Lattice simulations of SU(2) technicolor models

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Useful review: E. Neil, arXiv:1205.4706.

Outline

Technicolor

A (very) brief introduction to technicolor:

- Techniquarks ψ charged under a new gauge symmetry, which becomes strong at EW scale.
- $\overline{\psi}\psi$ plays role of Higgs field.
- Chiral symmetry of techniquarks breaks spontaneously.
- $\langle\overline{\psi}\psi\rangle$ becomes nozero and breaks EW symmetry.
- Technibaryons can be dark matter.

Extended technicolor

Technicolor gives masses to W and Z bosons, but not to SM fermions. Need *Extended Technicolor*.

- New gauge bosons at a scale M_{ETC} couple SM fermions to techniquarks. Give mass to SM fermions:

$$m \sim \frac{\langle \overline{\psi}\psi \rangle}{M_{ETC}^2}$$

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$$m \sim \frac{\langle \overline{\psi}\psi \rangle}{M_{ETC}^2}$$

- Require $M_{ETC} \sim 10\ \mathrm{TeV}$ to get right SM fermion masses.
- But to avoid flavour-changing neutral currents, need $M_{ETC} \gtrsim 1000\ \mathrm{TeV}$!

Walking technicolor

$$m \sim \frac{\langle \overline{\psi}\psi \rangle_{ETC}}{M_{ETC}^2}$$

Walking technicolor

$$m \sim \frac{\langle \overline{\psi}\psi \rangle_{ETC}}{M_{ETC}^2}$$

But $\langle \overline{\psi}\psi \rangle$ is determined at TC scale, and runs to ETC scale:

$$\langle \overline{\psi}\psi \rangle_{ETC} = \langle \overline{\psi}\psi \rangle_{TC} \, \exp \int_{\Lambda_{TC}}^{M_{ETC}} \frac{d\mu}{\mu} \gamma(\mu)$$

Can running be enhanced?

- Need $\gamma(\mu)$ large over a large range of scales.
- Unlike QCD, where $\gamma(\mu)$ falls rapidly above Λ_{QCD}.
- Need "walking": coupling runs slowly above Λ_{TC} so $\gamma(\mu)$ can be large.

Lattice methods

Minimal requirements for a walking technicolor model:

- Chiral symmetry breaking
- Walking
- Large anomalous dimension

Do any such theories exist? If so, what are their properties?

Nonperturbative question.

- Only known way to address it, controlling all systematic errors, is lattice field theory.

Aims

Aims of lattice calculations:

- Determine phase diagram.
- Search for walking theories.
- Measure γ.
- For promising theories: Measure masses, S-parameter, ...

Space of theories:

- N_c, N_f.
- Representation (can have more than one).
- 4-fermion operators?

This talk: $N_c = 2$.

Systematics

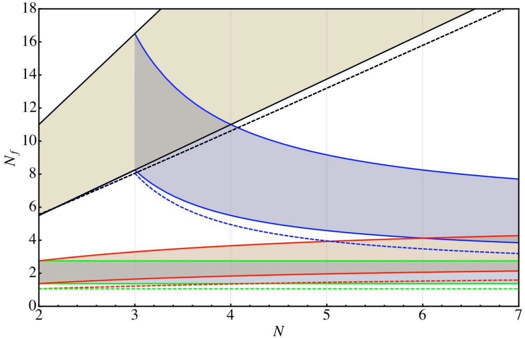
To give conclusive answers, need to control all systematic errors on the lattice. These include:

- Finite mass effects.
- Finite volume effects.
- Discretisation effects.

More subtly, don't know answers, unlike in QCD.

Phase diagram

Phase diagram:



Dietrich and Sannino, Phys Rev D 75 (2007) 085018.

- For $SU(2)$, only need to consider adjoint and fundamental representations.

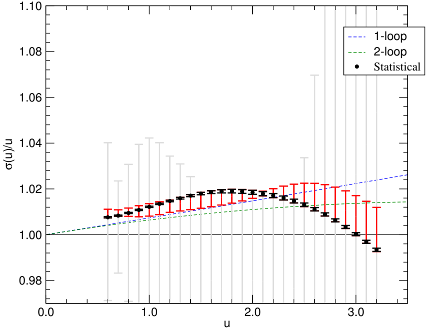
Adjoint fermions

$SU(2)$ with $N_f = 2$ adjoint fermions is "minimal walking technicolor".
Many lattice studies of this model.

- Appears to be conformal.
- Evidence from running of Schrödinger Functional coupling and scaling of spectrum.
- Anomalous dimension also measured.

Running coupling

The β function has been measured in the Schrödinger Functional scheme.



F.B., Del Debbio, Keegan, Pica, Pickup, arXiv: 0910.4535.

- Consistent with similar calculation of Hietanen, Rummukainen & Tuominen, arXiv:0904.0864.

Spectrum

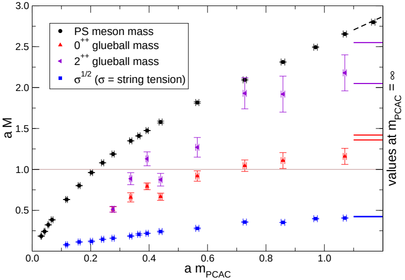
Introduce a small techniquark mass m_q, and measure technihadron masses while taking $m \to 0$.

- χSB: $m_{PS} \propto m_q^{1/2}$, $m_{\text{everything else}} \to \text{finite}$.
- Conformal: $m_{\text{everything}} \propto m_q^{1/(1+\gamma)}$.
- In particular, ratios like $\frac{m_V}{m_{PS}}$ are useful.

In practice, have to make sure results are not contaminated by finite-volume or discretisation effects.

Spectrum

PS meson, glueballs, and string tension:

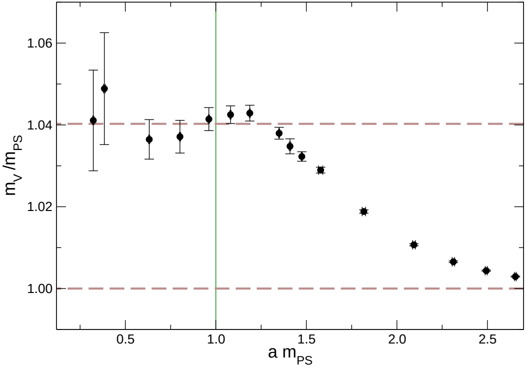


F. B, Del Debbio, Henty, Kerrane, Lucini, Patella, Pica, Pickup & Rago, arXiv:1104.4301.

Very unlike QCD: glueball is lighter than PS.

Ratios

Ratio of vector and PS masses:



Looks conformal. . . .

Anomalous dimension

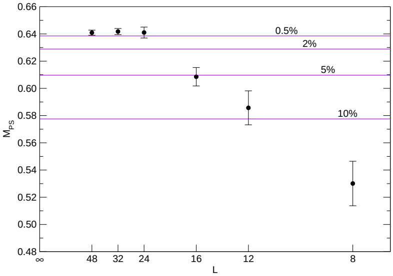
The anomalous dimension has also been measured, by several methods:

- Step-scaling of Z_P: $0.05 < \gamma < 0.56$ [arXiv: 0910.4535], $\gamma = 0.31(6)$ [arXiv:1102.2843].
- Spectrum: Fits are difficult, $\gamma \ll 1$ [arXiv:1011.0607].
- Finite size scaling: $\gamma = 0.22(6)$ [arXiv:1004.3206], $\gamma = 0.51(16)$ [arXiv:1201.6262].
- Spectral density of Dirac operator: $\gamma = 0.371(20)$ [A. Patella, arXiv:1204.4432].
- Monte Carlo renormalisation group: $-0.6 < \gamma < 0.6$ [arXiv:1108.3794].

Consistently much lower than 1, and anyway looks like no χSB. So probably this theory is not very useful for model-building.

Finite volume

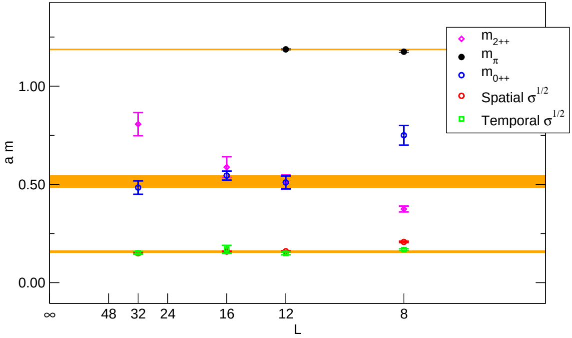
Use this theory to learn how to study conformal / near-conformal theories on the lattice.
E.g. Finite-size errors for PS mass:



Need $m_{PS}L$ at least 12-14. Worse than QCD, where $m_{PS}L = 4 - 5$ is enough.

Gluonic observables

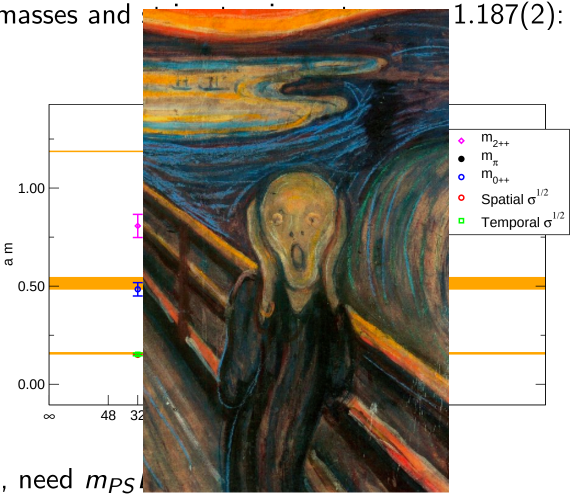
Glueball masses and string tensions at $m_{PS} = 1.187(2)$:



For $m_{2^{++}}$, need $m_{PS}L \geq 30$!

Gluonic observables

Glueball masses and … 1.187(2):



For $m_{2^{++}}$, need m_{PS} …

$SU(2)$ fundamental

For $SU(2)$ fundamental, lose asymptotic freedom at $N_f = 11$.

- Vary N_f to find edge of conformal window, and maybe walking.
- Is γ large, for any N_f?
- $N_f = 2, 4, \ldots, 10$ have been studied.

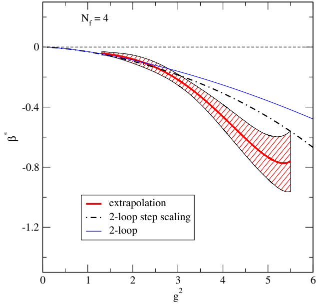
$N_f = 2$

Expect $N_f = 2$ to be QCD-like, with chiral symmetry breaking.

- Studied by Lewis, Pica and Sannino [arXiv:1109.3513].
- See expected pattern of χSB: $SU(4) \to Sp(4)$, with five Goldstone bosons.

$N_f = 4$

The β function has been measured in the Schrödinger Functional scheme.

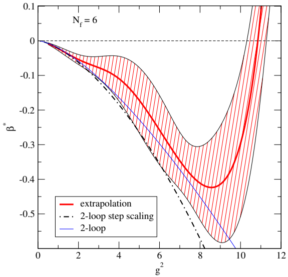


Karavirta, Rantaharju, Rummukainen, & Tuominen, arXiv:1111.4104.

No sign of a fixed point. Presumably still below conformal window.

$N_f = 6$

Running of the Schrödinger Functional coupling:



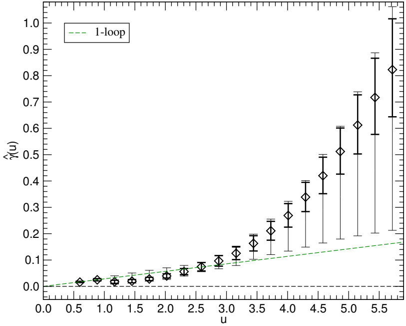
Karavirta, Rantaharju, Rummukainen, & Tuominen, arXiv:1111.4104.

Running slows down. Fixed point? Walking?

- Also measured by F.B., Del Debbio, Keegan, Pica & Pickup [arXiv:1007.3067]. Also see slowing down, again inconclusive.

Anomalous dimension for $N_f = 6$

γ has been measured using step-scaling of Z_P:

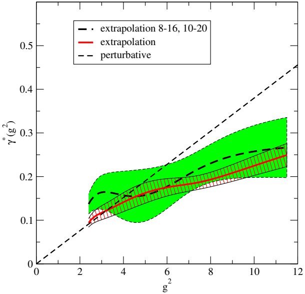


F.B., Del Debbio, Keegan, Pica & Pickup, arXiv:1007.3067.

$0.135 < \gamma < 1.03$. Depends strongly on location (and existence!) of fixed point.

Anomalous dimension for $N_f = 6$

However. . .

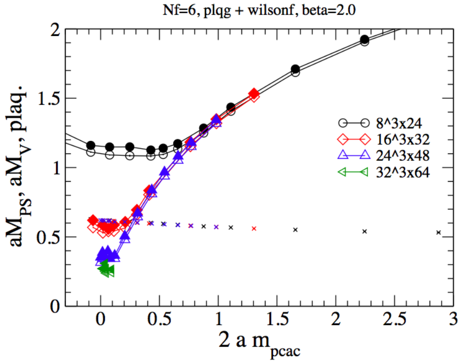


Karavirta, Rantaharju, Rummukainen, & Tuominen, arXiv:1111.4104.

Much lower than previous result. Because of different continuum extrapolations?

Spectrum for $N_f = 6$

Try to use spectrum to determine if $N_f = 6$ is conformal.

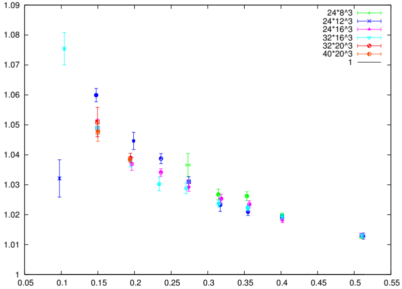


Yamada et al., talk at SGCT12mini, March 2012.

Too early to tell what happens in chiral limit.

Spectrum for $N_f = 6$

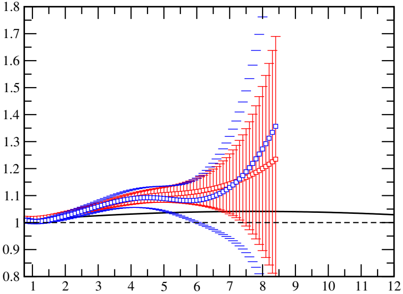
Preliminary results for m_V/m_{PS}:



- Doesn't look conformal: no plateau.
- Not like QCD either.
- Finite volume effects large again. Need $m_{PS}L \geq 12$.

$N_f = 8$

Only one study: running of coupling, in scheme defined using Wilson loops.

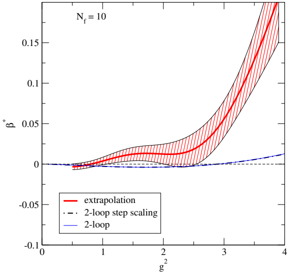


H. Ohki et al., arXiv:1011.0373.

Again, still inconclusive. No measurements of γ at all.

$N_f = 10$

Expect $N_f = 10$ to be conformal, with perturbative fixed point.
Running of the Schrödinger Functional coupling:



Karavirta, Rantaharju, Rummukainen, & Tuominen, arXiv:1111.4104.

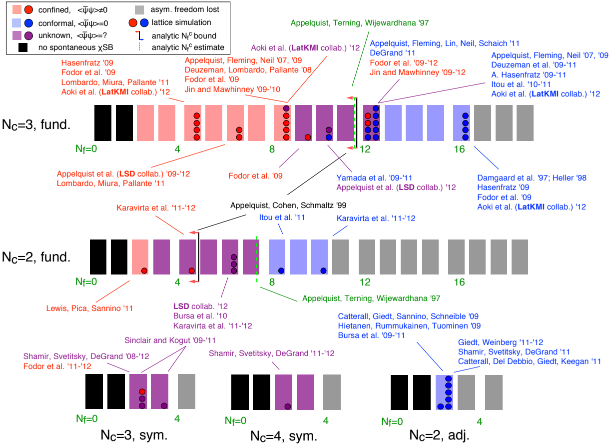
Must be negative at small coupling, so there must be a fixed point.

Conclusions

Technicolor on the lattice is making good progress.

- $SU(2)$ with two adjoint fermions is probably conformal, and has small γ anyway.
- For fundamental fermions, the conformal window starts at $N_f = 6$ or $N_f = 8$. γ *might* be large.
- Finite volume effects often big, and finding a walking theory needs high accuracy.
- It turns out lattice QCD was easy!

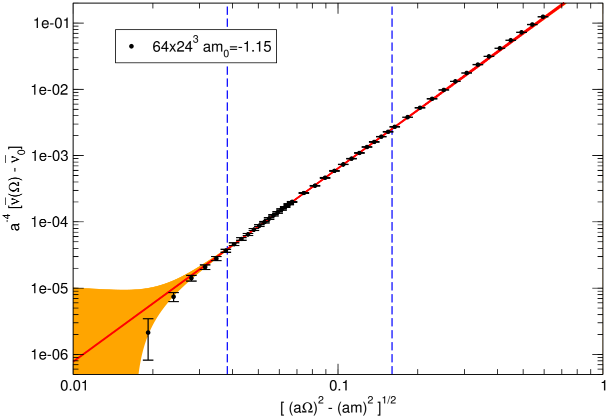
Bonus materials



E. Neil, arXiv:1205.4706.

Bonus materials

Anomalous dimension from spectral density:



$$\nu(\Omega) - \nu_0 \propto (\Omega^2 - m^2)^{\frac{2}{1+\gamma}}.$$