Baxter Q-operators for the nearest-neighbor spin chains

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Common Trends in Gauge Fields, Strings and Integrable Systems
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arXiv:1005.3261	V. Bazhanov, TL, C. Meneghelli, M. Staudacher
arXiv:1010.3699	V. Bazhanov, R. Frassek, TL, C. Meneghelli, M. Staudacher
arXiv:1012.6021	R. Frassek, TL, C. Meneghelli, M. Staudacher
arXiv:1112.3600	R. Frassek, TL, C. Meneghelli, M. Staudacher

Motivation

- From the Dima's talk:

$$
\begin{array}{cccc}
 & & \text{TBA} & \\
 & & \downarrow & \\
Y = \frac{TT}{TT} & \uparrow & \text{Y-system} & \downarrow \\
 & \text{easy} & \downarrow & \text{hard} \\
T = \det Q & \uparrow & \text{T-system} & \downarrow \\
 & \uparrow & \downarrow & \downarrow \\
 & \uparrow & \text{Q-system} & \downarrow \\
 & & \downarrow & \\
 & & \text{FINLIE} &
\end{array}
$$

- Analytic structure of T-functions and Y-functions are inherited from the analytic structure of Q-functions
- No construction of the Q-system is known

Why Q-operators are important?

- From the Sébastien's talk: no physical interpretation of the T-system for finite coupling (Y-functions are pseudorapidities of particles)
  - for strong coupling known up to the first semiclassical correction [Benichou 12]
  - for weak coupling known only for 1-loop – spin-chain transfer matrices

  Not clear what is a relation between these two T-systems
- It would be even better to have a construction of the Q-system - it was not known in neither of cases
- The famous example of such construction was given by Bazhanov, Lukyanov and Zamolodchikov in their solution of the KdV model
- The idea: use quantum inverse scattering method and construct **Q-operators** as traces of monodromies over the **oscillator** auxiliary space

Claim

Analytic properties of Q-functions follow directly from the construction of Q-operators, the former being just eigenvalues of the latter.

More motivation

- We focus on the weak coupling expansion of the AdS/CFT spectral problem. The 1-loop problem is described by the integrable spin-chain with the $\mathfrak{su}(2,2|4)$ symmetry
- Recently, the tree level scattering amplitudes were related to the 1-loop dilatation operator by Zwiebel. Better understanding of the weak coupling spectral problem may help in understanding amplitudes
- In general, the Yangian, which will play a crucial role in our construction, appears as a symmetry of various objects inside the AdS/CFT correspondence
- Not all states can be described by the TBA equations [Gleb's talk]
    - one has to consider twisted TBA equations
    - twist appears naturally when Q-operators are constructed

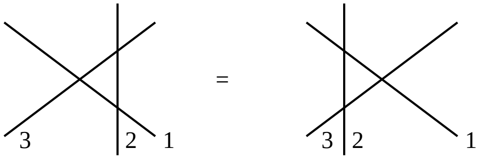
Basic ingredients

- Symmetry: $[J_A, J_B] = f_{AB}^C J_C$

$$[R_{12}^{\text{fund}}(z), J_1 + J_2] = 0$$

- Integrability – Yang-Baxter equation (YBE)

$$R_{12}(z_1 - z_2)R_{13}(z_1)R_{23}(z_2) = R_{23}(z_2)R_{13}(z_1)R_{12}(z_1 - z_2)$$



- Hilbert space!
- 'mild' assumptions about the analytic structure

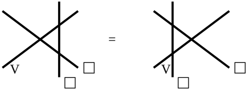
Plan of the talk

- Definition of the spin-chain
- Quantum inverse scattering method
- What are Q-functions for spin-chains?
- Construction of Q-operators
  - new solutions to YBE
  - analytic properties of Q-operators
- Nearest-neighbor Hamiltonians for spin-chains
- Summary and outlook

Definition of the spin chain

- Symmetry: $\mathfrak{gl}(n|m)$ algebra (we can take any real form – $\mathfrak{su}(n_1, n_2|m_1, m_2)$)
- Integrability $\longrightarrow$ Yang-Baxter equation

  $$R_{12}(z_1 - z_2)T_1(z_1)T_2(z_2) = T_2(z_2)T_1(z_1)R_{12}(z_1 - z_2)$$

  with the rational fundamental R-matrix

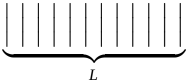
  $$R_{12}(z) = z\,\mathbf{I} + \mathbf{P}_{12}\,, \qquad R_{12}(z) \in \mathbb{C}^n \otimes \mathbb{C}^n \to \mathbb{C}^n \otimes \mathbb{C}^n$$

  and

  $$T(z) = \sum_{i,j=1}^{n} \sum_{r=0}^{\infty} e_{ij} \otimes t_{ji}^{(r)}\, z^{-r}$$

  RTT relation together with the definition of R_{12} presented above define a quadratic algebra which we call Yangian*. The choice of R-matrix defines the class of models we consider
- Quantum space:

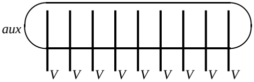
  $$\mathcal{V} = \underbrace{V \otimes \ldots \otimes V}_{L},$$

  where the space V is some representation space of the Yangian*
- Nearest-neighbor interaction (?)

Transfer matrices

- Quantum inverse scattering method tells us that we should focus on the trasfer matrices defined in the following way



$$T_{\mathcal{V},aux}(z) = \mathrm{Tr}_{aux}\left[R_{V,aux}(z) \otimes_{\mathcal{V}} \ldots \otimes_{\mathcal{V}} R_{V,aux}(z)\right]$$

- Again, the spaces V and aux are some representation spaces of the Yangian*
- One can prove that transfer matrices form a commuting family of operators

$$[T_{\mathcal{V},aux}(z), T_{\mathcal{V},aux'}(z')] = 0$$

- For $V = aux$ we can extract the local conserved charges

$$T_{V^{\otimes L},V}(z) = U + z\,H + \ldots\,, \qquad H = \frac{d}{dz}\log T_{V^{\otimes L},V}(z)\Big|_{z=0}$$

- Additionaly, for the rectangular representations of the $\mathfrak{gl}(n|m)$ algebra in the auxiliary space transfer matrices $T_{a,s}(z)$ satisfy the Hirota equation

Q-functions vs Bethe equations (I)

- Diagonalization of transfer matrices via the Algebraic Bethe Ansatz leads to the Bethe equations (we show a $\mathfrak{su}(2)$ example)

$$\left(\frac{z_k+\frac{1}{2}}{z_k-\frac{1}{2}}\right)^L=\prod_{l\neq k}\frac{z_k-z_l+1}{z_k-z_l-1}$$

- We introduce Q-function which vanishes on each Bethe root

$$Q(z)=\prod_k(z-z_k)$$

- Bethe equations (on-shell formulation) can be replaced by the Baxter equation (off-shell formulation)

$$T_{1/2}(z)Q(z)=T_0(z+\tfrac{1}{2})Q(z-1)+T_0(z-\tfrac{1}{2})Q(z+1)$$

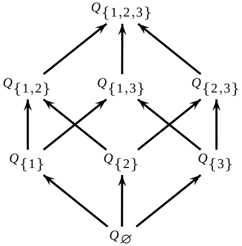
$T_{1/2}(z)$ and $T_0(z)=z^L$ are eigenvalues of the transfer matrices defined above

- Baxter equation is a second-order difference equation – two solutions: $Q_+(z)$ and $Q_-(z)$
- Baxter equation solved by

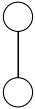
$$T_j(z)=Q_+(z+j+\tfrac{1}{2})Q_-(z-j-\tfrac{1}{2})-Q_+(z-j-\tfrac{1}{2})Q_-(z+j+\tfrac{1}{2})$$

Q-functions vs Bethe equations (II)

- Two Q-functions are the building blocks for the infinitely many T-functions. In the general case of $\mathfrak{gl}(n)$ we have 2^n Q-functions [Tsuboi]
- They are organized in the so-called Hasse diagram



Dynkin diagram



Bethe equations

$$\left(\frac{z_k^{\{23\}}+\frac{1}{2}}{z_k^{\{23\}}-\frac{1}{2}}\right)^L = \prod_{j\neq k}\frac{z_k^{\{23\}}-z_j^{\{23\}}+1}{z_k^{\{23\}}-z_j^{\{23\}}-1}\prod_j\frac{z_k^{\{23\}}-z_j^{\{2\}}-\frac{1}{2}}{z_k^{\{23\}}-z_j^{\{2\}}+\frac{1}{2}}$$

$$1 = \prod_{j\neq k}\frac{z_k^{\{2\}}-z_j^{\{2\}}+1}{z_k^{\{2\}}-z_j^{\{2\}}-1}\prod_j\frac{z_k^{\{2\}}-z_j^{\{23\}}-\frac{1}{2}}{z_k^{\{2\}}-z_j^{\{23\}}+\frac{1}{2}}$$

together with the QQ-relations

$$Q_{I\cup a\cup b}(z)Q_I(z) = Q_{I\cup a}(z+\tfrac{1}{2})Q_{I\cup b}(z-\tfrac{1}{2}) - Q_{I\cup a}(z-\tfrac{1}{2})Q_{I\cup b}(z+\tfrac{1}{2})$$

- Transfer matrices (for the compact representations Λ)

$$T_\Lambda(z) = \det_{a,b} Q_a(z_b)\,, \qquad z_{b+1}-z_b-1=\lambda_b$$

From Q-functions to Q-operators

Our goal

We want to construct a commuting family of **operators** acting on the quantum space of the spin-chain which satisfy the same bilinear relations (QQ-relations, TQ-relations) as Q-functions

- Other approaches:
  - Derkachov, Korchemsky, Manashov, ...
  - Kazakov, Laurent, Tsuboi, Vieira
- If we want to understand the 1-loop spectral problem of AdS/CFT then the construction of Q-operators has to work for both compact and non-compact representations of algebras $\mathfrak{gl}(n)$ ($\mathfrak{gl}(n|m)$)
- We provide a representation-independent construction of Q-operators – the final answer is written purely in terms of the generators of algebra

Solving Yang-Baxter equation (I) - defining relation

$$R_{12}(z_1 - z_2)T_1(z_1)T_2(z_2) = T_2(z_2)T_1(z_1)R_{12}(z_1 - z_2)$$

- It is enough to restrict to the evaluation representation: $t_{ij}^{(r)} \equiv 0$ for $r > 1$. Then

$$T_{ij}(z) = \frac{1}{z}(z\, t_{ij}^{(0)} + t_{ij}^{(1)})$$

- RTT relation tells us that $t_{ij}^{(0)}$ commute with all other generators and they serve as the structure constants for commutation of $t_{ij}^{(1)}$

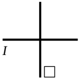
$$[t_{ij}^{(1)}, t_{kl}^{(1)}] = t_{kj}^{(0)} t_{il}^{(1)} - t_{il}^{(0)} t_{kj}^{(1)}$$

- Classification of solutions
    - using the symmetry of the R-matrix we have that $\tilde{T} = FTG$ also satisfy YBE. It leads to

$$t_{ij}^{(0)} = t_i^{(0)} \delta_{ij}\,, \qquad t_i^{(0)} = \begin{cases} 1, & i \in I \\ 0, & i \notin I \end{cases}$$

    - general solution

$$L_{I=\{1,\ldots,p\}}(z) = \begin{pmatrix} z + j_{rs} + \bar{a}_{r\dot{r}} a_{\dot{r}s} & \bar{a}_{r\dot{s}} \\ -a_{\dot{r}s} & \delta_{\dot{r}\dot{s}} \end{pmatrix}, \qquad \begin{cases} r, s = 1, \ldots, p\,, \\ \dot{r}, \dot{s} = p+1, \ldots, n \end{cases}$$



    - other L_I with the same $|I|$ are related by the Weyl symmetry

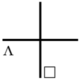
Solving Yang-Baxter equation (II) - defining relation

- Every solution gives the homomorphism

$$Y^*(\mathfrak{gl}(n)) \to \mathfrak{gl}(I) \otimes \mathcal{H}^{(I,\bar{I})} \equiv \mathcal{A}^I$$

- Every representation of $\mathcal{A}^I$ defines a representation of the Yangian*
- Pure $\mathfrak{gl}(n)$ representations ($|I| = n$) are also representations of Yangian*

$$\mathcal{L}(z) = z + e_{ij} \otimes J_{ji}$$



where the commutation relations for J_{ij} reads

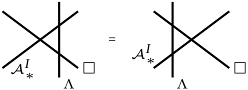
$$[J_{ij}, J_{kl}] = \delta_{kj} J_{il} - \delta_{il} J_{kj}$$

It means that they can serve as elements of the quantum space as well as the auxiliary space

- New types of representations - infinite dimensional (oscillator) representations. We use these new representations in the auxiliary space only

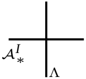
- We would like to intertwine any two Yangian* representations - in general a very difficult task
- We focus only on the R-matrices intertwining pure $\mathfrak{gl}(n)$ representations with pure oscillator representations $\mathcal{A}^I_* = \cdot \otimes \mathcal{H}^{(I,\bar{I})}$ (minimal representations), where by $\cdot$ we denote the singlet representation of $\mathfrak{gl}(n)$

$$\mathcal{L}(z_1 - z_2) L_I(z_1) \mathcal{R}_I(z_2) = \mathcal{R}_I(z_2) L_I(z_1) \mathcal{L}(z_1 - z_2)$$



- A linear equation for $\mathcal{R}_I$ which leads to the solution

$$\mathcal{R}_I(z) = \rho_I(z)\, e^{\bar{a}_{r\dot{s}} J_{\dot{s}r}} \prod_{k=1}^{n-p} \Gamma(z - \hat{\ell}^{\bar{I}}_k) e^{-\bar{a}_{\dot{r}s} J_{s\dot{r}}}$$



- Technical tool: non-commutative version of the Cayley-Hamilton theorem

$$p(\lambda) = \det(\lambda \mathbf{I} - \mathbf{M})$$

$$p(\mathbf{M}) = 0!$$

Solving Yang-Baxter equation (IV) - intertwining relation

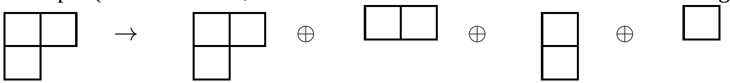
- What are these operators $\hat{\ell}_k^{\bar{I}}$ appearing in the solution?
- They are related to the Casimirs of $\mathfrak{gl}(\bar{I})$ subalgebra

$$c_n = J_{i_n}^{i_1} J_{i_1}^{i_2} \dots J_{i_{n-1}}^{i_n}\,, \qquad i_1, \dots, i_n \in \bar{I}$$

by the well-known formula

$$c_n = \sum_k \prod_{l \neq k} \left(1 + \frac{1}{\hat{\ell}_k^{\bar{I}} - \hat{\ell}_l^{\bar{I}}} \right) (\hat{\ell}_k^{\bar{I}})^n$$

- Operators $\hat{\ell}_k^{\bar{I}}$ are just an operatorial realization of the shifted weights of the subalgebra
- An example ($\ell_i = s_i - i + 1$, s_i is the number of boxes in i-th row of the Young tableaux)

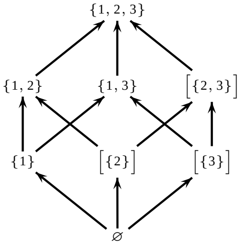


8	$\to$	**2**	$+$	**3**	$+$	**1**	$+$	**2**
ℓ_1		2		2		1		1
ℓ_2		0		-1		0		-1

- Direct construction of $\hat{\ell}_k$ as matrices not known.

$$\mathcal{R}_I(z) = \rho_I(z)\, e^{\bar{a}_{r\dot{s}}\, J_{\dot{s}r}} \prod_{k=1}^{n-p} \Gamma(z - \hat{\ell}_k^{\bar{I}}) e^{-\bar{a}_{\dot{r}s}\, J_{s\dot{r}}}$$

- compact representations – there exists a normalization $\rho_I(z)$ such that all $\mathcal{R}_I$ are polynomials because all spectra of $\hat{\ell}$ are bounded
- non-compact representations of the highest weight type – some spectra of $\hat{\ell}$ are bounded from below and others are bounded from above $\longrightarrow$ some of $\mathcal{R}_I$ can be normalize to be polynomials, the other are complicated meromorphic functions



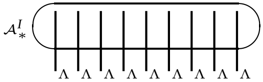
$\mathfrak{su}(1,2)$

- non-compact representations without the highest weight state – all $\hat{\ell}$ are unbounded $\longrightarrow$ no $\mathcal{R}_I$ can be a polynomial – they are complicated meromorphic functions

Q-operators - definition and what they really are

- Q-operators as traces of monodromies ($\mathbb{D}$ for convergence of the trace!)

$$Q_I(z) = e^{i\,z \sum_{r \in I} \Phi_r}\, \mathrm{Tr}_{\mathcal{A}^I_*} \left[\mathbb{D}\, \mathcal{R}_I(z) \otimes \ldots \otimes \mathcal{R}_I(z) \right]$$



- The analytic structure of Q-operators the same as for $\mathcal{R}_I(z)$
- Comparing different Q-operators
  - spin $= \frac{1}{2}$, $L = 2$

states	$Q_1(z)$	$Q_2(z)$
$\uparrow\uparrow$	$z^2 - z\cot\frac{\phi}{2} + \frac{1}{2\sin^2\frac{\phi}{2}} - \frac{1}{4}$	1
$\uparrow\downarrow + \downarrow\uparrow$	$z - \frac{1}{2}\cot\frac{\phi}{4}$	$z + \frac{1}{2}\cot\frac{\phi}{4}$
$\uparrow\downarrow - \downarrow\uparrow$	$z + \frac{1}{2}\tan\frac{\phi}{4}$	$z - \frac{1}{2}\tan\frac{\phi}{4}$
$\downarrow\downarrow$	1	$z^2 + z\cot\frac{\phi}{2} + \frac{1}{2\sin^2\frac{\phi}{2}} - \frac{1}{4}$

  - spin = $-\frac{1}{2}$, states = $D^{s_1} Z\, D^{s_2} Z + \ldots$
    - Q_1 is a polynomial counting excitations above the Z^L vacuum
    - Q_2 is a meromorphic function counting excitations above the '$D^\infty Z^L$ vacuum'

Bilinear relations for Q-operators (I)

- One has to check now if the objects which we constructed are really Q-operators – commute with transfer matrices and satisfy QQ- and TQ- relations
- Fusion relations – as a result of fusion of oscillators one can get any representation of $\mathfrak{gl}(n)$

$$L_1(z+j+\tfrac{1}{2})\,L_2(z-j-\tfrac{1}{2}) = S\,\mathcal{L}_j^+\,G\,S^{-1}$$

$$\begin{pmatrix} z+j+\bar{a}_1 a_1 & \bar{a}_1 \\ -a_1 & 1 \end{pmatrix}\begin{pmatrix} 1 & \bar{a}_2 \\ a_2 & z-j-\bar{a}_2 a_2 \end{pmatrix} = e^{\bar{a}_1 a_2}\begin{pmatrix} z+j-\bar{a}_1 a_1 & \bar{a}_1(2j-\bar{a}_1 a_1) \\ a_1 & z-j+\bar{a}_1 a_1 \end{pmatrix}\begin{pmatrix} 1 & \bar{a}_2 \\ 0 & 1 \end{pmatrix} e^{-\bar{a}_1 a_2}$$

- Matrix $\mathcal{L}_j^+(z)$ looks like $\mathcal{L}(z) = \begin{pmatrix} z+J_{11} & J_{21} \\ J_{12} & z+J_{22} \end{pmatrix}$. Generators J_{ij} in $\mathcal{L}_j^+(z)$ are in the infinite dimensional Holstein-Primakoff representation we denote by π_j^+.
- In order to get a finite-dimensional representation we use relation between Verma modules

$$\pi_j^+ = \pi_j \oplus \pi_{-j-1}^+\,, \qquad 2j+1 \in \mathbb{N}$$

Bilinear relations for Q-operators (II)

- We construct transfer matrices

$$
\begin{aligned}
Q_1(z) &= \mathrm{Tr}_{osc_1} L_1(z) \otimes \ldots \otimes L_1(z) \\
Q_2(z) &= \mathrm{Tr}_{osc_2} L_2(z) \otimes \ldots \otimes L_2(z) \\
T_j^+(z) &= \mathrm{Tr}_{\pi_j^+} \mathcal{L}_j^+(z) \otimes \ldots \otimes \mathcal{L}_j^+(z) \\
1 &= \mathrm{Tr}_{osc_2} G \otimes \ldots \otimes G
\end{aligned}
$$

  and get

$$T_j^+(z) = Q_1(z + j + \tfrac{1}{2}) Q_2(z - j - \tfrac{1}{2})$$

- Using the relation between modules we get a formula for transfer matrices with finite-dimensional auxiliary spaces

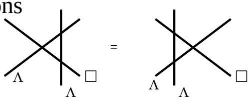
$$T_j(z) = T_j^+(z) - T_{-j-1}^+(z) = Q_1(z + j + \tfrac{1}{2}) Q_2(z - j - \tfrac{1}{2}) - Q_1(z - j - \tfrac{1}{2}) Q_2(z + j + \tfrac{1}{2})$$

  which is a desired formula

- QQ-relations can be proven with use of very similar methods

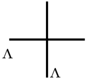
Hamiltonians (I)

- Can we find a simple form for the nearest-neighbor Hamiltonians for our spin-chain?
- One has to solve the YBE intertwining two Λ representations



$$\mathcal{R}(z_1 - z_2)\mathcal{L}(z_1)\mathcal{L}(z_2) = \mathcal{L}(z_2)\mathcal{L}(z_1)\mathcal{R}(z_1 - z_2)$$

- R-matrix $\mathcal{R}(z)$ can be written in terms of the projectors



$$\mathcal{R}(z) = \sum_{\lambda} \rho_\lambda(z) P_\lambda$$

where $\rho_\lambda(z)$ are simple functions of second Casimirs of the tensor product decomposition

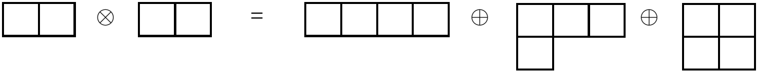
$$V_\Lambda \otimes V_\Lambda = \bigoplus_{\lambda} V_\lambda$$

if the decomposition is multiplicity-free

- For $\mathfrak{gl}(n)$ algebras: rectangular representation $\Longleftrightarrow$ multiplicity-free representations
- We found more explicit form for the rectangular representations $(\underbrace{s,\ldots,s}_{a},\underbrace{0,\ldots,0}_{n-a})$

$$\mathcal{R}(z) = \rho(z)\prod_{k=1}^{a}\Gamma\left(z + \frac{\hat{A}_k - \hat{A}_{\bar{k}} + 1}{2}\right)\Gamma\left(z - \frac{\hat{A}_k - \hat{A}_{\bar{k}} + 1}{2} + 1\right), \qquad \bar{k} = 2a - k + 1$$

- Again, $\hat{A}_i$ are operatorial shifted weights of the tensor product representation



3	$\times$	**3**	=	**5**	+	**3**	+	**1**
	A_1			4		3		2
	A_2			-1		0		1

- One can extract the nearest-neighbor Hamiltonian from the R-matrix

$$\mathcal{H} = \frac{d}{dz} \log \mathcal{R}(z)\Big|_{z=0} = -2 \sum_{k=1}^{a} \left[\psi\left(\frac{\hat{A}_k - \hat{A}_{\bar{k}} + 1}{2} \right) - \psi\left(\frac{\hat{A}_{\bar{k}} + \hat{A}_k + 1}{2} + 2a - k \right) \right]$$

- It is a generalization of the well-known formula for $\mathfrak{su}(2)$

$$\mathcal{H}_{\mathfrak{su}(2)} = 2(\psi(\mathbb{J}+1) - \psi(1))\,, \qquad \mathbb{J}(\mathbb{J}+1) = (J_1 + J_2)^2$$

- The harmonic action more efficient in calculations but known only for 1-row representations [Beisert]

Supersymmetric case

All results very similar with only small modifications:

- one has to replace oscillators with superoscillators $[\xi, \bar{\xi}\} = 1$
- minus signs appear in many places (making you cry while trying to keep track of them)
- fusion works a bit different compare to the bosonic case – one gets two types of QQ-relations
- Bethe equations differs for different paths on the Hasse diagrams – it reflects the freedom in choosing fermionic nodes for the Dynkin diagram.

  Still, the spectra one gets from different sets of Bethe equations agree.

Summary and outlook - lessons for the all-loop problem

- Analytic properties of Q-functions follow directly from the construction of Q-operators
- Q-operators for spin-chains can be constructed as traces of monodromy matrices over infinite-dmensional (oscillator) auxiliary space
- Bilinear relations are completely determined by the fusion rules in the auxiliary space! We have the same Hasse diagram for 1-loop spin chain as for the Q-system in the FINLIE approach. It means that if it is possible to construct Q-operators for finite-coupling then we know which auxiliary spaces we have to consider. An important question remains what kind of the quantum space we should take?

Question:

What is a proper quantum space suitable for the 2-, 3-, . . . loop description of the spectral problem at weak coupling? It seems that the spin-chain picture breaks down because of the length fluctuations and wrapping corrections

Thank you!

- Drinfeld realization of the Yangian $Y(\mathfrak{gl}(n))$:

$$[J_A, J_B] = f_{AB}^C\, J_C\,, \qquad [J_A, \hat{J}_B] = f_{AB}^C\, \hat{J}_C\,, \qquad + \text{Serre relations}$$

- Drinfeld definition is equivalent to the RTT relation with the additional assumption

$$t_{ij}^{(0)} = \delta_{ij}$$

- For the evaluation representation we get the homomorphism

$$Y(\mathfrak{gl}(n)) \to \mathfrak{gl}(n)$$

  We cannot get the oscillator representations for the standard Yangian

- Yangian* is not even a Hopf algebra because there is no co-unit

Another question:

What does Yangian* mean for the scattering amplitudes?