Infrared Modifications of Gravity

Felix Berkhahn
ASC @ LMU Munich

in collaboration with
Dennis D. Dietrich (CP3)
Stefan Hofmann (LMU)
Florian Kühnel (LMU)
Parvin Moyassari (LMU)
Florian Niedermann (LMU)
Robert Schneider (LMU)

JCAP 1011 (2010) 018

Phys. Rev. Lett. 106 (2011) 191102

JCAP 09 (2011) 024

Phys.Rev.Lett. 108 (2012) 131102

arXiv:1205.6801 (hep-th), accecpted to published in Physical Review D

Motivation

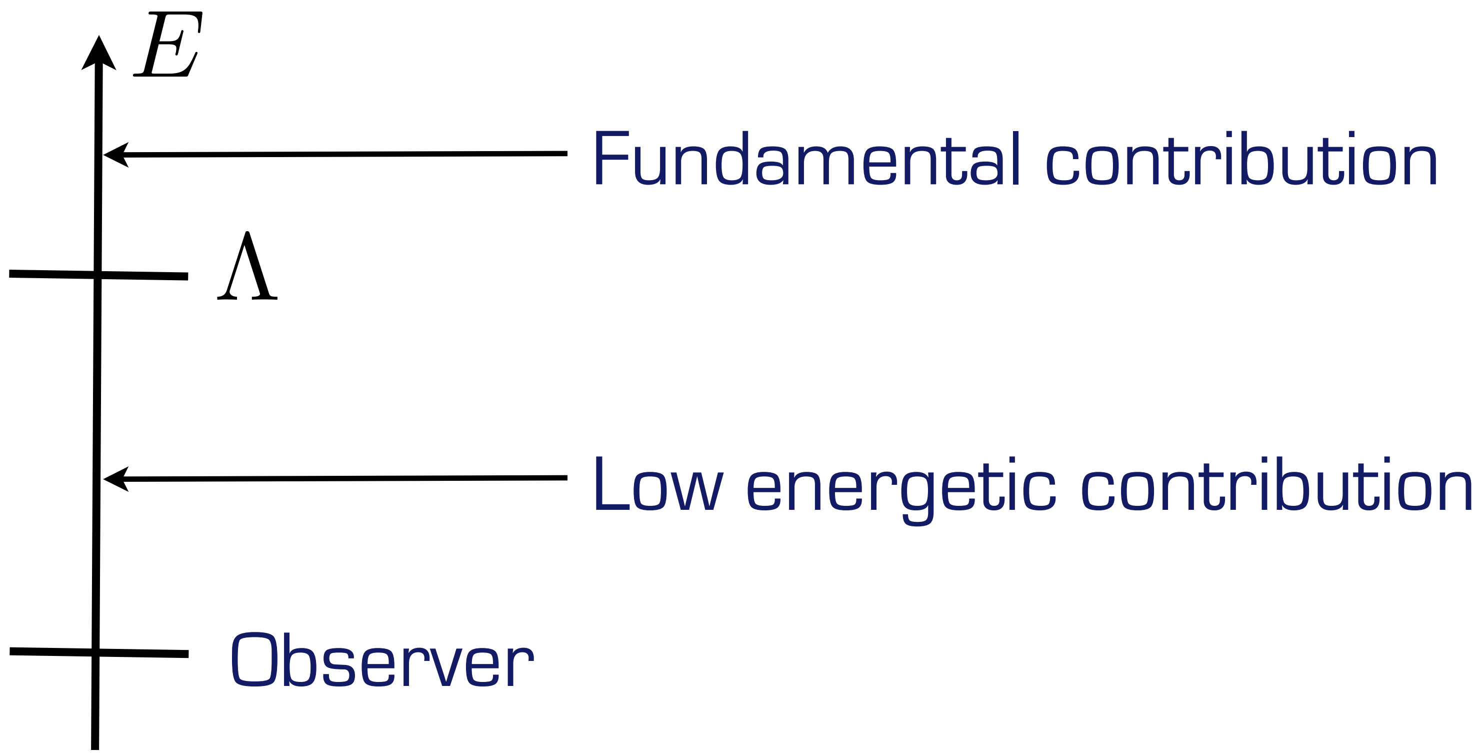
Point Of View

- Cosmological constant problem is a challenge of technical naturalness.

- Cannot be solved in a technical natural way by new high energy physics.

- Cosmological constant is only inferred gravitationally. Maybe the resolution resides in the gravitational sector?

- Massive Gravity and Brane Induced Gravity are examples of such theories.



New dofs in the IR?

Brane Induced Gravity

Brane Induced Gravity

- $$S = \int d^{4+n}x \left(\sqrt{-G} M_{4+n}^{n+2} R[G] + \sqrt{-g} M_{Pl}^2 \delta^{(n)}(x) R[g] + \sqrt{-g}\delta^{(n)}(x) \mathcal{L}(\Psi) \right)$$

- This effectively weakens gravity at large distances because of graviton leackage into the bulk.

- Crossover between 4dim- and (4+n)dim-gravity. $$r_c^2 = (M_4^2/M_{4+n}))(\epsilon^{2-n}/M_{4+n}^{n-2})$$

- For n > 1: CC on the brane curves the bulk and leaves the brane curvature untouched.

→ Very interesting with respect to the cosmological constant problem.

Ghost or no Ghost?

- Former claims in the literature suggested that BIG with n>1 contains a linear **ghost**.

- However, we **do not expect** a ghost **physically**:

Ghost or no Ghost?

- Former claims in the literature suggested that BIG with n>1 contains a linear **ghost**.

- However, we **do not expect** a ghost **physically**:

Ghost or no Ghost?

★ Former claims in the literature suggested that BIG with n>1 contains a linear **ghost**.

★ However, we **do not expect** a ghost **physically**:

$$\boxed{\sqrt{-G}\,M_6^4 R_6[G]}$$

Bulk

Brane

Ghost or no Ghost?

★ Former claims in the literature suggested that BIG with n>1 contains a linear **ghost**.

★ However, we **do not expect** a ghost **physically**:

$$\sqrt{-G}M_6^4 R_6[G]$$

Bulk

$$\boxed{\sqrt{-g}\delta^{(2)}(x)\mathcal{L}(\Psi)}$$

Brane

Ghost or no Ghost?

- ★ Former claims in the literature suggested that BIG with n>1 contains a linear **ghost**.

- ★ However, we **do not expect** a ghost **physically**:

$$\boxed{\int \mathcal{D}\Psi_{k^2>\Lambda} \exp\left(i \int d^4x \sqrt{-g}\mathcal{L}(\Psi) \right)}$$

$$\sqrt{-G}M_6^4 R_6[G]$$

$$\sqrt{-g}\delta^{(2)}(x)\mathcal{L}(\Psi)$$

Bulk

Brane

Ghost or no Ghost?

- ★ Former claims in the literature suggested that BIG with n>1 contains a linear **ghost**.

- ★ However, we **do not expect** a ghost **physically**:

$$\int \mathcal{D}\Psi_{k^2>\Lambda} \exp\left(i\int d^4x\sqrt{-g}\mathcal{L}(\Psi)\right)$$

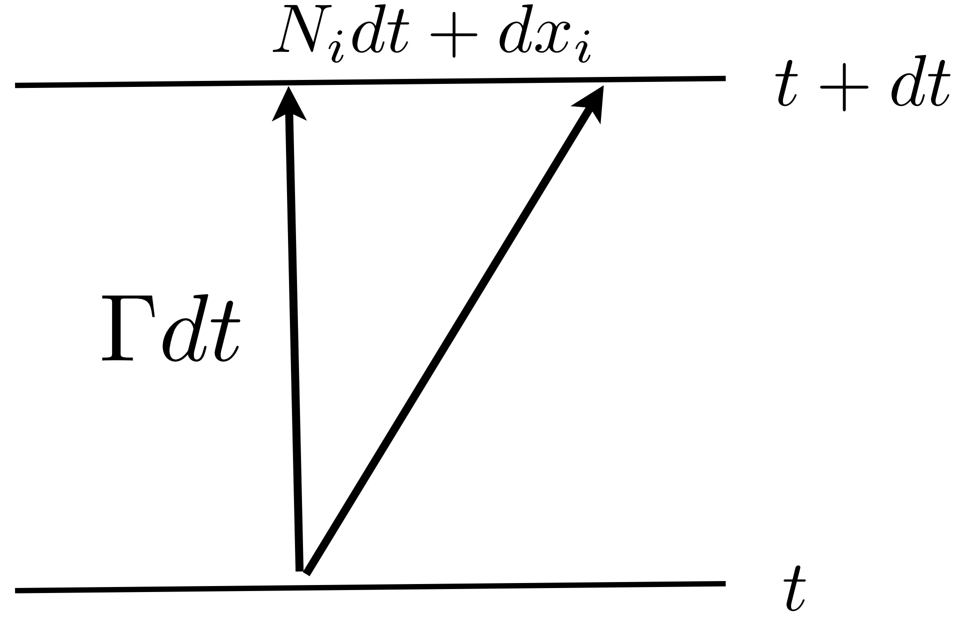
$$\sqrt{-G}M_6^4 R_6[G] + \sqrt{-g}\delta^{(2)}(x)M_{Pl}^2 R[g] + \sqrt{-g}\delta^{(2)}(x)\mathcal{L}(\Psi)$$

Bulk

Brane

Hamiltonian Analysis

- Successive ADM splits:



- $$g_{AB} = \begin{pmatrix} \lambda_{\mu\nu} & \Lambda_\mu \\ \Lambda_\nu & \Lambda^2 + \Lambda_\lambda \Lambda^\lambda \end{pmatrix}$$

- $$\lambda_{\mu\nu} = \begin{pmatrix} \omega_{\alpha\beta} & \Omega_\alpha \\ \Omega_\beta & \Omega^2 + \Omega_\gamma \Omega^\gamma \end{pmatrix}$$

- $$\omega_{\alpha\beta} = \begin{pmatrix} -\Gamma^2 + \Gamma_i \Gamma^i & \Gamma_i \\ \Gamma_j & \gamma_{ij} \end{pmatrix}$$

- $$\sqrt{-g} R^{(d)} = \sqrt{-g} \Big\{ R^{(d-1)} + (\hat{n}_d \cdot \hat{n}_d) \Big[(\mathrm{Tr} K_d)^2 - \mathrm{Tr} K_d^2 + 2(\nabla \cdot ((\hat{n}_d \cdot \nabla) \hat{n}_d) - \nabla \cdot (\hat{n}_d (\nabla \cdot \hat{n}_d))) \Big] \Big\} .$$

Constraints

- Inversion of $\Pi_l = \dfrac{\partial \mathcal{L}}{\partial \dot\rho_l}(\rho_l, \dot\rho_l)$ yields 6 primary constraints $\phi_a^{(1)}$

- Canonical consistency gives $\phi_a^{(2)} = \dot\phi_a^{(1)} = \{H, \phi_a^{(1)}\}$

  6 secondary constraints

- Secondary constraints are conserved $\dot\phi_a^{(2)} = \{H, \phi_a^{(2)}\} \simeq 0$

- First class system $\forall\, p, p', a, a' : \ \{\phi_a^{(p)}, \phi_{a'}^{(p')}\} \simeq 0$

$\longrightarrow$ Gauge freedom $\delta\Theta = \xi\{\Theta, \phi_a^{(p)}\}$

- Same number of constraints as in higher dimensional GR

SO(2) Symmetry in Extra Dimensions

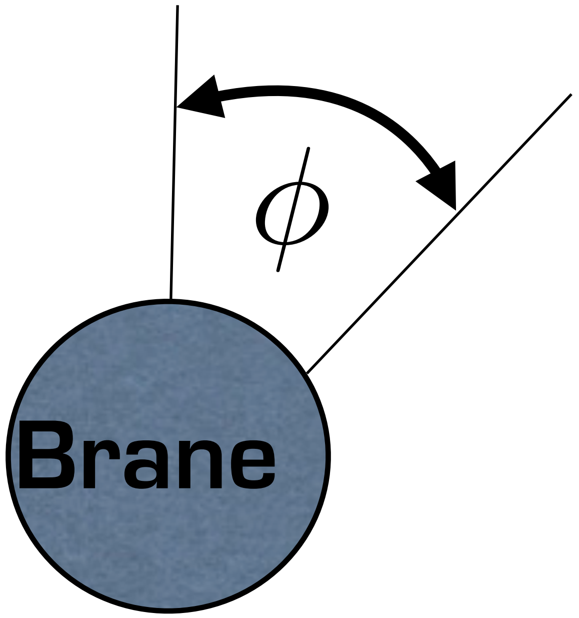
- Gravitational sources respect SO(2) Symmetry

  → Components of graviton field h are SO(2) symmetric.

- Implement this symmetry, for example

$$\begin{aligned} N_i &= \tilde{N}_i \cos\varphi \\ L_i &= \tilde{N}_i \sin\varphi \end{aligned} \quad \text{where} \quad \begin{aligned} N_i &= \delta\Omega_i \\ L_i &= \delta\Lambda_i \end{aligned}$$

are the extra-dimensional shift functions.



- Implementation before performing the ADM split allows to generalize to arbitrary n (in preparation with L. Eglseer and F. Niedermann)

Degrees of freedom

- ★ Index symmetry of h_{AB} yields 21 independent entries

  → 42 phase space degrees of freedom (psdof)

- ★ 12 first class constraints + SO(2) symmetry

  → Reduction by 24 + 8 = 32 psdof. Gives 42 - 32 = 10 psdof.

→ 5 physical degrees of freedom.
Same number as in Massive Gravity!
(Generic for every n)

- ★ However, only 2 are sourced by 4-dim. source $h_{\mu\nu}T^{\mu\nu}$

No Ghost!

★ Using the constraints and SO(2) Symmetry, we obtain the Hamiltonian on the constraint surface:

$$\mathcal{H} = \frac{1}{M_6^4}\Pi^{(T)}_{(R)ij}\Pi^{(T)ij}_{(R)} + \frac{1}{M_4^2}\,\delta_y^{(2)}\,\Pi^{(T)}_{(I)ij}\Pi^{(T)ij}_{(I)} + \frac{1}{4M_6^4}\Pi_N^2 + \frac{1}{2M_6^4}\tilde{\Pi}_i\tilde{\Pi}^i + \frac{1}{4}M_6^4\tilde{F}_{ij}\tilde{F}^{ij}$$
$$+\frac{1}{4}M_6^4\partial_a h^{(tt)}_{ij}\partial^a h^{(tt)ij} + \frac{1}{4}\left(M_6^4 + M_4^2\,\delta_y^{(2)}\right)\partial_k h^{(tt)}_{ij}\partial^k h^{(tt)ij} + 2M_6^4\partial_a N\partial^a N$$

→ **Manifestly positive definite!**

→ **We conclude that BIG in higher codimensions does not contain a ghost**

Covariant approach

★ Former treatments diagnosed the ghost in the scalar mode S

$$h_{\alpha\beta} \quad = \quad D^{(\mathrm{tt})}{}_{\alpha\beta} + P^{(\parallel)}{}_{\alpha\beta}\, B + \eta_{\alpha\beta}\, S$$

★ In some kinematic regime, S has the wrong sign in front of its kinetic operator.

→ Brane-to-brane propagator suggests the exchange of a ghost degree of freedom

$$G^{(\mathrm{S})}(p^2) = \frac{2}{(\frac{n+2}{n-1})\kappa_n^{-1} g_n^{-1}(p^2) - 2\kappa_0^{-1} p^2}$$

For example for codim 2 $\quad g_n(p^2) \propto \ln\left(1 + \frac{\kappa_2^{-1/2}}{p^2}\right)$

Possible Resolution

★ Former treatments did not consider the 00-Einstein constraint

$$\left[\partial^i\partial^j-\delta^{ij}(\Delta_3+\Delta_n)\right] D^{(\mathrm{tt})}_{ij}+\frac{n+2}{n-1}\left[\Delta_n P^{(\|)}{}_i^{\;i}+\Delta_3\right]S$$
$$=\delta^n(x)\kappa_n\left\{2t_{00}+\kappa_0^{-1}\left(2\Delta_3 S-\left(\partial^i\partial^j-\delta^{ij}\Delta_3\right)D^{(tt)}_{ij}\right)\right\}$$

→ S is a constrained degree of freedom

★ Conventional brane-to-brane propagator bad diagnostic tool (maybe use of Dirac brackets?)

Cosmology solution

Metric ansatz

★ Most general metric ansatz consistent with spatial homogeneity and isotropy along the 3 brane directions

$$ds^2 = -n^2(\tilde{r},\tilde{t})d\tilde{t}^2 + c^2(\tilde{r},\tilde{t})d\tilde{r}d\tilde{t} + a^2(\tilde{r},\tilde{t})\delta_{ij}dx^i dx^j + b^2(\tilde{r},\tilde{t})d\tilde{r}^2 + d^2(\tilde{r},\tilde{t})\tilde{r}^2 d\Omega$$

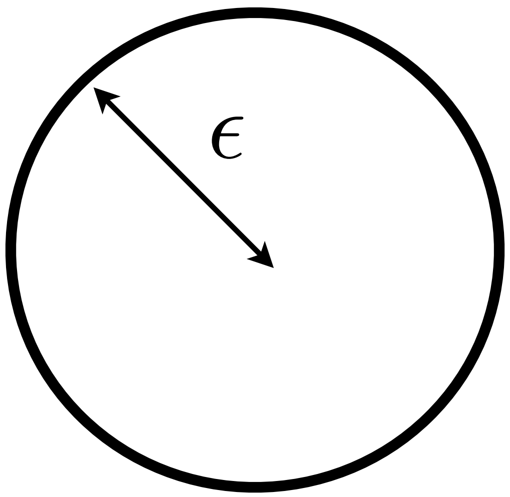
★ Use gauge symmetry $\begin{array}{c}\tilde{t} \to \tilde{t}(t,r) \\ \tilde{r} \to \tilde{r}(t,r)\end{array}$ to implement $\begin{array}{rcl} c(r,t) & = & 0 \\ b(r,t) & = & d(r,t)\end{array}$

$$ds^2 = -n^2(r,t)dt^2 + a^2(r,t)\delta_{ij}dx^i dx^j + b^2(r,t)(dr^2 + r^2 d\Omega)$$

★ Residual gauge symmetry $t \to t(t')$ allows to set $n(r_0,t) = 1$

Regularization

- ★ Brane width ϵ necessary to obtain a modification
- ★ Regularize the brane width using a hollow cylinder
  - ★ Technical advantage, use same approach as Deffayet (jump, mean etc)
  - ★ Minimal regularization scheme
  - ★ Results will not depend on regularization scheme



- ★ $$\delta^{(n)}(y) \to \frac{\delta(r-\epsilon)}{\epsilon^{n-1}} \frac{\Gamma(n/2)}{2\pi^{n/2}}$$

From PDE to ODE (I)

- 5 independent Einstein equations (00, ij, rr, tr, $\phi\phi$)
- For example, the 00-Einstein equation reads

$$M_6^4(-3\frac{\dot{a}^2}{a^2n^2} - 6\frac{\dot{a}\dot{b}}{abn^2} - \frac{\dot{b}^2}{b^2n^2} + 3\frac{a'}{ab^2r} + 3\frac{a'^2}{a^2b^2} + \frac{b'}{b^3r} - \frac{b'^2}{b^4} + 3\frac{a''}{ab^2} + \frac{b''}{b^3})$$
$$+\frac{1}{2\pi\epsilon b}\delta(r-\epsilon)\left(M_4^2(-3\frac{\dot{a}^2}{a^2n^2}) - \frac{\rho}{n^2}\right) = 0$$

Goal: Elimination of r derivatives

- Delta functions induce a kink: $a'' = \hat{a}'' + \delta(r-\epsilon)[a']$
- With the jump $[a'] = \lim_{\delta\to 0}\Big(a'(\epsilon+\delta,t) - a'(\epsilon-\delta,t)\Big)$ and regular part $\hat{a}''$

→ 3 delta function matching conditions

From PDE to ODE (II)

★ Additional information: Mean and Jump of Einstein equations

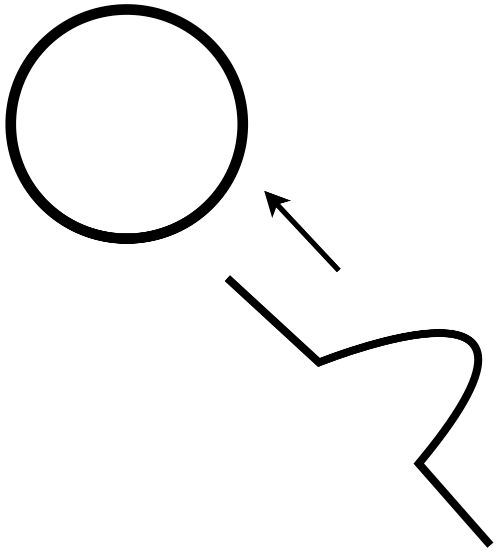
$$[G_{00}] = 0 \ , \ \ \#G_{00}\# = 0 \ , \ \text{etc.}$$

$$\#a'\# = \frac{1}{2}\lim_{\delta\to 0}\Big(a'(\epsilon+\delta,t) + a'(\epsilon-\delta,t)\Big)$$

★ 10 additional equations (8 independent), for 15 variables $\rho, a, b, \ \#a'\#, [a'], \ldots$

★ Together with matching conditions 8 + 3 = 11 equations.

→ 4 are missing.



No surprise:
Initial conditions on extra-dimensional hypersurface impact brane evolution

Embedded in a Minkowski Bulk

- Embed in a Minkowski bulk $$R_{\mu\nu\alpha\beta} = 0$$

→ No dependence on initial conditions

- Taking the limes from outside $$\lim_{\delta\to 0}\Big(R_{\mu\nu\alpha\beta}(\epsilon+\delta,t)\Big) = 0$$

gives exactly 4 more independent equations

Modified second Friedman equation

$$aH\frac{dH}{da} = -\frac{3}{2}\sum_i \Omega_i(a) - b\,\Omega_\epsilon\left(\frac{\sum_i \Omega_i(a)}{H} - 3H\right)$$
$$\pm\frac{1}{2}\sqrt{\left(2b\,\Omega_\epsilon\frac{\sum_i \Omega_i(a)}{H} - 6b\,\Omega_\epsilon H\right)^2 + 6b\,\Omega_\epsilon\left(\sum_i \Omega_i(a)\right)\left(2\,\frac{\sum_i \Omega_i(a)}{H} - 6\,H\right) + 8b\,\Omega_\epsilon H\left(-\sum_i \Omega_i(a) + 4H^2\right)}$$

Summary BIG

- **No Ghost** in $n>1$ models
- Possible to derive modified Friedman equations as it was for $n=1$
- Outlook: Confront theory with supernova data

Massive Gravity

Degravitation

- **Massless spin-2 field:**

$$\epsilon^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} = \Lambda \eta_{\mu\nu}$$

$$h_{\mu\nu} = \frac{\Lambda}{6} x_\mu x_\nu$$

- **Massive spin-2 field:**

$$\epsilon^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} - m^2 (h_{\mu\nu} - \eta_{\mu\nu} h) = \Lambda \eta_{\mu\nu}$$

$$h_{\mu\nu} = \frac{\Lambda}{3m^2} \eta_{\mu\nu}$$

- **Flat space is a solution:**

$$g_{\mu\nu} = \left(1 + \frac{\Lambda}{3m^2}\right) \eta_{\mu\nu}$$

General „Massive“ Deformations

- We are addressing cosmological questions.

→ Consider linear theory in a cosmological background.

$$S = \frac{1}{2}\int_M d^4x \, \sqrt{|g_0|} \left(h_{\mu\nu} \left[\mathcal{E}^{\alpha\beta\mu\nu}(g_0, \nabla) + \mathcal{M}(g_0)^{\alpha\beta\mu\nu} \right] h_{\alpha\beta} + T^{\mu\nu} h_{\mu\nu} \right)$$

$g_0^{\mu\nu}$ standard FRW metric.

- Question: Is there a unique choice for $\mathcal{M}$

like for a Minkowski background?

General „Massive“ Deformations

★ We are addressing cosmological questions.

→ Consider linear theory in a cosmological background.

$$S = \frac{1}{2}\int_M d^4x\,\sqrt{|g_0|}\,\left(h_{\mu\nu}\left[\mathcal{E}^{\alpha\beta\mu\nu}(g_0,\nabla) + \mathcal{M}(g_0)^{\alpha\beta\mu\nu}\right]h_{\alpha\beta} + T^{\mu\nu}h_{\mu\nu}\right)$$

Linearized Einstein Hilbert.

$g_0^{\mu\nu}$ standard FRW metric.

★ Question: Is there a unique choice for $\mathcal{M}$

like for a Minkowski background?

General „Massive“ Deformations

★ We are addressing cosmological questions.

→ Consider linear theory in a cosmological background.

Deformation term.

$$S = \frac{1}{2}\int_M d^4x\,\sqrt{|g_0|}\,\left(h_{\mu\nu}\,\left[\mathcal{E}^{\alpha\beta\mu\nu}(g_0,\nabla) + \mathcal{M}(g_0)^{\alpha\beta\mu\nu}\right]h_{\alpha\beta} + T^{\mu\nu}h_{\mu\nu}\right)$$

Linearized Einstein Hilbert.

$g_0^{\mu\nu}$ standard FRW metric.

★ Question: Is there a unique choice for $\mathcal{M}$

like for a Minkowski background?

Stability Analysis

Naive FP

HIGUCHI BOUND

- The most simple ansatz would be the naive FP

$$\mathcal{M}^{\mu\nu\alpha\beta} = m^2 \left(g_0^{\mu\nu} g_0^{\alpha\beta} - g_0^{\mu\beta} g_0^{\nu\alpha} \right)$$

- On a deSitter background, Higuchi has shown that

$$m^2 > H^2 = const.$$

to guarantee the absence of negative norm states.

- On FRW: $H \to H(t)$ Implications?

RESULTS OF NAIVE FP IN FRW I

★ At high energies the action can be diagonalized:

$$\mathcal{L} \supset A(t)\dot{\phi}^2 + B(t)(\vec{\nabla}\phi/a)^2$$

1. Sign of $A(t)$ determines the norm in Fock-space:

→ $$\left[a(\mathbf{k}), a^\dagger(\mathbf{k'})\right] = \mathrm{sign(A)}\delta^{(3)}(\mathbf{k} - \mathbf{k'})$$

→ Unitarity bound: $m^2 > H^2 + \dot{H}$

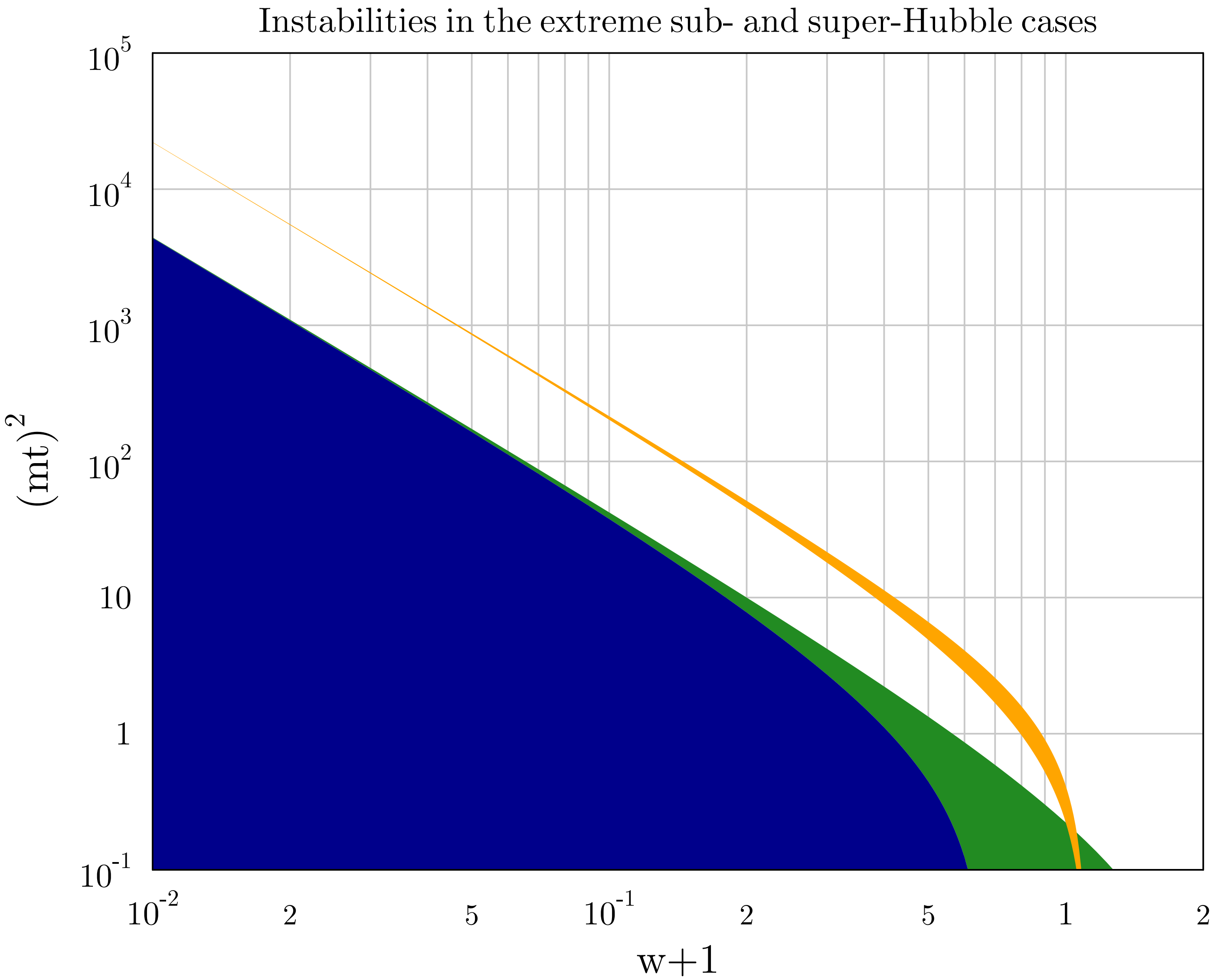
2. Sign of $B(t)$ implies classical (in)stability.

→ Stability bound: $m^2 > H^2 + \frac{1}{3}\dot{H}$

Results of Naive FP in FRW II

Additionally, we performed a complete cosmological perturbation analysis.

- Valid at all energies.
- Incorporates all degrees of freedom.



Orange: Classically unstable for zero momentum.

Green: Classically unstable for high momenta.

Blue: Unitarity violating.

SELF-PROTECTION

- The stability bound is stronger than the unitarity bound for non-phantom matter . $\dot{H} < 0$

  → System self-protects from direct unitarity violation.

- Violation of stability bound

  → Large fluctuations.

  → Formation of a new background.

- How to avoid the classical instability?

  Try $m \to m(t)$! Or even more general ....

General case

The "Deformation Matrix"

- Covariance and symmetry constrain the IR leading terms of the deformation matrix as:

$$\mathcal{M}^{\mu\nu\alpha\beta} = (m^2 + \alpha R_0)\left(g_0^{\mu\nu} g_0^{\alpha\beta} - g_0^{\mu\beta} g_0^{\nu\alpha}\right)$$

$$+ \beta \left(R_0^{\mu[\nu} g_0^{\beta]\alpha} + R_0^{\alpha[\beta} g_0^{\nu]\mu}\right)$$

$$+ \gamma R_0^{\mu\alpha\nu\beta}$$

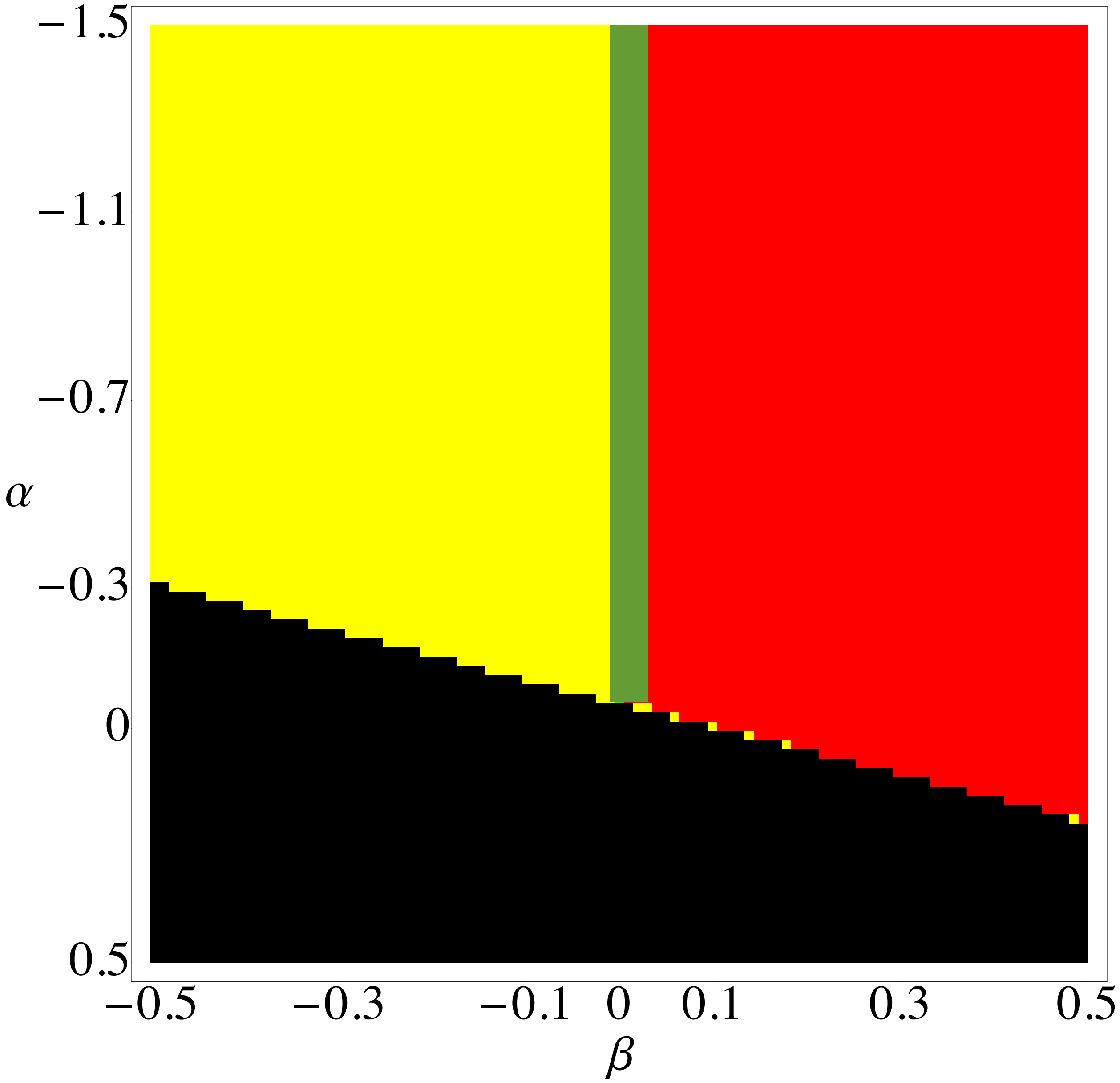
$$S = \frac{1}{2}\int_M d^4x \sqrt{|g_0|}\left(h_{\mu\nu}\left[\mathcal{E}^{\alpha\beta\mu\nu}(g_0, \nabla) + \mathcal{M}(g_0)^{\alpha\beta\mu\nu}\right] h_{\alpha\beta} + T^{\mu\nu} h_{\mu\nu}\right)$$

- On FRW: $\gamma = 0$ (vanishing Weyl tensor)

Stability Analysis: General Case

Bounds in the general case $\alpha \neq 0$, $\beta \neq 0$ are much more complicated.

$$m = 0$$



Green: Classically stable and unitary.

Yellow: Self-Protection.

Red: Unitarity violation.

Black: No stability or unitarity today.

Stability Analysis: Conclusion

★ ONLY the “running mass“ deformation

$$\mathcal{M}^{\mu\nu\alpha\beta} = \left(m_0^2 + \alpha R_0\right)\left({g_0}^{\mu\nu}{g_0}^{\alpha\beta} - {g_0}^{\mu\beta}{g_0}^{\nu\alpha}\right)$$

will yield a stable theory.

★ Absolute stability requires proper covariantization of the deformation matrix!

★ α must be sufficiently negative.

★ The form of the theory is constrained UNIQUELY like in Minkowski!