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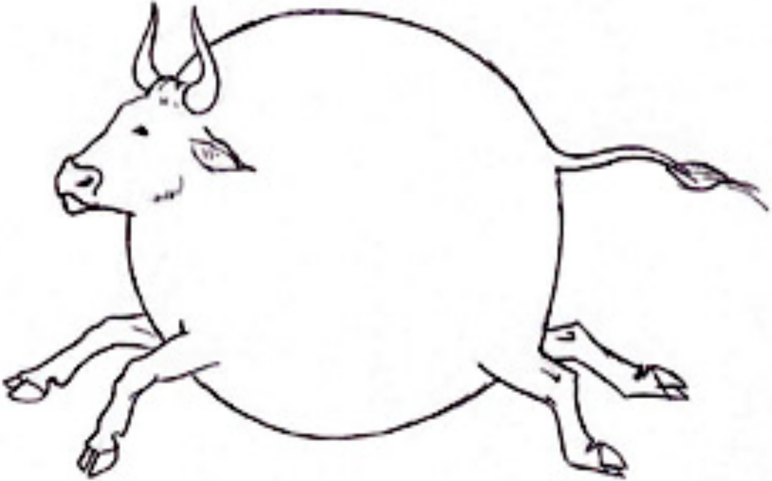

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$$s(r) := \begin{cases} \sinh(kr)/k & \sigma = -1 \\ r & \sigma = 0 \\ \sin(kr)/k & \sigma = +1 \end{cases}, \quad k > 0,$$

$$c(r) := s'(r) = \begin{cases} \cosh(kr) & \sigma = -1 \\ 1 & \sigma = 0 \\ \cos(kr) & \sigma = +1 \end{cases} \quad \bar{c}(t) := \begin{cases} \cos(kt) & \sigma = -1 \\ 1 & \sigma = 0 \\ \cosh(kt) & \sigma = +1 \end{cases}$$

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and isometry groups: $O(2,3)$, $O(1,3) \ltimes \mathbb{R}^4$, and $O(1,4)$.

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A few Lie brackets:

$$\begin{array}{llll}
[R_x, R_y] = -R_z, & [R_x, T_z] = T_y, & [T_y, T_z] = -\sigma k^2 R_x, & [R_z, T_z] = 0, \\
[L_y, L_z] = R_x, & [R_x, L_z] = L_y, & [R_z, L_z] = 0, & [T_x, L_z] = 0, \\
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May set $b = 1$, "cosmic time t", or set $b = a$, "conformal time η".

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Appreciate: The three isometry groups for the three families $\sigma = \pm 1,\ 0$ are isomorphic.
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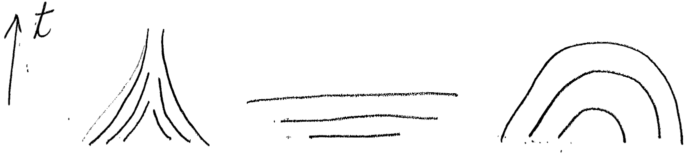
Robertson-Walker spacetimes admit foliations by $t = t_0$ with 3-dimensional space-like leaves of maximal symmetry.



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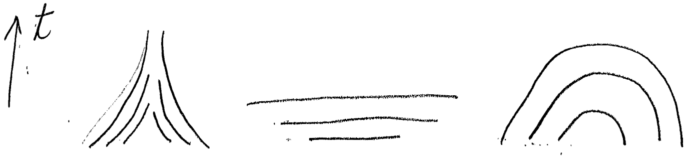
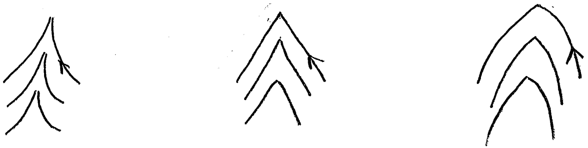


Want: spacetimes admitting foliations by $u = u_0$ with 3-dimensional light-like leaves of maximal symmetry.



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Lemma: In the coordinates χ, ψ, θ, φ,

$$\chi := \sqrt{\bar{s}^2(t(\eta(u,v))) - \bar{c}^2(t)\, s^2(r(u,v))},\ \psi := \mathrm{Ar}\tanh\, \frac{\bar{c}(t)\, s(r)}{\bar{s}(t)}\,,$$

all 6 generators, R_i, L_i are independent of σ.

For example

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Put the initial $\sigma = 0$ and redefine $\chi = \sqrt{t^2 - r^2} = \sqrt{2uv}$ such that $a(\chi) = \chi\, b(\chi)$. Then

$$\mathrm{d}\tau^2 = b^2(\sqrt{2uv})\left[2\,\mathrm{d}u\,\mathrm{d}v - \tfrac{1}{2}(u-v)^2\,\mathrm{d}\theta^2 - \tfrac{1}{2}(u-v)^2\sin^2\theta\,\mathrm{d}\varphi^2\right].$$

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Another herecy? Did you say our past light-cones?

$\Omega_k := -\sigma(a_0 H_0)^{-2} > 0$. Attention minus-sign: $\Omega_m + \Omega_\Lambda + \Omega_k = 1$.

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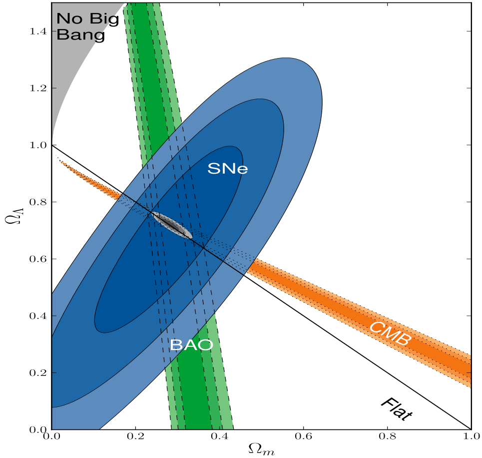

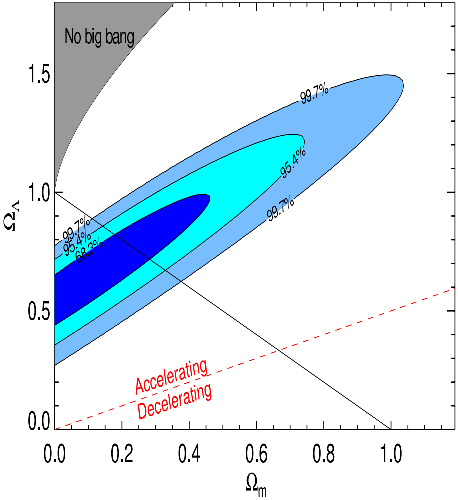

Experiments	Errors, 1σ	Reference
SNe+BAO+CMB+H_0	$\Omega_k = 0.002 \pm 0.005$	[1]
SNLS+WMAP7+BAO	$\Omega_k = -0.002 \pm 0.006$	[2]
SNe+BAO+CMB+H_0+w_Λ	$\Omega_k = 0.027^{+0.012}_{-0.011}$	[1]

[1] N. Suzuki et al., *The Hubble Space Telescope Cluster Supernova Survey: V. Improving the Dark Energy Constraints Above $z > 1$ and Building an Early-Type-Hosted Supernova Sample*, ApJ 746 (2012) 85.

[2] M. Sullivan et al., *SNLS3: Constraints on Dark Energy Combining the Supernova Legacy Survey Three Year Data with Other Probes*, Astrophys. J. 737 (2011) 102.