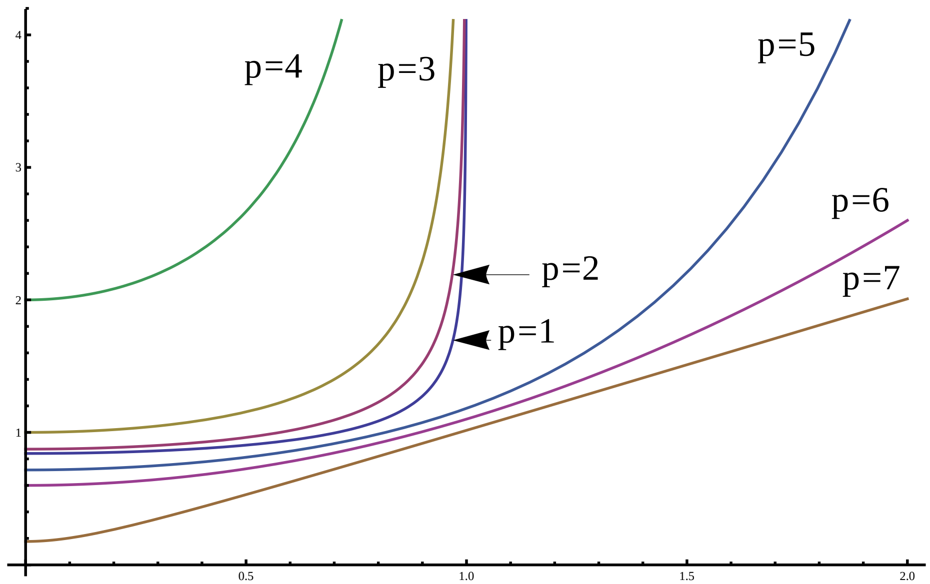
Wilson Loops in Five-Dimensional SYM: A new perspective on the question of UV-finiteness



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Outline

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  - ⋆ Recent results suggesting equivalence with $d=5$ SYM.
  - ⋆ Review of Wilson loops in the gauge-gravity duality.
  - ⋆ Review of Dp-brane geometries.
- Wilson loops in $d=5$ SYM: string theory
  - ⋆ $1/4$-BPS circular Wilson loops in the Dp-brane geometries.
  - ⋆ Generalizing to non-SUSY loops, renormalization properties.
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- Wilson loops in $d=5$ SYM: gauge theory
  - ⋆ Analysis of planar ladder/rainbow diagrams.
  - ⋆ Exponentiation of leading divergence.
  - ⋆ 1-loop analysis.
- Conclusions

Part I:
$(2,0)$ CFT review

The $(2,0)$ CFT in $d = 6$

The study of IIB on $\mathbb{R}^6 \times K3$ at special points in moduli space (singularities with an ADE classification) led to the discovery of the "self-dual" strings in $d = 6$, which can be made tensionless and therefore to decouple from gravity.

The spectrum of the self-dual strings fits into $d = 6$ tensor multiplets consisting of a two-form B_2 with self-dual field strength $H_3 = dB_2 = *H_3$, five scalar fields Φ^I, and compensating fermions.

The self-duality implies equality of the electric and magnetic charges, which implies that the coupling is ~ 1.

No known Lagrangian description. No coupling constant, no tunable parameters.

Later understood as the worldvolume theory of multiple M5-branes (for the A-series), and at large-N to be the CFT dual of $AdS_7 \times S^4$. Self-dual strings are M2-branes ending on the M5-stack.

Compactification of CFT on T^2 yields $\mathcal{N} = 4$ SYM in $d = 4$. Geometrical symmetries of T^2 are the S-duality of $\mathcal{N} = 4$ SYM.

The $(2,0)$ CFT in $d=6$ and MSYM in $d=5$

Compactification of CFT on S^1 yields MSYM in $d = 5$. Self-dual 2-form is equivalent to a one-form in $d = 5$, so $B_2^{d=6} \rightarrow A_1^{d=5}$, then there are the 5 scalars and the SUSY completion. One obtains a single new scale (and no new parameters)

$$\text{radius of } S^1 = R_6 = \frac{g_{YM}^2}{8\pi^2}.$$

But recently evidence has been found that higher modes present in $(2,0)$ CFT on $\mathbb{R}^5 \times S^1$ are **also** present in $d = 5$ MSYM.

Could it be true that

$$(2,0) \text{ CFT on } \mathbb{R}^5 \times S^1 \equiv \text{MSYM in } d = 5 \ \ \forall R_6,$$

and therefore that $d = 5$ MSYM is a finite theory?

5-d "instanton number" = 6-d KK-momentum

The self-dual strings which wrap the M-theory circle reduce upon compactification to 5-d particles:

- no momentum in the compact direction $\implies$ perturbative particle spectrum of 5-d MSYM.
- KK-modes with momentum k $\implies$ states with "instanton #" k, sometimes called "instantonic particles".

A way to see this is that the 6-d stress tensor has a component

$$T_{06} = H_0^{ij} H_{6ij} = \varepsilon^{0ijkl} F_{ij} F_{kl}$$

where i, j, k, l are 4-d indices. It can also be seen by comparing the 6-d and 5-d SUSY algebra.

Continuous parameter relating to instanton size has been a mystery, as there is no analog on the 6-d side.

Unwrapped self-dual strings are 5-d monopole strings

The EM-dual to a particle in 5-d is a string. The unwrapped self-dual strings give rise to these monopole strings under compactification.

- no momentum in the compact direction $\Longrightarrow$ equivalent dual-2-form description of the perturbative particles.

- KK-modes with momentum k $\Longrightarrow$ magnetic states with instanton-# k.

Both these string and particle states have been found in SU(2) 5-d SYM with a scalar VEV turned on [Lambert, Papageorgakis, Schmidt-Sommerfeld, 2011]. They are like 4-d instantons and 't Hooft-Polyakov monopoles, respectively. The spectrum has been shown to match that coming from a SUSY analysis of the 6-d theory.

Mystery of N^3 d.o.f.

The story for general N is much less clear. From M-theory we know there should be N^3 degrees of freedom. Of course at first glance 5-d SYM has only N^2 at high temperature. Attempts at resolution:

- Instantons are made up of N partons [Collie, Tong, (2010)], [Bolognesi, Lee, (2011)].
- 1/4 BPS 3-string junctions may play an important role [Bolognesi, Lee, (2011)].

We suffer from not having a non-abelian action for the $(2,0)$ theory...

Part II:
Wilson loops review

Maldacena-Wilson loops in d-dim. SYM

The Wilson loop is given by

$$W = \frac{1}{N} \underset{R}{\mathrm{Tr}}\, P \exp\left[\oint_C d\tau \left(i\dot{x}_\mu(\tau)\, A_\mu(x) + |\dot{x}(\tau)|\theta^I(\tau)\, \Phi^I(x) \right)\right]$$

with $\theta^I \theta^I = 1$.

- $x_\mu(\tau)$ describes a closed path in d-dim. spacetime.
- There are $10 - d$ scalars Φ^I which are coupled to the Wilson loop in order to preserve local SUSY.
- In general, one may consider a scalar coupling which varies along the loop: $\theta^I(\tau)$.
- $\theta^I(\tau)$ may be thought of as a path on S^{9-d}.

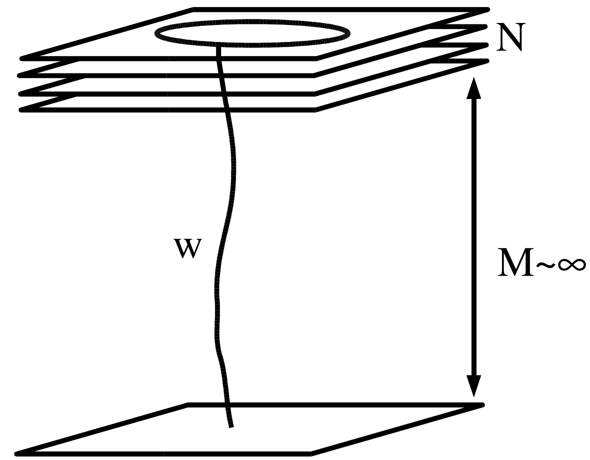
Brane construction of the Wilson loop

The construction for $\theta^I = \text{const.}$ may be understood as the holonomy of a heavy, fundamental W-boson:

The scalars of the $SU(N+1)$ theory are given a VEV:

$$\hat{\Phi}^I = \begin{pmatrix} \Phi^I & w^I \\ w^{I\dagger} & M\theta^I \end{pmatrix}$$

this gives the w^I (fundamental rep.) a mass M.



One can then show that

$$\int dy \langle w(x)\, w^\dagger(x)\, w(y)\, w^\dagger(y)\rangle \sim \int \mathcal{D}x_\mu \int \mathcal{D}A_\mu \mathcal{D}\Phi^I e^{-S_{SU(N)} - ML(x_\mu)} W(x_\mu)$$

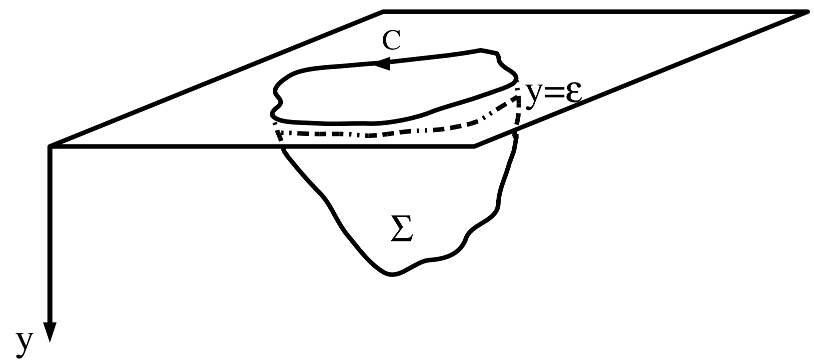
Wilson loop at strong coupling

At strong coupling the Wilson loop is given by the semi-classical partition function for a string in the appropriate geometry:

$$Z = \int \mathcal{D}X^\mu \, \mathcal{D}Y^I \, \mathcal{D}h_{ab} \mathcal{D}\vartheta^\alpha \exp\left(-\frac{\sqrt{\lambda}}{4\pi} \int_\Sigma d^2\sigma \sqrt{h} \, h_{ab} \frac{\partial_a X^\mu \partial_b X^\mu + \partial_a Y^I \partial_b Y^I}{Y^2} + \text{fermions} \right)$$

$$X^\mu|_{\partial\Sigma} = x_\mu(\tau), \qquad Y^I|_{\partial\Sigma} = \theta^I(\tau) Y|_{\partial\Sigma}, \qquad Y|_{\partial\Sigma} = 0$$

The saddle-point is obtained when the string worldsheet Σ describes a surface of minimal area $\mathcal{A}$



$$\frac{\sqrt{\lambda}}{2\pi} \int_\Sigma d^2\sigma \frac{1}{Y^2} \sqrt{\det\left(\partial_a X^\mu \partial_b X^\mu + \partial_a Y^I \partial_b Y^I\right)}$$

$$= \mathcal{A}_{\text{reg.}} + \frac{L(C)}{\varepsilon}$$

And so $1/\varepsilon$ corresponds to M, the mass of the heavy boson. We then have,

$$\langle W \rangle = e^{-\frac{\sqrt{\lambda}}{2\pi} \mathcal{A}_{\text{reg.}}}$$

Zarembo loops

There is a special class of supersymmetric Wilson loops, due to Zarembo, which have

$$\theta^I(\tau) = \frac{\dot{x}_\mu(\tau)}{|\dot{x}(\tau)|} M^I_\mu, \qquad \text{where} \qquad M^I_\mu M^I_\nu = \delta_{\mu\nu}$$

For these loops, the loop-to-loop propagator always vanishes,

$$\begin{aligned}\Big\langle (i\dot{x}_\mu A_\mu + |\dot{x}|\theta\cdot\Phi)(\tau_1)(i\dot{x}_\mu A_\mu + |\dot{x}|\theta\cdot\Phi)(\tau_2)\Big\rangle \\ = \frac{-\dot{x}(\tau_1)\cdot\dot{x}(\tau_2) + M^I_\mu M^I_\nu \dot{x}_\mu(\tau_1)\dot{x}_\nu(\tau_2)}{4\pi^2\left(x(\tau_1)-x(\tau_2)\right)^2} \\ = 0\end{aligned}$$

and leads to the statement

$$\langle W_{\text{Zarembo}}\rangle = 1$$

These loops can also be constructed in maximally supersymmetric Yang-Mills theories in arbitrary dimension [Agarwal, DY, 2009].

$d=4$: 1/2 BPS Circular loop and matrix model

The circular Wilson loop is given by $x_\mu = (\cos\tau, \sin\tau, 0, 0)$, $\theta^I = \text{const.}$

The loop-to-loop propagator on the circle is a constant:

$$\left\langle \left(i\dot{x}_\mu A_\mu + \theta\cdot\Phi\right)\left(i\dot{x}_\mu A_\mu + \theta\cdot\Phi\right)\right\rangle = \frac{1-\cos\tau_1\cos\tau_2 - \sin\tau_1\sin\tau_2}{4\pi^2\left[(\cos\tau_1-\cos\tau_2)^2+(\sin\tau_1-\sin\tau_2)^2\right]} = \frac{1}{8\pi^2}$$

This means that ladder diagrams may summed via a matrix model (ESZ-DG)

$$\langle W_{\text{circle}}\rangle = \frac{1}{Z}\int DM \frac{1}{N}\operatorname{Tr}\exp M \exp\left(-\frac{2N}{\lambda}\operatorname{Tr} M^2\right)$$

$$= \frac{1}{N} L^1_{N-1}\left(-\sqrt{\lambda}/4N\right) e^{-\lambda/8N} = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) + \frac{\lambda}{48N^2} I_2(\sqrt{\lambda}) + \ldots$$

In fact this statement is **exact** for all N and λ, as has been proven using localization [Pestun, 2007].

Part III:
String-side analysis

Dp-brane geometries

The gravity duals to $p+1$-dim. MSYM at large-N:

$$ds^2 = \alpha' \left(\frac{U^{(7-p)/2}}{C_p} dx_{\parallel}^2 + \frac{C_p}{U^{(7-p)/2}} dU^2 + C_p \, U^{(p-3)/2} \, d\Omega_{8-p}^2 \right),$$

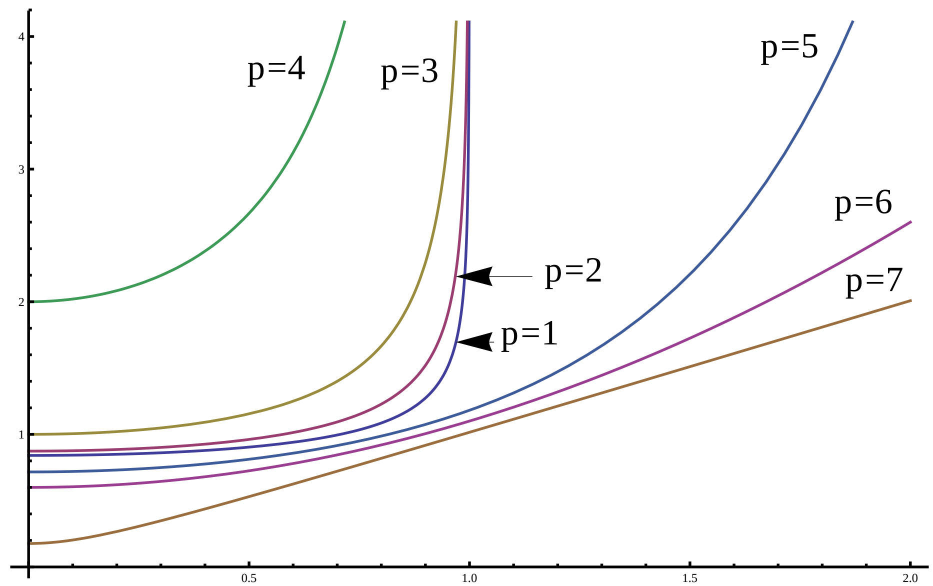
$$e^{\phi} = (2\pi)^{1-p} g^2 \left(\frac{C_p^2}{U^{7-p}} \right)^{(3-p)/4}, \; C_p^2 = g^2 N \, 2^{6-2p} \pi^{(9-3p)/2} \, \Gamma\left(\frac{7-p}{2} \right).$$

Strong coupling in the UV (IR) for $p > 3$ ($p < 3$), i.e. $d > 4$ ($d < 4$), leading to a break-down of the validity of the description.

In the IIA case (even-p, odd-d) an M-theory description takes over at strong coupling (for large enough N).

Zarembo circles in the d-dim. MSYM

Solutions found in [Agarwal, DY, 2009]:



$$r(U) = \begin{cases} \sqrt{\frac{2\,C_p^2}{5-p}}\sqrt{U_{\text{min.}}^{p-5} - U^{p-5}}, & p < 5 \\ \sqrt{\frac{2\,C_p^2}{p-5}}\sqrt{U^{p-5} - U_{\text{min.}}^{p-5}}, & p > 5 \\ \sqrt{2\,C_5^2\,\log\frac{U}{U_{\text{min.}}}}, & p = 5 \end{cases}, \qquad R = \begin{cases} \sqrt{\frac{2\,C_p^2}{5-p}}\sqrt{U_{\text{min.}}^{p-5}}, & p < 5 \\ \sqrt{\frac{2\,C_p^2}{p-5}}\sqrt{U_{\text{max.}}^{p-5} - U_{\text{min.}}^{p-5}}, & p > 5 \\ \sqrt{2\,C_5^2\,\log\frac{U_{\text{max.}}}{U_{\text{min.}}}}, & p = 5 \end{cases},$$

$$\sin\theta = \frac{U_{\text{min.}}}{U}.$$

Non-BPS circles

The Zarembo circles are rather special: they have zero regularized area, are 1/4 BPS. Let us consider a generic non-BPS circle sitting at a point on the S^{8-p}. One finds, for large-U (i.e. in the UV)

$$r(U) = R - \frac{1}{5-p}\frac{C_p^2}{R}\frac{1}{U^{5-p}} + \ldots, \qquad p \leq 4,$$

$$r(U) = R - \frac{C_5^2}{R}\log\frac{U_{\text{max.}}}{U} + \ldots, \qquad p = 5,$$

i.e. at $p \geq 5$ the solution depends explicitly on a UV cut-off $U_{\text{max.}}$. We might have expected this already at $p = 4$, if 5-d MSYM requires a cut-off to be defined.

Regularization of worldsheet area

The leading divergence in the area is removed via a Legendre transformation [Drukker, Gross, Ooguri, 1999]. Define coordinates

$$\frac{dU^2}{U^2} + d\Omega_{8-p}^2 = \frac{d\mathcal{Y}^I d\mathcal{Y}^I}{\mathcal{Y}^2}, \quad \mathcal{Y}^I = U\hat{\theta}^I, \quad \hat{\theta}^I\hat{\theta}^I = 1, \quad I = 1, \ldots, 9-p,$$

then,

$$S_{\text{reg.}} = S - \int d\tau\, d\sigma\, \partial_\sigma \left(\mathcal{Y}^I \frac{\delta \mathcal{L}}{\delta \partial_\sigma \mathcal{Y}^I} \right) = S - \int d\tau\, \mathcal{Y}^I \frac{\delta \mathcal{L}}{\delta \partial_\sigma \mathcal{Y}^I} \bigg|_{\partial\Sigma}.$$

One obtains, for the circles

$$S_{\text{reg.}} = \int_{U_{\text{min.}}}^{U_{\text{max.}}} dU\, r \sqrt{1 + \frac{U^{7-p}}{C_p^2} r'^2} - \frac{U\, r}{\sqrt{1 + \frac{U^{7-p}}{C_p^2} r'^2}} \Bigg|_{U=U_{\text{max.}}}.$$

Regularization of worldsheet area cont'd

As a direct consequence of the UV behaviour of the solutions we saw previously, the leading divergence is not removed by the Legendre transformation for $\boxed{p \geq 5}$. But it **does work** for $p = 4$, specifically

$$S = \int^{U_{\mathrm{max.}}} dU \left(R - \frac{C_4^2}{2RU} + \ldots \right) = R\, U_{\mathrm{max.}} - \frac{C_4^2}{2R} \log U_{\mathrm{max.}} + \mathrm{finite},$$

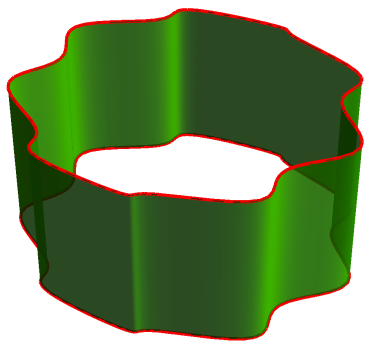
leaving a log divergence.

Again, a property one might have expected to find for the last time at $p = 3$ persists at $p = 4$.

$$\langle W_{\mathrm{circle}} \rangle = \mathcal{V}\, e^{-S_{\mathrm{reg.}}} = \mathcal{V} \exp\left(\frac{g^2 N}{16\pi R} \log U_{\mathrm{max.}} \right) \cdot (\mathrm{finite}),$$

M-theory lift, Graham-Witten Anomaly

For strong coupling $g^2 U \gg N^{1/3}$, D4-brane geometry is replaced by $AdS_7 \times S^4$ with boundary direction periodically identified $x_6 \sim x_6 + 2\pi R_6$, $R_6 = g^2/(8\pi^2)$.



Wilson loop contour takes on the topology $S^1 \times S^1$.

Analogous analysis of the probe M2-brane gives precisely the same log divergence. Analogous Legendre transformation removes leading divergence.

This is a "Wilson surface" in the $(2,0)$ CFT: why in a scale-invariant theory do we find a scale dependent log divergence?

The answer was provided by Graham & Witten: submanifold CFT observables of even dimension suffer an anomaly. Here

$$\text{coefficient of log div.} = S_{\text{rigid}} = \frac{N}{4\pi} \int d^2\tau \sqrt{h} \left(\nabla^2 X^i\right)^2 .$$

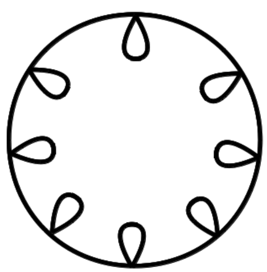
Part IV:
Gauge theory analysis

Gauge-theory perspective: rainbow/ladders

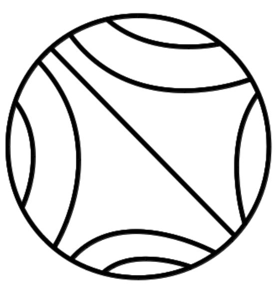
We cheat and use dimensional reduction from $\mathcal{N} = 1$ SYM in $d = 2\omega = 10$. This is **not** an honest UV-regulation scheme, it is an analytic continuation from the convergent region at $d < 4$ to $d = 5$.

Rainbow/ladders are finite in $d = 5 - \varepsilon$:

$$\left\langle \left(i\dot{x}^\mu A^a_\mu + |\dot{x}|\Theta^I \Phi^a_I\right)(x_1)\left(i\dot{x}^\mu A^b_\mu + |\dot{x}|\Theta^I \Phi^b_I\right)(x_2)\right\rangle = \frac{g^2\Gamma(\omega-1)}{\pi^\omega R^{2\omega-4}}\frac{\delta_{ab}\,2^{1-2\omega}}{\sin^{2\omega-4}\frac{\tau_1-\tau_2}{2}}$$

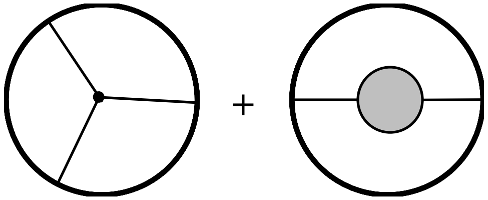


$\Longrightarrow$ exponentiates to $\exp\left(\frac{g^2 N}{16\pi R}\begin{cases}\frac{1}{\varepsilon} \\ -\log\delta\end{cases}\right) +$ subleading



$\Longrightarrow$ subleading (contribution to prefactor $\mathcal{V}$?)

Gauge-theory perspective: leading loop effects



$$= -\frac{g^4 N^2}{4} \oint d\tau_1\, d\tau_2\, d\tau_3\, \varepsilon(\tau_1\, \tau_2\, \tau_3) \left[D(\tau_1, \tau_3)\, \dot{x}_2 \cdot \partial_{x_1} G - \partial_{\tau_1} \Big(D(\tau_1, \tau_3)\, G \Big) \right],$$

where

$$G(x_1, x_2, x_3) = \frac{\Gamma(2\omega - 3)}{2^6 \pi^{2\omega}} \int_0^1 d\alpha\, d\beta\, d\gamma\, (\alpha\beta\gamma)^{\omega-2} \delta(1 - \alpha - \beta - \gamma)$$
$$\times \left[\alpha\beta(x_1 - x_2)^2 + \beta\gamma(x_2 - x_3)^2 + \alpha\gamma(x_1 - x_3)^2 \right]^{-2\omega+3},$$

and $D(\tau_1, \tau_2) = |\dot{x}_1||\dot{x}_2| - \dot{x}_1 \cdot \dot{x}_2$. For smooth closed curves this integral is **finite** for $d < 6$.

Note that in $d = 6$ two problems arise which are absent in $d = 5$:

1. Rainbow/ladders are not ε-away from finiteness.

2. (Analytic continuation of) dimensional reduction does not yield a finite answer at leading loop level.

Part V:
Conclusions

Conclusions and questions

- Another thing 5-d MSYM knows about the $d = 6$ $(2, 0)$ CFT: conformal anomaly. Captured entirely by ladder diagrams.

- Wilson loop is sensible without cut-off in $d = 5$ but not for $d \geq 6$: evidence on both sides of the gauge-gravity duality.

- Consistent with the idea that $d = 5$ MSYM could be finite.

- Are interacting diagrams finite at higher loops? If so all divergences are found in the ladders/rainbow diagrams. Seems interacting diagrams must at least be subleading in divergence.

- Exponentiated divergence is very reminiscent of renormalization of cusped Wilson loops in $d = 4$. Does there exist some scheme to give physical meaning to the finite part?

- Can we make anything out of the connection here to $\mathcal{N} = 4$ SYM 1/2 BPS circle (i.e. through compactification of $(2, 0)$ theory on T^2)?

- Any hope of comparing correlators with local operators between string and gauge theory?

- What are the effects of the non-perturbative degrees of freedom dual to the KK-modes on the Wilson loop VEV? Any chance for matching?