Reconstruction **theorem** (Connes, hep-th/9603053): There is a one-to-one correspondence between *commutative* even real spectral triples

$$(\mathcal{A}, \mathcal{H}, \not{\partial}, J, \chi)$$

and even-dimensional compact Riemannian spin manifolds M.

$$\mathcal{A} = \mathcal{C}^\infty(M),$$

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$$\not{\partial} = i\gamma^\mu(\partial/\partial x^\mu + \tfrac{1}{4}\omega_{\mu ab}\gamma^a\gamma^b),$$

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The spectral action (Chamseddine & Connes hep-th/9606001)

$$S_{CC}[\not{\partial}] := \operatorname{tr} f(\not{\partial}^2/\Lambda^2)$$

counts the (discrete) eigenvalues λ of $\not{\partial}$ with $|\lambda| \leq \Lambda$ for a cut-off Λ.

For large Λ and torsion $= 0$

$$
\begin{aligned}
S_{CC} \quad = \quad & \int_M \tfrac{2\Lambda_c}{16\pi G}\ \mathrm{d}vol \\
& -\tfrac{1}{16\pi G}\int_M \left[R_g \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \right]\ \mathrm{d}vol \\
& +a\left(\tfrac{44\pi^2}{9}\mathrm{Euler}(M) - \int_M \left[|\mathrm{Weyl}_g|^2 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \right]\ \mathrm{d}vol\right) \\
& + O(\Lambda^{-2}),
\end{aligned}
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$$
\Lambda_c = \tfrac{6f_0}{f_2}\Lambda^2, \qquad G = \tfrac{3\pi}{f_2}\Lambda^{-2}, \qquad a = \tfrac{f_4}{320\pi^2},
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f_0 := \int_0^\infty uf(u)\mathrm{d}u, \quad f_2 := \textstyle\int_0^\infty f(u)\mathrm{d}u, \quad f_4 = f(0).
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For large Λ and torsion $= T + V$:
(Hanisch, Pfäffle & Stephan and Iochum, Levy & Vassilevich)

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S_{CC} \;=\; & \int_M \tfrac{2\Lambda_c}{16\pi G}\,\mathrm{d}vol \\
& -\tfrac{1}{16\pi G}\int_M \left[R_g + 18\,\mathrm{div}_g V - 54\,|V|^2 - 9|T|^2\right]\,\mathrm{d}vol \\
& + a\left(\tfrac{44\pi^2}{9}\mathrm{Euler}(M) - \int_M \left[|\mathrm{Weyl}_g|^2 + \tfrac{27}{10}|\delta T|^2 + \tfrac{27}{10}|\mathrm{d}V|^2\right]\,\mathrm{d}vol\right) \\
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Other goodies:

- The spectral action produces the boundary terms added by hand to the Einstein-Hilbert action in order to get a Hamiltonian formulation. (Chamseddine & Connes '07 and Iochum, Levy & Vassilevich)

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$$S^L_{CC}[\not{\partial}] := \mathrm{tr}\, f\left(\frac{1-\gamma_5}{2}\,\frac{\not{\partial}^2}{\Lambda^2}\right)$$

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- In the almost commutative case, the spectral action produces a non-minimal coupling between gravity and the Higgs scalar $\xi(R_g \qquad\qquad)|\varphi|^2$ with $\xi = \frac{1}{12}$, that makes its kinetic term conformally invariant. (Chamseddine & Connes '96), experimentally from Higgs desintegration at LHC: $\xi < 1.3 \cdot 10^{15}$ (Atkins & Calmet).

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S_{CC} is non-local $= \Sigma$ local terms

$f_0\Lambda^4 a_0 + f_2\Lambda^2 a_2 + f_4\Lambda^0 a_4 + f_6\Lambda^{-2} a_6 + f_8\Lambda^{-4} a_8 + f_{10}\Lambda^{-6} a_{10} ..., \;\; f_{10} = -f^{(3)}(0).$

For Eucidean Robertson-Walker on $S^1 \times S^3$ Chamseddine & Connes '11 find:

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For Eucidean Robertson-Walker on $S^1 \times S^3$ Chamseddine & Connes '11 find:

$$
\begin{aligned}
a_{10} = {} & \tfrac{1}{665280a(t)^6}(-11700a'(t)^8a''(t) + 11700a(t)a'(t)^7a^{(3)}(t) \\
& + 3a'(t)^6(5a''(t)(-165 + 5096a(t)a''(t)) - 2046a(t)^2a^{(4)}(t)) \\
& + 3a(t)a'(t)^5((825 - 34628a(t)a''(t))a^{(3)}(t) + 746a(t)^2a^{(5)}(t)) \\
& + 3a(t)a'(t)^4(3476a''(t)^2 - 54054a(t)a''(t)^3 + 14440a(t)^2a''(t)a^{(4)}(t) \\
& + a(t)(10448a(t)a^{(3)}(t)^2 - 429a^{(4)}(t) - 217a(t)^2a^{(6)}(t))) \\
& + 3a(t)^2a'(t)^3(78902a(t)a''(t)^2a^{(3)}(t) - 2a''(t)(2222a^{(3)}(t) + 2127a(t)^2a^{(5)}(t)) \\
& + a(t)(-7992a(t)a^{(3)}(t)a^{(4)}(t) + 154a^{(5)}(t) + 55a(t)^2a^{(7)}(t))) \\
& + a(t)^2a'(t)^2(-11880a''(t)^3 + 111378a(t)a''(t)^4 - 68664a(t)^2a''(t)^2a^{(4)}(t) \\
& + 33a(t)(113a^{(3)}(t)^2 + 124a(t)^2a^{(4)}(t)^2 + 192a(t)^2a^{(3)}(t)a^{(5)}(t) - 4a(t)a^{(6)}(t)) \\
& + 3a(t)a''(t)(-31973a(t)a^{(3)}(t)^2 + 1738a^{(4)}(t) + 968a(t)^2a^{(6)}(t)) - 43a(t)^4a^{(8)}(t)) \\
& + a(t)^3a'(t)(-117600a(t)a''(t)^3a^{(3)}(t) + 66a''(t)^2(211a^{(3)}(t) + 172a(t)^2a^{(5)}(t)) \\
& - 2a(t)a''(t)(-19701a(t)a^{(3)}(t)a^{(4)}(t) + 693a^{(5)}(t) + 238a(t)^2a^{(7)}(t)) \\
& + a(t)(-2640a^{(3)}(t)a^{(4)}(t) - 2a(t)^2(778a^{(4)}(t)a^{(5)}(t) + 537a^{(3)}(t)a^{(6)}(t)) \\
& + 33a(t)(271a^{(3)}(t)^3 + a^{(7)}(t)) + 18a(t)^3a^{(9)}(t))) + a(t)^3(2354a''(t)^4 \\
& - 3a(t)a''(t)(3446a''(t)^4 + 1243a^{(3)}(t)^2 + 924a''(t)a^{(4)}(t)) + 66a(t)^2(331a''(t)^2a^{(3)}(t)^2 \\
& + 160a''(t)^3a^{(4)}(t) + 6a^{(4)}(t)^2 + 9a^{(3)}(t)a^{(5)}(t) + 4a''(t)a^{(6)}(t)) - a(t)^3(331a^{(3)}(t)^2a^{(4)}(t) \\
& + 482a''(t)a^{(4)}(t)^2 + 1110a''(t)a^{(3)}(t)a^{(5)}(t) + 448a''(t)^2a^{(6)}(t) + 11a^{(8)}(t)) \\
& - 3a(t)^4(74a^{(5)}(t)^2 + 116a^{(4)}(t)a^{(6)}(t) + 49a^{(3)}(t)a^{(7)}(t) + 6a''(t)a^{(8)}(t)) \\
& + 3a(t)^5a^{(10)}(t)))
\end{aligned}
$$

Applications of the spectral action in cosmology:

Marcolli & Pierpaoli suppose that $\Lambda_c(E)$, $G(E)$ and the Yukawa couplings are specific functions of energy motivated by renormalization and use the Higgs potential for inflation. They find:

- $m_H = 158$ GeV,
- modified propagation of gravitational waves,
- anti-gravity in the early universe,
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Marcolli, Pierpaoli & Teh compute the Higgs potential in the spectral action after a Poisson resummation for locally but not globally maximally symmetric cosmologies with positve and zero curvature and including non-trivial spin structures. They find that spectral indices and tensor to scalar ratios do not distinguish different topologies (same curvatures), but the amplitudes of the power spectra do.

Applications of the spectral action in cosmology:

Mairi Sakellariadou & Buck, Fairbairn, Nelson, Ochoa find:

- The spectral action does not modify Schwarzschild's and Friedmann's solutions.
- The spectral action modifies the propagation of gravitational radiation. The observed energy loss from binary pulsars yields the constraint:

$$B := (32\pi Ga)^{-1/2} > 7.55 \cdot 10^{-13}\,\mathrm{m}^{-1}.$$

- Higgs inflation using the conformal coupling of the scalar field to curvature in the spectral action is ruled out phenomenologically.