The quantum measurement problem in inflation

Patrick Peter

Institut d'Astrophysique de Paris

GRεCO

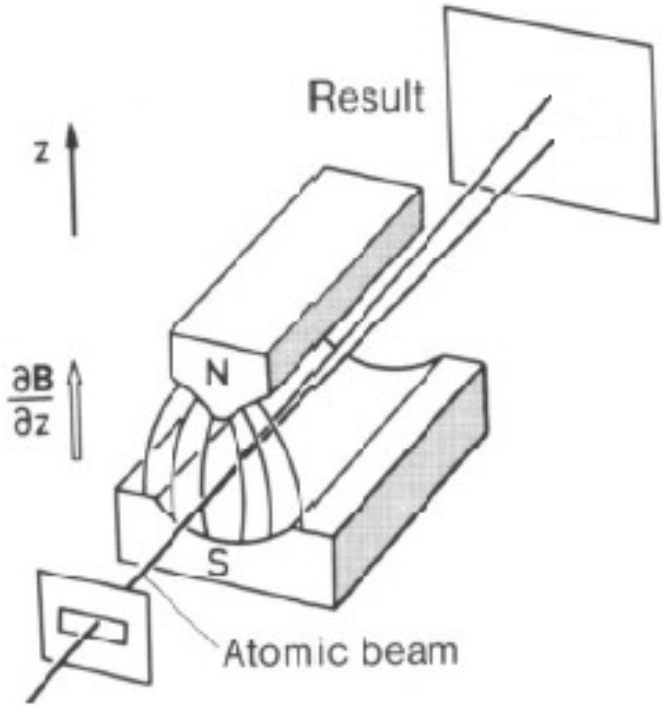
Based on J.Martin, V.Vennin & P.P., hep-th 1207.2086 (PR **D** – nov. 2012)

cnrs dépasser les frontières

UPMC SORBONNE UNIVERSITÉS

NORDITA

The measurement Problem in Quantum Mechanics



The Stern & Gerlach experiment

$$|\Psi_{\rm in}\rangle = \frac{1}{\sqrt{2}}\big(|\uparrow\rangle + |\downarrow\rangle\big) \otimes |SG_{\rm in}\rangle$$

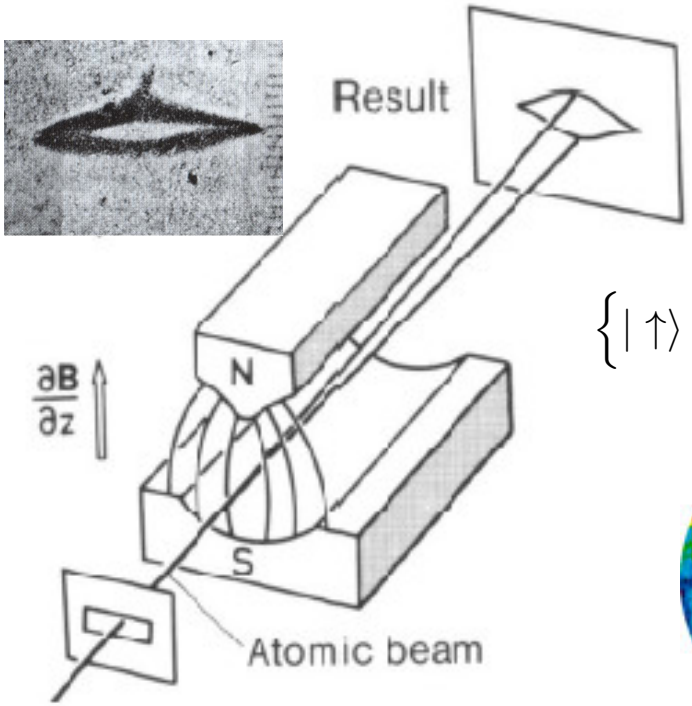
unitary / deterministic Schrödinger evolution

$$|\Psi_{\rm f}\rangle = T\Big[\exp\int_{t_{\rm in}}^{t_{\rm f}} \hat{H}(t')\mathrm{d}t'\Big]\ |\Psi_{\rm in}\rangle$$

$$= \frac{1}{\sqrt{2}}\big(|\uparrow\rangle \otimes |SG_{\uparrow}\rangle + |\downarrow\rangle \otimes |SG_{\downarrow}\rangle\big)$$

Problem: How to reach $|\Psi_{\rm f}\rangle = |\uparrow\rangle \otimes |SG_{\uparrow}\rangle$ or $|\Psi_{\rm f}\rangle = |\downarrow\rangle \otimes |SG_{\downarrow}\rangle$?

The measurement Problem in Quantum Mechanics



The Stern & Gerlach experiment

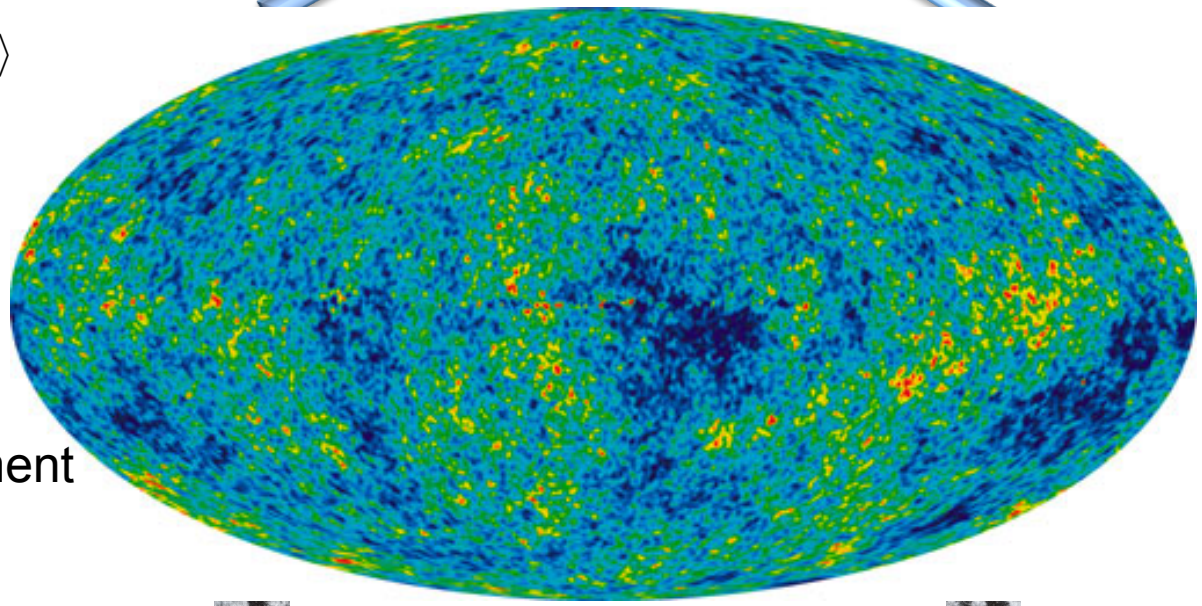
$\{|\uparrow\rangle$

Statistical mixture

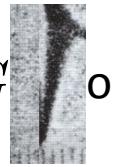
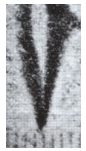
What about the Universe itself ?

$$\left\{|\uparrow\rangle \otimes |SG_{\uparrow}\rangle\right\} \cup \left\{|\downarrow\rangle \otimes |SG_{\downarrow}\rangle\right\}$$

reversibility ?



What about physical situations where only a single realization is at hand?

Problem: How to reach $|\Psi_{\rm f}\rangle = |\uparrow\rangle \otimes |SG$  or $|\Psi_{\rm f}\rangle = |\downarrow\rangle \otimes |SG_{\downarrow}\rangle$? 

The quantum mechanics of closed systems

- Copenhagen interpretation and the measurement problem

Physical system = Hilbert space of configurations

State vectors

Observables = self-adjoint operators

Measurement = eigenvalue

$$A|a_n\rangle = a_n|a_n\rangle$$

Evolution = Schrödinger equation (time translation invariance)

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$$

Hamiltonian

Born rule $\mathrm{Prob}[a_n;t] = |\langle a_n|\psi(t)\rangle|^2$

Collapse of the wavefunction $|\psi(t)\rangle$ before measurement $|a_n\rangle$ after

Schrödinger equation = linear (superposition principle) / unitary evolution

Wavepacket reduction = non linear / stochastic

Mutually incompatible

+ External observer

- EPR “element of reality”

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

$\Longrightarrow$ Entanglement - Bell’s inequality - cryptography - non locality ...

OCTOBER 15, 1935 PHYSICAL REVIEW VOLUME 48

Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

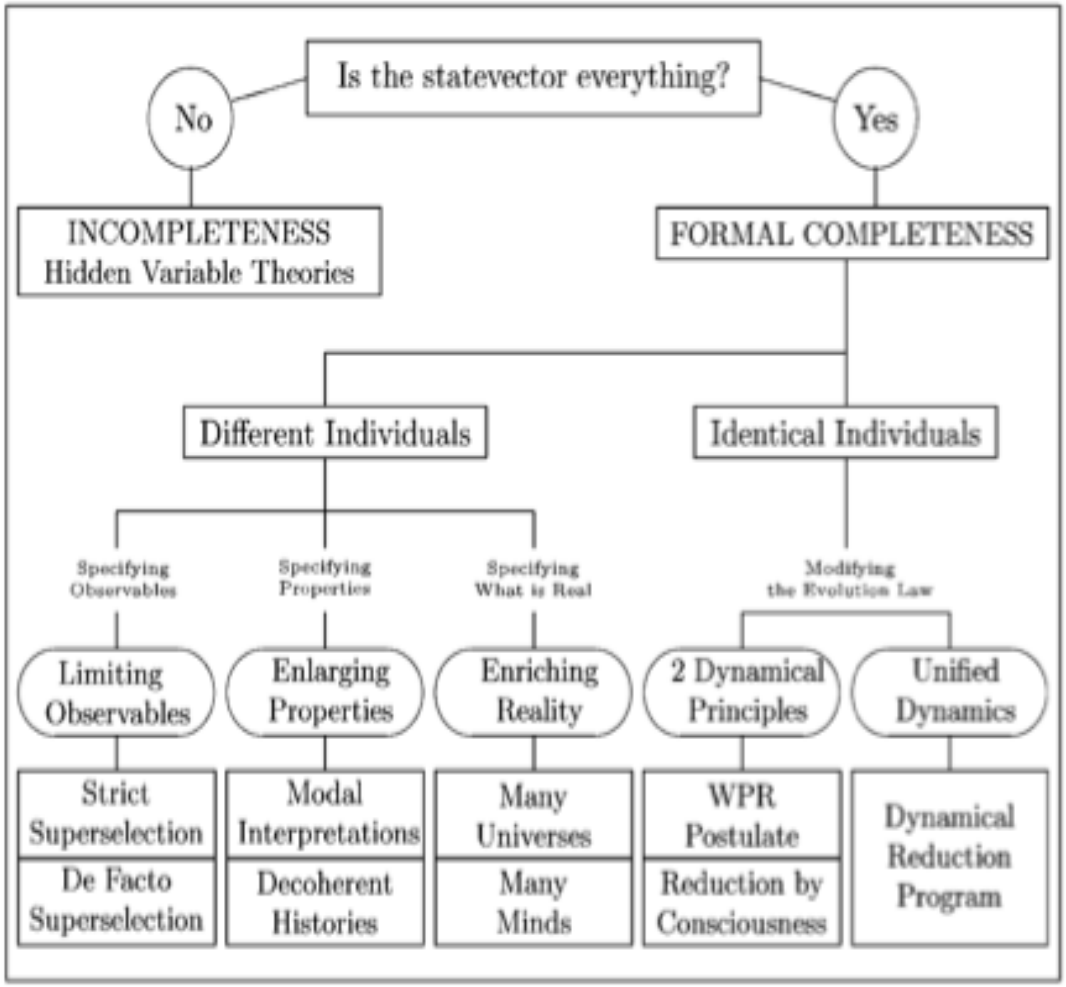
N. BOHR, *Institute for Theoretical Physics, University, Copenhagen*

(Received July 13, 1935)

It is shown that a certain "criterion of physical reality" formulated in a recent article with the above title by A. Einstein, B. Podolsky and N. Rosen contains an essential ambiguity when it is applied to quantum phenomena. In this connection a viewpoint termed "complementarity" is explained from which quantum-mechanical description of physical phenomena would seem to fulfill, within its scope, all rational demands of completeness.

Philosophical discussion, argument?

- Possible solutions and a criterion: the Born rule



A. Bassi, G.C. Ghirardi / Physics Reports 379 (2003) 257–426

- ***Superselection rules***
- ***Modal interpretation***
- ***Decoherent histories***
- ***Many worlds / many minds***

- ***Hidden variables***
- ***Modified Schrödinger dynamics***

} Born rule not put by hand!

Inflationary Cosmology

Quantized fluctuations evolved over an expanding, homogeneous and isotropic background

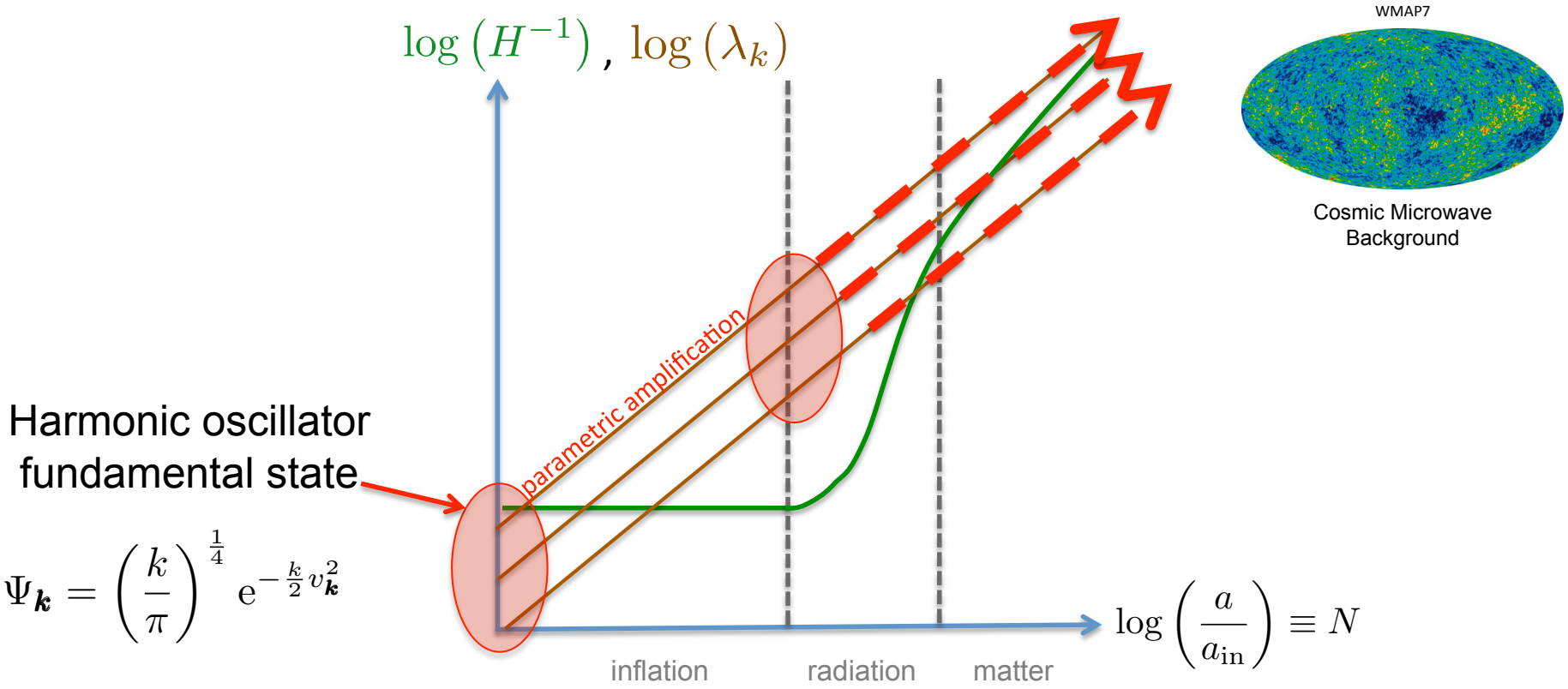
Such fluctuations are:

- Combined perturbations v of the metric and of the inflaton scalar field
- Of quantum nature $\hat{v}$
- Amplified through parametric oscillations
- The seeds of the CMB temperature fluctuations

$$\frac{\delta \hat{T}}{T} \propto \hat{\zeta} \propto \hat{v}$$

Inflationary Cosmology

Quantized fluctuations evolved over an expanding, homogeneous and isotropic background



Classicalization of Cosmological Perturbations

Predictions of the theory:

Calculated by quantum average $\langle \Psi | \hat{O} | \Psi \rangle$

Usually in a lab:
repeat the experiment

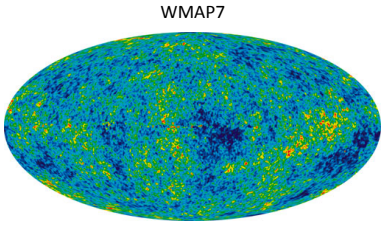
Ensemble average over experiments ⟷ Quantum average

Here one has a single experiment (a single universe)

→ Ergodicity

Spatial average over directions in the sky ⟷ Quantum average

Classicalization of Cosmological Perturbations



Cosmic Microwave Background

$$\frac{\delta\hat{T}}{T} = \frac{\hat{\zeta}}{5} = \frac{1}{5a\sqrt{2\epsilon_1}}\frac{\hat{v}}{M_{\rm Pl}}$$

Temperature Fluctuations

Curvature Perturbation

Background Function

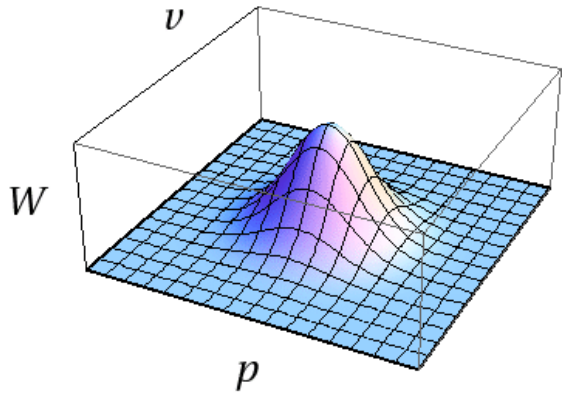
Mukhanov-Sasaki Variable

Quantization in the Schrödinger picture (reciprocal space)

$$\Psi(\eta, v_{\boldsymbol{k}}^{\rm R,I}) = \left[\frac{2\,\Re{\rm e}\Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1/4} {\rm e}^{-\Omega_{\boldsymbol{k}}(\eta)\left(v_{\boldsymbol{k}}^{\rm R,I}\right)^2}$$

Wigner Function

$$W(v_{\boldsymbol{k}}, p_{\boldsymbol{k}}) = \int \frac{{\rm d}x}{2\pi^2}\Psi^*(v_{\boldsymbol{k}} - \frac{x}{2})\ {\rm e}^{-ip_{\boldsymbol{k}}x}\ \Psi(v_{\boldsymbol{k}} + \frac{x}{2})$$



$$\xrightarrow[\text{large squeezing}]{} \Psi\Psi^*\delta(p_{\boldsymbol{k}} + k\tan\phi_{\boldsymbol{k}}\ v_{\boldsymbol{k}})$$

Stochastic distribution of classical processes

Ergodicity

$$\left\langle \frac{\delta T(\xi, \boldsymbol{e})}{T} \right\rangle_\xi = \left\langle \frac{\delta T(\xi, \boldsymbol{e})}{T} \right\rangle_{\boldsymbol{e}}$$

ξ labels the realization

$\boldsymbol{e}$ labels the spatial direction

Classicalization of Cosmological Perturbations

Decoherence

Density matrix $\hat{\rho}_{\boldsymbol{k}} = |\Psi_{\boldsymbol{k}}\rangle\langle\Psi_{\boldsymbol{k}}|$

$$\mathrm{Tr}_{\mathrm{environment}}\left(\hat{\rho}_{\boldsymbol{k}}\right) \to \sum_{v} p(v)\ |\delta(v_{\boldsymbol{k}} - v)\rangle\,\langle\delta(v_{\boldsymbol{k}} - v)|$$

Statistical mixture

Questions:

- Statistical mixture … of what?
- Still, how do we get a specific outcome?
  (Quantum Measurement Problem)

Classicalization of Cosmological Perturbations

Decoherence and the quantum measurement probem

- S. L. Adler, quant-ph/0112095: *"Why Decoherence has not solved the Measurement Problem"*

- Joos, 1999: "*Does decoherence solve the measurement problem? Clearly not...*"

- Rebuttals to the claim that decoherence solves the measurement problem have been given in the books of Albert (1992), Barrett (1999) and Bub (1997).

- Arnold Neumaier: "*Many physicist nowadays think that decoherence provides a fully satisfying answer to the measurement problem. But this is an illusion.*"

- M. Schlosshauer: " *...note that the formal identification of the reduced density matrix with a mixed state density matrix is easily misinterpreted as implying that the state of the system can be viewed as mixed too... the total composite system is still described by a superposition, it follows from the rules of quantum mechanics that no individual definite state can be attributed to one of (the parts) of the system ...*"

- Joos & Zeh, 2003: *Decoherence by itself does not yet solve the measurement problem (. . .). This argument is nonetheless found wide-spread in the literature. (. . .) It does seem that the measurement problem can only be resolved if the Schrödinger dynamics (. . .) is supplemented by a nonunitary collapse (. . .).*

The GRW collapse dynamical model

Ghirardi-Rimini-Weber

Modified Schrödinger equation with collapse towards $\hat{C}$ eigenstates

$$\mathrm{d}|\Psi\rangle = -i\hat{H}\,|\Psi\rangle \mathrm{d}t$$

regular Hamiltonian term

$$\langle \hat{C} \rangle = \langle \Psi | \hat{C} | \Psi \rangle$$

non linearities

$$\mathbb{E}(\mathrm{d}W_t) = 0$$
$$\mathbb{E}(\mathrm{d}W_t\,\mathrm{d}W_{t'}) = \mathrm{d}t\,\mathrm{d}t'\,\delta(t-t')$$

Stochasticity (Wiener process)

The GRW dynamical collapse model

Ghirardi-Rimini-Weber

Modified Schrödinger equation with collapse towards $\hat{C}$ eigenstates

$$\mathrm{d}|\Psi\rangle = -i\hat{H}\,|\Psi\rangle \mathrm{d}t$$

regular Hamiltonian term

This new theory is:

- non-linear, to break the superposition principle at the macroscopic level and ensure the localization of the wave function of macro objects.
- stochastic, to explain why the outcomes occur randomly in measurement-like situations, and why they are distributed according to the Born probability rule.
- endowed with an amplification mechanism, so that the new terms have negligible effects on the dynamics of microscopic systems and very strong effects on large macroscopic objects, in order to recover their classical-like behavior.

The GRW collapse dynamical model

Year	first author [ref.]	interfering object	m/m_p	τ	d	in GRW $\lambda <$	in GRW $\lambda/\sigma^2 <$	in CSL $\lambda <$	in CSL $\lambda/\sigma^2 <$
1927	Davisson [13]	electron	5×10^{-4}	N/A	2×10^{-10}m	10^{14} s^{-1}	3×10^{33} m^{-2}s^{-1}	10^{17} s^{-1}	5×10^{36} m^{-2}s^{-1}
1930	Estermann [15]	He	4	N/A	4×10^{-10}m	10^{11} s^{-1}	6×10^{29} m^{-2}s^{-1}	3×10^{10} s^{-1}	10^{29} m^{-2}s^{-1}
1959	Möllenstedt [28]	electron	5×10^{-4}	3×10^{-9} s	2×10^{-6} m	7×10^{11} s^{-1}	10^{23} m^{-2}s^{-1}	10^{15} s^{-1}	3×10^{26} m^{-2}s^{-1}
1987	Tonomura [37]	electron	5×10^{-4}	10^{-8} s	10^{-4} m	2×10^{11} s^{-1}	2×10^{19} m^{-2}s^{-1}	4×10^{14} s^{-1}	4×10^{22} m^{-2}s^{-1}
1988	Zeilinger [40]	neutron	1	10^{-2} s	10^{-4} m	2×10^{2} s^{-1}	2×10^{10} m^{-2}s^{-1}	2×10^{2} s^{-1}	2×10^{10} m^{-2}s^{-1}
1991	Carnal [9]	He	4	6×10^{-4} s	10^{-5} m	4×10^{2} s^{-1}	4×10^{12} m^{-2}s^{-1}	10^{2} s^{-1}	10^{12} m^{-2}s^{-1}
1999	Arndt [4]	C_{60}	720	6×10^{-3} s	10^{-7} m	2×10^{-1}s^{-1}	2×10^{13} m^{-2}s^{-1}	3×10^{-4} s^{-1}	3×10^{10} m^{-2}s^{-1}
2001	Nairz [29]	C_{70}	840	10^{-2} s	3×10^{-7} m	10^{-1}s^{-1}	10^{12} m^{-2}s^{-1}	10^{-4} s^{-1}	10^{9} m^{-2}s^{-1}
2004	Hackermüller [24]	C_{70}	840	2×10^{-3} s	10^{-6} m	10^{0} s^{-1}	10^{12} m^{-2}s^{-1}	10^{-3} s^{-1}	10^{9} m^{-2}s^{-1}
2007	Gerlich [17]	$C_{30}H_{12}F_{30}N_2O_4$	10^3	10^{-3} s	3×10^{-7} m	10^{0} s^{-1}	10^{13} m^{-2}s^{-1}	10^{-3} s^{-1}	10^{10} m^{-2}s^{-1}
2011	Gerlich [18]	$C_{60}[C_{12}F_{25}]_{10}$	7×10^3	10^{-3} s	3×10^{-7} m	10^{-1}s^{-1}	10^{12} m^{-2}s^{-1}	10^{-5} s^{-1}	10^{8} m^{-2}s^{-1}
	Proposed future experiments								
	Romero-Isart [35]	$[SiO_2]_{150,000}$	10^7	10^{-1} s	4×10^{-7} m	10^{-6}s^{-1}	6×10^{6} m^{-2}s^{-1}	10^{-13}s^{-1}	6×10^{-1} m^{-2}s^{-1}
	Nimmrichter [30]	$Au_{500,000}$	10^8	6×10^{0} s	10^{-7} m	2×10^{-9}s^{-1}	2×10^{5} m^{-2}s^{-1}	2×10^{-17}s^{-1}	2×10^{-3} m^{-2}s^{-1}

Table 1: Bounds on σ, λ obtained from different diffraction experiments. For each experiment, m = mass of the interfering object, m_p = proton mass, τ = time of flight between grating and image plane, d = period of grating (or transverse coherence length in [37]), N/A = not applicable. For each theory (GRW or CSL), two bounds are obtained. This table is the basis for Fig. 3.

Feldmann & Tumulka 2011

« Macro objectification » without impacting much on the micro processes

γ must lie in some range

The GRW collapse dynamical model

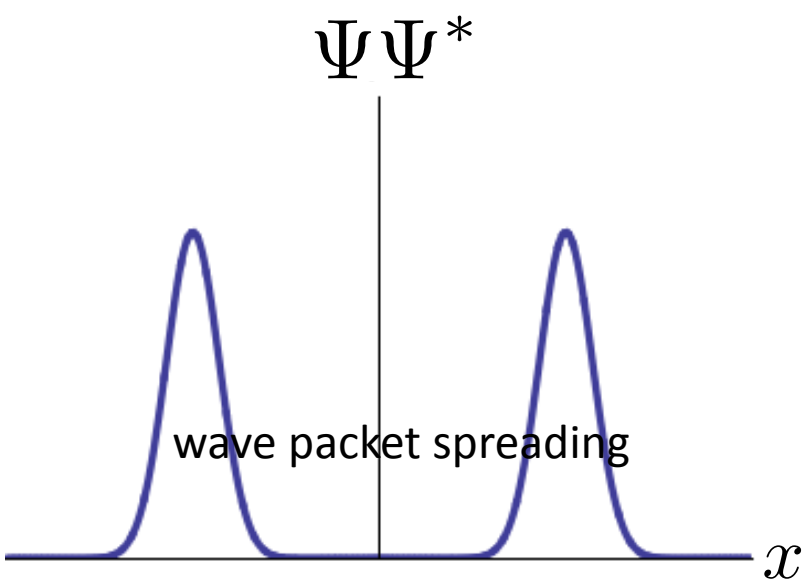
Ghirardi-Rimini-Weber

Example: free particle

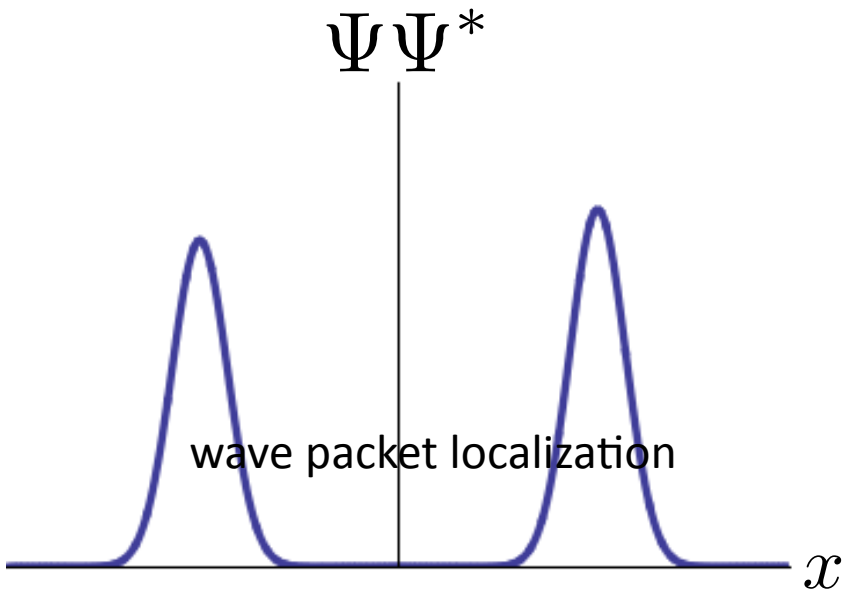
$$\hat{H} = \frac{\hat{p}^2}{2m}$$

$$\hat{C} = \hat{x}$$

Standard Free Particle



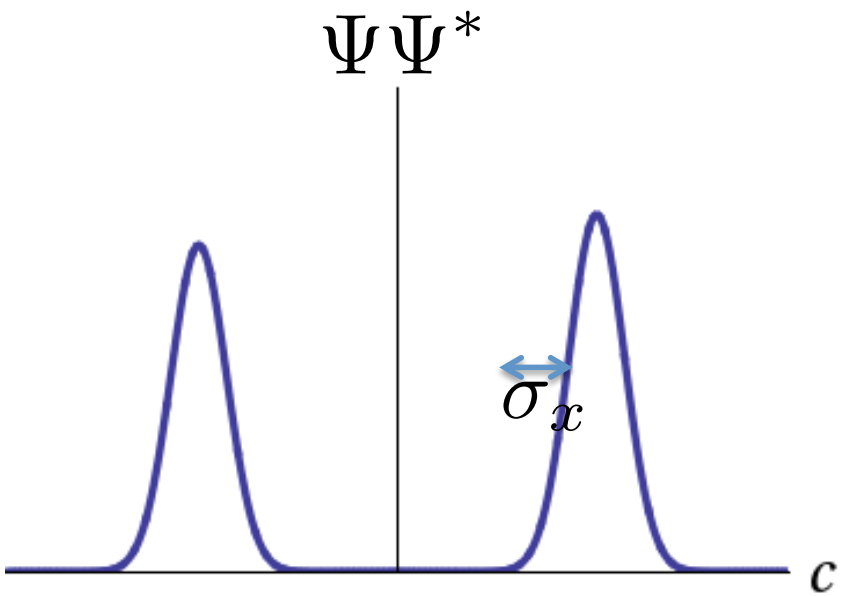
Modified (macro)
Free Particle
with collapse operator



Born Rules recovered !
(and accounted for)

The GRW collapse dynamical model

Ghirardi-Rimini-Weber



Amplification mechanism: $\gamma \propto$ number of particles

$$\sigma_x(\infty) = \frac{\sqrt{2}}{2}\left(\frac{\hbar}{m\gamma}\right)^{\frac{1}{4}} =$$

$4.7\ \mathrm{cm}$ for a proton

$4.6 \times 10^{-14}\ \mathrm{m}$ for a 1g object

$5.9 \times 10^{-28}\ \mathrm{m}$ for the Earth

The GRW collapse dynamical model

Constraining this (falsifiable) new theory

« Macro objectification » without impacting much on the micro processes → γ must lie in some range

Motivations for studying cosmological perturbations collapse:

- Atomic energy levels
- Nuclear energy levels
- Diffraction Experiments
- Proton Decay
- Spontaneous Xray emission
- Spontaneous IGM warming
- Dissociation of cosmic H
- Decay of supercurrents
- Latent image formation
- Thermalized spectral distorsions
- Neutrino and kaon oscillations

Primordial Power Spectrum

Standard case

Quantization in the Schrödinger picture (functional representation)

$$i\frac{\mathrm{d}|\Psi_{\boldsymbol{k}}\rangle}{\mathrm{d}\eta} = \hat{\mathcal{H}}_{\boldsymbol{k}}\,|\Psi_{\boldsymbol{k}}\rangle$$

with

$$\hat{\mathcal{H}}_{\boldsymbol{k}} = \frac{\hat{p}_{\boldsymbol{k}}^2}{2} + \omega^2(\boldsymbol{k},\eta)\hat{v}_{\boldsymbol{k}}^2$$

$$\hat{v}_{\boldsymbol{k}} = v_{\boldsymbol{k}}$$

$$\hat{p}_{\boldsymbol{k}} = i\frac{\partial}{\partial v_{\boldsymbol{k}}}$$

and

$$\omega^2(\boldsymbol{k},\eta) = k^2 - \frac{(a\sqrt{\epsilon_1})''}{a\sqrt{\epsilon_1}}$$

$$= k^2 - \frac{\beta(\beta+1)}{\eta^2}$$

$$a(\eta) = \ell_0(-\eta)^{1+\beta}$$

$$\beta \lesssim -2$$

(de Sitter: $\beta = -2$)

Parametric Oscillator System

Primordial Power Spectrum

Standard case

Quantization in the Schrödinger picture
(functional representation)

$$\Psi_{\boldsymbol{k}}(\eta, v_{\boldsymbol{k}}) = \left[\frac{2\,\Re\mathrm{e}\Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1/4} \mathrm{e}^{-\Omega_{\boldsymbol{k}}(\eta)\, v_{\boldsymbol{k}}^2}$$

$$i\frac{\mathrm{d}|\Psi_{\boldsymbol{k}}\rangle}{\mathrm{d}\eta} = \hat{\mathcal{H}}_{\boldsymbol{k}}\,|\Psi_{\boldsymbol{k}}\rangle$$

with

$$\hat{\mathcal{H}}_{\boldsymbol{k}} = \frac{\hat{p}_{\boldsymbol{k}}^2}{2} + \omega^2(\boldsymbol{k},\eta)\hat{v}_{\boldsymbol{k}}^2$$

$$\Omega'_{\boldsymbol{k}} = -2i\Omega_{\boldsymbol{k}}^2 + \frac{i}{2}\omega^2(\eta,\boldsymbol{k})$$

$$\Omega_{\boldsymbol{k}} = -\frac{i}{2}\frac{f'_{\boldsymbol{k}}}{f_{\boldsymbol{k}}}$$

$$f''_{\boldsymbol{k}} + \omega^2(\boldsymbol{k},\eta) f_{\boldsymbol{k}} = 0$$

Primordial Power Spectrum

Standard case

Two physical scales

Hubble radius $H^{-1} = \frac{a^2}{a'} \underset{\beta \sim -2}{\simeq} \ell_0$

wavelength $\lambda = \frac{a}{k} \underset{\beta \sim -2}{\simeq} \frac{\ell_0}{-k\eta}$

Sub-Hubble regime

$\lambda \ll H^{-1}$

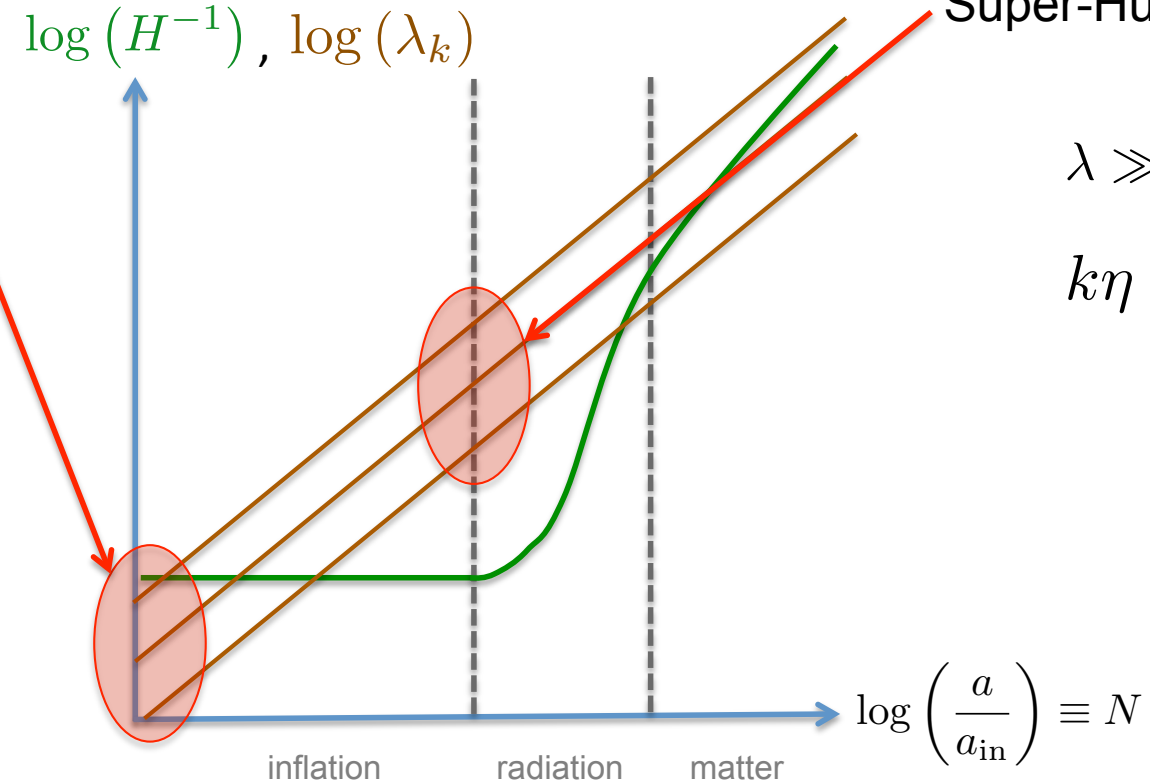
$k\eta \to -\infty$

$\omega \simeq k$

harmonic oscillator

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2} v_{\boldsymbol{k}}^2}$$

Bunch Davis vacuum



Super-Hubble regime

$\lambda \gg H^{-1}$

$k\eta \to 0^-$

sets initial conditions $f_{\boldsymbol{k}}(k\eta \to -\infty) = \mathrm{e}^{ik\eta}/\sqrt{2k}$

Primordial Power Spectrum

Standard case

$\boxed{f_{\boldsymbol{k}}'' + \omega^2(\boldsymbol{k},\eta) f_{\boldsymbol{k}} = 0}$ with $\omega^2(\boldsymbol{k},\eta) = k^2 - \dfrac{\beta(\beta+1)}{\eta^2}$ and $f_{\boldsymbol{k}}(k\eta \to -\infty) = \mathrm{e}^{ik\eta}/\sqrt{2k}$

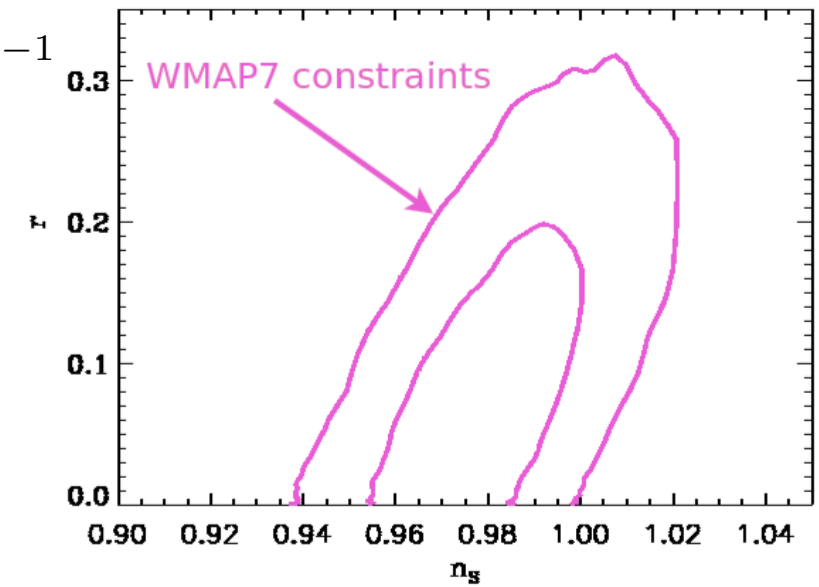
Uniquely determines $f_{\boldsymbol{k}}$ $\xrightarrow{\Omega_{\boldsymbol{k}} = -\frac{i}{2}\frac{f_{\boldsymbol{k}}'}{f_{\boldsymbol{k}}}}$ $\Re\mathrm{e}\,\Omega_{\boldsymbol{k}} = \langle \hat{v}_{\boldsymbol{k}}^2 \rangle - \langle \hat{v}_{\boldsymbol{k}} \rangle^2$

Evaluated at the end of inflation$(k\eta \to 0^-)$, this gives $P_v(k) = \dfrac{k^3}{2\pi^3}\left(\langle \hat{v}_{\boldsymbol{k}}^2 \rangle - \langle \hat{v}_{\boldsymbol{k}} \rangle^2\right)$

and eventually $P_\zeta(k) = \dfrac{1}{2a^2 M_{\mathrm{Pl}}^2 \epsilon_1} P_v(k) = A_S k^{n_{\mathrm{S}}-1}$

with $n_{\mathrm{S}} = 2\beta + 5 \underset{\beta\sim -2}{\simeq} 1$

WMAP7: $1 - n_{\mathrm{S}} = 0.018^{+0.019}_{-0.02}$



Primordial Power Spectrum

Modified Theory

Modified Schrödinger equation

$$\mathrm{d}|\Psi_{\boldsymbol{k}}\rangle = -i\hat{\mathcal{H}}_{\boldsymbol{k}}|\Psi\rangle\mathrm{d}\eta + \sqrt{\gamma}\left(\hat{\mathcal{O}}_{\boldsymbol{k}} - \langle\hat{\mathcal{O}}_{\boldsymbol{k}}\rangle\right)\mathrm{d}W_\eta\,|\Psi_{\boldsymbol{k}}\rangle - \frac{\gamma}{2}\left(\hat{\mathcal{O}}_{\boldsymbol{k}} - \langle\hat{\mathcal{O}}_{\boldsymbol{k}}\rangle\right)^2\mathrm{d}\eta\,|\Psi_{\boldsymbol{k}}\rangle$$

Extended Gaussian wave function

$$\Psi_{\boldsymbol{k}}(\eta, v_{\boldsymbol{k}}) = \left[\frac{2\,\Re\mathrm{e}\Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1/4} \exp\left\{-\Re\mathrm{e}\Omega_{\boldsymbol{k}}(\eta)\left[v_{\boldsymbol{k}} - \bar{v}_{\boldsymbol{k}}(\eta)\right]^2 + i\sigma_{\boldsymbol{k}}(\eta) + i\chi_{\boldsymbol{k}}(\eta)v_{\boldsymbol{k}} - i\Im\mathrm{m}\Omega_{\boldsymbol{k}}(\eta)\left(v_{\boldsymbol{k}}\right)^2\right\}$$

Modified equation of motion

$$\Omega'_{\boldsymbol{k}} = -2i\Omega_{\boldsymbol{k}}^2 + \frac{i}{2}\omega^2(\eta, \boldsymbol{k}) + \gamma \xrightarrow{\Omega_{\boldsymbol{k}} = -\frac{i}{2}\frac{f'_{\boldsymbol{k}}}{f_{\boldsymbol{k}}}} f''_{\boldsymbol{k}} + \left[\omega^2(\eta, k) - 2i\gamma\right] f_{\boldsymbol{k}} = 0$$

Primordial Power Spectrum

Modified Theory

$$f_{\boldsymbol{k}}'' + \left[k^2 - \frac{\beta(\beta+1)}{\eta^2} - 2i\gamma\right] f_{\boldsymbol{k}} = 0$$

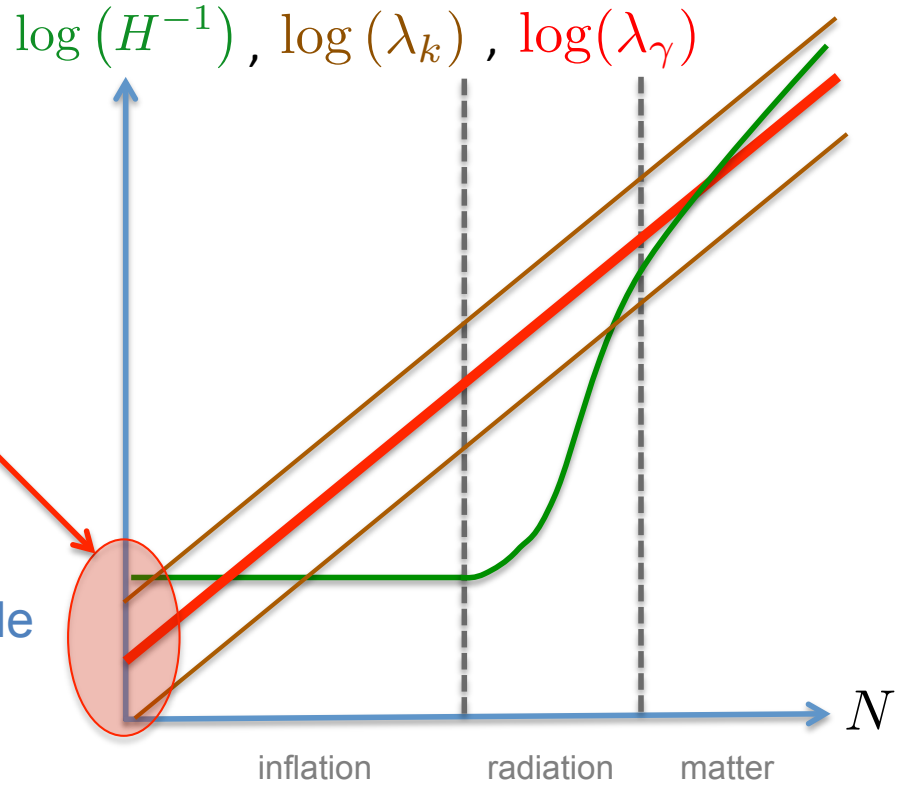
Ability to fix Bunch Davis vacuum as an initial condition ?

$$\omega(\eta, k) \underset{k\eta \to -\infty}{\longrightarrow} k^2 - 2i\gamma$$

$$f_{\boldsymbol{k}} \underset{k\eta \to -\infty}{=} A_{\boldsymbol{k}} e^{i\omega\eta} + \cancel{B_{\boldsymbol{k}} e^{-i\omega\eta}}$$

leads to non normalizable wavefunction

Bunch Davis vacuum $(\Re e\, \Omega_{\boldsymbol{k}} < 0)$



Primordial Power Spectrum

Modified Theory

$$f_{\boldsymbol{k}}'' + \left[k^2 - \frac{\beta(\beta+1)}{\eta^2} - 2i\gamma\right] f_{\boldsymbol{k}} = 0$$

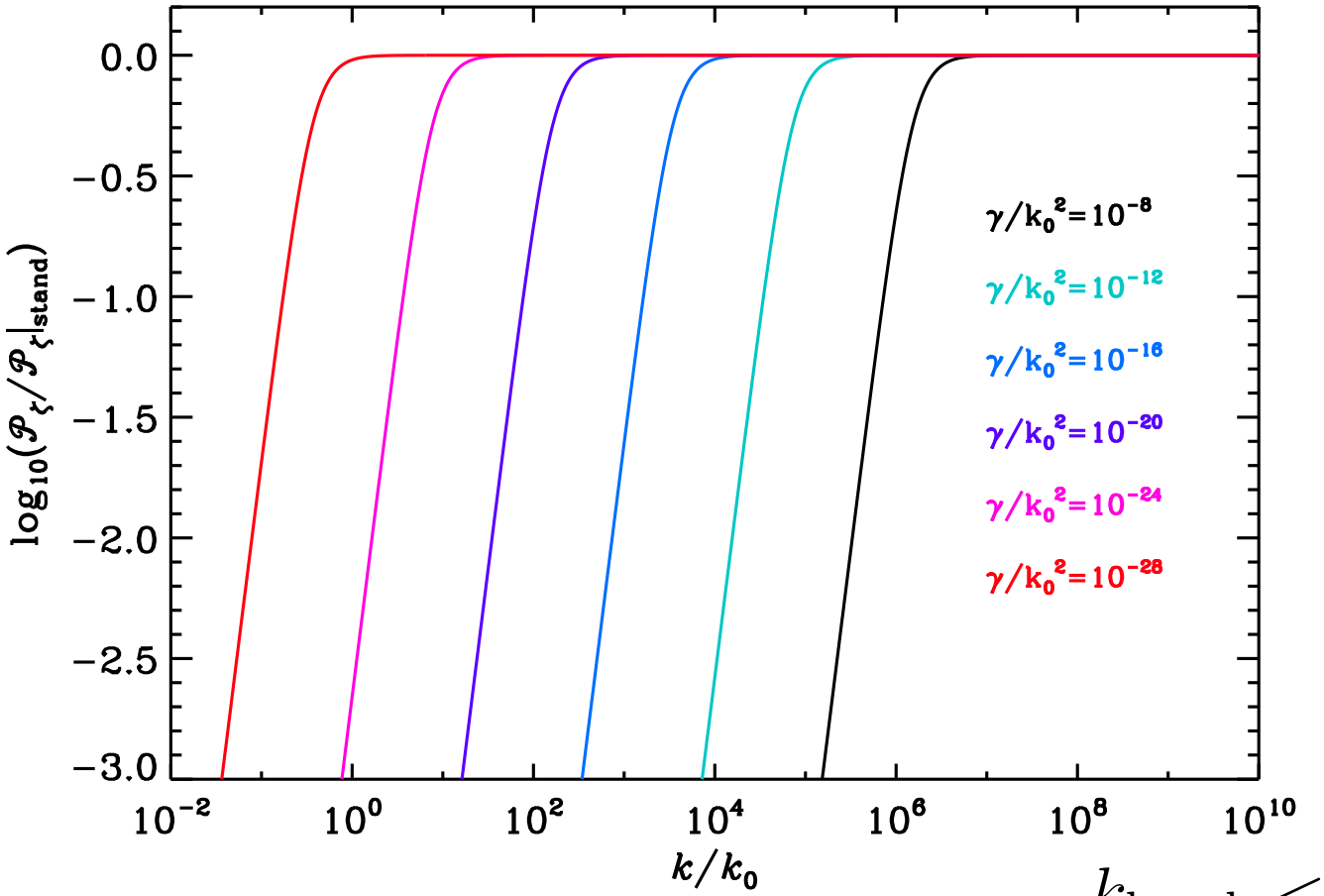
$$\Re\mathrm{e}\Omega_{\boldsymbol{k}}(\eta)_{k\eta\to 0^-} = -\frac{k}{2(\beta+3/2)}\frac{\gamma}{k^2}(-k\eta)$$

$$+\frac{|D_{\boldsymbol{k}}|}{|C_{\boldsymbol{k}}|}|z_{\boldsymbol{k}}|^{-(2\beta+1)}\cos\left[\pi\beta + (2\beta+1)\,\theta_{\boldsymbol{k}} - \theta_d + \theta_c\right]\frac{k\,\pi\,2^{2\beta+1}}{\Gamma^2(-\beta-1/2)\cos(\pi\beta)}(-k\eta)^{\overset{\simeq 2}{-2\beta-2}} + \cdots$$

$$\mathcal{P}_\zeta(k) = g_\gamma(k,\beta)\left[1 - \frac{\gamma}{k^2}g_\gamma(k,\beta)f(\beta)\frac{(-k\eta)^{\overset{\simeq -1}{2\beta+3}}}{\beta+3/2}\right]^{-1}\mathcal{P}_\zeta(k)\Big|_{\mathrm{stand}}$$

Primordial Power Spectrum

Modified Theory



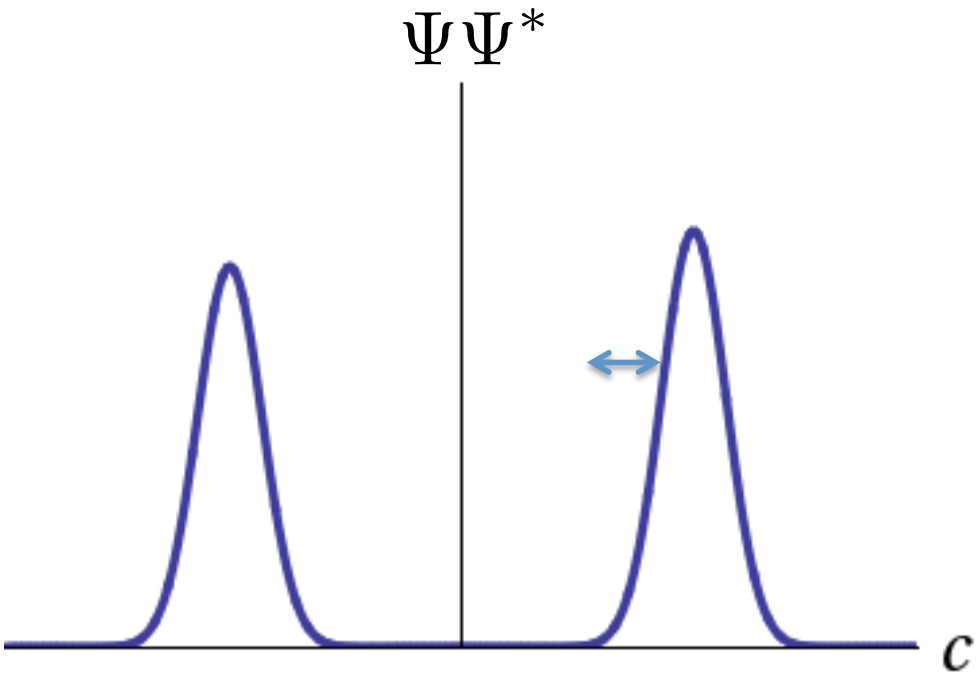
$$k_{\rm break} < k_0$$

$$k < k_{\rm break} : n_{\rm S} = 4$$

$$k > k_{\rm break} : n_{\rm S} = 2\beta + 5 \simeq 1$$

$$\frac{\gamma}{k_0^2} \ll {\rm e}^{-\Delta N_*} \simeq 10^{-28}$$

Efficiency of the Collapse



How much time does the collapse take ?

How precise is the collapse (final spread) ?

Efficiency of the Collapse

Collapse Time

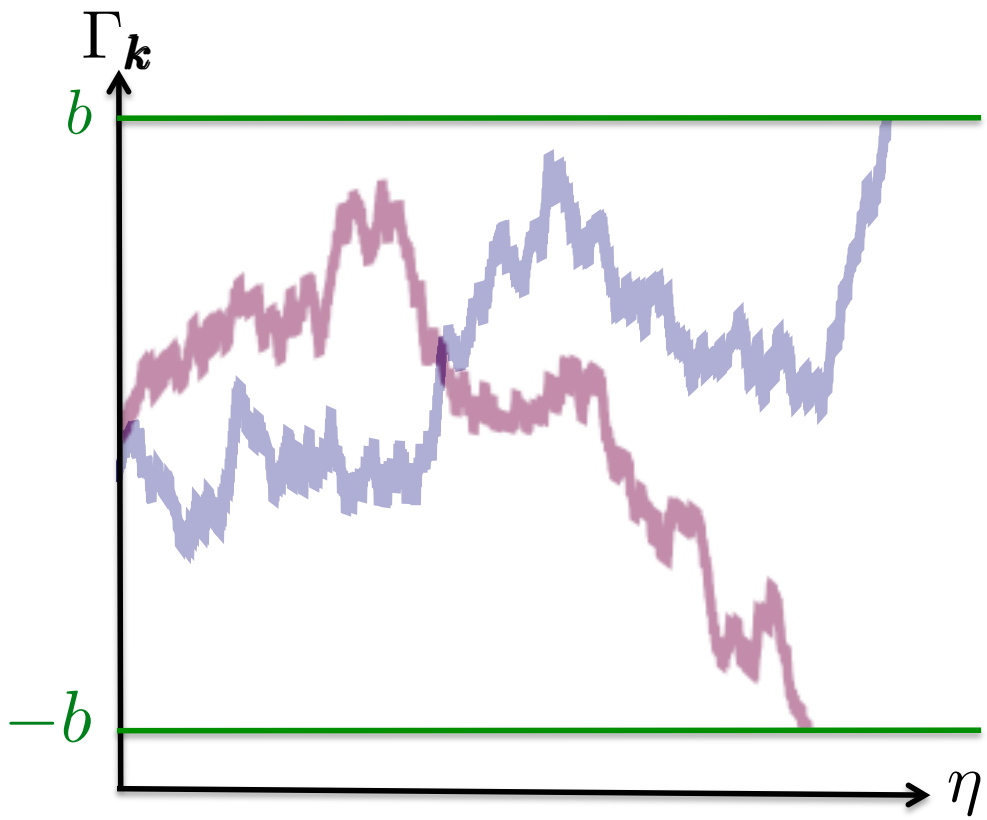
Double Gaussian wavefunction

$$\Psi_{\boldsymbol{k}}(\eta, v_{\boldsymbol{k}}) = |N_{\boldsymbol{k}}^{(1)}(\eta)| \exp\left\{-\Re\mathrm{e}\Omega_{\boldsymbol{k}}^{(1)}(\eta)\left[v_{\boldsymbol{k}} - \bar{v}_{\boldsymbol{k}}^{(1)}(\eta)\right]^2 + i\sigma_{\boldsymbol{k}}^{(1)}(\eta) + i\chi_{\boldsymbol{k}}^{(1)}(\eta)v_{\boldsymbol{k}} - i\Im\mathrm{m}\Omega_{\boldsymbol{k}}^{(1)}(\eta)(v_{\boldsymbol{k}})^2\right\}$$
$$+ |N_{\boldsymbol{k}}^{(2)}(\eta)| \exp\left\{-\Re\mathrm{e}\Omega_{\boldsymbol{k}}^{(2)}(\eta)\left[v_{\boldsymbol{k}} - \bar{v}_{\boldsymbol{k}}^{(2)}(\eta)\right]^2 + i\sigma_{\boldsymbol{k}}^{(2)}(\eta) + i\chi_{\boldsymbol{k}}^{(2)}(\eta)v_{\boldsymbol{k}} - i\Im\mathrm{m}\Omega_{\boldsymbol{k}}^{(2)}(\eta)(v_{\boldsymbol{k}})^2\right\}$$

$$\Gamma_{\boldsymbol{k}} = \log\left|\frac{N_{\boldsymbol{k}}^{(2)}}{N_{\boldsymbol{k}}^{(1)}}\right|$$

$$\eta_{\mathrm{collapse}} = \inf\{\eta\,;\ |\Gamma_{\boldsymbol{k}}(\eta)| > b\}$$

b: collapse criterion



Efficiency of the Collapse

If the two Gaussian are well separated $X_{\boldsymbol{k}}^2 \equiv \left[\bar{v}_{\boldsymbol{k}}^{(2)} - \bar{v}_{\boldsymbol{k}}^{(1)}\right]^2 \gg 1/\Re\mathrm{e}\Omega_{\boldsymbol{k}}$

$$\frac{\mathrm{d}\Gamma_{\boldsymbol{k}}}{\mathrm{d}\eta} = \sqrt{\gamma} X_{\boldsymbol{k}} \frac{\mathrm{d}W_\eta}{\mathrm{d}\eta} + \gamma X_{\boldsymbol{k}}^2 \tanh\left(\Gamma_{\boldsymbol{k}}\right) \ , \quad \text{with } X_{\boldsymbol{k}}(\eta) \propto \begin{cases} \mathrm{e}^{-\frac{\gamma}{k^2} k\eta} & \text{(sub Hubble)} \\ (-k\eta)^{\beta+1} & \text{(super Hubble)} \end{cases}$$

Sub Hubble regime

occurs if $b \ll k X_{\boldsymbol{k},\mathrm{ini}}^2$

$$N_{\mathrm{col}} \simeq \underbrace{\frac{2b}{k X_{\mathrm{ini}}^2}}_{\ll 1} \underbrace{\frac{k^2}{\gamma}}_{\gtrsim 10^{28}} \underbrace{\mathrm{e}^{-N_{\mathrm{subH}}}}_{\sim \mathrm{e}^{-10^2, -10^6}}$$

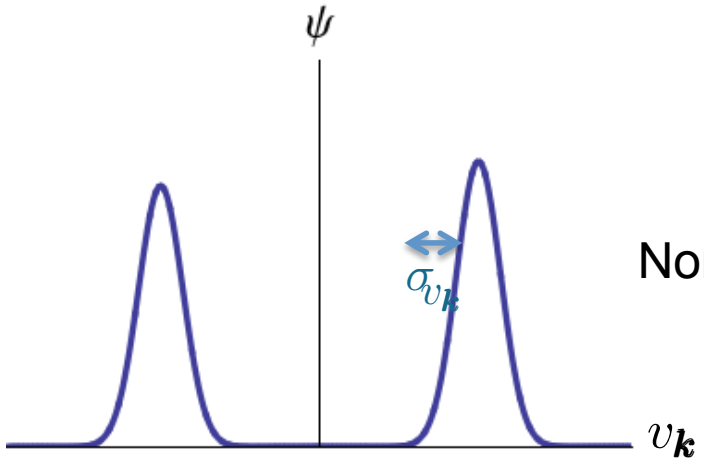
Super Hubble regime

if $b \gg k X_{\boldsymbol{k},\mathrm{ini}}^2$

$$N_{\mathrm{col}} \simeq \underbrace{\ln\left(\frac{k^2}{\gamma}\right)}_{\gtrsim 28} + \underbrace{\ln\left(\frac{b}{k X_{\boldsymbol{k},\mathrm{ini}}^2}\right)}_{\text{a few}}$$

Efficiency of the Collapse

Collapse Precision



Scale invariant modes: $k\,\sigma_{v_{\boldsymbol{k}}} \simeq \exp\left(2\,\Delta N_{*,\boldsymbol{k}}\right)$

Non scale invariant modes: $k\,\sigma_{v_{\boldsymbol{k}}} < \dfrac{k^2}{\gamma}\exp\left(\Delta N_{*,\boldsymbol{k}}\right)$

The observed power spectrum scale invariance constrain γ to be fined tuned to such small values that the wave function is eventually very badly localized

Limitations and Prospects

Choice of the collapse operator $\hat{C}_{\boldsymbol{k}} = \hat{v}_{\boldsymbol{k}}$

$$\hat{C}_{\boldsymbol{k}} = f(a)\hat{v}_{\boldsymbol{k}} \ ?$$

function of the background ⟷ time gauge issue

$$f(a) = a^p$$

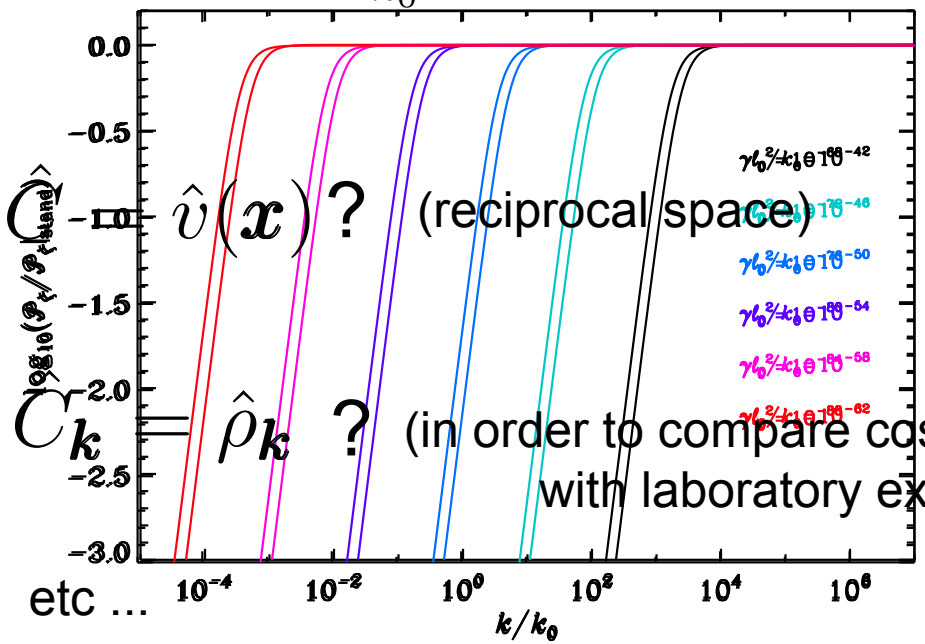
$p < 0$ — impossible to set the initial Bunch Davis vacuum

$p = 0$ — $\dfrac{\gamma}{k_0^2} \ll \mathrm{e}^{-\Delta N_*}$

$p = 1$ — $\dfrac{\gamma \ell_0}{k_0} \ll \mathrm{e}^{-2\Delta N_*}$

$p = 2$ — $\gamma \ell_0^2 \ll \mathrm{e}^{-3\Delta N_*}$

etc ...



$\hat{v}(\boldsymbol{x})$? (reciprocal space)

$\hat{C}_{\boldsymbol{k}} = \hat{\rho}_{\boldsymbol{k}}$? (in order to compare cosmological constraints with laboratory experiments ones)

etc ...

Limitations and Prospects

Need for a fields-collapse theory

Non relativistic equation applied to a relativistic context

Usually: preferred timelike direction invoked

Time dependent Minkowski approximation

Here: would solve the time gauge ambiguity for example…

Constraints from the vacuum energy increase

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\langle\hat{\mathcal{H}}_{\boldsymbol{k}}\rangle = \omega\omega'\langle\hat{v}_{\boldsymbol{k}}^2\rangle + \frac{\gamma}{2}$$

on small scales

Back reaction problem ?

Propagate P_ζ through (p)reheating

Parametric resonance ➔ complex k and instability ?

Laboratory parametric resonance ?

etc ...

Conclusions

- GRW scenario addresses the measurement problem in quantum mechanics and is falsifiable. Confronting it with Cosmological Inflation is a priori relevant to explain how the wave-packet reduction took place in the early universe, and to put new constraints on the modified theory.

- In order to preserve the scale invariance of the power spectrum, it is necessary to fine-tune the parameter $\propto \gamma$ which controls the amplitude of the GRW corrections to $\exp(-\text{a few } \Delta N_*)$.

- The time available during the inflationary phase is sufficient in order for the perturbations wavefunction to collapse.

- Due to the smallness of γ, the spread of the final wavefunction is too important, rendering the collapse process not sufficiently efficient.

Under the (**debatable**) assumptions made, the collapse theory does not seem to solve the inflationary "macro-objectification" question.

Quantum cosmology

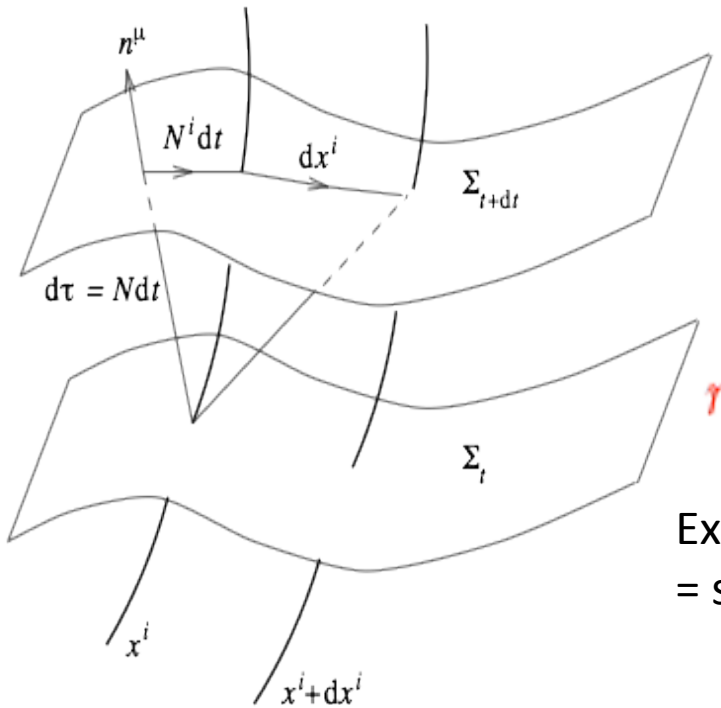
- Hamiltonian GR

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\mathcal{N}^2 dt^2 + h_{ij}\left(dx^i + \mathcal{N}^i dt\right)\left(dx^j + \mathcal{N}^j dt\right)$$

Shift vector

Lapse function

Intrinsic metric
= first fundamental form



n^μ Normal to Σ_t

Intrinsic curvature tensor ${}^3R^i{}_{jkl}(h)$

Extrinsic curvature
= second fundamental form

$$K_{ij} \equiv -\nabla_j n_i = -\Gamma^0{}_{ij} n_0 = \frac{1}{2\mathcal{N}}\left(\nabla_j \mathcal{N}_i + \nabla_i \mathcal{N}_j - \frac{\partial h_{ij}}{\partial t}\right)$$

Action: $$\mathcal{S} = \frac{1}{16\pi G_N}\left[\int_{\mathcal{M}} d^4x\sqrt{-g}\left({}^4R - 2\Lambda\right) + 2\int_{\partial\mathcal{M}} d^3x\sqrt{h}K^i{}_i\right] + \mathcal{S}_{\text{matter}}$$

In 3+1 expansion: $$S \equiv \int \mathrm{d}t\, L = \frac{1}{4\kappa^2} \int \mathrm{d}t\, \mathrm{d}^3x \mathcal{N} \sqrt{h} \left(K_{ij} K^{ij} - K^2 + {}^3R - 2\Lambda \right) + S_{\text{matter}}$$

Canonical momenta

$$\pi^{ij} \equiv \frac{\delta L}{\delta \dot{h}_{ij}} = -\frac{\sqrt{h}}{4\kappa^2} \left(K^{ij} - h^{ij} K \right),$$

$$\pi_\Phi \equiv \frac{\delta L}{\delta \dot{\Phi}} = \frac{\sqrt{h}}{\mathcal{N}} \left(\dot{\Phi} - \mathcal{N}^i \Phi_{,i} \right),$$

$$\left.\begin{aligned} \pi^0 &\equiv \frac{\delta L}{\delta \dot{\mathcal{N}}} = 0, \\ \pi^i &\equiv \frac{\delta L}{\delta \dot{\mathcal{N}}_i} = 0. \end{aligned}\right\} \text{Primary constraints}$$

Hamiltonian $$H \equiv \int \mathrm{d}^3x \left(\pi^0 \dot{\mathcal{N}} + \pi^i \dot{\mathcal{N}}_i + \pi^{ij} \dot{h}_{ij} + \pi_\Phi \dot{\Phi} \right) - L = \int \mathrm{d}^3x \left(\pi^0 \dot{\mathcal{N}} + \pi^i \dot{\mathcal{N}}_i + \mathcal{N}\mathcal{H} + \mathcal{N}_i \mathcal{H}^i \right)$$

$$\left.\begin{aligned} &\text{Variation wrt lapse} \quad \mathcal{H} = 0 \quad \text{Hamiltonian constraint} \\ &\text{Variation wrt shift} \quad \mathcal{H}^i = 0 \quad \text{momentum constraint} \end{aligned}\right\} \text{Secondary (dynamical) constraints}$$

$\Longrightarrow$ Classical description

- Superspace & canonical quantization

Relevant configuration space?

matter fields

$$\mathrm{Riem}\,(\Sigma) \equiv \{h_{ij}(x), \Phi(x) \mid x \in \Sigma\}$$

parameters

GR $\Longrightarrow$ invariance / diffeomorphisms $\Longrightarrow \mathrm{Conf} = \dfrac{\mathrm{Riem}(\Sigma)}{\mathrm{Diff}_0(\Sigma)}$ superspace

Wave functional $\Psi\left[h_{ij}(x), \Phi(x)\right]$

Dirac canonical quantization

$$\pi^{ij} \to -i\frac{\delta}{\delta h_{ij}}, \quad \pi_\Phi \to -i\frac{\delta}{\delta \Phi}, \quad \pi^0 \to -i\frac{\delta}{\delta \mathcal{N}}, \quad \pi^i \to -i\frac{\delta}{\delta \mathcal{N}_i}\,.$$

Primary constraints

$$\hat{\pi}\Psi = -i\frac{\delta\Psi}{\delta\mathcal{N}} = 0$$

$$\hat{\pi}^i\Psi = -i\frac{\delta\Psi}{\delta\mathcal{N}_i} = 0$$

Momentum constraint $\hat{\mathcal{H}}^i\Psi = 0 \quad \Rightarrow \quad i\left[\frac{\delta\Psi}{\delta h_{ij}}\right]_{|j} = 2\kappa^2\hat{T}^{\hat{0}\hat{i}}\Psi$

$\Longrightarrow$ Ψ is the same for configurations $\{h_{ij}(x), \Phi(x)\}$ related by a coordinate transformation

Hamiltonian constraint

$$\boxed{\hat{\mathcal{H}}\Psi = \left[-4\kappa^2\mathcal{G}_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} + \frac{\sqrt{h}}{4\kappa^2}\left(-{}^3R + 2\Lambda + 4\kappa^2\hat{T}^{\hat{0}\hat{0}}\right)\right]\Psi = 0}$$

Wheeler - De Witt equation

- Minisuperspace

Restrict attention from an infinite dimensional configuration space to 2 dimensional space = mini - superspace

$$h_{ij}\mathrm{d}x^i\mathrm{d}x^j = a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right)\right] \qquad \Phi(x) = \phi(t)$$

WDW equation becomes Schrödinger-like for $\Psi\left[a(t), \phi(t)\right]$

Conceptual and technical problems:

Infinite number of dof $\longrightarrow$ a few: mathematical consistency?
Freeze momenta? Heisenberg uncertainties?
MQ = minisuperspace of QFT

Hidden Variable Theories

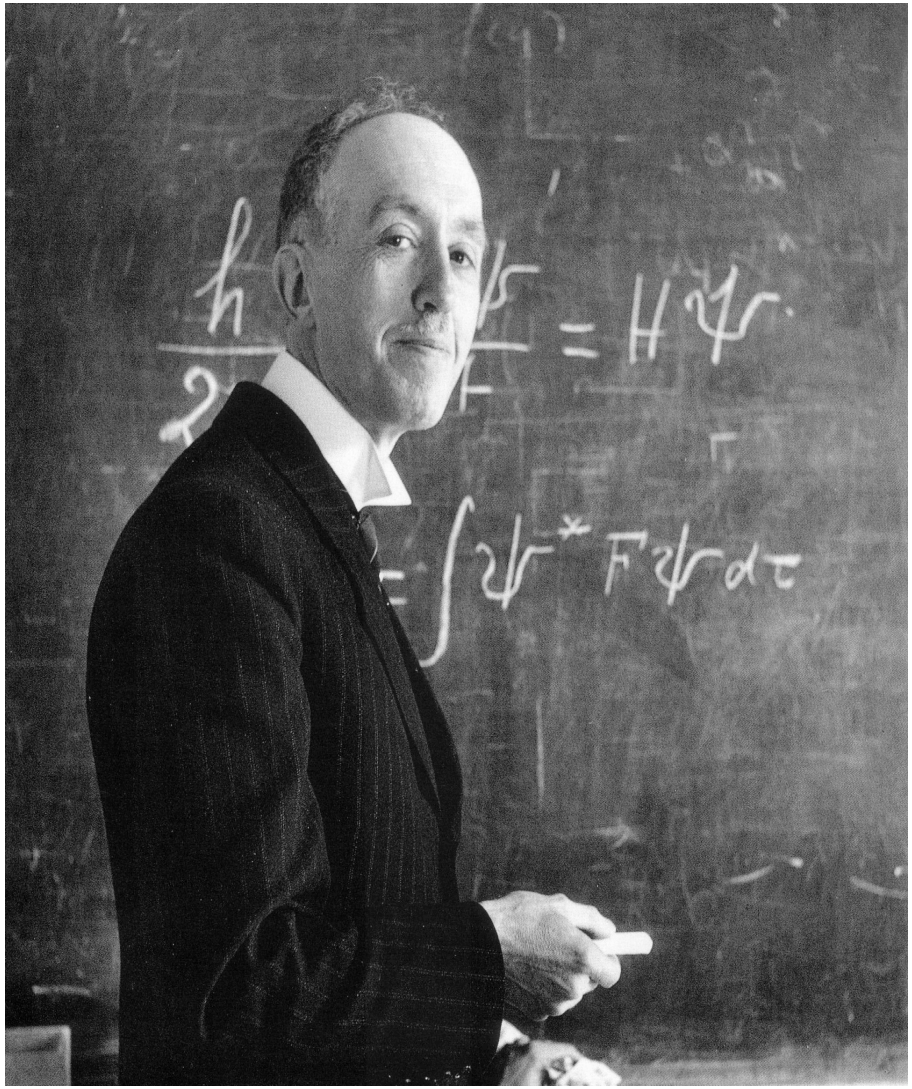
Schrödinger $$i\frac{\partial \Psi}{\partial t} = \left[-\frac{\nabla^2}{2m} + V(\boldsymbol{r})\right]\Psi$$

Polar form of the wave function $$\Psi = A(\boldsymbol{r},t)\,\mathrm{e}^{iS(\boldsymbol{r},t)}$$

Hamilton-Jacobi $$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V(\boldsymbol{r}) + Q(\boldsymbol{r},t) = 0$$

quantum potential $$\equiv -\frac{1}{2m}\frac{\nabla^2 A}{A}$$

Ontological interpretation (BdB)



Louis de Broglie

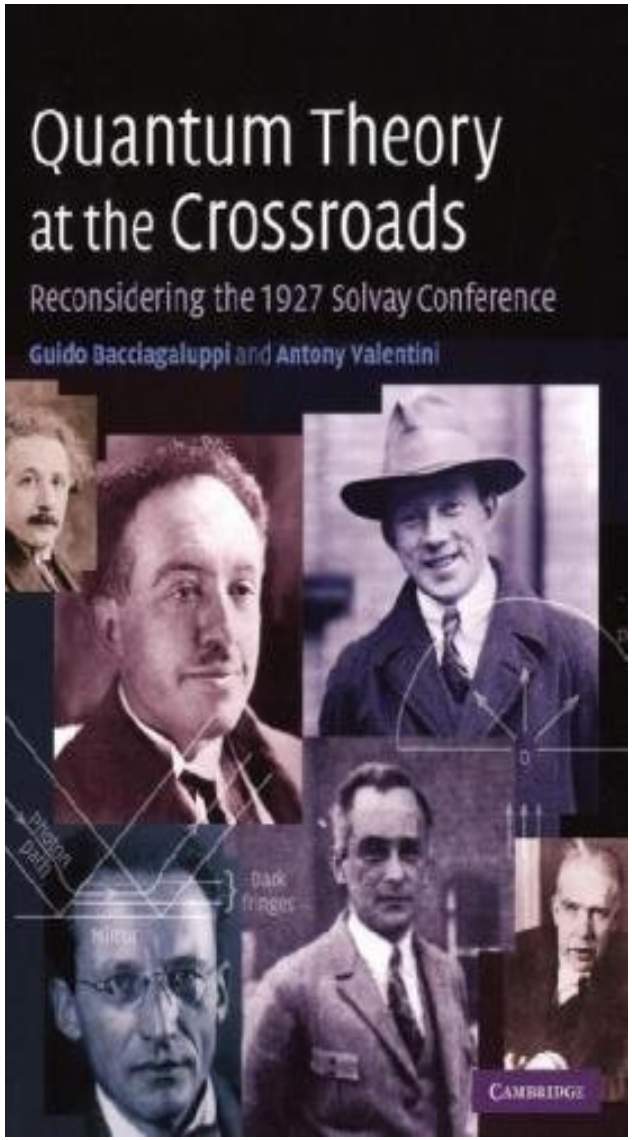
David Bohm

1927 Solvay meeting and von Neuman mistake ... *'In 1952, I saw the impossible done'* (J. Bell)

(aside):

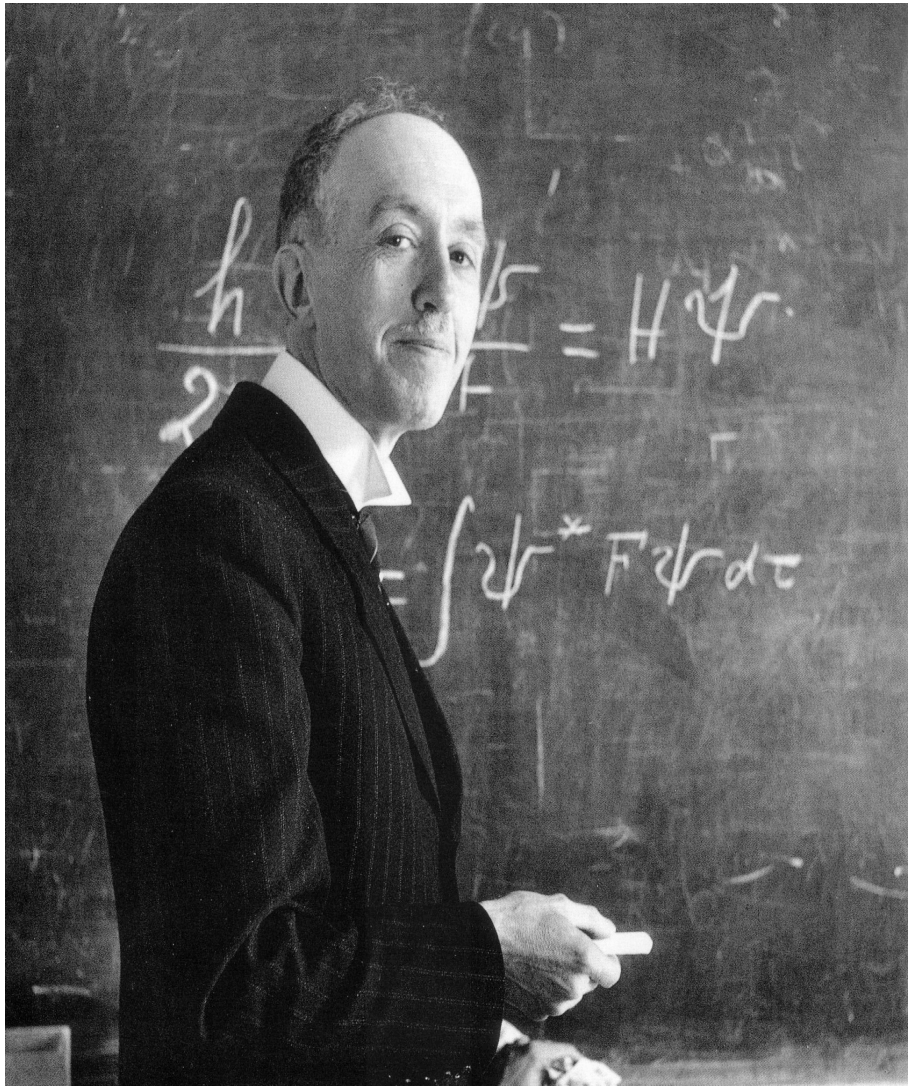
Quantum Theory at the Crossroads: Reconsidering the 1927 Solvay Conference

Guido Bacciagaluppi & Antony Valentini


Quantum Theory at the Crossroads
Reconsidering the 1927 Solvay Conference
Guido Bacciagaluppi and Antony Valentini
CAMBRIDGE

arXiv:quant-ph/0609184

Ontological interpretation (BdB)

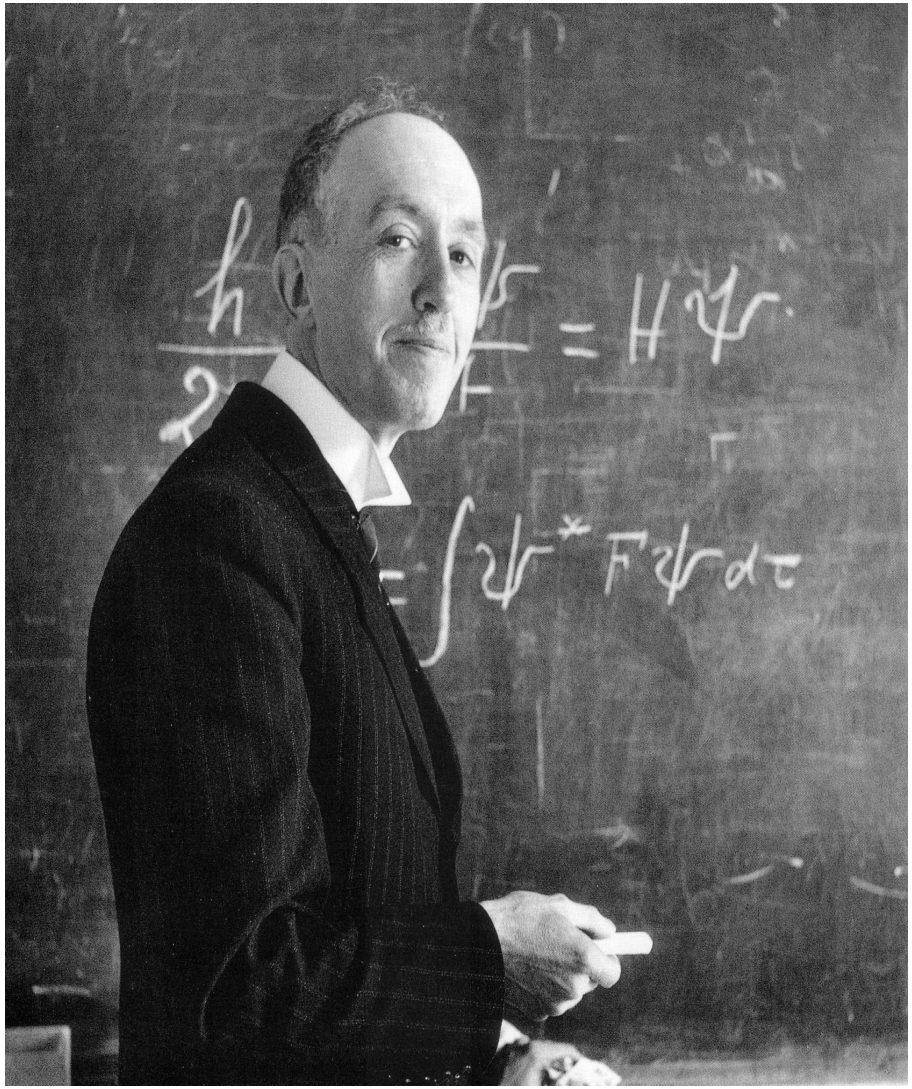


Louis de Broglie

David Bohm

1927 Solvay meeting and von Neuman mistake ... '*In 1952, I saw the impossible done*' (J. Bell)

Ontological interpretation (BdB)

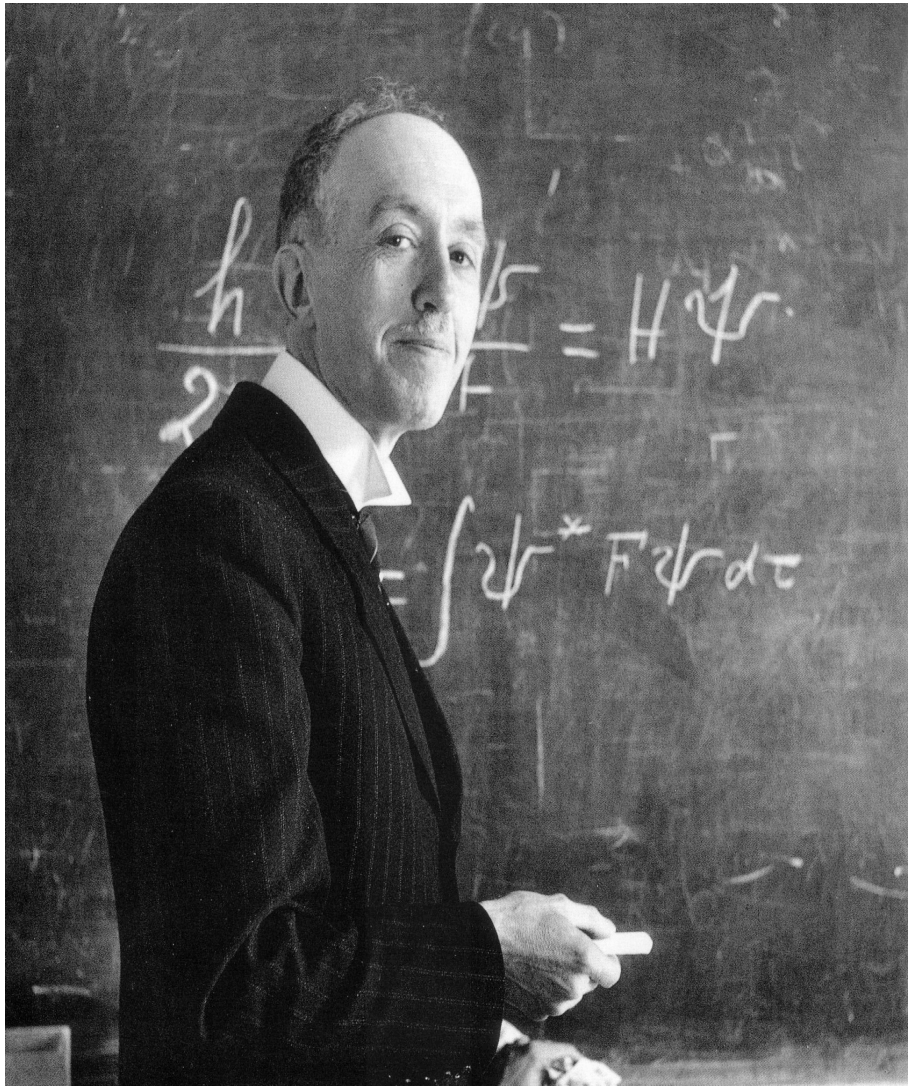


Louis de Broglie (Prince, duke ...)

David Bohm

1927 Solvay meeting and von Neuman mistake ... '*In 1952, I saw the impossible done*' (J. Bell)

Ontological interpretation (BdB)



Louis de Broglie (Prince, duke ...)

David Bohm (Communist)

1927 Solvay meeting and von Neuman mistake ... '*In 1952, I saw the impossible done*' (J. Bell)

Hidden Variable Theories

Schrödinger $$i\frac{\partial \Psi}{\partial t} = \left[-\frac{\nabla^2}{2m} + V(\boldsymbol{r})\right]\Psi$$

Polar form of the wave function $$\Psi = A(\boldsymbol{r},t)\,\mathrm{e}^{iS(\boldsymbol{r},t)}$$

Hamilton-Jacobi $$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V(\boldsymbol{r}) + Q(\boldsymbol{r},t) = 0$$

quantum potential $$\equiv -\frac{1}{2m}\frac{\nabla^2 A}{A}$$

Ontological interpretation (dBB) $\exists\, x(t)$

$$\Psi = A\left(\boldsymbol{r}, t\right) \mathrm{e}^{iS(\boldsymbol{r},t)}$$

Trajectories satisfy (de Broglie)

$$m\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t} = \hbar\,\Im\mathrm{m}\,\frac{\Psi^*\boldsymbol{\nabla}\Psi}{|\Psi(\boldsymbol{x})|^2} = -\hbar\boldsymbol{\nabla}S$$

Ontological interpretation (BdB)

$$\exists\, x(t)$$

$$\Psi = A\left(\boldsymbol{r}, t\right) \mathrm{e}^{iS(\boldsymbol{r},t)}$$

Trajectories satisfy (Bohm)

$$m\frac{\mathrm{d}^2 x(t)}{\mathrm{d}t^2} = -\nabla\left(V + Q\right)$$

Ontological interpretation (dBB) $\exists\ x(t)$ $\Psi = A(\boldsymbol{r},t)\,\mathrm{e}^{iS(\boldsymbol{r},t)}$

Trajectories satisfy (de Broglie) $$m\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t} = \hbar\,\Im\mathrm{m}\,\frac{\Psi^*\boldsymbol{\nabla}\Psi}{|\Psi(\boldsymbol{x})|^2} = -\hbar\boldsymbol{\nabla}S$$

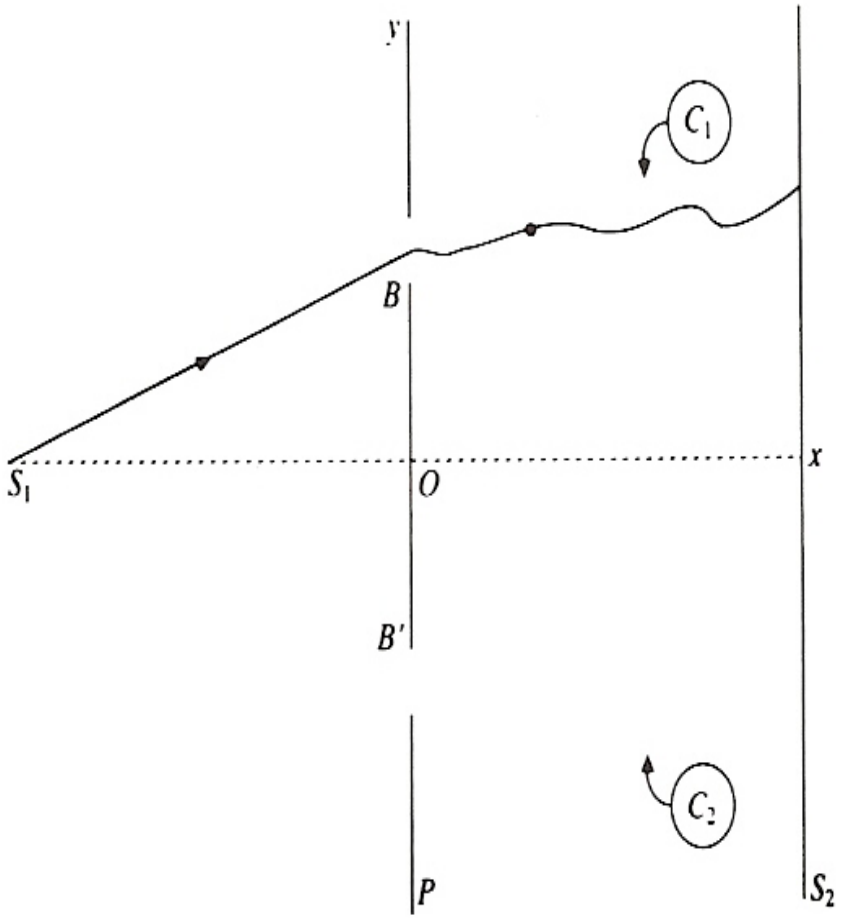
<u>Properties:</u>

- strictly equivalent to Copenhagen QM
  - probability distribution (attractor)

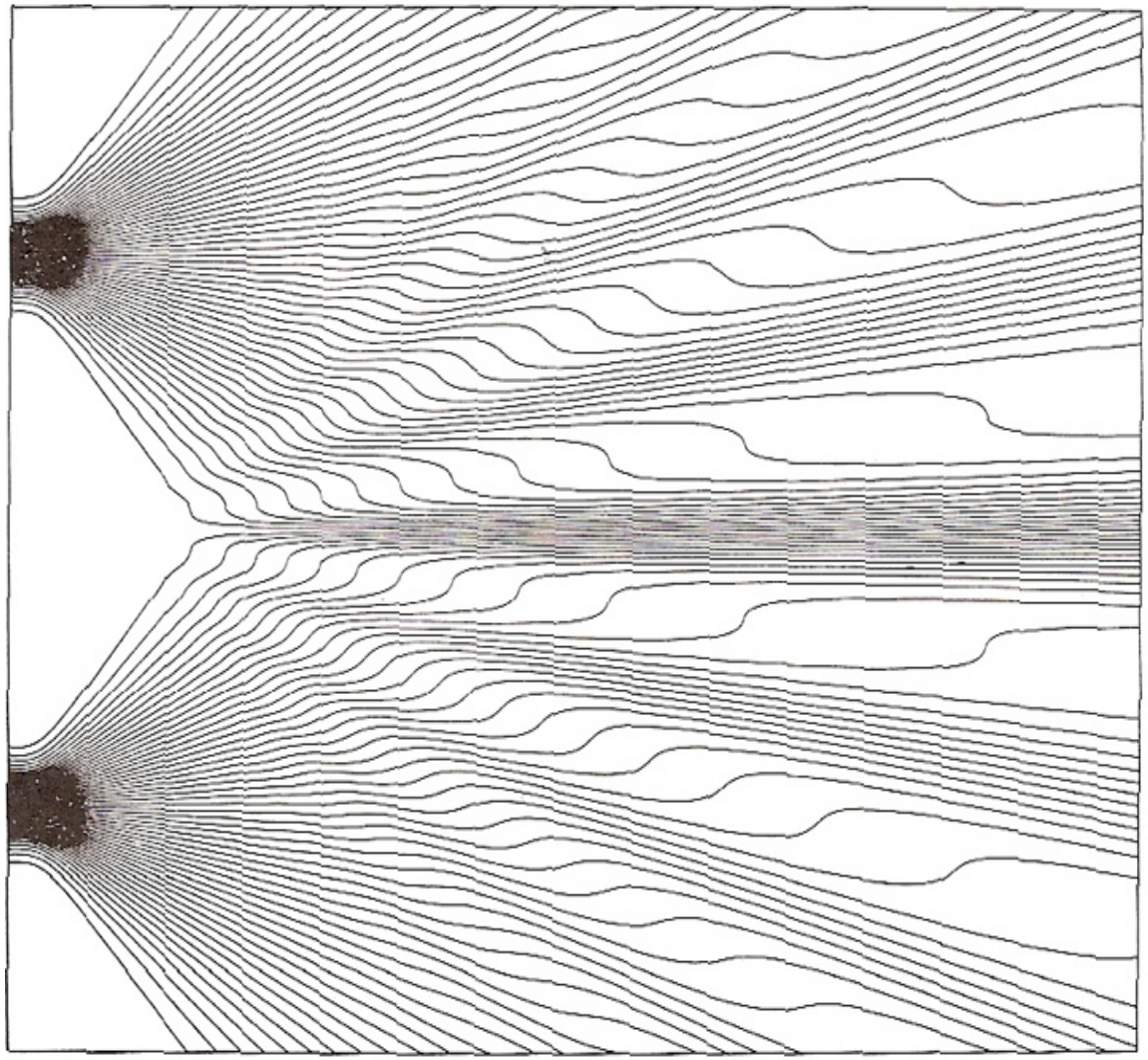
$$\exists t_0;\, \rho(\boldsymbol{x},t_0) = |\Psi(\boldsymbol{x},t_0)|^2$$

- classical limit well defined $Q \longrightarrow 0$
- state dependent
- $\exists$ intrinsic reality
  - non local …
- no need for external classical domain!

The two-slit experiment:



Trajectories in the two-slit experiment

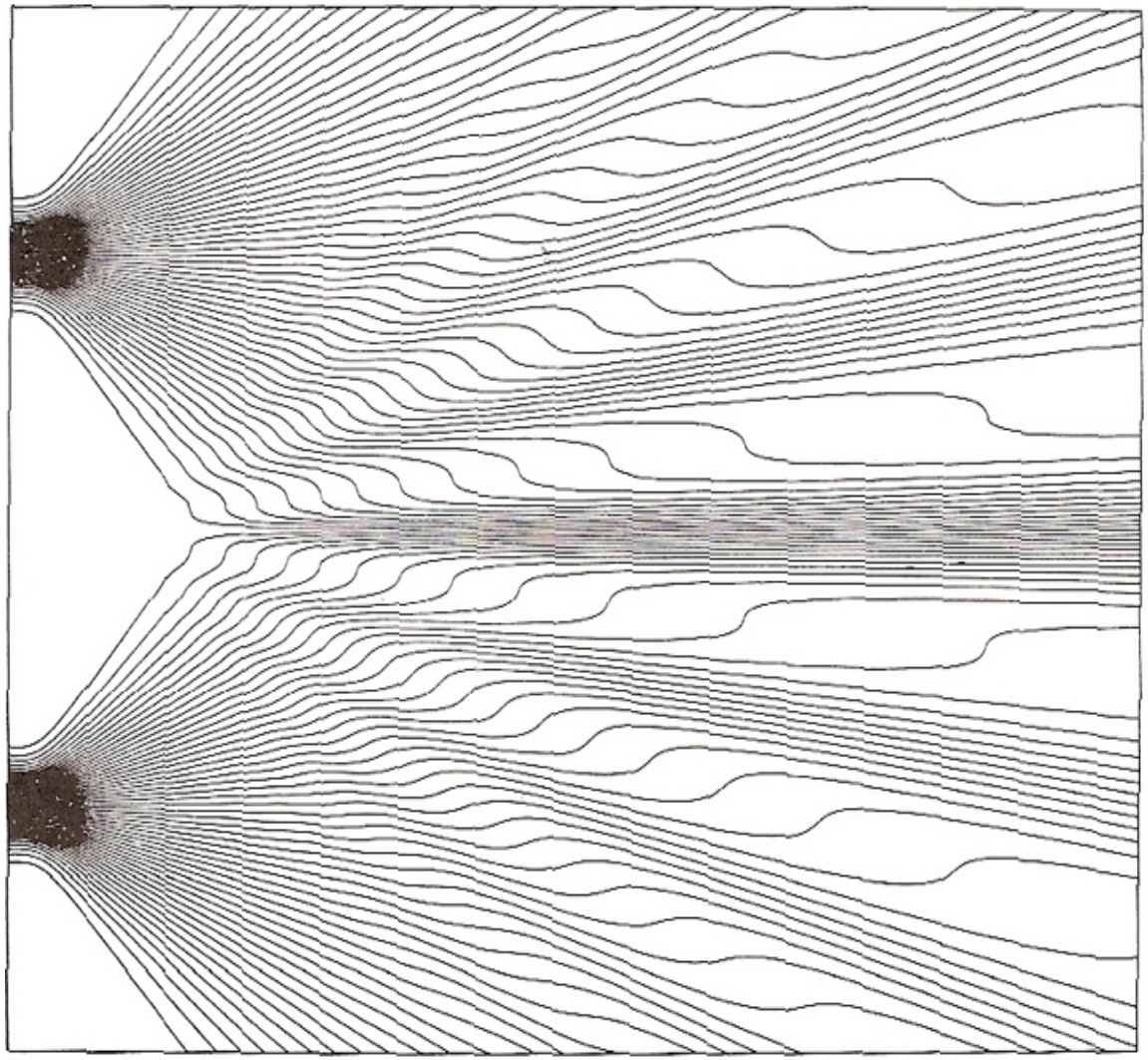


Surrealistic trajectories?

↓

Non straight in vacuum...

Trajectories in the two-slit experiment



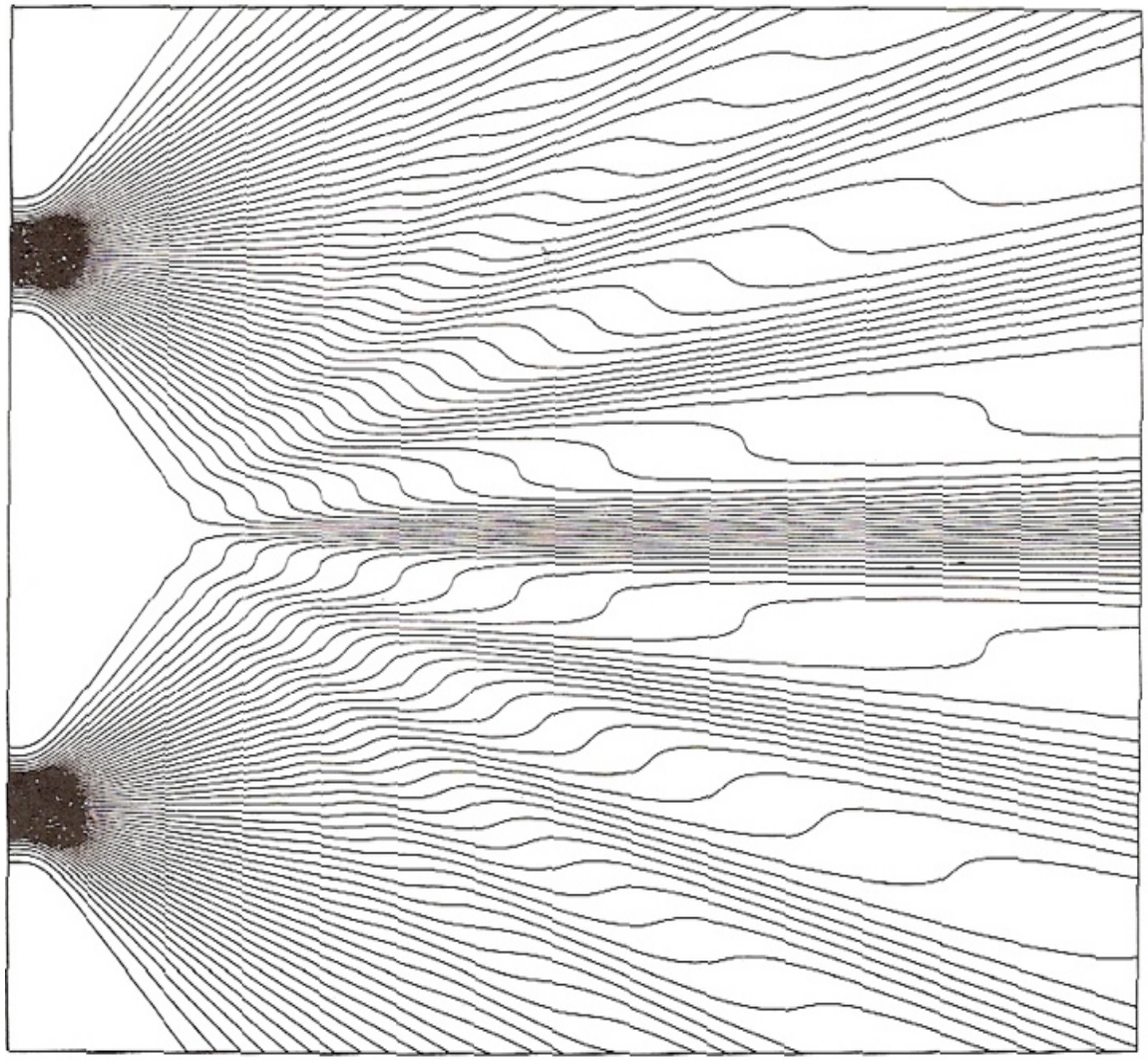
Surrealistic trajectories?

↓

Non straight in vacuum...

$$m\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} = -\nabla\left(V+Q\right)$$

Trajectories in the two-slit experiment

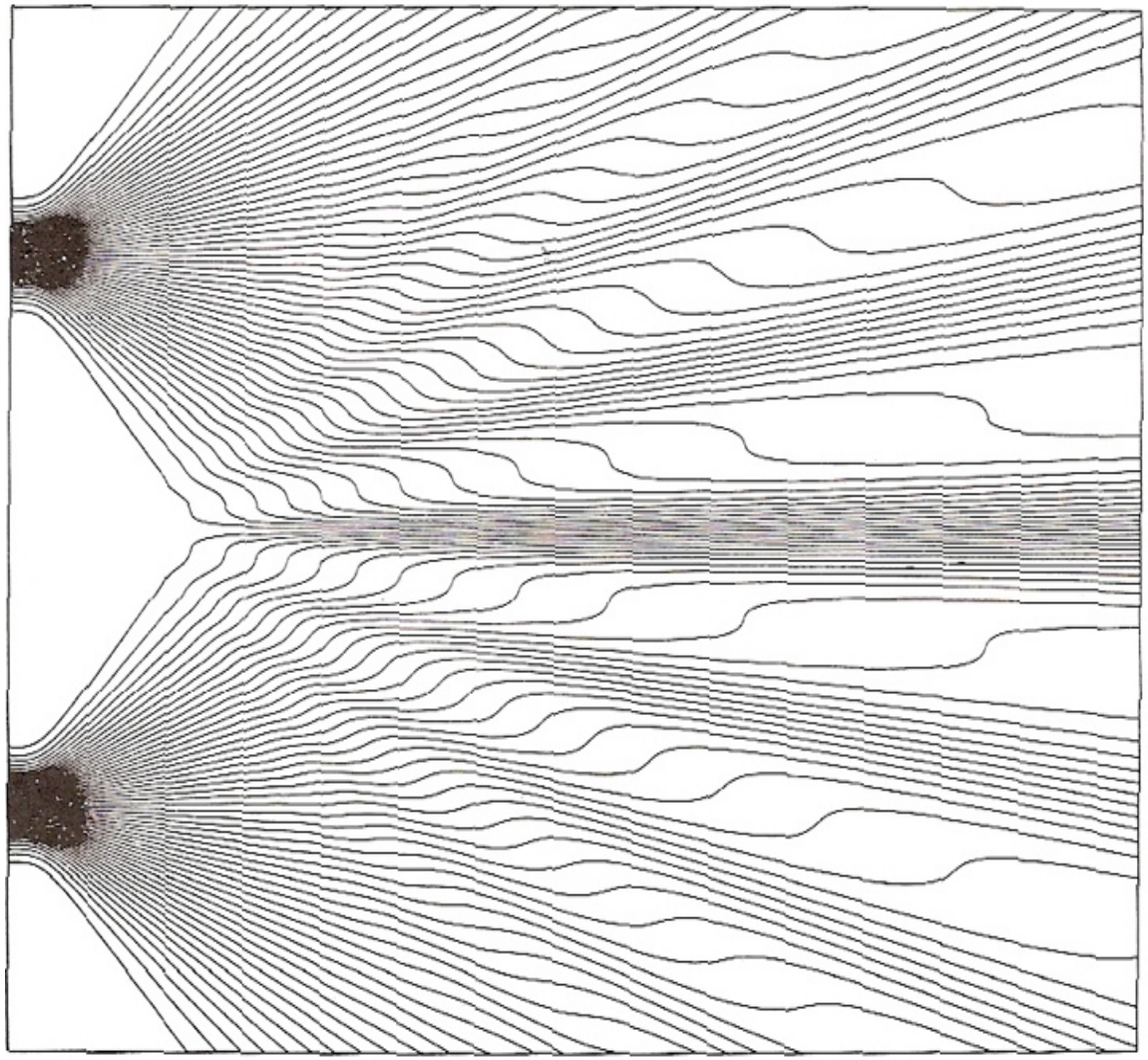


Surrealistic trajectories?

Non straight in vacuum...

$$m\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} = -\nabla(\cancel{V} + Q)$$

Trajectories in the two-slit experiment

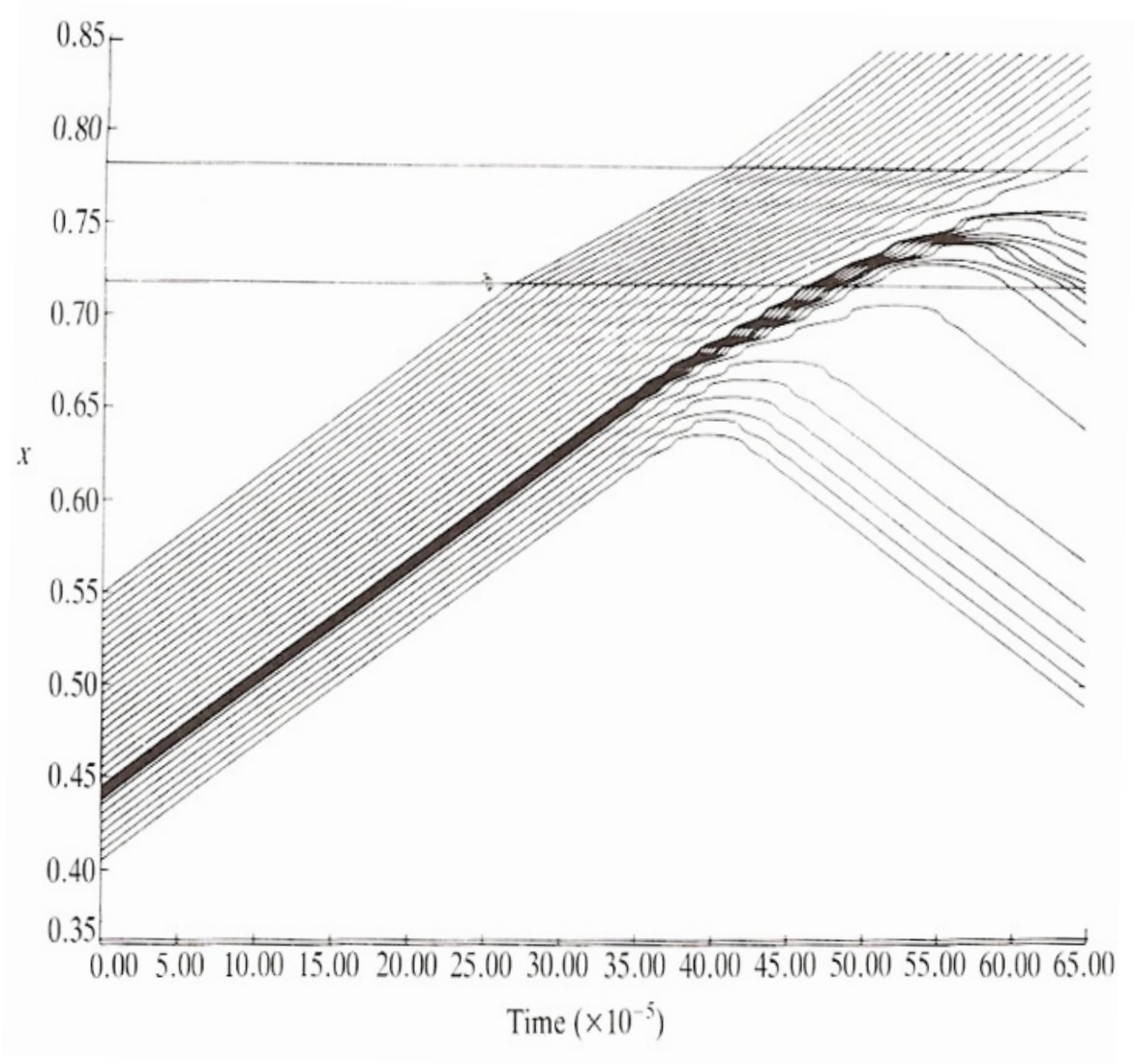


Surrealistic trajectories?

Non straight in vacuum...

$$m\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} = -\nabla(\cancel{V} + Q)$$

Diffraction by a potential simple understanding of tunnelling ...



The Universe is a closed system

- Quantum cosmology of a perfect fluid

$$\mathrm{d}s^2 = N^2(\tau)\mathrm{d}\tau - a^2(\tau)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

Perfect fluid: Schutz formalism ('70)

$$p = p_0 \left[\frac{\dot{\varphi} + \theta\dot{s}}{N(1+\omega)}\right]^{\frac{1+\omega}{\omega}}$$

$(\varphi, \theta, s) =$ Velocity potentials

canonical transformation: $T = -p_s \mathrm{e}^{-s/s_0} p_\varphi^{-(1+\omega)} s_0 \rho_0^{-\omega}$ ...

+ rescaling (volume...) + units... : simple Hamiltonian:

$$H = \left(-\frac{p_a^2}{4a} - \mathcal{K}a + \frac{p_T}{a^{3\omega}}\right) N$$

$$a^{3\omega}$$

Quantum cosmology $\mathrm{d}s^2 = N^2(\tau)\mathrm{d}\tau - a^2(\tau)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j$

+ canonical transformation

+ rescaling (volume ...) = a simple Hamiltonian:

+ units

$$H = \left(-\frac{p_a^2}{4a} - \mathcal{K}a + \frac{p_T}{a^{3\omega}}\right) N$$

$a^{3\omega}$

Wheeler-De Witt $H\Psi = 0$

+ Technical trick: $\mathcal{K} = 0 \Longrightarrow \chi \equiv \frac{2a^{3(1-\omega)/2}}{3(1-\omega)} \Longrightarrow \boxed{i\frac{\partial\Psi}{\partial T} = \frac{1}{4}\frac{\partial^2\Psi}{\partial\chi^2}}$

space defined by $\chi > 0$ $\longrightarrow$ constraint $\bar{\Psi}\frac{\partial\Psi}{\partial\chi} = \Psi\frac{\partial\bar{\Psi}}{\partial\chi}$

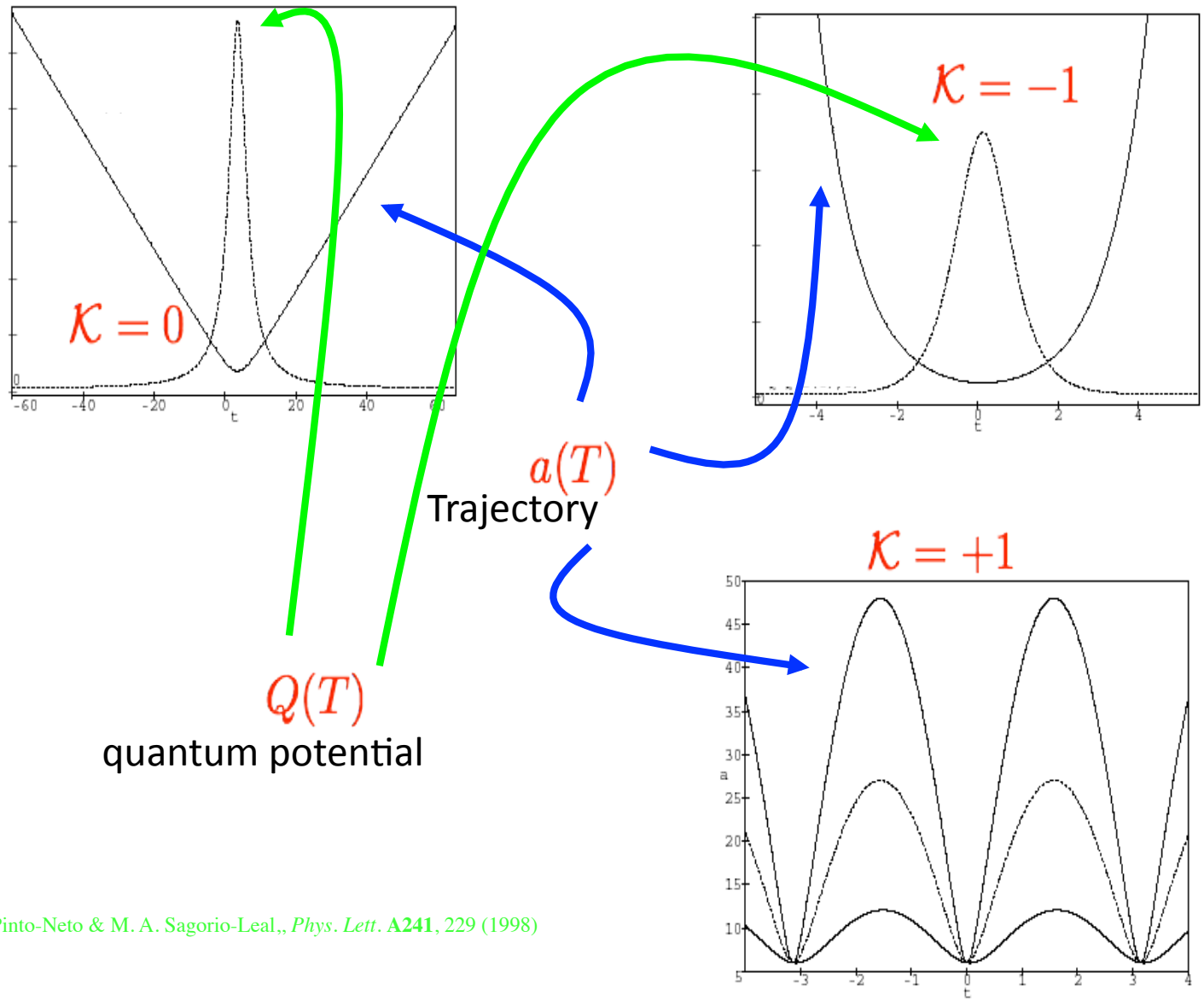
- Scale factor trajectories

Gaussian wave packet

$$\Psi = \left[\frac{8T_0}{\pi\left(T_0^2+T^2\right)^2}\right]^{\frac{1}{4}} \exp\left(-\frac{T_0\chi^2}{T_0^2+T^2}\right) \mathrm{e}^{-iS(\chi,T)}$$

phase $$S = \frac{T\chi^2}{T_0^2+T^2} + \frac{1}{2}\arctan\frac{T_0}{T} - \frac{\pi}{4}$$

Bohmian trajectory $$a = a_0\left[1+\left(\frac{T}{T_0}\right)^2\right]^{\frac{1}{3(1-\omega)}}$$



J. Acacio de Barros, N. Pinto-Neto & M. A. Sagorio-Leal,, *Phys. Lett.* **A241**, 229 (1998)

- Both background and perturbations are quantum

Usual treatment of the perturbations?

Einstein-Hilbert action up to 2nd order

$$S_{\mathrm{E-H}} = \int \mathrm{d}^4x \left[R^{(0)} + \delta^{(2)} R \right]$$

Classical Quantum

Bardeen (Newton) gravitational potential

$$\mathrm{d}s^2 = a^2(\eta) \left\{ (1+2\Phi)\, \mathrm{d}\eta^2 - \left[(1-2\Phi)\, \gamma_{ij} + h_{ij} \right] \mathrm{d}x^i \mathrm{d}x^j \right\}$$

conformal time $\mathrm{d}\eta = a(t)^{-1} \mathrm{d}t$

$$\Delta\Phi = -\frac{3\ell_{\mathrm{Pl}}^2}{2} \sqrt{\frac{\rho + p}{\omega}}\, a \frac{\mathrm{d}}{\mathrm{d}\eta} \left(\frac{v}{a} \right)$$

Mukhanov-Sasaki variable

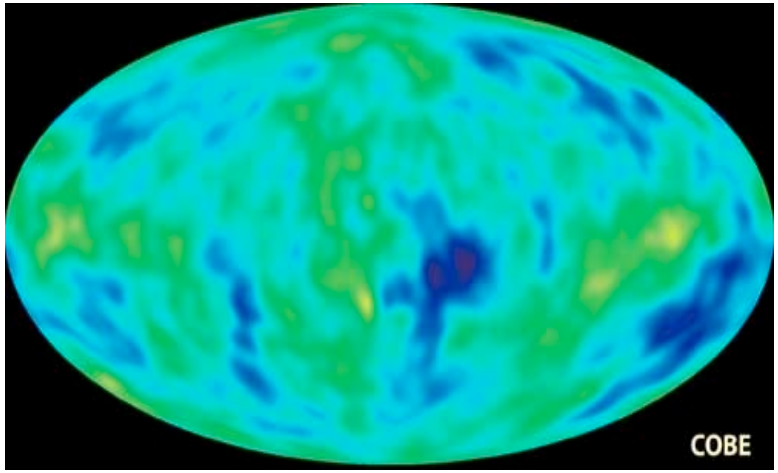
$$\int \mathrm{d}^4x\, \delta^{(2)} \mathcal{L} = \frac{1}{2} \int \sqrt{\gamma}\, \mathrm{d}^3\boldsymbol{x}\, \mathrm{d}\eta \left[(\partial_\eta v)^2 - \gamma^{ij} \partial_i v \partial_j v + \frac{z''}{z} v^2 \right]$$

V. F. Mukhanov, H. A. Feldman & R. H. Brandenberger, *Phys. Rep.* **215**, 203 (1992)

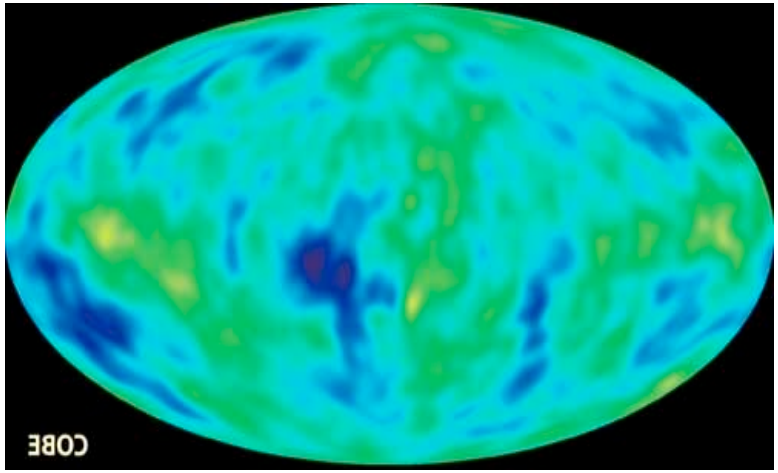
Simple scalar field with varying mass in Minkowski space!!!

$$z = z[a(\eta)]$$

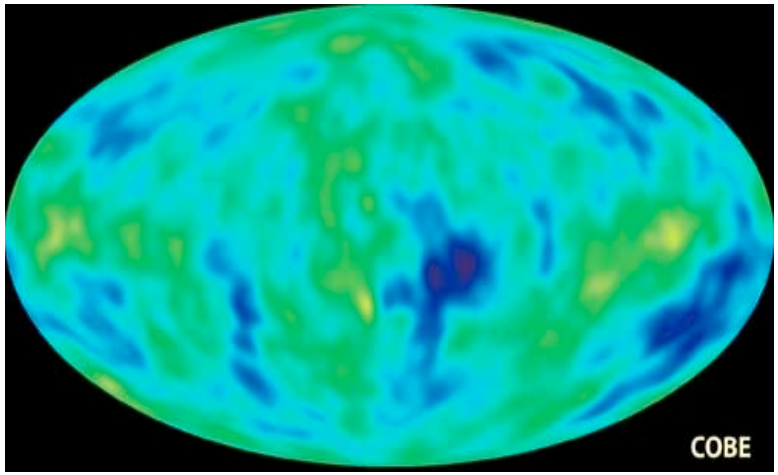
Wave function? No question about it ...



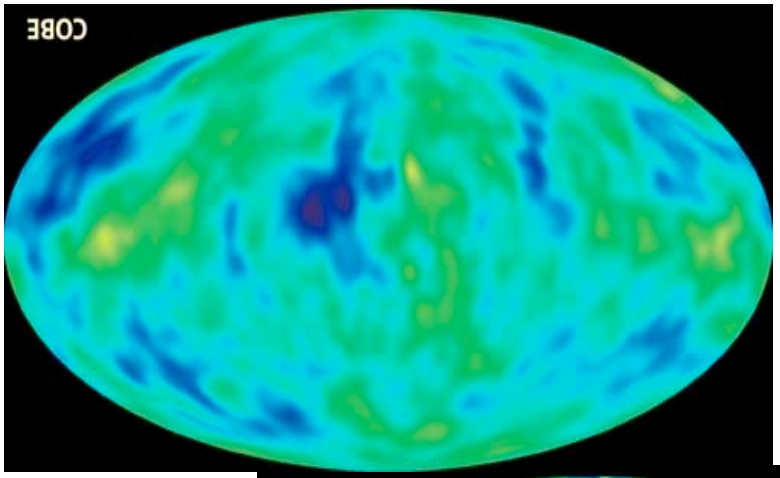
+



+



+

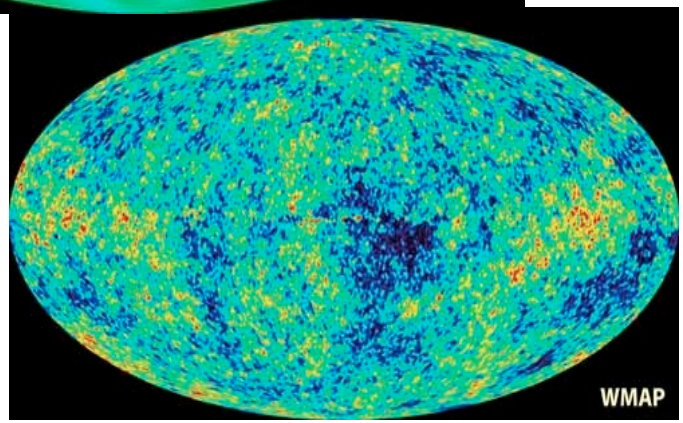


+ ...

Superposition

Collapse in 1992 ???

Further collapse in 2003
on smaller scales???



Decoherence ...

Self-consistent treatment of the perturbations?

Hamiltonian up to 2nd order $H = H_{(0)} + H_{(2)} + \cdots$

Bardeen (Newton) gravitational potential

$$\mathrm{d}s^2 = a^2(\eta)\left\{(1+2\Phi)\,\mathrm{d}\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$$

conformal time $\mathrm{d}\eta = a^{3\omega-1}\mathrm{d}T$

$$\Delta\Phi = -\frac{3\ell_{\mathrm{Pl}}^2}{2}\sqrt{\frac{\rho+p}{\omega}}\,a\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{v}{a}\right)$$

factorization of the wave function

$$\Psi = \Psi_{(0)}\left(a,T\right)\Psi_{(2)}\left[v,T;a\left(T\right)\right]$$

comes from 0th order

Need dBB !!!

More conclusions?