Real clocks: a toy model for non-locality

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Non-locality:

Aspects and Consequences

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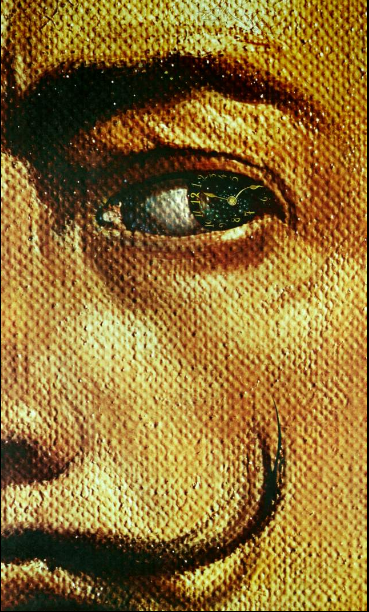

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- Non-ideal clocks
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Pictures by 

Evolution according to ideal clocks

- s: ideal Schrödinger time
- Ideal Hamiltonian evolution:

$$\partial_s \varrho(s) = -i\left[H, \varrho(s)\right] := -i\mathscr{L}\varrho(s)$$


non-ideal clocks

Non-ideal clocks

- Any clock is prone to errors
- Degree of randomness in the measure of time
- Sources: quantum, temperature, imperfections…

Perform N experiments:

$t=0$	$\cdots$	t
$\|\psi_0\rangle$	$\cdots$	A_1
$\cdots$	$\cdots$	$\cdots$
$\|\psi_0\rangle$	$\cdots$	A_N

➜

•	Prob(A)
∘	quantum
∘	lack of knowledge of exact Schrödinger time

Functional approach

- Relative error $\alpha(t)$: $\dfrac{\mathrm{d}s}{\mathrm{d}t} = 1 + \alpha(t)$ (Langevin eq.)

  Absolute error: $s = t + \Delta(t)$, $\Delta(t) = \int \mathrm{d}t\, \alpha(t)$

- Clock described by the probability functional $\mathscr{P}[\alpha]$.

  - Alternative: probability function $P(t,s)$:

    $$P(t,s) = \int \mathscr{D}\alpha \mathscr{P}[\alpha] \delta(t + \Delta(t) - s)$$

  - No systematic drift: $\langle \alpha(t) \rangle = 0 \quad \Leftrightarrow \quad \langle s \rangle_t = t$

  - The clock should always behave in the same:

    $\mathscr{P}[\alpha(t)]$ should be stochastically stationary

- For good clocks, the relative errors should always be small:

  - ⋆ Correlation function:

$$\langle \alpha(t')\alpha(t) \rangle := c(t'-t) \leq c(0)$$

  - ⋆ Correlation time: $\vartheta \equiv \frac{1}{c(0)} \int \mathrm{d}t\, c(t)$

  - ⋆ Small relative errors: $c(0) := \boxed{\tau/\vartheta \ll 1}$

- Microcausality $\Rightarrow$ $\alpha \geq 0$

Evolution according to real clocks

Evolution of $\varrho(s)$ $\Longrightarrow$ evolution of $\rho(t) = \langle \varrho(s) \rangle$

Steps to obtain the evolution equation in clock time t:

1. Hamiltonian evolution of $\varrho(s)$: $\partial_s \varrho(s) = -i\mathscr{L}\varrho(s)$

2. For each stochastic process α,

$$s = t + \Delta(t) \quad \Rightarrow \quad \partial_t = (1+\alpha)\partial_s, \quad \varrho_\alpha(t) := \varrho(t+\Delta)$$

$$\Rightarrow \quad \partial_t \varrho_\alpha = -i(1+\alpha)\mathscr{L}\varrho_\alpha$$

3. ∘ Use interaction picture: $\varrho^{\mathrm{I}}_{\alpha}(t) = e^{it\mathscr{L}}\varrho_{\alpha}(t)$,

$$\dot{\varrho}^{\mathrm{I}}_{\alpha}(t) = -\alpha\mathscr{L}\varrho^{\mathrm{I}}_{\alpha}(t)$$

∘ Expand in powers of α (integrate and substitute)

∘ Average over α with $\mathscr{P}[\alpha(t)]$

∘ Undo interaction picture

$$\partial_t\rho(t) = -i\mathscr{L}\rho(t) - \int_0^t \mathrm{d}t'\, c(t')\mathscr{L}^2\rho(t-t') + \iint c^2\mathscr{L}^4\rho$$

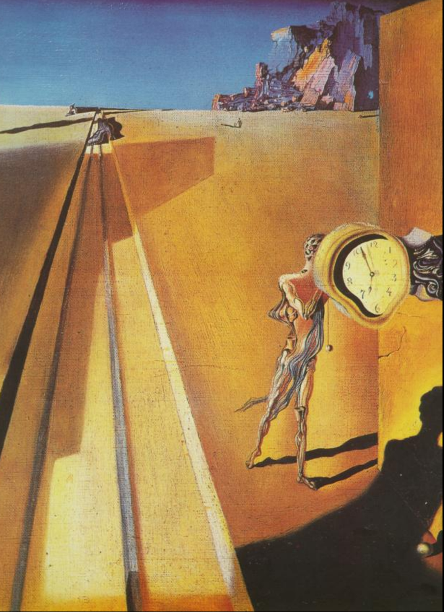
4. Good clock

  - Intrinsically: $\tau \ll \vartheta$ (small correlations)
  - For the system: $\vartheta \ll \zeta$, where
    $\zeta \equiv 1/\Delta\omega_{\max}$ is the characteristic evolution time

Then, $\iint c^2 \mathscr{L}^4 \sim \tau^4/\vartheta^2\zeta^2 \ll \tau^2/\vartheta\zeta \sim \int c\mathscr{L}^2$ ✓

Second order expansion is fine

4. Markov approximation: $\vartheta \ll \zeta, \quad \Rightarrow \quad \rho(t-t') \sim \rho(t)$



Quantum evolution
according to a
real clock:

$$\partial_t \rho = -i \mathscr{L} \rho - \tau \mathscr{L}^2 \rho$$

Loss of coherence

- Exact solution (in the energy basis)

$$\rho_{nm}(t) = \rho_{nm}(0)e^{-i\omega_{nm}t}e^{-\tau(\omega_{nm})^2 t}$$

- Energy conservation: $\langle H\rangle = \mathrm{Tr}(H\rho) = \text{constant}$

- Decoherence: off-diagonal terms decay

- Decoherence time: $T \sim \zeta^2/\tau \gg \zeta$

Non-local description

- Master equation:
  - evolution with a free Hamiltonian H *plus*
  - classical noise with interaction Hamiltonian αH.
- Path integral formalism

- Qualitatively, the idea is simple:
  - Path integral for this system $\qquad Q := (q, p)$

$$\int \mathscr{D}\alpha \mathscr{P}[\alpha] \int \mathscr{D}Q e^{iS_0[Q]-i\int \mathrm{d}t \alpha H(Q(t))}$$

  - $\mathscr{P}[\alpha]$ Gaussian for simplicity:

$$\mathscr{P}[\alpha] = e^{-\int \mathrm{d}t_1 \mathrm{d}t_2 \alpha(t_1)\alpha(t_2)/2c(t_1-t_2)}$$

  - Integrate over α (Gaussian)

$$\int \mathscr{D}Q e^{iS_0[Q]} e^{-\frac{1}{2}\int \mathrm{d}t_1 \mathrm{d}t_2 c(t_1-t_2) H(Q(t_1)) H(Q(t_2))}$$

- Technically, it is a bit more sophisticated →

  influence functional

  - Evolution operator $\rho(t) = \$(t)\rho(0)$
  - Factorization of \$ $\Rightarrow$ unitary evolution:

$$\rho(t) = \$(t)\rho(0) = U(t)\rho(0)U(t)^{-1} \quad \Rightarrow$$
$$\Rightarrow \quad \mathrm{Tr}\rho(t)^2 = \mathrm{Tr}\rho(0)^2$$

In other words

$$\$(t) = \int \mathscr{D}Q\mathscr{D}Q' e^{iS_0[Q;t]} e^{-iS_0[Q';t]}$$

- Non-factorizability controlled by the influence functional W

$$\$(t) = \int \mathscr{D}Q\mathscr{D}Q' e^{-iS_0[Q;t]} e^{iS_0[Q';t]} e^{W[Q,Q';t]}$$

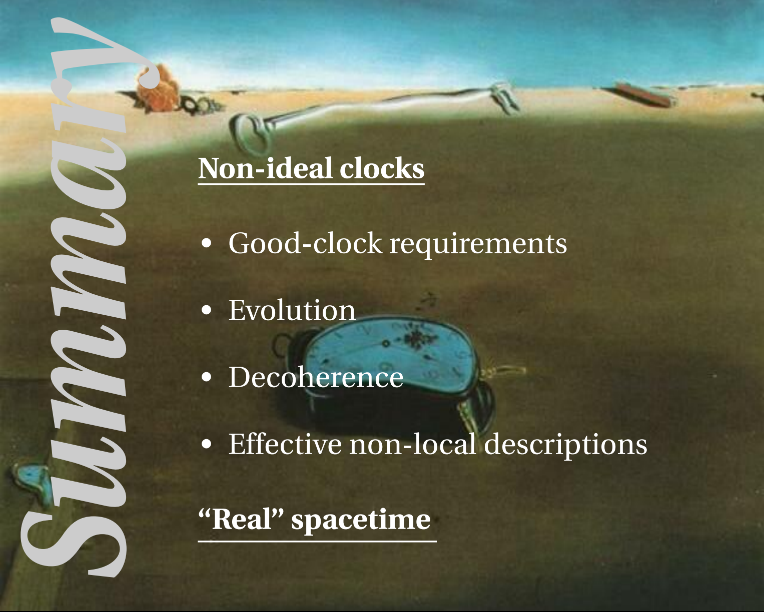
where

$$\begin{aligned} W[Q,Q';t] = & -\frac{1}{2}\int \mathrm{d}t_1 \mathrm{d}t_2 c(t_1 - t_2)\times \\ & \times [H(Q(t_1)) - H(Q'(t_1))]\times \\ & \times [H(Q(t_2)) - H(Q'(t_2))] \end{aligned}$$

Spacetime fluctuations

- Inaccuracies in time $\rightsquigarrow$ inaccuracies in spacetime
- In a semiclassical picture, spacetime topological (or quantum) fluctuations could be modelled by an effective flat spacetime plus non-local interactions just as for time errors and clocks:
  - Influence functional
  - Master equation

- Energy (and momentum, etc.) conservation need not be incompatible with loss of coherence
  - Interactions that commute with the bare evolution
  - Relational evolution
  - Non-Markovian effects at very small scales
- Since non-localities are localised, asymptotic dynamics enforce conservation

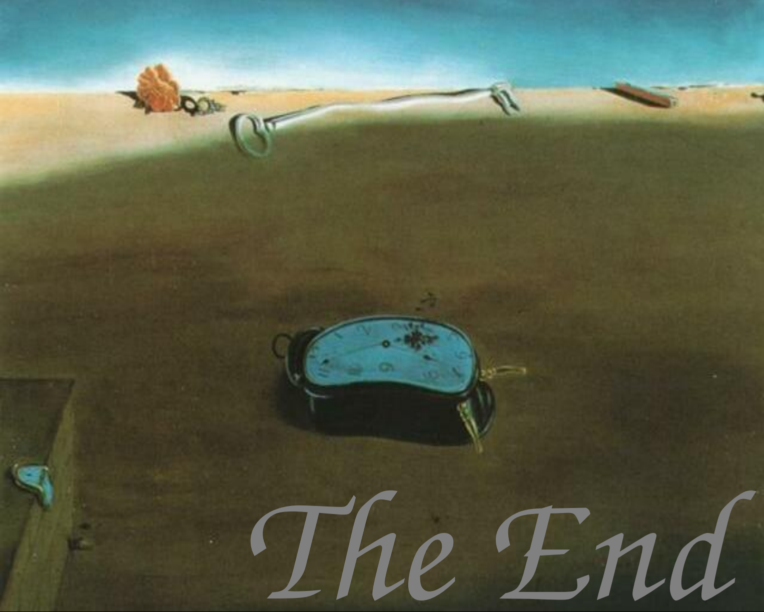


Summary

Non-ideal clocks

- Good-clock requirements
- Evolution
- Decoherence
- Effective non-local descriptions

“Real” spacetime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