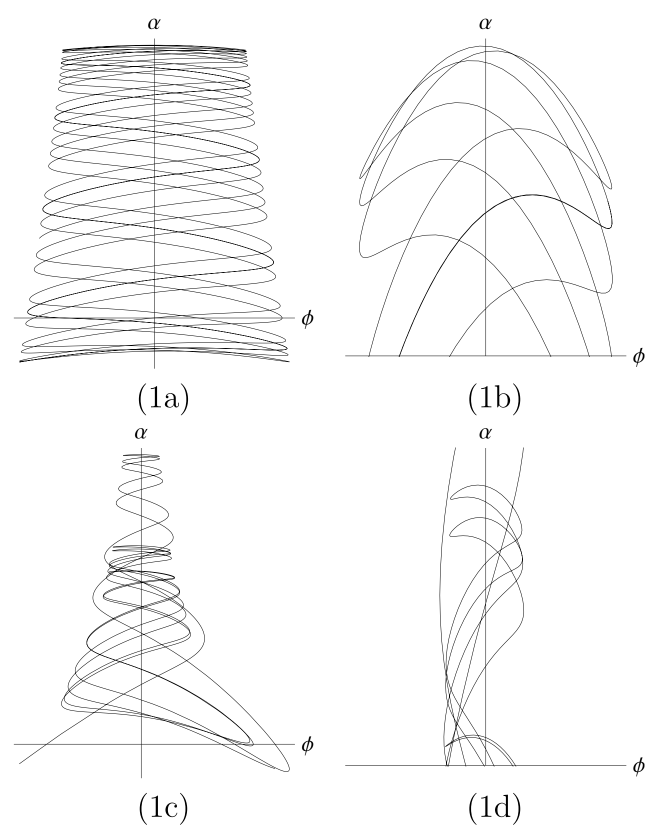

Figure 1 (1a), (1c) show evolution up to maximal expansion, (1b) close–up of (1a) near α_{max}, (1d) close–up on intermediate section of (1c).

Sensitivity to initial conditions is prominent - trajectories arbitrarily close to each other experience uncorrelated fates.

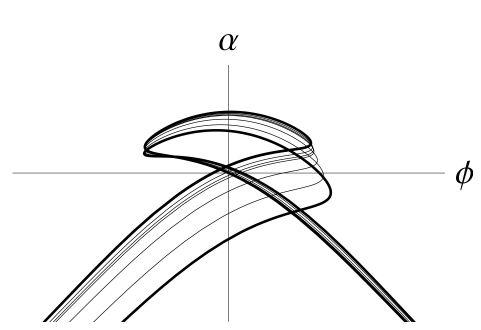

Figure 2 Defocussing of trajectories, caustics develop along extrema of ϕ.

Defocussing of trajectories as ultimate cause of generic breakdown of semiclassicality and relational evolution.

References

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Effective relational dynamics

Evolve the system by **switch**ing between ***Zeitgeist***er near their turning points, **patch up** trajectories and **plot** the evolution.

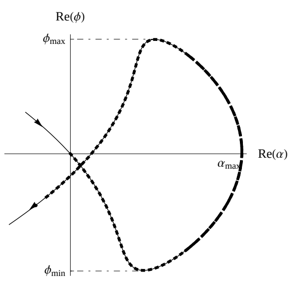

Figure 3 (3) Classical (dotted) and patched up effective trajectory (solid/dashed).

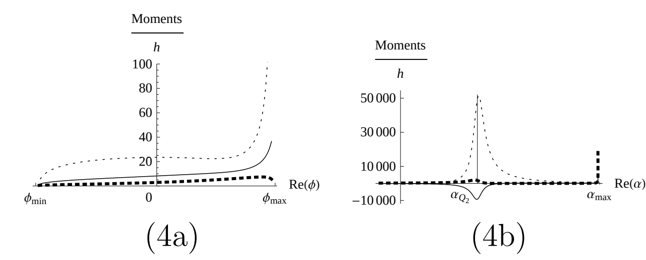

Figure 4 (4a) ϕ–time: $(\Delta\alpha)^2$ (thick, dashed), $(\Delta p_\alpha)^2$ (thin, dashed), $\Delta(\alpha p_\alpha)$ (solid). (4b) α–time: $(\Delta\phi)^2$ (thick, dashed), $(\Delta p_\phi)^2$ (thin, dashed), $\Delta(\phi p_\phi)$ (solid). α_{Q_2} is point where clock becomes 'slow'.

Trajectory with **more** than one local **extrema** in α – turning points in α and ϕ too close, system gets unstable, **no clock change** between α– and ϕ–*Zeitgeister* **can be performed**.

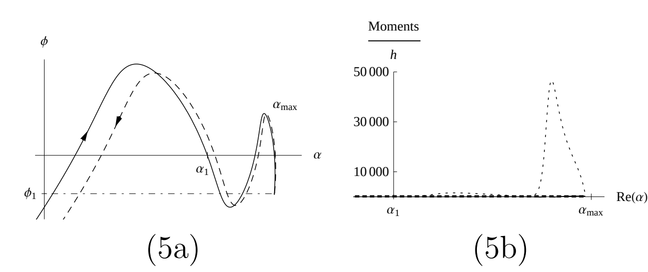

Figure 5 (5a) Trajectory Fig. 3 with different initial conditions: incoming branch (solid), outgoing branch (dashed).(5b) α–time moments of (5a): $(\Delta\phi)^2$ (thick, dashed), $(\Delta p_\phi)^2$ (thin, dashed), $\Delta(\phi p_\phi)$ (solid).

Conclusion

Systematic method for switching between different clocks in quantum theory is available.

Using this method it is **possible to obtain semiclassical solutions** in this model which follow classical trajectory **if state is initially sufficiently sharply peaked** and corresponding classical trajectory sufficiently benign.

In generic case effective relational dynamics breaks down in the region of maximal expansion on account of wealth of structure on all scales in this chaotic model and **no change of *Zeitgeist* can remedy this.**

Generically, a '**good relational evolution**' appears to be only a **transient** and **semiclassical phenomenon.**