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Superstrings on non-maximally supersymmetric AdS superbackgrounds and integrability

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Abstract

This talk is based on results obtained in collaboration with Linus Wulff, Arkady Tseytlin, Kostya Zarembo and Alessandra Cagnazzo and published in arXiv: 1009.3498, 1104.1793, 1111.4197. I would like to discuss peculiarities of Green–Schwarz superstrings on non-maximally supersymmetric backgrounds and, in particular the problem of how the lack of supersymmetry affects (or deforms) the integrable structure of the string sigma–models in these cases.

1 Plan of the talk

- Explain what is the **difference from the maximally supersymmetric** $AdS_5 \times S^5$ **case** which makes the structure of the less supersymmetric sigma–models much more complicated - **presence** on the string worldsheet of fermionic degrees of freedom associated with broken supersymmetries (**broken susy fermionic modes**)

- Consider general problem: what are generic conditions on the structure of a GS sigma-model which make it classically integrable, i.e. in which superbackgrounds one manages to construct a zero-curvature Lax connection.

- Examples: The form of the Lax connection of the superstrings on $AdS_4 \times CP^3$ and $AdS_2 \times S^2 \times T^6$ in which the conventional super–coset Lax connection gets deformed by the presence of non–susy fermionic degrees of freedom

- Open issues which arise due to the presence of these non-susy fermionic modes.

2 Green–Schwarz superstrings and supercosets

We will consider the Green–Schwarz formulation of string theory, since it is a suitable framework for the description of strings in backgrounds supported by Ramond–Ramond fluxes, which are the most of the AdS backgrounds of interest in connection with different instances of AdS/CFT correspondence.

In the GS formulation a string propagates in a superspace parametrized by the bosonic coordinates $X^M(\xi^i)$ coordinates and fermionic (Grassmann–odd) coordinates $\Theta(\xi)$

$$X^M(\xi^i) \quad M = 0, 1, ..., 9 \qquad \Theta^\alpha(\xi) \quad \alpha = 1, ..., 32$$

where ξ^i $(i = 0, 1)$ are two coordinates parametrizing the string worldsheet. We are interested in type II superstrings in D=10, so the vector index takes ten values and the number of the spinorial coordinates is 32.

Let me remind that in the GS formulation there is a local worldsheet fermionic symmetry which transforms Θ and X as follows

$$\delta_\kappa \Theta = P_{16} \kappa(\xi), \qquad \delta_\kappa X^M = i \Theta \Gamma^M \delta_\kappa \Theta,$$

where $\kappa(\xi)$ is the spinorial parameter and P_{16} is a projector matrix of rank 16. Therefore using the kappa–symmetry one can gauge away 16 components of Θ. The remaining 16 fermions are physical degrees of freedom of the string.

Note also that the supersymmetry transformations in the bulk act on Θ^α as local shifts (or diffeomorphisms)

$$\delta \Theta^\alpha = \epsilon^\alpha(X, \Theta).$$

If some of the supersymmetries are broken, the number N of independent parameters ϵ is less than 32, and Θ split accordingly into two sets of variables

$$\Theta = (\vartheta, \upsilon) \qquad \vartheta = \mathcal{P}_N \Theta, \qquad \upsilon = (1 - \mathcal{P}_N)\Theta$$

N variables ϑ are associated with N unbroken supersymmetries, and $N - 32$ modes υ correspond to the broken ones. There is always a projector of rank N which is associated with this splitting. The structure of this projector is determined by the form of the RR fluxes in the bulk.

The generic form of the GS action is know in arbitrary supergravity background (Grisaru et. al. 1985), but its explicit form has been derived only for some specific backgrounds including the supercoset spaces with Z_4–grading.

3 $AdS_5 \times S^5$ superstring

Let us now consider the well–known maximally supersymmetric example of the string propagating in $AdS_5 \times S^5$ superspace. So all the Θ are associated with bulk supersymmetries which generate the isometry group $SU(2,2|4)$. The unique superspace with this isometry which has 32 fermionic directions and whose bosonic subspace is $AdS_5 \times S^5$ is the supercoset $\frac{SU(2,2|4)}{SO(1,4)\times SO(5)}$. The geometry of this supercoset space satisfies the type IIB supergravity constraints and hence is the proper superspace description of the $AdS_5 \times S^5$ background. This geometry is characterized by the $SU(2,2|4)$ Cartan forms pulled–back on the supercoset $g(X,\Theta)$. The Z_4–grading decomposition of the Cartan form looks as follows

$$g^{-1}dg(X,\Theta) = \Omega_0(X,\Theta) + E_2(X,\Theta) + E_1(X,\Theta) + E_3(X,\Theta)\,, \tag{1}$$

where the indices (0,1,2,3) label the Z_4–grading of the Cartan form components.

Ω_0 has Z_4 grading zero and describes the connection of the coset space taking value in the stability algebra $SO(1,4) \times SO(5)$. E_2 has grading 2 (i.e. it changes the sign under the action of the Z_4 automorphism).

E_2 is associated with 10 generators of the bosonic translations in the coset superspace and have the geometrical meaning of the bosonic (super)vielbeins.

$$E_2 = E^A(X,\Theta)P_A.$$

E_1 and E_3 have, respectively, Z_4–grading 1 and 3. They are associated with the 32 supersymmetry generators and have the meaning of fermionic supervielbeins

$$E_1 + E_3 = E^\alpha(X,\Theta)Q_\alpha.$$

The pull–backs on the string worldsheet of the Cartan form components E_2, E_1 and E_3 are used to construct the GS superstring sigma–model action on the coset superspace. It has the following form:

$$S = \frac{1}{2}\int d^2\xi\, \mathrm{Str}(\sqrt{-h}h^{ij}E_{i2}E_{j2} + \varepsilon^{ij}E_{i1}E_{j3}) = \frac{1}{2}\int \mathrm{Str}(*E_2E_2 + E_1E_3)\,.$$

Note that this form of the action is generic for all the supercoset sigma–models with Z_4–grading.

The equations of motion can be written as the conservation of the $SU(2,2|4)$ Noether current

$$d * J = \partial_i\left[g\left(E_2^i - \frac{1}{2}\varepsilon^{ij}(E_{j1} - E_{j3})\right)g^{-1}\right] = 0.$$

The integrability of this model is unraveled by constructing a zero–curvature Lax connection using the prescription by Bena, Polchinski and Roiban in 2003. The Lax connection has the following form

$$L_{coset} = \frac{1}{2}\Omega_0 + (1+\alpha_1)E_2 + \alpha_2 * E_2 + \beta_1E_1 + \beta_2E_3\,, \qquad dL - LL = 0\,.$$

Numerical parameters $\alpha_{1,2}$ and $\beta_{1,2}$ are certain functions of a single spectral parameter z:

$$\alpha_1 = \frac{2\mathsf{z}^2}{1-\mathsf{z}^2}, \qquad \alpha_2 = \frac{2\mathsf{z}}{1-\mathsf{z}^2} \qquad \beta_1 = -\frac{i\mathsf{z}}{\sqrt{\mathsf{z}^2-1}}, \qquad \beta_2 = \frac{i}{\sqrt{\mathsf{z}^2-1}}.$$

Z_4–invariance of the Lax connection

$$\mathcal{Z}(L_{coset}(\mathsf{z})) = \mathcal{Z}^{-1}L_{coset}(\frac{1}{\mathsf{z}})\,\mathcal{Z} = L_{coset}(\mathsf{z}),$$

where $\mathcal{Z}$ is the Z_4–automorphism transfomation which acts on the graded components of the Cartan form as follows.

$$\mathcal{Z}^{-1}\Omega_0\,\mathcal{Z} = \Omega_0, \qquad \mathcal{Z}^{-1}E_2\,\mathcal{Z} = -E_2, \qquad \mathcal{Z}^{-1}E_1\,\mathcal{Z} = iE_1 \qquad \mathcal{Z}^{-1}E_3\,\mathcal{Z} = -iE_3.$$

This property is important for studying the Bethe equations, Y– and T–systems etc., as have been explained e.g. in the talk by Sebastian Leurent.

4 Less supersymmetric cases

Let us now pass to less supersymmetric cases

- Type IIA string on $AdS_4 \times CP^3$. Preserves 24 (of 32) supersymmetries governed by the isometry group $OSp(6|4)$. The relevant supercoset space is $\frac{OSp(6|4)}{U(3)\times SO(1,3)}$. It has 24 Grassmann–odd direction and thus cannot accomodate additional 8 fermionic string degrees of freedom corresponding to broken supersymmetries. Complete type IIA superspace with $OSp(6|4)$ isometry and 32 fermionic directions was constructed in [7]. It is not a supercoset and has much more complicated structure. So the complete GS superstring action on $AdS_4 \times CP^3$ contains contributions of non–coset fermions, which make it much more complicated. Using kappa-symmetry one can gauge away the non–coset fermions and reduce the GS action to the supercoset sigma–model. But such a gauge is inadmissible for some classical string solutions, e.g. when the string moves only on AdS_4.

- Type IIB strings on $AdS_3 \times S^3 \times T^4$, which preserve 16 (of 32) supersymmetries generating $PSU(1,1|2) \times PSU(1,1|2)$ isometries.

- Type IIA and IIB strings on $AdS_2 \times S^2 \times T^6$ preserving 8 (of 32) supersymmetries. Their underlying symmetry is $PSU(1,1|2) \times U(1)^6$. The corresponding supercoset is $\frac{PSU(1,1|2)}{SO(1,1)\times U(1)}$. It has only 8 fermionic directions and therefore describes only a subsector of the complete string theory. 24 non–susy fermionic modes of the string are again not captured by the coset model. Moreover the 16 parameters of kappa–symmetry are not enough to get rid of them. At least 8 fermionic modes of this kind are physical degrees of freedom of the string. **An important issue is that in the Lagrangian they do not decouple from the supercoset degrees of freedom.**

- Note that the $AdS_4 \times CP^3$ and $AdS_2 \times S^2 \times T^6$ case are in some sense dual (or complimentary) to each other, since the 32 fermions are split into 8+24, though their meaning gets interchanged. This “duality” is of great help and allows to study the two cases simultaneously.

Now the natural question arises what happens with integrability when the non-coset degrees of freedom are taken into account. The supercoset Lax connection will not have zero curvature anymore, since the string equations of motion will acquire contributions that involve the non–coset fermions. One should therefore find a deformation of the supercoset Lax connection containing the v–fermions which would make its curvature vanish.

5 General conditions on the superbackground to be integrable

One may pose this problem in a more general context and formulate the question as follows:

- Suppose we have a Green–Schwarz string action in a superbackground which has some isometry and which may preserve or not some supersymmetry. **What are the conditions this superbackground should satisfy to make the string sigma–model integrable.**

- First of all, when all the fermionic modes are switched off, **the purely bosonic part of the sigma–model should be integrable.** This requirement already heavily restricts the choice of the bosonic backgrounds, mainly to the class of the bosonic symmetric spaces G/H, to which the AdS spaces belong.

- Then, since **we do not, a priori, assume that the extension of these symmetric spaces to superspaces should have the supercoset structure**, we do not have supercoset Cartan superforms at our disposal to construct a Lax connection. We should find other objects in the theory from which the Lax connection can be constructed. **The natural objects are the conserved currents of the isometries.**

To simplify the analysis we start with the GS action truncated to the second order in fermions. This action was derived in an arbitrary background by Cvetic et. al. in 1999. In a type IIA background with Ramond–Ramond fluxes it has the following form

$$S = \int d^2\xi \left[\sqrt{-h} h^{ij} \left(\frac{1}{2} e_i^A e_{jA}(X) + i e_i^A \, \Theta \Gamma_A \mathcal{D}_j \Theta \right) - i \varepsilon^{ij} e_i^A \, \Theta \Gamma_A \Gamma^{11} \, \mathcal{D}_j \Theta \right] ,$$

where

$$\mathcal{D}\Theta = (\nabla - \frac{1}{8} e^A \, \not{F} \Gamma_A) \Theta \, .$$

and

$$\not{F} = e^{\phi} \left\{ -\frac{1}{2} \Gamma^{AB} \Gamma_{11} F_{AB} + \frac{1}{4!} \Gamma^{ABCD} F_{ABCD} \right\} .$$

Note that when fluxes take appropriate values, like in the cases which we have discussed, $\not{F} = \mathcal{P}_N$ becomes the projector which singles out the number of preserved supersymmetries and corresponding fermionic modes.

From this action we can derive the Noether currents associated with the bulk isometries. For this we should consider the variation of S under the following transformations of the coordinates X^M and Θ. Under the bosonic isometries and under supersymmetry (if preserved)

$$\delta X^M e_M^A(X) = K^A(X) + i\Theta\Gamma^A\Xi(X), \qquad \delta\Theta = \Theta\Gamma^{AB}[K_A, K_B] + \Xi(X)\,, \qquad \mathcal{D}\Xi = 0$$

where $K_A(X)$ are the Killing vectors associated with the bosonic isometries and $\Xi(X)$ are the Killing spinors associated with the supersymmetries. Note that under the bosonic isometry the spinor Θ is transformed by a corresponding induced transformation of the stability subgroup H of G.

The bosonic and supersymmetry Noether currents have the following structure

$$J_{\mathcal{B}}(X,\Theta) = j(X) + J_1^A(X,\Theta)K_A + J_2^{AB}(X,\Theta)[K_A,K_B]\,,$$
$$J_{susy} = J^\alpha(X,\Theta)\Xi_\alpha(X)\,.$$

More precisely the currents look as follows:

$$j(X) = e^A(X)K_A(X)\,,$$
$$J_1^A = i\Theta\Gamma^A\mathcal{D}\Theta - \frac{i}{8}e^B\,\Theta\Gamma^A\not{F}\Gamma_B\Theta + i\Theta\Gamma^A\Gamma_{11} * \mathcal{D}\Theta - \frac{i}{8} * e^B\,\Theta\Gamma^A\Gamma_{11}\not{F}\Gamma_B\Theta\,,$$
$$J_2^{AB} = -\frac{i}{4}e^C\,\Theta\Gamma^{AB}{}_C\Theta + \frac{i}{4} * e^C\,\Theta\Gamma^{AB}{}_C\Gamma_{11}\Theta\,,$$
$$J_{susy} = \frac{i}{2R}(e^A\,\Theta\Gamma_A\Xi - *e^A\,\Theta\Gamma_A\Gamma_{11}\Xi)\,.$$

The Lax connection is constructed with the pieces of the currents taken with arbitrary coefficients. Then one fixes the values of these coefficients by trying to satisfy the zero–curvature condition using the current conservation and algebraic properties of the Killing vectors and spinors. Then one arrives at the following expression for the Lax connection

$$\mathcal{L} = \alpha_1\, j + \alpha_2 * J_{\mathcal{B}} + \alpha_2^2\, J_2 + \alpha_1\alpha_2 * J_2 - \alpha_2(\beta_1 J_{susy} - \beta_2 * J_{susy}) + \mathcal{O}(\Theta^3),$$

where $\alpha_{1,2}(z)$ and $\beta_{1,2}(z)$ turn out to be the same as in the supercoset case. Note that the parameters α_1 and α_2 are already fixed by the zero–curvature condition of the purely bosonic limit of the Lax connection. The Lax connection has zero curvature if the currents satisfy the following relations:

$$dJ_{susy} = -2(J_{\mathcal{B}} \wedge J_{susy} + J_{susy} \wedge J_{\mathcal{B}})\,,$$
$$(\nabla J_2^{AB} - J_1^A \wedge j^B)[K_A,K_B] = -J_{susy} \wedge J_{susy}\,.$$

It should be stressed that these relations do not follow from the equations of motion. They are additional conditions (imposed by the zero–curvature requirement) on the form of the background and, in particular, on the values of the RR fluxes. Note that their form reminds the Maurer–Cartan equations of the supercoset case. These conditions on the Noether currents restrict possible superbackgrounds in which the superstring sigma–model is integrable. All the supersymmetric backgrounds listed in the Introduction are of this kind.

If a background is not supersymmetric the components J_1 and J_2 of the bosonic isometry current should satisfy the relation

$$(\nabla J_2^{AB} - J_1^A \wedge j^B)[K_A,K_B] = 0\,. \qquad (2)$$

Then, a natural question arises whether there exist integrable string sigma–models in superbackgrounds in which target–space supersymmetry is completely broken. The obvious examples to check are non–supersymmetric $AdS \times M$ backgrounds which are obtained from the supersymmetric ones by changing the sign of a supporting gauge field flux. It turns out, however, that for these backgrounds the condition (2) is not satisfied. The only example of the integrable superstring in non–supersymmetric background which is known so far is a $D = 4$, $\mathcal{N} = 2$ superstring in AdS_4 with completely broken supersymmetry [1]. This model is a consistent truncation to $D = 4$ of the $AdS_4 \times CP^3$ superstring in which only 8 non–supercoset fermionic modes are kept. The Lax connection for this model has been constructed in [1] to all orders in the string fermionic modes in a particular kappa–symmetry gauge of [14].

Now one would like to explicitly see how the supercoset Lax connection, e.g. in the $AdS_4 \times CP^3$ and $AdS_2 \times S^2 \times T^6$ cases gets modified by the non–coset fermions.

It can be shown that this Lax connection (when the non-coset fermions are put to zero) is related by an isometry group transformation to the supercoset Lax connection. Applying this gauge transformation to the Lax connection containing the υ–fermions we can get the corrections to the Bena–Polchinski–Roiban Lax connection. For the $AdS_4 \times CP^3$ and $AdS_2 \times S^2 \times T^6$ cases (due to their similarity) the Lax connection has a similar form which looks as follows (to all orders in the coset fermions ϑ and to the second order in the υ–fermions)

$$\begin{aligned}
L &= \frac{1}{2}\Omega_0(X,\vartheta) + (1+\alpha_1)E^A P_A + \alpha_2 * E^A P_A + Q(\beta_2 + \beta_1\Gamma_{11})E(X,\vartheta) \qquad \text{[Supercoset Lax]} \\
&\quad -\frac{i\alpha_2}{R}Q\gamma_\star\left[*(E^A + 2i\upsilon\Gamma^A E)\,\Gamma_A V\upsilon - (E^A + 2i\upsilon\Gamma^A E)\,\Gamma_A\Gamma_{11}V\upsilon\right] \\
&\quad -\frac{i\alpha_2}{R}Q\gamma_\star\left[i(\upsilon\Gamma^A\Gamma_{11}E)\,\Gamma_A V\upsilon + i(\upsilon\Gamma^A E)\,\Gamma_A\Gamma_{11}V\upsilon\right] \\
&\quad + \alpha_2(2i\upsilon\Gamma^A * E + i\upsilon\Gamma^A * \nabla\upsilon - \frac{2}{R} * E^B\,\upsilon\Gamma^A\mathcal{P}\gamma_\star\Gamma_B\upsilon)\,P_A \\
&\quad + \alpha_2(2i\upsilon\Gamma^A\Gamma_{11}E + i\upsilon\Gamma^A\Gamma_{11}\nabla\upsilon - \frac{2}{R}E^B\,\upsilon\Gamma^A\Gamma_{11}\mathcal{P}\gamma_\star\Gamma_B\upsilon)\,P_A \\
&\quad + \frac{i\alpha_2}{8}(*E^C\,\upsilon\Gamma_C{}^{DE}V^2\upsilon - E^C\,\upsilon\Gamma_C{}^{DE}\Gamma_{11}V^2\upsilon)\,R_{DE}{}^{AB}\,M_{AB}\,,
\end{aligned}$$

where $\mathcal{P}$ is the supersymmetry projector

$$V = \beta_2 + \beta_1\Gamma_{11}, \qquad \gamma_\star = \gamma^5\ (AdS_4 \times CP^3), \qquad \gamma_\star = \Gamma^{23}\Gamma^{11}\ (AdS_2 \times S^2 \times T^6)$$

Such a generalization of the supercoset Lax connection preserves [15] its important property to be Z_4–invariant.

Note that in the $AdS_2 \times S^2 \times T^6$ case, for a generic classical configuration of the string we can use kappa–symmetry and completely remove all the super–coset fermions ϑ. Then the Lax connection reduces to

$$\begin{aligned}
L &= \frac{1}{2}\Omega_0(X) + (1+\alpha_1)e^a(x)P_a + \alpha_2 * e^a(x)P_a \\
&\quad -\frac{i\alpha_2}{R}Q\Gamma^{23}\Gamma^{11}\left[*e^{a'}(y)\,\Gamma_{a'}(\beta_2 + \beta_1\Gamma_{11})\upsilon - e^{a'}(y)\,\Gamma_{a'}(\beta_1 + \beta_2\Gamma_{11})\upsilon\right] \\
&\quad + i\alpha_2(\upsilon\Gamma^a * \nabla\upsilon + \upsilon\Gamma^a\Gamma_{11}\nabla\upsilon)\,P_a \\
&\quad + \frac{i\alpha_2}{8}\,\upsilon\left(*e^A(X)\Gamma_A{}^{cd}(1+\alpha_1-\alpha_2\Gamma_{11}) - e^A(X)\,\Gamma_A{}^{cd}\Gamma_{11}(1+\alpha_1-\alpha_2\Gamma_{11})\right)\upsilon R_{cd}{}^{ab}\,M_{ab}\,,
\end{aligned}$$

where $a, b = 0, 1, 2, 3$ are tangent space indices of $AdS_2 \times S^2$ parametrized by the coordinates x^m and $a', b' = 4, 5, 6, 7$ label T^6 parametrized by $y^{a'}$.

6 Discussion

We have reviewed some features of superstring theories in AdS superbackgrounds with a particular emphasis on the cases with less supersymmetries in which the string has physical fermionic modes associated with broken symmetries that do not admit the supercoset interpretation. The presence of the non–coset fermions makes the proof of the classical integrability of the theory much more difficult.

Having at hand Lax connections which include the contribution of noncoset worldsheet modes one can

- construct the monodromy matrix and address the problem of whether and how these modes modify the algebraic curve and Bethe ansatz equations for the full superstring theory in these backgrounds.
- analogous problem seems still remain open in the case of integrable 2d supersymmetric sigma–models on S^N and CP^N.
- Or maybe the contributions containing the non-coset modes factor out somehow.
- In the GS action these modes do not decouple from the supercoset sector. And it is not obvious how this decoupling might occur. Moreover the two sectors cannot completely decouple from each other since they are linked together by the Virasoro constraint.
- One should mention that there is an alternative description of a string in the $AdS_2 \times S^2 \times T^6$ background by Berkovits with collaborators []. It is called the hybrid model because it is the direct sum of an $AdS_2 \times S^2$ supercoset model and a free RNS string on T^6. So in the hybrid model the non-coset modes are completely decoupled (modulo the Virasoro constraint). However, the relation of the hybrid model to the GS superstring is still unclear. If exists, it should involve highly non–trivial and probably non–local transformation of fields from one model to another. It would be interesting to explore this relation.
- It would be also interesting to understand what are the CFT dual counterparts of the worldsheet degrees of freedom associated with broken supersymmetries.

References

[1] Sorokin D., Wulff L. // Evidence for the classical integrability of the complete AdS(4) x CP(3) superstring. JHEP. 2010. V. 11. P. 143.

[2] Sorokin D. et. al. // Superstrings in $AdS(2) \times S(2) \times T(6)$. J. Phys. A. . 2011 V. 44. P. 275401.

[3] Maldacena J. M. // The Large N limit of superconformal field theories and supergravity. Adv. Theor. Math. Phys.. 1998. V. 2. P. 231–252.

[4] Grisaru M. T. et. al. // N=2 Superstrings in a Supergravity Background. Phys.Lett. B. 1985. V. 162. P. 116.

[5] Russo J.,Tseytlin A. A. // Green-Schwarz superstring action in a curved magnetic Ramond-Ramond background. JHEP. 1998. V. 9804. P. 014.

[6] Metsaev R. R., Tseytlin A. A. // Type IIB superstring action in AdS(5) x S(5) background. Nucl. Phys. B. 1998. V. 533, P. 109–126.

[7] Gomis J., Sorokin D., Wulff L. // The complete AdS(4) x CP(3) superspace for the type IIA superstring and D-branes. JHEP. 2009. V. 03. P. 015.

[8] Akulov V. P., Bandos I. A., Zima,V. G. // Nonlinear realization of extended superconformal symmetry. Theor. Math. Phys.. 1983. V. 56. P. 635–642.

[9] Arutyunov G., Frolov S. // Superstrings on $AdS_4 \times CP^3$ as a Coset Sigma-model. JHEP. 2008. V. 09. P. 129.

[10] Stefanski Jr. B. // Green-Schwarz action for Type IIA strings on $AdS_4 \times CP^3$. Nucl. Phys. B. 2008. V. 808. P. 80–87.

[11] Cagnazzo A., Sorokin D., Wulff L. // String instanton in AdS(4)xCP(3). JHEP. 2010. V. 05. P. 009.

[12] Bena I., Polchinski J., Roiban R. // Hidden symmetries of the $AdS_5 \times S^5$ superstring. Phys. Rev. D. 2004. V. 69. P. 046002.

[13] Eichenherr H. Forger M. // On the Dual Symmetry of the Nonlinear Sigma Models. Nucl. Phys. B. 1979. V. 155 P. 381.

[14] Grassi P. A., Sorokin D., and Wulff L. // Simplifying superstring and D-brane actions in AdS(4) x CP(3) superbackground. JHEP. 2009. V. 08. P. 060.

[15] Cagnazzo A., Sorokin D., Wulff L. // More on integrable structures of superstrings in $AdS(4) \times CP(3)$ and $AdS(2) \times S(2) \times T(6)$ superbackgrounds. arXiv:1111.4197 [hep-th].

[16] Berkovits N., et. al.//Superstring theory on AdS(2) x S**2 as a coset supermanifold, Nucl. Phys.B. 2000, V.567. P. 61.