Non-local charges, curved momentum space and fractal space-time

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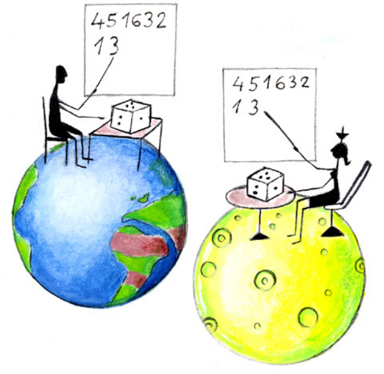
Non-locality vs. Leibniz



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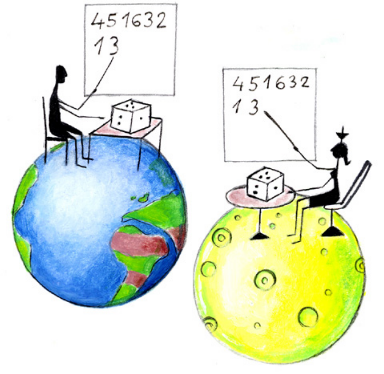

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An impossible co-existence in quantum field theory?

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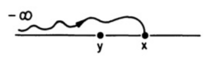

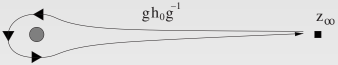

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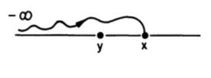

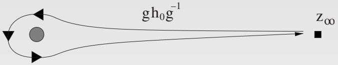

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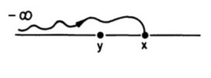

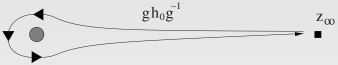

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their existence in QFT relies on a *generalization of the Leibniz rule* for their action on composite quantum states.

Outline

- **Beyond Leibniz in 2d**
- **“Bending” phase space in 3d gravity: group valued momenta and NC-fields**
- **NC heat kernel: running spectral dimension**
- **4d case: de Sitter momentum space and κ-deformed symmetries**
- **κ-Fock space: “hidden entanglement” at the Planck scale**

Quantum mechanics and Leibniz

Given an action of a *symmetry generator* g on the space of states of a quantum system (Hilbert space) $\mathcal{H}$

- action of g on *composite system* e.g. $\mathcal{H} \otimes \mathcal{H}$ *(Leibniz rule)*

$$g(\psi_1 \otimes \psi_2) \equiv g(\psi_1) \otimes \psi_2 + \psi_1 \otimes g(\psi_2)$$

- action on observables

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These basic facts *are the key to implement symmetries in QM*... do they admit generalizations/modifications?

YES...

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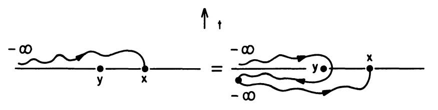

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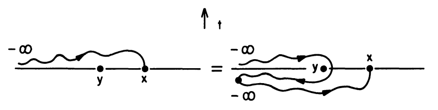

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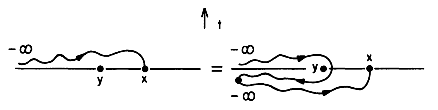

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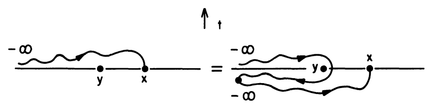

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The non-locality of the currents leads to a generalized **"non-Leibniz"** *action of the* **(internal)** *symmetry generators on fields*

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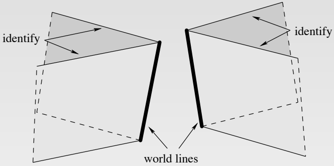

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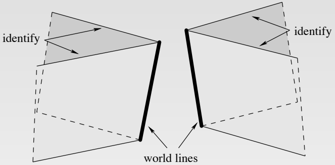

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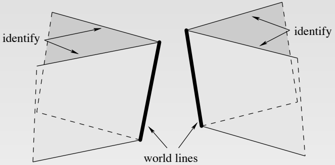

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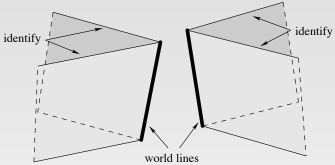

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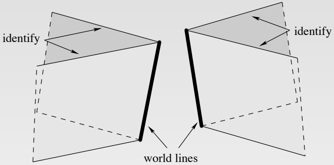

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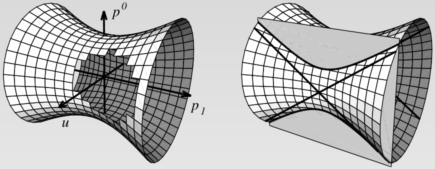
Momenta become coordinate functions on a non-abelian group!

Group valued momenta and deformed mass-shell

The components $\vec{p}$ are *coordinates on a group manifold*

$$u^2 - 16\pi^2 G^2 \vec{p}^{\,2} = 1$$



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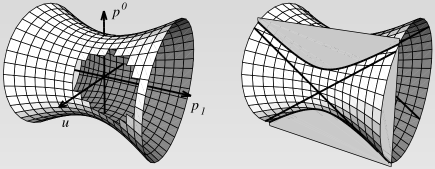

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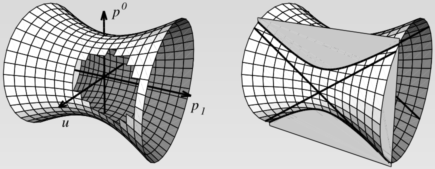

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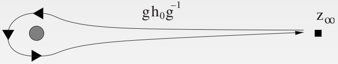
- **Mass-shell**: *holonomies* representing a rotation by $8\pi Gm \Rightarrow \vec{p}^{\,2} = -\frac{\sin^2(4\pi Gm)}{16\pi^2 G^2}$



$$\vec{p} = \frac{\kappa}{2i}\mathrm{Tr}(h\vec{\gamma}) \quad \text{where} \quad h = gh_0g^{-1} \text{ and } \kappa = (4\pi G)^{-1}$$

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Fourier transform maps fields *on the group manifold* to fields on a *dual "spacetime"*

$$\mathcal{F}(f)(x) = \int d\mu_H(\mathbf{P}) f(\mathbf{P})\, e_{\mathbf{P}}(x)\,,$$

where: $e_{\mathbf{P}}(x) = e^{\frac{i}{2\kappa}\mathrm{Tr}(\mathbf{xP})} = e^{i\vec{p}\cdot\vec{x}}$ with $\mathbf{x} = x^i\sigma_i$

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Plane waves = eigenfunctions of *translation generators* P_a

$$\Downarrow$$

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$$\vec{p}_1 \oplus \vec{p}_2 = p_0(\vec{p}_2)\,\vec{p}_1 + p_0(\vec{p}_2)\,\vec{p}_2 + \frac{1}{\kappa}\vec{p}_1 \wedge \vec{p}_2 = \vec{p}_1 + \vec{p}_2 + \frac{1}{\kappa}\vec{p}_1 \wedge \vec{p}_2 + \mathcal{O}(1/\kappa^2) \neq \vec{p}_2 \oplus \vec{p}_1$$

Plane waves = eigenfunctions of *translation generators* P_a

$\Downarrow$

non-abelian composition of momenta = **non-Leibniz action on product of plane waves**

$$P_a(e_{\mathbf{P}_1} \otimes e_{\mathbf{P}_2}) = P_a(e_{\mathbf{P}_1}) \otimes e_{\mathbf{P}_2} + e_{\mathbf{P}_1} \otimes P_a(e_{\mathbf{P}_2}) + \frac{1}{\kappa}\,\epsilon_{abc} P_b(e_{\mathbf{P}_1}) \otimes P_c(e_{\mathbf{P}_2}) + \mathcal{O}(1/\kappa^2)$$

the *smoking gun* of symmetry deformation...P_a belong to a deformed algebra with κ as a **deformation parameter**!

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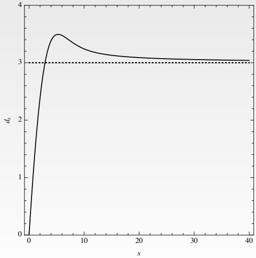

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The non-abelian composition of momenta in "flat slicing" coordinates

$$\begin{aligned}
\eta_0(p_0, \mathbf{p}) &= \kappa \sinh p_0/\kappa + \frac{\mathbf{p}^2}{2\kappa}\, e^{p_0/\kappa}, \\
\eta_i(p_0, \mathbf{p}) &= p_i\, e^{p_0/\kappa}, \\
\eta_4(p_0, \mathbf{p}) &= \kappa \cosh p_0/\kappa - \frac{\mathbf{p}^2}{2\kappa}\, e^{p_0/\kappa}.
\end{aligned}$$

reads $\quad p \oplus q = (p^0 + q^0; \; p^j \; e^{-\frac{q^0}{\kappa}} + q^j)$

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in the limit $\kappa \longrightarrow \infty$ recover ordinary Poincaré algebra

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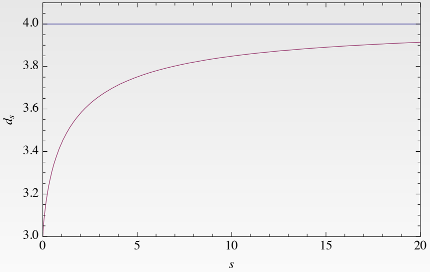

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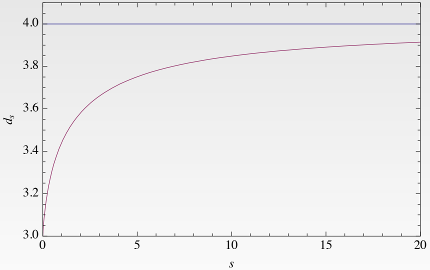

formulate proper Green's function/heat kernel for the theory (work in progress with T. Trezsniewski)

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given n-different modes one has $n!$ different n-particle states, one for each permutation of the n modes $\mathbf{k_1}\,, \mathbf{k_2} \ldots \mathbf{k_n}$

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- e.g. the state superposition of two total "classical" energies $\epsilon_A = \epsilon(\mathbf{k_1}_A) + \epsilon(\mathbf{k_2}_A)$ and $\epsilon_B = \epsilon(\mathbf{k_1}_B) + \epsilon(\mathbf{k_2}_B)$ can be entangled with the additional hidden modes e.g.

$$|\Psi\rangle = 1/\sqrt{2}(|\epsilon_A\rangle \otimes |\uparrow\rangle + |\epsilon_B\rangle \otimes |\downarrow\rangle)$$

...possible consequences for phenomenology?

(MA., D. Benedetti, [arXiv:0809.0889 [hep-th]]. MA., A. Marciano, [arXiv:0707.1329 [hep-th]]. MA, A. Hamma, S. Severini, [arXiv:0806.2145 [hep-th]].)

Conclusions

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