Common Trends in Gauge Fields, Strings and Integrable Systems

Thermal string probes in AdS and finite temperature Wilson loops

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Thermal brane probes

- **G. G., T. Harmark, A. Marini, N. A. Obers and M. Orselli**, "Heating up the BIon", JHEP **1106**, 058 (2011) [arXiv:1012.1494 [hep-th]]
- **G. G., T. Harmark, A. Marini, N. A. Obers and M. Orselli**, "Thermodynamics of the hot BIon", Nucl. Phys. B **851**, 462 (2011) [arXiv:1101.1297 [hep-th]]
- **G. G., T. Harmark, A. Marini, N. A. Obers and M. Orselli**, "Thermal string probes in AdS and finite temperature Wilson loops", [arXiv:1201.4862 [hep-th]]

Outline

Motivations and Overview

- Subject of talk: New method to describe D-brane and F-string probes in thermal backgrounds.
- - F-string probes $\Longrightarrow$ Wilson loops.
  - D3-brane/D5-brane probes $\Longrightarrow$ Wilson loops in large sym/antisym rep
  - D3-brane probes on S^3 in AdS_5 or S^5 $\Longrightarrow$ Large operators in gauge theory (Giant gravitons)
  - Quark-gluon plasma (*e.g.* energy loss of a heavy quark).
- Holographic duals to finite temperature gauge theory?
  $AdS_5 \Longrightarrow$ BH in AdS_5
- We need to put also the probe branes and strings at finite temperature
- $\Longrightarrow$ Method of this talk can be used to find new finite temperature effects in AdS/CFT

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Zero-temperature backgrounds

D-brane and string probes

1. The action for the string world-sheet is the usual Nambu-Goto action

$$S = \frac{1}{2\pi l_s^2} \int d\tau d\sigma \sqrt{\gamma}$$

$\gamma = \det \gamma_{ab}$.

2. Low energy effective theory for a single D-brane is DBI action (valid for $g_s \ll 1$)

$$I_{\mathrm{DBI}} = -T_{\mathrm{D_p}} \int_{\mathrm{w.v.}} d^p\sigma \sqrt{-\det(\gamma_{ab} + 2\pi l_s^2 F_{ab})} - T_{\mathrm{D_p}} \int_{\mathrm{w.v.}} e^F \wedge C_{(4)}$$

$$\gamma_{ab} = g_{\mu\nu} \partial_a X^\mu \partial_b X^\nu$$

γ_{ab} is the induced metric on the string or brane world-sheet, $g_{\mu\nu}$ the spacetime metric, F_{ab} the two-form field strength living on the D-brane, $C_{(4)}$ is the pull-back to the world-volume of the RR-four form gauge field of the background, T_{Dp} the Dp-brane tension and l_s the string length.

Finite-temperature backgrounds

What about the DBI and Nambu-Goto actions as probes of finite temperature backgrounds? Standard method employed in the last 14 years: Use the (Wick rotated) classical DBI or NG action in Euclidean background

- In this talk: We want to study the thermal generalization of the string solution dual to a rectangular Wilson loop that provides the energy of a quark anti-quark pair of $\mathcal{N} = 4$ SYM in 4D at finite temperature
- in previous papers: BIon solution $\Longrightarrow$ the first example of a solution of the DBI action that contains full non-linear dynamics of the DBI action $\Longrightarrow$ new phenomena introduced.
- Thermal fundamental string probes: a new term in the potential between static quarks which for sufficiently low temperatures is the leading correction to the Coulomb force potential.

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1. Motivations and Overview
2. Wilson loops at finite temperature: standard method
3. Wilson loop at finite temperature: new method
4. Physics of the rectangular Wilson loop
5. Summary and Outlook

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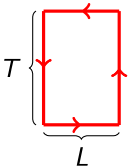
Wilson loops at finite temperature

- The Wilson loops in $\mathcal{N}=4$ SYM include a coupling to the scalar fields

$$W\left(\mathcal{C}\right)=\frac{1}{N}\mathrm{Tr}\mathcal{P}\exp\left[\oint\left(A_\mu\dot{x}^\mu+|\dot{x}|\theta^I\Phi_I\right)ds\right]$$

- The potential between very massive quarks is necessary to compute Wilson loops in the $U(N)$ theory. For a pair of antiparallel lines (rectangular Wilson loop)



$$\langle W\left(\mathcal{C}\right)\rangle\approx\exp\left[-TV(L,\lambda)\right]$$

- Explicit calculations at weak and strong couplings give

$$V(L,\lambda)=\begin{cases}-\frac{\lambda}{4\pi L}+\frac{\lambda^2}{8\pi^2 L}\ln\frac{T}{L}+\dots, & \lambda\ll 1;\\ -\frac{4\pi^2\sqrt{\lambda}}{\Gamma\left(\frac{1}{4}\right)^4 L}\left(1-\frac{1.3359\ldots}{\sqrt{\lambda}}+\dots\right), & \lambda\gg 1\end{cases}$$

Wilson loop at strong coupling

- In the AdS/CFT correspondence a Wilson loop is dual to a fundamental string (F-string) probe with its world-sheet extending into the bulk of Anti-de Sitter space (AdS) and ending at the location of the loop on the boundary of AdS [Rey, Yee; Maldacena(1998)].
- The action for the string world-sheet is the usual Nambu-Goto action

$$S = \frac{1}{2\pi l_s^2} \int d\tau d\sigma \sqrt{\gamma} \qquad \gamma_{ab} = g_{\mu\nu} \partial_a X^\mu \partial_b X^\nu$$

  $\gamma = \det \gamma_{ab}$, γ_{ab} is the induced metric on the string world-sheet and $g_{\mu\nu}$ the spacetime metric.
- Standard method at finite temperature: Wick rotated classical Nambu-Goto action in Euclidean background [Brandhuber et al.,Rey et al.(1998)].

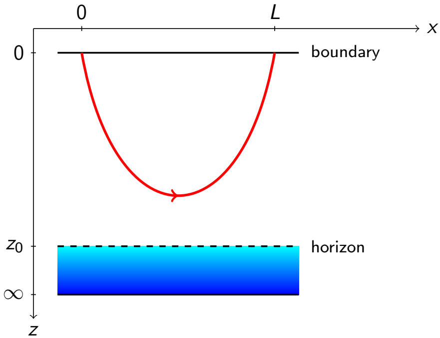
Background and induced metric

- We want to study the rectangular Wilson loop in $\mathcal{N} = 4$ SYM on $S^1 \times \mathbb{R}^3$, the S^1 meaning that it is at finite temperature.
- For the finite temperature case the background we want to probe is the AdS black hole in the Poincaré patch times a five-sphere.
- The metric of this background is

$$ds^2 = \frac{R^2}{z^2}(-fdt^2 + dx^2 + dy_1^2 + dy_2^2 + f^{-1}dz^2) + R^2 d\Omega_5^2 \ , \quad f(z) = 1 - \frac{z^4}{z_0^4}$$

  where R is the AdS radius, the boundary of AdS is at $z = 0$ and the event horizon is at $z = z_0$. The temperature of the black hole as measured by an asymptotic observer is $T = 1/(\pi z_0)$.

- Ansatz for the embedding of the F-string probe: $t = \tau \equiv \sigma^0$, $z = \sigma \equiv \sigma^1$, $x = x(\sigma)$
- Static string probe between the point charge Q at $(x, y_1, y_2) = (0, 0, 0)$ and the point charge $\bar{Q}$ at $(x, y_1, y_2) = (L, 0, 0)$ on the boundary of AdS at $z = 0$.



The induced metric and redshift factor for this embedding are

$$\gamma_{ab} d\sigma^a d\sigma^b = \frac{R^2}{\sigma^2} \left[-f d\tau^2 + \left(f^{-1} + x'^2 \right) d\sigma^2 \right] \ , \quad R_0(\sigma) = \frac{R}{\sigma} \sqrt{f(\sigma)}$$

- The redshift factor R_0 induces a local temperature $T_{\rm local} = T/R_0$ which is the temperature that the static string probe locally is subject to, T is the global temperature of the background space-time as measured by an asymptotic observer (Tolman Law (1930)).

- With this embedding the Nambu-Goto action becomes

$$S = \frac{R^2}{2\pi l_s^2}\int \frac{d\sigma}{\sigma^2}\sqrt{1+f(\sigma)x'(\sigma)^2}$$

- x is a cyclic variable, its conjugate momentum is conserved

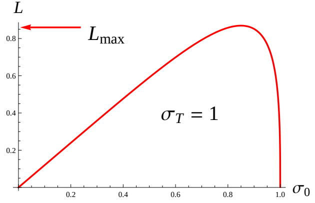
$$\frac{f(\sigma)x'(\sigma)}{\sigma^2\sqrt{1+f(\sigma)x'(\sigma)^2}} = \color{red}{const} = \frac{\sqrt{f(\sigma_0)}}{\sigma_0^2}$$

- x can now be expressed as a function of σ

$$x(\sigma) = \frac{\sigma_0\sqrt{\sigma_T^4-\sigma_0^4}}{\sigma_T^2}\int_0^{\sigma/\sigma_0}\frac{y^2 dy}{\sqrt{(1-y^4)\left(1-\frac{\sigma_0^4}{\sigma_T^4}y^4\right)}} \qquad \sigma_T = \frac{1}{\pi T}$$

The integration constant σ_0 can be related to L, the distance between the quark and the anti-quark

$$L = \frac{2\sigma_0\sqrt{\sigma_T^4-\sigma_0^4}}{\sigma_T^2}\int_0^1\frac{y^2 dy}{\sqrt{(1-y^4)\left(1-\frac{\sigma_0^4}{\sigma_T^4}y^4\right)}}$$
$$= \frac{(2\pi)^{3/2}\sigma_0\sqrt{\sigma_T^4-\sigma_0^4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{5}{4};\frac{\sigma_0^4}{\sigma_T^4}\right)}{\sigma_T^2\Gamma\left(\frac{1}{4}\right)^2}$$

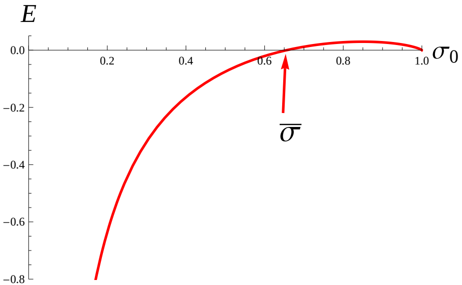


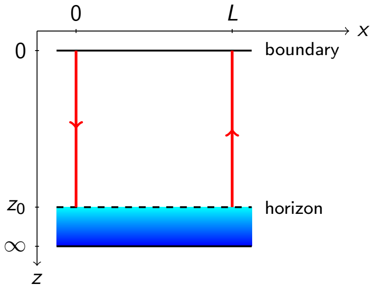
There is a maximum length L_{max}!

Energy

To obtain a finite result from the action S computed on the solution, subtract the (infinite) mass of the quarks which corresponds to two strings stretched between the boundary at $z = 0$ and the horizon at $z = z_0 \Longrightarrow \sigma = \sigma_T$

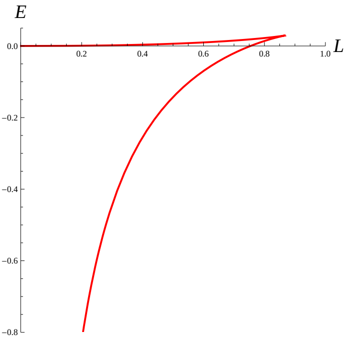
$$E = \frac{\sqrt{2\pi} R^2 \left(\sigma_0^4 - \sigma_T^4\right) \, {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{1}{4}; \frac{\sigma_0^4}{\sigma_T^4}\right)}{l_s^2 \sigma_0^4 \sigma_T^4 \Gamma\left(\frac{1}{4}\right)^2} + \frac{R^2}{\pi l_s^2 \sigma_T}$$

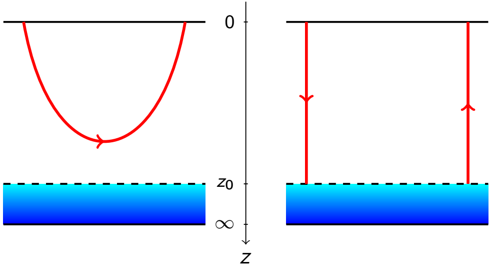




There is a critical $\overline{\sigma}$ at which the energy of the string configuration is the same as that of a pair of free quark and anti-quark with asymptotically zero force between them.
$\overline{\sigma}$ is reached before the L reaches its maximal value L_{max} value.

Static energy $E(L, T)$ between the "quark" and the "anti-quark" $\Longrightarrow$ eliminate σ_0 in terms of L and T.





Small temperatures $\Longrightarrow$ solve perturbatively for σ_0 as a function of L and get (the first term is the zero temperature Maldacena result)

$$E = -\frac{\sqrt{\lambda}}{L}\left(\frac{4\pi^2}{\Gamma\left(\frac{1}{4}\right)^4} + \frac{3}{160}(LT)^4\Gamma\left(\frac{1}{4}\right)^4 + O\left((LT)^8\right)\right)$$

Physical picture: for a given temperature T we encounter two region with different behavior. For $L \ll 1/T \Longrightarrow$ Coulomb like potential. For $L \gg 1/T$ the quarks become free due to screening by the thermal bath.

Outline

Thermal F-string probe

The new method is based on the blackfold approach, a general framework to describe black branes in the probe approximation [Emparan,Harmark,Niarchos,Obers, 2009].
Thermal F-string probe $\Longrightarrow$ SUGRA solution of k coincident black F-strings in type IIB SUGRA in 10D Minkowski space.
Energy-momentum tensor

$$T_{00} = Ar_0^6(7 + 6\sinh^2\alpha) \ , \qquad T_{11} = -Ar_0^6(1 + 6\sinh^2\alpha)$$

with $A = \Omega_7/(16\pi G)$. Temperature and charge

$$T = \frac{3}{2\pi r_0 \cosh\alpha} \ , \qquad Q = kT_{\mathrm{F1}} = 6Ar_0^6 \cosh\alpha \sinh\alpha$$

with $T_{\mathrm{F1}} = 1/(2\pi l_s^2)$

Equations of motion for any probe brane $\Longrightarrow$ Carter equation

$$\boxed{K_{ab}{}^{\rho} T^{ab} = J \cdot F^{\rho}}$$

T_{ab} is the world-sheet energy-momentum tensor for the string, $K_{ab}{}^{\rho}$ is the extrinsic curvature given by the embedding geometry and the right hand side, $J \cdot F^{\rho}$, represents possible external forces arising from having a charged brane that couples to an external field.

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- In the applications of the Nambu-Goto action as a probe of thermal backgrounds the string is treated as if the temperature of the background does not affect the physics on the string $\Longrightarrow$ the EM tensor that enters in the Carter equation is the same as in the zero-temperature case. Nambu-Goto EOM$\equiv$ Carter equation.
- DOFs living on the string that are "warmed up" by the temperature of the thermal background. The thermal background acts as a heat bath for the F-string probe, the system attains thermal equilibrium when the string probe gains the same temperature as the background.
- This changes the EM tensor of the string $\Longrightarrow$ change the EOMs for the probe string.
- For an AdS black hole this can make the classical NG action an increasingly inaccurate description as one approaches the event horizon since by the Tolman Law the local temperature for a static string probe is redshifted towards infinity.

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Action principle, equation of motion and solution

The action principle for a stationary blackfold uses the free energy as the action: the first law of thermodynamics is equivalent to the EOMs. The EOMs are the Carter equations.
Free energy for a SUGRA F-string probe in a general background with redshift factor R_0

$$\mathcal{F} = A \int dV_{(1)} R_0 r_0^6 (1 + 6 \sinh^2 \alpha)$$

Using the background defined before the free energy for the thermal F-string probe with the same ansatz for the embedding is

$$\mathcal{F} = A \left(\frac{3}{2\pi T} \right)^6 \int d\sigma \sqrt{1 + f(\sigma) x'(\sigma)^2}\, G(\sigma) \, , \quad G(\sigma) \equiv \frac{R^8}{\sigma^8} f(\sigma)^3 \frac{1 + 6 \sinh^2 \alpha(\sigma)}{\cosh^6 \alpha(\sigma)}$$

Varying with respect to $x(\sigma)$ this gives the EOM

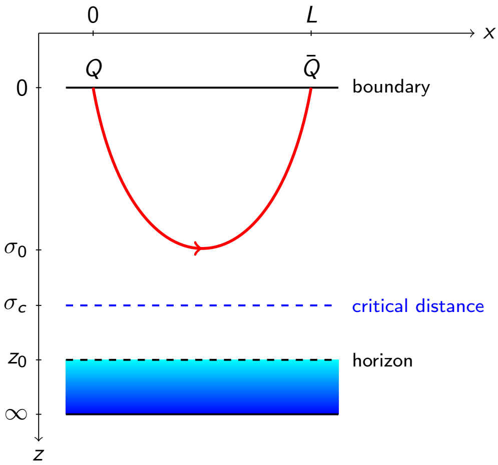
$$\left(\frac{f(\sigma) x'(\sigma)}{\sqrt{1 + f(\sigma) x'^2(\sigma)}} G(\sigma) \right)' = 0$$

Since $x' \to \infty$ for $\sigma \to \sigma_0 \Longrightarrow$ general solution

$$x'(\sigma) = \left(\frac{f(\sigma)^2 G(\sigma)^2}{f(\sigma_0) G(\sigma_0)^2} - f(\sigma) \right)^{-\frac{1}{2}}$$

The full solution goes from $(x, z) = (0, 0)$ to $(x, z) = (L/2, \sigma_0)$ and back to $(x, z) = (L, 0)$.

$$L = 2\int_0^{\sigma_0} d\sigma \left(\frac{f(\sigma)^2 G(\sigma)^2}{f(\sigma_0) G(\sigma_0)^2} - f(\sigma)\right)^{-\frac{1}{2}}$$



Q corresponds to the symmetric representation of k quarks.

Introduce the dimensionless parameter $\hat{\sigma} = \frac{\sigma}{\sigma_T} = \pi T \sigma$. In terms of this

$$LT = \frac{2}{\pi} \int_0^{\hat{\sigma}_0} d\hat{\sigma} \left(\frac{f(\hat{\sigma})^2 H(\hat{\sigma})^2}{f(\hat{\sigma}_0) H(\hat{\sigma}_0)^2} - f(\hat{\sigma}) \right)^{-\frac{1}{2}}$$

with $f(\hat{\sigma}) = 1 - \hat{\sigma}^4$, $\hat{\sigma}_0 = \pi T \sigma_0$,

$$H(\hat{\sigma}) = \frac{f(\hat{\sigma})^3}{\hat{\sigma}^8} \frac{1 + 6\sinh^2\alpha(\hat{\sigma})}{\cosh^6\alpha(\hat{\sigma})} \ , \quad \kappa \equiv \frac{2^5 k T_{\mathrm{F1}}}{3^7 A R^6} = \frac{f(\hat{\sigma})^3}{\hat{\sigma}^6} \frac{\sinh\alpha(\hat{\sigma})}{\cosh^5\alpha(\hat{\sigma})}$$

- LT depends only on the dimensionless quantities κ and $\hat{\sigma}_0$.
- The equation for κ enforces the charge conservation and can be used to find $\alpha(\hat{\sigma})$.
- In terms of the gauge theory variables k, λ and N, κ is

$$\boxed{\kappa = \frac{2^7}{3^6} \frac{k\sqrt{\lambda}}{N^2}}$$

There is another type of solution. These solutions trivially solve the EOM with $x'(\sigma) = 0$ and represent a pair of free quark and anti-quark with asymptotically zero force between them.

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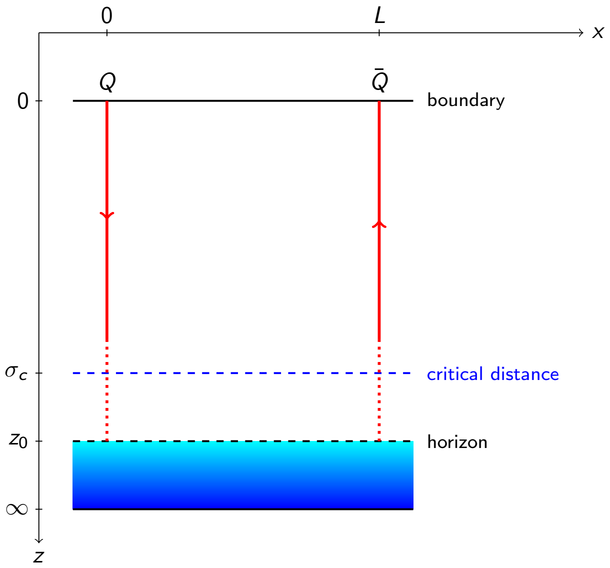


Critical distance

From the equation for κ and the fact that

$$\frac{\sinh\alpha}{\cosh^5\alpha} \le \frac{2^4}{5^{5/2}}$$

for a given κ the equation can only be satisfied provided

$$\hat{\sigma} \le \hat{\sigma}_c\ , \qquad \text{with} \qquad \hat{\sigma}_c^2 = \sqrt{1 + \frac{5^{5/3}}{2^{14/3}}\kappa^{\frac{2}{3}}} - \frac{5^{5/6}}{2^{7/3}}\kappa^{\frac{1}{3}}$$

- $\hat{\sigma}_c < 1 \implies$ one reaches the critical distance $\hat{\sigma}_c$ before reaching the horizon.
- Physical interpretation: the F-string probe is in thermal equilibrium with the background, the SUGRA F-string has a maximal temperature for a given k , the Tolman law means that the local temperature goes to infinity approaching the black hole $\implies$ critical distance.
- This is a qualitatively new effect which means that the probe description breaks down beyond the critical distance $\hat{\sigma}_c$.

details

Outline

LT for small $\hat{\sigma}_0 = \pi T\sigma_0$

Length L between the Q and $\bar{Q}$ charges for small $\hat{\sigma}_0 = \pi T\sigma_0$ for $\kappa \ll 1$

$$LT = \frac{2\sqrt{2\pi}}{\Gamma(\frac{1}{4})^2}\hat{\sigma}_0 + \left(\frac{\sqrt{2\pi}}{3\Gamma(\frac{1}{4})^2} - \frac{1}{6}\right)\sqrt{\kappa}\hat{\sigma}_0^4 - \frac{2\sqrt{2\pi}}{5\Gamma(\frac{1}{4})^2}\hat{\sigma}_0^5 + \mathcal{O}(\hat{\sigma}_0^7)$$

- The leading term is the same as for an extremal F-string probing the zero temperature AdS background.
- Appears the term proportional to $\sqrt{\kappa}\hat{\sigma}_0^4$ in the expansion. In general the expansion of LT has terms of the form $\kappa^{m/2}\hat{\sigma}_0^{3m+4n+1}$.
- The terms with κ marks an important departure from the results obtained using the extremal F-string to probe the AdS black hole background, which here would correspond to setting $\kappa = 0$, thus ignoring the thermal excitations of the F-string.

Free energy of the rectangular Wilson loop

Divergent free energy of the string extended between the Q and $\bar{Q}$ charges

$$\mathcal{F} = \sqrt{\lambda} k T \int_0^{\hat{\sigma}_0} \frac{d\hat{\sigma}}{\hat{\sigma}^2} \left(\tanh \alpha + \frac{1}{6 \cosh \alpha \sinh \alpha} \right) \sqrt{1 + f x'^2}$$

Subtract the "bare" free energy of the two Polyakov loops: same ultraviolet divergences. To do this in a controlled way, introduce an infrared cutoff at $z = \sigma_{\text{cut}}$ near the event horizon with $\sigma_{\text{cut}} \leq \sigma_c$

$$\mathcal{F}_{\text{sub}} = \sqrt{\lambda} k T \int_0^{\hat{\sigma}_{\text{cut}}} \frac{d\hat{\sigma}}{\hat{\sigma}^2} \left(\tanh \alpha + \frac{1}{6 \cosh \alpha \sinh \alpha} \right)$$

The difference between the free energies of the rectangular Wilson and the two Polyakov loops

$$\Delta \mathcal{F} = \mathcal{F}_{\text{loop}} - 2 \mathcal{F}_{\text{charge}}$$

$$\mathcal{F}_{\text{loop}}(T, L, k, \lambda) = \sqrt{\lambda} k T \left[-\frac{1}{\hat{\sigma}_0} + \int_0^{\hat{\sigma}_0} \frac{d\hat{\sigma}}{\hat{\sigma}^2} \left(\tanh \alpha + \frac{1}{6 \cosh \alpha \sinh \alpha} \right) \left(\sqrt{1 + f x'^2} - 1 \right) \right]$$

$$\mathcal{F}_{\text{charge}}(T, k, \lambda, \sigma_{\text{cut}}) = -\frac{1}{2} \sqrt{\lambda} k T \left(\frac{1}{\hat{\sigma}_{\text{cut}}} + \int_0^{\hat{\sigma}_{\text{cut}}} \frac{d\hat{\sigma}}{\hat{\sigma}^2} \left(1 - \tanh \alpha - \frac{1}{6 \cosh \alpha \sinh \alpha} \right) \right)$$

- $\mathcal{F}_{\text{loop}} \Longrightarrow$ regularized free energy of the rectangular Wilson loop.
- $\mathcal{F}_{\text{charge}} \Longrightarrow$ regularized free energy for each of the Polyakov loops.

Free energy of rectangular Wilson loop for small LT

Regularized free energy of the rectangular Wilson loop in powers of LT

$$\mathcal{F}_{\rm loop} = -\frac{\sqrt{\lambda}k}{L}\left(\frac{4\pi^2}{\Gamma(\frac{1}{4})^4} + \frac{\Gamma(\frac{1}{4})^4}{96}\sqrt{\kappa}(LT)^3 + \frac{3\Gamma(\frac{1}{4})^4}{160}(LT)^4 + \cdots\right)$$

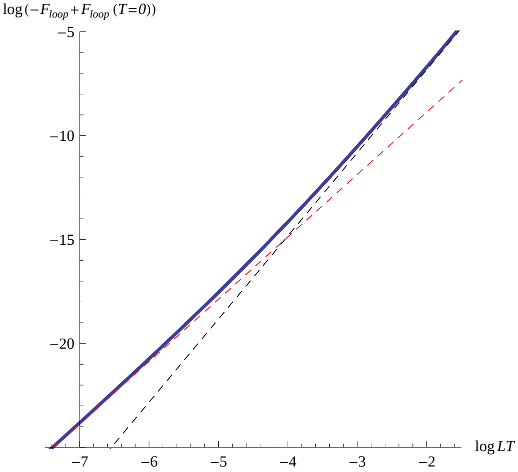
- The leading term is the well-known Coulomb force potential found by probing $AdS_5 \times S^5$ in the Poincaré patch with an extremal F-string [Rey,Yee;Maldacena].
- For $\kappa = 0$ we regain the results found previously in the literature by using an extremal F-string probe in the AdS black hole background. [Brandhuber et al.,Rey et al.].
- The higher order term suppressed as $\sqrt{\kappa}(LT)^3$ compared to the leading term is a new term that appears as consequence of including the thermal excitations of the F-string probe, and demanding that the F-string probe is in thermal equilibrium with the background!

- Compare this to the term suppressed as $(LT)^4$. Since $\kappa \propto k\sqrt{\lambda}/N^2$ we see that the new term is the dominant correction to the Coulomb force potential provided

$$LT \ll \frac{\sqrt{k}\lambda^{1/4}}{N}$$

- For sufficiently small temperatures the leading correction to the Coulomb force potential can only be seen by including the thermal excitations of the F-string probe.
- This is a very striking consequence of our new thermal F-string probe technique which means that one misses important physical effects by ignoring the thermal excitations of the F-string probe and merely using the extremal F-string as probe of a thermal background.

As an illustration of this new effect, we have depicted the value $\mathcal{F}_{loop} - \mathcal{F}_{loop}(T = 0)$ as a function of LT in a logarithmic scale. We compare this line with the $(LT)^3$ (red dashed line) and the $(LT)^4$ (black dashed line) terms



For each solution $\Longrightarrow$ $\exists!$ σ_0 that measures the closest proximity of the F-string probe to the black hole horizon, at the point $(z, x) = (\sigma_0, L/2)$.

Plot LT as a function of $\hat{\sigma}_0 = \pi T \sigma_0$ for various values of κ

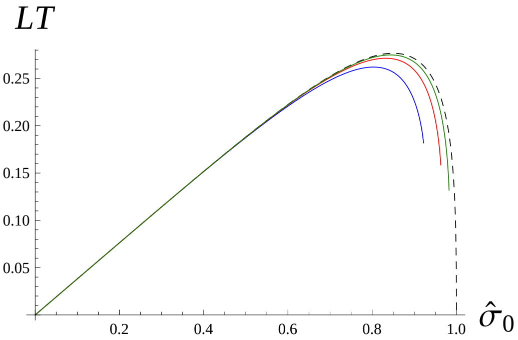


Figure: $\kappa = 0.01$ (blue line), $\kappa = 0.001$ (red line), $\kappa = 0.0001$ (green line), extremal probe (dashed line). For a given κ there exists solutions in the range from $\hat{\sigma}_0 = 0$ to $\hat{\sigma}_0 = \hat{\sigma}_c(\kappa)$. $\hat{\sigma}_c(\kappa)$ is the critical distance from the event horizon for which the F-string probe reaches the maximal temperature. The quantity LT always has a maximum value $(LT)_{\max}$ for a given value of κ.

For a given κ there exists solutions in the range from $\hat{\sigma}_0 = 0$ to $\hat{\sigma}_0 = \hat{\sigma}_c(\kappa)$. $\hat{\sigma}_c(\kappa)$ is the critical distance from the event horizon for which the F-string probe reaches the maximal temperature. For a given κ we write $(LT)_c$ as the value of LT for the solution with $\hat{\sigma}_0 = \hat{\sigma}_c(\kappa)$. Consider what happens for given values of κ and LT

- For $LT < (LT)_c$ we find one solution.
- For $(LT)_c \leq LT < (LT)_{\max} \Rightarrow$ two branches of solutions. Then for $LT = (LT)_{\max}$ we reach the maximal possible value of LT for a given value of κ.
- For $LT > (LT)_{\max}$ we do not have any available solutions corresponding to the F-string probe stretching between Q and $\bar{Q}$.
- The extremal F-string probe corresponding to $\kappa = 0$ is also plotted

$$LT|_{\kappa=0} = \frac{2\sqrt{2\pi}}{\Gamma\left(\frac{1}{4}\right)^2}\hat{\sigma}_0\sqrt{1-\hat{\sigma}_0^4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{5}{4};\hat{\sigma}_0^4\right)$$

- This matches with the result found in [Brandhuberet al., Rey et al.] using the NG string probe in the AdS Poincaré black hole background.
- LT also exhibits a maximum in this case: Its value is $(LT)_{\max} \simeq 0.277$ and it is reached for $\hat{\sigma}_0 \simeq 0.85$.

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- The SUGRA F-string probe induces both qualitative and quantitative differences.
- Qualitative difference: while for the extremal probe one has two solutions available for given LT, for the thermal F-string probe there is only one solution when LT is sufficiently small, *i.e.* for $LT < (LT)_c$. Taking into account the validity of the probe approximation the values of LT for which there are two solutions are even more restricted.
- Quantitative difference in the dependence of LT on $\hat{\sigma}_0$ as well as the value of $(LT)_{\max}$. The value of $(LT)_{\max}$ receives a $\mathcal{O}(\sqrt{\kappa})$ correction for small κ, which is thus a $1/N$ effect that is missed by the less accurate extremal F-string probe.

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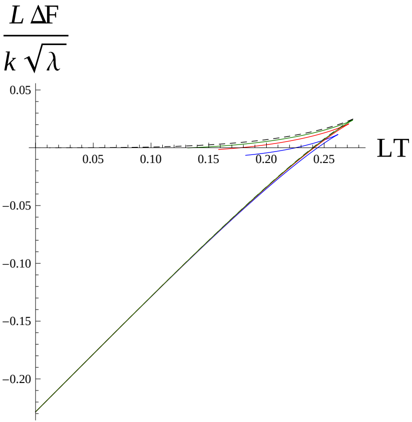
Free energy of the Wilson loop and its comparison to that of two Polyakov loops (two straight strings stretching towards the horizon from the charge Q and $\bar{Q}$, $\exists$ for all values of LT).

If $\Delta\mathcal{F}$ is less then zero the Wilson loop is thermodynamically preferred in the canonical ensemble.



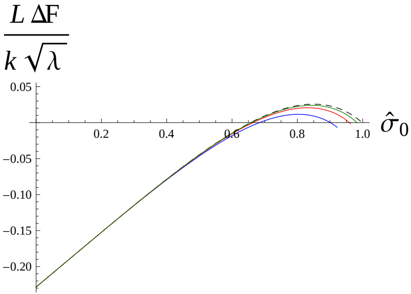
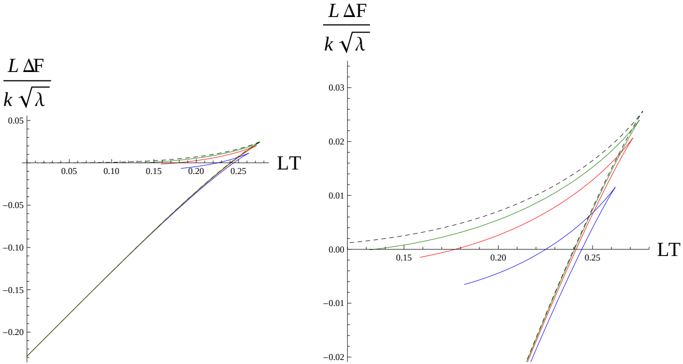


Figure: $L\Delta\mathcal{F}/(k\sqrt{\lambda})$ as a function of LT (left) and $\hat{\sigma}_0$ (right) for various values of κ: $\kappa = 0.01$ (blue line), $\kappa = 0.001$ (red line), $\kappa = 0.0001$ (green line), extremal probe (dashed line)



- Intersections with the horizontal axis: where the thermodynamically preferred configuration is that of 2 straight lines $\Longrightarrow$ the phase transition can be interpreted as Debye screening of the Q-$\bar{Q}$ pair.
- As one moves to non-zero values of $\kappa \Longrightarrow$ the onset moves to higher values of $LT \Longrightarrow$ using the thermal F-string probe the pair is less easily screened compared to the extremal F-string probe.
- The cusp of the swallow tail is where one reaches $(LT)_{\max}$ (corresponding to $\hat{\sigma}_{0,\max}$)
- The lower part of the swallow tail corresponds to $0 < \hat{\sigma}_0 < \hat{\sigma}_{0,\max}$.
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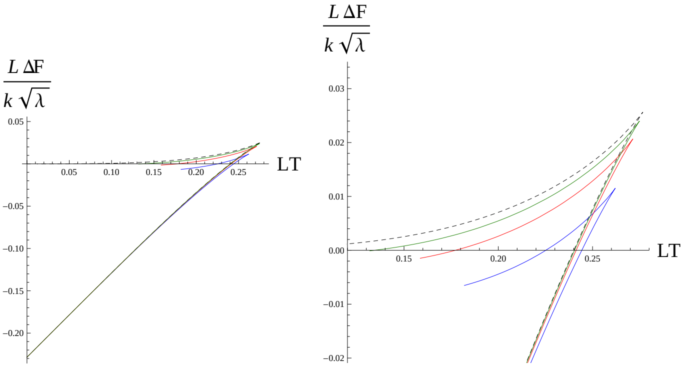
- The plots show that one never reaches $(LT)_{\max}$, since $\hat{\sigma}_0|_{\Delta\mathcal{F}=0} < \hat{\sigma}_{0,\max}$, so that the quarks are screened before reaching this distance.
- The intersection of the curves with the vertical axis corresponds to the Coulomb potential term.



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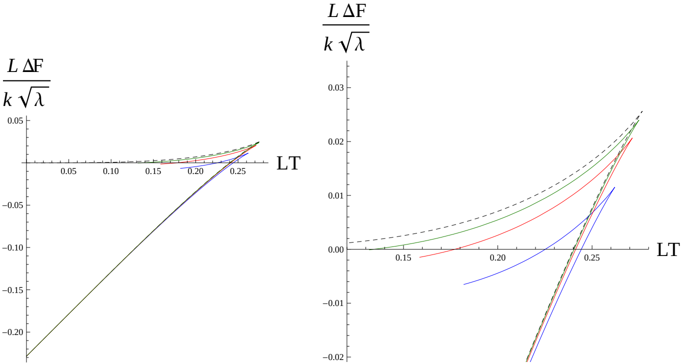

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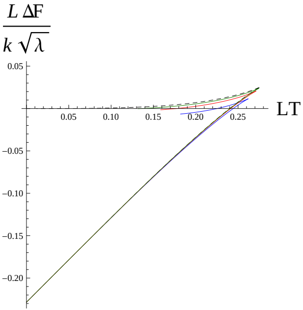

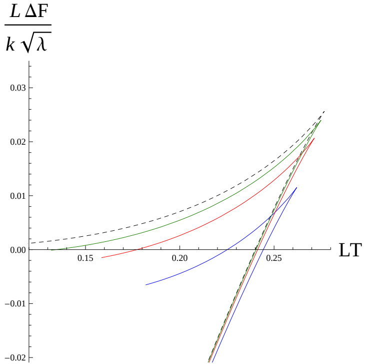

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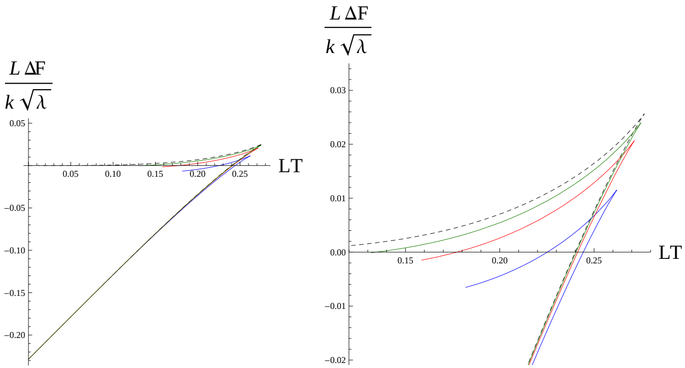

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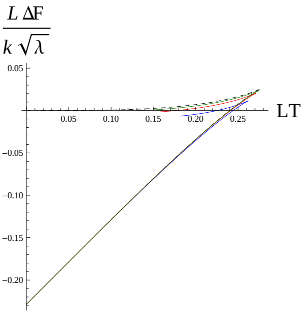

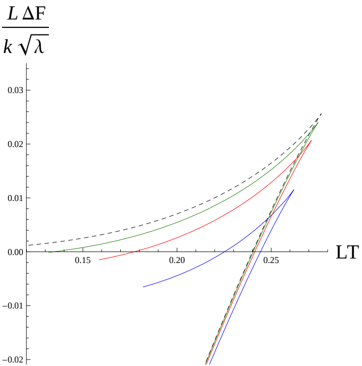

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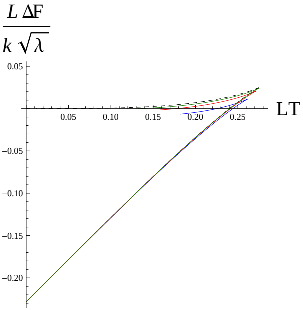

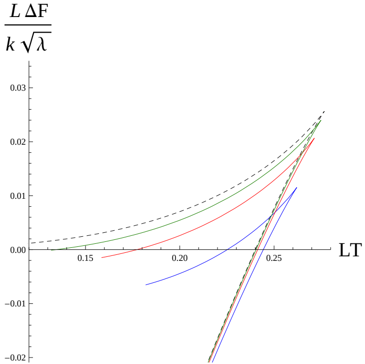

- Intersections with the horizontal axis: where the thermodynamically preferred configuration is that of 2 straight lines $\Longrightarrow$ the phase transition can be interpreted as Debye screening of the Q-$\bar{Q}$ pair.
- As one moves to non-zero values of $\kappa \Longrightarrow$ the onset moves to higher values of $LT \Longrightarrow$ using the thermal F-string probe the pair is less easily screened compared to the extremal F-string probe.
- The cusp of the swallow tail is where one reaches $(LT)_{\max}$ (corresponding to $\hat{\sigma}_{0,\max}$)
- The lower part of the swallow tail corresponds to $0 < \hat{\sigma}_0 < \hat{\sigma}_{0,\max}$.
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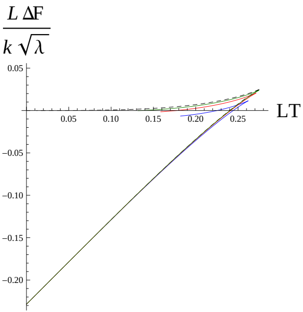

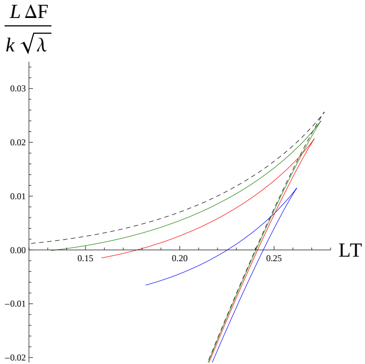

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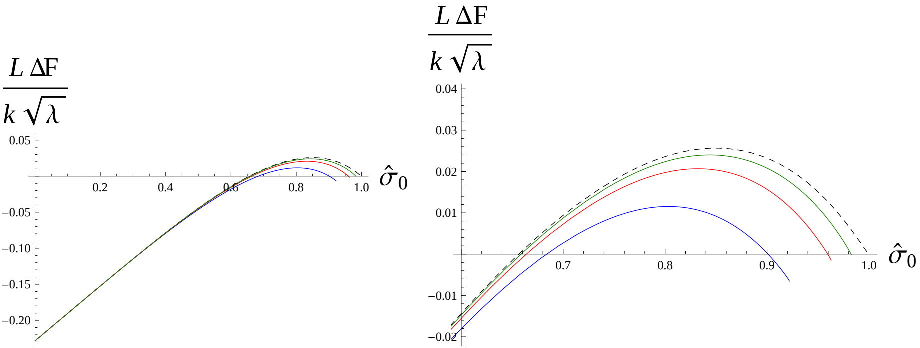
The extremal F-string probe yields for the free energy difference,

$$\Delta\mathcal{F}|_{\kappa=0} = \sqrt{\lambda}kT\left[1+\frac{\sqrt{2}\pi^{3/2}\left(\hat{\sigma}_0^4-1\right)}{\hat{\sigma}_0\Gamma\left(\frac{1}{4}\right)^2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{1}{4};\hat{\sigma}_0^4\right)\right],$$



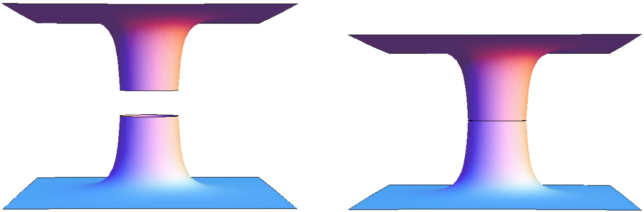
- Intersections with the $\hat{\sigma}_0$ axis: where the thermodynamically preferred config. is that of 2 straight lines.
- For larger values of κ one finds that the onset moves to larger values of $\hat{\sigma}_0$.
- Using the thermal probe string, the pair is less easily screened and it sets in later as κ increases.
- $\hat{\sigma}_0|_{\Delta\mathcal{F}=0}$ = onset for quark screening $\Longrightarrow$ $\hat{\sigma}_0|_{\Delta\mathcal{F}=0} < \hat{\sigma}_{0,\max} < \hat{\sigma}_c$ is true for any value of κ.

- The SUGRA F-string probe induces both qualitative and quantitative differences.
- For the onset of quark screening we find for small κ

$$(LT)|_{\Delta\mathcal{F}=0} \simeq 0.240038 + 0.0379706\sqrt{\kappa}$$

  a $\sqrt{\kappa}$ correction to the result obtained using the extremal F-string probe.
- For a finite value of LT, the quantitative difference consists in having $\sqrt{\kappa}$ corrections to the extremal probe results.
- For the gauge theory this corresponds to a $\sqrt{k}\lambda^{1/4}/N$ correction to the previously obtained results for the potential between the charges, as well as to the critical value $(LT)|_{\Delta\mathcal{F}=0}$ of LT where the charges become screened.

Heating up the BIon



- The thermal effects observed for thermal string probes in AdS, bear some resemblance to those found for the thermal BIon [G.G., Harmark, Marini, Obers, Orselli,2011]: a thermal D3-F1 brane probe in hot flat space. Finite temperature generalization of the BIon configuration consisting of a D-brane and a parallel anti-D-brane connected by a wormhole with F-string charge.
- Qualitative difference: for a given separation between the D-brane and anti-D-brane there are either one or three phases available, while at zero temperature there are two phases.
- Analysis of the free energy of the finite temperature generalization of the BIon shows a similar swallow tail structure as found above.

Outline

Summary and Outlook

- A method that provides a new tool for thermal probes in finite temperature backgrounds. It keeps into account the fact that for extended probes the internal degrees of freedom should be in thermal equilibrium with the background.
- The application of this method to the study of Wilson loops in finite temperature $\mathcal{N} = 4$ SYM using thermal F-string probes in the AdS black hole background, confirms that there are both qualitative and quantitative differences.

Outlook

- Examine other holographic aspects of quark-gluon plasma physics, such as the energy loss of a heavy quark moving through the plasma.
- Revisit the thermal generalization of the Wilson loop in higher representations, in the regime where it involves a "blown-up" version: D3-brane (symmetric representation) or a D5-brane (antisymmetric representation). This may be interesting in view of the discrepancies between gauge theory and gravity results found for the symmetric representation: SUGRA black D3-brane probe in the AdS black hole background [Hartnoll,Kumar;G.G.,Karczmarek,Semenoff].

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Validity of the probe approximation

- The probe approximation means that we should be able to piece the string probe together out of small pieces of SUGRA F-strings in hot flat ten-dimensional space-time. For this to work, the local length scale of the string probe, *i.e.* the "thickness" of the string, should be much smaller than the length scale of each of the pieces of F-string in hot flat space.
- We want to consider the branch of the SUGRA F-string connected to the extremal F-string $\Longrightarrow$ the thickness of the F-string is the charge radius

$$r_c = r_0(\cosh\alpha \sinh\alpha)^{1/6}$$

- Using this one finds $r_c \propto \kappa^{1/6} R$.
- In the AdS black hole background R and $z_0 \propto 1/T$ are the two length scales in the metric. Since we need to require that the sizes of the pieces of F-string should be smaller than the length scales of the metric we need that $r_c \ll R$ and $r_c \ll 1/T$. This gives the conditions

$$\boxed{\kappa \ll 1\,, \quad RT \ll \kappa^{-\frac{1}{6}}}$$

- Given that $\kappa \ll 1$, the condition $RT \ll \kappa^{-1/6}$ is easily fulfilled as it gives a weak upper bound on how high the asymptotic temperature T can be.
- The condition $\kappa \ll 1 \Longrightarrow$ critical distance $\hat{\sigma}_c$ is very close to the horizon: $1 - \hat{\sigma}_c \propto \kappa^{1/3}$ for small κ.
- The regime of validity of the SUGRA F-string is $1 \ll k \ll N$ and $\lambda^2 k \gg N^2$. Instead, the probe approximation requires $\kappa \ll 1$. We see that this is consistent with the regime of validity of the SUGRA F-string provided that $\lambda \ll N^2$. This translates to $g_s \ll N$ which is trivially satisfied since we assume weak string coupling $g_s \ll 1$.
- A relevant quantity is the local temperature $T_{\mathrm{local}} = T/R_0$. Need that the local temperature varies over sufficiently large length scales such that we can regard the probe locally as a SUGRA F-string in hot flat space of temperature $T_{\mathrm{local}} \Longrightarrow$ we need in particular that $r_c T'_{\mathrm{local}}(\sigma)/T \ll 1$, *i.e.* that the variation of the local temperature is small over the length scale of the F-string probe.

$$\boxed{\frac{r_c T'_{\mathrm{local}}}{T} = \frac{r_c}{R\sqrt{f}} + \frac{2r_c(\pi T \sigma)^4}{Rf^{3/2}} \ll 1}$$

- As a check we can see that for $\sigma \gg z_0$ the condition $r_c T'_{\rm local}/T \ll 1$ reduces to $r_c \ll R$.
- Considering instead the near horizon region $z_0 - \sigma \ll z_0$ we find in the $\kappa \to 0$ limit that $r_c T'_{\rm local}/T \ll 1$ requires $z_0 - \sigma \gg \kappa^{1/9} z_0$. When we reach

$$\boxed{z_0 - \sigma \sim \kappa^{1/9} z_0}$$

the probe approximation breaks down. For $\sigma \simeq \sigma_c$ the probe approximation is not valid, indeed $r_c T'_{\rm local}/T \sim \kappa^{-1/3}$.
- The probe approximation in fact breaks down before we reach the critical distance $\sigma = \sigma_c$ where the local temperature reaches the maximal possible temperature of the SUGRA F-string.
- We should also consider the extrinsic curvature of the solution. Since we already took into account the variation of the background, the easiest way to analyze the extrinsic curvature is to neglect the derivatives of the metric. Doing this we should require $z_0 - \sigma \gg \kappa^{1/3} z_0$. However, this is already guaranteed by the stronger condition $z_0 - \sigma \gg \kappa^{1/9} z_0$ found above.

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