Relative locality in 3 dimension

Jerzy Kowalski-Glikman

Non-locality: Aspects and Consequences, June 27, 2012

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Relative locality

Special Relativity

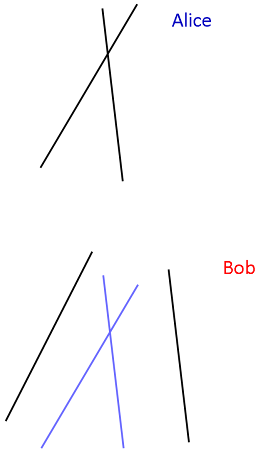
In Special Relativity locality is absolute: all observers (translated, rotated, boosted) agree that an event is local i.e., taking place in a single spacetime point.

Relative Locality

In a RL theory locality is not absolute:

- Only local observers agree that an event is local;
- In general an event might be local to one observer and non-local to another.

For example, wordlines of two particles that cross for one observer, may not cross for another.



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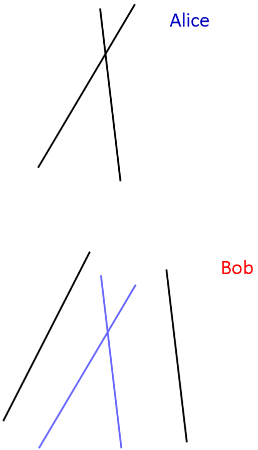

Curved momentum space and relative locality

Special Relativity

In Special Relativity both spacetime and momentum space are flat; the particle Lagrangian is:

$$L = \dot{x}^a\, p_a + \dots$$

General Relativity

In General Relativity momentum space is flat and spacetime is curved; the particle Lagrangian is:

$$L = \dot{x}^\mu\, e_\mu{}^a(x)\, p_a + \dots$$

Relative Locality

In Relative Locality momentum space is curved while spacetime is flat; the particle Lagrangian is:

$$L = -x^a\, E^\mu{}_a(p)\, \dot{p}_a + \dots$$

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If momentum space is curved the particle action is still invariant under translations, but with translation parameter becoming momentum dependent.

Special Relativity

$$L = \dot{x}^a\, p_a + \ldots$$

$$\delta x^a = \epsilon^a\,, \ \delta p_a = 0$$

$$\delta L = \text{total derivative}$$

Relative Locality

$$L = -x^a\, E^\mu{}_a(p)\, \dot{p}_a + \ldots$$

$$\delta x^a = \epsilon^\mu\, E^a{}_\mu(p) = \epsilon^a(p)\,, \ \delta p_\mu = 0$$

$$\delta L = \text{total derivative}$$

Therefore, in Relative Locality, wordlines of particles with different momenta are translated by different (momentum dependent) amounts. If the wordlines meet for one observer, they may not meet for the translated (boosted, rotated) one.

Relative locality regime

Is relative locality physical? Does it exist a regime of (quantum) gravity coupled to particles and/or fields, which can be understood in terms of (approximated by) relative locality?

Dimensional analysis

The momentum space tetrad $E^{\mu}{}_{a}(p)$ is dimensionless, and thus in order to make it nontrivial we must have in our disposal a mass scale. This scale is to be defined by some "master" theory (e.g., quantum gravity.)

Quantum gravity in 4 dimensions

Quantum gravity is characterized by two scales: of length ℓ_{Pl} and mass M_{Pl}. If in some process ℓ_{Pl} is negligibly small, while M_{Pl} remains finite we can hope that the process is described by RL of some sort. In other words we may consider a limit of QG:

$$\hbar,\, G \to 0\,, \qquad \sqrt{\hbar/G}\ \text{finite.}$$

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In fact we can follow the Gauss' footsteps to claim:

If there is a scale, you may expect a nontrivial geometry.

Everything is curved, unless it cannot be (because the scale is not available.)

If a theory predicts existence of the mass (momentum) scale, the space of momenta should be curved.

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This leads us to 3D gravity . . .

3D gravity as a topological field theory

In 3D Ricci-flat is equivalent to Riemann-flat, and therefore the vacuum Einstein equations tell us that spacetime is (locally) a flat Minkowski space. There are no local gravitational interactions, gravitational waves, etc. The theory of gravity is topological.

Particles coupling

The equation governing a massive particle coupling to gravity has the form

$$\mathrm{Riemann}(x) \sim \sum_i GM_i \, \delta(x - x_i(\tau))$$

so that the curvature has a delta-like singularity at the particles' wordlines.

Relative locality in 3D?

In 3D the Newton's constant G has the dimension of inverse mass. Could gravity in 3D be interpreted as a relative locality type of a theory? Yes it could!

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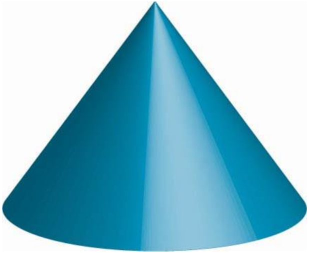
Curved momentum space from topological degrees of freedom

Point particle metric

The solution of Einstein equations describing a particle of mass M is (in the units in which $G = 1/4\pi$)

$$ds^2 = -dt^2 + dr^2 + \left(1 - \frac{M}{\pi}\right)^2 r^2 d\phi^2 ,$$

which describes a cone with $\alpha = 2M$ opening.



Curved momentum space from topological degrees of freedom

Gravity as a gauge theory of $ISO(2,1)$

Instead of the metric formulation, one can use the formulation of gravity as a gauge theory of $ISO(2,1)$, with connection

$$A_\mu = e_\mu^a\, P_a + \omega_\mu^{ab}\, L_{ab}\,,$$

with P_a, L_{ab} being generators of the 3D Poincare group $ISO(2,1)$.

Field equations

The field equations tell that the curvature of this connection vanishes away from the particle position; thus in any simply connected region

$$\omega_\mu = g^{-1}\partial_\mu\, g\,, \quad e_\mu = g^{-1}\partial_\mu f\, g\,, \quad g \in SO(2,1)\,, f \in so(2,1) \approx R^3\,.$$

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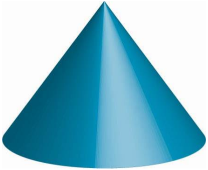
Curved momentum space from topological degrees of freedom

Decomposition of the manifold

We remove the singularity by cutting off the tip of the cone; then by cutting along the lint $\phi = 0$ we form a simply connected manifold (with boundaries.) The appropriate boundary conditions must be imposed.



Curved momentum space from topological degrees of freedom

Boundary conditions

To make contact with the previous situation we must impose the following boundary conditions:

- The circumference of the upper boundary must be zero, thus $e_\phi(r=0,\phi)=0$;
- The gauge field must be continuous across the cut:

$$\omega_\mu(r,\phi=0)=\omega_\mu(r,\phi=2\pi)\,,$$

$$e_\mu(r,\phi=0)=e_\mu(r,\phi=2\pi)\,;$$

- The parallel transport of a vector around the (would be) particle position must produce the deficit angle $\alpha=2M$.

Curved momentum space from topological degrees of freedom

Continuity conditions

$$\omega_\mu(r,\phi=0)=\omega_\mu(r,\phi=2\pi)\,,$$

$$e_\mu(r,\phi=0)=e_\mu(r,\phi=2\pi)\,;$$

This means that

$$g(r,\phi=0)=U\,g(r,\phi=2\pi)\,,\quad U\in SO(2,1)\,,$$

$$f(r,\phi=0)=U^{-1}[f(r,\phi=2\pi)-v]\,U\,,\quad v\in R^3\,.$$

The result

It turns out that the elements $U(t)=p_3+p_a\,T^a\in SO(2,1)$ and $v(t)\in R^3$ characterize completely the dynamics of the particles.

Curved momentum space from topological degrees of freedom

Topological degrees of freedom

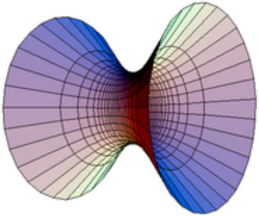
The $U \in SO(2,1)$ element encompass the topological degrees of freedom of gravity in the situation when the massless particle is present. It describes the (deformed) particle momentum.

Curved momentum space

The momenta p_a are coordinates on the curved group manifold

$$U(t) = p_3 + p_a\,T^a \in SO(2,1) \to p_3^2 - p_a p^a = 1\,,$$

so that the momentum space is the 3D anti de Sitter space.



Effective Lagrangian for one particle

The Lagrangian

The effective deformed Lagrangian for one particle is

$$L = -\frac{1}{2}\,\mathrm{Tr}\left[U^{-1}\frac{d}{d\tau}U T_a\right] x^a + N\left(\mathrm{Tr}[U] - \cos M\right)$$

which can be expressed in the Relative Locality form

$$L = -x^a E_a^b(p)\dot{p}_b + N\left(p_a p^a + \sin^2 M\right)$$

with

$$E_a^b = \delta_b^a p_3 - \epsilon_a{}^{bc}\, p_c - \frac{1}{p_3}\, p_a p^b\,.$$

Translations

The Lagrangian is invariant under the RL-type translations

$$\delta x^a = p_3\,\epsilon^a - \epsilon^{abc}\,\epsilon_b\, p_c\,.$$

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Summary

What has happened?

- We started with a free particle moving in its own gravitational field. The particle's momentum space was a structureless R^3, which could be identified with the Lie algebra $so(2,1)$.
- Since the gravitational field has only a finite number of (topological) degrees of freedom, we could take the backreaction into account exactly. As a result we get the deformed particle, with momentum space being a curved manifold of the Lie group $SO(2,1)$.

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