Quantum Gravity and Quantum Cosmology

A General Introduction

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Contents

Max Planck, Über irreversible Strahlungsvorgänge, *Sitzungsberichte der königlich-preußischen Akademie der Wissenschaften zu Berlin, phys.-math. Klasse*, Seiten 440–80 (1899)

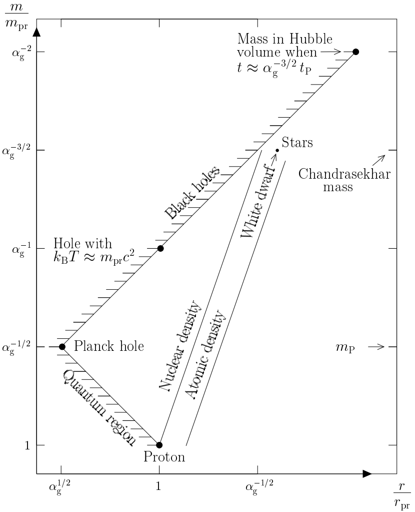
Planck units

$$
\begin{aligned}
l_{\mathrm{P}} &= \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-33}\ \mathrm{cm} \\
t_{\mathrm{P}} &= \frac{l_{\mathrm{P}}}{c} = \sqrt{\frac{\hbar G}{c^5}} \approx 5.40 \times 10^{-44}\ \mathrm{s} \\
m_{\mathrm{P}} &= \frac{\hbar}{l_{\mathrm{P}} c} = \sqrt{\frac{\hbar c}{G}} \approx 2.17 \times 10^{-5}\ \mathrm{g} \approx 1.22 \times 10^{19}\ \mathrm{GeV}/c^2
\end{aligned}
$$

Max Planck (1899):

Diese Grössen behalten ihre natürliche Bedeutung so lange bei, als die Gesetze der Gravitation, der Lichtfortpflanzung im Vacuum und die beiden Hauptsätze der Wärmetheorie in Gültigkeit bleiben, sie müssen also, von den verschiedensten Intelligenzen nach den verschiedensten Methoden gemessen, sich immer wieder als die nämlichen ergeben.

Structures in the Universe



$$\alpha_{\mathrm{g}} = \frac{Gm_{\mathrm{pr}}^2}{\hbar c} = \left(\frac{m_{\mathrm{pr}}}{m_{\mathrm{P}}}\right)^2 \approx 5.91 \times 10^{-39}$$

Meaning of the Planck scale?

- “Yet another example of choosing a basic system is provided by Planck’s natural units . . . ” (Gamow, Ivanenko, Landau 1927); cf. Stoney (1881)
- Compton wavelength $\sim$ Schwarzschild radius, that is, the curvature of a quantum object of Planck size cannot be neglected
- “Quantum foam”: huge fluctuations of curvature and topology?
- Planck length as the smallest possible length?

Why quantum gravity?

- Unification of all interactions
- Singularity theorems
  - Black holes
  - ‘Big Bang’
- Problem of time
- Absence of viable alternatives

Richard Feynman 1957:

. . . if you believe in quantum mechanics up to any level then you have to believe in gravitational quantization in order to describe this experiment. . . . It may turn out, since we've never done an experiment at this level, that it's not possible . . . that there is something the matter with our quantum mechanics when we have too much *action* in the system, or too much mass—or something. But that is the only way I can see which would keep you from the necessity of quantizing the gravitational field. It's a way that I don't want to propose. . . .

Background independence

Wolfgang Pauli (1955):

Es scheint mir . . . , daß nicht so sehr die Linearität oder Nichtlinearität Kern der Sache ist, sondern eben der Umstand, daß hier eine allgemeinere Gruppe als die Lorentzgruppe vorhanden ist . . . .

Matvei Bronstein (1936):

The elimination of the logical inconsistencies connected with this requires a radical reconstruction of the theory, and in particular, the rejection of a Riemannian geometry dealing, as we see here, with values unobservable in principle, and perhaps also the rejection of our ordinary concepts of space and time, modifying them by some much deeper and nonevident concepts. *Wer's nicht glaubt, bezahlt einen Taler*.

The problem of time

- Absolute time in quantum theory:

$$\mathrm{i}\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi$$

- Dynamical time in general relativity:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

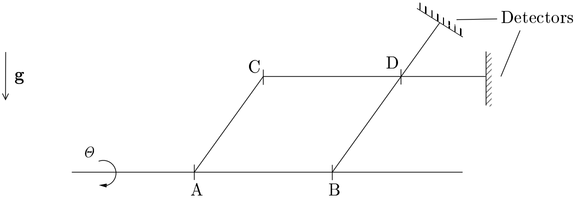
QUANTUM GRAVITY?

Steps towards quantum gravity

- Interaction of micro- and macroscopic systems with an external gravitational field
- Quantum field theory on curved backgrounds (or in flat background, but in non-inertial systems)
- Full quantum gravity

Quantum systems in external gravitational fields

Neutron and atom interferometry



Experiments:

- Neutron interferometry in the field of the Earth
  (Colella, Overhauser, and Werner (‘COW’) 1975)
- Neutron interferometry in accelerated systems
  (Bonse and Wroblewski 1983)
- Discrete neutron states in the field of the Earth
  (Nesvizhevsky *et al*. 2002)
- Neutron whispering gallery
  (Nesvizhevsky *et al*. 2009)
- Atom interferometry
  (e.g. Peters, Chung, Chu 2001: measurement of g with accuracy $\Delta g/g \sim 10^{-10}$)

Non-relativistic expansion of the Dirac equation yields

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} \approx H_{\mathrm{FW}}\psi$$

mit

$$\begin{aligned} H_{\mathrm{FW}} &= \underbrace{\beta mc^2}_{\text{rest mass}} + \underbrace{\frac{\beta}{2m}\mathbf{p}^2}_{\text{kinetic energy}} - \underbrace{\frac{\beta}{8m^3c^2}\mathbf{p}^4}_{\text{SR correction}} + \underbrace{\beta m(\mathbf{a}\,\mathbf{x})}_{\text{COW}} \\ &\quad - \underbrace{\boldsymbol{\omega}\mathbf{L}}_{\text{Sagnac effect}} - \underbrace{\boldsymbol{\omega}\mathbf{S}}_{\text{`Mashhoon effect'}} \\ &\quad + \frac{\beta}{2m}\mathbf{p}\frac{\mathbf{a}\,\mathbf{x}}{c^2}\mathbf{p} + \frac{\beta\hbar}{4mc^2}\vec{\Sigma}(\mathbf{a}\times\mathbf{p}) + \mathcal{O}\left(\frac{1}{c^3}\right) \end{aligned}$$

Black-hole radiation

Black holes radiate with a temperature proportional to $\hbar$

('Hawking temperature'):

$$T_{\mathrm{BH}} = \frac{\hbar\kappa}{2\pi k_{\mathrm{B}} c}$$

Schwarzschild case:

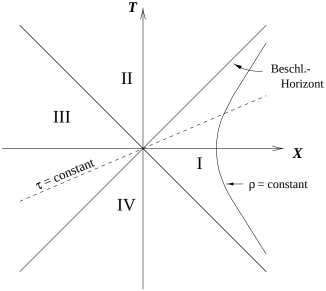
$$\begin{aligned} T_{\mathrm{BH}} &= \frac{\hbar c^3}{8\pi k_{\mathrm{B}} G M} \\ &\approx 6.17 \times 10^{-8} \left(\frac{M_\odot}{M}\right) \mathrm{K} \end{aligned}$$

Black holes also have an entropy

('Bekenstein–Hawking entropy'):

$$S_{\mathrm{BH}} = k_{\mathrm{B}} \frac{A}{4 l_{\mathrm{P}}^2} \overset{\text{Schwarzschild}}{\approx} 1.07 \times 10^{77} k_{\mathrm{B}} \left(\frac{M}{M_\odot}\right)^2$$

Analogous effect in flat spacetime



Accelerated observer in the Minkowski vacuum experiences thermal radiation with temperature

$$T_{\mathrm{DU}} = \frac{\hbar a}{2\pi k_{\mathrm{B}} c} \approx 4.05 \times 10^{-23}\, a \left[\frac{\mathrm{cm}}{\mathrm{s}^2}\right] \mathrm{K}\ .$$

('Davies–Unruh temperature')

Is thermodynamics more fundamental than gravity?

Possible tests of Hawking and Unruh effect

- Search for primordial black holes (e.g. by the Fermi Gamma-ray Space Telescope)
- Production of small black holes at the LHC in Geneva?
- Signatures of the Unruh effect via high-power, short-pulse lasers? (Thirolf *et al.* 2009)

Main approaches to quantum gravity

No question about quantum gravity is more difficult than the question, “What is the question?”
(John Wheeler 1984)

- Quantum general relativity
  - Covariant approaches (perturbation theory, path integrals, ...)
  - Canonical approaches (geometrodynamics, connection dynamics, loop dynamics, ...)
- String theory
- Fundamental discrete approaches
  (quantum topology, causal sets, group field theory, ...);
  have partially grown out of the other approaches

Covariant quantum gravity

Perturbation theory:

$$\boxed{g_{\mu\nu} = \bar{g}_{\mu\nu} + \sqrt{\frac{32\pi G}{c^4}} f_{\mu\nu}}$$

- $\bar{g}_{\mu\nu}$: classical background
- Perturbation theory with respect to $f_{\mu\nu}$ (Feynman rules)
- 'Particle' of quantum gravity: graviton (massless[1] spin-2 particle)

$\boxed{\text{Perturbative non-renormalizability}}$

[1] $m_{\rm g} \lesssim 10^{-29}$ eV

Divergences in perturbative quantum gravity

- Quantum general relativity: divergences at two loops (Goroff and Sagnotti 1986)
- $N = 8$ supergravity (maximal supersymmetry!) is finite up to four loops (explicit calculation!) and there are arguments that it is finite also at five and six loops (and perhaps up to eight loops) (Bern *et al.* 2009) – new symmetry?
- There are theories that exist at the non-perturbative level, but are perturbatively non-renormalizable (e.g. non-linear σ model for $D > 2$)
- Approach of asymptotic safety (see below)

Path integrals

$$\boxed{Z[g] = \int \mathcal{D}g_{\mu\nu}(x)\ \mathrm{e}^{\mathrm{i}S[g_{\mu\nu}(x)]/\hbar}}$$

In addition: sum over all topologies?

- Euclidean path integrals
  (e.g. for Hartle–Hawking proposal or Regge calculus)
- Lorentzian path integrals
  (e.g. for dynamical triangulation)

Effective field theory

One-loop corrections to the non-relativistic potentials obtained from the scattering amplitude by calculating the non-analytic terms in the momentum transfer

- Quantum gravitational correction to the Newtonian potential

$$V(r) = -\frac{Gm_1m_2}{r}\left(1 + \underbrace{3\frac{G(m_1+m_2)}{rc^2}}_{\mathrm{GR-correction}} + \underbrace{\frac{41}{10\pi}\frac{G\hbar}{r^2c^3}}_{\mathrm{QG-correction}}\right)$$

(Bjerrum-Bohr *et al*. 2003)

- Quantum gravitational effects to the Coulomb potential (scalar QED)

$$V(r) = \frac{Q_1Q_2}{r}\left(1 + 3\frac{G(m_1+m_2)}{rc^2} + \frac{6}{\pi}\frac{G\hbar}{r^2c^3}\right) + \dots$$

(Faller 2008)

Beyond perturbation theory?

Example: self-energy of a thin charged shell

Energy of the shell using the bare mass m_0 is

$$m(\epsilon) = m_0 + \frac{Q^2}{2\epsilon} \ ,$$

which diverges for $\epsilon \to 0$. But the inclusion of gravity leads to

$$m(\epsilon) = m_0 + \frac{Q^2}{2\epsilon} - \frac{Gm^2(\epsilon)}{2\epsilon} \ ,$$

which leads for $\epsilon \to 0$ to a finite result,

$$m(\epsilon) \stackrel{\epsilon \to 0}{\longrightarrow} \frac{|Q|}{\sqrt{G}} \ .$$

The sigma model

Non-linear σ model: N-component field ϕ_a satisfying $\sum_a \phi_a^2 = 1$

- is non-renormalizable for $D > 2$
- exhibits a non-trivial UV fixed point at some coupling g_c ('phase transition')
- an expansion in $D - 2$ and use of renormalization-group (RG) techniques gives information about the behaviour in the vicinity of the non-trivial fixed point

Example: superfluid Helium

The specific heat exponent α was measured in a space shuttle experiment (Lipa *et al*. 2003): $\alpha = -0.0127(3)$, which is in excellent agreement with three calculations in the $N = 2$ non-linear σ-model:

- $\alpha = -0.01126(10)$ (4-loop result; Kleinert 2000);
- $\alpha = -0.0146(8)$ (lattice Monte Carlo estimate; Campostrini *et al.* 2001);
- $\alpha = -0.0125(39)$ (lattice variational RG prediction; cited in Hamber 2009)

Asymptotic Safety

Weinberg (1977): A theory is called asymptotically safe if all essential coupling parameters g_i of the theory approach for $k \to \infty$ a non-trivial fix point

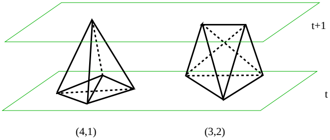
Preliminary results:

- Effective gravitational constant vanishes for $k \to \infty$?
- Effective gravitational constant increases with distance? (simulation of Dark Matter?)
- Small positive cosmological constant as an infrared effect? (Dark Energy?)
- Spacetime appears two-dimensional on smallest scales

(H. Hamber *et al*., M. Reuter *et al*.)

Dynamical triangulation

- makes use of *Lorentzian* path integrals
- edge lengths of simplices remain fixed; sum is performed over all possible combinations with equilateral simplices
- Monte-Carlo simulations



Preliminary results:

- Hausdorff dimension $H = 3.10 \pm 0.15$
- Spacetime two-dimensional on smallest scales (cf. asymptotic-safety approach)
- positive cosmological constant needed
- continuum limit?

(Ambjørn, Loll, Jurkiewicz from 1998 on)

A brief history of early covariant quantum gravity

- L. Rosenfeld, Über die Gravitationswirkungen des Lichtes, *Annalen der Physik* (1930)
- M. P. Bronstein, Quantentheorie schwacher Gravitationsfelder, *Physikalische Zeitschrift der Sowjetunion* (1936)
- S. Gupta, Quantization of Einstein's Gravitational Field: Linear Approximation, *Proceedings of the Royal Society* (1952)
- C. Misner, Feynman quantization of general relativity, *Reviews of Modern Physics* (1957)
- R. P. Feynman, Quantum theory of gravitation, *Acta Physica Polonica* (1963)
- B. S. DeWitt, Quantum theory of gravity II, III, *Physical Review* (1967)

Canonical quantum gravity

Central equations are constraints:

$$\boxed{\hat{H}\Psi = 0}$$

Different canonical approaches

- Geometrodynamics –
  metric and extrinsic curvature
- Connection dynamics –
  connection (A_a^i) and coloured electric field (E_i^a)
- Loop dynamics –
  flux of E_i^a and holonomy for A_a^i

Erwin Schrödinger 1926:

We know today, in fact, that our classical mechanics fails for very small dimensions of the path and for very great curvatures. Perhaps this failure is in strict analogy with the failure of geometrical optics . . . that becomes evident as soon as the obstacles or apertures are no longer great compared with the real, finite, wavelength. . . . Then it becomes a question of searching for an undulatory mechanics, and the most obvious way is by an elaboration of the Hamiltonian analogy on the lines of undulatory optics.[2]

---

[2] *wir wissen doch heute, daß unsere klassische Mechanik bei sehr kleinen Bahndimensionen und sehr starken Bahnkrümmungen versagt*. Vielleicht ist dieses Versagen eine volle Analogie zum Versagen der geometrischen Optik . . . , das bekanntlich eintritt, sobald die 'Hindernisse' oder 'Öffnungen' nicht mehr groß sind gegen die wirkliche, endliche Wellenlänge. . . . Dann gilt es, eine 'undulatorische Mechanik' zu suchen – und der nächstliegende Weg dazu ist wohl die wellentheoretische Ausgestaltung des Hamiltonschen Bildes.

Hamilton–Jacobi equation

$\boxed{\text{Hamilton–Jacobi equation} \longrightarrow \text{guess a wave equation}}$

In the vacuum case, one has

$$\begin{aligned} 16\pi G\, G_{abcd}\frac{\delta S}{\delta h_{ab}}\frac{\delta S}{\delta h_{cd}} - \frac{\sqrt{h}}{16\pi G}\left({}^{(3)}R - 2\Lambda\right) &= 0\,, \\ D_a \frac{\delta S}{\delta h_{ab}} &= 0 \end{aligned}$$

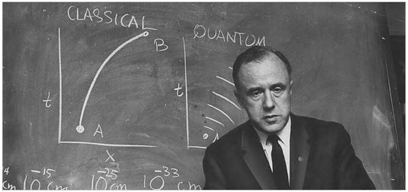
(Peres 1962)

Find wave equation which yields the Hamilton–Jacobi equation in the semiclassical limit:

$$\text{Ansatz}: \quad \Psi[h_{ab}] = C[h_{ab}]\exp\left(\frac{\mathrm{i}}{\hbar}S[h_{ab}]\right)$$

The dynamical gravitational variable is the three-metric h_{ab}! It is the argument of the wave functional.

Quantum geometrodynamics



In the vacuum case, one has

$$\hat{H}\Psi \equiv \left(-2\kappa\hbar^2 G_{abcd}\frac{\delta^2}{\delta h_{ab}\delta h_{cd}} - (2\kappa)^{-1}\sqrt{h}\left({}^{(3)}R - 2\Lambda\right)\right)\Psi = 0,$$

$$\kappa = 8\pi G$$

Wheeler–DeWitt equation

$$\hat{D}^a\Psi \equiv -2\nabla_b\frac{\hbar}{\mathrm{i}}\frac{\delta\Psi}{\delta h_{ab}} = 0$$

quantum diffeomorphism (momentum) constraint

Problem of time

- no external time present; spacetime has *disappeared*!
- local intrinsic time can be defined through local hyperbolic structure of Wheeler–DeWitt equation ('wave equation')
- related problem: *Hilbert-space problem* –
which inner product, if any, to choose between wave functionals?
  - Schrödinger inner product?
  - Klein–Gordon inner product?
- Problem of *observables*

Recovery of quantum field theory in an external spacetime

An expansion of the Wheeler–DeWitt equation with respect to the Planck mass leads to the functional Schrödinger equation for non-gravitational fields in a spacetime that is a solution of Einstein's equations
(Born–Oppenheimer type of approximation)

(Lapchinsky and Rubakov 1979, Banks 1985, Halliwell and Hawking 1985, Hartle 1986, C.K. 1987, . . .)

Quantum gravitational corrections

Next order in the Born–Oppenheimer approximation gives

$$\hat{H}^{\mathrm{m}} \to \hat{H}^{\mathrm{m}} + \frac{1}{m_{\mathrm{P}}^2}\,(\text{various terms})$$

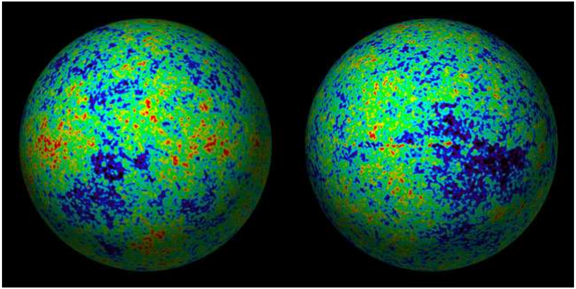
(C.K. and Singh 1991; Barvinsky and C.K. 1998)

- Quantum gravitational correction to the trace anomaly in de Sitter space:

$$\delta\epsilon \approx -\frac{2G\hbar^2 H_{\mathrm{dS}}^6}{3(1440)^2\pi^3 c^8}$$

  (C.K. 1996)
- Possible contribution to the CMB anisotropy spectrum (C.K. and Krämer 2012)

Does the anisotropy spectrum of the Cosmic Background Radiation contain information about quantum gravity?



Path Integral satisfies constraints

- Quantum mechanics: path integral satisfies Schrödinger equation
- Quantum gravity: path integral satisfies Wheeler–DeWitt equation and diffeomorphism constraints

The full path integral with the Einstein–Hilbert action (if defined rigorously) should be equivalent to the constraint equations of canonical quantum gravity

A brief history of early quantum geometrodynamics

- F. Klein, *Nachrichten von der Königlichen Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-physikalische Klasse*, **1918**, 171–189:
  first four Einstein equations are ‘Hamiltonian’ and ‘momentum density’ equations
- L. Rosenfeld, *Annalen der Physik*, 5. Folge, **5**, 113–152 (1930):
  general constraint formalism; first four Einstein equations are constraints; consistency conditions in the quantum theory (‘Dirac consistency’)

- P. Bergmann and collaborators (from 1949 on): general formalism (mostly classical); notion of observables
  Bergmann (1966): $H\psi = 0$, $\partial\psi/\partial t = 0$
  ("To this extent the Heisenberg and Schrödinger pictures are indistinguishable in any theory whose Hamiltonian is a constraint.")
- P. Dirac (1951): general formalism; Dirac brackets
- P. Dirac (1958/59): application to the gravitational field; reduced quantization
  ("I am inclined to believe from this that four-dimensional symmetry is not a fundamental property of the physical world.")
- ADM (1959–1962): lapse and shift; rigorous definition of gravitational energy and radiation by canonical methods

- B. S. DeWitt, Quantum theory of gravity. I. The canonical theory. *Phys. Rev.*, **160**, 1113–48 (1967):
  general Wheeler–DeWitt equation; configuration space; quantum cosmology; semiclassical limit; conceptual issues, ...
- J. A. Wheeler, Superspace and the nature of quantum geometrodynamics. In *Battelle rencontres* (ed. C. M. DeWitt and J. A. Wheeler), pp. 242–307 (1968):
  general Wheeler–DeWitt equation; superspace; semiclassical limit; conceptual issues; ...

Ashtekar's new variables

- new momentum variable: densitized version of triad,

  $E^a_i(x) := \sqrt{h(x)}e^a_i(x)$;

- new configuration variable: 'connection' ,

  $GA^i_a(x) := \Gamma^i_a(x) + \beta K^i_a(x)$

$$\boxed{\{A^i_a(x), E^b_j(y)\} = 8\pi\beta\delta^i_j\delta^b_a\delta(x,y)}$$

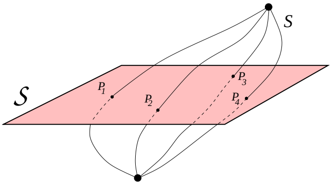
Loop quantum gravity

- new configuration variable: holonomy,

  $U[A, \alpha] := \mathcal{P} \exp \left(G \int_\alpha A \right)$;

- new momentum variable: densitized triad flux

  $E_i[\mathcal{S}] := \int_{\mathcal{S}} \mathrm{d}\sigma_a \, E^a_i$



Quantization of area:

$$\hat{A}(\mathcal{S})\Psi_S[A] = 8\pi\beta l_{\mathrm{P}}^2 \sum_{P \in S \cap \mathcal{S}} \sqrt{j_P(j_P+1)}\Psi_S[A]$$

On space and time in string theory

String theory contains general relativity; therefore, the above arguments apply: the Wheeler–DeWitt equation should approximately be valid away from the Planck scale

'Problem of time' is the same here; new insight is obtained for the concept of space

- *Matrix models*: finite number of degrees of freedom connected with the description of M-theory; fundamental scale is the 11-dimensional Planck scale
- *AdS/CFT correspondence*: non-perturbative string theory in a background spacetime that is asymptotically anti-de Sitter (AdS) is dual to a conformal field theory (CFT) defined in a flat spacetime of one less dimension (Maldacena 1998).

AdS/CFT correspondence

Often considered as a mostly background-independent definition of string theory (background metric enters only through boundary conditions at infinity).

Realization of the ‘holographic principle’:
laws including gravity in $d = 3$ are equivalent to laws excluding gravity in $d = 2$.

In a sense, space has here vanished, too!

Why Quantum Cosmology?

Gell-Mann and Hartle 1990:
Quantum mechanics is best and most fundamentally understood in the framework of quantum cosmology.

- Quantum theory is universally valid:
  Application to the Universe as a whole as the only closed quantum system in the strict sense
- Need quantum theory of gravity, since gravity dominates on large scales

Quantization of a Friedmann Universe

Closed Friedmann–Lemaître universe with scale factor a, containing a homogeneous massive scalar field ϕ (two-dimensional *minisuperspace*)

$$\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\mathrm{d}\Omega_3^2$$

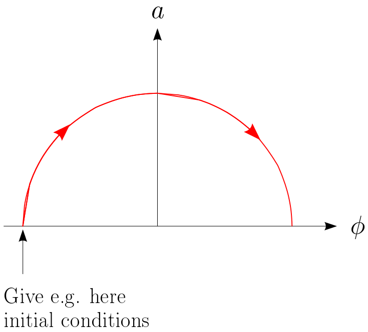
The Wheeler–DeWitt equation reads (with units $2G/3\pi = 1$)

$$\frac{1}{2}\left(\frac{\hbar^2}{a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) - \frac{\hbar^2}{a^3}\frac{\partial^2}{\partial \phi^2} - a + \frac{\Lambda a^3}{3} + m^2 a^3 \phi^2\right)\psi(a,\phi) = 0$$

Factor ordering chosen in order to achieve covariance in minisuperspace

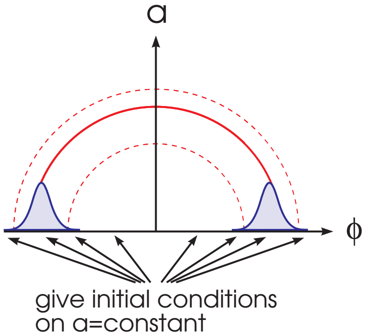
Determinism in classical and quantum theory

Classical theory



Recollapsing part is deterministic successor of expanding part

Quantum theory

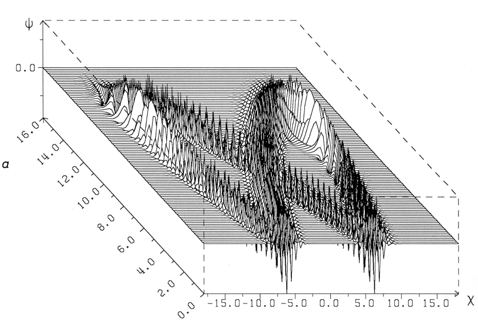


'Recollapsing' wave packet must be present 'initially'

Example

Indefinite Oscillator

$$\hat{H}\psi(a,\chi) \equiv (-H_a + H_\chi)\psi \equiv \left(\frac{\partial^2}{\partial a^2} - \frac{\partial^2}{\partial \chi^2} - a^2 + \chi^2\right)\psi = 0$$



C.K. (1990)

Validity of Semiclassical Approximation?

Closed universe: 'Final condition' $\psi \xrightarrow{a\to\infty} 0$

$$\Downarrow$$

wave packets in general disperse

$$\Downarrow$$

WKB approximation not always valid

Solution: Decoherence (see next talk)

More details in

C.K., *Quantum Gravity*, third edition (Oxford 2012).