Regge Limit of Gauge Theories & Gravity

with Links to Integrability

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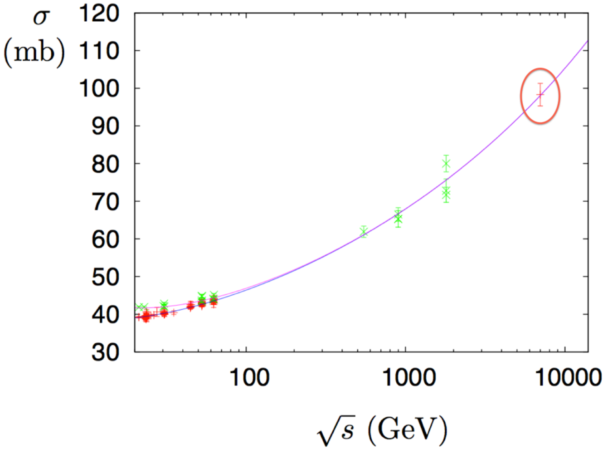
  - Multi-Regge Kinematics
  - Beyond Multi-Regge Kinematics
- Effective Vertices in QCD & Gravity
- Amplitudes & Monte Carlo Techniques
- BFKL & Spin Chains
- Graviton Reggeization

Multi-Regge kinematics

Hadron-hadron total cross-sections rise @ $\sqrt{s} = 7$ TeV @ LHC:
$\sigma^{\mathrm{TOT}} = 98.3 \pm 0.2\,(\mathrm{stat}) \pm 2.8\,(\mathrm{syst})$ mb

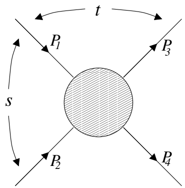


Consistent with Regge theory: $\sigma^{\mathrm{TOT}} \simeq s^{0.08}$

Multi-Regge kinematics

Regge theory is pre-QCD

Microscopic picture in terms of quarks & gluons?



Elastic amplitude with Mandelstam invariants

$$s = (P_1 + P_2)^2 \qquad t = (P_1 - P_3)^2$$

Regge limit in perturbative QCD: $s \gg t, Q^2$

Hard scale $Q^2 \gg \Lambda^2_{\rm QCD}$ allows perturbative expansion $\alpha_s(Q^2) \ll 1$

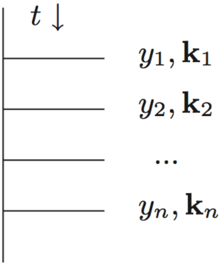
Multi-Regge kinematics

All-orders resummation of $\alpha_s(Q^2) \log\left(\frac{s}{Q^2}\right)$ terms: How?

Multi-Regge limit: $s \gg s_i \gg t_i \sim Q^2$ (Regge limit in all sub-channels)

Equivalent to strong ordering in rapidity of emitted particles:

$$Y \sim \log s \qquad y_i \gg y_{i-1} \qquad \mathbf{k}_i^2 \sim \mathbf{k}_{i-1}^2 \sim Q^2$$



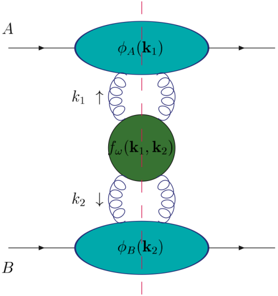
Allows for resummation of longitudinal phase space:

$$\alpha_s^n(Q^2) \int_0^Y dy_1 \int_0^{y_1} dy_2 \ldots \int_0^{y_{n-1}} dy_n \sim \frac{\left(\alpha_s(Q^2) Y\right)^n}{n!}$$

Multi-Regge kinematics

$$\sigma(Q_1, Q_2, s) = \int d^2\mathbf{k}_1 d^2\mathbf{k}_2 \, \phi_A(Q_1, \mathbf{k}_1) \, \phi_B(Q_2, \mathbf{k}_2) \, s^{\alpha_s(\mathbf{k}_1\mathbf{k}_2)\varphi(\mathbf{k}_1,\mathbf{k}_2)}$$

$$\simeq \left(\frac{s}{Q_1 Q_2}\right)^{\mathcal{C}\cdot\alpha_s(Q_1 Q_2)} \mathcal{G}\,(Q_1, Q_2)$$



$$\mathcal{C} \cdot \alpha_s(Q_1 Q_2) \simeq 0.5\,(\mathrm{LO}) - 0.3\,(\mathrm{NLO})$$

$\phi_{A,B}$ bring external scales Q_1, Q_2

Cut diagram $\rightarrow$ Multi-jet events (High multiplicity)
Uncut diagram $\rightarrow$ Diffractive events (Pomeron exchange)

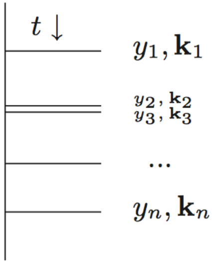
Beyond Multi-Regge kinematics

Regge limit in perturbative QCD: $s \gg t, Q^2$

Multi-Regge limit: Regge limit in all sub-channels [Only gluons]

Quasi-multi-Regge limit: one sub-channel with no limit [Quarks appear]
No ordering in rapidity in two nearby emitted particles:

$$Y \sim \log \frac{s}{s_0} \qquad y_i \gg y_{i-1} \qquad y_l \sim y_{l+1} \qquad \mathbf{k}_i^2 \sim \mathbf{k}_{i-1}^2 \sim Q^2$$



Fixes "factorization scale" s_0; $b \sim \log \frac{s_0'}{s_0}$:

$$\alpha_s^n(Q^2) \int_0^{Y'+b} dy_1 \int_0^{y_1} dy_2 \ldots \int_0^{y_{n-1}} dy_n \sim \underbrace{\frac{\left(\alpha_s(Q^2)Y'\right)^n}{n!}}_{\text{LO}} + \underbrace{b\frac{\alpha_s^n(Q^2)}{(n-1)!}Y'^{n-1}}_{\text{NLO}} + \cdots$$

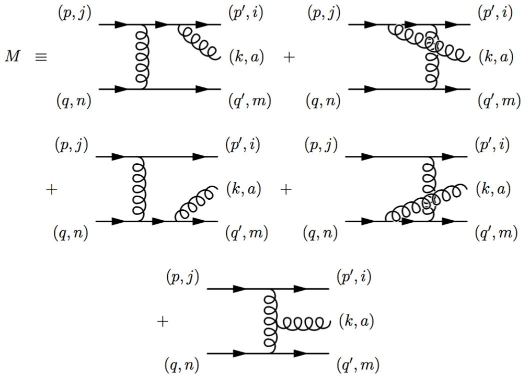
Effective Vertices in QCD & Gravity

"Graviton emission in Einstein-Hilbert gravity", arXiv:1112.4494
with M A VÁZQUEZ MOZO, E SERNA CAMPILLO (Salamanca)

- QCD reggeon-reggeon-gluon vertex
- Gravity reggeon-reggeon-graviton vertex
- Gravity $\simeq$ Gauge $\times$ Gauge?

 QCD $QQ' \to QQ'g$

$$s = (p+q)^2, \qquad t = (p-p')^2, \qquad s' = (p'+q')^2, \qquad t' = (q-q')^2.$$

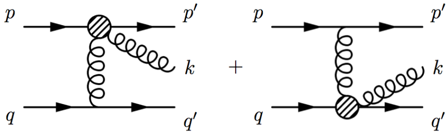


$$M = M_1 + M_2 + M_3 + M_4 + M_5$$

$$M_5 = \frac{t}{t-t'} M_5 + \frac{t'}{t'-t} M_5 \equiv M_5' + M_5''.$$

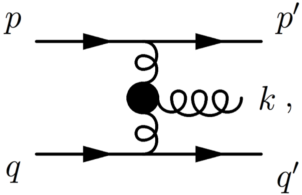
 QCD Lipatov Vertex

$$M_1 + M_2 + M_5' \qquad\qquad M_3 + M_4 + M_5''$$



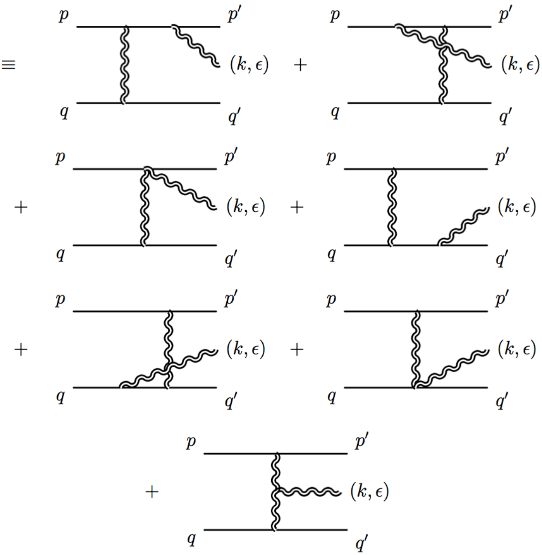
$$t = k_1^2, \qquad t' = k_2^2.$$

$$\begin{aligned} k_1^\mu &= \alpha_1 p^\mu + \beta_1 q^\mu + k_{1,\perp}^\mu, \\ k_2^\mu &= \alpha_2 p^\mu + \beta_2 q^\mu + k_{2,\perp}^\mu, \end{aligned} \qquad 1 \gg \alpha_1 \gg \alpha_2 = \frac{-t'}{s}, \qquad 1 \gg |\beta_2| \gg |\beta_1| = \frac{-t}{s},$$



$$, \; \Gamma^\nu_{\mu\sigma} = i g \eta_{\mu\sigma} \left[\left(\alpha_1 - \frac{2t}{s\beta_2} \right) p^\nu + \left(\beta_2 - \frac{2t'}{s\alpha_1} \right) q^\nu - (k_1 + k_2)^\nu_\perp \right]$$

$\mathcal{M} \equiv$



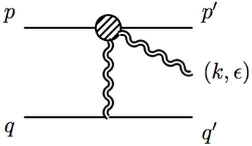
$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \mathcal{M}_4 + \mathcal{M}_5 + \mathcal{M}_6 + \mathcal{M}_7$$

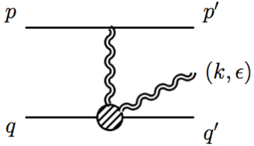
New Exact E-H Vertices

$$
\begin{aligned}
\mathcal{M} &= [\varepsilon\cdot(k_{\perp 1}+k_{\perp 2})][\varepsilon\cdot(k_{\perp 1}+k_{\perp 2})]A_{kk} + [\varepsilon\cdot(k_{1\perp}+k_{2\perp})](\varepsilon\cdot p)A_{kp}\\
&+ [\varepsilon\cdot(k_{1\perp}+k_{2\perp})](\varepsilon\cdot q)A_{kq} + (\varepsilon\cdot p)(\varepsilon\cdot p)A_{pp} + (\varepsilon\cdot q)(\varepsilon\cdot q)A_{qq}\\
&+ (\varepsilon\cdot p)(\varepsilon\cdot q)A_{pq}.
\end{aligned}
$$

A_{ii} in agreement with Steinmann constraints

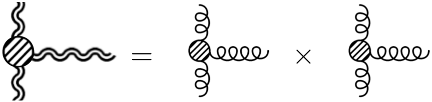
$$
\begin{aligned}
\mathcal{M} &= \left[\mathcal{M}_1+\mathcal{M}_2+\frac{t}{t-t'}(\mathcal{M}_3+\mathcal{M}_7)\right]_{\text{gauge invariant}} \to \mathcal{M}_\uparrow\\
&+ \left[\frac{t'}{t'-t}\mathcal{M}_3+\frac{t}{t-t'}\mathcal{M}_6\right] \to 0\\
&+ \left[\mathcal{M}_4+\mathcal{M}_5+\frac{t'}{t'-t}(\mathcal{M}_6+\mathcal{M}_7)\right]_{\text{gauge invariant}} \to \mathcal{M}_\downarrow
\end{aligned}
$$

$\mathcal{M}_\uparrow =$ 

$\mathcal{M}_\downarrow =$ 

$$t = k_1^2, \qquad t' = k_2^2.$$

$$\begin{aligned} k_1^\mu &= \alpha_1 p^\mu + \beta_1 q^\mu + k_{1,\perp}^\mu, \\ k_2^\mu &= \alpha_2 p^\mu + \beta_2 q^\mu + k_{2,\perp}^\mu, \end{aligned} \qquad 1 \gg \alpha_1 \gg \alpha_2 = \frac{-t'}{s}, \qquad 1 \gg |\beta_2| \gg |\beta_1| = \frac{-t}{s},$$



$$+ \; 4\beta_1\alpha_2 \left(\frac{p^\mu p^\nu}{\beta_2^2} + \frac{q^\mu q^\nu}{\alpha_1^2} + \frac{p^\mu q^\nu + q^\mu p^\nu}{\alpha_1 \beta_2} \right)$$

New subtraction term (from exact calculation) relevant for Steinman relations (no simultaneous singularities in overlapping channels)

- Agreement with Lipatov's calculation in MRK
- New exact Effective Vertices will simplify multi-loop calculations

Amplitudes & Monte Carlo Techniques

"Adjoint representation of BFKL Green function", arXiv:1112.4162
with G CHACHAMIS (PSI)

- Iteration directly in momentum-rapidity representation
- Access to exclusive information using Monte Carlo integration
- Valid for QCD (phenomenology), SUSY & gravity
- Both for forward (cut) and non-forward (uncut, pomeron)

Amplitudes & Monte Carlo Techniques

In dimension regularisation the equation reads

$$\omega f_\omega\left(\mathbf{k}_a, \mathbf{k}_b\right) \quad = \quad \delta^{(2+2\epsilon)}\left(\mathbf{k}_a-\mathbf{k}_b\right)+\int d^{2+2\epsilon} \mathbf{k}^{\prime}\, \mathcal{K}\left(\mathbf{k}_a, \mathbf{k}^{\prime}\right) f_\omega\left(\mathbf{k}^{\prime}, \mathbf{k}_b\right)$$

with kernel

$$\mathcal{K}\left(\mathbf{k}_a, \mathbf{k}\right)=2\, \omega^{(\epsilon)}\left(\mathbf{k}_a^2\right)\, \delta^{(2+2\epsilon)}\left(\mathbf{k}_a-\mathbf{k}\right)+\mathcal{K}_r\left(\mathbf{k}_a, \mathbf{k}\right)$$

integration in terms of emitted momenta:

$$\omega f_\omega\left(\mathbf{k}_a, \mathbf{k}_b\right)=\delta^{(2+2\epsilon)}\left(\mathbf{k}_a-\mathbf{k}_b\right)+\int d^{2+2\epsilon} \mathbf{k}\, 2\, \omega^{(\epsilon)}\left(\mathbf{k}_a^2\right) \delta^{(2+2\epsilon)}\left(\mathbf{k}_a-\mathbf{k}\right) f_\omega\left(\mathbf{k}, \mathbf{k}_b\right)$$

$$+\int d^{2+2\epsilon} \mathbf{k}\, \mathcal{K}_r^{(\epsilon)}\left(\mathbf{k}_a, \mathbf{k}_a+\mathbf{k}\right) f_\omega\left(\mathbf{k}_a+\mathbf{k}, \mathbf{k}_b\right)+\int d^2 \mathbf{k}\, \widetilde{\mathcal{K}}_r\left(\mathbf{k}_a, \mathbf{k}_a+\mathbf{k}\right) f_\omega\left(\mathbf{k}_a+\mathbf{k}, \mathbf{k}_b\right)$$

Introduce a phase space slicing parameter λ ...

The NLL BFKL equation then reads

$$
\begin{aligned}
\left(\omega-\omega_\lambda\left(\mathbf{k}_a\right)\right) f_\omega\left(\mathbf{k}_a, \mathbf{k}_b\right) &= \delta^{(2)}\left(\mathbf{k}_a-\mathbf{k}_b\right) \\
&+\int d^2\mathbf{k}\left(\frac{\Gamma_{\text{cusp}}\left(\mathbf{k}^2\right)}{\pi\mathbf{k}^2}\theta\left(\mathbf{k}^2-\lambda^2\right)+\widetilde{\mathcal{K}}_r\left(\mathbf{k}_a, \mathbf{k}_a+\mathbf{k}\right)\right) f_\omega\left(\mathbf{k}_a+\mathbf{k}, \mathbf{k}_b\right)
\end{aligned}
$$

where $\Gamma_{\text{cusp}}(\mathrm{X}) = \bar{\alpha}_s + \frac{\bar{\alpha}_s^2}{4}\left(\frac{4}{3}-\frac{\pi^2}{3}+\frac{5}{3}\frac{\beta_0}{N_c}-\frac{\beta_0}{N_c}\ln\frac{\mathrm{X}}{\mu^2}\right)$

In this notation:

$$
\omega_\lambda(\mathbf{q}) = -\int_{\lambda^2}^{\mathbf{q}^2}\frac{d\mathbf{k}^2}{\mathbf{k}^2}\Gamma_{\text{cusp}}\left(\mathbf{k}^2\right)+\text{constant}
$$

Ensuring the λ–independence of the equation...

... the NLL BFKL gluon Green's function reads

$$f(\mathbf{k}_a, \mathbf{k}_b, \mathrm{Y}) = e^{\omega_\lambda(\mathbf{k}_a)\mathrm{Y}} \left\{ \delta^{(2)}(\mathbf{k}_a - \mathbf{k}_b) \right.$$

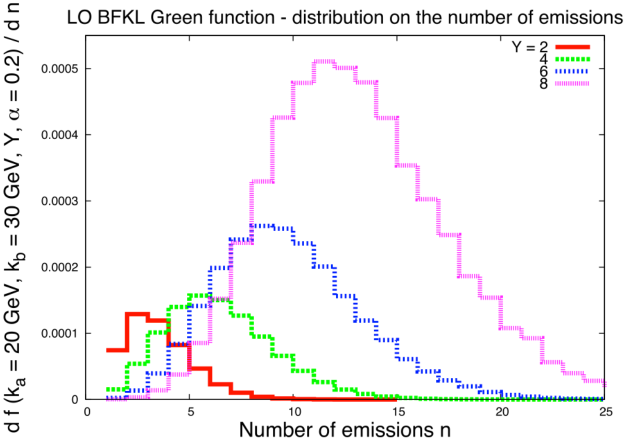
$$+ \sum_{n=1}^{\infty} \prod_{i=1}^{n} \int d^2\mathbf{k}_i \left[\frac{\theta\left(\mathbf{k}_i^2 - \lambda^2\right)}{\pi \mathbf{k}_i^2} \Gamma_{\mathrm{cusp}}\left(\mathbf{k}_i^2\right) + \widetilde{\mathcal{K}}_r \left(\mathbf{k}_a + \sum_{l=0}^{i-1} \mathbf{k}_l, \mathbf{k}_a + \sum_{l=1}^{i} \mathbf{k}_l \right) \right]$$

$$\left. \times \int_0^{y_{i-1}} dy_i \; e^{\left(\omega_\lambda\left(\mathbf{k}_a + \sum_{l=1}^{i} \mathbf{k}_l\right) - \omega_\lambda\left(\mathbf{k}_a + \sum_{l=1}^{i-1} \mathbf{k}_l\right)\right) y_i} \delta^{(2)} \left(\sum_{l=1}^{n} \mathbf{k}_l + \mathbf{k}_a - \mathbf{k}_b \right) \right\}$$

with $y_0 \equiv \mathrm{Y}$.

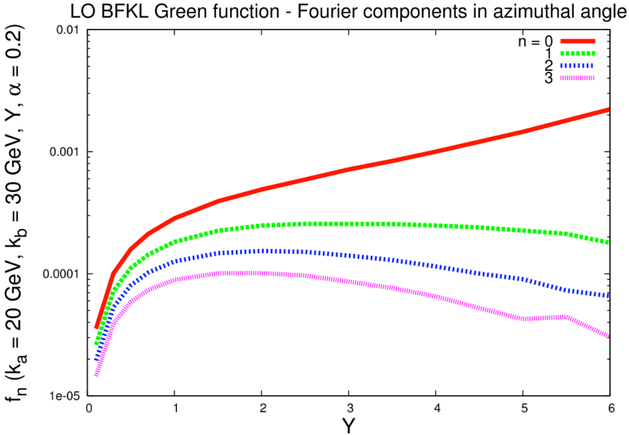
Exponentials are Reggeized gluon propagators
The rest are emission kernels (RRg,RRgg,RRqq ...)

How many gluons are really emitted for a given energy (rapidity) Y?



The area of these figures is the gluon Green function

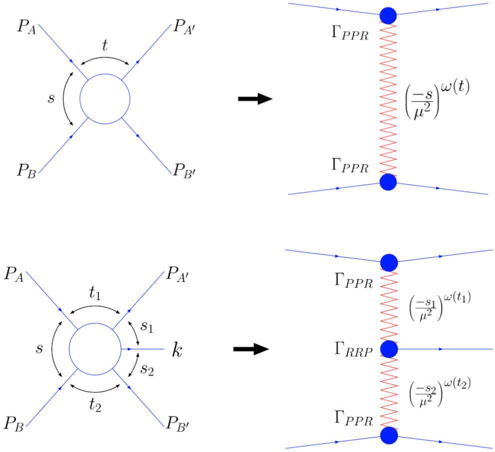
Let us explore $f_n(k_a, k_b, Y) \simeq \int d\theta f(k_a, k_b, \theta, Y) cos(n\theta)$



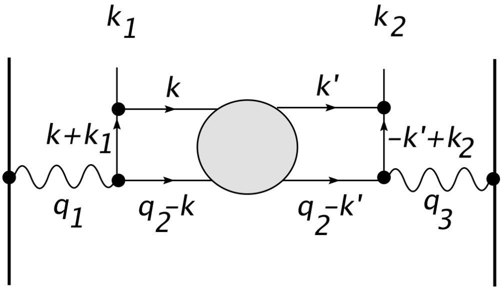
In the non-forward case n are conformal spins of $SL(2, C)$

For the singlet $n = 0$ dominates at high energy

4- & 5-point MHV ($N = 4$ SUSY, planar) amplitudes are Regge-exact.

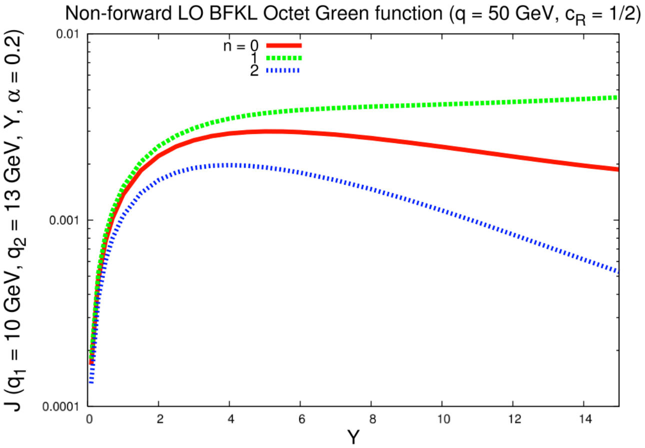


6-point MHV ($N = 4$ SUSY, planar) amplitudes is not Regge-exact
Regge limit based on non-forward octet BFKL gluon Green function



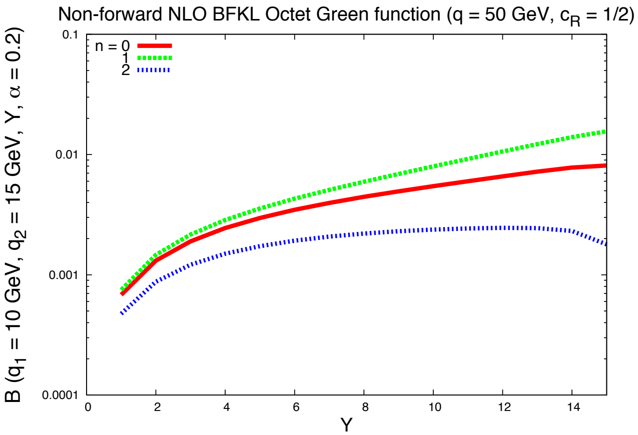
In Regge limit remainder function $R_6^{(2)}$ (not in BDS) found analytically.

Exploring $f_n(k_a, k_b, Y) \simeq \int d\theta f(k_a, k_b, \theta, Y) cos(n\theta)$



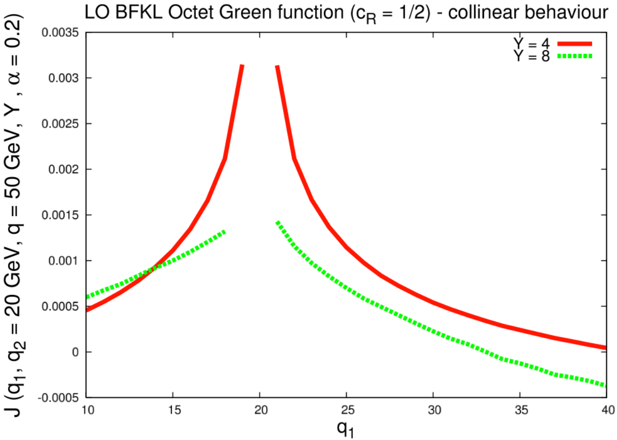
For the octet at LO now $n = 1$ dominates at high energy

Exploring $f_n(k_a, k_b, Y) \simeq \int d\theta f(k_a, k_b, \theta, Y) cos(n\theta)$



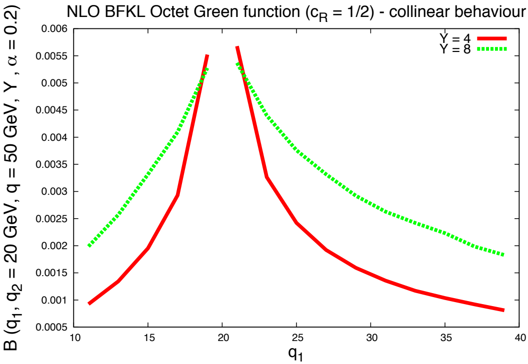
For the octet at NLO $n = 1$ dominates, but $n = 0$ also contributes

Collinear limit contains the information about anomalous dimensions



In the octet these are quite different to the singlet.

We are investigating this at NLO



BFKL & Spin Chains

"A hidden BFKL / $XXX_{\frac{-1}{2}}$ spin chain mapping", arXiv:1111.4553
with A ROMAGNONI (Madrid UAM/IFT)

- UV/IR diffusion of BFKL evolution is related to XXX spin chains?
- Take the forward limit (cut pomeron)
- Eliminate information on azimuthal angles, only keep anomalous dim
- Discretize in virtuality space $\rightarrow$ matrix representation of BFKL
- Connection to Beisert's representation of $XXX_{-\frac{1}{2}}$ spin chain?

BFKL & Spin Chains

(Minahan/Zarembo 2003)

Dilatation operator of planar $\mathcal{N} = 4$ SYM $\leftrightarrow$ XXX spin chain Hamiltonian

(Beisert 2004)

Hamiltonian for 1-loop AD of spin $S-1$ operators of sl(2) closed sector:

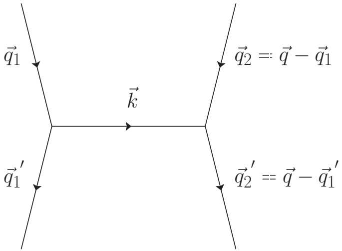
$$\mathcal{H}_{1,2}^{\mathrm{sl}(2)}\theta(S-N)(a_1^\dagger)^{N-1}(a_2^\dagger)^{S-N}|00\rangle \quad = \quad -\lambda\sum_{l=1}^{\infty}\left(\frac{(1-\delta_l^N)}{|l-N|} - (h(N-1)+h(S-N))\,\delta_l^N\right)\theta(S-l)(a_1^\dagger)^{l-1}(a_2^\dagger)^{S-l}|00\rangle$$

$$\lambda \;=\; \frac{g^2 N_c}{8\pi^2} \qquad h(N) \;=\; \sum_{l=1}^{N}\frac{1}{l} \;=\; \psi(N+1)-\psi(1)$$

$(a^\dagger)^n|0\rangle = \frac{1}{n!}(\mathcal{D})^n\Phi \quad \rightarrow \quad$ 1 site in a 1-dim lattice $\quad \rightarrow \quad$ n-th excited state

(with $a|0\rangle = 0$, $\left[a, a^\dagger\right] = 1$)

Excitations classified in $s = -\frac{1}{2}$ representation of sl(2)



"Reggeized Propagators" $\simeq$

$$g^2 N_c \delta^{(2)}\left(\vec{q}_1 - \vec{q}_1{}'\right)\delta^{(2)}\left(\vec{q}_2 - \vec{q}_2{}'\right) \times \left(\int d^2\vec{r}\frac{\vec{q}_1{}^2}{\vec{r}^{\,2}(\vec{q}_1-\vec{r})^2} + \int d^2\vec{r}\frac{\vec{q}_2{}^2}{\vec{r}^{\,2}(\vec{q}_2-\vec{r})^2}\right)$$

$$\text{“Emission”} \simeq \delta^{(2)}\left(\vec{q}_1 + \vec{q}_2 - \vec{q}_1{}' - \vec{q}_2{}'\right)\frac{g^2 N_c}{\vec{q}_1{}^2\vec{q}_2{}^2} \times \left(\frac{\vec{q}_1{}^2\vec{q}_2{}'^2 + \vec{q}_2{}^2\vec{q}_1{}'^2}{\vec{k}^2} - (\vec{q}_1 + \vec{q}_2)^2\right)$$

SL(2,$\mathbb{C}$) invariance (Lipatov 1986)

Forward case: $\vec{q} = 0$ $\rightarrow$ "Emission" $\simeq 2\frac{g^2 N_c}{\vec{k}^2} = 2\frac{g^2 N_c}{(\vec{q}_1 - \vec{q}_1{}')^2}$

Integrate over azimuthal angle between $\vec{q}_1$ & $\vec{q}_1{}'$, BFKL equation:

$$\frac{\partial \varphi(Q^2, Y)}{\alpha \partial Y} = \int_0^\infty \frac{dq^2}{|q^2 - Q^2|}\left\{\varphi(q^2, Y) - \frac{2\min(q^2, Q^2)}{q^2 + Q^2}\varphi(Q^2, Y)\right\}$$

Discretize virtuality space: $l^2 = n\,\Delta$, $Q^2 = N\,\Delta$, $\phi_n \equiv \varphi(l^2, Y)$

$$\frac{\partial \phi_N}{\alpha \partial Y} = \sum_{n=1}^{N-1} \frac{\phi_n}{N-n} + \sum_{n=N+1}^{\infty} \frac{\phi_n}{n-N} - 2h(N-1)\phi_N$$

$$\vec{\phi} \equiv (\phi_1, \dots, \phi_N)^t \qquad \vec{\phi}_\infty \equiv (\phi_1, \dots, \phi_N, \dots)^t \qquad \frac{\partial \vec{\phi}}{\alpha \partial Y} = \hat{\mathcal{H}}_N \cdot \vec{\phi}_\infty$$

$$\hat{\mathcal{H}}_N = \begin{pmatrix} -2h(0) & 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \cdots \\ 1 & -2h(1) & 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \cdots \\ \frac{1}{2} & 1 & -2h(2) & 1 & \frac{1}{2} & \frac{1}{3} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{1}{N-1} & \frac{1}{N-2} & \cdots & 1 & -2h(N-1) & 1 & \cdots \end{pmatrix}$$

$$\hat{\mathcal{H}}_N = -\sum_{n=N}^{\infty} \frac{\hat{\mathcal{S}}_{\rm IR}^n}{n} - \log\left(1 - \hat{\mathcal{S}}_{\rm IR}\right) - \log\left(1 - \hat{\mathcal{S}}_{\rm UV}\right) + \hat{\mathcal{G}}, \quad \left(\hat{\mathcal{G}}\right)_{i,j} = -2h(i-1)\delta_i^j$$

Shift operators to IR & UV: $\left(\hat{\mathcal{S}}_{\rm IR}\right)_{i,j} = \delta_i^{j+1} \quad \left(\hat{\mathcal{S}}_{\rm UV}\right)_{i,j} = \delta_{i+1}^j$

$XXX_{-\frac{1}{2}}$ Hamiltonian for $S = 2N - 1$ acts on two $(N-1)$-th excited states:

$$\mathcal{H}^{\mathbf{sl}(2)}_{1,2}\theta(N-1)(a_1^+)^{N-1}(a_2^+)^{N-1}|00\rangle \quad = \\ -\lambda\sum_{l=1}^{\infty}\left(\frac{(1-\delta_l^N)}{|l-N|} - 2h(N-1)\delta_l^N\right)\theta(2N-1-l)(a_1^+)^{l-1}(a_2^+)^{2N-1-l}|00\rangle$$

BFKL equation can be written as

$$\mathcal{H}^{\mathrm{BFKL}}\phi_N \quad = \quad \alpha\sum_{l=1}^{\infty}\left(\frac{(1-\delta_l^N)}{|l-N|} - 2h(N-1)\delta_l^N\right)\phi_l$$

Map discretized Green function & double harmonic oscillator?

$$\phi_l \quad \leftrightarrow \quad \theta(2N-1-l)(a_1^+)^{l-1}(a_2^+)^{2N-1-l}|00\rangle$$

It should be valid only in the $N \to \infty$ limit ...

Graviton Reggeization (no time - sorry!)

Coming soon:
All-orders resummation in SUGRA & Einstein-Hilbert
with J BARTELS (Hamburg), L N LIPATOV (St Petersburg)

$$
\begin{aligned}
\mathcal{A}_4^{\mathcal{N}=8} &= \left(\hat{\kappa}^2 s\right)\left(\frac{s}{-t}\right) \\
&\times \Bigg\{ 1 + \frac{1}{2}\left(\frac{\hat{\kappa}^2 s}{4\pi^2}\right)\left(\frac{-t}{s}\right)\Bigg[\underbrace{-\ln^2\left(\frac{s}{-t}\right)}_{\text{Double Log}} \underbrace{-\ln\left(\frac{-t}{\mu^2}\right)\ln\left(\frac{s}{-t}\right)}_{\text{Reggeization}} \\
&+ i\pi \left(\underbrace{\left(\frac{s}{-t}\right)\ln\left(\frac{-t}{\mu^2}\right)}_{\text{Eikonal}} + \ln\left(\frac{s}{-t}\right)\right) \underbrace{-\ln\left(\frac{-t}{\mu^2}\right)}_{\text{Vertex}} \\
&+ \sum_{n=1}^{\infty}\left(\ln\left(\frac{s}{-t}\right) - i\pi + \frac{1}{n+1}\ln\left(\frac{-t}{\mu^2}\right)\right)\frac{1}{n}\left(\frac{-t}{s}\right)^n \Bigg]\Bigg\}
\end{aligned}
$$

- Effective Vertices in QCD & Gravity
- Amplitudes & Monte Carlo Techniques
- BFKL & Spin Chains
- Graviton Reggeization

... still many things to be learnt from this limit