

pseudo-force transformation simpler force

centrifugal, rotations 0
Coriolis

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magnetic Lorentz electric

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gravitational general coordinate 0

pseudo-force	transformation	simpler force	geometry
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centrifugal, Coriolis	rotations	0	Euclid
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magnetic	Lorentz	electric	Minkowski
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gravitational	general coordinate	0	Riemann
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pseudo-force	transformation	simpler force	geometry	time
centrifugal, Coriolis	rotations	0	Euclid	absolute
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elect.-magn., weak, strong	gauge	gravitational	NCG	$\Delta\tau \sim 10^{-41}$ s

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new physics	discrete spectra	

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Maxwell
Oskar Klein
Gordon
Dirac
Weyl
Elie Cartan
Majorana
Yukawa
Brout
Englert

G

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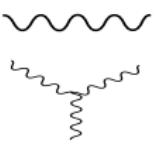
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$$\varphi\in \mathcal{H}_S$$

$$\textcolor{blue}{g},\lambda,\mu\in\mathbb{R}_+$$

$$\textcolor{blue}{g}_Y\in\mathbb{C}$$

$$\mathcal{L}[A, \psi, \varphi] = \frac{1}{2} \text{tr} (\partial_\mu A_\nu \partial^\mu A^\nu - \partial_\mu A_\nu \partial^\nu A^\mu)$$



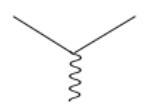
$$+ g \text{ tr} (\partial_\mu A_\nu [A^\mu, A^\nu])$$



$$+ g^2 \text{ tr} ([A_\mu, A_\nu] [A^\mu, A^\nu])$$



$$+ \bar{\psi} \not{\partial} \psi$$



$$+ ig \bar{\psi} (\tilde{\rho}_L \oplus \tilde{\rho}_R)(A_\mu) \gamma^\mu \psi$$

$$A_\mu \in \text{Lie}(\textcolor{red}{G})^\mathbb{C}$$



$$+ \frac{1}{2} \partial_\mu \varphi^* \partial^\mu \varphi$$

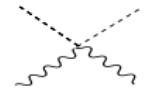
$$\psi \in \mathcal{H}_L \oplus \mathcal{H}_R$$



$$+ \frac{1}{2} g \{ (\tilde{\rho}_S(A_\mu)\varphi)^* \partial^\mu \varphi + \partial_\mu \varphi^* \tilde{\rho}_S(A_\mu)\varphi \}$$

$$\xrightarrow{\quad} \quad \xleftarrow{\quad}$$

$$+ \frac{1}{2} g^2 (\tilde{\rho}_S(A_\mu)\varphi)^* \tilde{\rho}_S(A^\mu)\varphi$$



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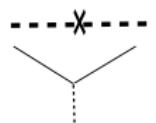


$$g, \lambda, \mu \in \mathbb{R}_+$$



$$g_Y \in \mathbb{C}$$

$$+ \lambda \varphi^* \varphi \varphi^* \varphi$$



$$- \frac{1}{2} \mu^2 \varphi^* \varphi$$

$$+ g_Y \bar{\psi} \varphi \psi + \bar{g}_Y \bar{\psi} \varphi^* \psi$$

Properties:

- ▶ For

$$G = U(1) \ni \exp A, \quad \mathcal{H}_L = \mathcal{H}_R = \mathbb{C} \ni \psi, \quad \mathcal{H}_S = \{0\},$$

with $\tilde{\rho}_L(A)\psi = \tilde{\rho}_R(A)\psi = qA\psi$, the Yang-Mills Lagrangian is Maxwell's Lagrangian describing the interaction of the photon A with a spinor ψ of electric charge qe and with coupling constant $g = e/\sqrt{\epsilon_0}$.

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- ▶ The Yang-Mills-Higgs Lagrangian defines a perturbatively renormalizable quantum field theory if the Yang-Mills anomaly

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g_Y 's \leadsto fermion masses and mixings.

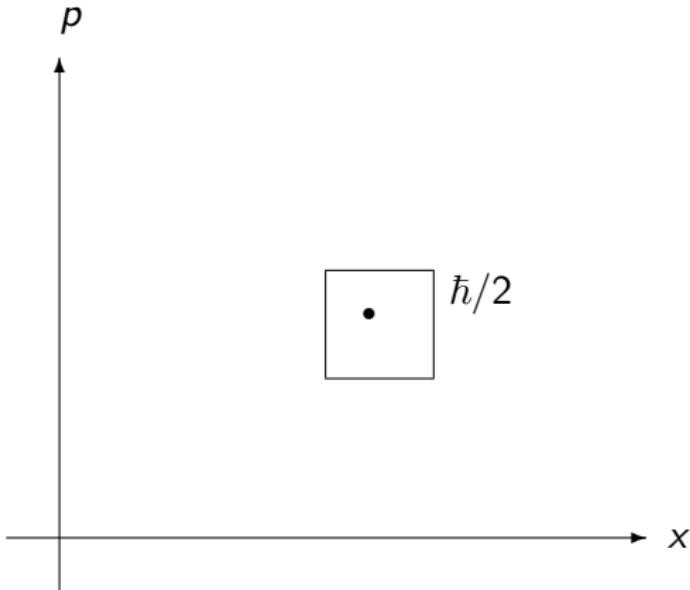
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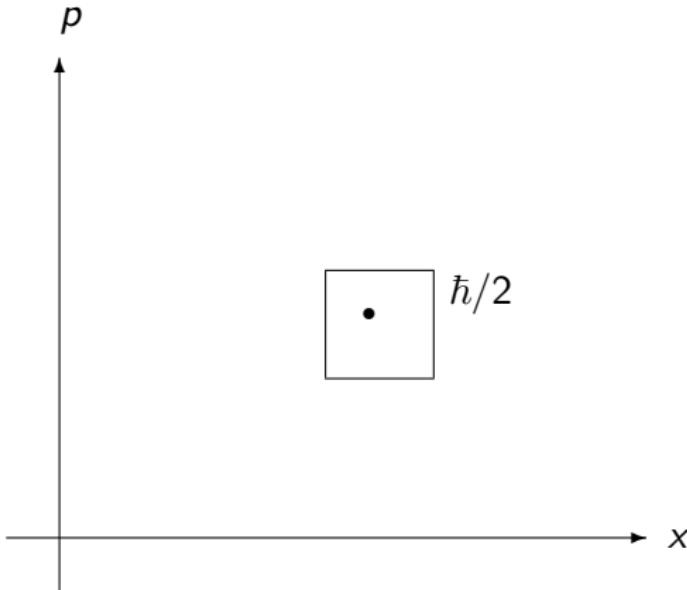
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\mathcal{A} = algebra of observables, \mathcal{H} = unitary representation, $\not{\partial}$ = Dirac operator

Reconstruction **theorem** (Connes, hep-th/9603053): There is a one-to-one correspondence between *commutative* (even) real spectral triples

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and (even-dimensional) compact Riemannian spin manifolds M .

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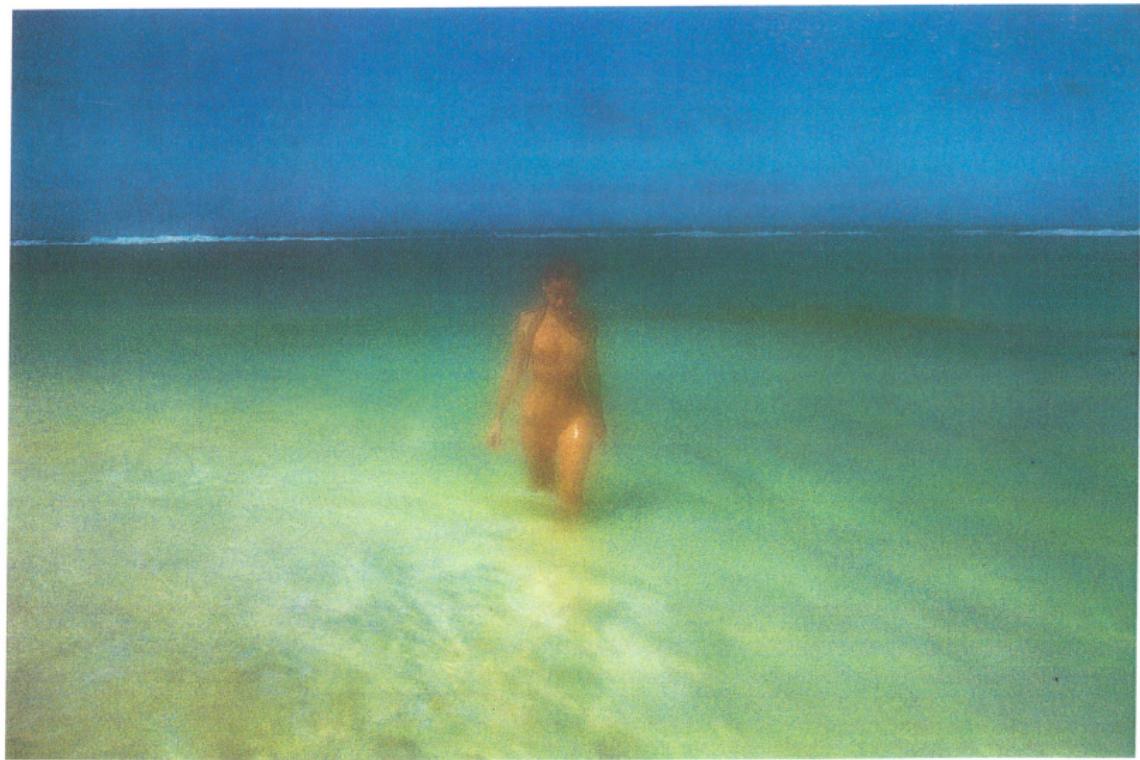
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Example (D. Hamilton, 1996)





noncommutative
geometry



??

Connes

almost
commutative
geometry

Riemannian
geometry

Connes,
 $(AC)^2$, mc^2



Einstein-Hilbert action
+ Weyl² + ...
+ Yang-Mills-Higgs ansatz
+ constraints

Einstein



Einstein-Hilbert action

Constraints on discrete parameters:



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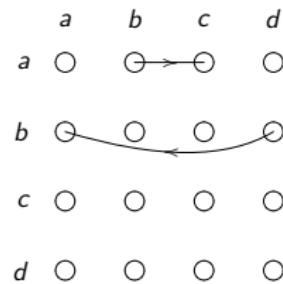
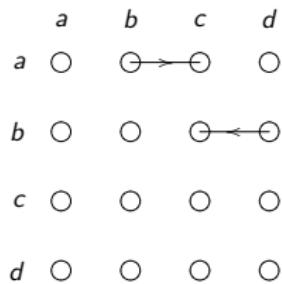
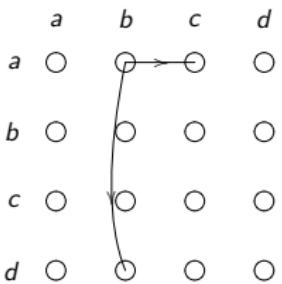
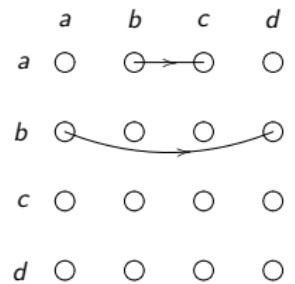


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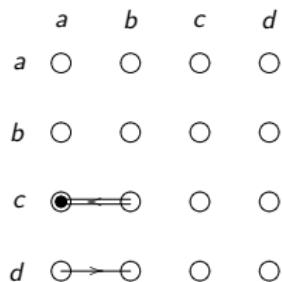
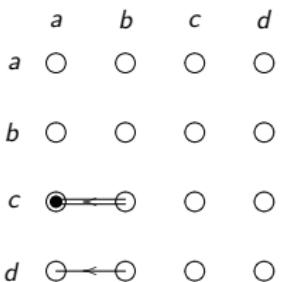
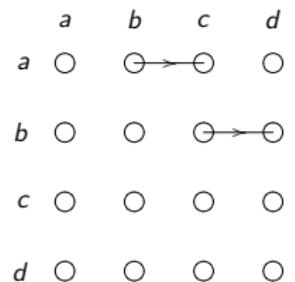


diag. 1

diag. 2

diag. 3

diag. 4



diag. 5

diag. 6

diag. 7

Jureit & Stephan 2007: the irreducible Krajewski diagrams with 4 or less simple algebras in KO dimension 6. Diag. 6 yields the standard model with one generation of fermions.

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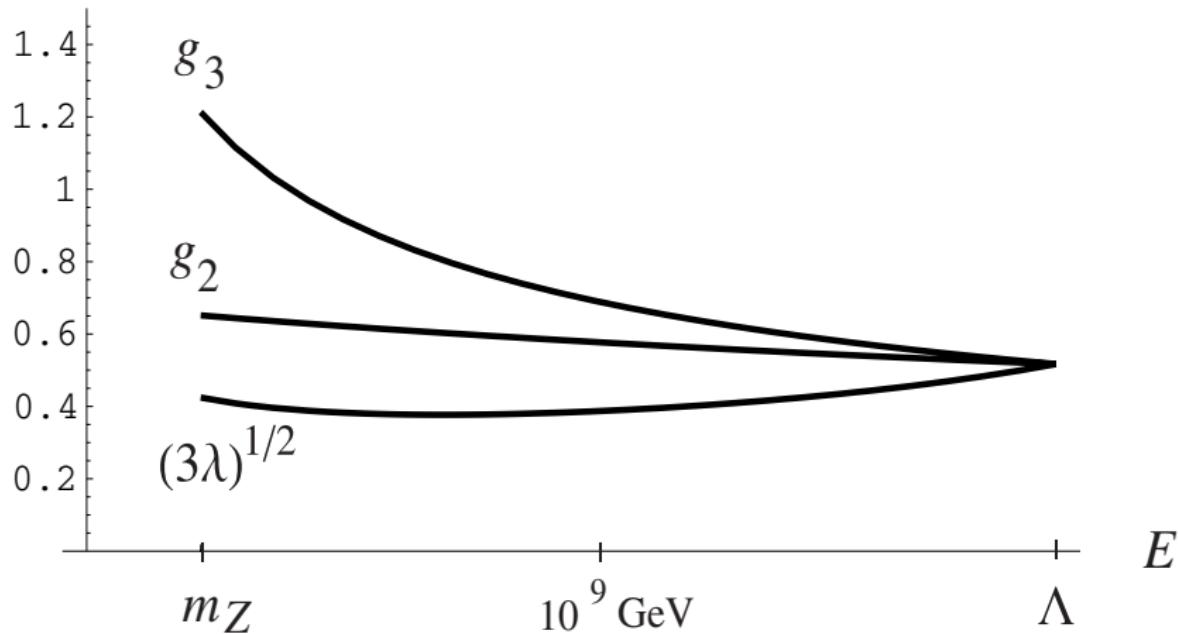
Constraints on continuous parameters:

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$$g_Y \rightsquigarrow \emptyset_{\text{internal}} \Rightarrow f(g, \lambda, g_Y) = 0.$$

$$g_2 = 0.6518 \pm 0.0003, \quad g_3 = 1.218 \pm 0.01 \quad \text{at} \quad E = m_Z$$

$$\Rightarrow \quad g_2 = g_3 = \sqrt{3\lambda} = \frac{1}{2}\sqrt{\sum g_Y^2} \quad \text{at} \quad E = \Lambda = 10^{17} \text{ GeV}.$$



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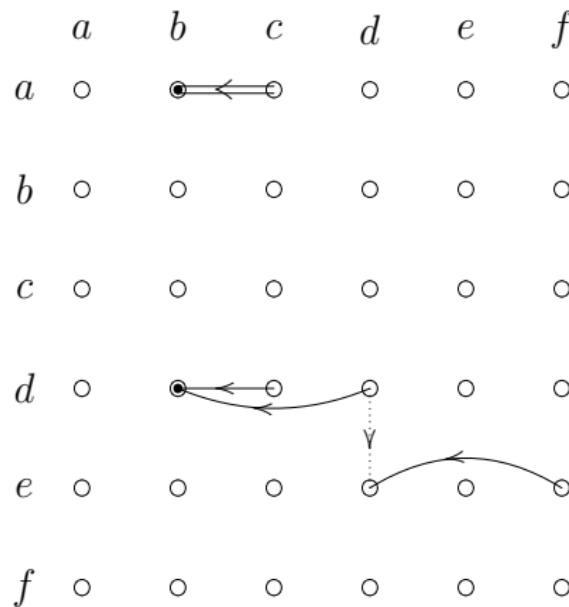
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$$m_H = 170 \pm 10 \text{ GeV} \quad (= 125.5 \pm 1.5 \text{ GeV}), \quad \text{PROBLEM!}$$

C. A. Stephan, "New Scalar Fields in Noncommutative Geometry,"
Phys. Rev. D **79** (2009) 065013 [arXiv:0901.4676 [hep-th]].





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- ▶ Two remaining physical scalars mix,

$$m_{H_1} \geq 120 \text{ GeV}, \quad m_{H_2} \geq 170 \text{ GeV},$$

depending on the masses of the X fermions.