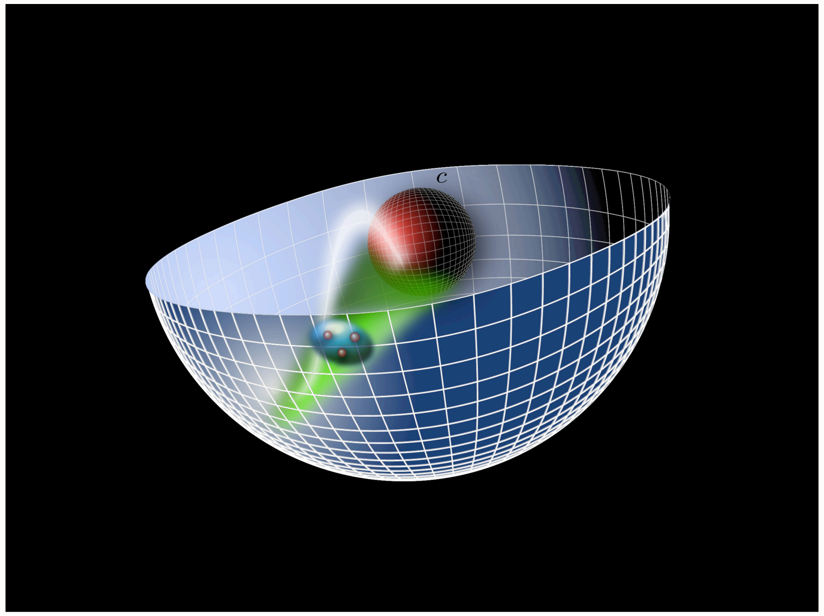
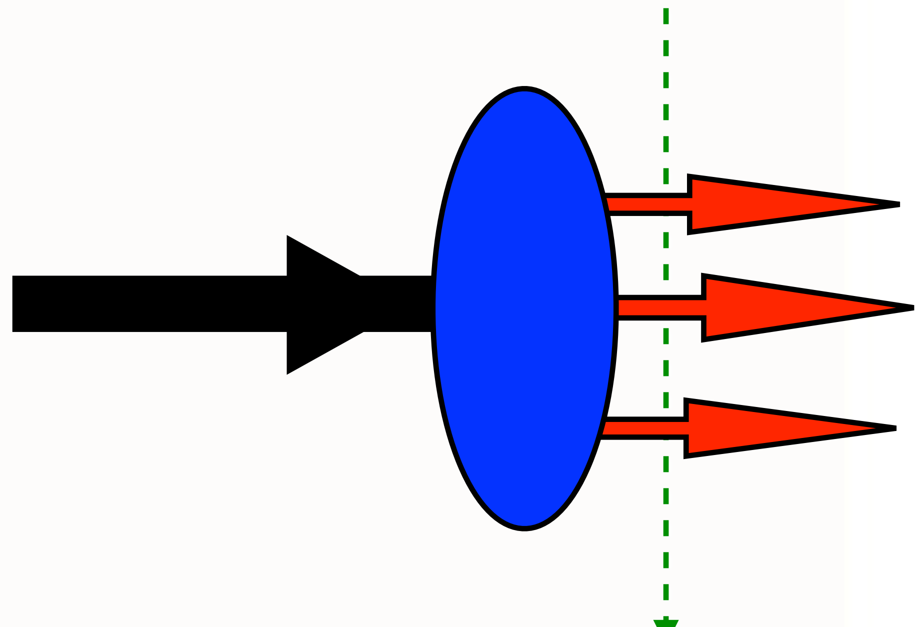
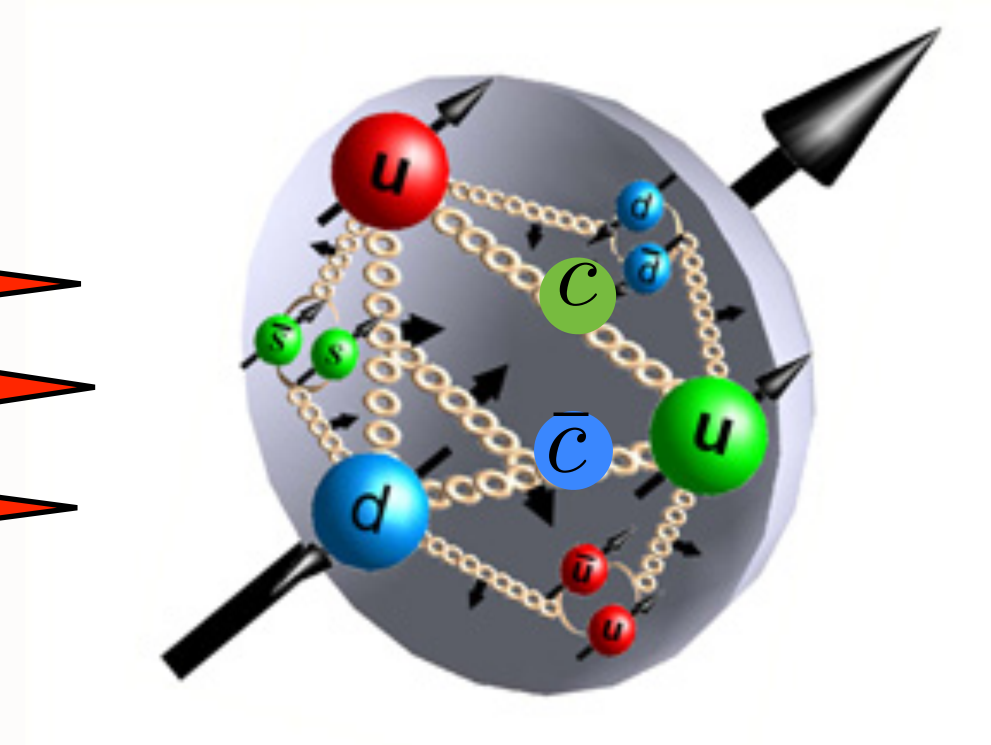
AdS/QCD and Light-Front Holography: New Perspectives for QCD and the Light-Front Vacuum





Fixed $\tau = t + z/c$



Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

NORDITA

Conference on the Origin of Mass 2012

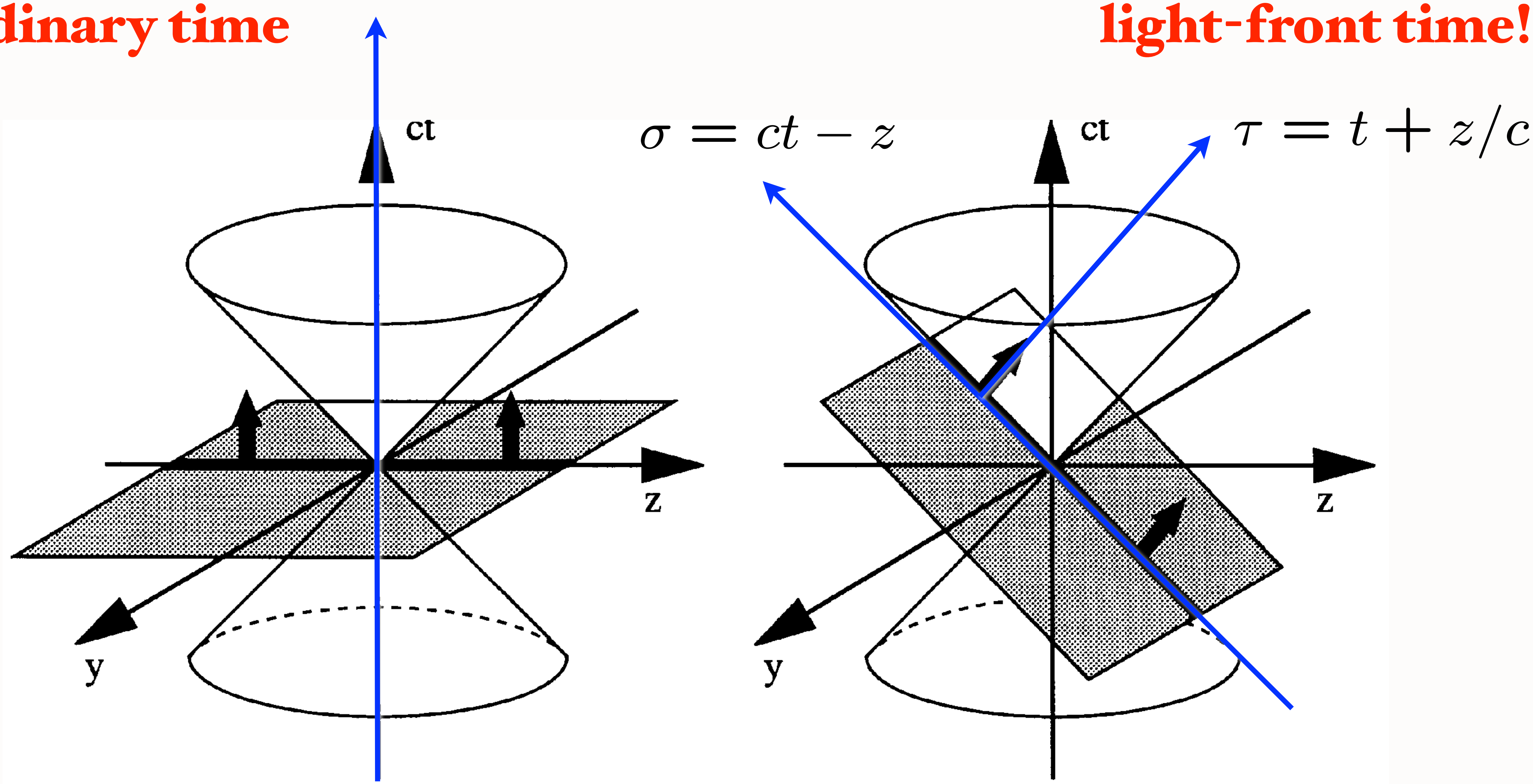
11-17 June 2012

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Dirac's Amazing Idea: The Front Form

Evolve in ordinary time

Evolve in light-front time!



Instant Form **Front Form**

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

Eigenstate -- independent of τ

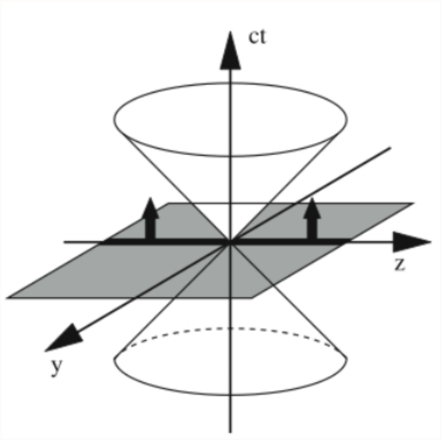
Measurements never at fixed time t

HELEN BRADLEY - PHOTOGRAPHY

- Different possibilities to parametrize space-time [Dirac (1949)]

- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different "times" and has its own Hamiltonian, but should give the same physical results

- *Instant form*: hypersurface defined by $t = 0$, the familiar one



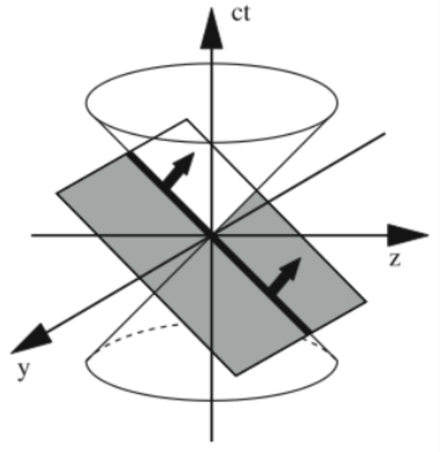
- *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$

$x^+ = x^0 + x^3$ light-front time

$x^- = x^0 - x^3$ longitudinal space variable

$k^+ = k^0 + k^3$ longitudinal momentum $(k^+ > 0)$

$k^- = k^0 - k^3$ light-front energy



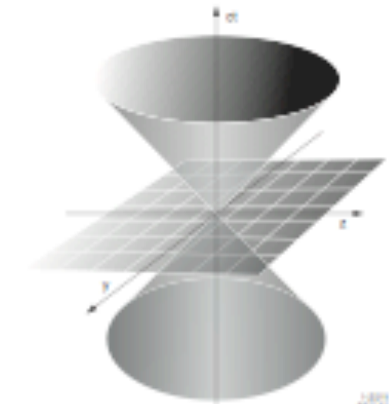
$$k \cdot x = \frac{1}{2}\left(k^+ x^- + k^- x^+\right) - \mathbf{k}_\perp \cdot \mathbf{x}_\perp$$

On shell relation $k^2 = m^2$ leads to dispersion relation $k^- = \frac{\mathbf{k}_\perp^2 + m^2}{k^+}$

Quantum chromodynamics and other field theories on the light cone.
Stanley J. Brodsky (SLAC), Hans-Christian Pauli (Heidelberg, Max Planck Inst.),
Stephen S. Pinsky (Ohio State U.). SLAC-PUB-7484, MPIH-V1-1997. Apr 1997. 203 pp.
Published in **Phys.Rept. 301 (1998) 299-486**
e-Print: **hep-ph/9705477**

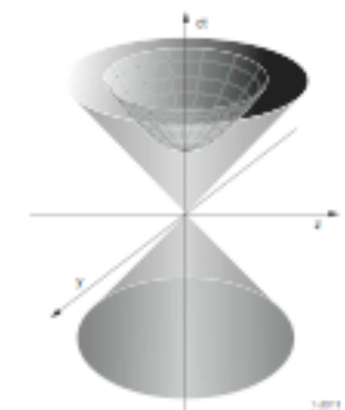
Instant Form vs. Front Form

- Different possibilities to parametrize space-time [Dirac (1949)]
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different "times" and has its own Hamiltonian, but should give the same physical results
- Forms of Relativistic Dynamics: dynamical *vs.* kinematical generators [Dirac (1949)]
- *Instant form*: hypersurface defined by $t = 0$, the familiar one



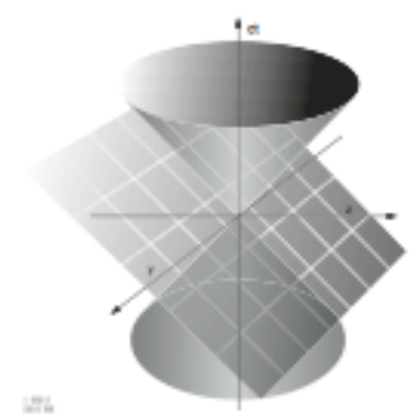
$$H,\ \mathbf{K}\ \text{dynamical},\qquad \mathbf{L},\ \mathbf{P}\ \text{kinematical}$$

- *Point form*: hypersurface is an hyperboloid



$$P^{\mu}\ \text{dynamical},\qquad M^{\mu\nu}\ \text{kinematical}$$

- *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$

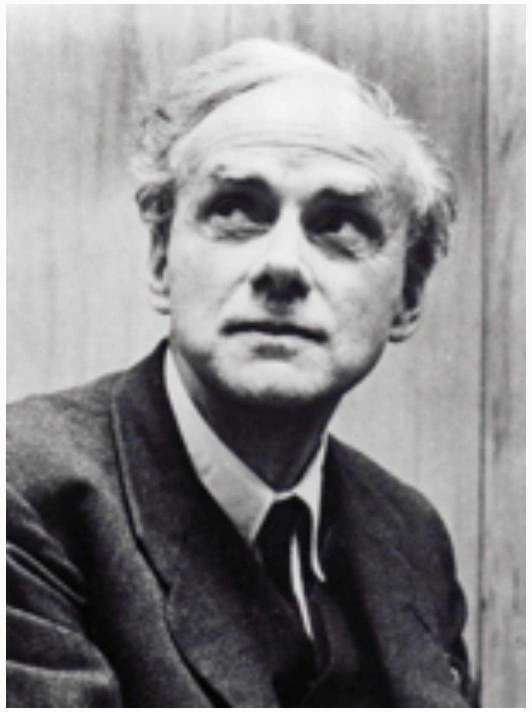


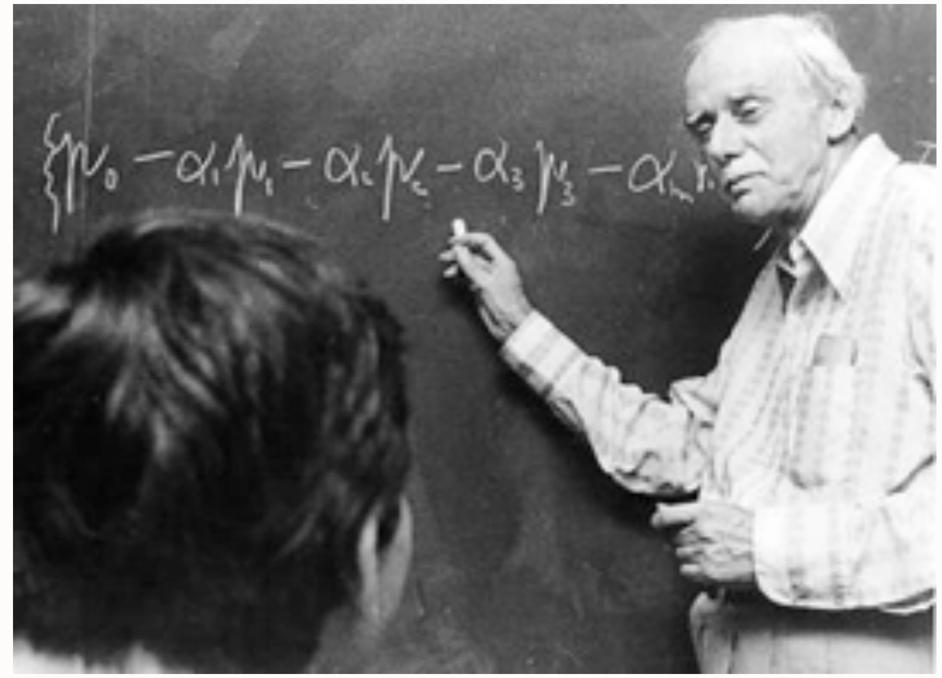
$$P^{-},\ L^{x},\ L^{y}\ \text{dynamical},\qquad P^{+},\ \mathbf{P}_{\perp},\ L^{z},\ \mathbf{K}\ \text{kinematical}$$

$P^{\pm} = P^{0} \pm P^{3}$

Causal!

States are eigenstates of invariant mass





"Working with a front is a process that is unfamiliar to physicists.

But still I feel that the mathematical simplification that it introduces is all-important.

I consider the method to be promising and have recently been making an extensive study of it.

It offers new opportunities, while the familiar instant form seems to be played out " - P.A.M. Dirac (1977)

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

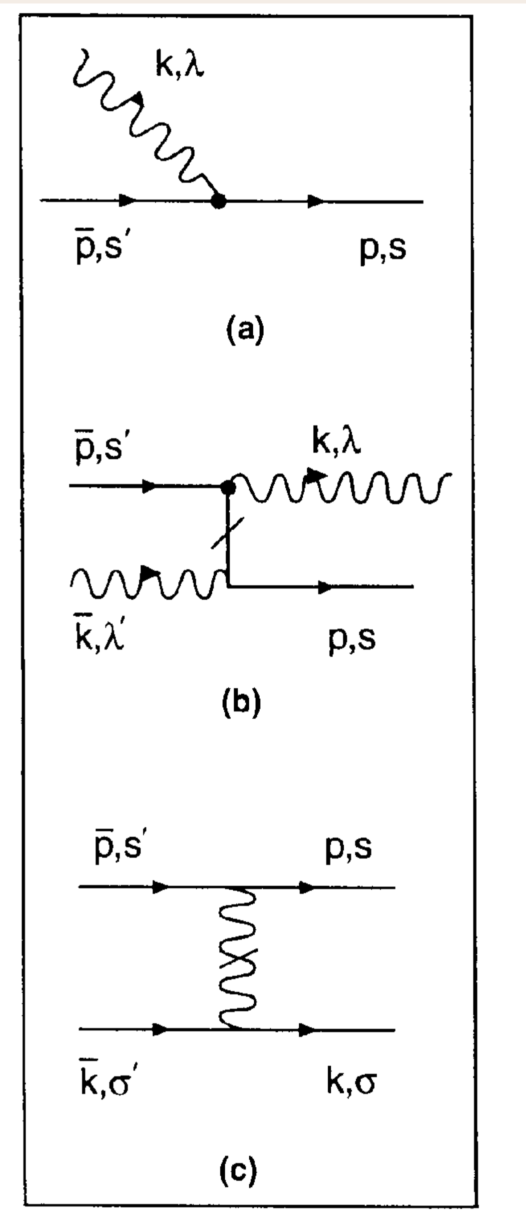
$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

H^{int}_{LF}: Matrix in Fock Space

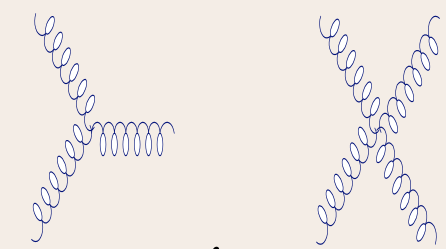
$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

$$|p, S_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions



LFWFs: Off-shell in P- and invariant mass



H^{int}_{LF}

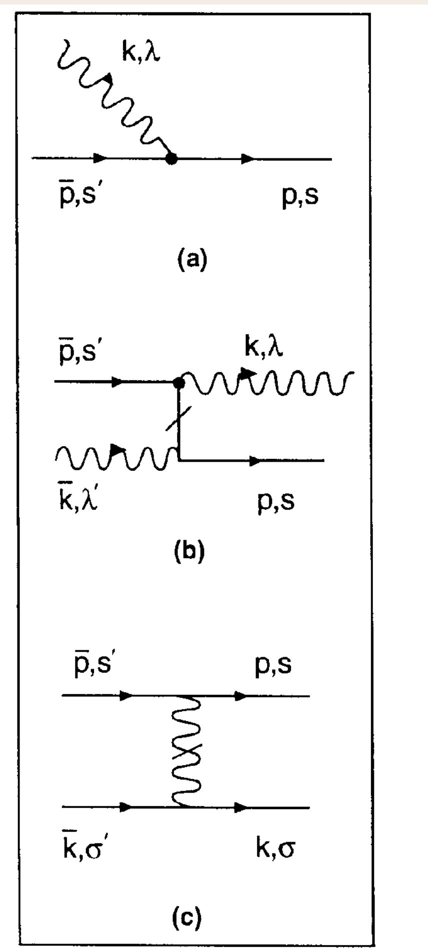
Light-Front QCD

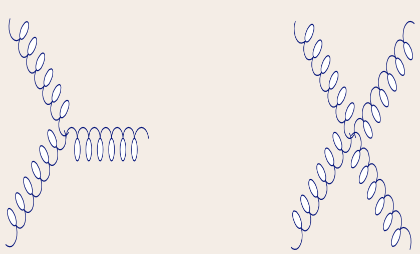
Heisenberg Matrix Formulation

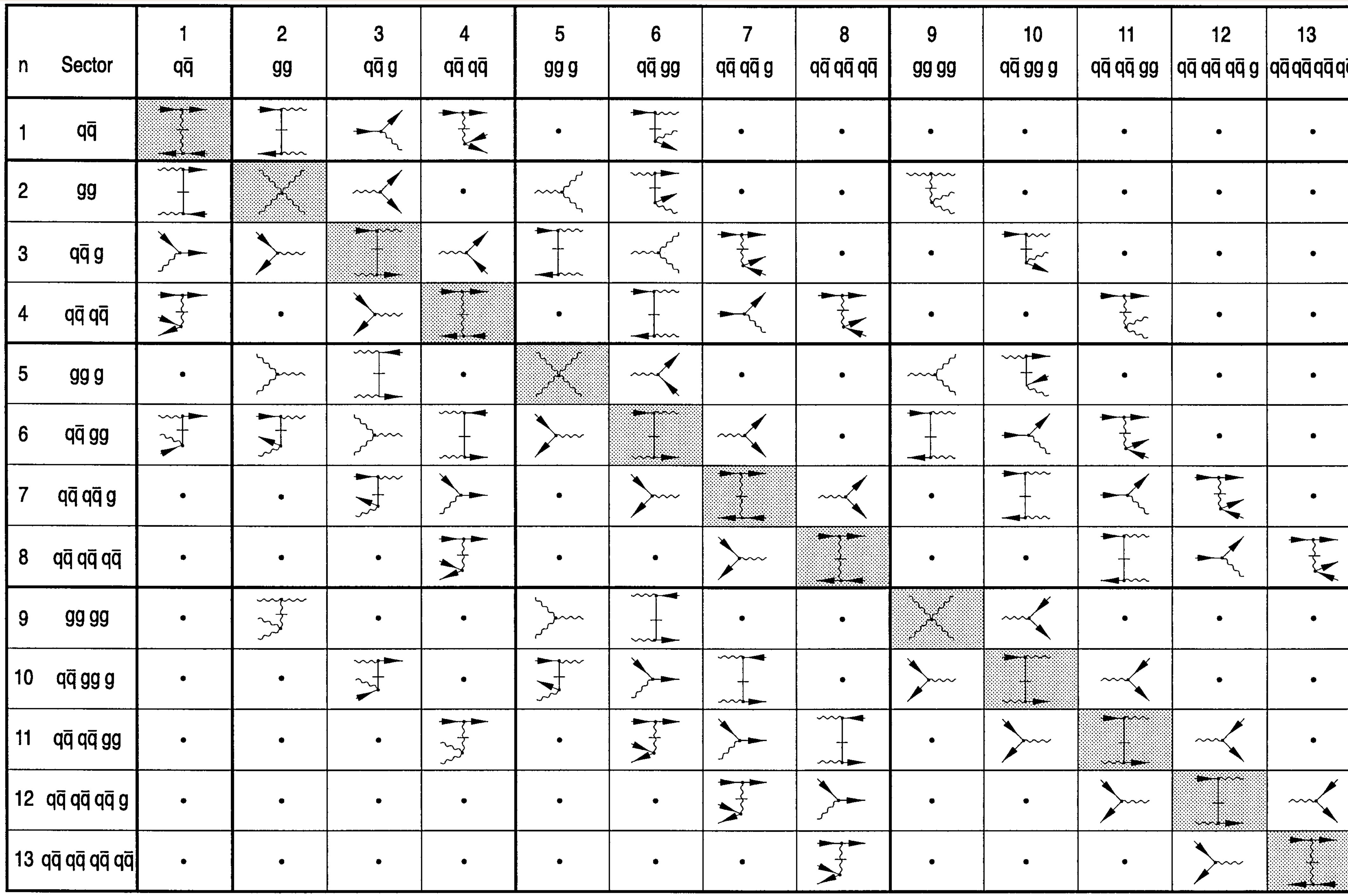
$$H^{QCD}_{LC}\,|\Psi_h\rangle = \mathcal{M}_h^2\,|\Psi_h\rangle$$

DLCQ

Discretized Light-Cone Quantization







Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

Pauli, Hornbostel & sjb

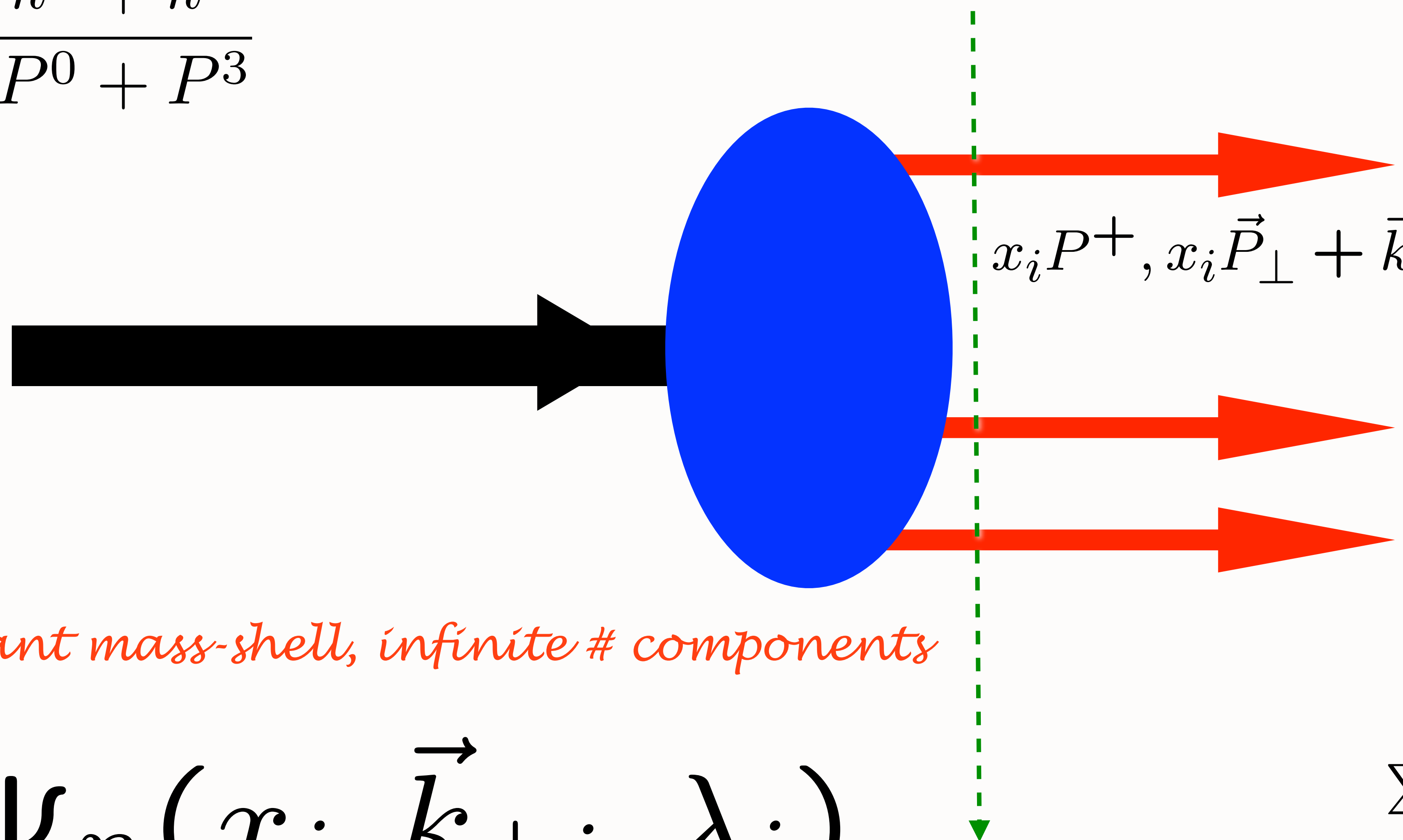
e.g. solve QCD(1+1): arbitrary color, flavor, quark mass

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$

$P^+, \vec{P}_\perp$



$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

LFWFs: off invariant mass-shell, infinite # components

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

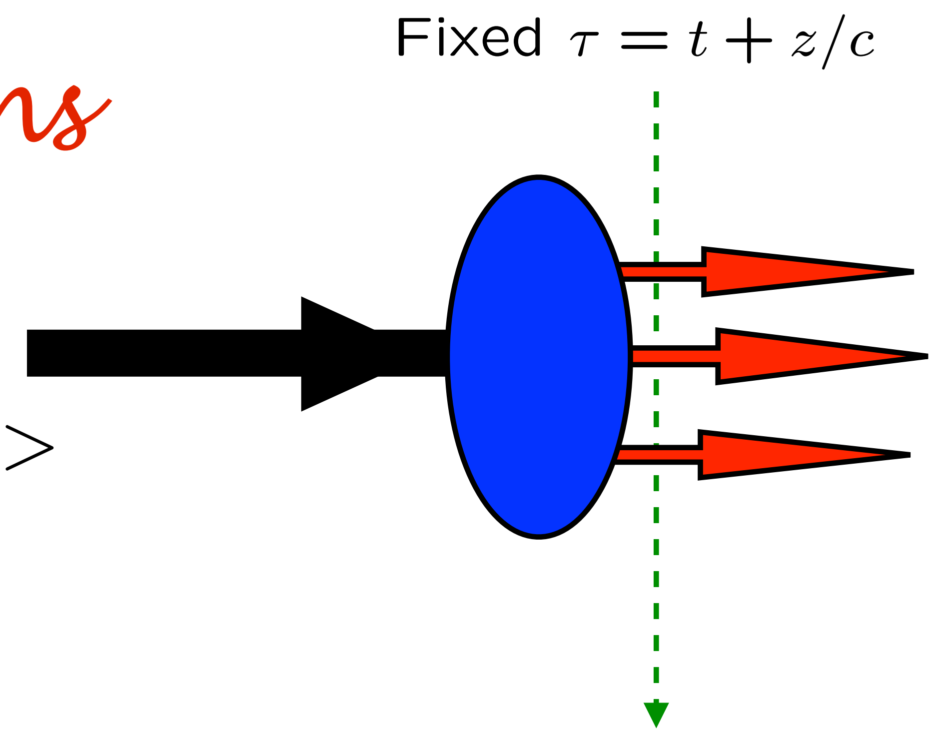
Conserved in each LF Fock state

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Nonzero Anomalous Moment -->Nonzero orbital angular momentum

Light-Front Wavefunctions



$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

$$|p, S_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

- **Eigenfunctions of the exact QCD LF Hamiltonian**
- **Boost invariant! Independent of P^+, $P_\perp$**
- **Compute all observables intrinsic to hadron from LFWFs**
- **Form factors, structure functions, GPDs, transverse momentum distributions**
- **DGLAP and ERBL Evolution Built In**
- **No renormalization scale ambiguity: "Principle of Maximal Conformality"**
- **LF Vacuum Trivial: In-Hadron Condensates -- Eliminate 10^{45} discrepancy with cosmological constant**
- **Pseudo-T-odd observables from Lensing**
- **Angular Momentum Sum Rule for each Fock state**

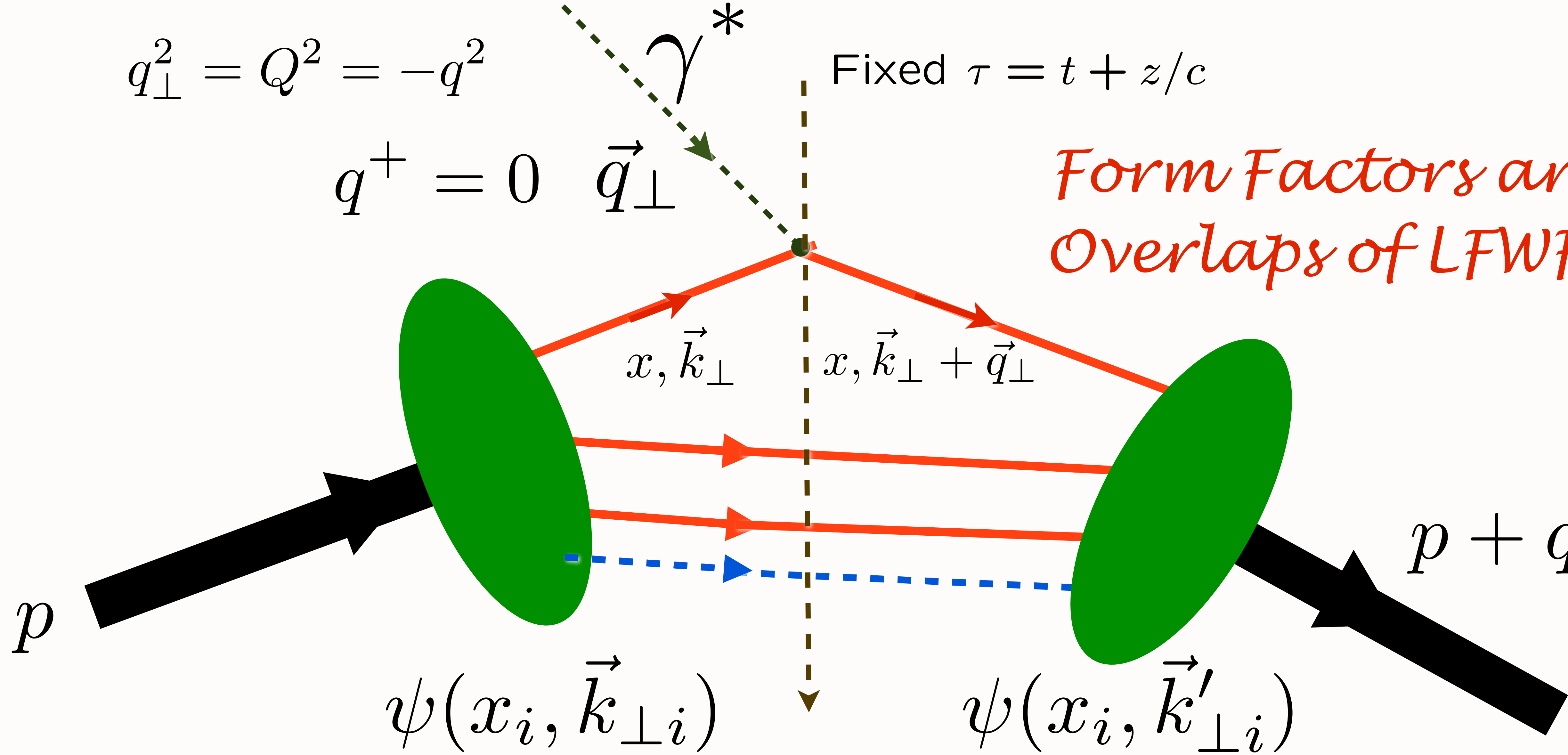
Non-Perturbative QCD: Diagonalize the LF Hamiltonian

- Frame-Independent
- No Fermion-Doubling
- Minkowski not Euclidian space
- Dynamical, positive-metric gluons
- No restriction on quark masses
- Complete spectrum
- Tested in color-confining low-dimension theories
- Simple Causal LF Vacuum
- Calculate observables from LFWFs

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Interaction picture

Form Factors are Overlaps of LFWFs



struck $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell, Yan; West

Exact LF Formula for Pauli Form Factor

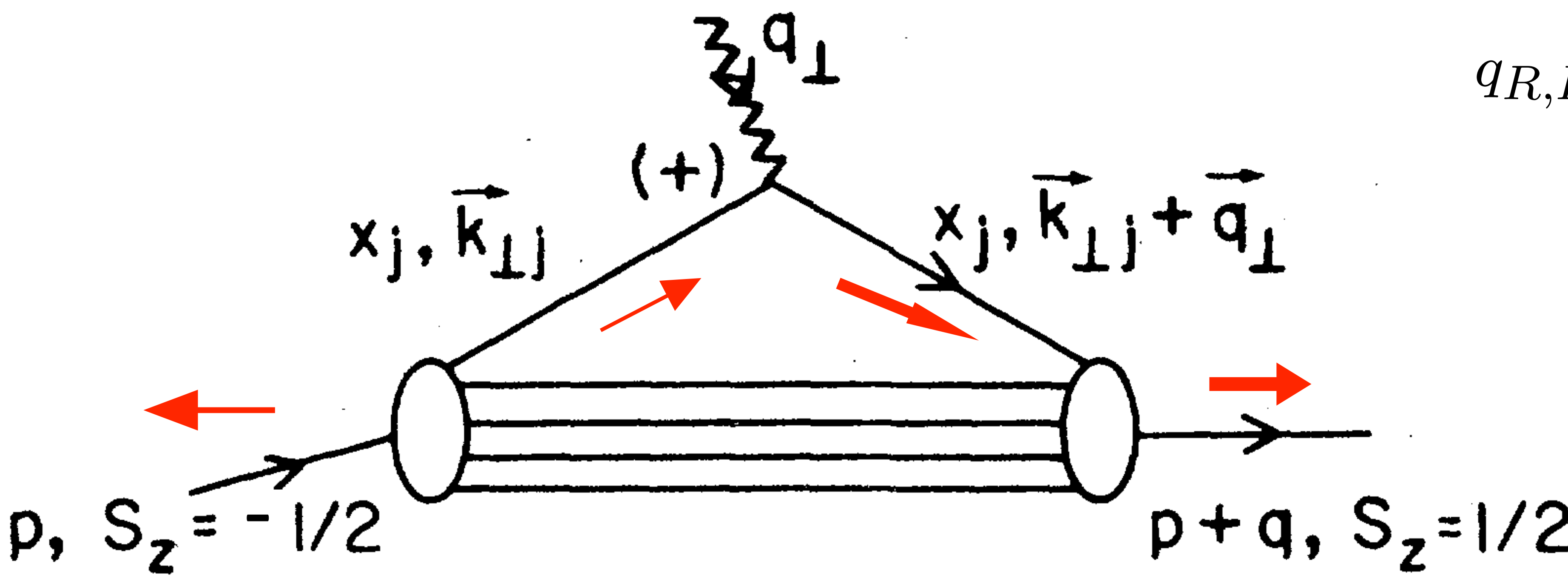
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$

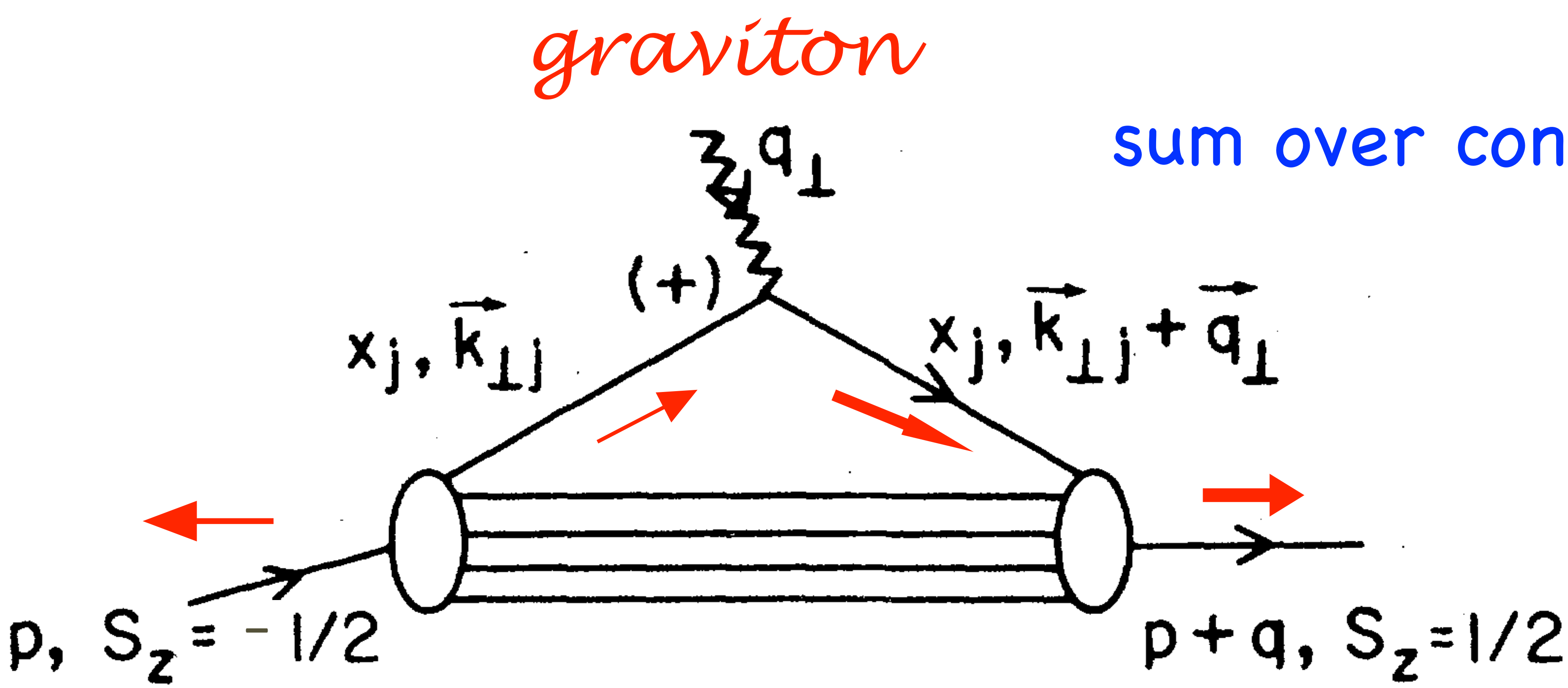


$$\boxed{\text{Must have } \Delta\ell_z = \pm 1 \text{ to have nonzero } F_2(q^2)}$$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Anomalous gravitomagnetic moment B(0)

Terayev, Okun, et al: *B(0) Must vanish because of Equivalence Theorem!*



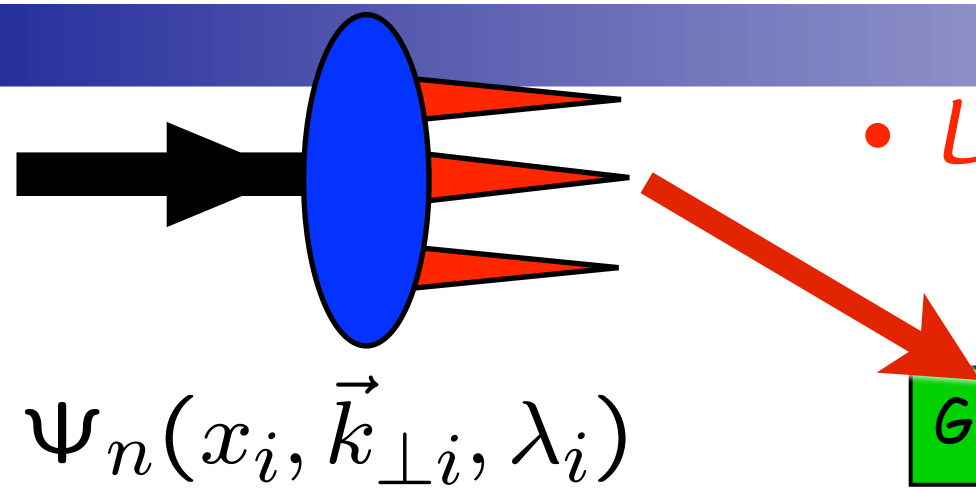
sum over constituents

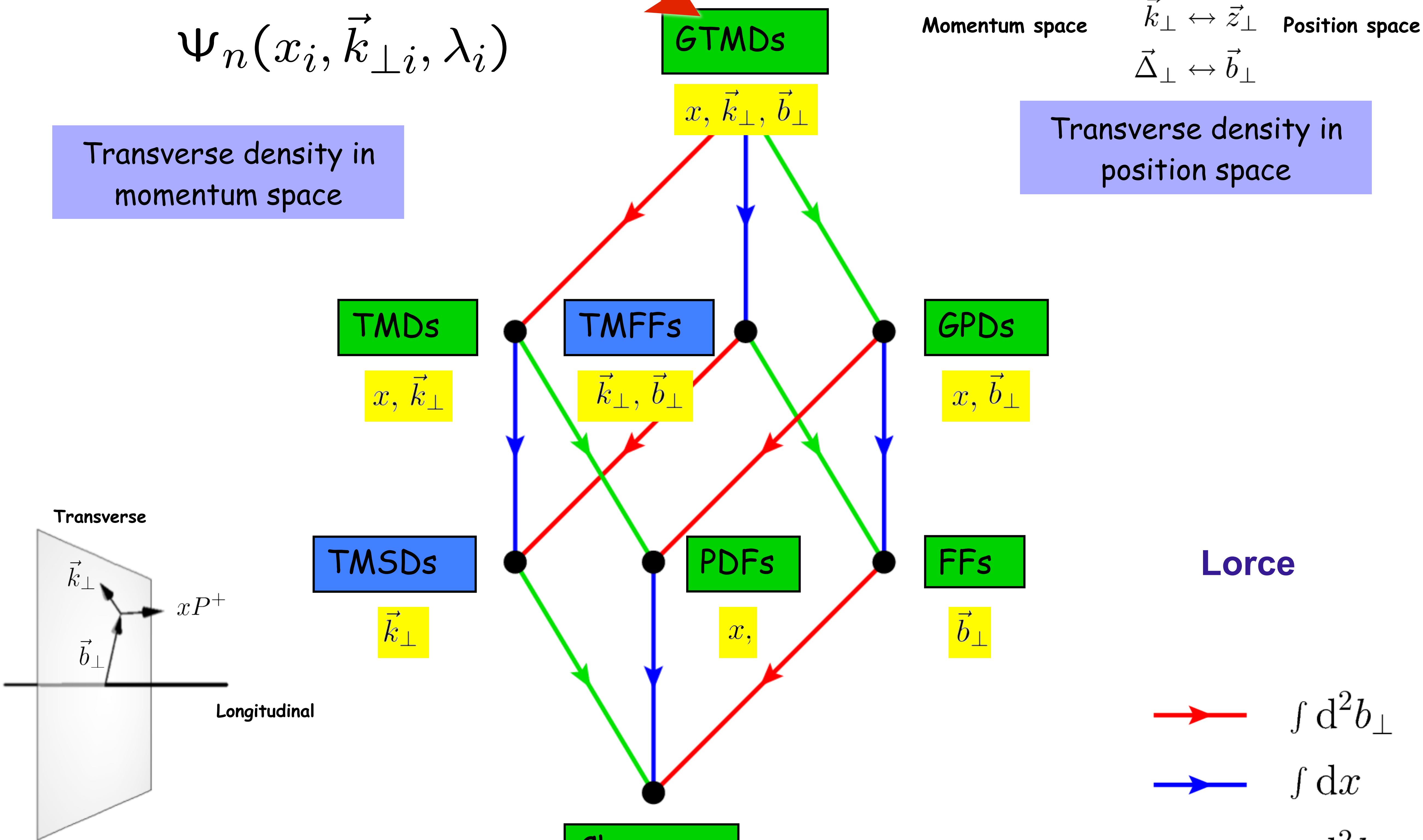
Hwang, Schmidt, Ma, sjb

$$\boxed{B(0) = 0}$$

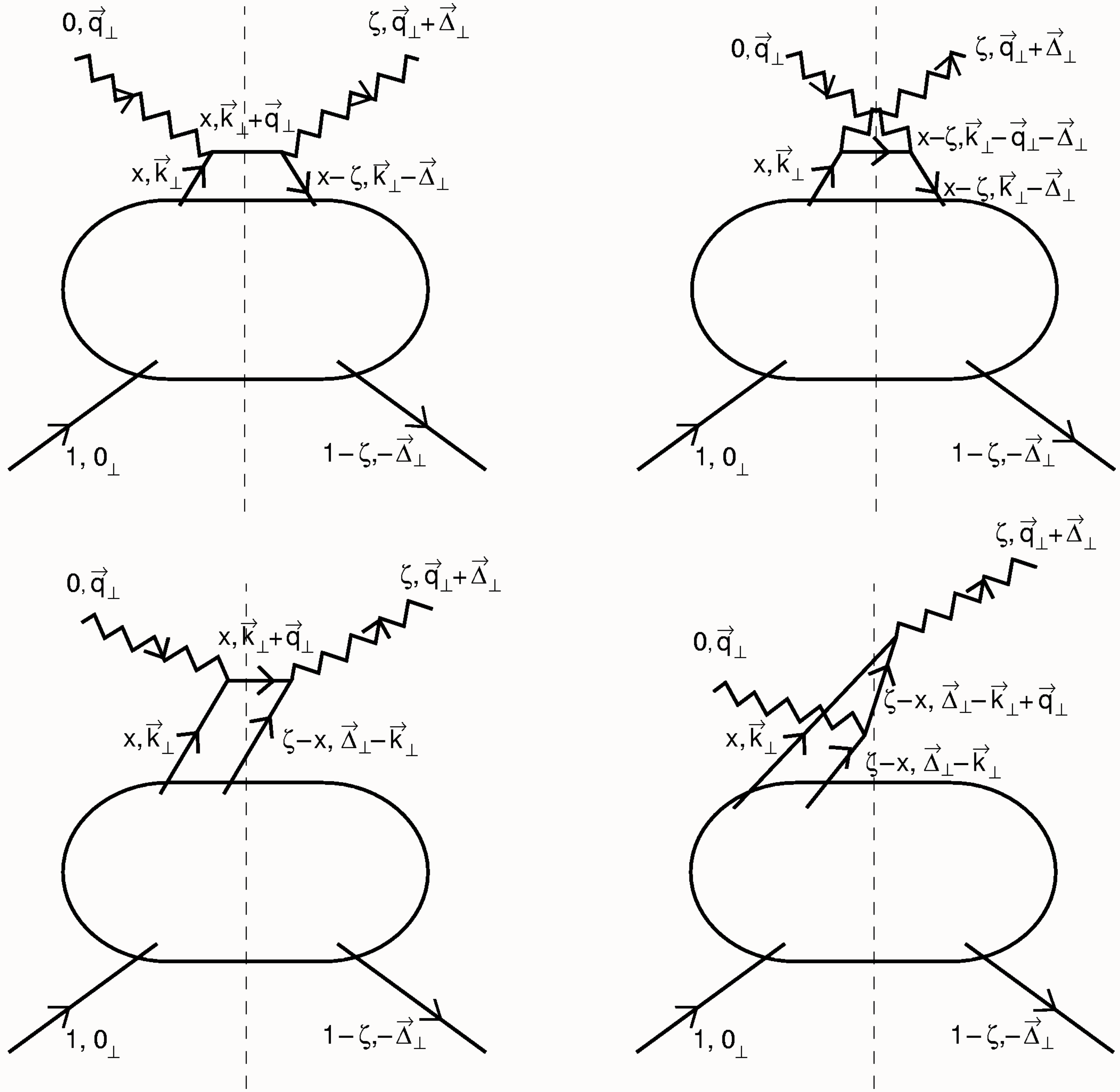
Each Fock State

• *Light Front Wavefunctions:*





Sivers, T-odd from lensing



Light-cone wavefunction representation of deeply virtual Compton scattering ☆

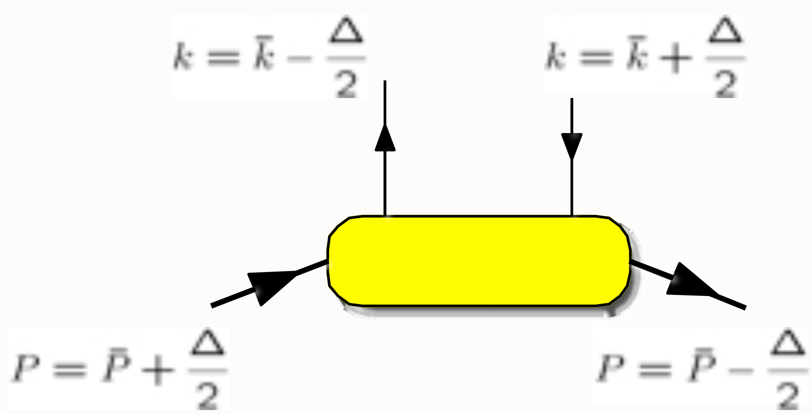
Stanley J. Brodsky [a], Markus Diehl [a,1], Dae Sung Hwang [b]

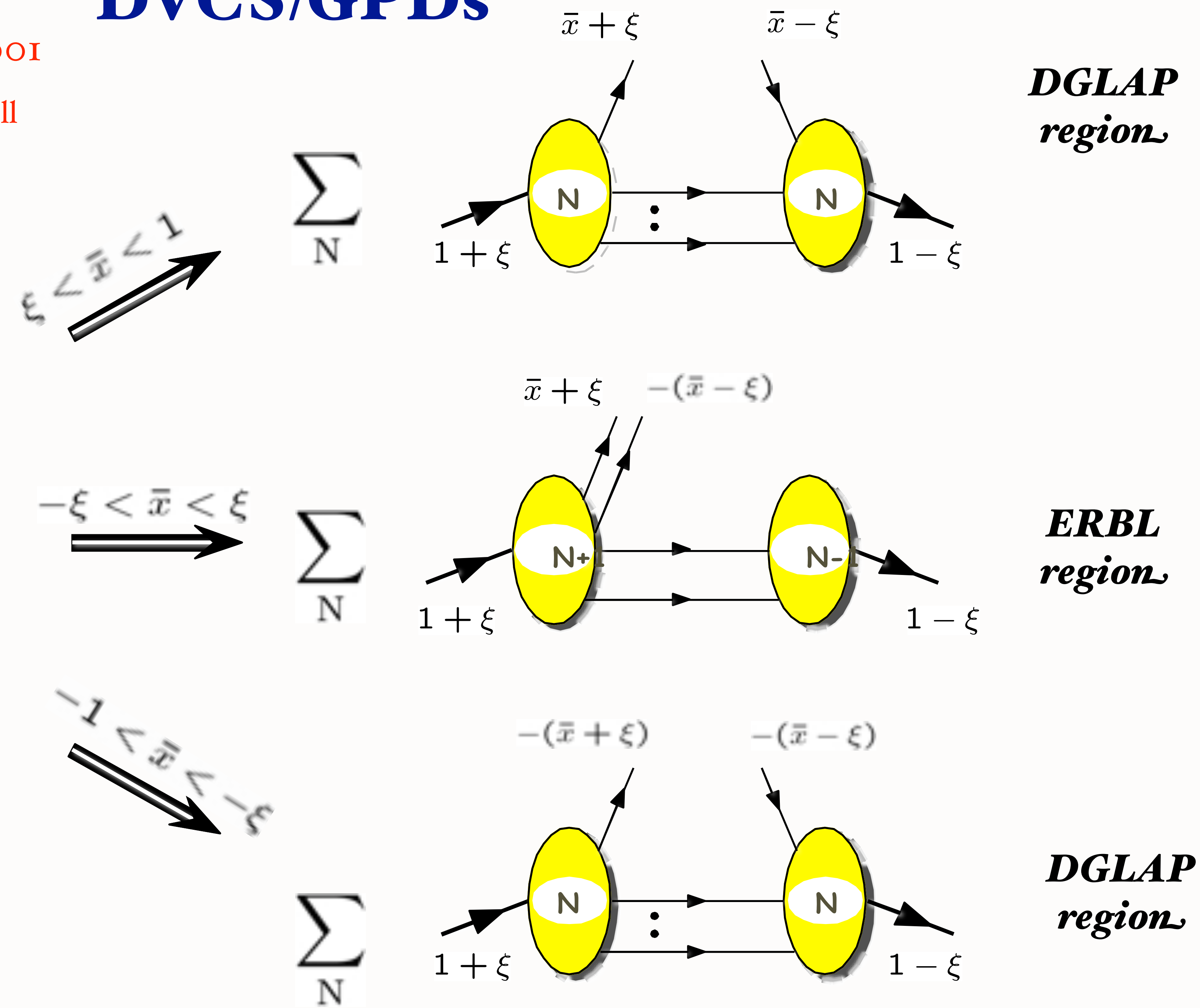
Light-Front Wave Function Overlap Representation

DVCS/GPDs

Diehl, Hwang, sjb, NPB596, 2001

See also: Diehl, Feldmann, Jakob, Kroll





Bakker & JI
Lorce

Example of LFWF representation of GPDs (n => n)

Diehl, Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}} \frac{\Delta^1 - i\,\Delta^2}{2M} E_{(n\to n)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{2-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3}\, 16\pi^3 \delta\left(1 - \sum_{j=1}^{n} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n} \vec{k}_{\perp j}\right)$$

$$\times\, \delta(x - x_1)\, \psi^{\uparrow *}_{(n)}(x'_i, \vec{k}'_{\perp i}, \lambda_i)\, \psi^{\downarrow}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i),$$

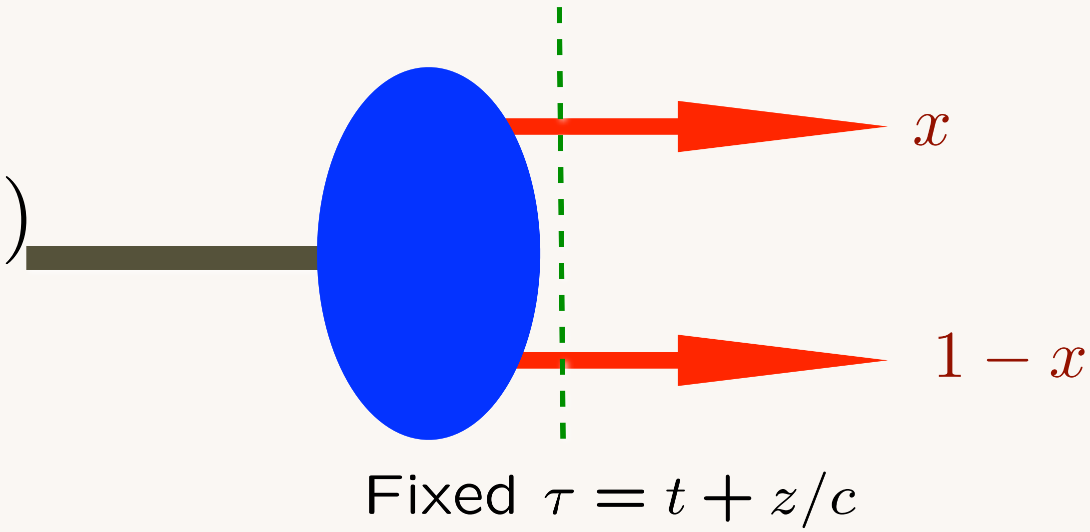
where the arguments of the final-state wavefunction are given by

$$x'_1 = \frac{x_1 - \zeta}{1-\zeta}, \qquad \vec{k}'_{\perp 1} = \vec{k}_{\perp 1} - \frac{1-x_1}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the struck quark,}$$

$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the spectators } i = 2, \ldots, n.$$

Hadron Distribution Amplitudes

$$\phi_M(x, Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x, \vec{k}_\perp)$$



$k_\perp^2 < Q^2$

$\sum_i x_i = 1$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

Lepage, sjb

- Evolution Equations from PQCD, OPE

Lepage, sjb
Efremov, Radyushkin
Sachrajda, Frishman Lepage, sjb

- Conformal Invariance

Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge

Single-spin asymmetries

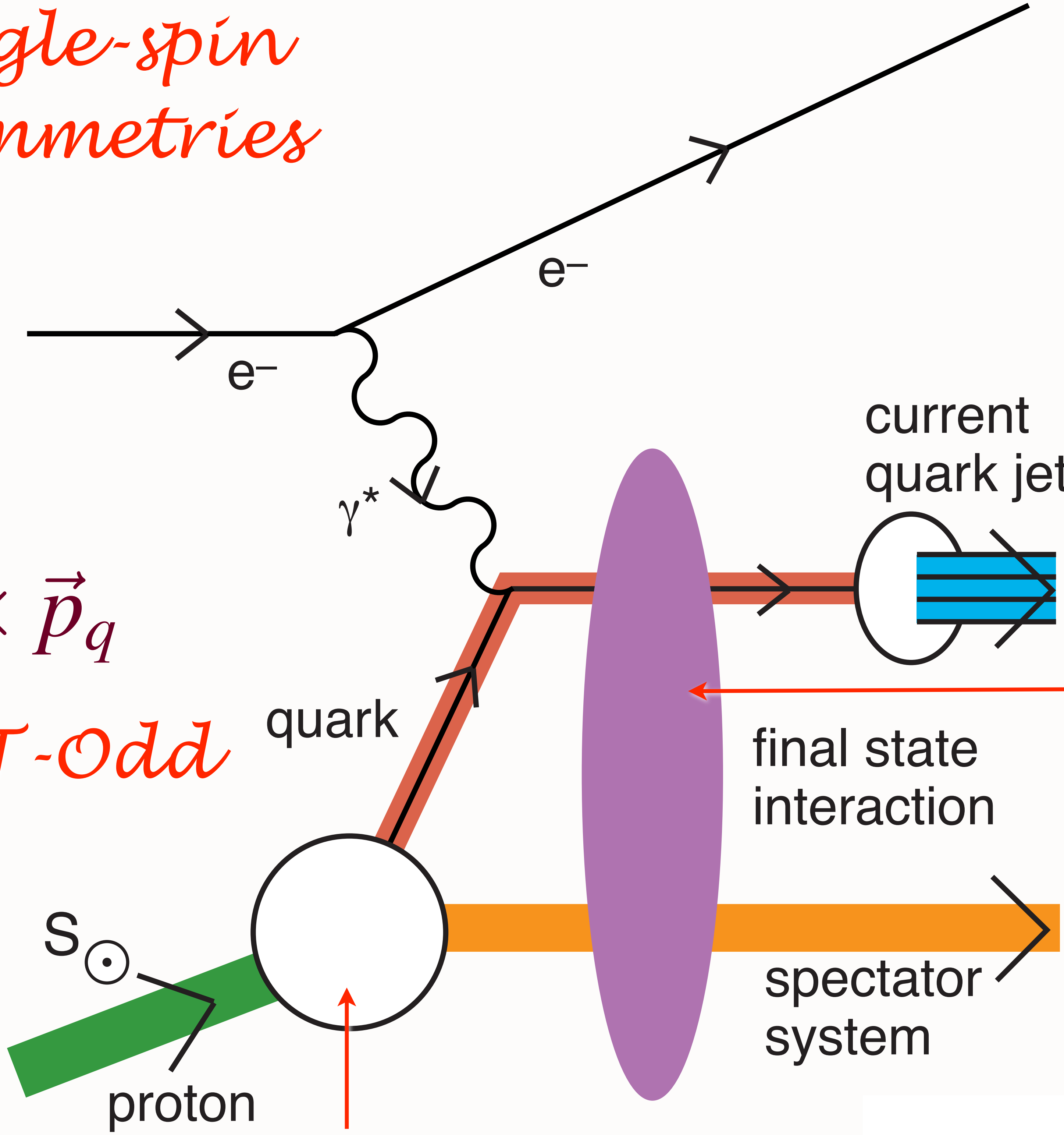
Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt, Ji, Yuan. Pasquini, ...

i $\vec{S}_p \cdot \vec{q} \times \vec{p}_q$

Pseudo- T-Odd



QCD S- and P-
Coulomb Phases
--Wilson Line

"Lensing Effect"

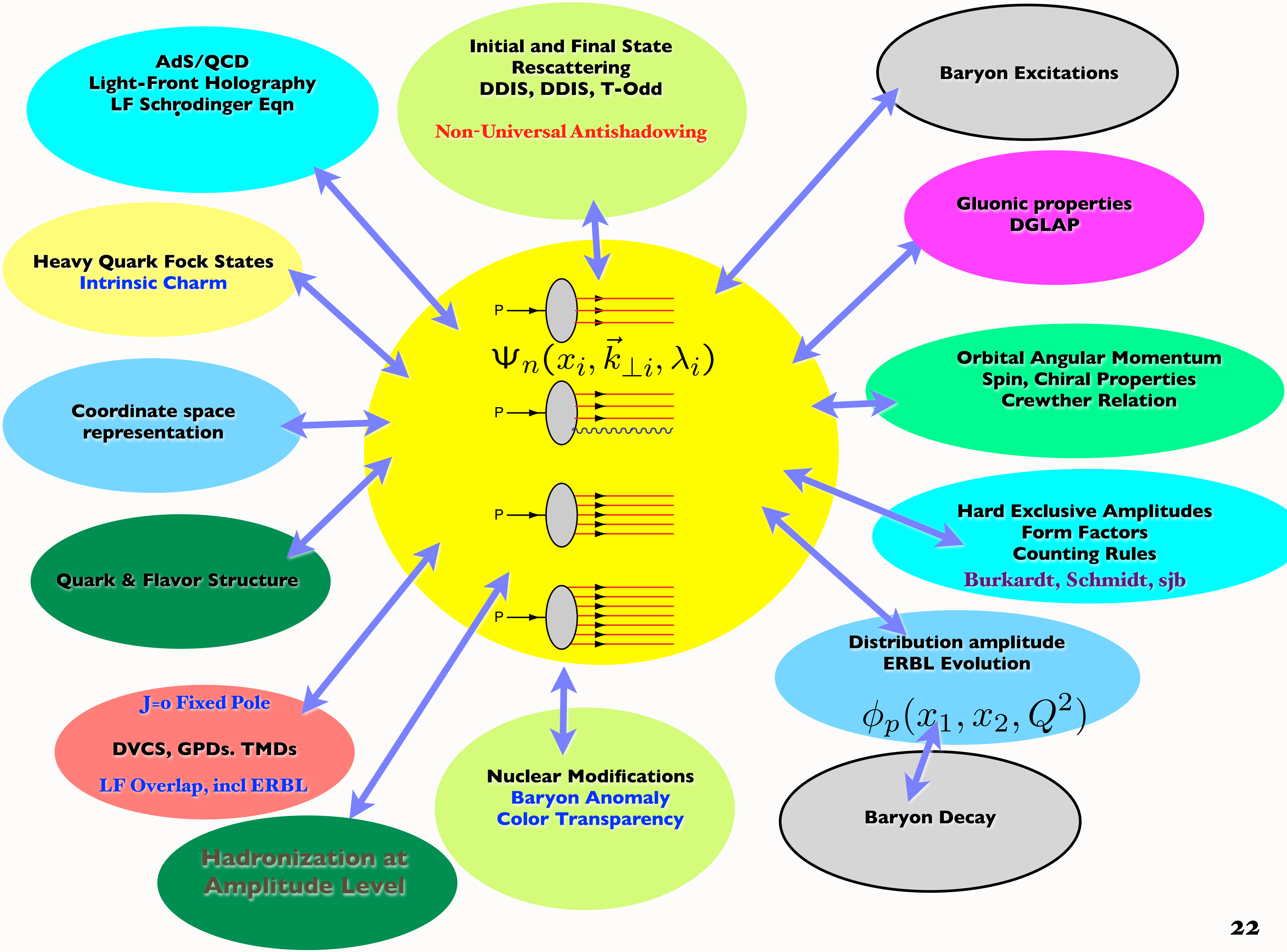
QED: Lensing involves soft scales

Light-Front Wavefunction
S and P- Waves!

Leading-Twist Rescattering Violates pQCD Factorization!

Sign reversal in DY!

QCD and LF Hadron Wavefunctions



Static | Dynamic

Static	Dynamic
Square of Target LFWFs	Modified by Rescattering: ISI & FSI
No Wilson Line	Contains Wilson Line, Phases
Probability Distributions	No Probabilistic Interpretation
Process-Independent	Process-Dependent - From Collision
T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
Sum Rules: Momentum and J^z	Sum Rules Not Proven
DGLAP Evolution; mod. at large x	DGLAP Evolution
No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS

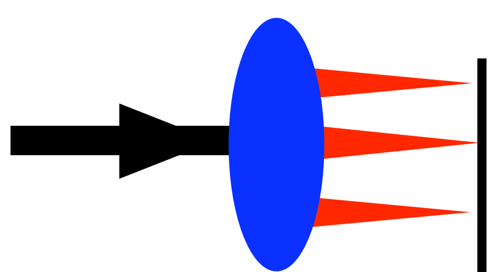
Hwang, Schmidt, sjb,

Mulders, Boer

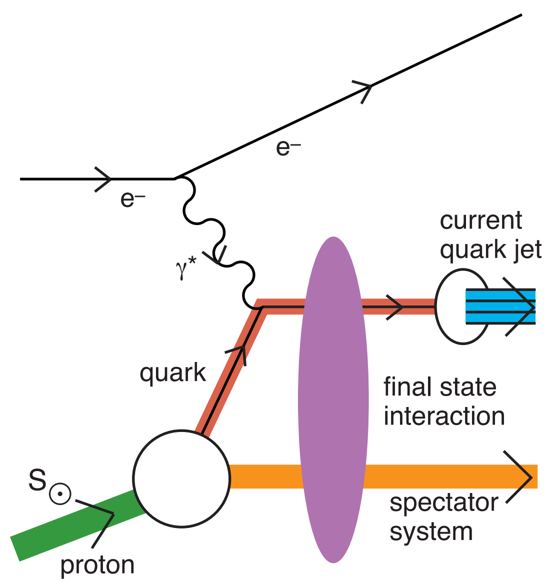
Qiu, Sterman

Collins, Qiu

Pasquini, Xiao, Yuan, sjb

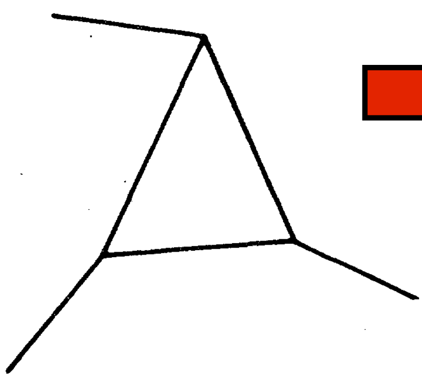


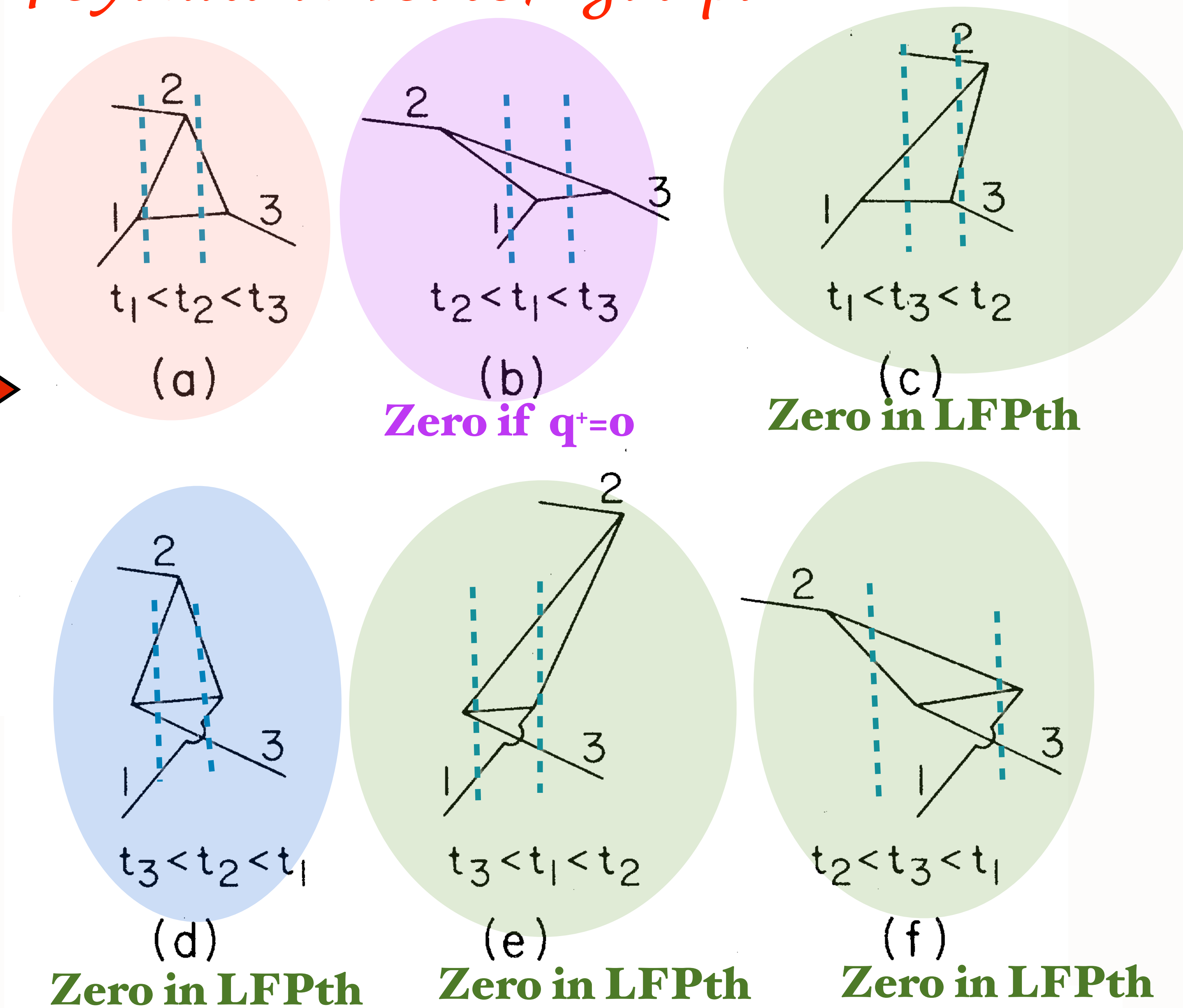
$$\left| \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \right|^2$$



The surviving LF time-ordered contributions to the Feynman vertex graph

Feynman



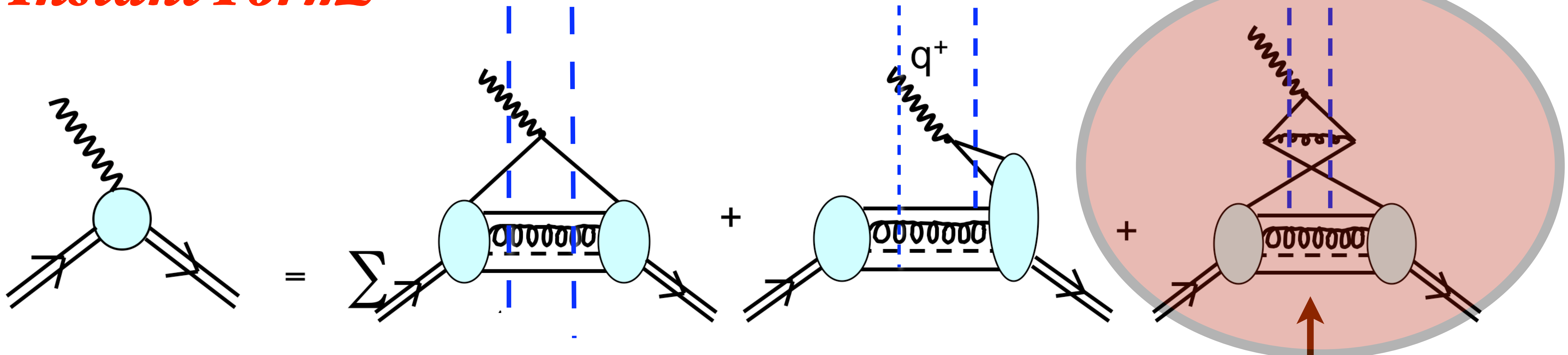


$$k^+ = k^0 + k^z \geq 0$$

Time flows from left to right

Calculation of Form Factors in Equal-Time Theory

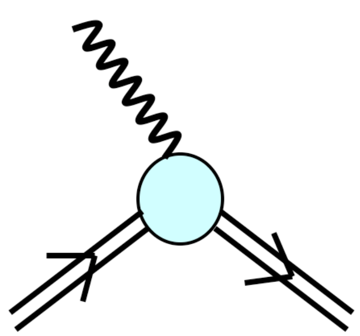
Instant Form

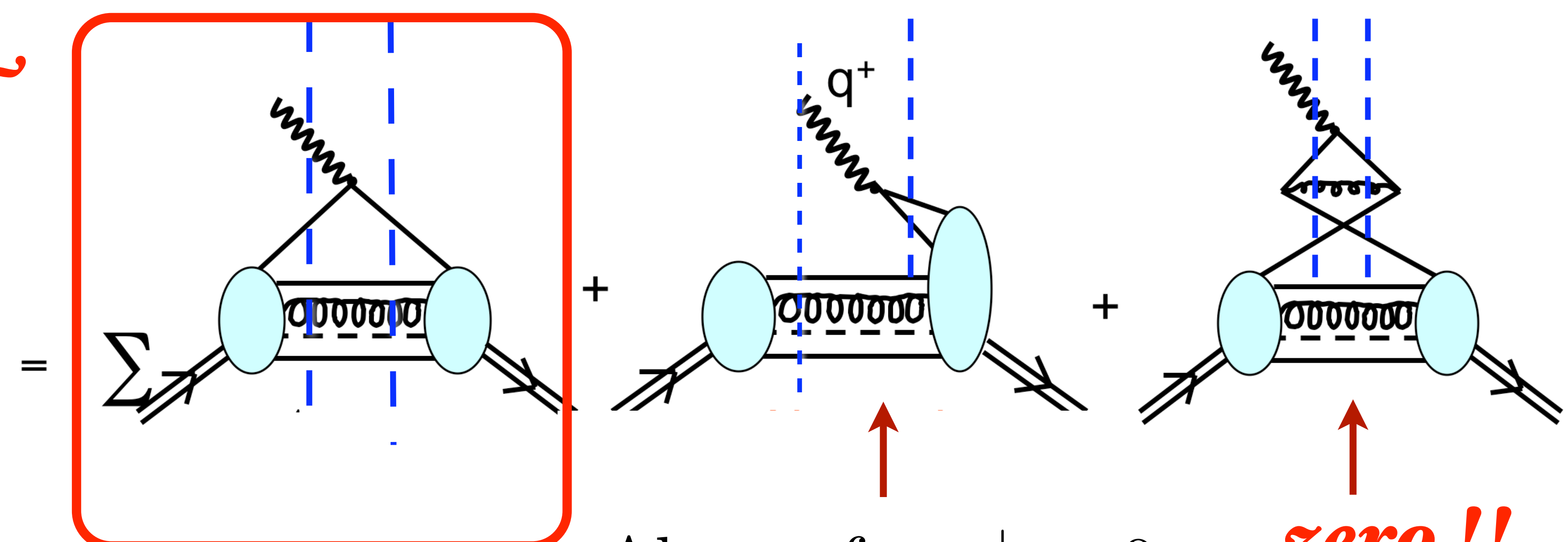


Need vacuum-induced currents!

Calculation of Form Factors in Light-Front Theory

Front Form





Complete Answer

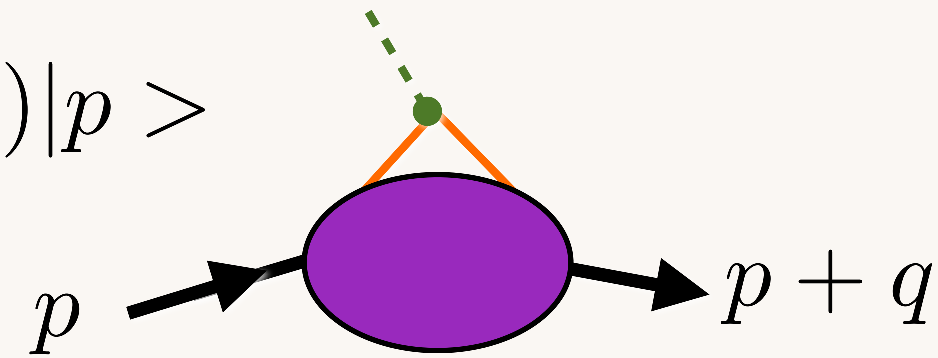
Absent for $q^+ = 0$

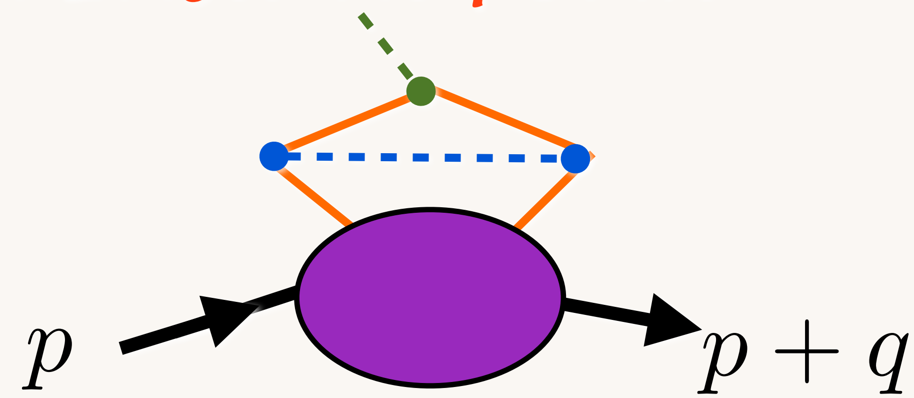
zero !!

No vacuum graphs

Calculation of proton form factor in Instant Form

$$< p+q | J^{\mu}(0) | p >$$





- **Need to boost proton instant form wavefunction from p to p+q: Extremely complicated dynamical problem; particle number changes**

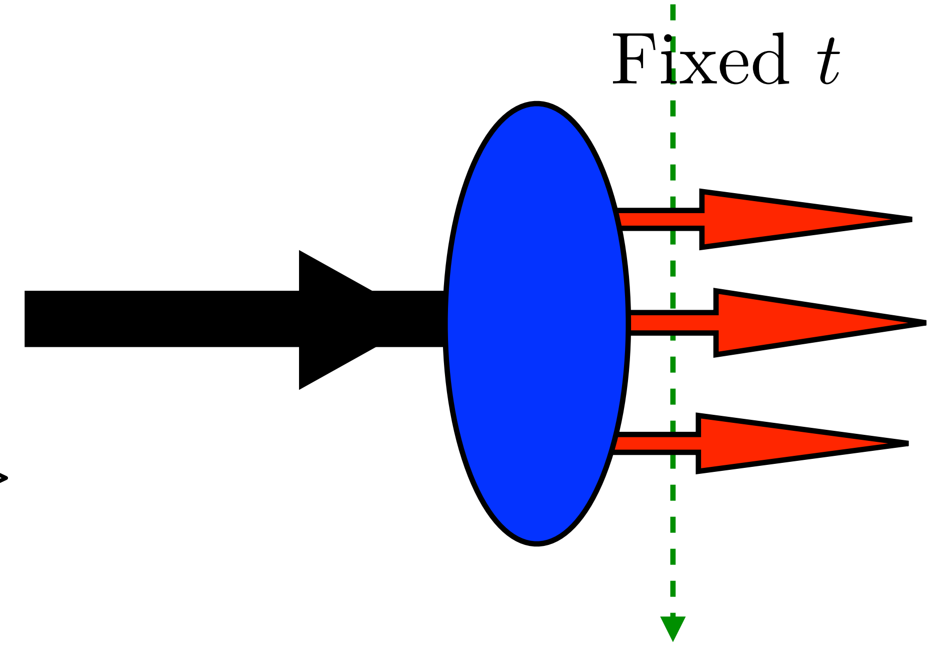
- **Need to couple to all currents arising from vacuum!**

- **Wavefunctions alone do not determine hadronic properties! Not even pdfs!**

- **Each time-ordered contribution is frame-dependent**

- **None of these problems occur in the front form!**

Observables cannot be computed from Usual Instant Form Wavefunctions



$$H_{IF}^{QCD}|\Psi_h>=E_h|\Psi_h>$$

$$|p,S_z>=\sum_{n=3}\psi_n(\vec{k}_i,\lambda_i)|n;\vec{k}_i,\lambda_i>$$

- **Eigenfunctions of the exact QCD IF Hamiltonian**
- **Boosts of IFWFs dynamical, complicated**
- **Require vacuum-induced currents to compute observables!**
- **Form factors, structure functions, GPDs, transverse momentum distributions cannot be computed from IFWFs alone!**
- **No Angular Momentum Sum Rule**
- **Vacuum Complicated -- Need Normal Ordering**

Instant-Form Vacuum

- Instant-Form Vacuum defined at fixed time t
- Acausal, Frame-dependent!
- State of minimum energy - frame dependent
- Non-Trivial in QED - must normal order!!
- Form Factors are not overlaps of WFs -- add vacuum-induced currents!!!
- Vacuum loops -- huge cosmological constant 10^{120} !!!!
- *Instant Form Vacuum is not a match to the causal universe!!*

Causal LF Vacuum

- Front-Form Vacuum defined at fixed light-front time τ
- $\tau = t + z/c$ reduces to ordinary time in NR limit
- Causal, Frame-independent
- State of minimum invariant mass
- Can Describe Empty Universe!
- Trivial in QED since $k^+ > 0$
- Form Factors are overlaps of LFWFs
- Dual to AdS/QCD using LF Holography
- In-Hadron Condensates: Zero Cosmological Constant from QCD, QED!

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \qquad x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

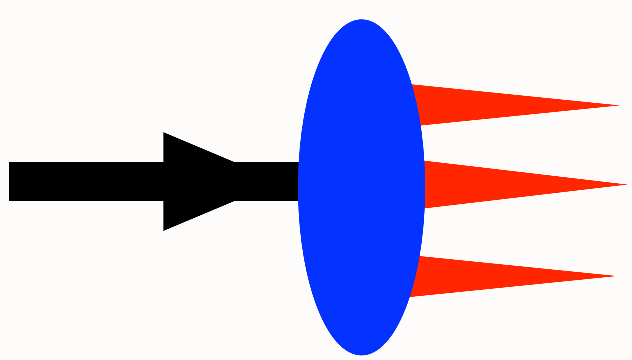
Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

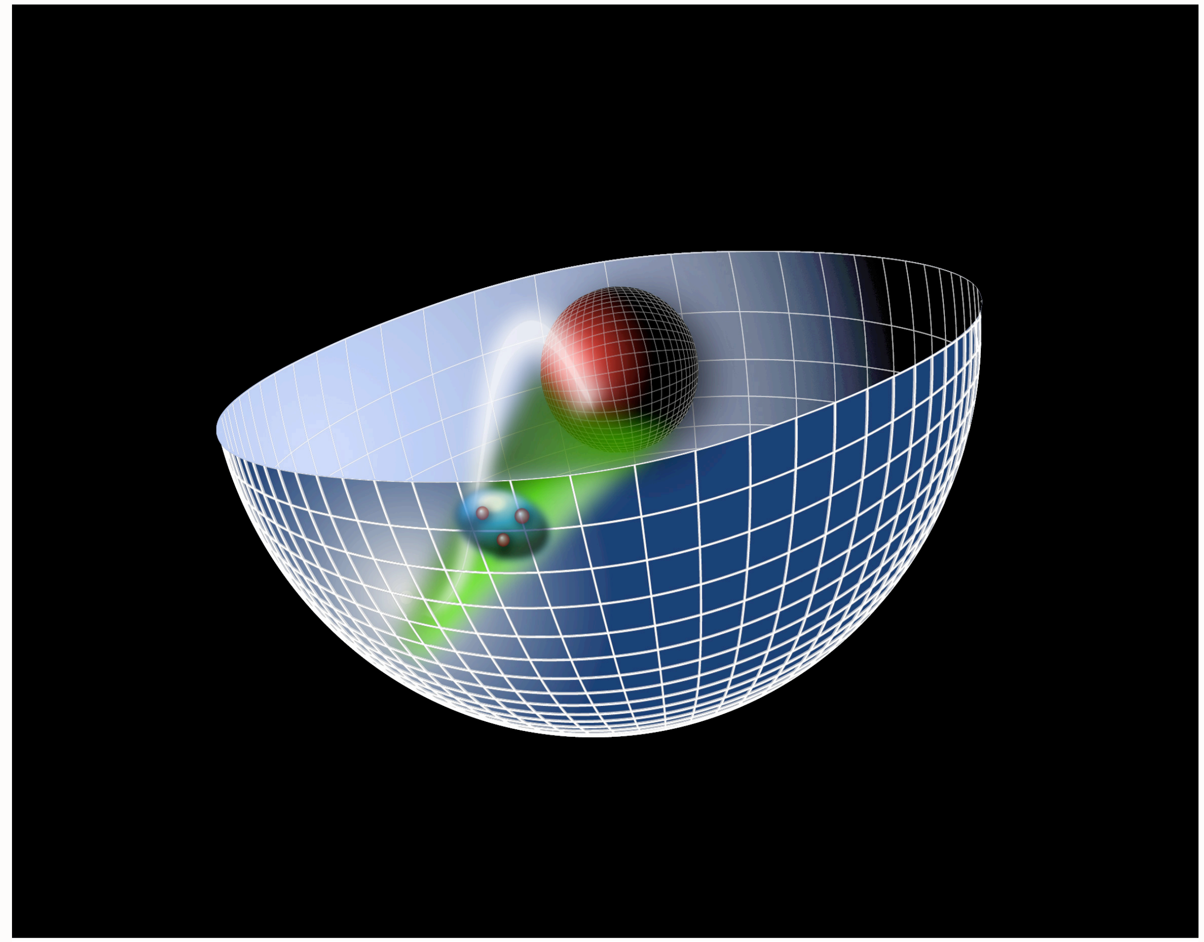
Light-Front Holography and Non-Perturbative QCD

Goal:
Use AdS/QCD duality to construct a first approximation to QCD

Hadron Spectrum
Light-Front Wavefunctions,
Running coupling in IR

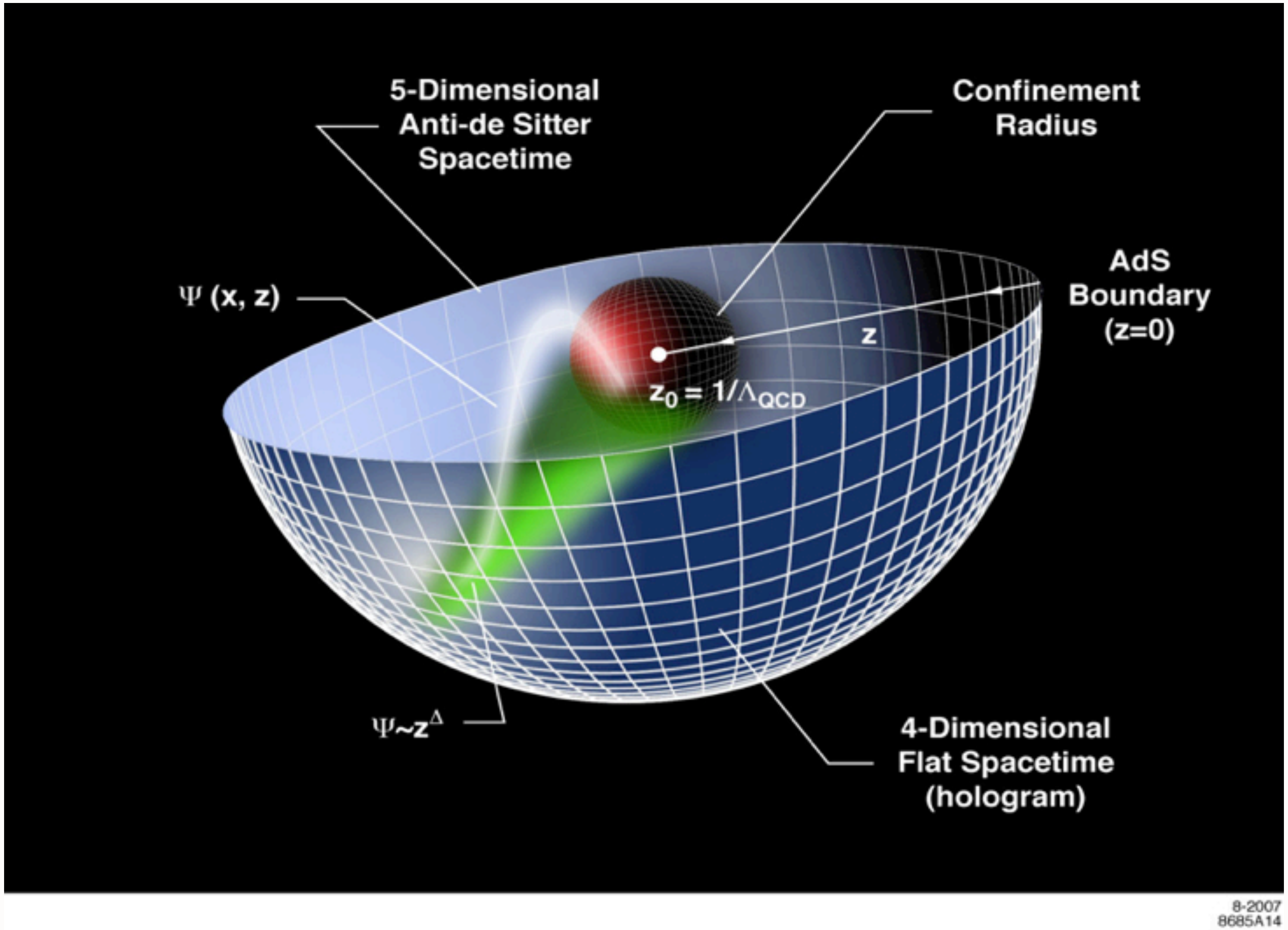


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



in collaboration with Guy de Teramond

Central problem for strongly-coupled gauge theories



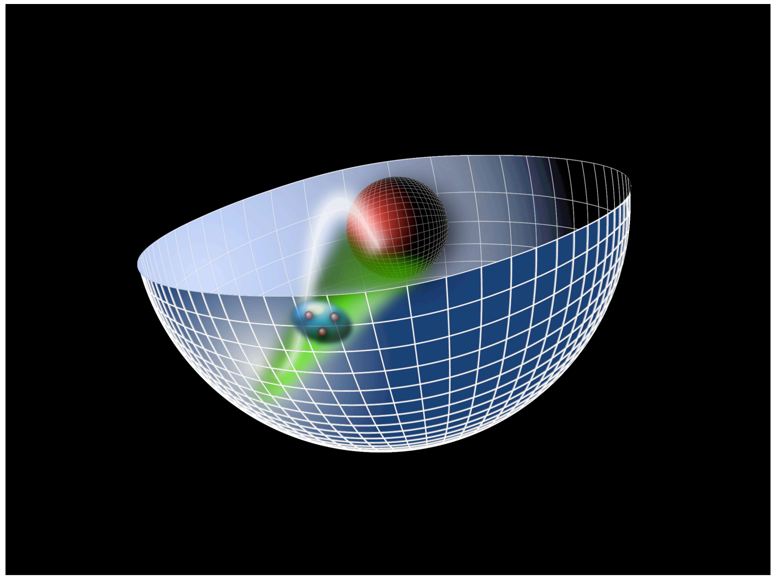
Changes in physical length scale mapped to evolution in the 5th dimension z

Light-Front Holography

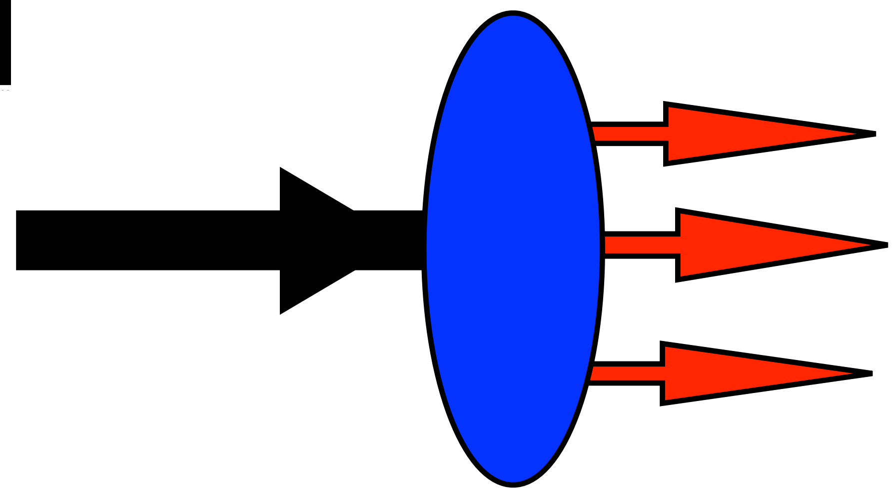
in collaboration with Guy de Teramond

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model)
  - Erlich, Karch, Katz, Son, Stephanov

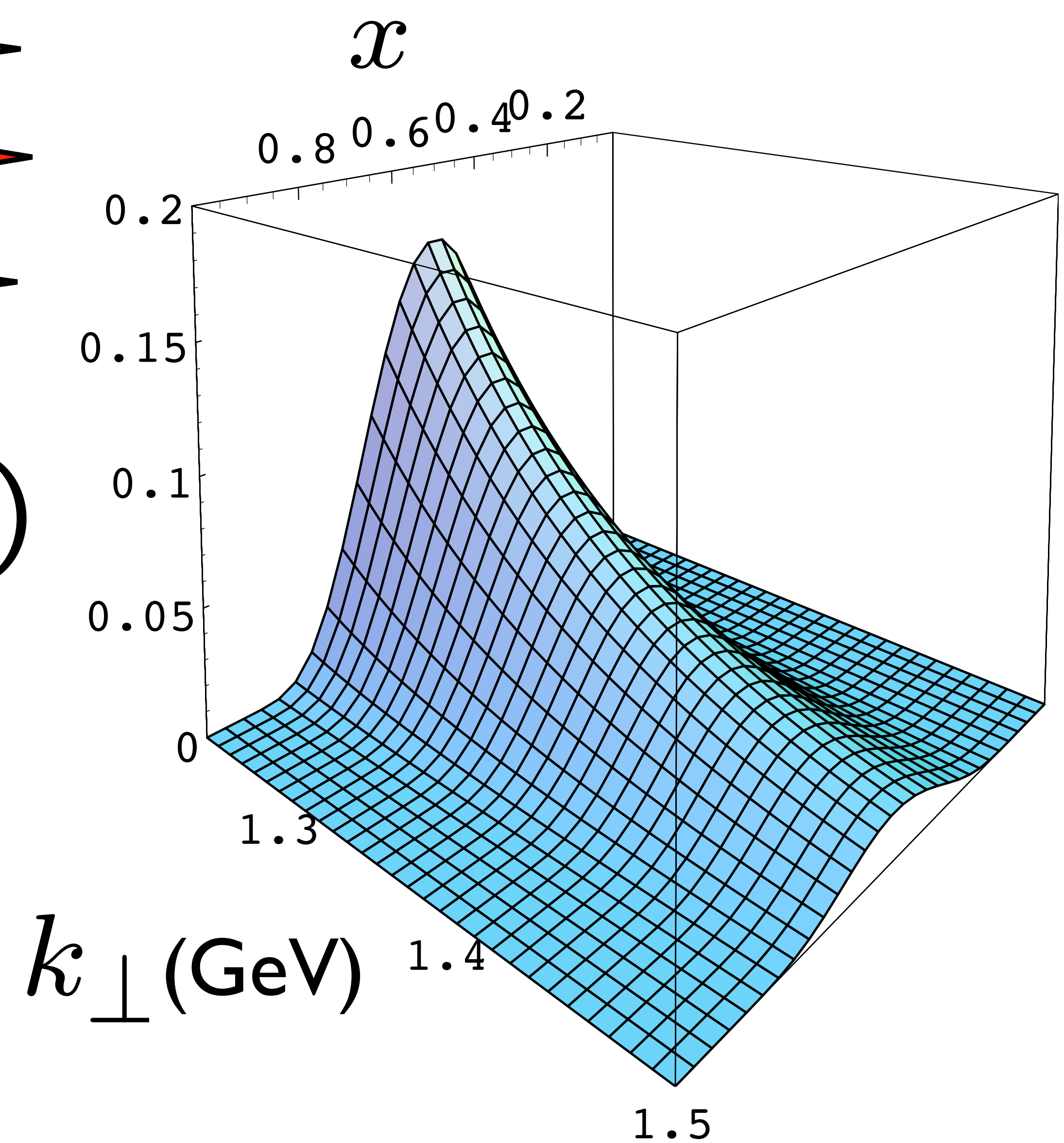
$\phi(z)$



- *Light-Front Holography*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



- *Light Front Wavefunctions:*

  Schrödinger Wavefunctions of Hadron Physics

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space $SO(1,5)$

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m}$.
- Action for massive scalar modes on AdS$_{d+1}$:

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \, \tfrac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g} \, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ-coordinates , $\Phi_P(x, z) = e^{-iP \cdot x} \, \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

$$\boxed{\left[z^2 \partial_z^2 - (d-1) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.}$$

- Solution: $\Phi(z) \to z^\Delta$ as $z \to 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta - d/2}(z\mathcal{M}) \qquad \Delta = \tfrac{1}{2} \left(d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \qquad d = 4 \qquad (\mu R)^2 = L^2 - 4$$

$$\text{Let } \Phi(z) = z^{3/2}\phi(z)$$

AdS Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2}\right]\phi(z) = \mathcal{M}^2\phi(z)$$

$L = L^z$: light-front orbital angular momentum

Derived from variation of Action in AdS_5

Hard wall model: truncated space

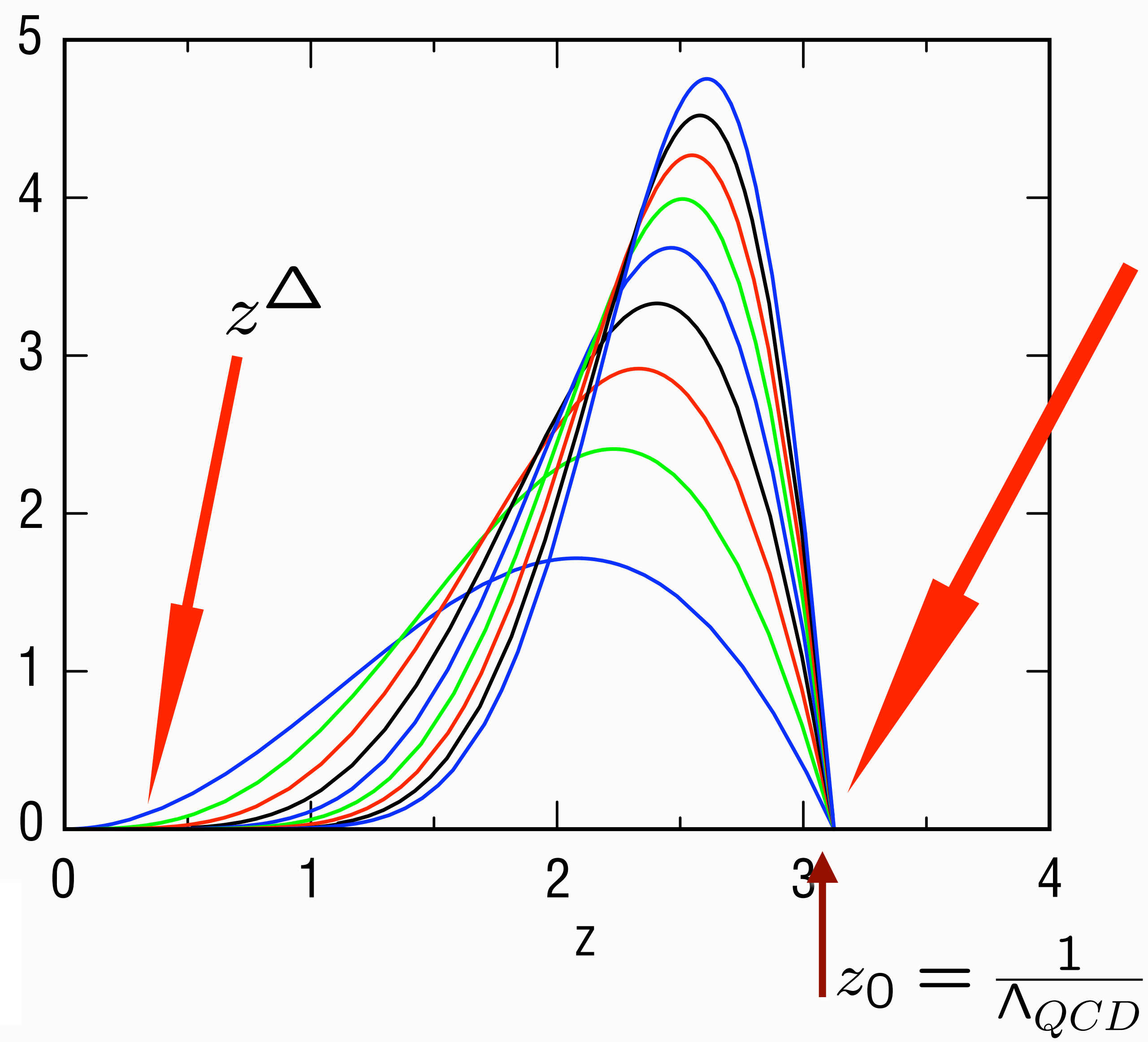
$$\phi(\mathrm{z} = \mathrm{z}_0 = \frac{1}{\Lambda_c}) = 0.$$

- Physical AdS modes $\Phi_P(x,z) \sim e^{-iP\cdot x}\,\Phi(z)$ are plane waves along the Poincaré coordinates with four-momentum P^μ and hadronic invariant mass states $P_\mu P^\mu = \mathcal{M}^2$.
- For small-z $\Phi(z) \sim z^\Delta$. The scaling dimension Δ of a normalizable string mode, is the same dimension of the interpolating operator $\mathcal{O}$ which creates a hadron out of the vacuum: $\langle P|\mathcal{O}|0\rangle \neq 0$.

$$\Delta = 2 + L$$

Twist dimension of meson

equivalent to dimensions of chiral superfields



Confinement in the 5th dimension

de Teramond, sjb

Identify hadron by its interpolating operator at z --> 0

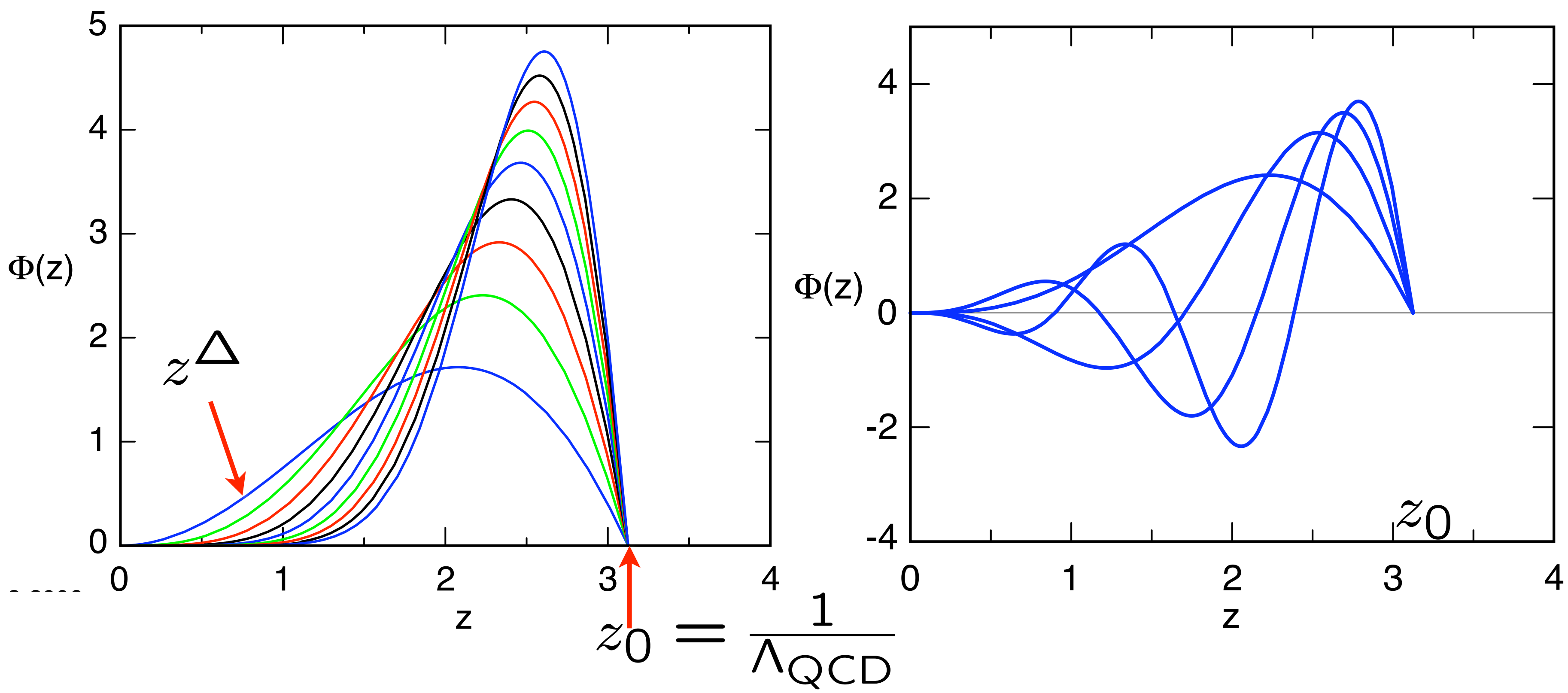
Match fall-off at small z to conformal twist-dimension at short distances

twist

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge). $\quad \Delta = 2 + L$

- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$

- Normalizable AdS modes $\Phi(z)$



$S = 0$ Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

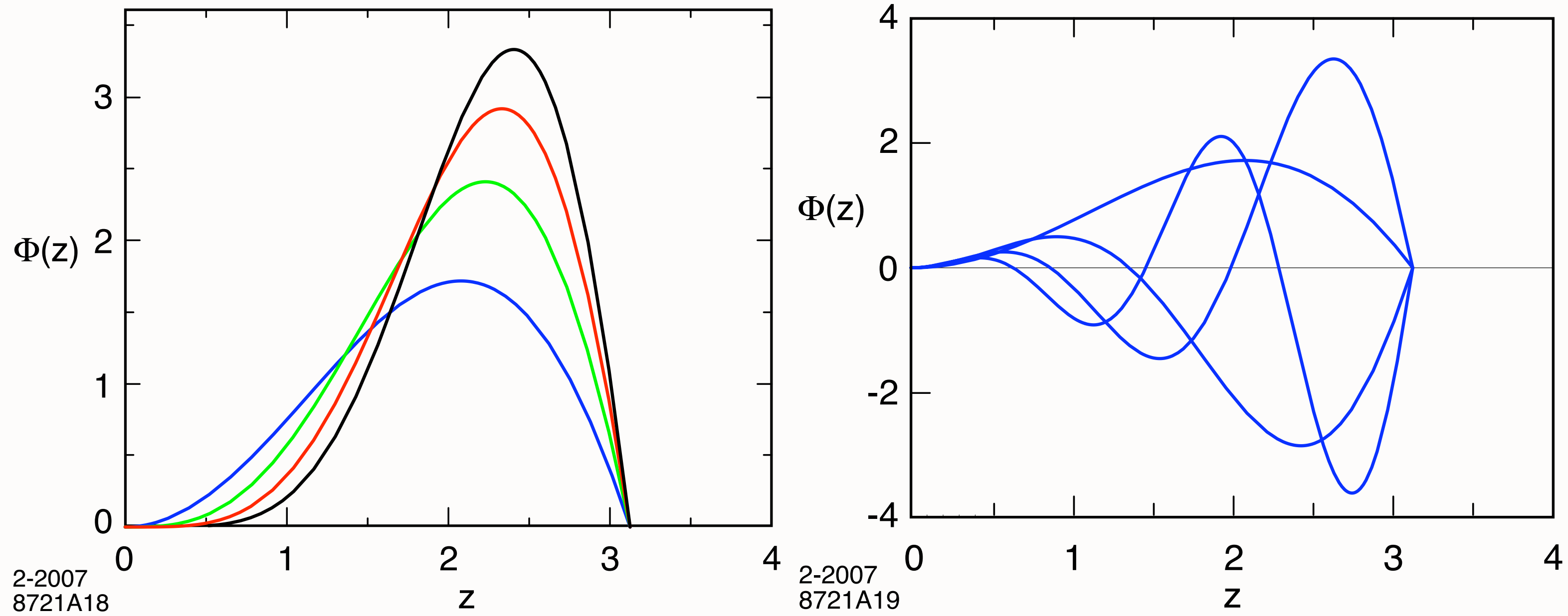


Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{\rm QCD}$ = 0.32 GeV .

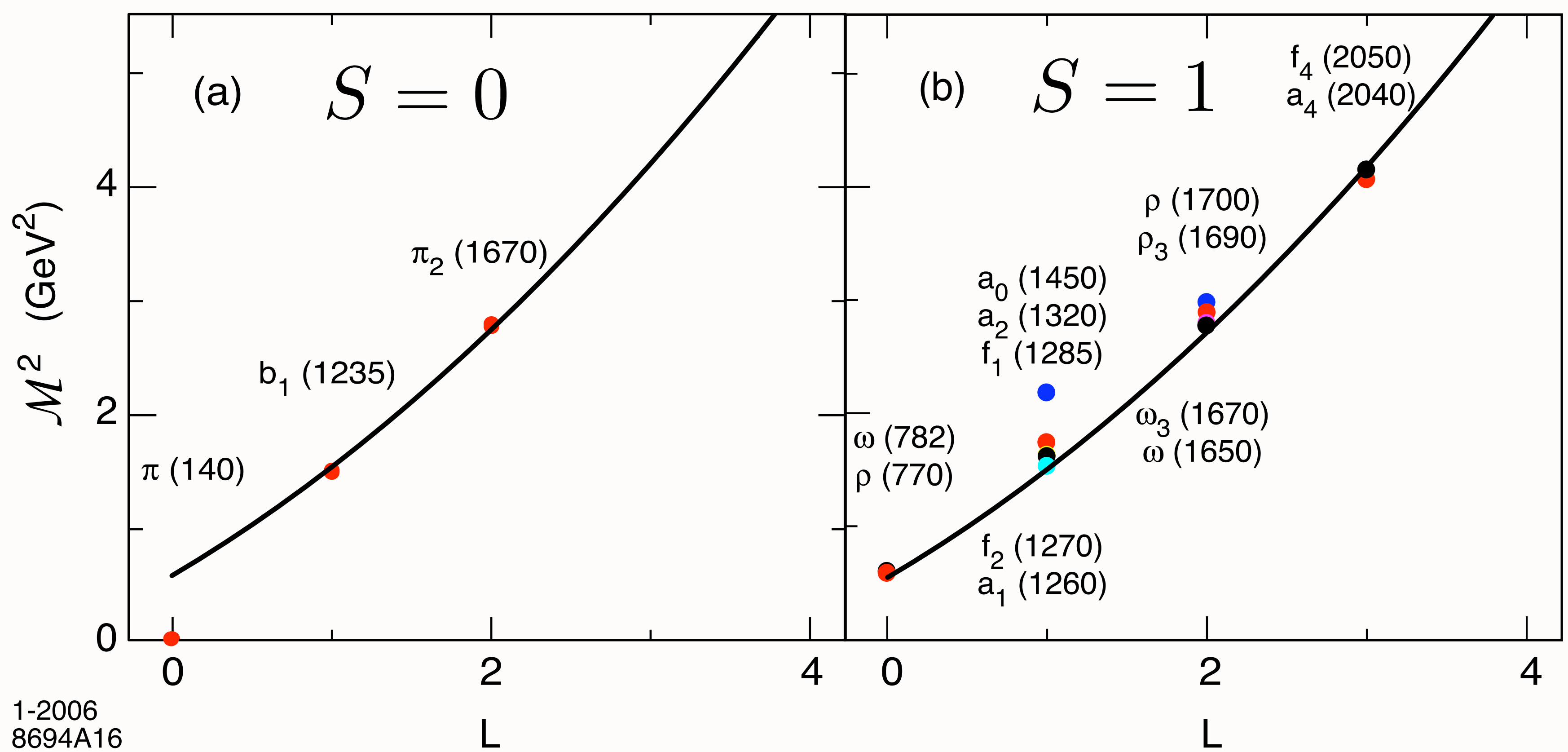


Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

- Nonconformal metric dual to a confining gauge theory

$$ds^2 = \frac{R^2}{z^2}\, e^{\varphi(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

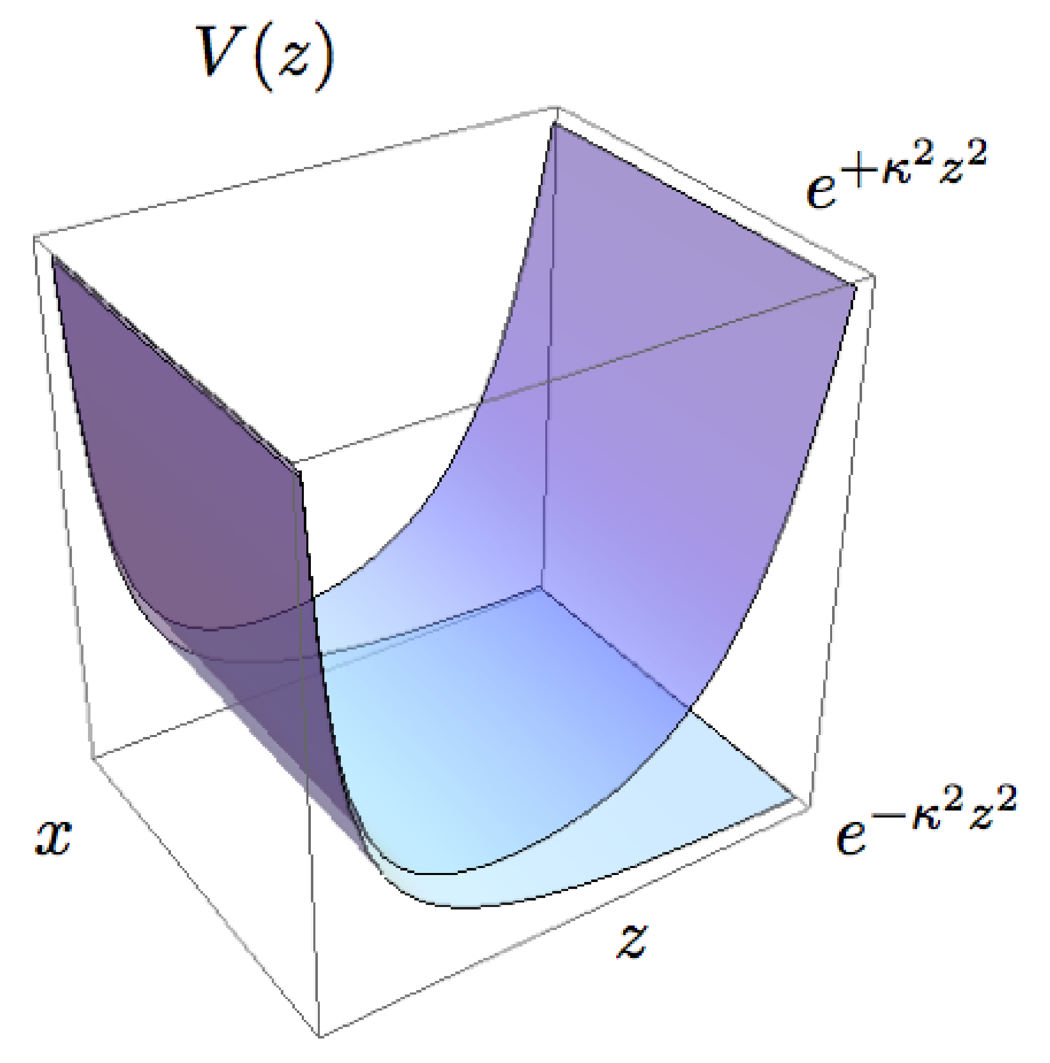
where $\varphi(z) \to 0$ at small z for geometries which are asymptotically AdS_5

- Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R\, \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm\kappa^2 z^2)$

- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$



Light-Front Holographic Mapping of Wave Equations

Higher Spin Modes in AdS Space

- Spin-J in AdS represented by totally symmetric rank J tensor field $\Phi_{M_1 \cdots M_J}$
- Action for spin-J field in AdS$_{d+1}$ in presence of dilaton background $\varphi(z)$ $\left(x^M = (x^\mu, z)\right)$

$$S = \frac{1}{2}\int d^d x\, dz\, \sqrt{g}\, e^{\varphi(z)} \Big(g^{NN'} g^{M_1 M_1'} \cdots g^{M_J M_J'} D_N \Phi_{M_1 \cdots M_J} D_{N'} \Phi_{M_1' \cdots M_J'} \\ - \mu^2 g^{M_1 M_1'} \cdots g^{M_J M_J'} \Phi_{M_1 \cdots M_J} \Phi_{M_1' \cdots M_J'} + \cdots \Big)$$

where D_M is the covariant derivative which includes parallel transport

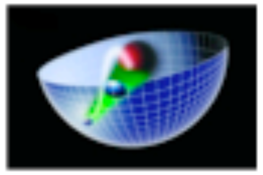
- Physical hadron has plane-wave and polarization indices along $3+1$ physical coordinates

$$\Phi_P(x,z)_{\mu_1 \cdots \mu_J} = e^{-iP\cdot x}\Phi(z)_{\mu_1 \cdots \mu_J}, \quad \Phi_{z\mu_2 \cdots \mu_J} = \cdots = \Phi_{\mu_1 \mu_2 \cdots z} = 0$$

with four-momentum P_μ and invariant hadronic mass $P_\mu P^\mu = M^2$

- Find AdS wave equation for spin J-mode $\Phi_J = \Phi_{\mu_1 \cdots \mu_J}$ and all indices along 3+1

$$\boxed{\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi}(z)}{z^{d-1-2J}} \partial_z \right) + \left(\frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)}$$



$$e^{\phi(z)} = e^{+\kappa^2 z^2}$$

- de Teramond, sjb

Positive-sign dilaton

AdS Soft-Wall Schrödinger Equation for bound state of two constituents:

$$[-\frac{d^2}{dz^2} + \frac{4L^2-1}{4z^2} + U(z)]\phi(z) = \mathcal{M}^2\phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action : Dilaton-Modified AdS_5

Matches the LF QCD Schrödinger Equation !

$$[-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{4\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = \mathcal{M}^2\ \psi_{LF}(\zeta)$$

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi >= E\ |\Psi >$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S},\vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r,S,\ell)]\ \psi(r) = E\ \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \to V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED

$$H^{LF}_{QCD}$$

QCD Meson Spectrum

$$(H^0_{LF} + H^I_{LF})|\Psi>= M^2|\Psi>$$

Coupled Fock states

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\text{eff}}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\boxed{[-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{4\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)}$$

invariant impact variable

$$\zeta^2 = x(1-x)b^2_\perp$$

Azimuthal Basis: ζ, ϕ

$$U(\zeta, S, L) = \kappa^4\zeta^2 + \kappa^2(L + S - 1/2)$$

Confining AdS/QCD potential

Semiclassical first approximation to QCD

Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\mathcal{M}^2 = \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left|\psi(x,\vec{k}_\perp)\right|^2 + \text{interactions}$$

$$= \int_0^1 \frac{dx}{x(1-x)} \int d^2\vec{b}_\perp \, \psi^*(x,\vec{b}_\perp) \left(-\vec{\nabla}^2_{\vec{b}_{\perp\ell}}\right) \psi(x,\vec{b}_\perp) + \text{interactions}.$$

Change variables

$$(\vec{\zeta},\varphi),\ \vec{\zeta} = \sqrt{x(1-x)}\vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta}\frac{d}{d\zeta}\left(\zeta\frac{d}{d\zeta}\right) + \frac{1}{\zeta^2}\frac{\partial^2}{\partial\varphi^2}$$

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}}$$

$$+ \int d\zeta \, \phi^*(\zeta) U(\zeta) \phi(\zeta)$$

$$= \int d\zeta \, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} + \boxed{\frac{4L^2-1}{4\zeta^2}} + U(\zeta)\right)\phi(\zeta)$$

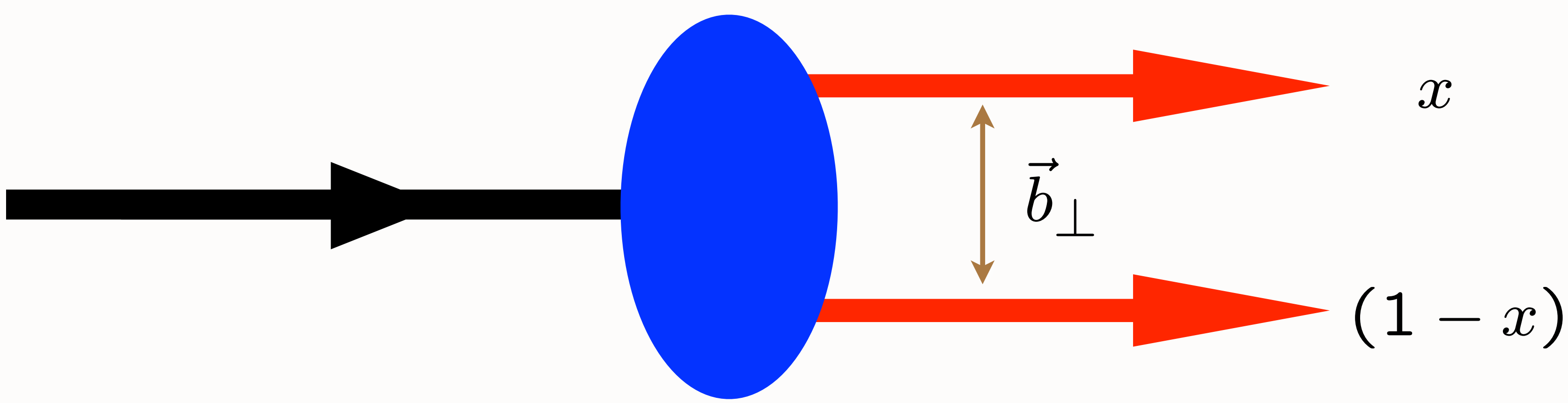
$$L = L^z$$

de Teramond, sjb

$LF(3+1)$ $\longleftrightarrow$ AdS_5

$\psi(x, \vec{b}_\perp)$ $\longleftrightarrow$ $\phi(z)$

$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$ $\longleftrightarrow$ z



$$\psi(x,\zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Light Front Holography: Identical mapping derived from equality of LF (DYW) and AdS formulas for current matrix elements

Dual QCD Light-Front Wave Equation

$$z \Leftrightarrow \zeta, \quad \Phi_P(z) \Leftrightarrow |\psi(P)\rangle$$

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

G. de Teramond
& sjb

- Upon substitution $z \to \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2}\, \Phi_J(\zeta)$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z\right) + \left(\frac{\mu R}{z}\right)^2\right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)$$

find LFWE $(d = 4)$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

with

$$U(\zeta) = \frac{1}{2}\varphi''(z) + \frac{1}{4}\varphi'(z)^2 + \frac{2J-3}{2z}\varphi'(z)$$

and $(\mu R)^2 = -(2-J)^2 + L^2$

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$
- Scaling dimension τ of AdS mode Φ_J is $\tau = 2 + L$ in agreement with twist scaling dimension of a two parton bound state in QCD and determined by QM stability condition

$$e^{\phi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

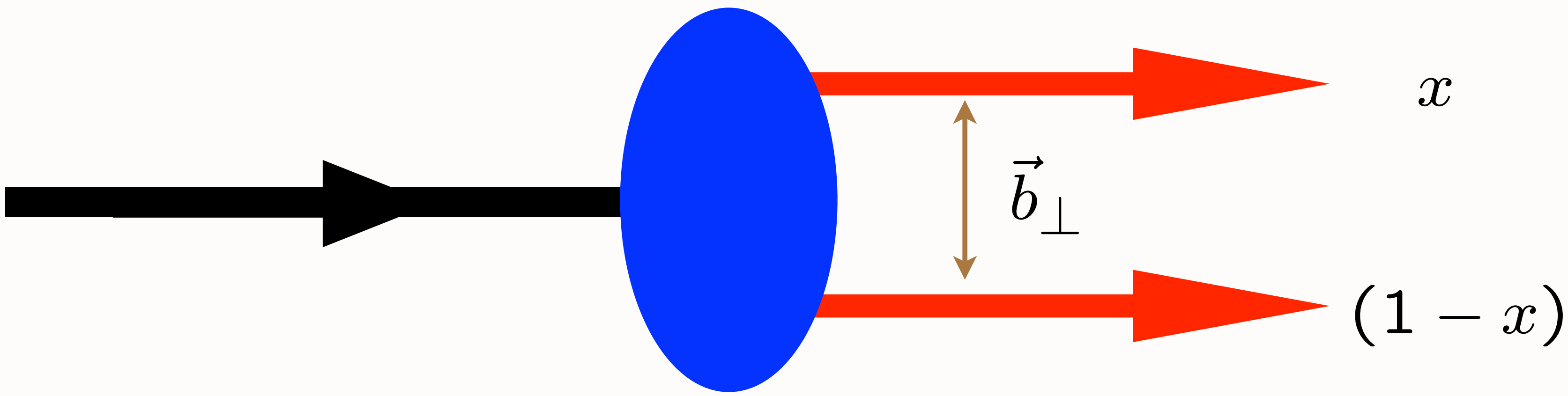
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2}+\frac{4L^2-1}{4\zeta^2}+U(\zeta,S,L)\right]\psi_{LF}(\zeta)=M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp .$$



$$U(\zeta)=\kappa^4\zeta^2+2\kappa^2(L+S-1)$$

soft wall confining potential:

G. de Teramond, sjb

Quark separation increases with L

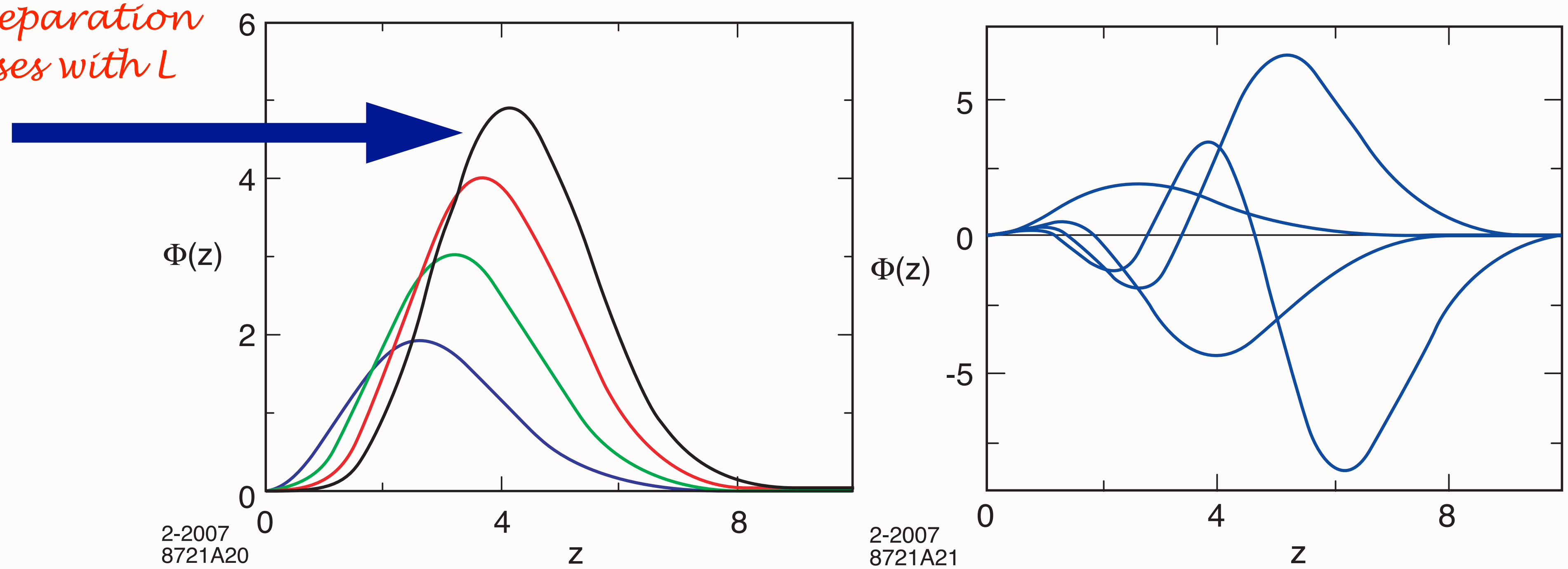
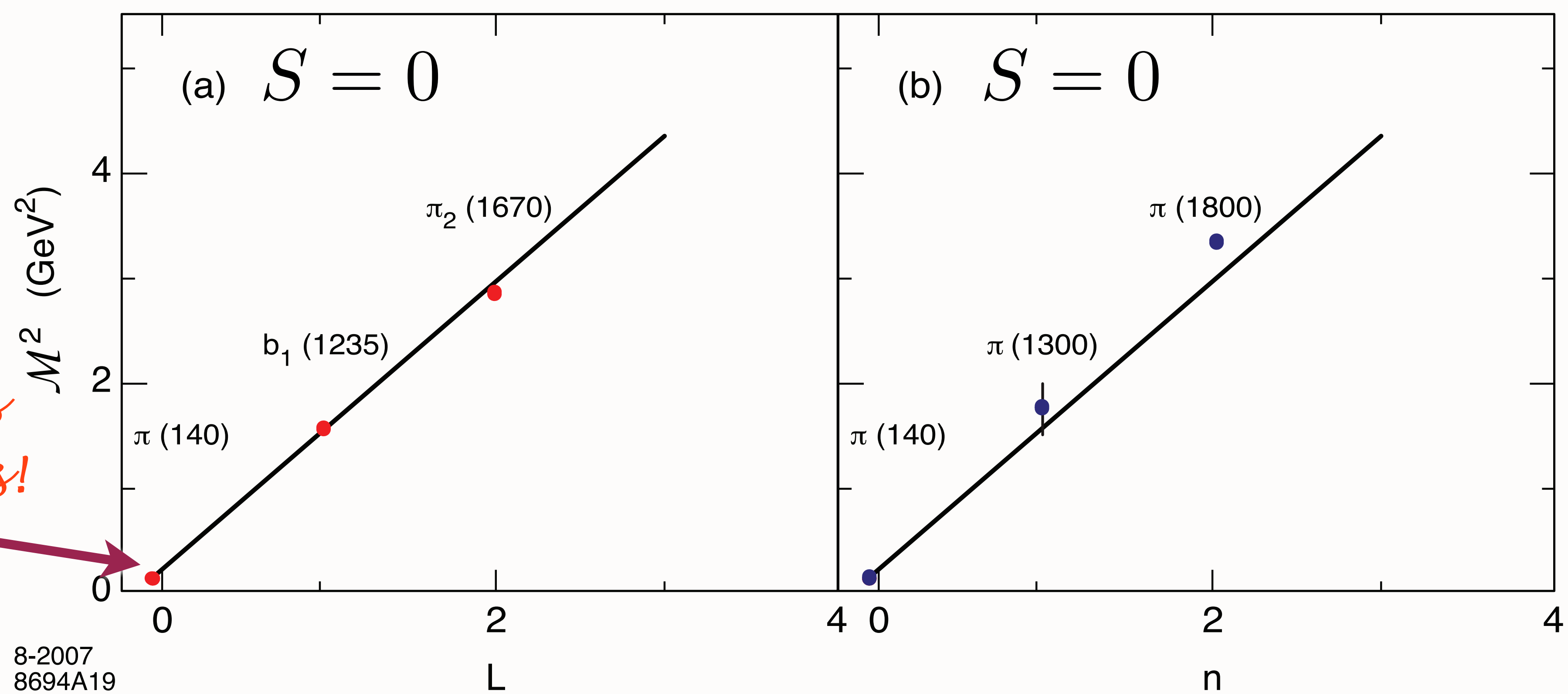


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model

Pion has zero mass!



Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Pion mass automatically zero!

$$m_q = 0$$

General-Spin Hadrons

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2 \partial_z^2 - \left(3 - 2J - 2\kappa^2 z^2\right) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

- Upon substitution $z \rightarrow \zeta$

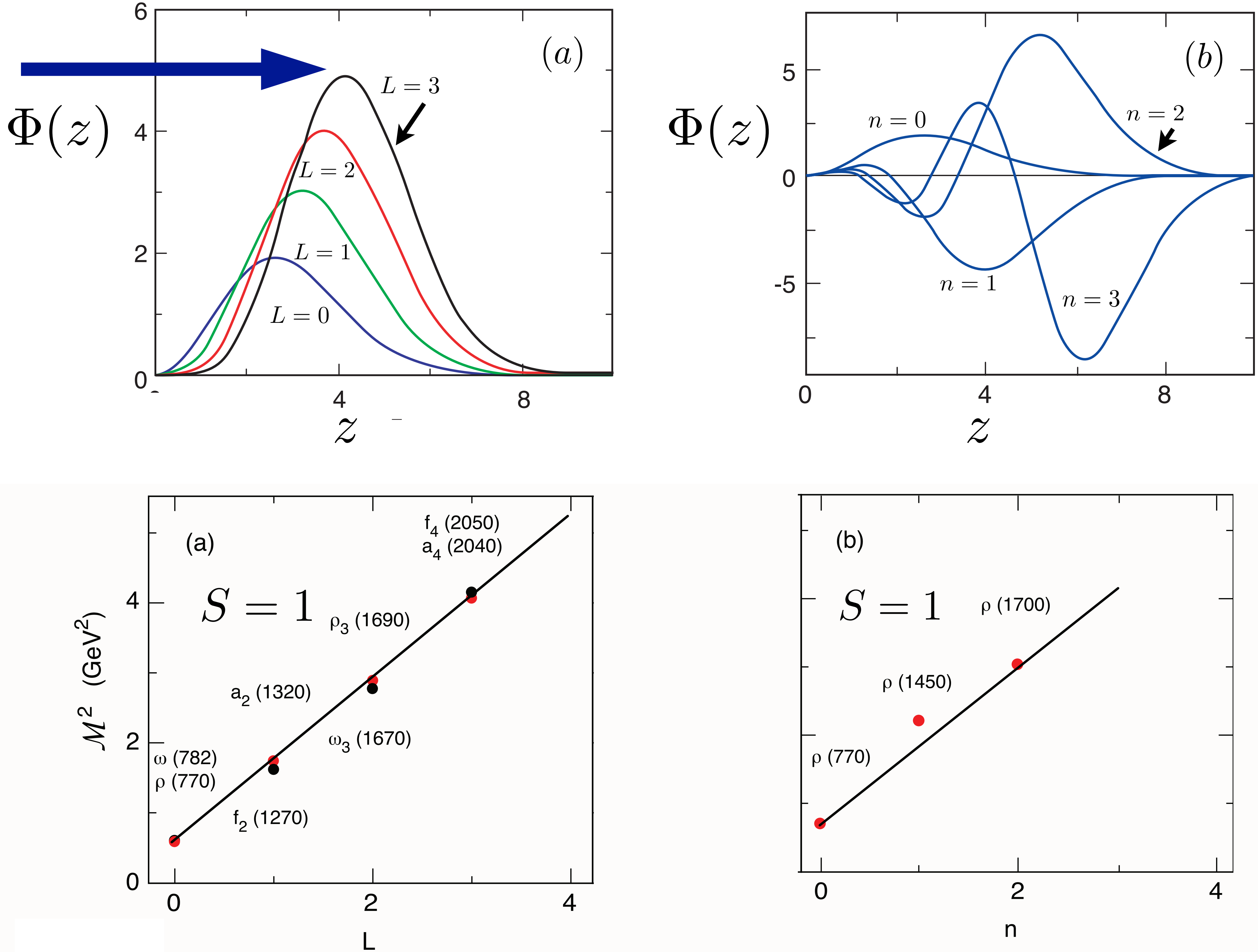
$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2}\, \Phi_J(\zeta)$$

we find the LF wave equation

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}}$$

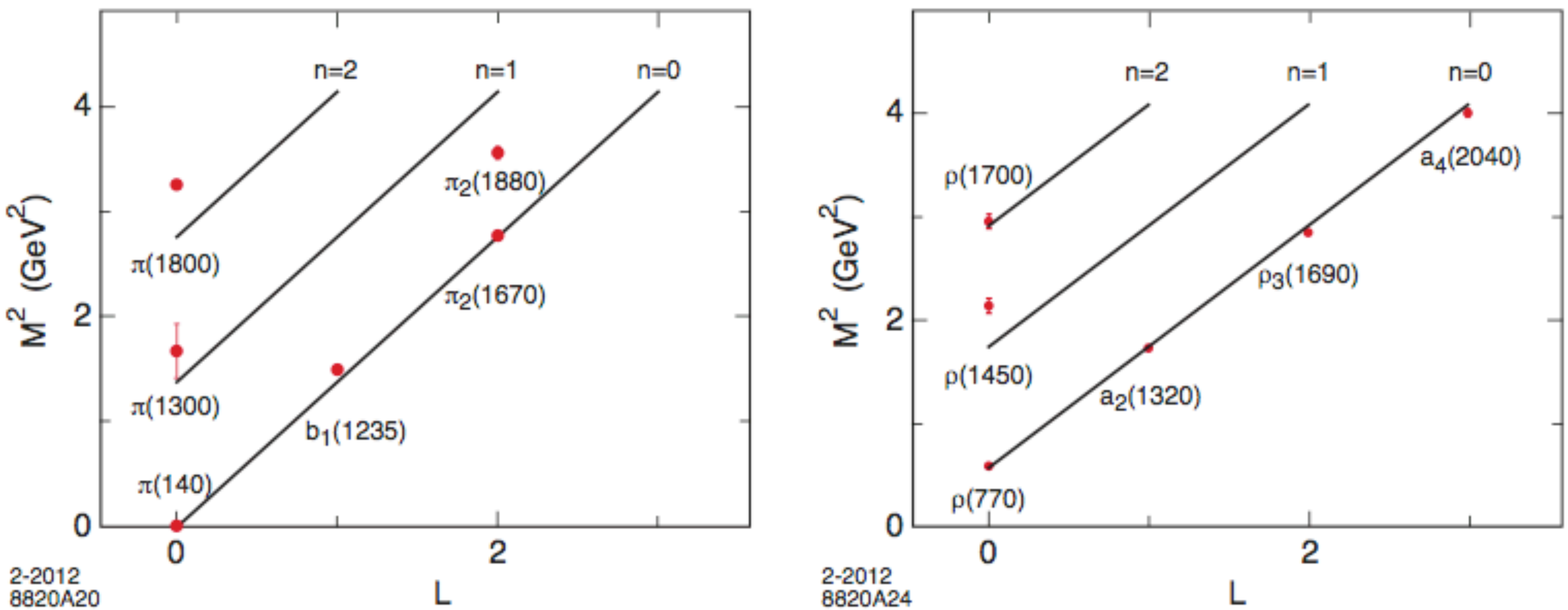
with $(\mu R)^2 = -(2 - J)^2 + L^2$

Quark separation increases with L



- $J = L + S, I = 1$ meson families $\mathcal{M}^2_{n,L,S} = 4\kappa^2 (n + L + S/2)$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$



I=1 orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ-meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1, L = 1, J = 0, 1, 2$, vector meson a-states

$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

Meson Spectrum in Soft Wall Model

- Linear Regge trajectories [Karch, Katz, Son and Stephanov (2006)]
- Dilaton profile $\varphi(z) = +\kappa^2 z^2$
- Effective potential: $U(z) = \kappa^4 \zeta^2 + 2\kappa^2(J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

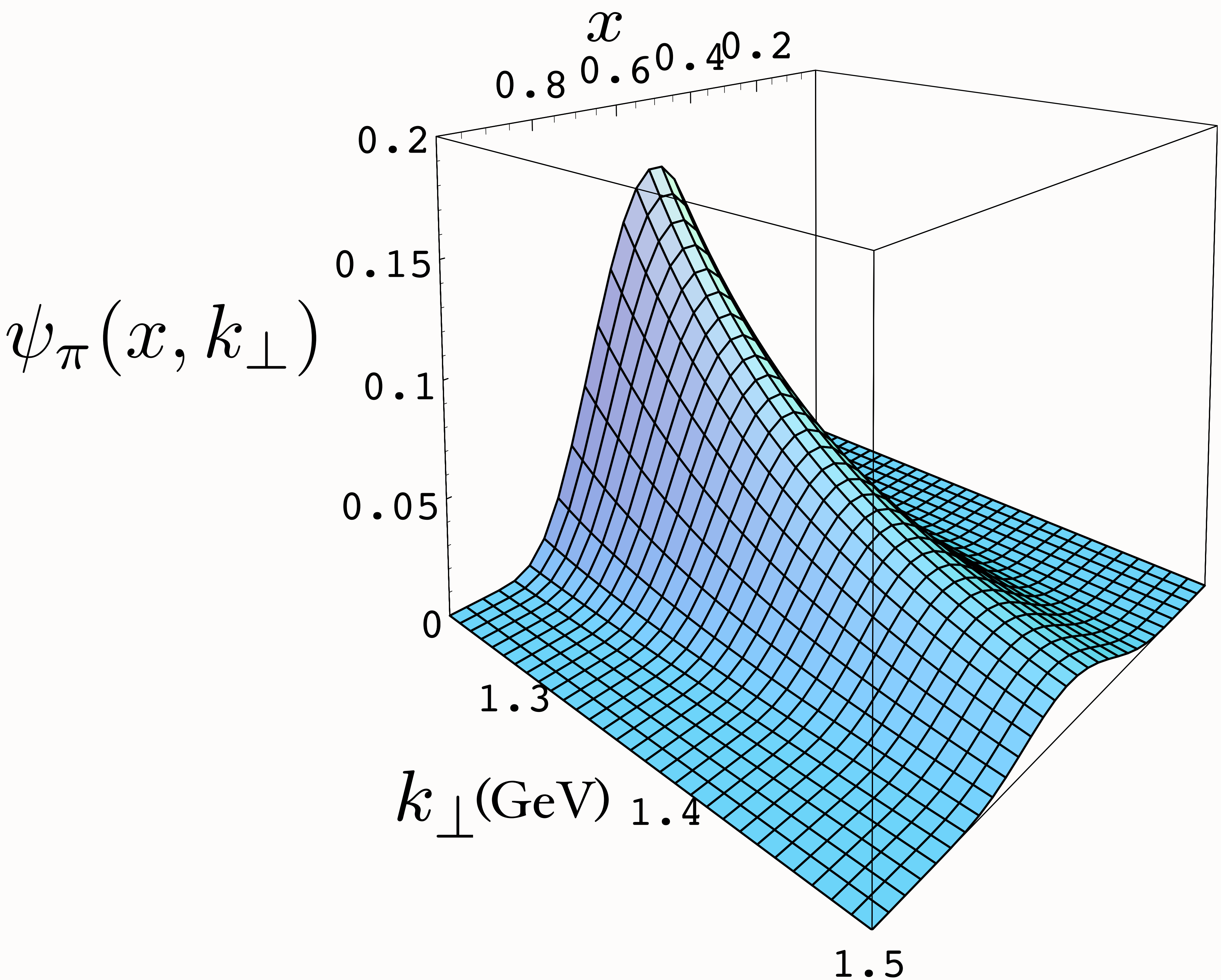
- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

Prediction from AdS/CFT: Pion Light-Front Wavefunction



Soft Wall Model

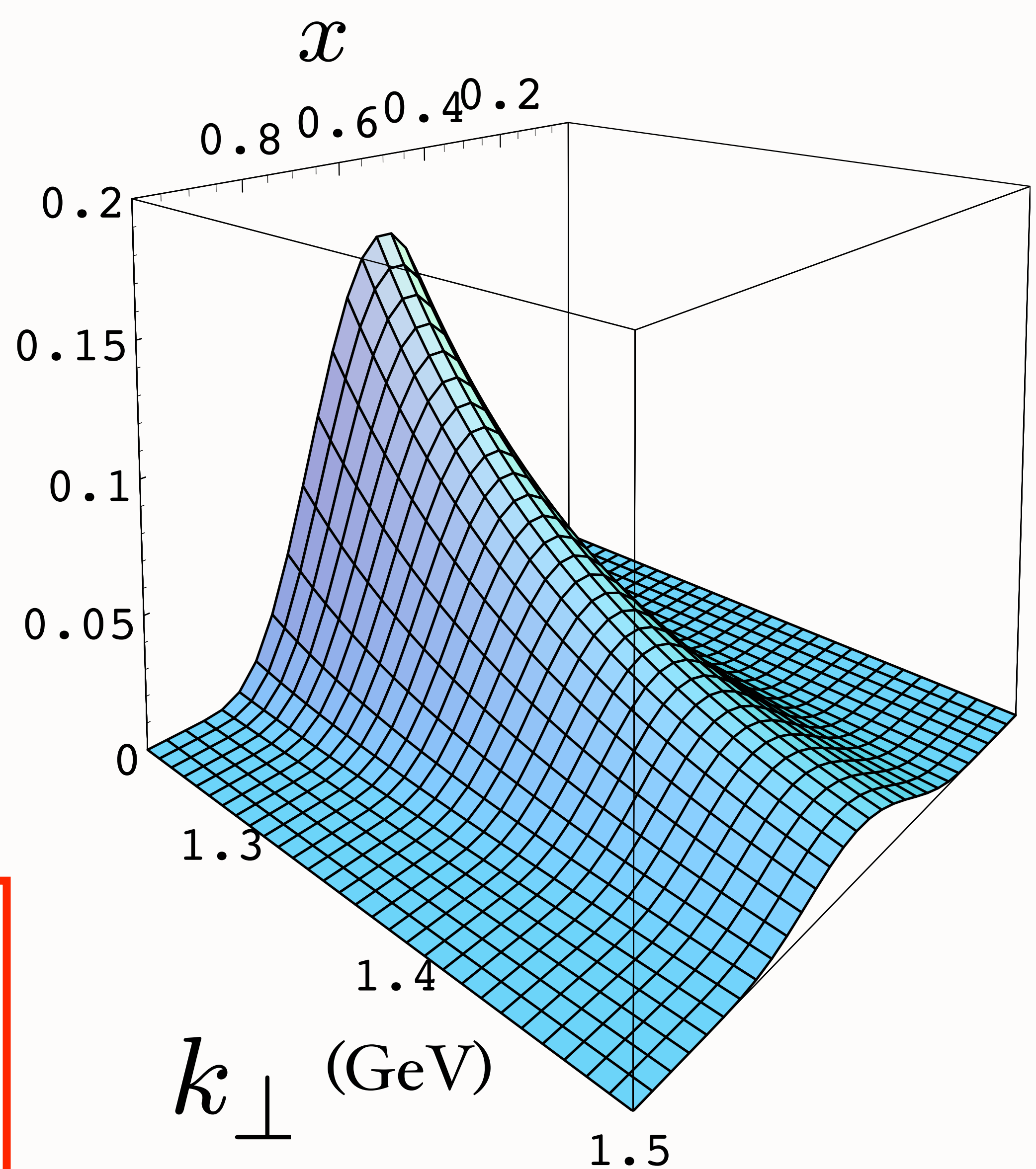
de Teramond, sjb

Increases PQCD prediction for $F_\pi(Q^2)$ by 16/9

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

$$\psi_M(x, k_\perp^2)$$



"Soft Wall" model

$\kappa = 0.375$ GeV

massless quarks

Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\boxed{\phi_M(x, Q_0) \propto \sqrt{x(1-x)}}$$

Connection of Confinement to TMDs

Second Moment of Pion Distribution Amplitude

$$<\xi^2>=\int_{-1}^{1} d\xi\ \xi^2\phi(\xi)$$

$$\xi = 1-2x$$

$$<\xi^2>_\pi = 1/5 = 0.20 \qquad \phi_{asympt} \propto x(1-x)$$

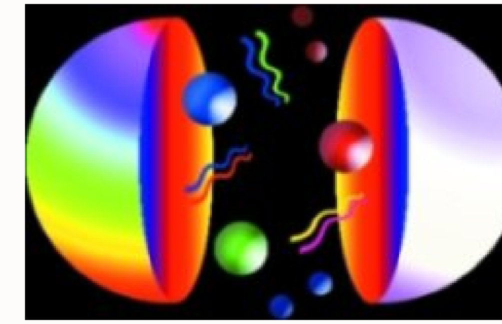
$$<\xi^2>_\pi = 1/4 = 0.25 \qquad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

Lattice (I) $<\xi^2>_\pi = 0.28 \pm 0.03$ Donnellan et al.

Lattice (II) $<\xi^2>_\pi = 0.269 \pm 0.039$ Braun et al.

- Baryons Spectrum in "bottom-up" holographic QCD

GdT and Brodsky: hep-th/0409074, hep-th/0501022.



From Nick Evans

Baryons in Ads/CFT

- Action for massive fermionic modes on AdS$_5$:

$$S[\overline{\Psi}, \Psi] = \int d^4x \, dz \, \sqrt{g} \, \overline{\Psi}(x, z) \left(i\Gamma^\ell D_\ell - \mu \right) \Psi(x, z)$$

- Equation of motion: $\left(i\Gamma^\ell D_\ell - \mu \right) \Psi(x, z) = 0$

$$\left[i \left(z\eta^{\ell m} \Gamma_\ell \partial_m + \frac{d}{2} \Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0$$

Hard Wall

- Solution $(\mu R = \nu + 1/2)$

$$\Psi(z) = C z^{5/2} \left[J_\nu(z\mathcal{M}) u_+ + J_{\nu+1}(z\mathcal{M}) u_- \right]$$

- Hadronic mass spectrum determined from IR boundary conditions $\psi_\pm \left(z = 1/\Lambda_{\rm QCD} \right) = 0$

$$\mathcal{M}^+ = \beta_{\nu,k} \, \Lambda_{\rm QCD}, \quad \mathcal{M}^- = \beta_{\nu+1,k} \, \Lambda_{\rm QCD}$$

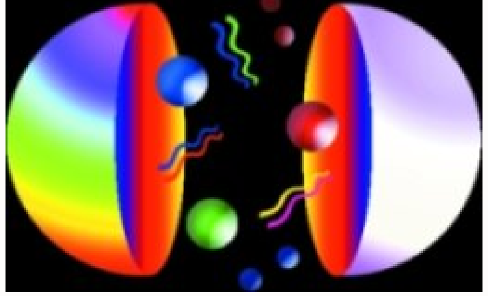
with scale independent mass ratio

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions

Fermionic Modes and Baryon Spectrum

GdT and sjb, PRL 94, 201601 (2005)

Yukawa interaction in 5 dimensions



From Nick Evans

- Action for Dirac field in AdS$_{d+1}$ in presence of dilaton background $\varphi(z)$ [Abidin and Carlson (2009)]

$$S = \int d^{d+1} \sqrt{g} e^{\varphi}(z) \left(i \overline{\Psi} e^M_A \Gamma^A D_M \Psi + h.c + \varphi(z) \overline{\Psi} \Psi - \mu \overline{\Psi} \Psi\right)$$

$$\phi(z) = e^{\kappa^2 z^2}$$

- Factor out plane waves along 3+1: $\Psi_P(x^\mu, z) = e^{-iP \cdot x} \Psi(z)$

$$\left[i\left(z\eta^{\ell m}\Gamma_\ell \partial_m + 2\Gamma_z\right) + \mu R + \kappa^2 z\right] \Psi(x^\ell) = 0.$$

- Solution $(\nu = \mu R - \frac{1}{2},\ \nu = L + 1)$

$$\Psi_+(z) \sim z^{\frac{5}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^\nu(\kappa^2 z^2), \quad \Psi_-(z) \sim z^{\frac{7}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^{\nu+1}(\kappa^2 z^2)$$

- Eigenvalues (how to fix the overall energy scale, see arXiv:1001.5193)

$$\mathcal{M}^2 = 4\kappa^2(n + L + 1)$$

positive parity

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions
- Large N_C: $\mathcal{M}^2 = 4\kappa^2(N_C + n + L - 2) \implies \mathcal{M} \sim \sqrt{N_C}\, \Lambda_{\rm QCD}$

Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\nu = L + 1$$

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

and its adjoint $\Pi^\dagger$, with commutation relations

Soft Wall

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu+1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

$$\begin{aligned}\psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2),\\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2).\end{aligned}$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$

- Excitation spectrum of nucleon represents formidable challenge to LQCD due to enormous computational complexity beyond ground state configuration

- LF Holographic nucleon modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\, \psi_+^2(\zeta) = \int d\zeta\, \psi_-^2(\zeta) = 1$$

Equal probability L=0,1 !

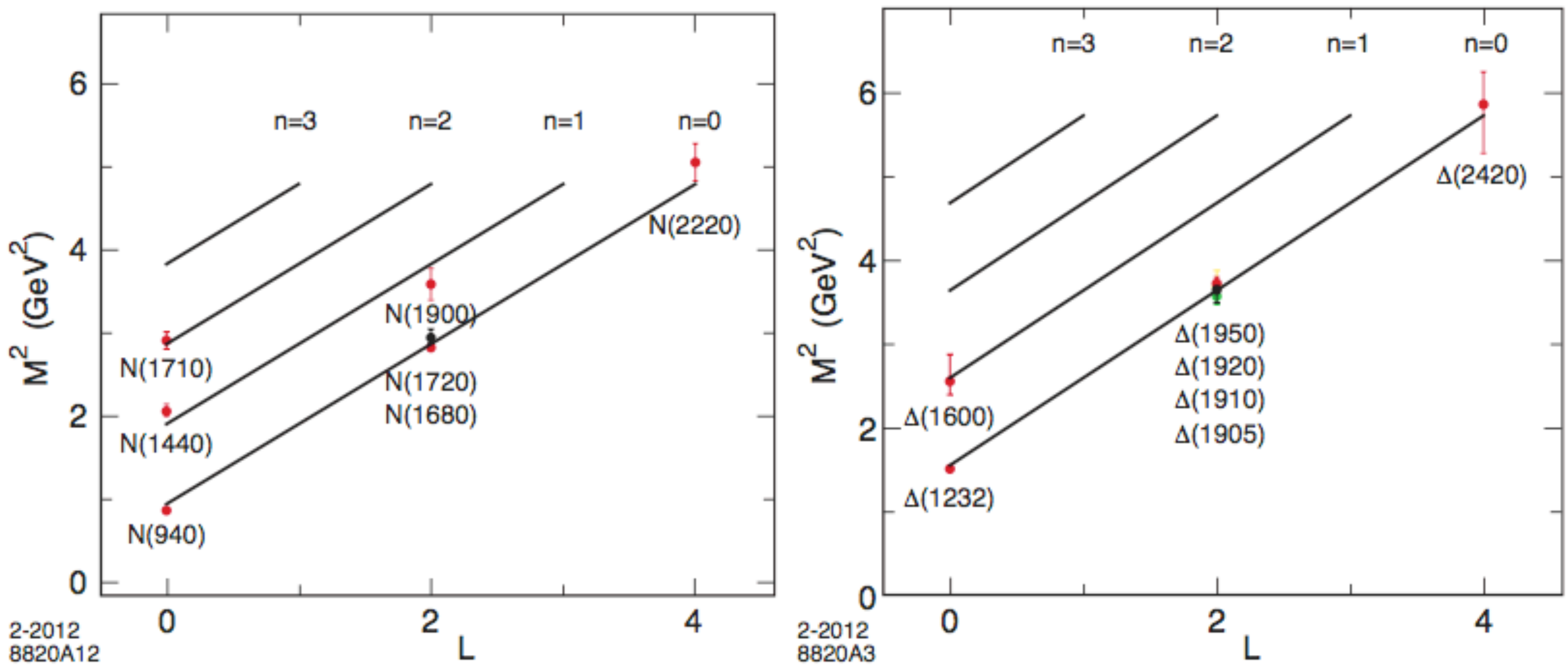
- Eigenvalues

$$\mathcal{M}^{2(+)}_{n,L,S} = 4\kappa^2\left(n+L+S/2+3/4\right)$$

$$\mathcal{M}^{2(-)}_{n,L,S} = 4\kappa^2\left(n+L+S/2+5/4\right)$$

- Same multiplicity of states for mesons and baryons!

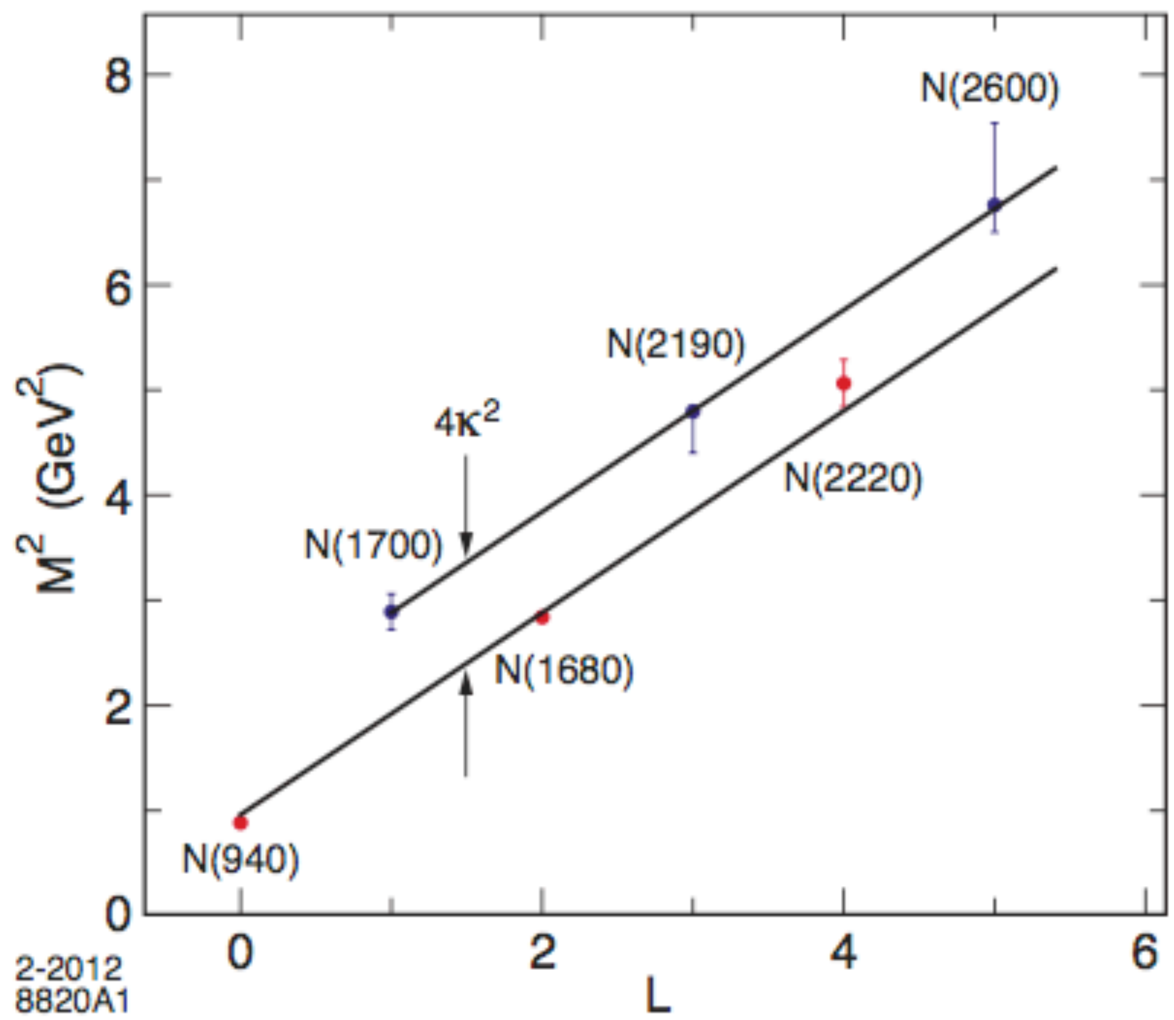
$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$



Orbital and radial excitations for positive parity N and Δ baryon families ($\kappa = 0.49 - 0.51$ GeV)

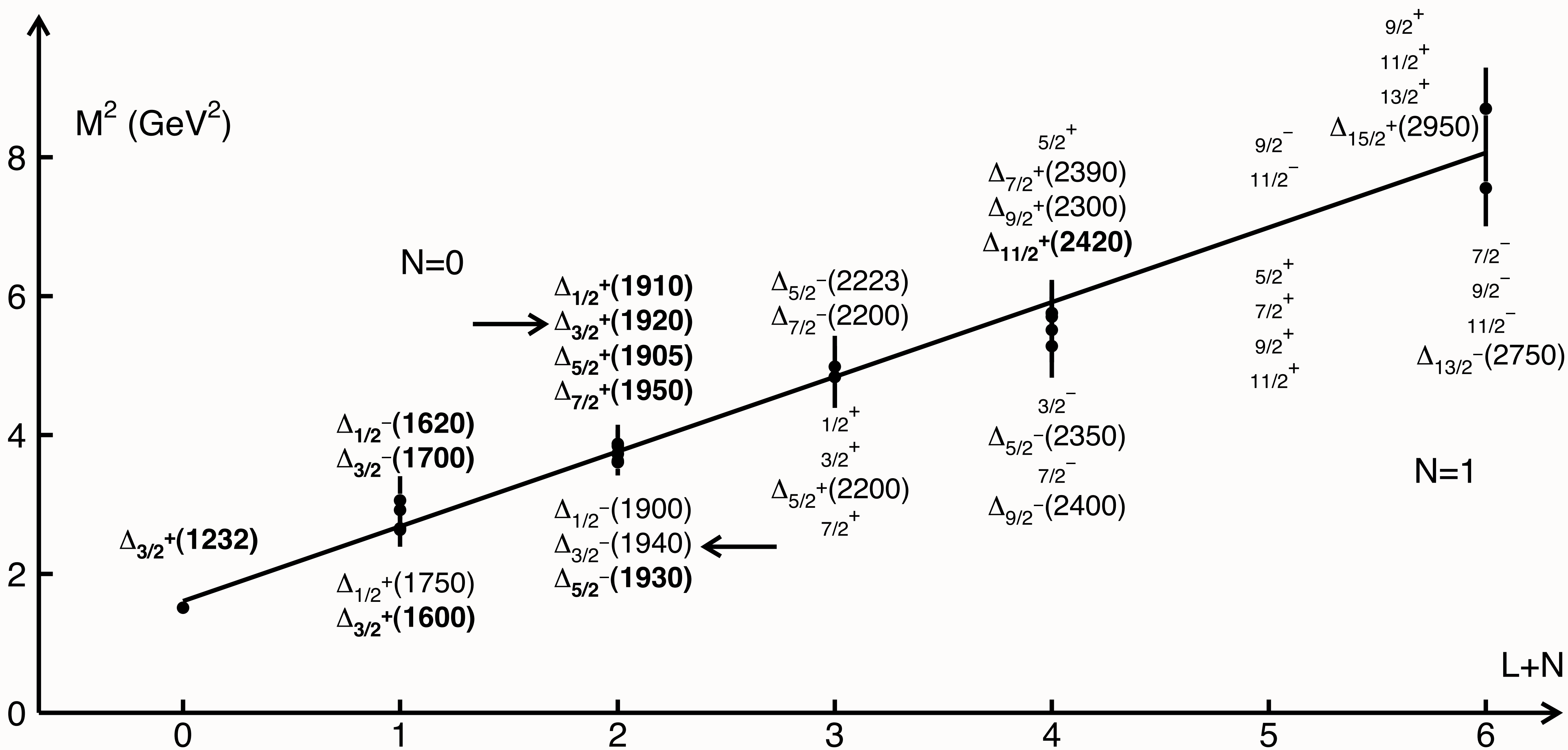
Same results for the Δ spectrum: H. Forkel, M. Beyer and T. Frederico, JHEP **0707**, 077 (2007)

- Gap scale $4\kappa^2$ determines trajectory slope and spectrum gap between plus-parity spin-$\frac{1}{2}$ and minus-parity spin-$\frac{3}{2}$ nucleon families for the branch solutions $L+1=\mu R-1/2$ and $L+1=\mu R+1/2$



Plus-minus nucleon spectrum gap for $\kappa = 0.49$ GeV

- **Δ spectrum identical to Forkel and Klempt, Phys. Lett. B 679, 77 (2009)**



E. Klempt *et al.*: Δ^* resonances, quark models, chiral symmetry and AdS/QCD

H. Forkel, M. Beyer and T. Frederico, JHEP **0707** (2007) 077.
H. Forkel, M. Beyer and T. Frederico, Int. J. Mod. Phys. E **16** (2007) 2794.

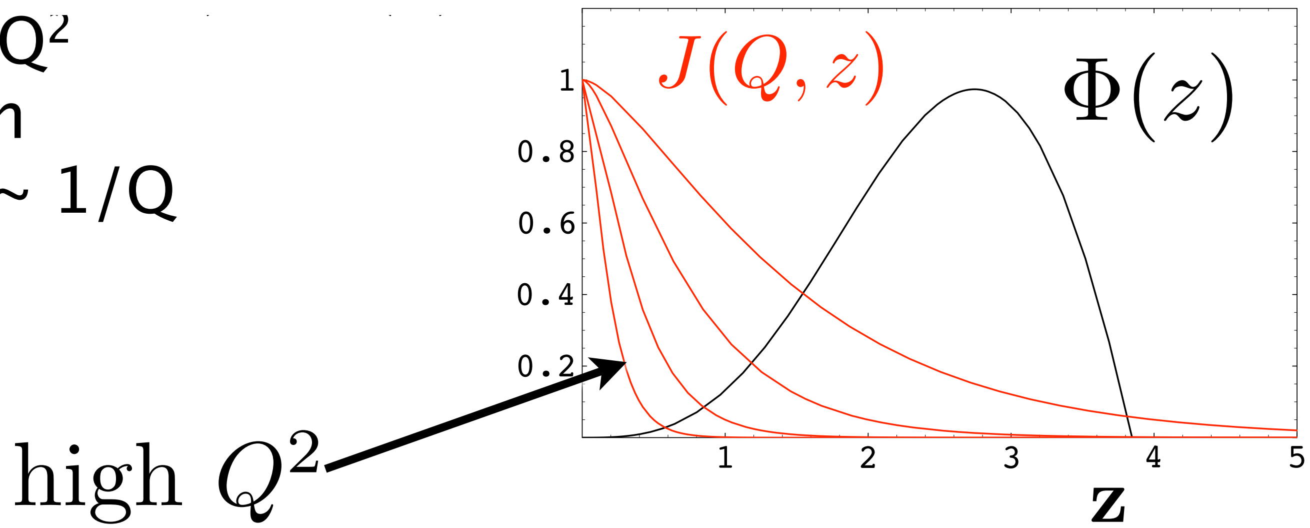
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High Q^2 from small $z \sim 1/Q$



Polchinski, Strassler de Teramond, sjb

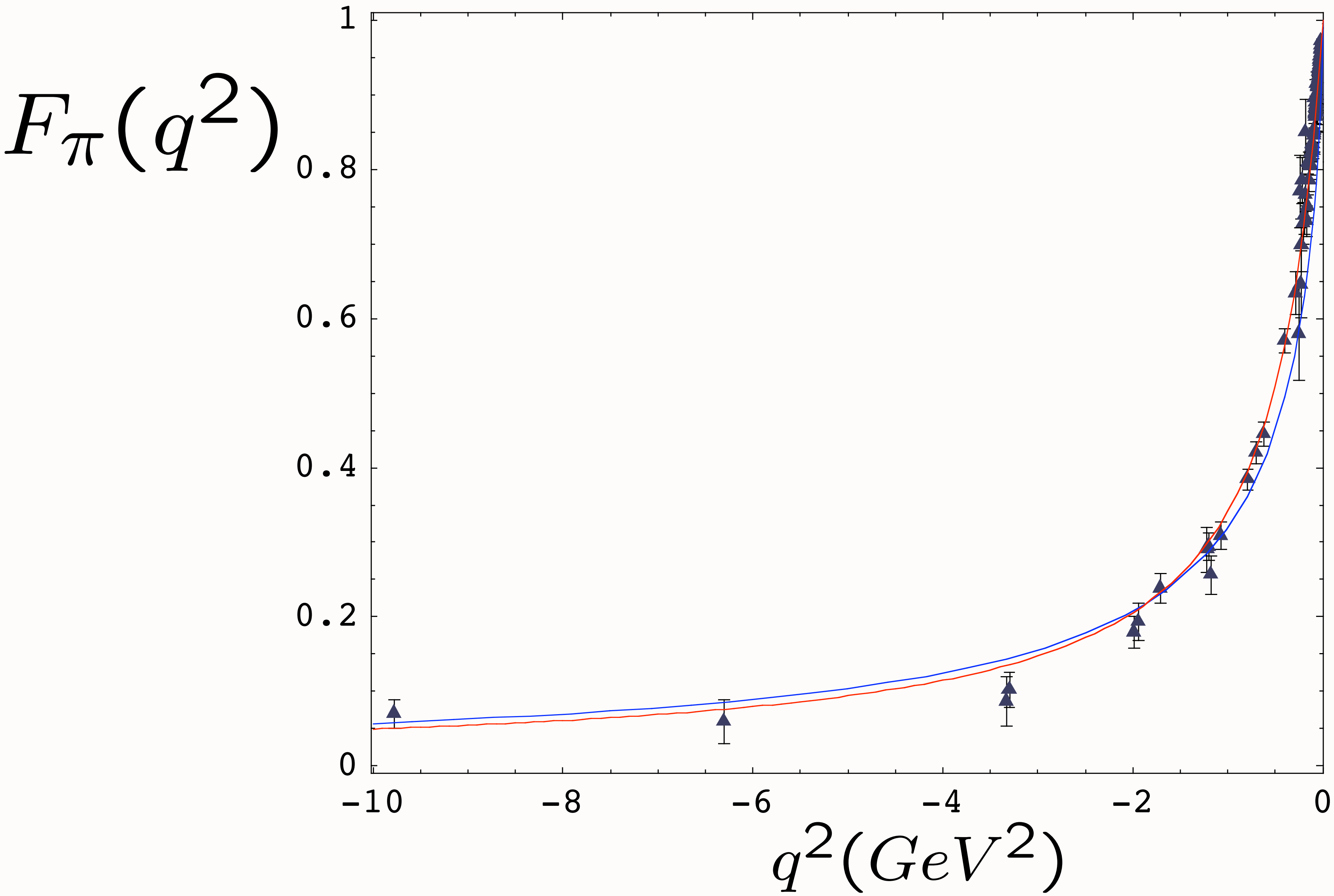
Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, Φ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rules: General result from AdS/CFT and Conformal Invariance

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n}\sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Spacelike pion form factor from AdS/CFT



Data Compilation Baldini, Kloe and Volmer

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3}\, H(Q^2, z)\, |\Phi_\pi(z)|^2 \,,$$

Abidin & Carlson

where $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x\, dx\, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x\, dx \int \frac{dz}{z^3}\, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right) |\Phi_\pi(z)|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary Q

$$\boxed{\left|\tilde{\psi}_{q\overline{q}/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi}\, x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4},}$$

Identical to LF Holography obtained from electromagnetic current

Current Matrix Elements in AdS Space (SW)

sjb and GdT
Grigoryan and Radyushkin

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q,z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q,z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

Soft Wall Model

where $U(a,b,c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a,b,z) = \int_0^\infty e^{-zt} t^{a-1}(1+t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

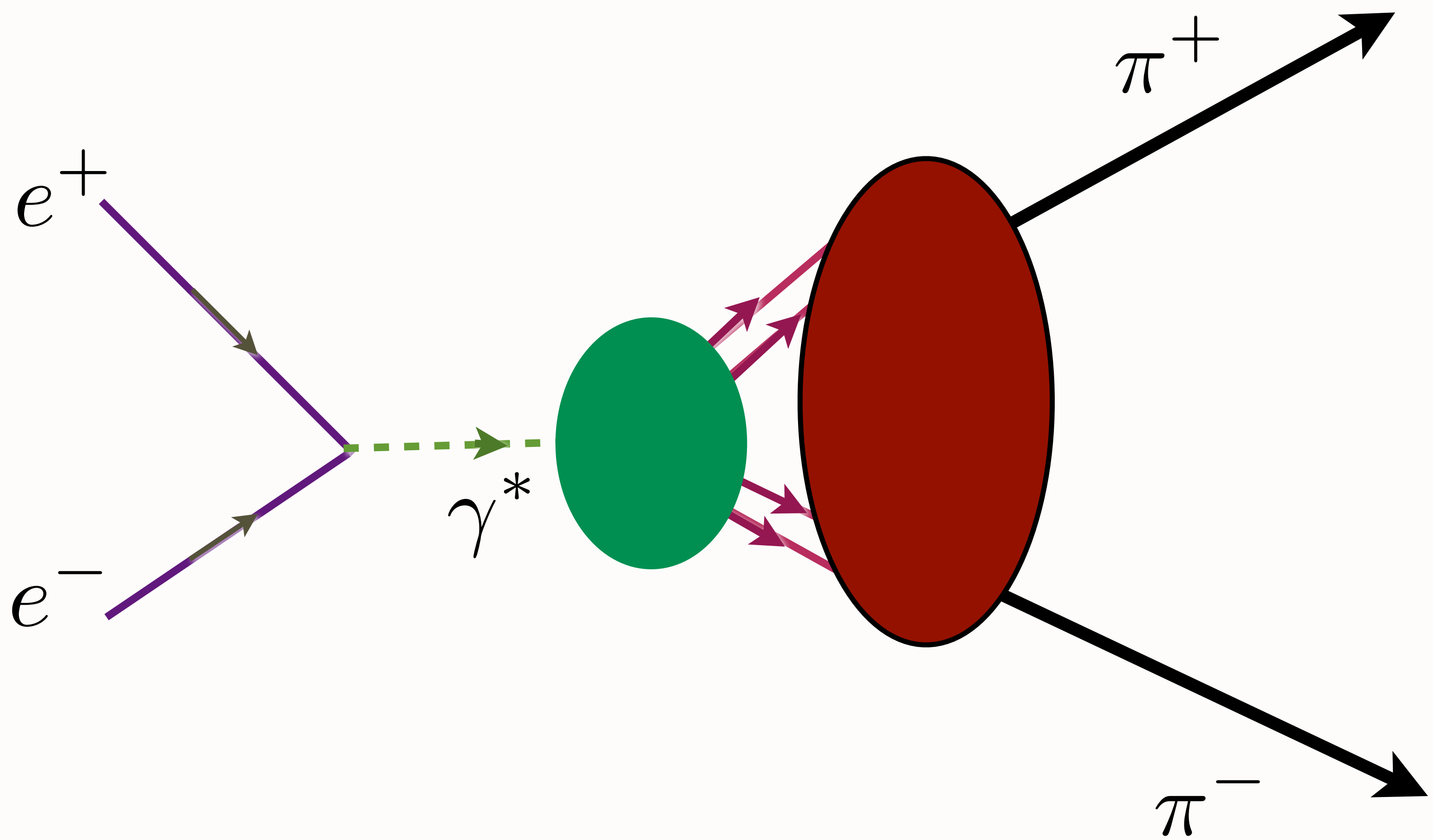
$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q,z) \Phi(z).$$

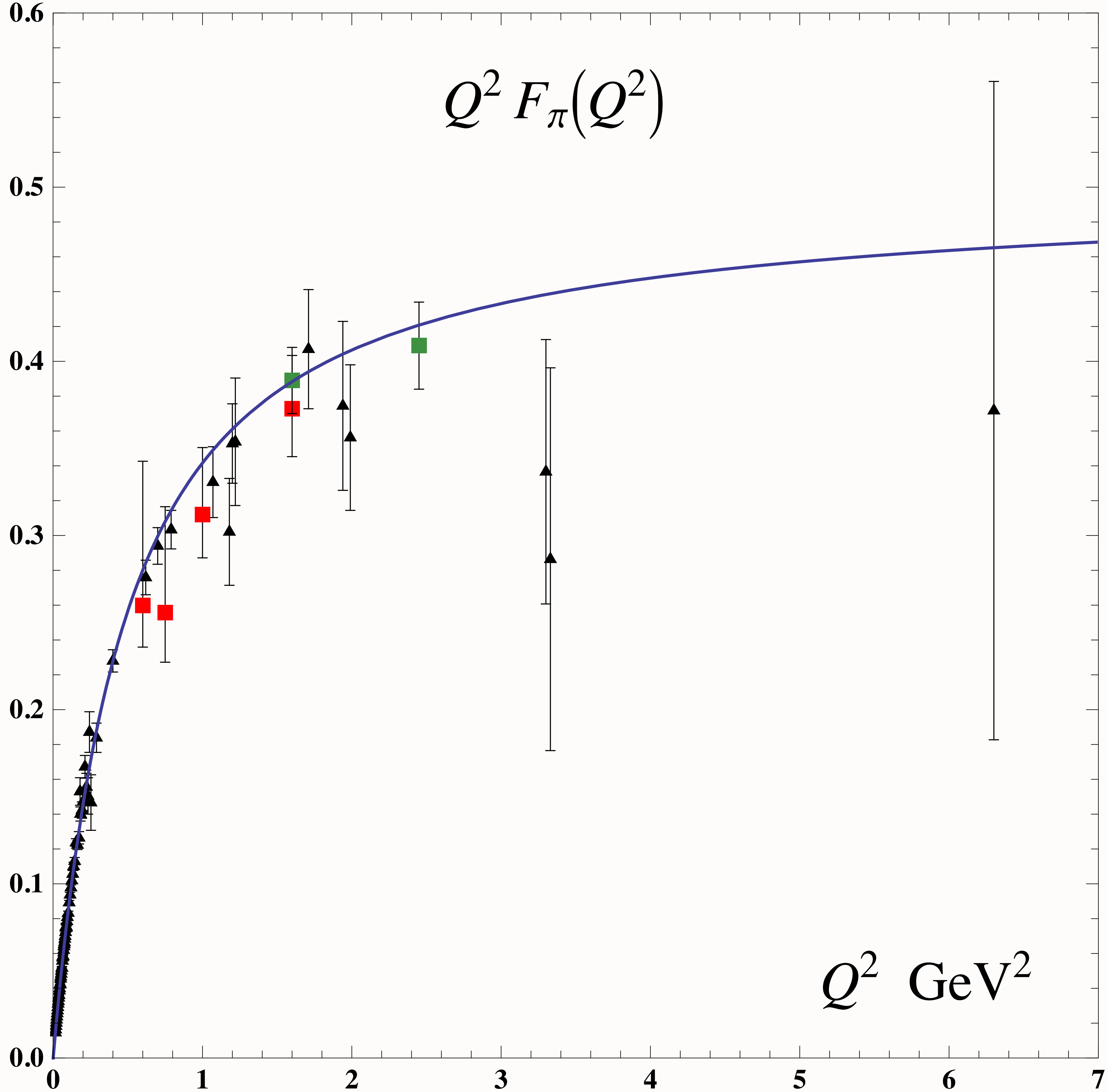
- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q,z) \to zQK_1(zQ) = J(Q,z),$$

the external current decouples from the dilaton field.

Dressed soft-wall current brings in higher Fock states and more vector meson poles




$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7

Spacelike and Timelike Pion Form Factor

de Teramond, sjb

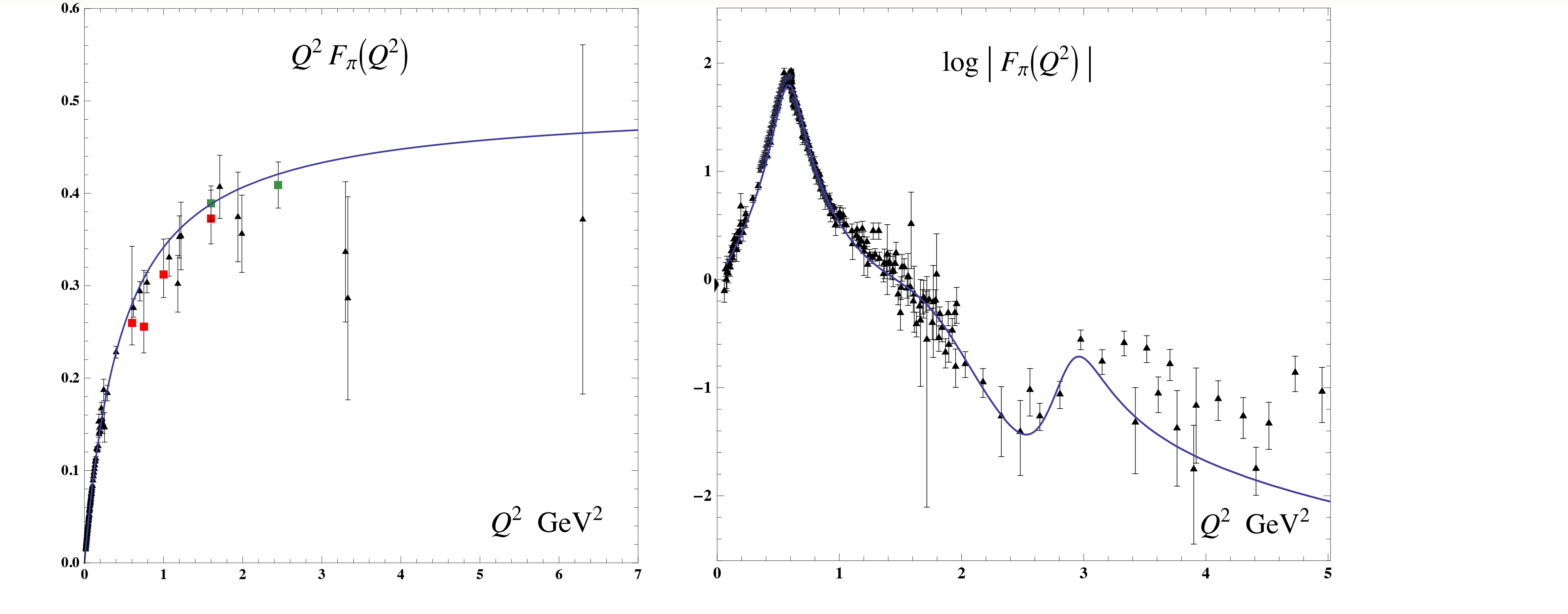
- Higher Fock components in pion LFWF

$$|\pi\rangle = \psi_{q\bar{q}/\pi}|q\bar{q}\rangle_{\tau=2} + \psi_{q\bar{q}q\bar{q}/\pi}|q\bar{q}q\bar{q}\rangle_{\tau=4} + \cdots$$

corresponding to interpolating operators $\mathcal{O} = \overline{\psi}\gamma^+\gamma^5\psi$ and $\mathcal{O} = \overline{\psi}\gamma^+\gamma^5\psi\psi\overline{\psi}$

- Expansion of LFWF up to twist 4

$$\kappa = 0.54 \text{ GeV}, \Gamma_\rho = 130,\ \Gamma_{\rho'} = 400,\ \Gamma_{\rho''} = 300 \text{ MeV}, P_{q\bar{q}q\bar{q}} = 13\%$$



Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$\begin{aligned} F_+(Q^2) &= g_+ \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2, \\ F_-(Q^2) &= g_- \int d\zeta\, J(Q,\zeta) |\psi_-(\zeta)|^2, \end{aligned}$$

  where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

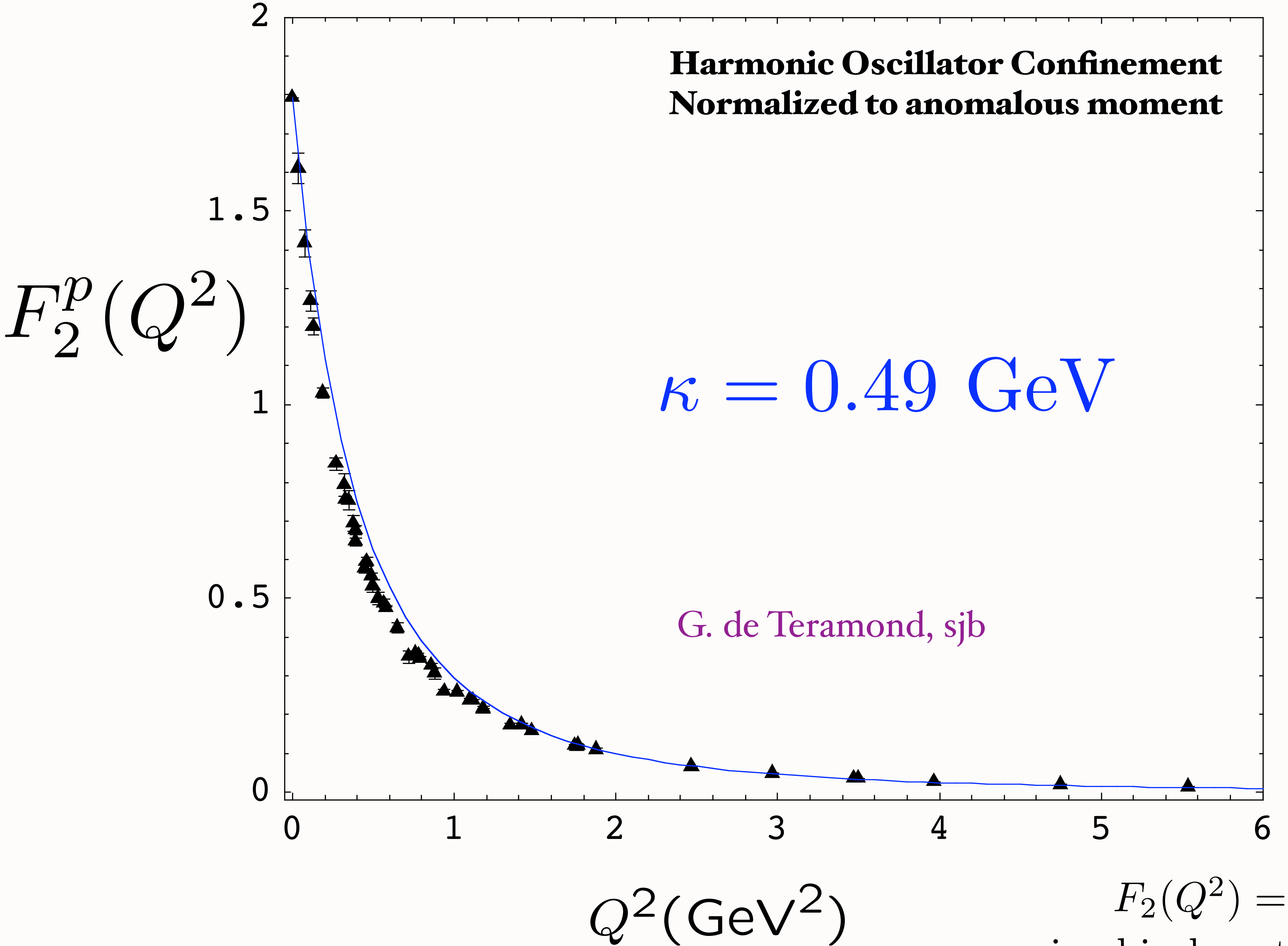
- For $SU(6)$ spin-flavor symmetry

$$\begin{aligned} F_1^p(Q^2) &= \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2, \\ F_1^n(Q^2) &= -\frac{1}{3} \int d\zeta\, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2\right], \end{aligned}$$

  where $F_1^p(0) = 1,\ F_1^n(0) = 0$.

Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs



AdS/QCD No chiral divergence!

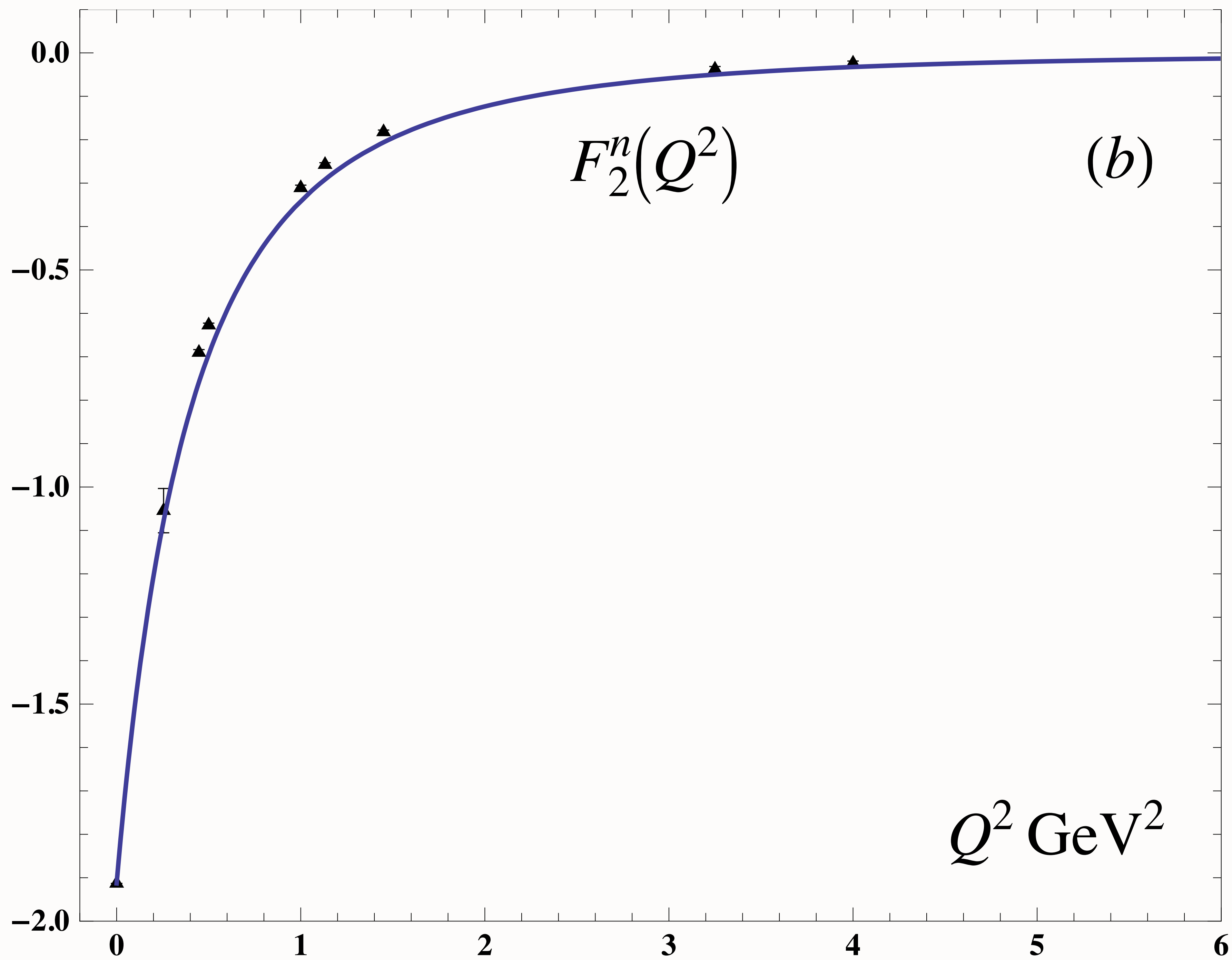
$$F_2(Q^2) = 1 + \mathcal{O}\frac{Q^2}{m_\pi m_p}$$

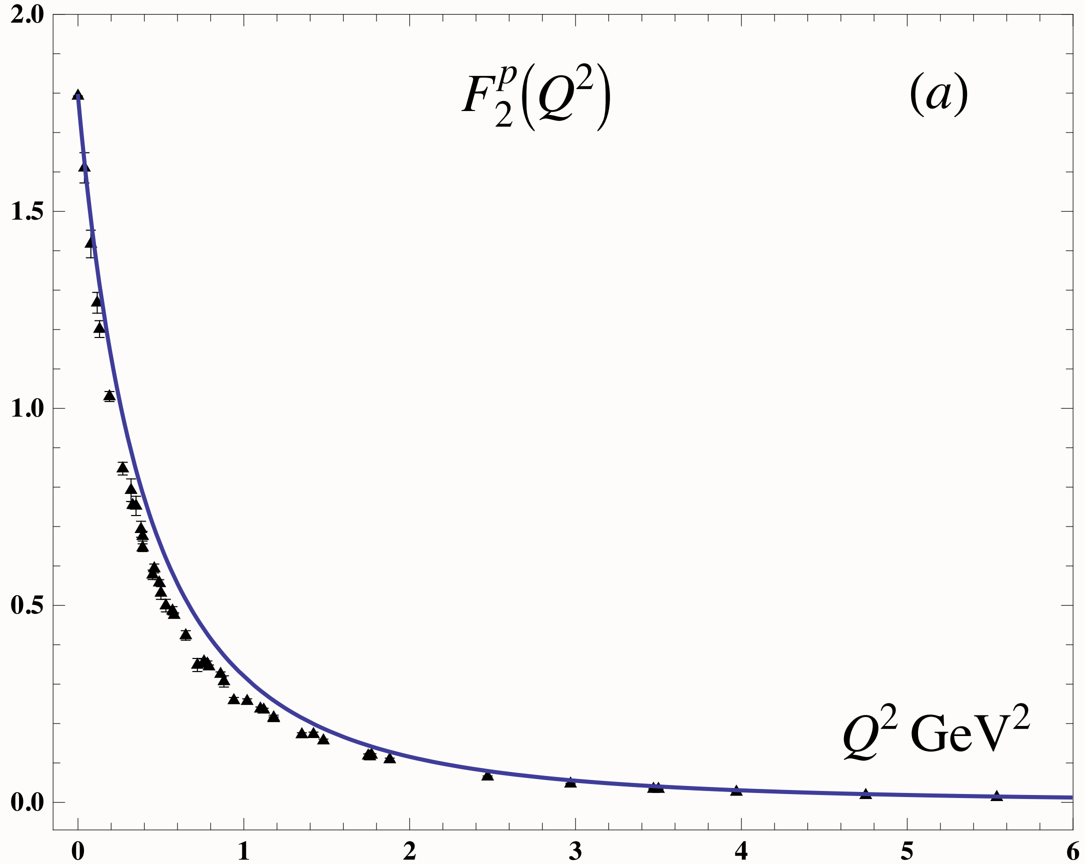
in chiral perturbation theory

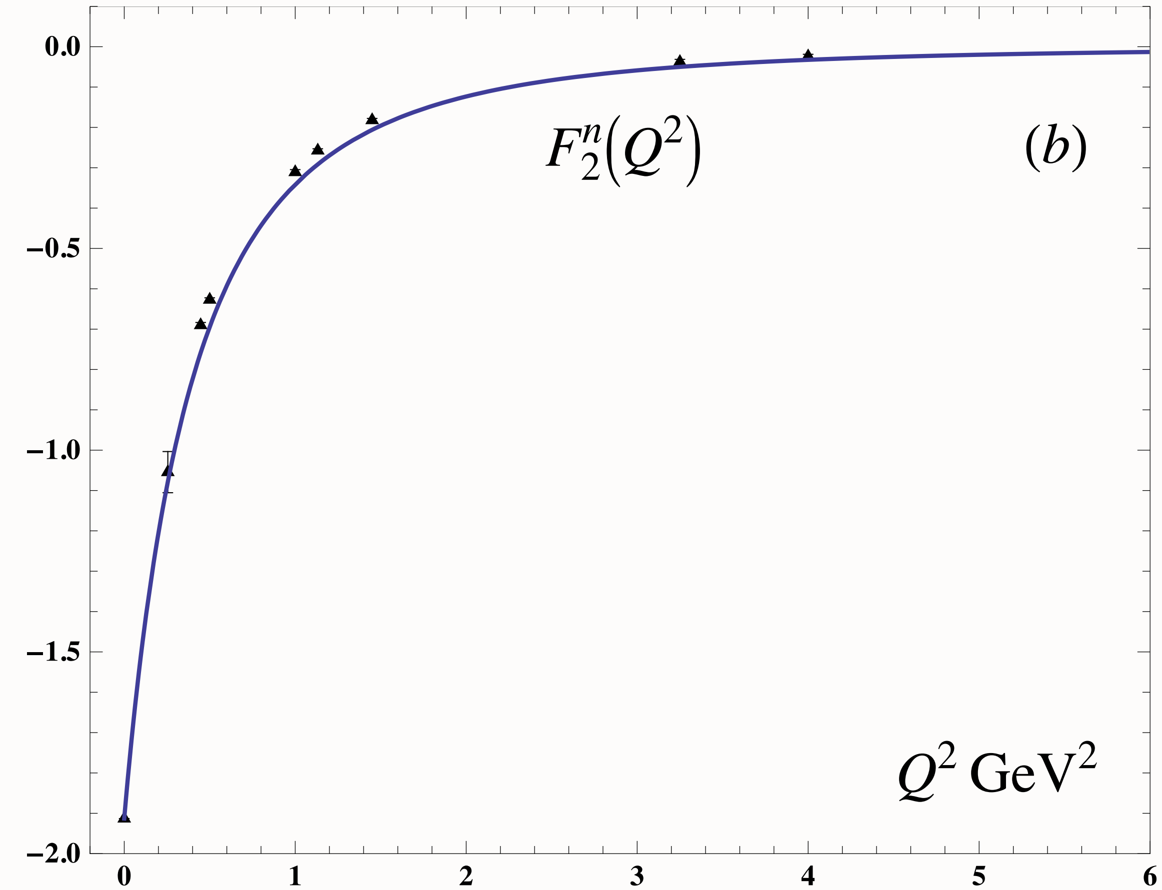
Spacelike Neutron Pauli Form Factor

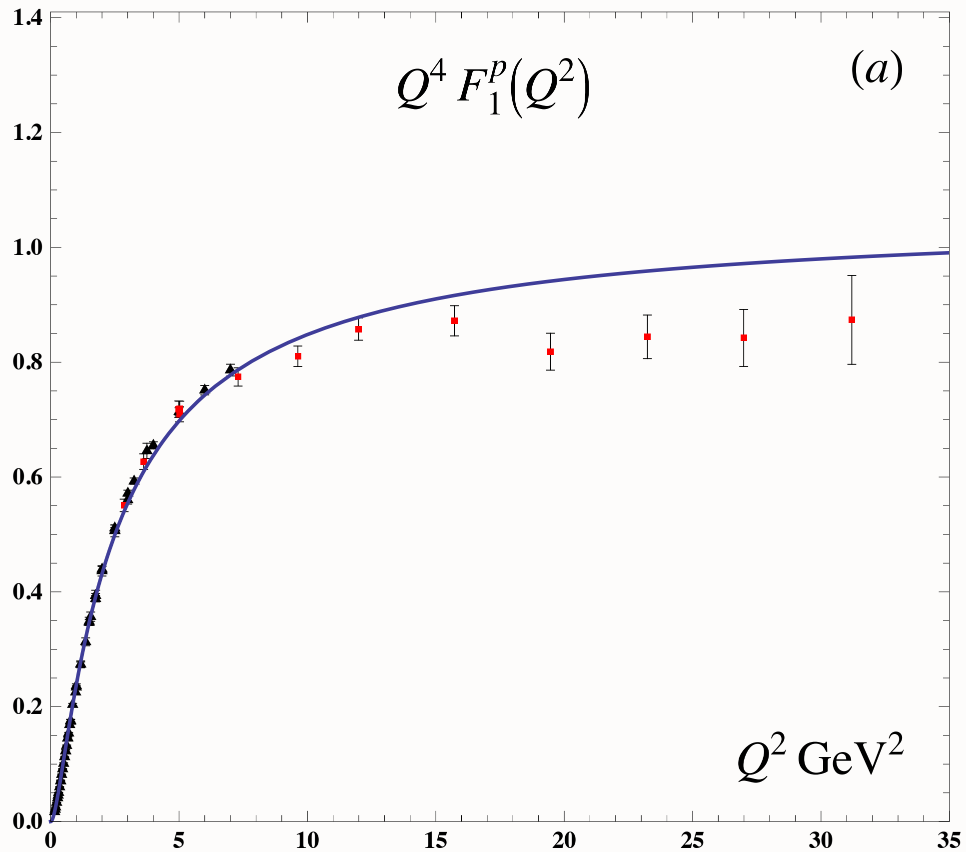
Preliminary

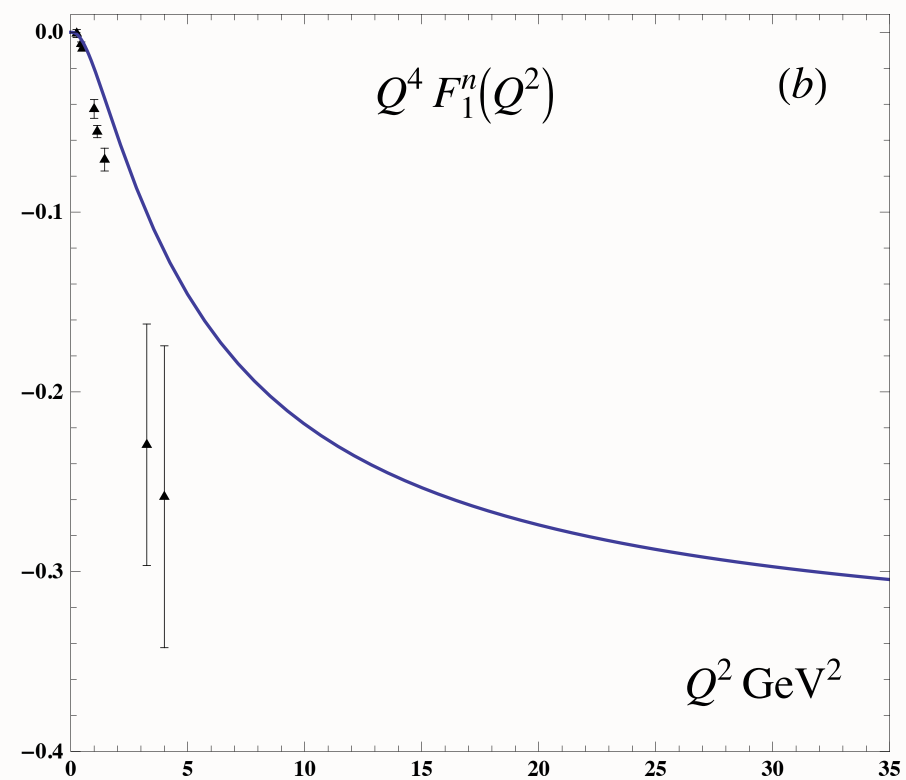
From overlap of $L = 1$ and $L = 0$ LFWFs




$F_2^p(Q^2)$
(a)
Q^2 GeV^2


$F_2^n(Q^2)$
(b)
Q^2 GeV^2


$Q^4 F_1^p(Q^2)$
(a)
Q^2 GeV^2


$Q^4 F_1^n(Q^2)$
(b)
Q^2 GeV^2

Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \to N^*(1440)$: $\Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$

- Transition form factor

$$F_1{}^p_{N \to N^*}(Q^2) = R^4 \int \frac{dz}{z^4} \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $\left(F_1{}^p_{N \to N^*}(0) = 0, \quad V(Q=0,z) = 1\right)$

$$R^4 \int \frac{dz}{z^4} \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

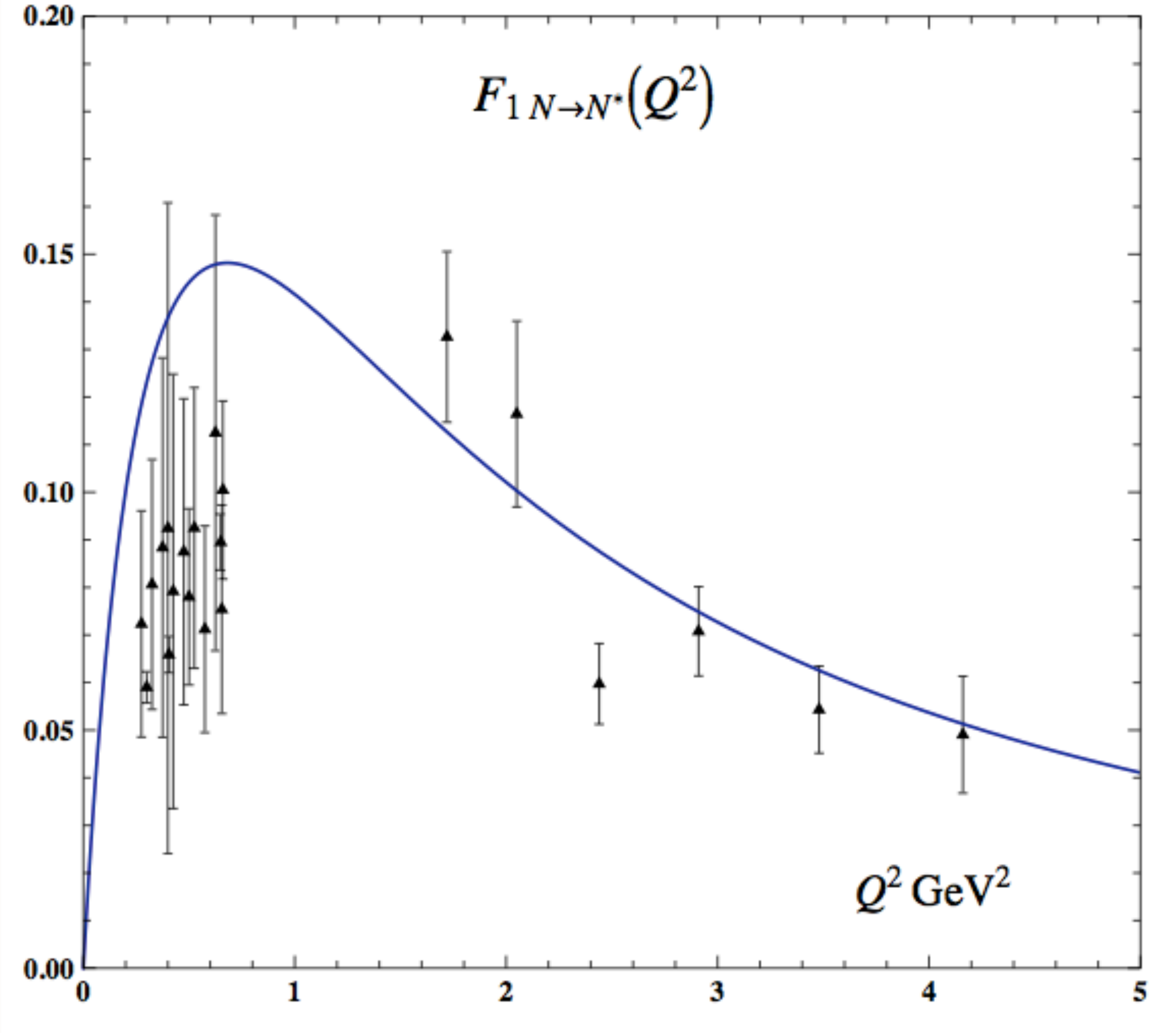
$$F_1{}^p_{N \to N^*}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

with $\mathcal{M}_{\rho_n}^2 \to 4\kappa^2(n + 1/2)$

de Teramond, sjb

Consistent with counting rule, twist 3

$N(940) \to N^*(1440)$: $\quad \Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$



Data from I. Aznauryan, *et al.* CLAS (2009)

$$\boxed{F_{1\,N\to N^*}^{\,p}(Q^2) = \frac{2\sqrt{2}}{3}\,\frac{\frac{Q^2}{M_P^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}}$$

with ${\mathcal{M}_{\rho_n}}^2 \to 4\kappa^2(n+1/2)$

Note: Analytical Form of Hadronic Form Factor for Arbitrary Twist

- Form factor for a string mode with scaling dimension τ, Φ_τ in the SW model

$$F(Q^2) = \Gamma(\tau)\frac{\Gamma\left(1+\frac{Q^2}{4\kappa^2}\right)}{\Gamma\left(\tau+\frac{Q^2}{4\kappa^2}\right)}.$$

- For $\tau = N, \quad \Gamma(N + z) = (N - 1 + z)(N - 2 + z)\dots(1 + z)\Gamma(1 + z).$

- Form factor expressed as $N - 1$ product of poles

$$\begin{aligned}
F(Q^2) &= \frac{1}{1+\frac{Q^2}{4\kappa^2}}, \quad N = 2, \\
F(Q^2) &= \frac{2}{\left(1+\frac{Q^2}{4\kappa^2}\right)\left(2+\frac{Q^2}{4\kappa^2}\right)}, \quad N = 3, \\
&\cdots \\
F(Q^2) &= \frac{(N-1)!}{\left(1+\frac{Q^2}{4\kappa^2}\right)\left(2+\frac{Q^2}{4\kappa^2}\right)\cdots\left(N-1+\frac{Q^2}{4\kappa^2}\right)}, \quad N.
\end{aligned}$$

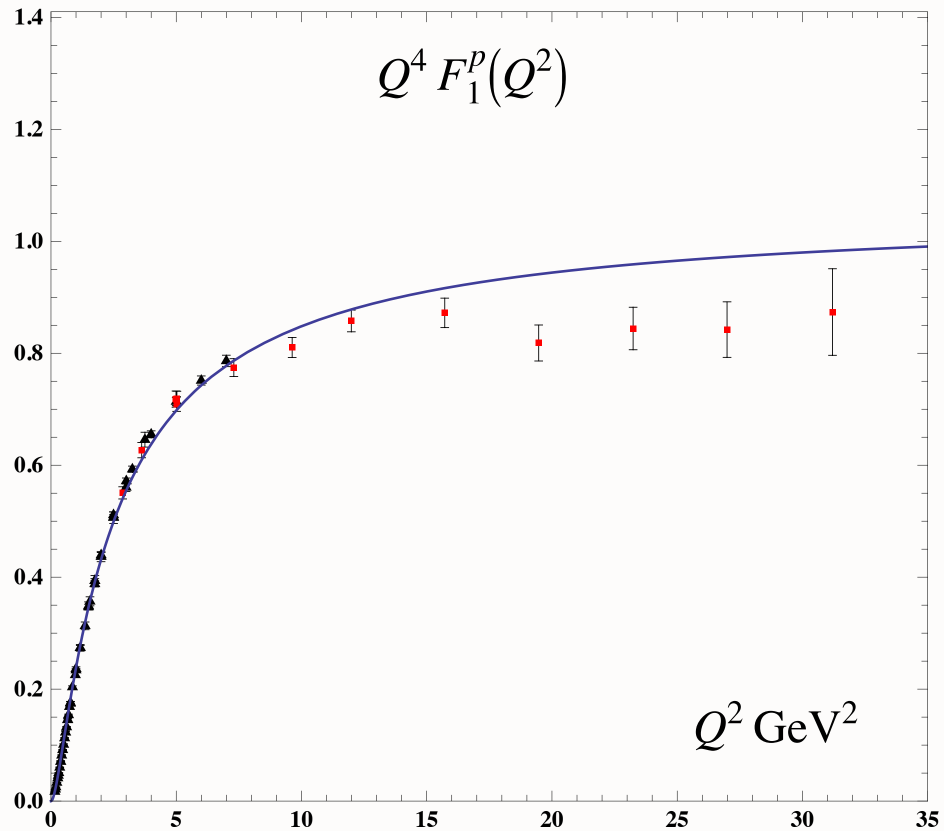
- For large Q^2:

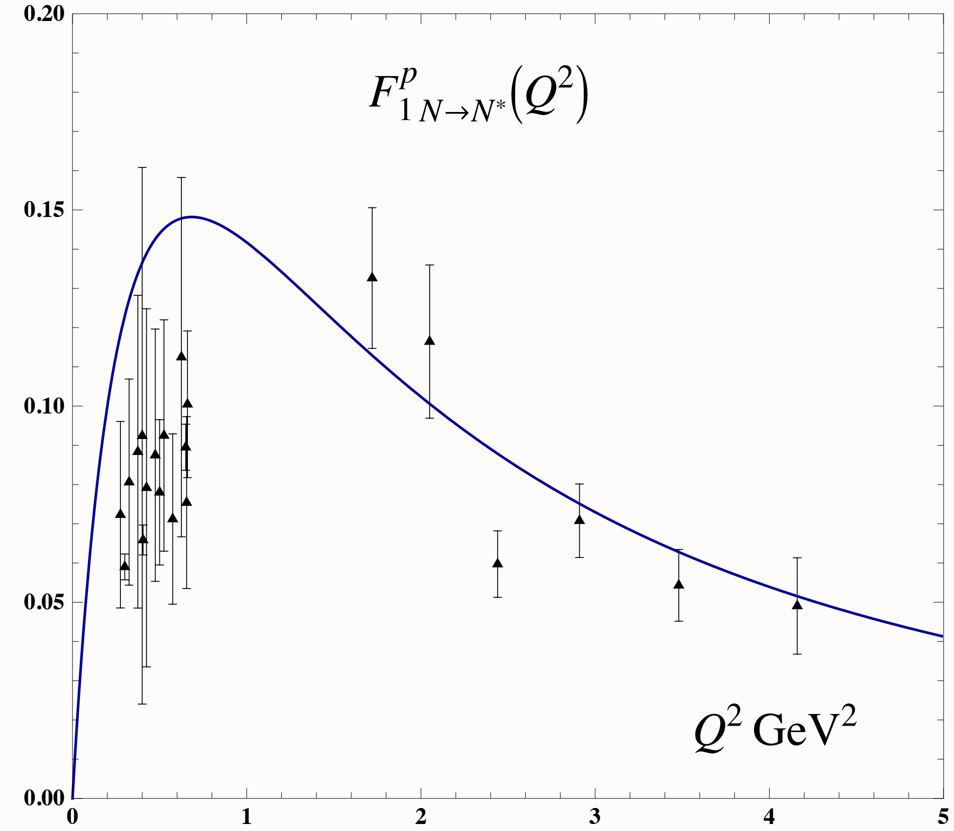
$$F(Q^2) \to (N-1)!\left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)}.$$

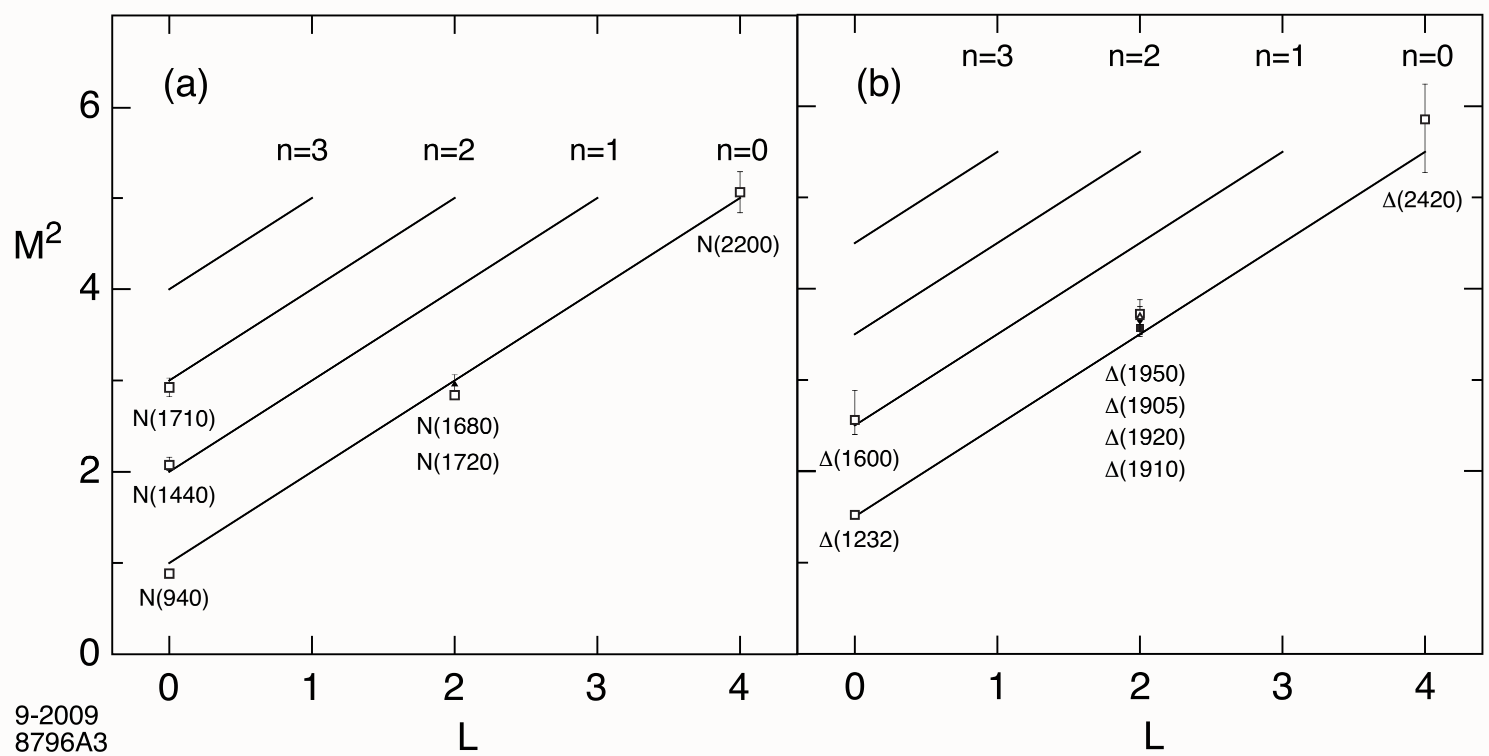
Predict hadron spectroscopy and dynamics

Excited Baryons in Holographic QCD

G. de Teramond & sjb

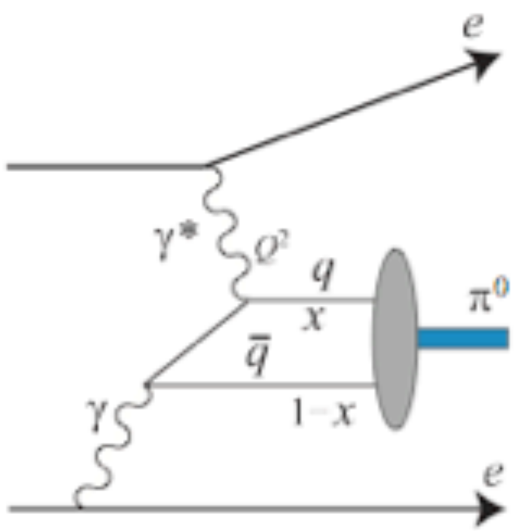






Pion Transition Form-Factor

[S. J. Brodsky, F.-G. Cao and GdT, arXiv:1005.39XX]



- Definition of $\pi - \gamma$ TFF from $\gamma^* \pi^0 \to \gamma$ vertex in the amplitude $e\pi \to e\gamma$

$$\Gamma^\mu = -ie^2 F_{\pi\gamma}(q^2)\epsilon_{\mu\nu\rho\sigma}(p_\pi)_\nu \epsilon_\rho(k) q_\sigma, \quad k^2 = 0$$

- Asymptotic value of pion TFF is determined by first principles in QCD:
  $Q^2 F_{\pi\gamma}(Q^2 \to \infty) = 2 f_\pi$ [Lepage and Brodsky (1980)]

- Pion TFF from 5-dim Chern-Simons structure [Hill and Zachos (2005), Grigoryan and Radyushkin (2008)]

$$\int d^4x \int dz\, \epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q$$

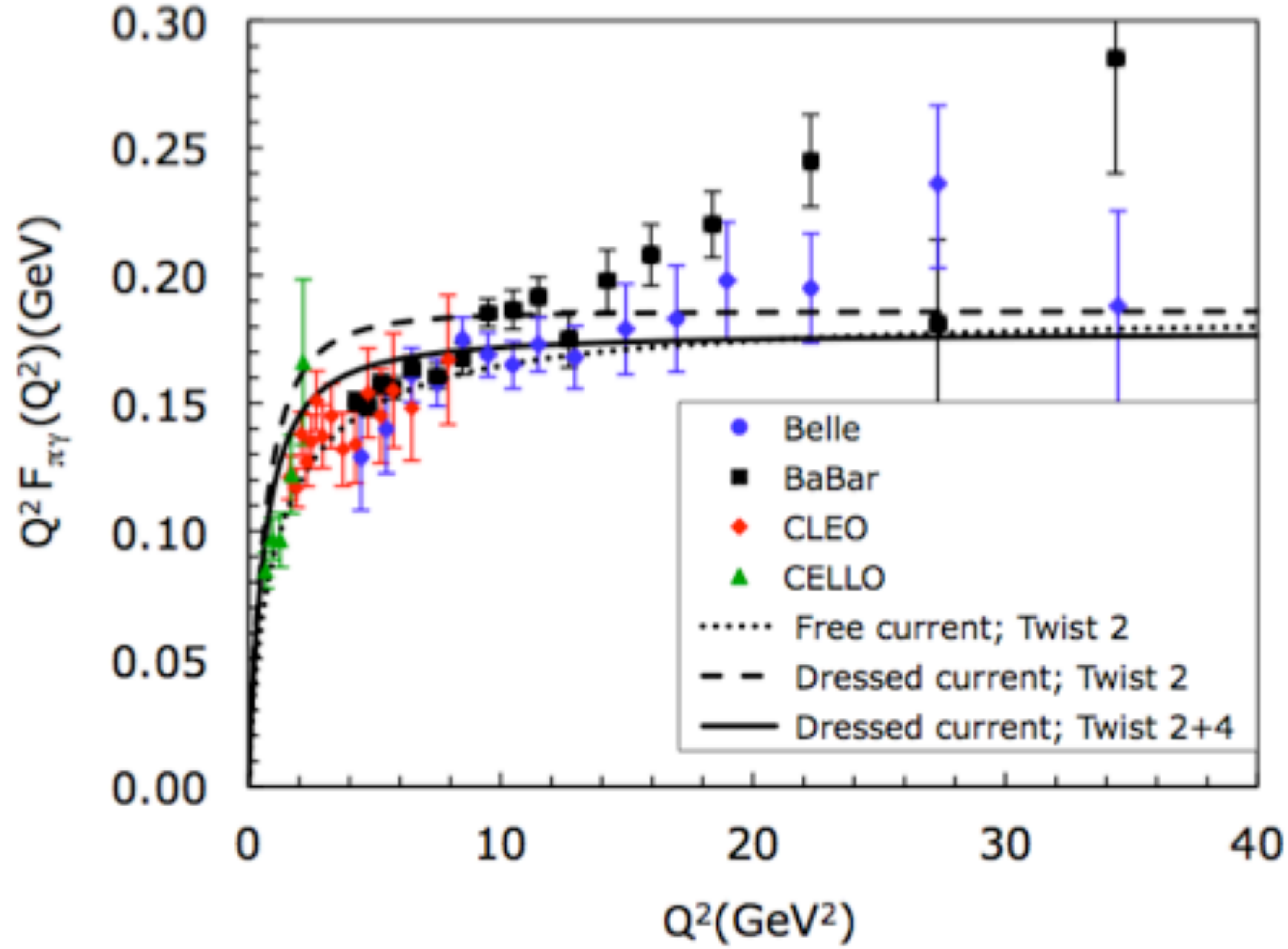
$$\sim (2\pi)^4 \delta^{(4)}\left(p_\pi + q - k\right) F_{\pi\gamma}(q^2) \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu(q)(p_\pi)_\nu \epsilon_\rho(k) q_\sigma$$

- Find for $A_z \propto \Phi_\pi(z)/z$

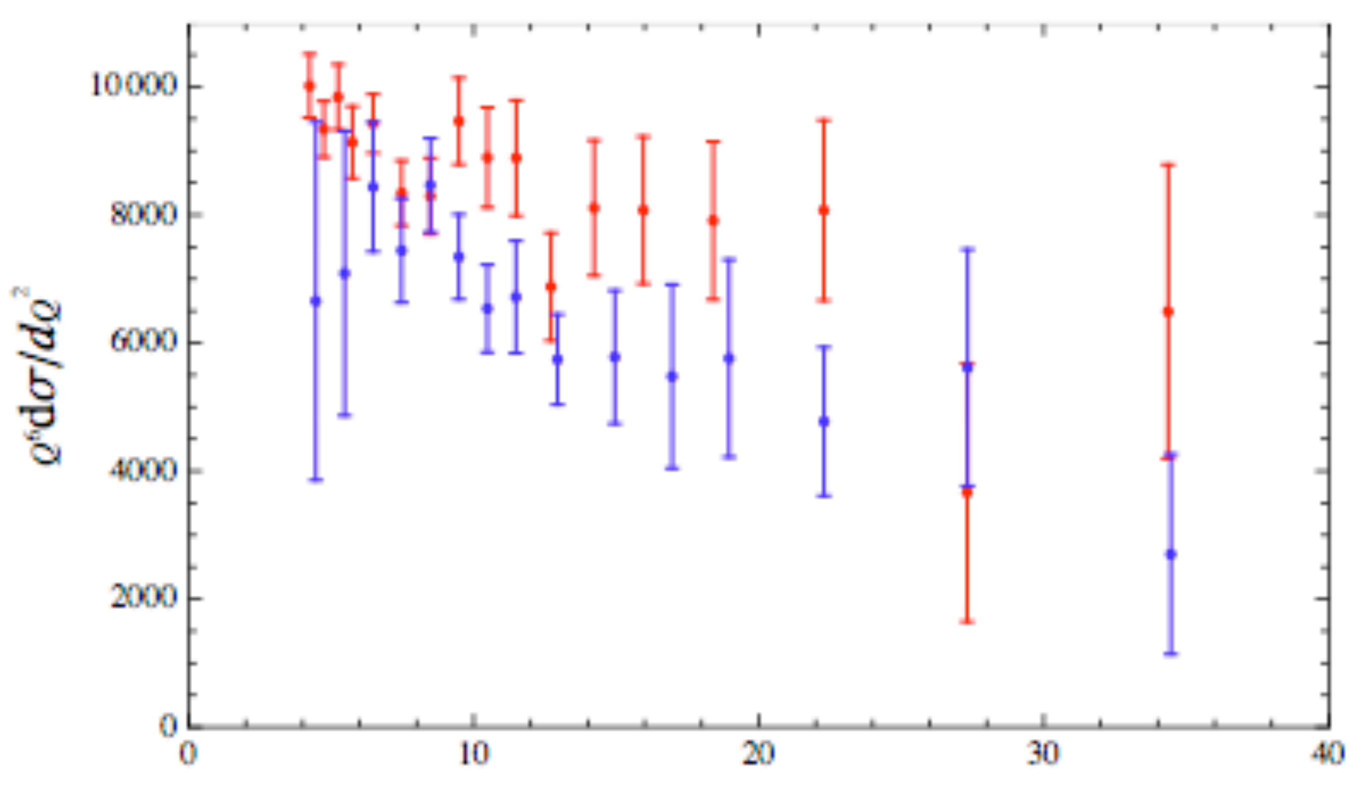
$$F_{\pi\gamma}(Q^2) = \frac{1}{2\pi} \int_0^\infty \frac{dz}{z}\, \Phi_\pi(z) V\left(Q^2, z\right)$$

  with normalization fixed by asymptotic QCD prediction

- $V\left(Q^2, z\right)$ bulk-to-boundary propagator of γ^*



0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
10
20
30
40
Q2 Fπγ(Q2)(GeV)
Q2(GeV2)
Belle
BaBar
CLEO
CELLO
Free current; Twist 2
Dressed current; Twist 2
Dressed current; Twist 2+4



10000
8000
6000
4000
2000
0
0
10
20
30
40

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \, \frac{1}{g_5^2} \, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \, \alpha_s^{AdS}(\zeta)$$

- Solution

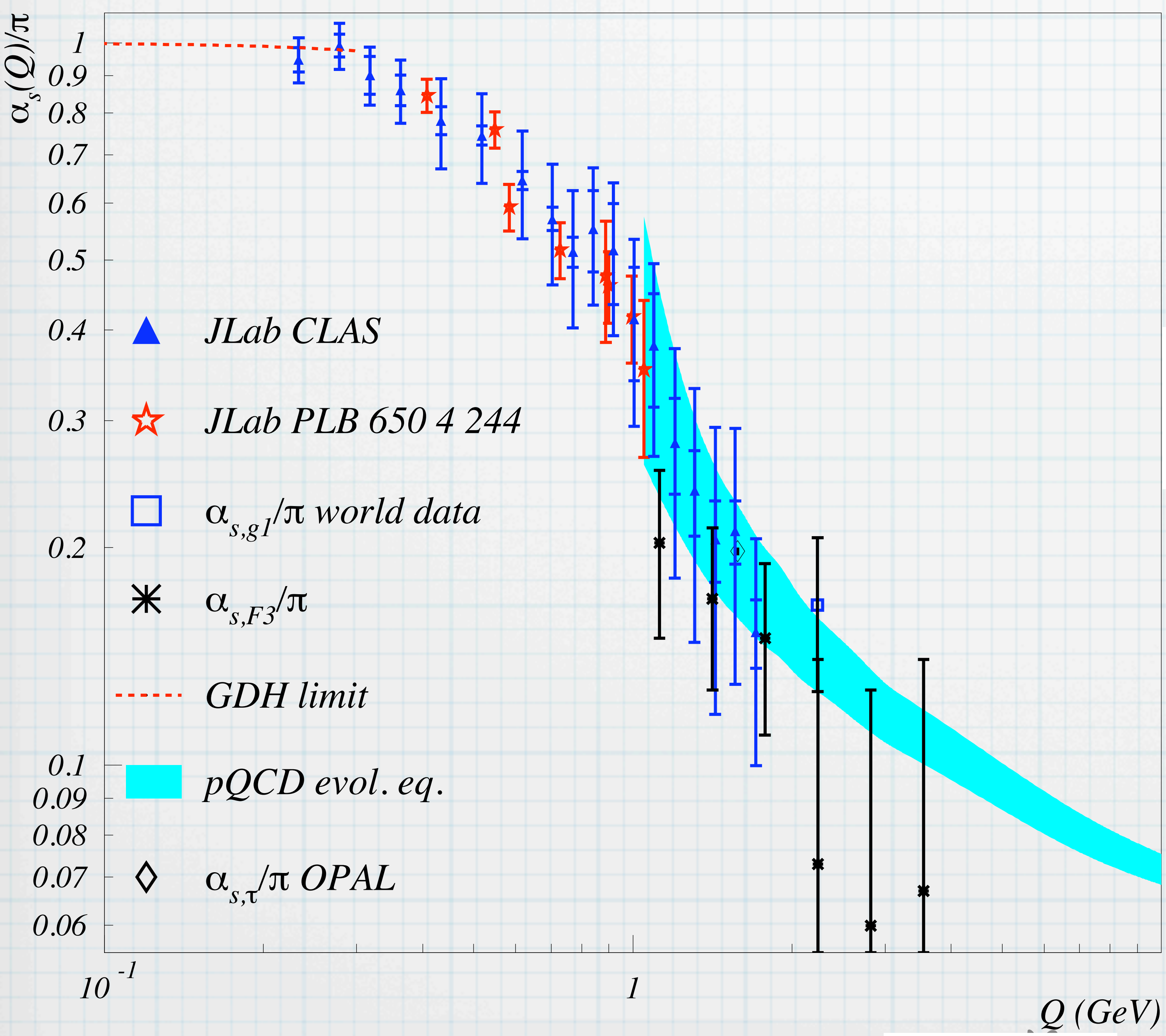
$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) \, e^{-Q^2/4\kappa^2}.$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Nearly conformal QCD?

Define α_s from Björkén sum,

$$\Gamma_1^{p-n} \equiv \int_0^1 dx\, \left(g_1^p(x,Q^2) - g_1^n(x,Q^2)\right) = \frac{1}{6} g_A \left(1 - \frac{\alpha_{s,g_1}}{\pi}\right)$$



g1 = spin dependent structure functio

JLab data from EG1(2008), CLAS, and Hall A

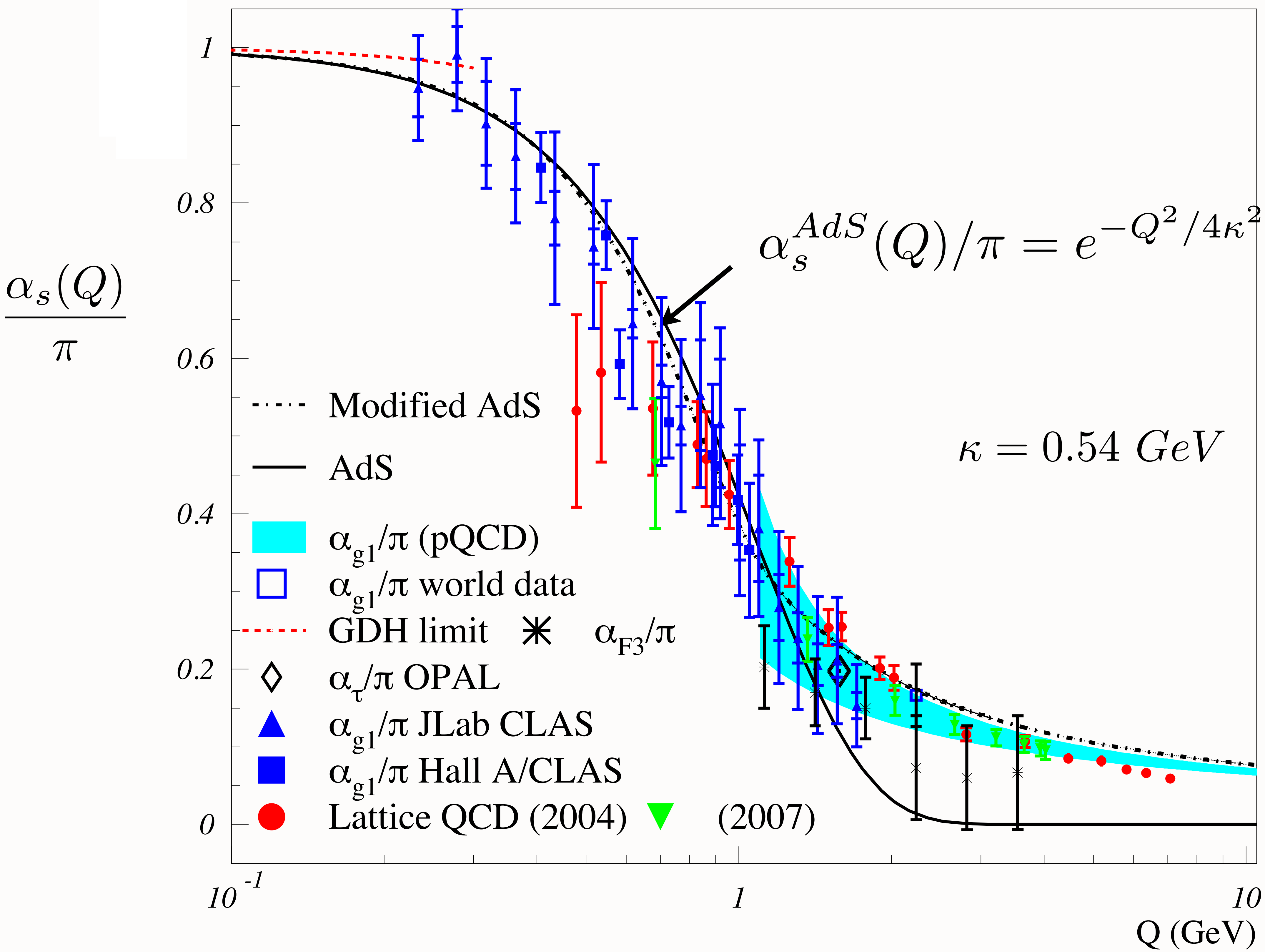
α_s runs only modestly at small Q^2

Fig. from 0803.4119, Duer et al.

Deur, de Teramond, sjb

Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point

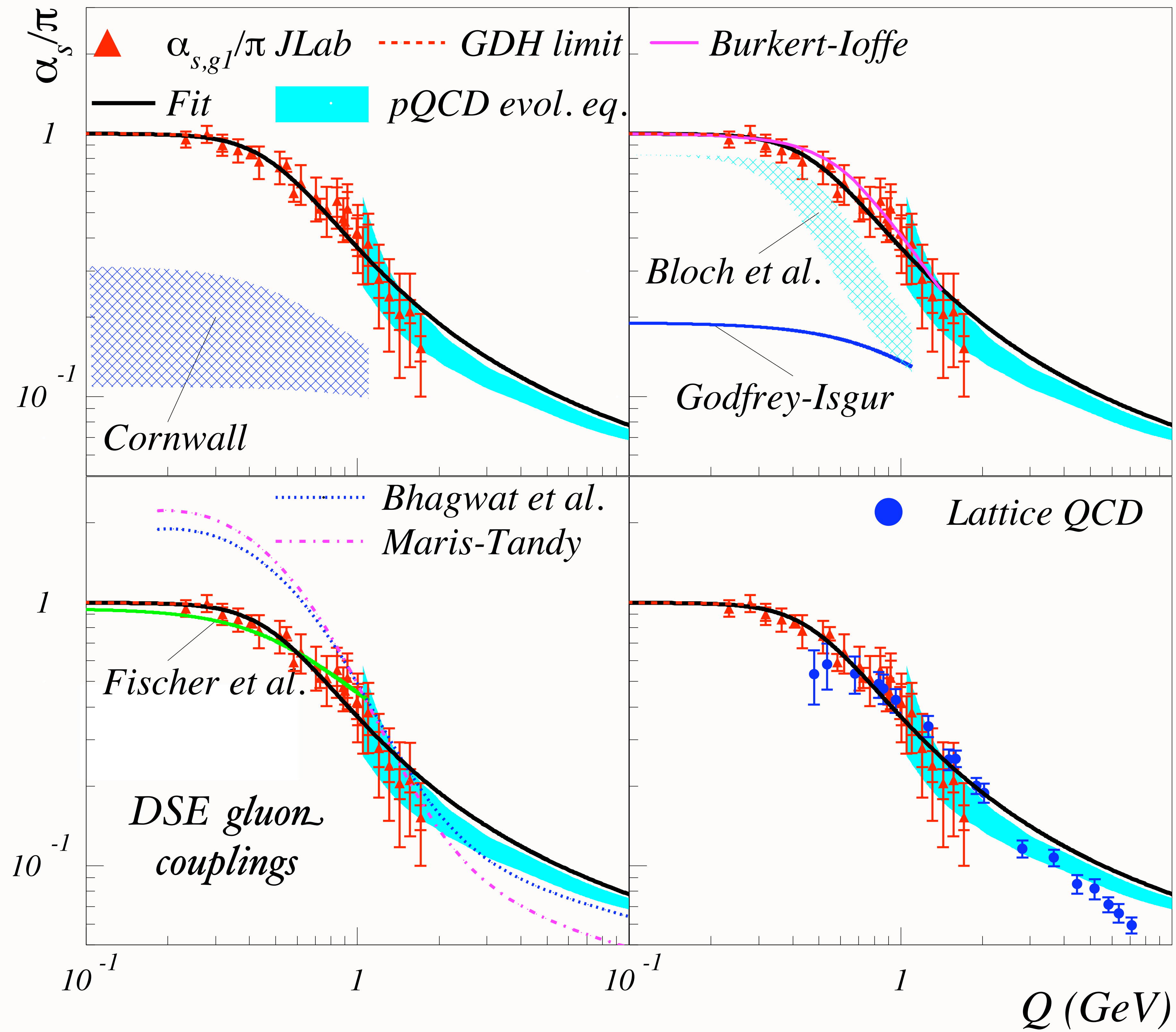


Deur, de Teramond, sjb

Sublimated Gluons

- AdS/QCD soft-wall model has confining potential . Gluon exchange absent.
- Coupling falls exponentially -- misses asymptotic freedom at large Q^2
- Interpretation: Gluons sublimated into potential below 1 GeV^2 virtuality
- Higher Fock states with extra quark-antiquark pairs, no gluons

Deur, Korsch, et al.

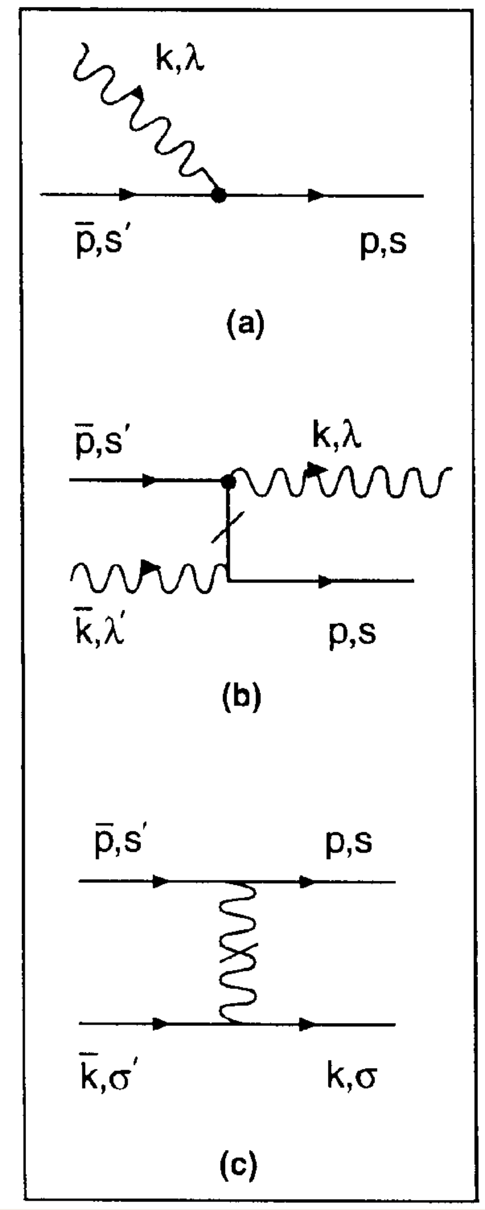


Light-Front QCD

Exact formulation of nonperturbative QCD

Physical gauge: $A^+ = 0$

$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

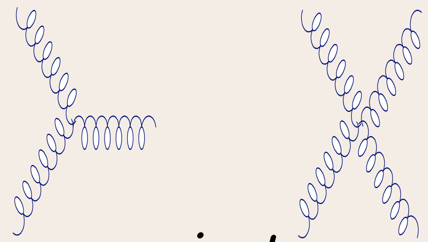


$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

H^{int}_{LF}: Matrix in Fock Space

$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

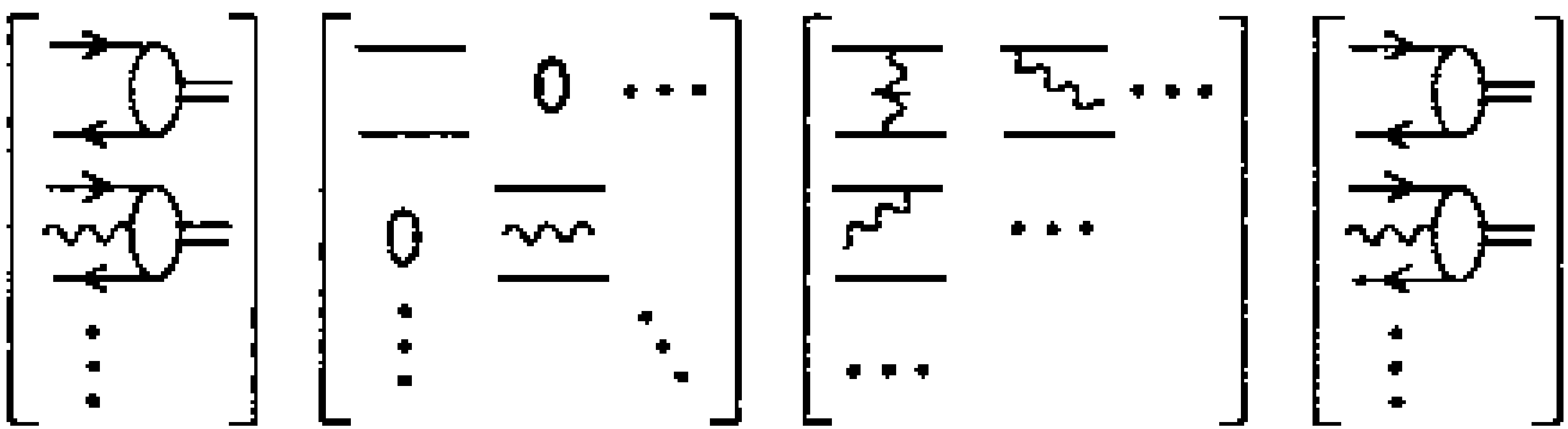


H^{int}_{LF}

LIGHT-FRONT SCHRODINGER EQUATION

Direct connection to QCD Lagrangian

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\, V \,|q\bar{q}\rangle & \langle q\bar{q}|\, V \,|q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\, V \,|q\bar{q}\rangle & \langle q\bar{q}g|\, V \,|q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$



$A^+ = 0$

G.P. Lepage, sjb

Eigensolutions of the LF Hamiltonian:

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

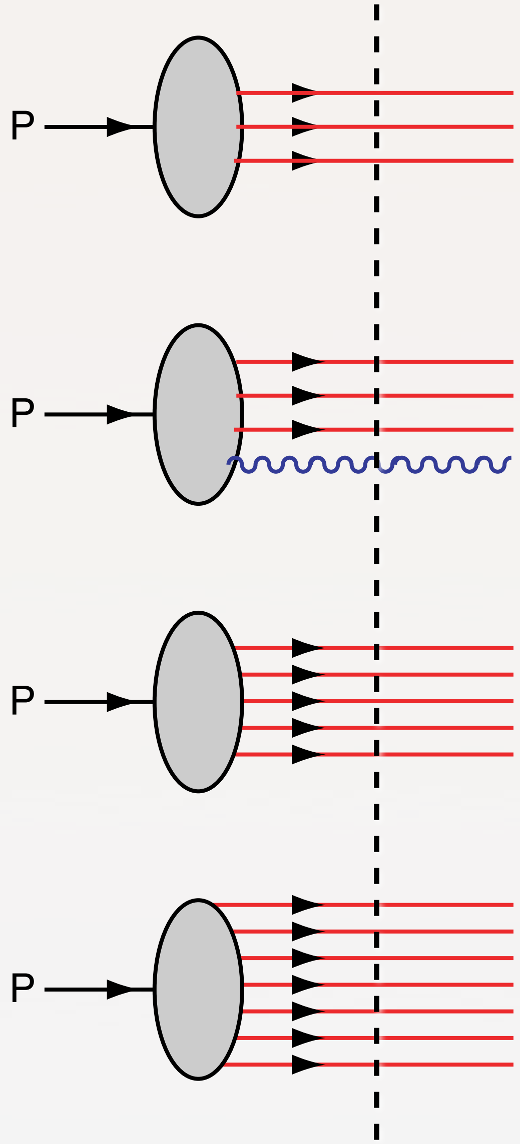
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \; \sum_i^n x_i = 1, \; \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time
Coupled. infinite set

Intrinsic heavy quarks
s(x), c(x), b(x) at high x !

$\bar{s}(x) \neq s(x)$
$\bar{u}(x) \neq \bar{d}(x)$

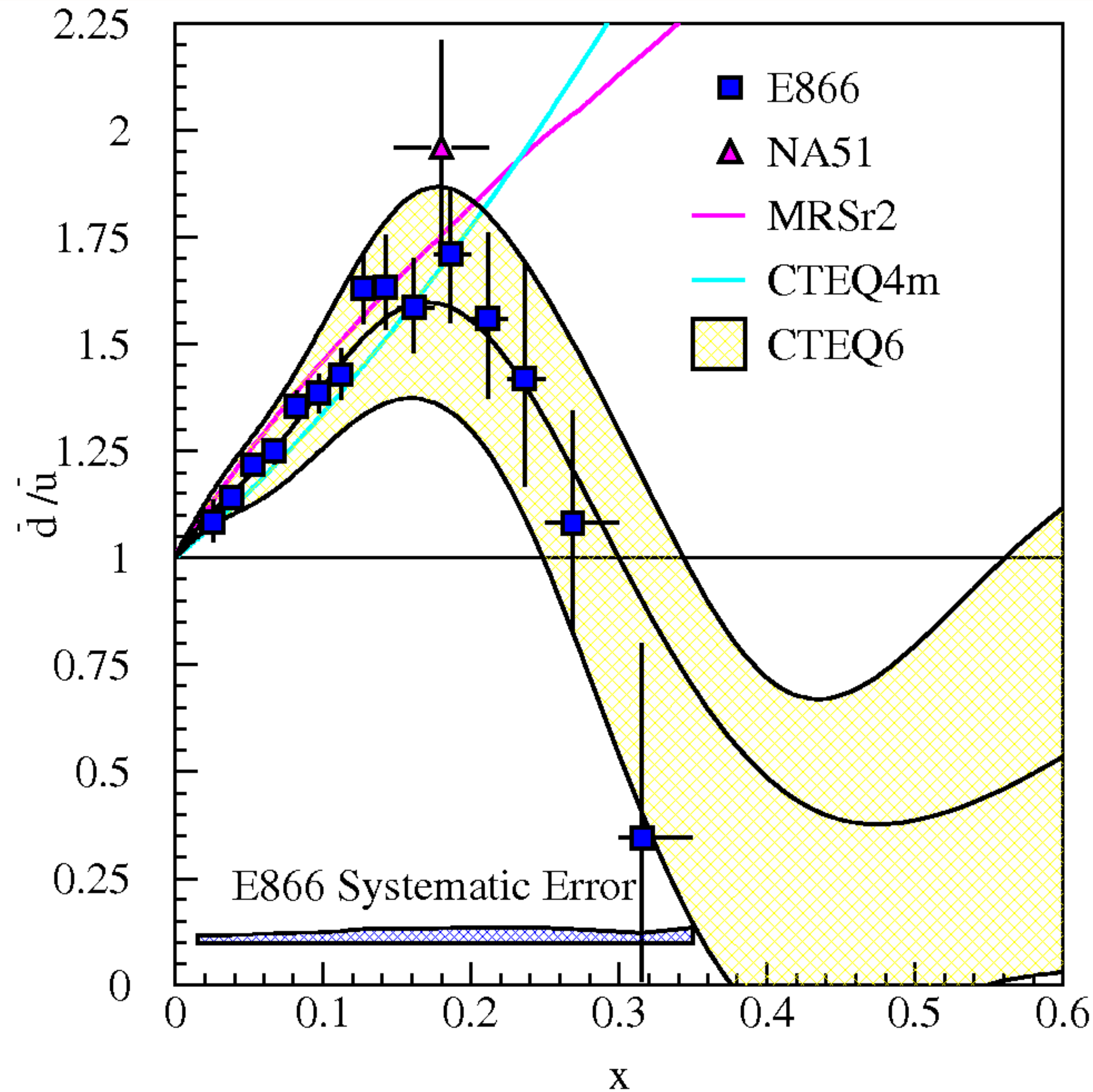
Nuclei: Hidden Color

Mueller: gluonic Fock states >> BFKL

$\bar{d}(x)/\bar{u}(x)$ for $0.015 \leq x \leq 0.35$

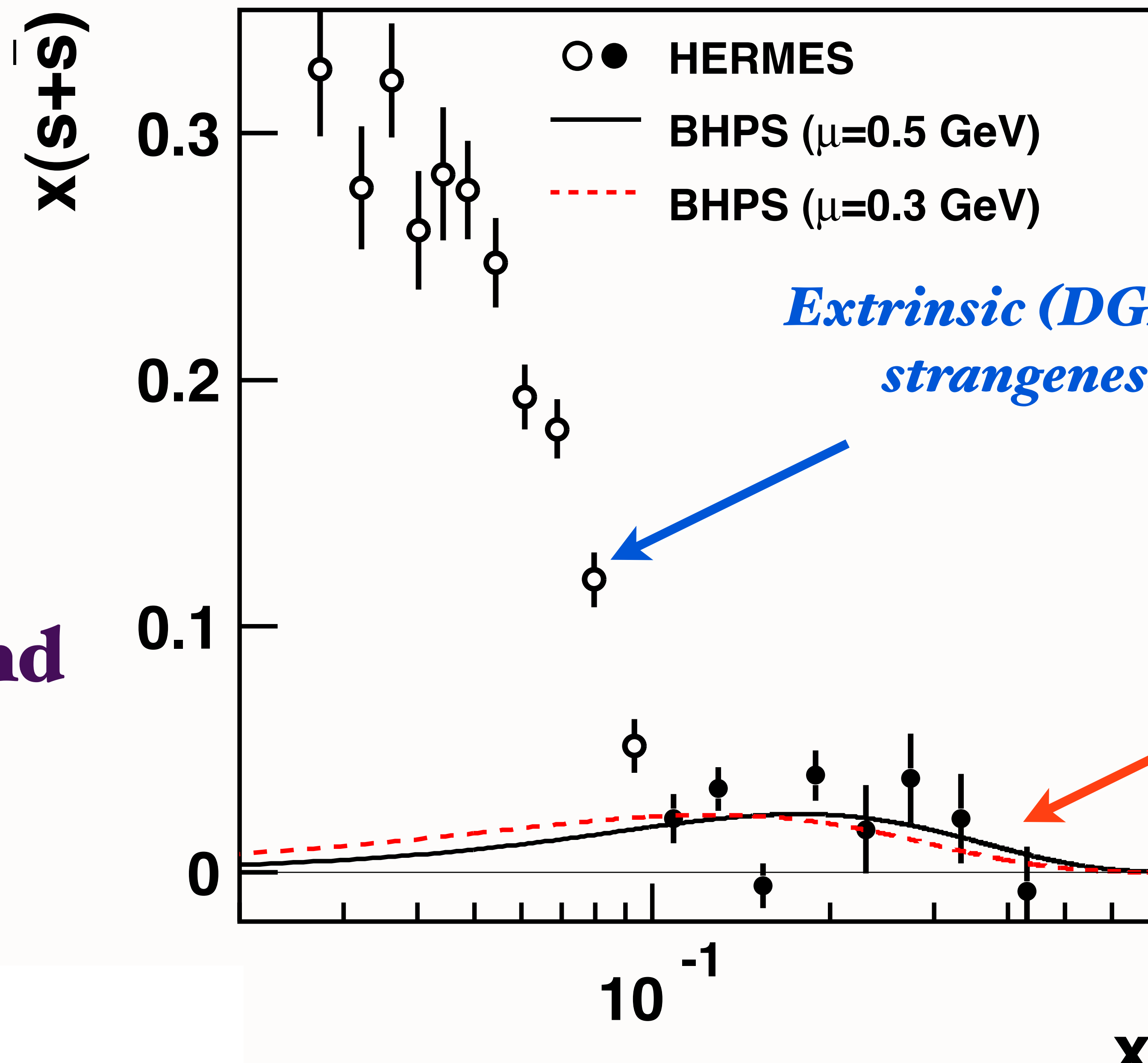
- E866/NuSea (Drell-Yan)

$$\bar{d}(x) \neq \bar{u}(x)$$



Intrinsic glue, sea, heavy quarks

HERMES: Two components to s(x,Q²)!



Intrinsic strangeness!

W. C. Chang and J.-C. Peng

arXiv:1105.2381

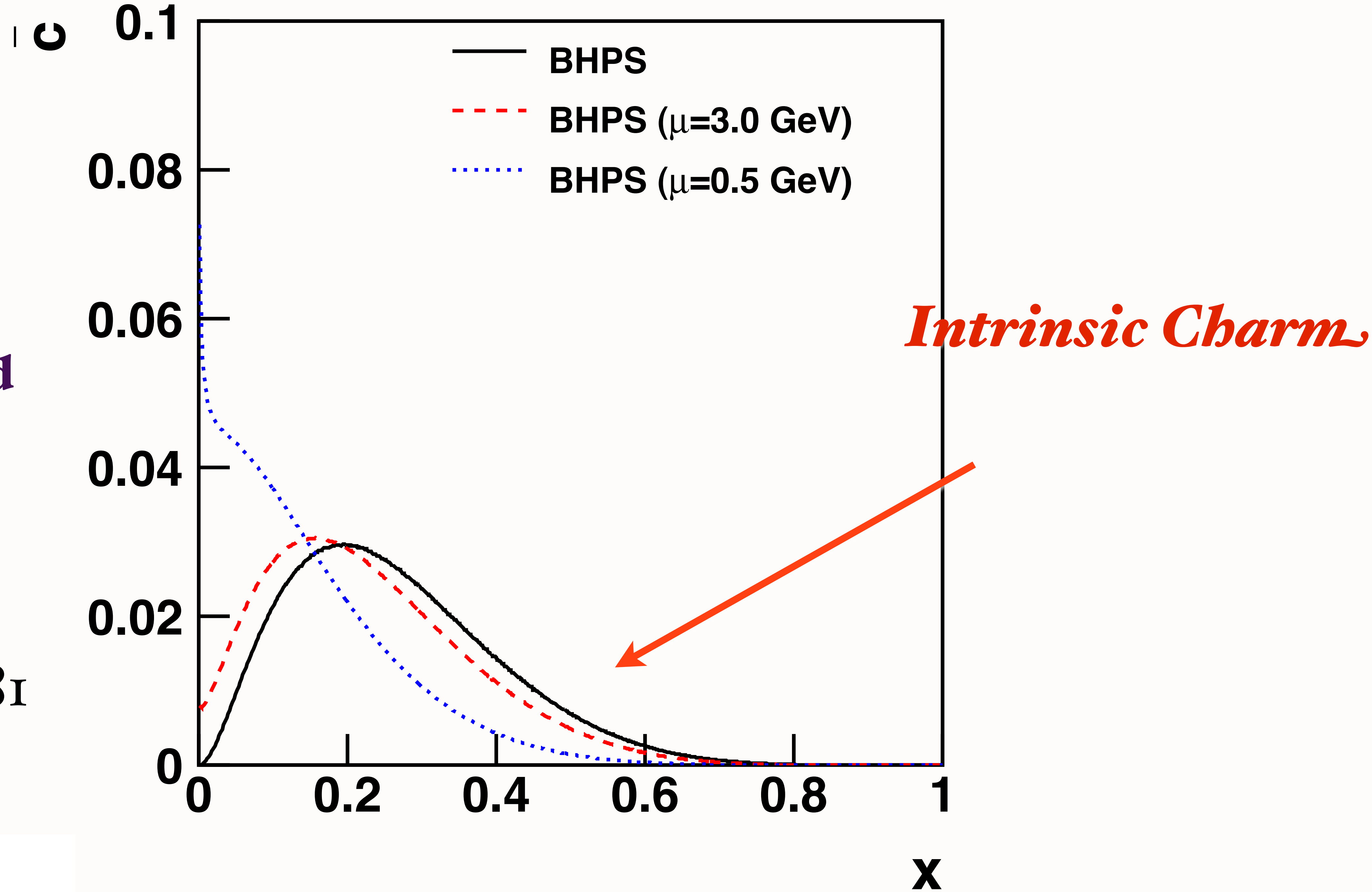
Consistent with intrinsic charm data

QCD: $\frac{1}{M_Q^2}$ scaling

Comparison of the HERMES $x(s(x)+\bar{s}(x))$ data with the calculations based on the BHPS model. The solid and dashed curves are obtained by evolving the BHPS result to $Q^2 = 2.5$ GeV^2 using $\mu = 0.5$ GeV and $\mu = 0.3$ GeV, respectively. The normalizations of the calculations are adjusted to fit the data at $x > 0.1$ with statistical errors only, denoted by solid circles.

$$s(x,Q^2) = s(x,Q^2)_{\text{extrinsic}} + s(x,Q^2)_{\text{intrinsic}}$$

QCD ($1/m_Q^2$) scaling: predict IC

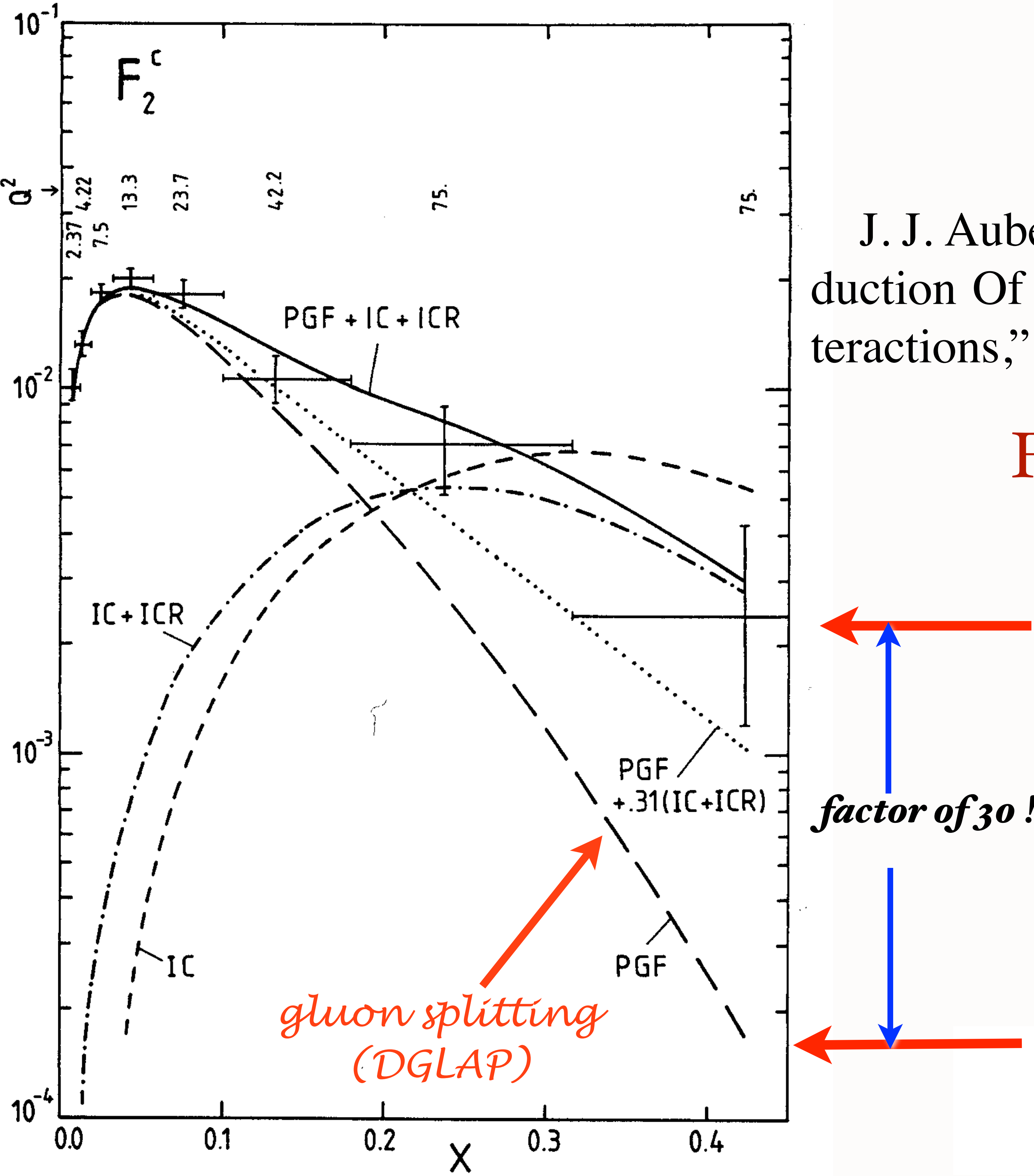


W. C. Chang and J.-C. Peng

arXiv:1105.2381

Figure 4. Calculations of the $\bar{c}(x)$ distributions based on the BHPS model. The solid curve corresponds to the calculation using Eq. 1 and the dashed and dotted curves are obtained by evolving the BHPS result to $Q^2 = 75$ GeV^2 using $\mu = 3.0$ GeV, and $\mu = 0.5$ GeV, respectively. The normalization is set at $\mathcal{P}_5^{c\bar{c}} = 0.01$.

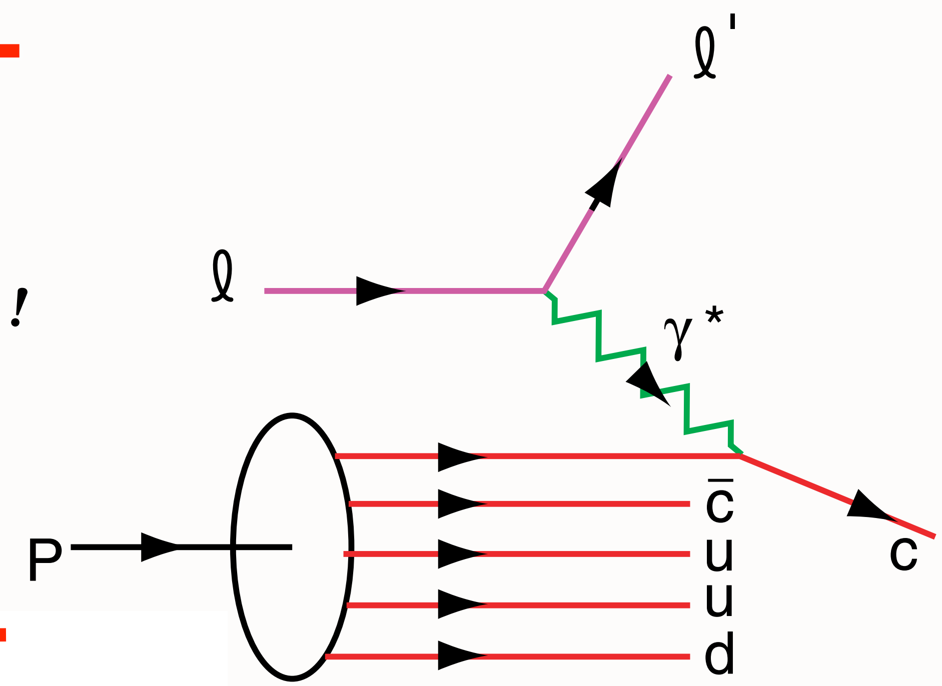
Consistent with EMC



Measurement of Charm Structure Function

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-Gev Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).

First Evidence for Intrinsic Charm



DGLAP / Photon-Gluon Fusion: factor of 30 too small

Two Components (separate evolution):

$$c(x,Q^2) = c(x,Q^2)_{\text{extrinsic}} + c(x,Q^2)_{\text{intrinsic}}$$

Do heavy quarks exist in the proton at high x?

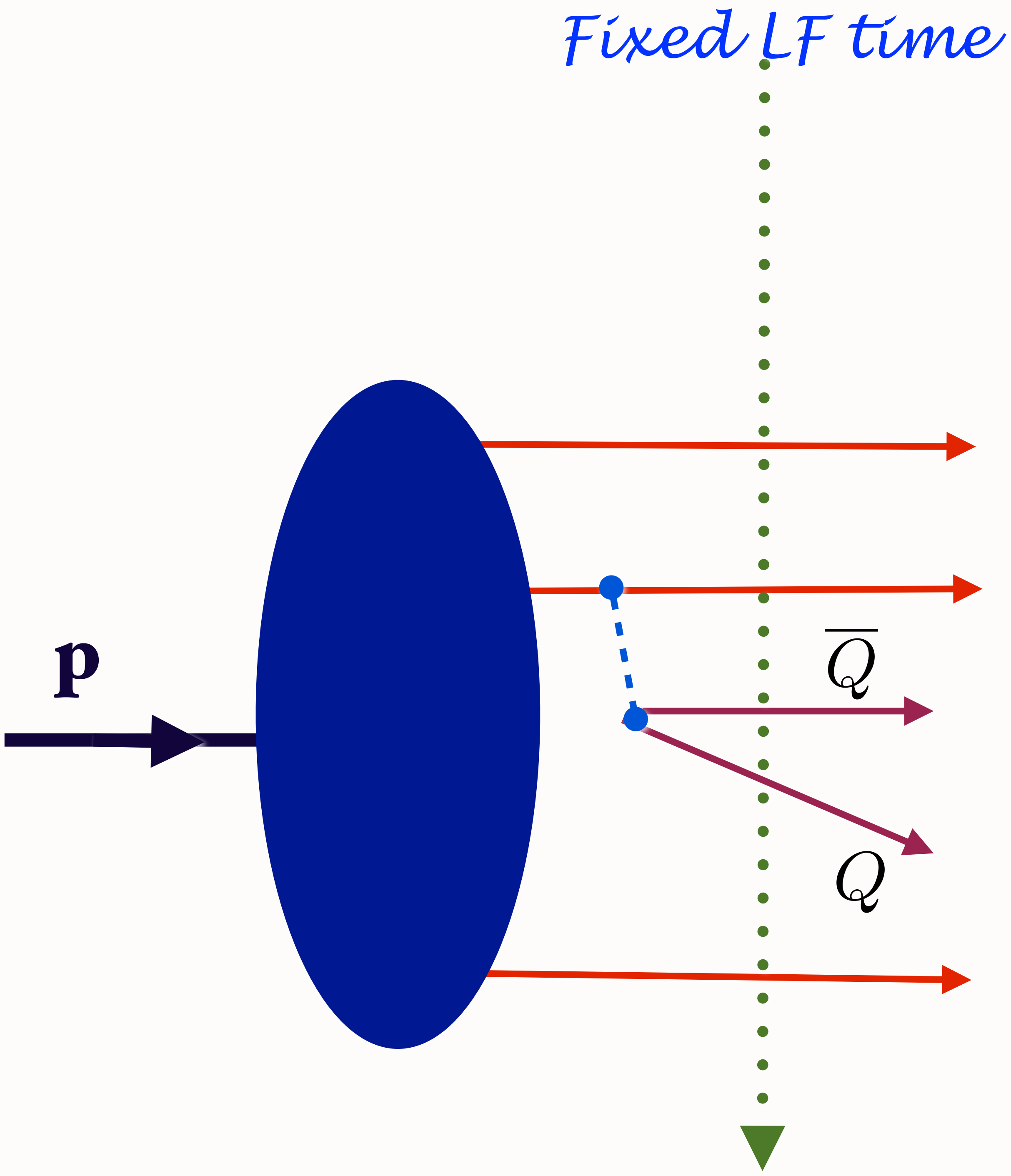
Conventional wisdom: impossible!

Heavy quarks generated only at low x
via DGLAP evolution
from gluon splitting

$$s(x, \mu_F^2) = c(x, \mu_F^2) = b(x, \mu_F^2) \equiv 0$$

$$\text{at starting scale } \mu_F^2$$

Conventional wisdom is wrong even in QED!



Proton's 5-quark Fock State from gluon splitting

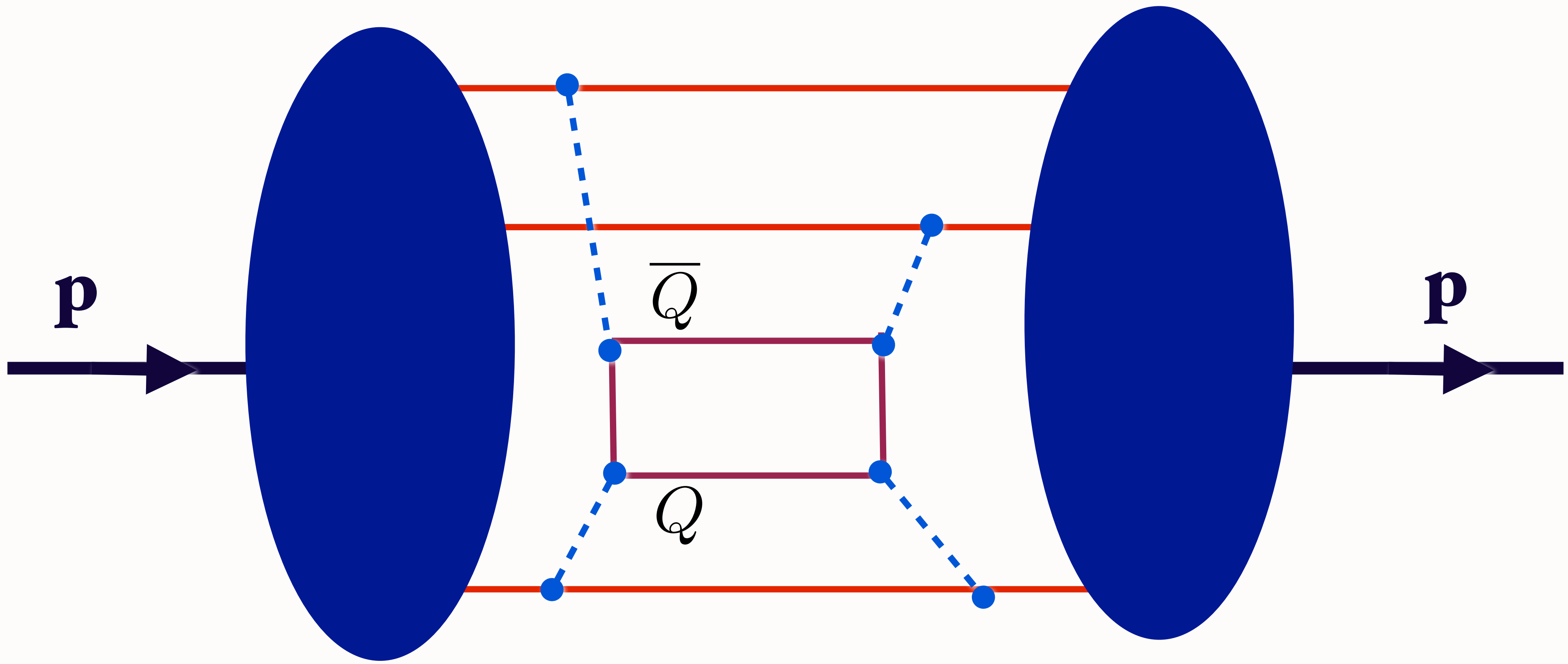
"Extrinsic" Heavy Quarks

$$s(x,Q^2)_{\text{extrinsic}} \sim (1-x)g(x,Q^2) \sim (1-x)^5$$

Proton Self Energy from gluon-gluon scattering

QCD predicts Intrinsic Heavy Quarks!

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$



$$\text{Probability (QED)} \propto \frac{1}{M_\ell^4}$$

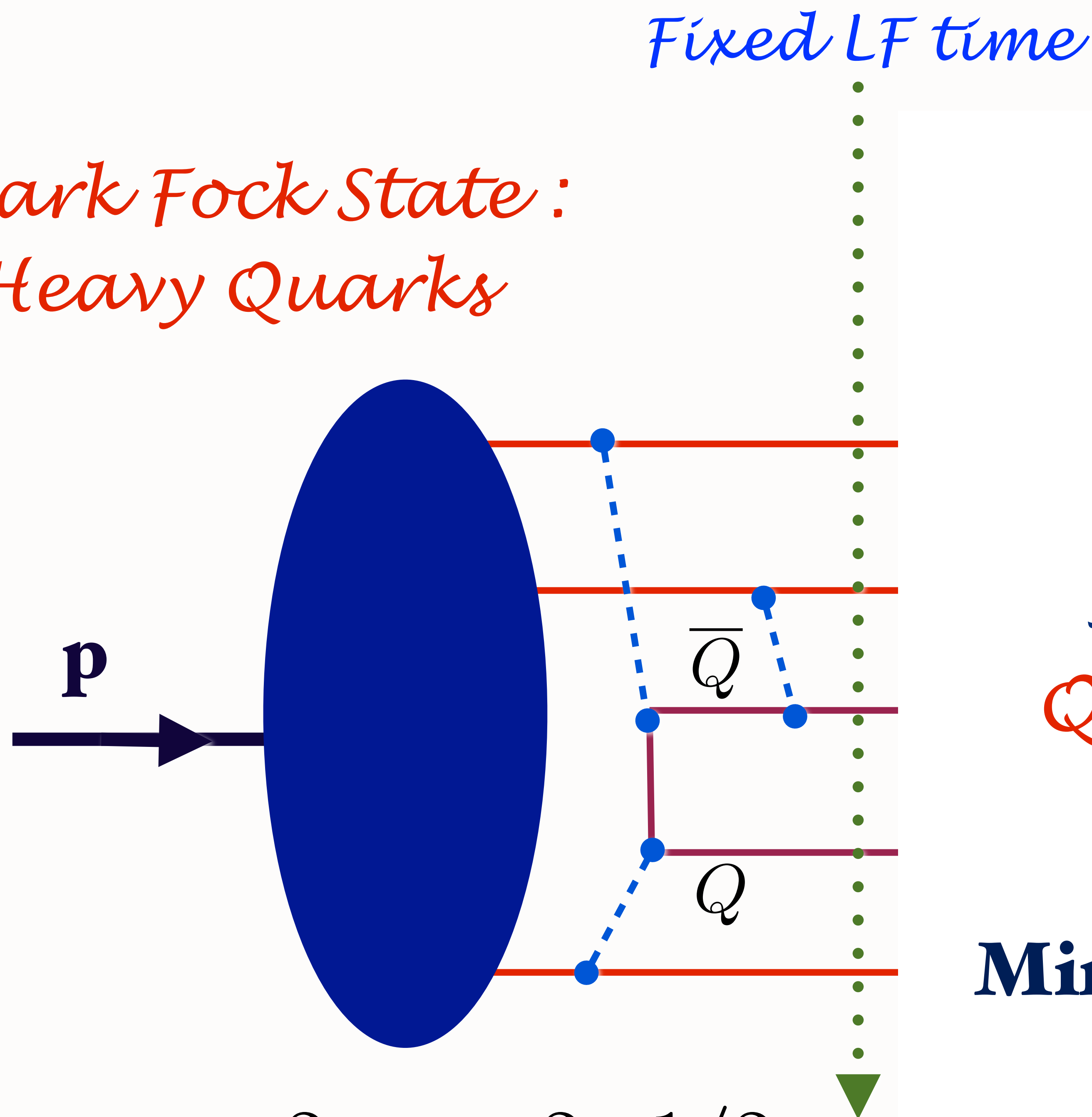
$$\text{Probability (QCD)} \propto \frac{1}{M_Q^2}$$

$$(g-2)_\mu \propto \frac{\alpha^3}{\pi^3} \log \frac{m_\mu^2}{m_e^2}$$

Collins, Ellis, Gunion, Mueller, sjb
M. Polyakov, et al.

from light-by-light scattering

Proton 5-quark Fock State :
Intrinsic Heavy Quarks



QCD predicts Intrinsic Heavy Quarks at high x!

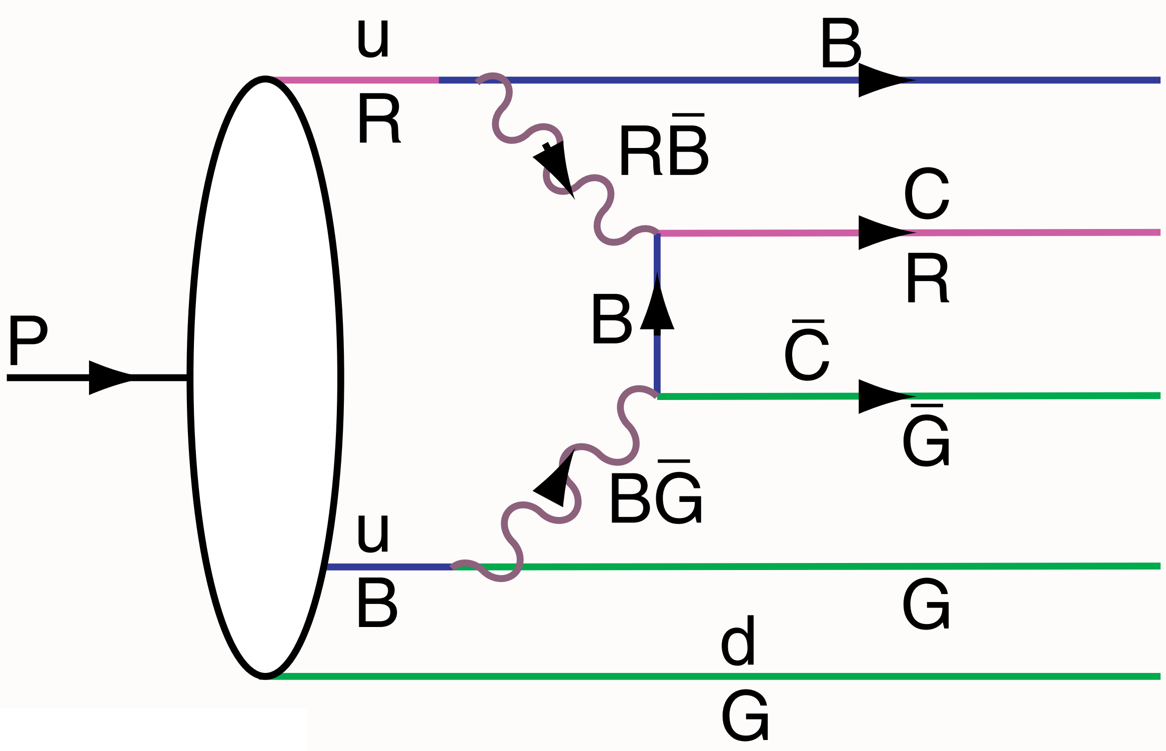
Minimal off-shellness

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

$$\text{Probability (QED)} \propto \frac{1}{M_\ell^4} \qquad \text{Probability (QCD)} \propto \frac{1}{M_Q^2}$$

Collins, Ellis, Gunion, Mueller, sjb
M. Polyakov

BHPS: Hoyer, Peterson, Sakai, sjb



$|uudcc >$ Fluctuation in Proton

QCD: Probability $\frac{\sim\Lambda^2_{QCD}}{M^2_Q}$

$|e^+e^-\ell^+\ell^- >$ Fluctuation in Positronium

QED: Probability $\frac{\sim(m_e\alpha)^4}{M^4_\ell}$

OPE derivation - M.Polyakov et al.

$< p|\frac{G^3_{\mu\nu}}{m^2_Q}|p >$ vs. $< p|\frac{F^4_{\mu\nu}}{m^4_\ell}|p >$

cc in Color Octet

$$\widehat{x}_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

Distribution peaks at equal rapidity (velocity)

Therefore heavy particles carry the largest momentum fractions

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

High x charm! *JLab: Charm at Threshold*

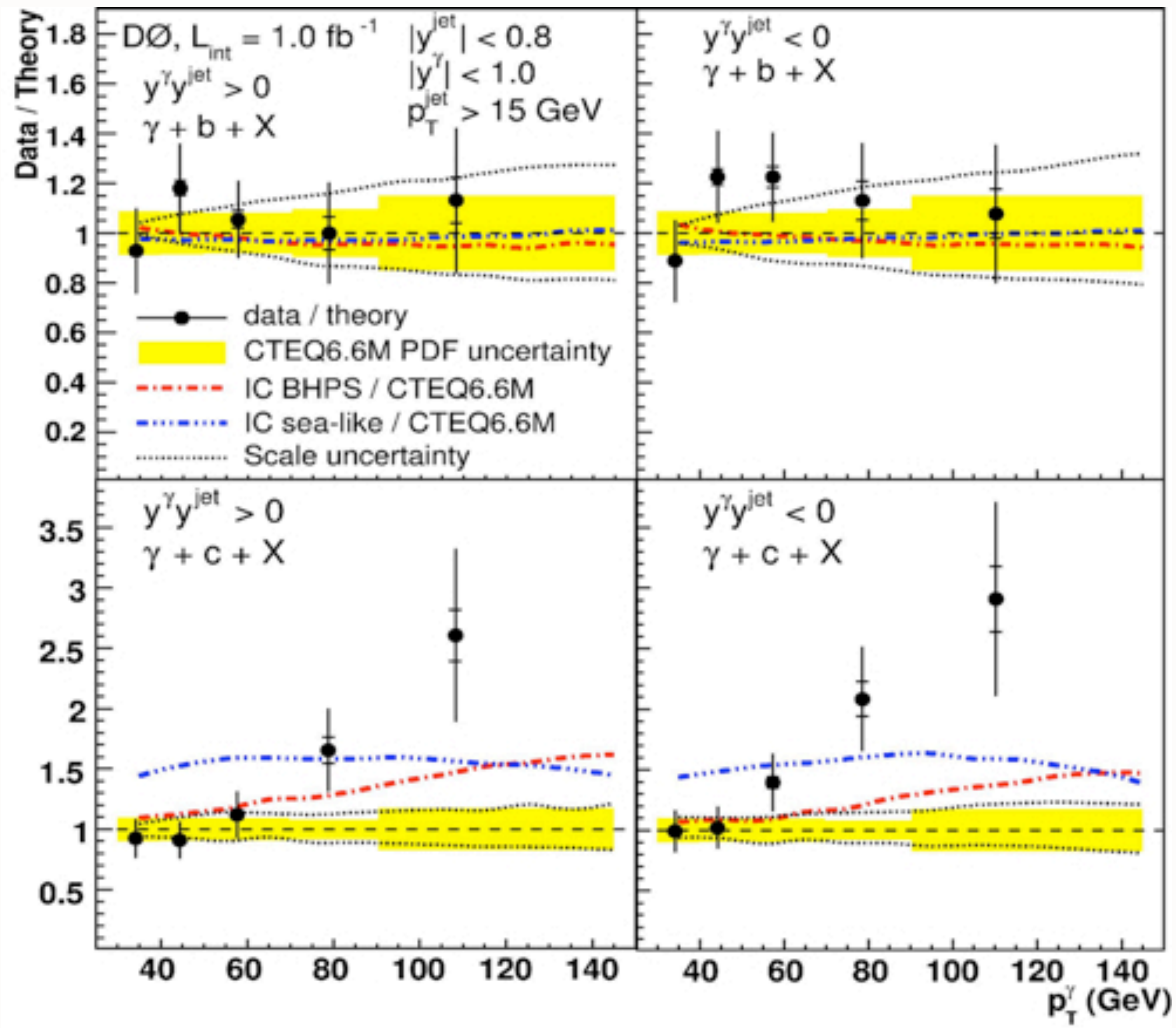
Action Principle: Minimum KE, maximal potential

PRL **102,** 192002 (2009) PHYSICAL REVIEW LETTERS week ending 15 MAY 2009

D0

Measurement of $\gamma + b + X$ and $\gamma + c + X$ Production Cross Sections in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV

$$p\bar{p} \to \gamma + Q + X$$



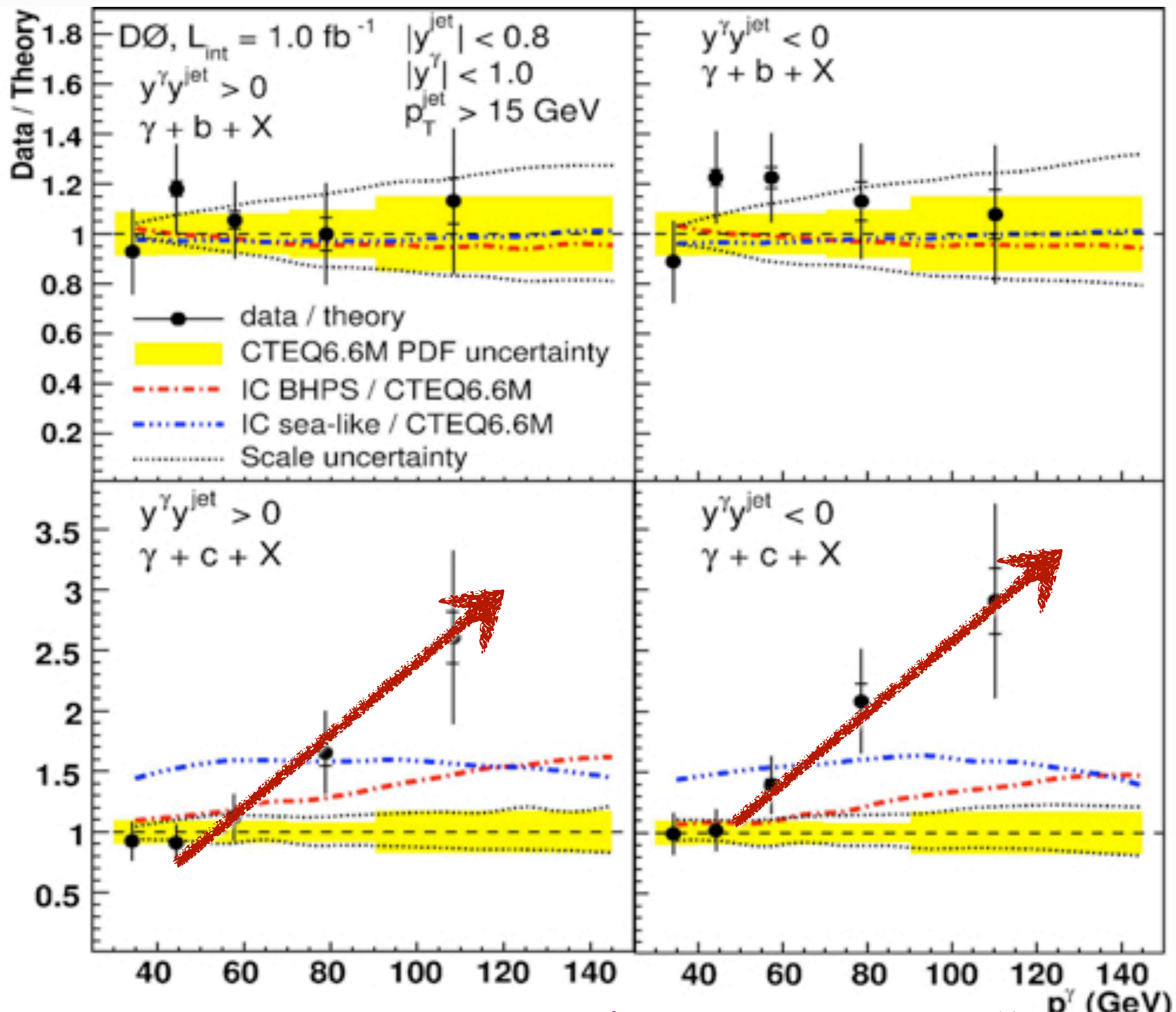
$$\frac{\Delta\sigma(\bar{p}p \to \gamma c X)}{\Delta\sigma(\bar{p}p \to \gamma b X)}$$

Ratio is insensitive to gluon PDF, scales

DO

Measurement of $\gamma + b + X$ and $\gamma + c + X$ Production Cross Sections in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV

$$p\bar{p} \to \gamma + Q + X$$



$$\frac{\Delta\sigma(\bar{p}p \to \gamma c X)}{\Delta\sigma(\bar{p}p \to \gamma b X)}$$

Ratio is insensitive to gluon PDF, scales

$$gc \to \gamma c$$

Signal for significant intrinsic charm at x > 0.1 ?

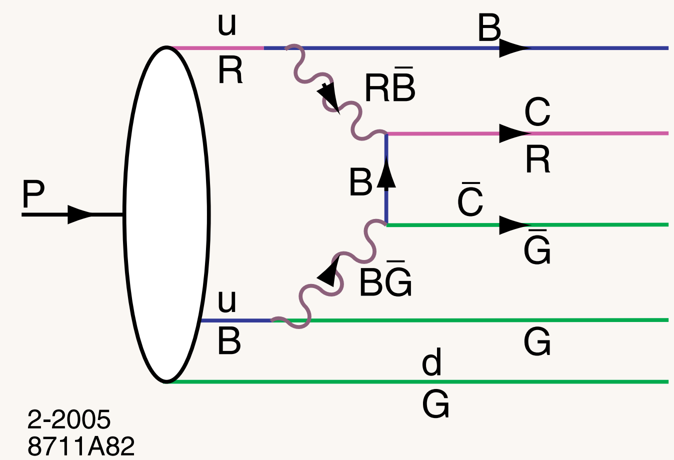
Two Components (separate evolution):

$$c(x,Q^2) = c(x,Q^2)_{\text{extrinsic}} + c(x,Q^2)_{\text{intrinsic}}$$

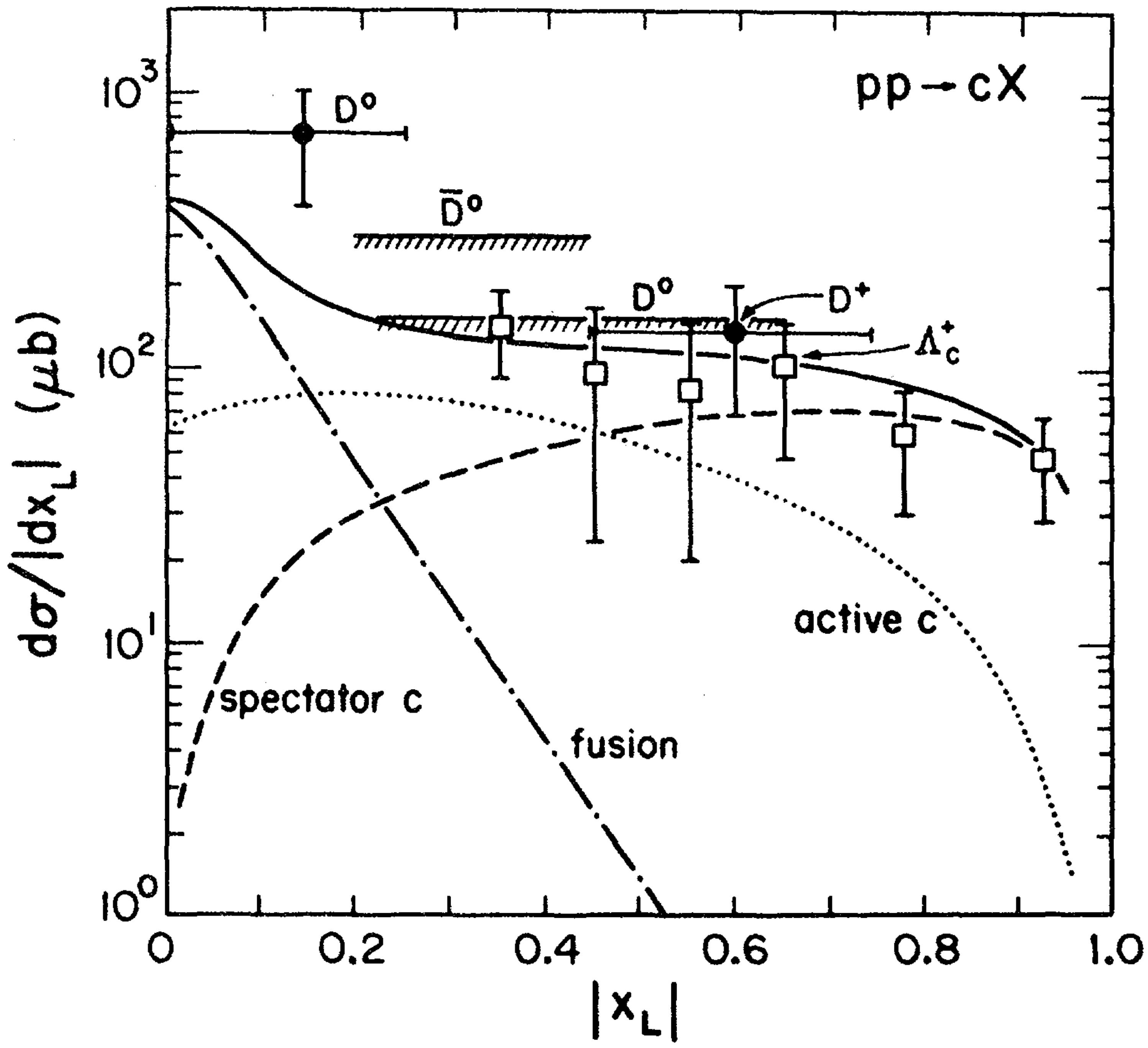
(Need to evolve IC with nonzero quark mass)

Hoyer, Peterson, Sakai, sjb

Intrinsic Heavy-Quark Fock States



- ***Rigorous*** prediction of QCD, OPE
- Color-Octet Color-Octet Fock State!
- Probability $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$ $P_{Q\bar{Q}Q\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$ $P_{c\bar{c}/p} \simeq 1\%$
- ***Large Effect at high x and at threshold!***
- Greatly increases kinematics of colliders such as Higgs production (**Kopeliovich, Schmidt, Soffer, Goldhaber, sjb**)
- Severely underestimated in conventional parameterizations of heavy quark distributions (**Except CTEQ**)
- Slow evolution compared to extrinsic quarks from gluon splitting!
- Many empirical tests at JLAB 12, COMPASS



Barger, Halzen, Keung

More evidence for charm at large x

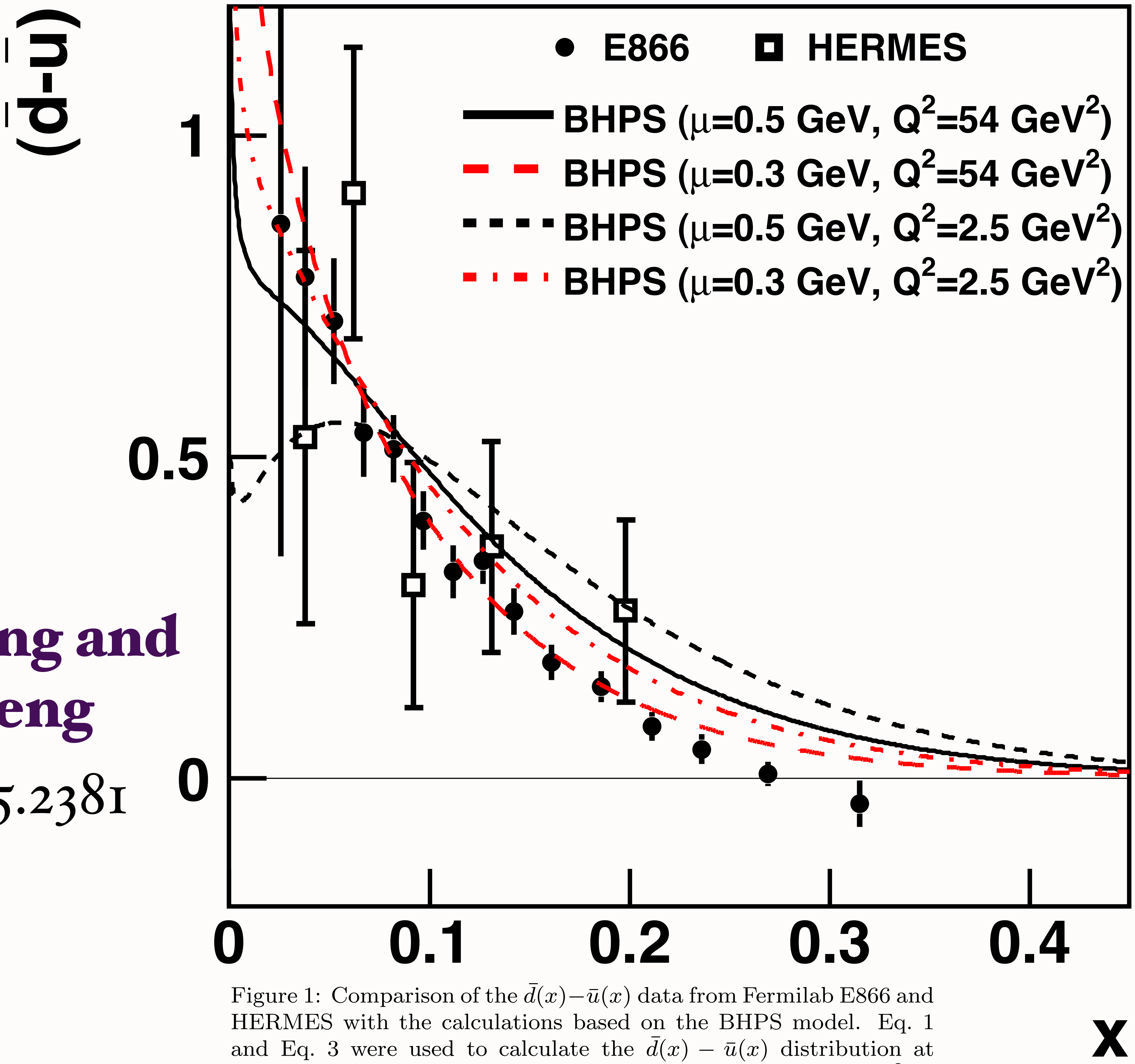
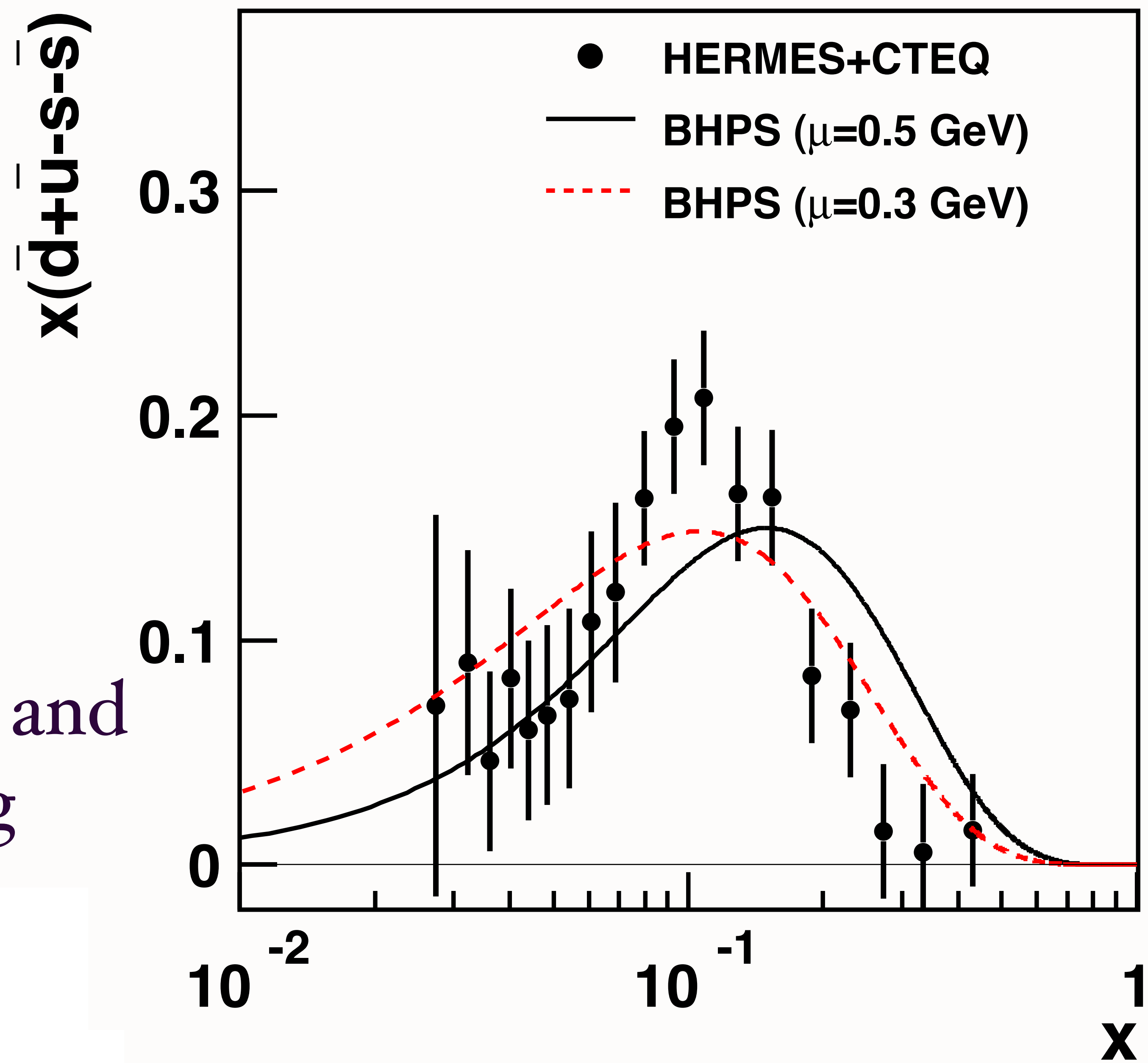


Figure 1: Comparison of the $\bar{d}(x) - \bar{u}(x)$ data from Fermilab E866 and HERMES with the calculations based on the BHPS model. Eq. 1 and Eq. 3 were used to calculate the $\bar{d}(x) - \bar{u}(x)$ distribution at the initial scale. The distribution was then evolved to the Q^2 of the experiments and shown as various curves. Two different initial scales, $\mu = 0.5$ and 0.3 GeV, were used for the E866 calculations in order to illustrate the dependence on the choice of the initial scale.

W. C. Chang and J.-C. Peng

arXiv:1105.2381

$$x[\bar{u}(x) + \bar{d}(x) - s(x) - \bar{s}(x)]$$

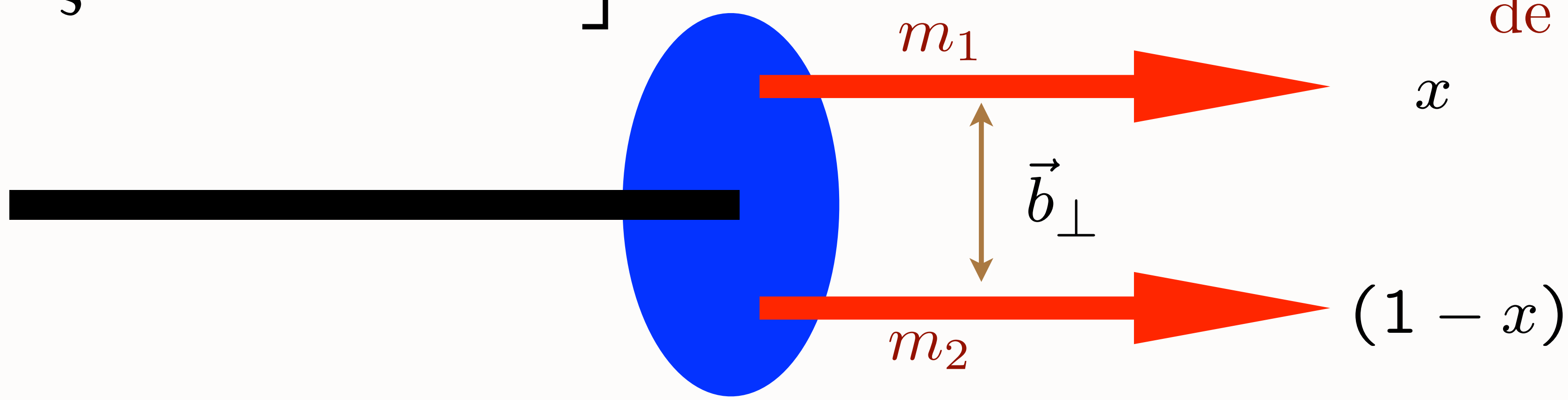


W. C. Chang and J.-C. Peng

Comparison of the $x(\bar{d}(x)+\bar{u}(x)-s(x)-\bar{s}(x))$ data with the calculations based on the BHPS model. The values of $x(s(x)+\bar{s}(x))$ are from the HERMES experiment [6], and those of $x(\bar{d}(x)+\bar{u}(x))$ are obtained from the PDF set CTEQ6.6 [11]. The solid and dashed curves are obtained by evolving the BHPS result to $Q^2 = 2.5$ GeV^2 using $\mu = 0.5$ GeV and $\mu = 0.3$ GeV, respectively. The normalization of the calculations are adjusted to fit the data.

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

de Teramond, sjb



$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

Holographic Variable

$$-\frac{d}{d\zeta^2} \equiv \frac{k_\perp^2}{x(1-x)}$$

LF Kinetic Energy in momentum space

Assume LFWF is a dynamical function of the quark-antiquark invariant mass squared

$$-\frac{d}{d\zeta^2} \to -\frac{d}{d\zeta^2} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \equiv \frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x}$$

Result: Soft-Wall LFWF for massive constituents

$$\psi(x, \mathbf{k}_\perp) = \frac{4\pi c}{\kappa\sqrt{x(1-x)}} e^{-\frac{1}{2\kappa^2}\left(\frac{\mathbf{k}_\perp^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right)}$$

LF WF in impact space: soft-wall model with massive quarks

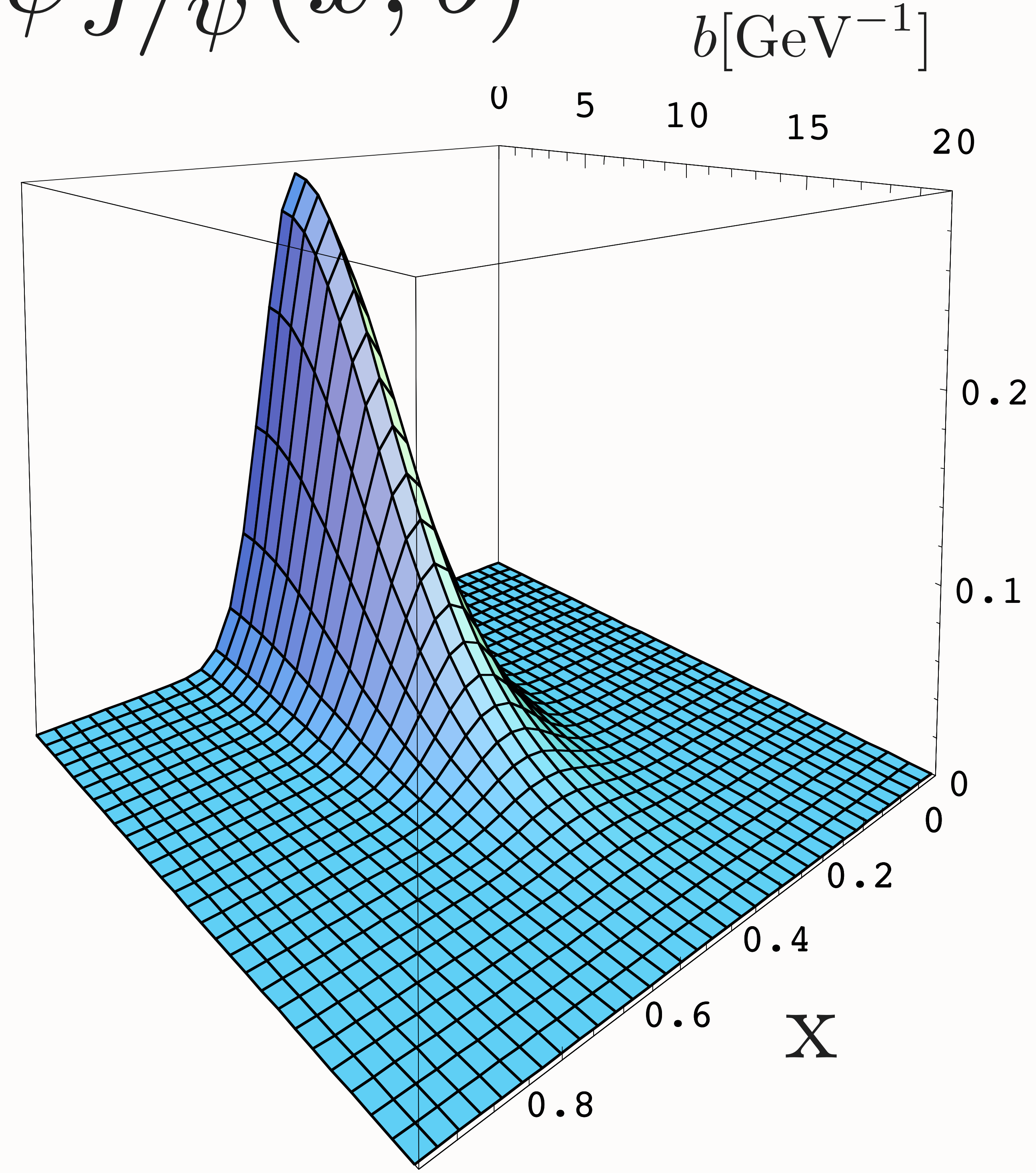
$$\psi(x, \mathbf{b}_\perp) = \frac{c\,\kappa}{\sqrt{\pi}} \sqrt{x(1-x)}\, e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2 - \frac{1}{2\kappa^2}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]}$$

$$z \to \zeta \to \chi$$

$$\chi^2 = b^2 x(1-x) + \frac{1}{\kappa^4}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]$$

J/ψ

$\psi_{J/\psi}(x, b)$



LFWF peaks at

$$x_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

where

$$m_{\perp i} = \sqrt{m^2 + k_\perp^2}$$

minimum of LF energy denominator

$$\kappa = 0.375 \text{ GeV}$$

$$m_a = m_b = 1.25 \text{ GeV}$$

"One of the gravest puzzles of theoretical physics"

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

Department of Physics, University of California, Santa Barbara, CA 93106, USA
Kavil Institute for Theoretical Physics, University of California,
Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu

$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

$$\Omega_\Lambda = 0.76(expt)$$

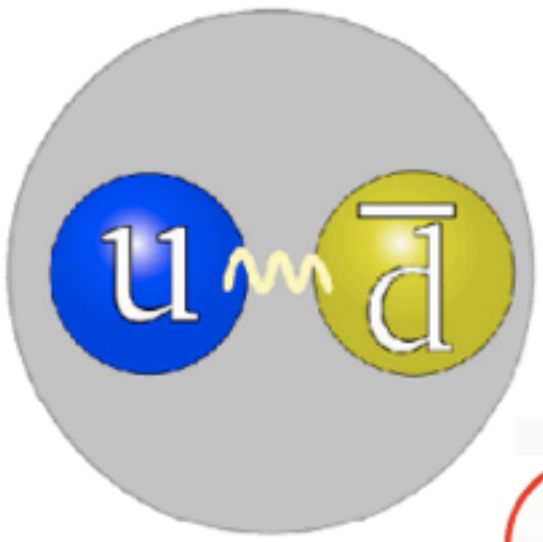
$$(\Omega_\Lambda)_{QCD} \propto < 0|q\bar{q}|0 >^4$$

QCD Problem Solved if quark and gluon condensates reside within hadrons, not vacuum!

R. Shrock, sjb Proc.Nat.Acad.Sci. 108 (2011) 45-50 "Condensates in Quantum Chromodynamics and the Cosmological Constant"

C. Roberts, R. Shrock, P. Tandy, sjb Phys.Rev. C82 (2010) 022201 "New Perspectives on the Quark Condensate"

What is the evidence for a nonzero vacuum quark condensate?



Gell-Mann - Oakes - Renner Relation (1968)

$$f_\pi^2 m_\pi^2 = -2\, m(\zeta) \langle \bar{q} q \rangle_0^\zeta$$

- Pion's leptonic decay constant, mass-dimensioned <u>observable</u> which describes rate of process $\pi^+ \to \mu^+ \nu$
- *Vacuum quark condensate*

ζ: renormalization scale

Derived in current algebra using an effective pion field

How is this modified in QCD for a composite pion?

Gell-Mann Oakes Renner Formula in QCD

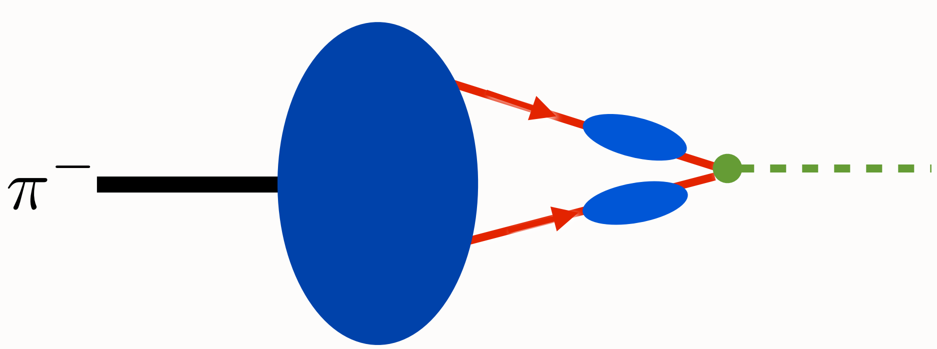
$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} < 0|\bar{q}q|0 >$$

current algebra: effective pion field

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi} < 0|i\bar{q}\gamma_5 q|\pi >$$

QCD: composite pion Bethe-Salpeter Eq.

vacuum condensate actually is an "in-hadron condensate"



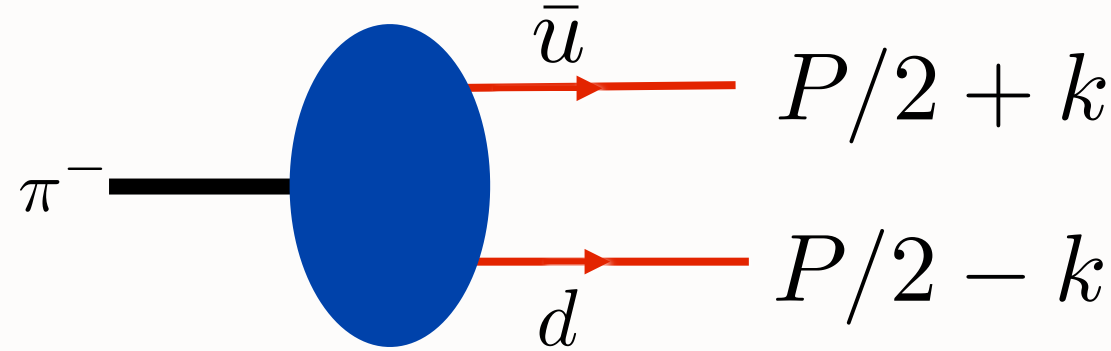
$$< 0|\bar{q}\gamma_5 q|\pi >$$

Maris, Roberts, Tandy

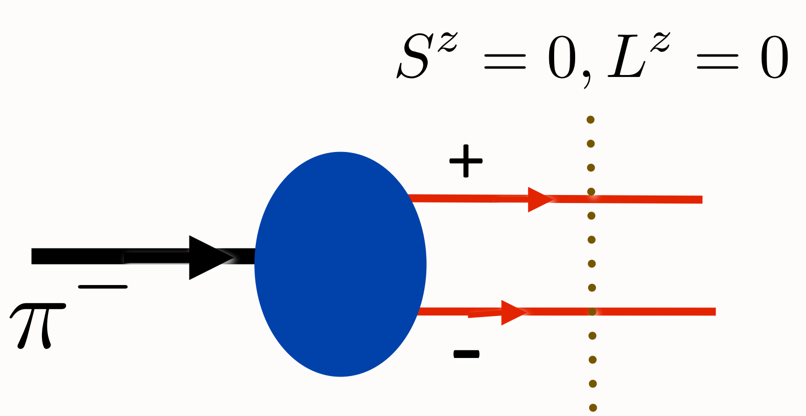
General Form of Bethe-Salpeter Wavefunction

$$\Gamma_\pi(k;P) = i\gamma_5 E_\pi(k,P) + \gamma_5\gamma\cdot P F_\pi(k;P)$$
$$+\gamma_5\gamma\cdot k G_\pi(k;P) - \gamma_5\sigma_{\mu\nu}k^\mu P^\nu H_\pi(k;P)$$

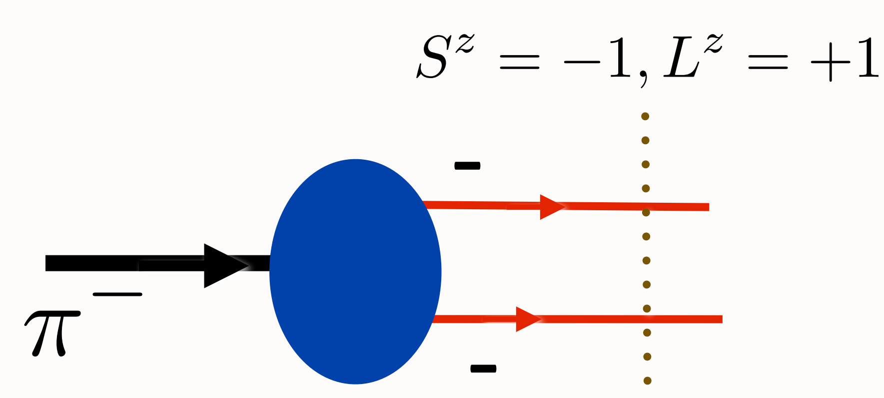
$\Gamma_\pi(k;P)$



Allows both $<0|\bar{q}\gamma_5\gamma_\mu q|\pi>$ and $<0|\bar{q}\gamma_5 q|\pi>$



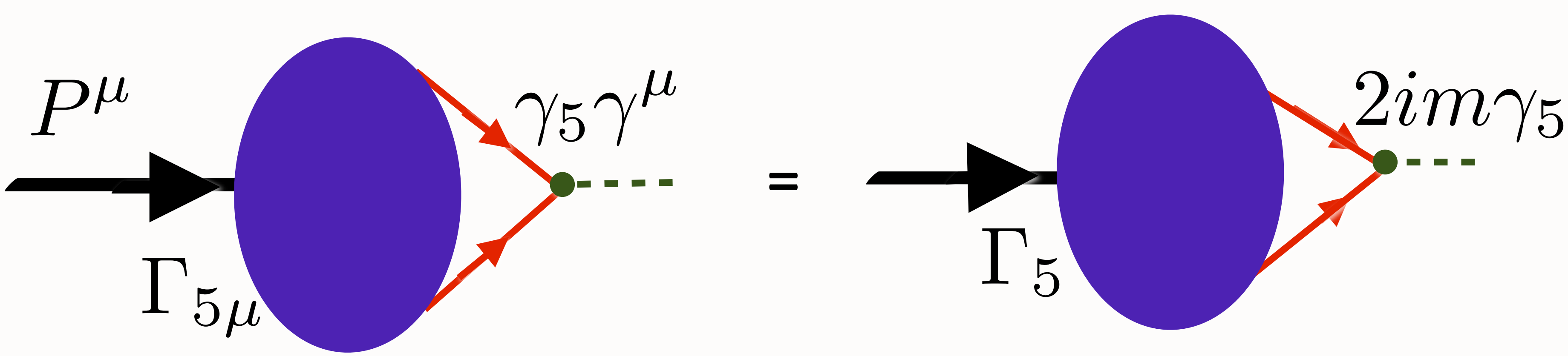
LFWFs



Ward-Takahashi Identity for axial current

$$P^\mu \Gamma_{5\mu}(k,P) + 2im\Gamma_5(k,P) = S^{-1}(k+P/2)i\gamma_5 + i\gamma_5 S^{-1}(k-P/2)$$

$$S^{-1}(\ell) = i\gamma\cdot\ell A(\ell^2) + B(\ell^2) \qquad m(\ell^2) = \frac{B(\ell^2)}{A(\ell^2)}$$



plus non-pole

Identify pion pole at $P^2 = m_\pi^2$

$$P^\mu < 0|\bar{q}\gamma_5\gamma^\mu q|\pi >= 2m < 0|\bar{q}i\gamma_5 q|\pi >$$

$$f_\pi m_\pi^2 = -(m_u + m_d)\rho_\pi$$

Gell-Mann Oakes Renner Formula in QCD

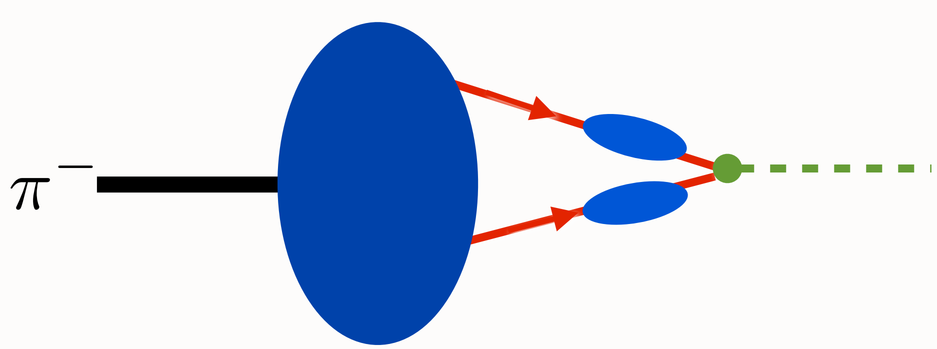
$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} < 0|\bar{q}q|0 >$$

current algebra: effective pion field

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi} < 0|i\bar{q}\gamma_5 q|\pi >$$

QCD: composite pion Bethe-Salpeter Eq.

vacuum condensate actually is an "in-hadron condensate"

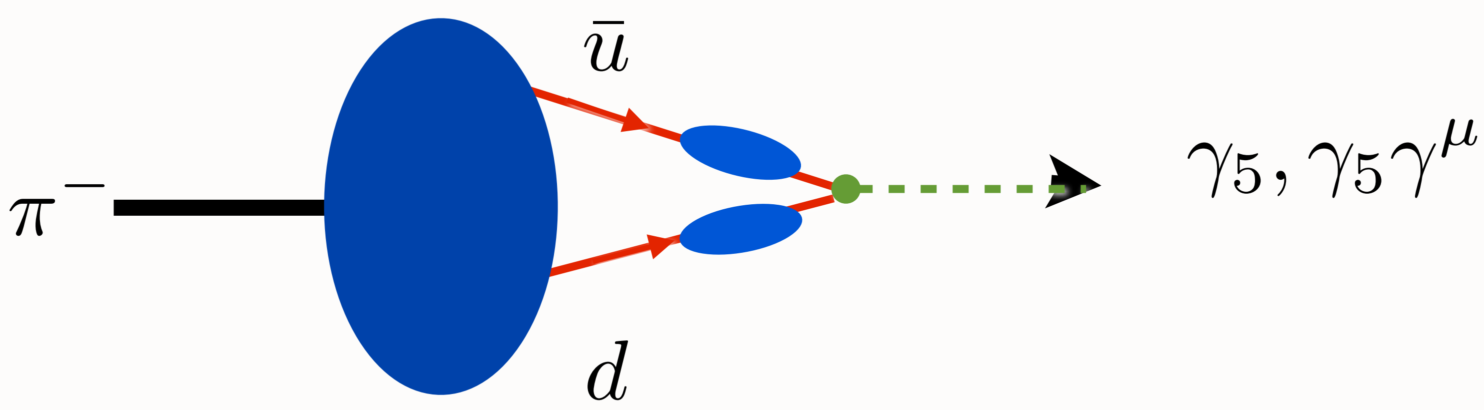


$$< 0|\bar{q}\gamma_5 q|\pi >$$

Maris, Roberts, Tandy

Bethe-Salpeter Analysis

$$f_H P^\mu = Z_2 \int^\Lambda \frac{d^4q}{(2\pi)^4}\ \frac{1}{2}\big[T_H \gamma_5 \gamma^\mu \mathcal{S}(\frac{1}{2}P+q))\Gamma_H(q;P)\mathcal{S}(\frac{1}{2}P-q))\big]$$



f_H Meson Decay Constant
T_H flavor projection operator,
$Z_2(\Lambda)$, $Z_4(\Lambda)$ renormalization constants
$S(p)$ dressed quark propagator
$\Gamma_H(q;P) = F.T.\langle H|\psi(x_a)\bar{\psi}(x_b)|0\rangle$
Bethe-Salpeter bound-state vertex amplitude.

$$i\rho^H_\zeta \equiv \frac{-\langle q\bar{q}\rangle^H_\zeta}{f_H} = Z_4 \int^\Lambda \frac{d^4q}{(2\pi)^4}\ \frac{1}{2}\big[T_H \gamma_5 \mathcal{S}(\frac{1}{2}P+q))\Gamma_H(q;P)\mathcal{S}(\frac{1}{2}P-q))\big]$$

renormalization scale ζ

$$\rho^H = - < 0|\bar{q}\gamma^5 q|H >$$

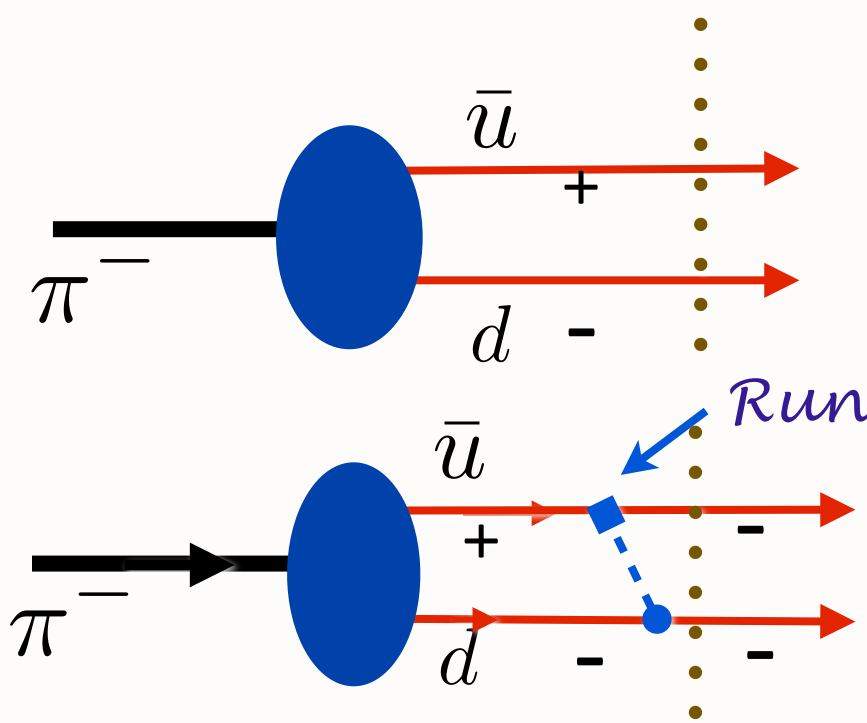
$$f_H m_H^2 = -\rho^H_\zeta \mathcal{M}_H \qquad \mathcal{M}_H = \sum_{q\in H} m_q$$

$$\rho_\pi \sim (0.4\ \mathrm{GeV})^2 \text{ at } \zeta = 1\ \mathrm{GeV}^2$$

Maris, Roberts, Tandy

Light-Front Pion Valence Wavefunctions

$S^z_{\bar u} + S^z_d = +1/2 - 1/2 = 0$



Running constituent mass at vertex

$S^z_{\bar u} + S^z_d = -1/2 - 1/2 = -1$

Couples to

$$L^z = 0, S^z = 0 \quad < \pi|\bar\gamma^\mu q \gamma_5 q|0 > \sim f_\pi$$

$$L^z = +1, S^z = -1 \quad < \pi|\bar q \gamma_5 q|0 > \sim \rho_\pi$$

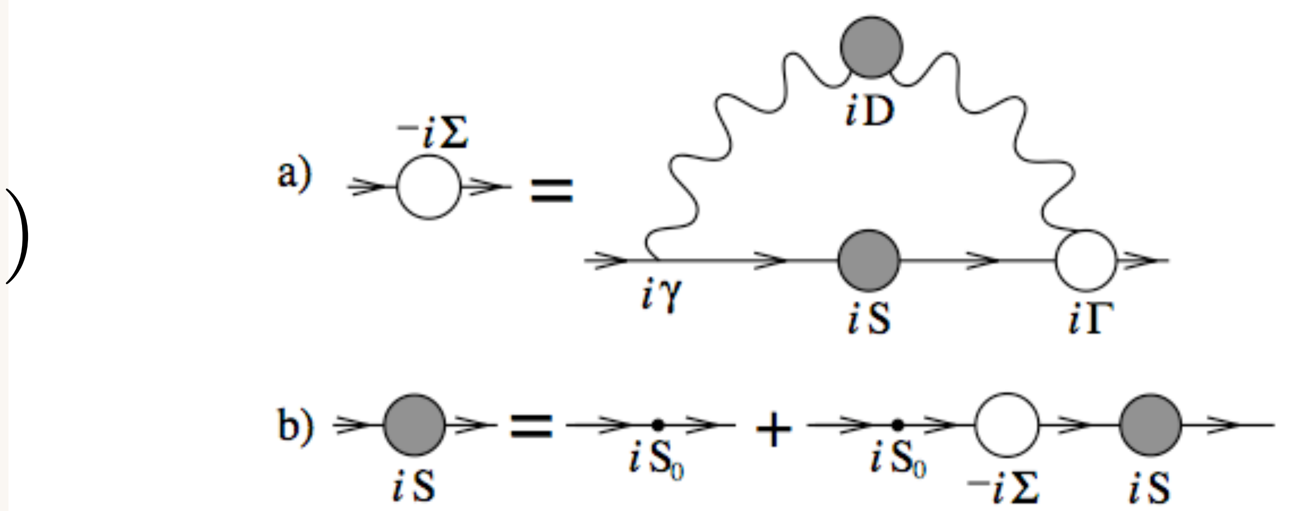
Angular Momentum Conservation

$$J^z = \sum_i^n S^z_i + \sum_i^{n-1} L^z_i$$

Running mass enhanced within Hadron Wavefunction

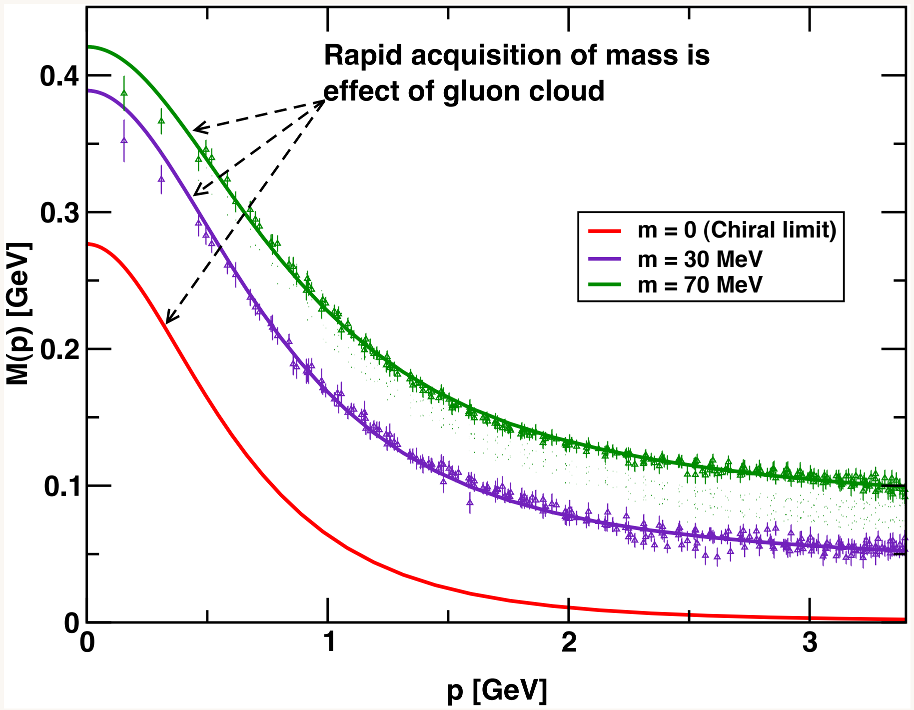
$$S^{-1}(p) = i\gamma \cdot p \; A(p^2) + B(p^2)$$

$$m(p^2) = \frac{B(p^2)}{A(p^2)}$$



- QCD gluon loop corrections increase running mass
- Dyson-Schwinger model predictions Alkofer, Roberts et al.
- Effects of higher Fock states: Casher & Susskind
  spontaneous chiral symmetry breaking
- All effects within confinement domain
- IR cutoff from confinement/bound state

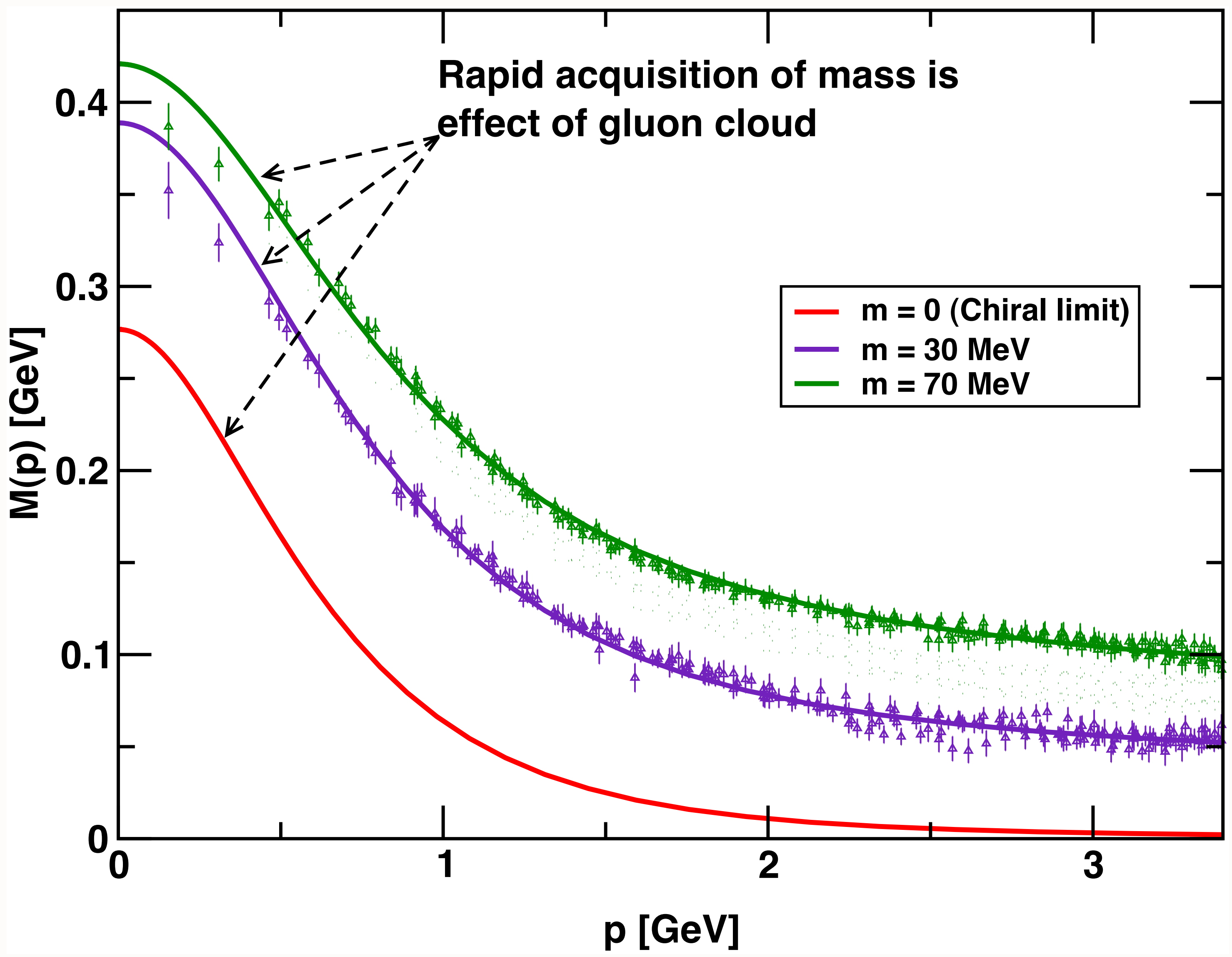
Shrock, sjb



Lei Chang, et al.

Running quark mass in QCD

$$S^{-1}(p) = i\gamma \cdot p\ A(p^2) + B(p^2) \qquad m(p^2) = \frac{B(p^2)}{A(p^2)}$$



Dyson-Schwinger

Chang, Cloet, El-Bennich Klahn, Roberts

Consistent with EW input at high p^2

Survives even at m=0!

Spontaneous Chiral Symmetry Breaking!

Summary on QCD `Condensates'

- Condensates do not exist as space-time-independent phenomena

- Property of hadron wavefunctions: Bethe-Salpeter or Light-Front: "In-Hadron Condensates"

- Find: $$\frac{<0|\bar{q}q|0>}{f_\pi} \rightarrow -<0|i\bar{q}\gamma_5 q|\pi>=\rho_\pi$$

$$<0|\bar{q}i\gamma_5 q|\pi> \text{ similar to } <0|\bar{q}\gamma^\mu\gamma_5 q|\pi>$$

- Zero contribution to cosmological constant! Included in hadron mass

- ρ_π survives for small m_q -- enhanced running mass from gluon loops / multiparton Fock states

PHYSICAL REVIEW C **82**, 022201(R) (2010)

New perspectives on the quark condensate

Stanley J. Brodsky,[1,2] Craig D. Roberts,[3,4] Robert Shrock,[5] and Peter C. Tandy[6]

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We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wave functions.

- **Eliminates 45 orders of magnitude conflict**

PHYSICAL REVIEW D VOLUME 9, NUMBER 2 15 JANUARY 1974

Chiral magnetism (or magnetohadrochironics)

Aharon Casher and Leonard Susskind

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon. Because of an instability of the chirally invariant vacuum, the real vacuum is "aligned" into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame. A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron's wave function and not to the vacuum.

Light-Front Formalism

Chiral Symmetry Breaking in AdS/QCD

We consider the action of the X field which encodes the effects of CSB in AdS/QCD:

$$S_X = \int d^4x dz \sqrt{g} \left(g^{\ell m} \partial_\ell X \partial_m X - \mu_X^2 X^2 \right), \tag{1}$$

Erlich, Katz, Son, Stephanov
Babington, Erdmenger, Evans, Kirsch, Guralnik, Thelfall

with equations of motion

$$z^3 \partial_z \left(\frac{1}{z^3} \partial_z X \right) - \partial_\rho \partial^\rho X - \left(\frac{\mu_X R}{z} \right)^2 X = 0. \tag{2}$$

The zero mode has no variation along Minkowski coordinates

$$\partial_\mu X(x, z) = 0,$$

thus the equation of motion reduces to

$$\left[z^2 \partial_z^2 - 3z\, \partial_z + 3 \right] X(z) = 0. \tag{3}$$

for $(\mu_X R)^2 = -3$, which corresponds to scaling dimension $\Delta_X = 3$. The solution is

$$X(z) = \langle X \rangle = Az + Bz^3, \tag{4}$$

where A and B are determined by the boundary conditions.

$$A \propto m_q \qquad\qquad B \propto < \bar\psi \psi >$$

Expectation value with z^3 taken inside hadron - not VEV!

Chiral Symmetry Breaking in AdS/QCD

- **Chiral symmetry breaking effect in AdS/QCD depends on weighted z^2 distribution, not constant condensate**

Erlich et al.

$$\delta M^2 = -2m_q < \bar{\psi}\psi > \times \int dz\ \phi^2(z) z^2$$

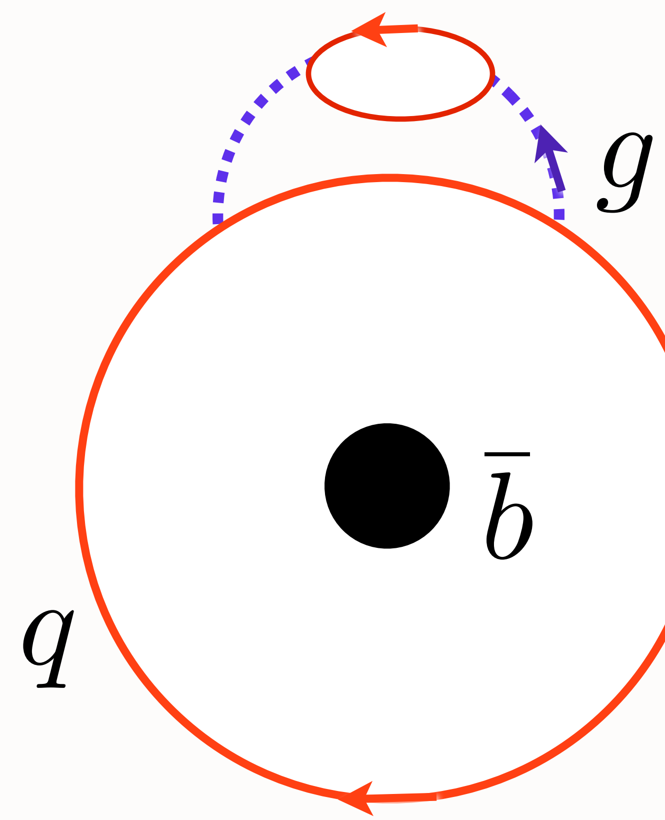
- **z^2 weighting consistent with higher Fock states at periphery of hadron wavefunction**
- **mass shift depends on hadron size, etc.**
- **AdS/QCD: confined condensate**
- **Consistent with "In-Hadron" Condensates**

Shrock, Roberts, Tandy, sjb

Confinement: Maximum wavelength of bound quarks and gluons

$$\lambda < \Lambda_{\rm QCD}$$

$$k > \frac{1}{\Lambda_{\rm QCD}}$$



B-Meson

Shrock, sjb

Use Dyson-Schwinger Equation for bound-state quark propagator: find confined condensate

$$< \bar{b}|\bar{q}q|\bar{b} > \text{ not } < 0|\bar{q}q|0 >$$

Quark and Gluon condensates reside within hadrons, not LF vacuum

- **Bound-State Dyson-Schwinger Equations**

  Maris, Roberts, Tandy

- **Spontaneous Chiral Symmetry Breaking within infinite-component LFWFs**

  Casher Susskind

- **Finite size phase transition - infinite # Fock constituents**

- **AdS/QCD Description -- CSB is in-hadron Effect**

- **Analogous to finite-size superconductor!**

- **Phase change observed at RHIC within a single-nucleus-nucleus collisions-- quark gluon plasma!**

- **Implications for cosmological constant**

"Confined QCD Condensates"

Pion mass **and decay constant.**
Pieter Maris, Craig D. Roberts (Argonne, PHY) , Peter C. Tandy (Kent State U.) . ANL-PHY-8753-TH-97, KSUCNR-103-97, Jul 1997. 12pp.
Published in **Phys.Lett.B420:267-273,1998**.
e-Print: **nucl-th/9707003**

Pi- and K meson Bethe-Salpeter amplitudes.
Pieter Maris, Craig D. Roberts (Argonne, PHY) . ANL-PHY-8788-TH-97, Aug 1997. 34pp.
Published in **Phys.Rev.C56:3369-3383,1997**.
e-Print: **nucl-th/9708029**

Concerning the quark condensate.
K. Langfeld (Tubingen U.) , H. Markum (Vienna, Tech. U.) , R. Pullirsch (Regensburg U.) , C.D. Roberts (Argonne, PHY & Rostock U.) , S.M. Schmidt (Tubingen U. & HGF, Bonn) . ANL-PHY-10460-TH-2002, MPG-VT-UR-239-02, Jan 2003. 7pp.
Published in **Phys.Rev.C67:065206,2003**.
e-Print: **nucl-th/0301024**

"In-Meson Condensate"

$$- \langle \bar{q} q \rangle_\zeta^\pi = f_\pi \langle 0 | \bar{q} \gamma_5 q | \pi \rangle \, .$$

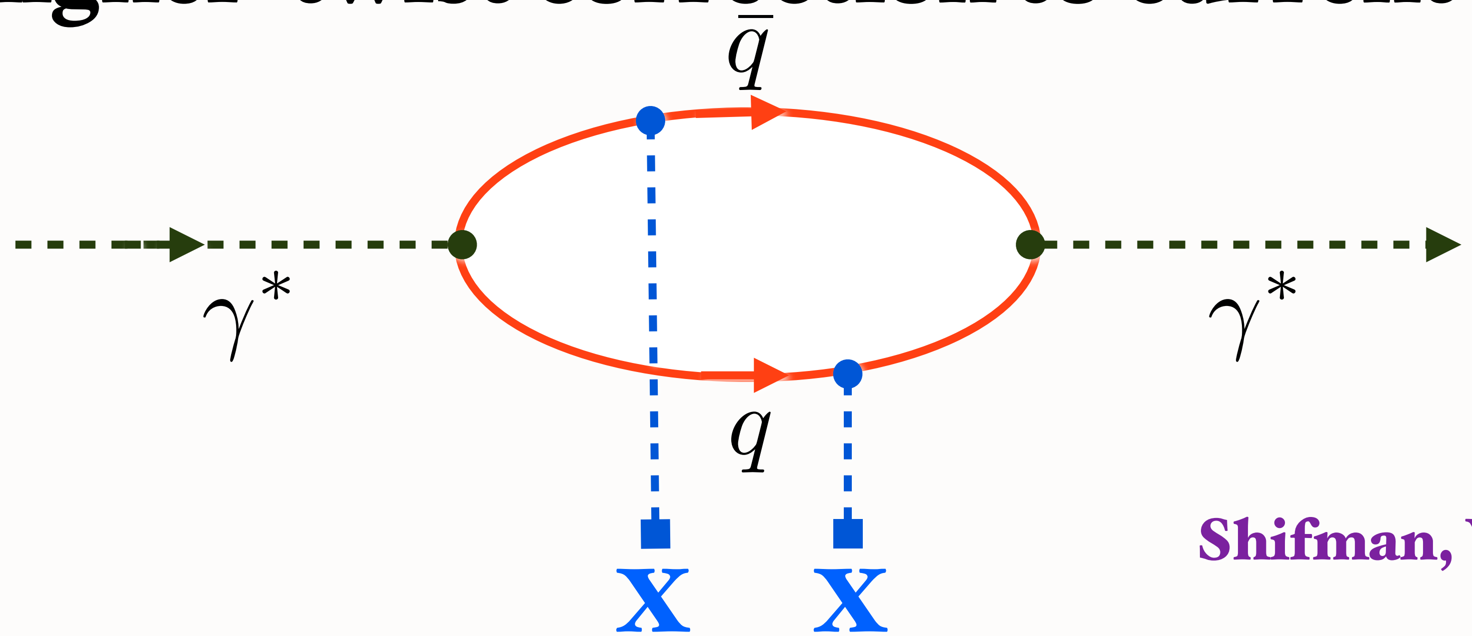
Valid even for $m_q \to 0$

f_π nonzero

Is there evidence for a gluon vacuum condensate?

$$< 0|\frac{\alpha_s}{\pi} G^{\mu\nu}(0) G_{\mu\nu}(0)|0 >$$

Look for higher-twist correction to current propagator



Shifman, Vainshtein, Zakharov

$$e^+e^- \to X,\ \tau \text{ decay},\ Q\bar{Q} \text{ phenomenology}$$

$$R_{e^+e^-}(s) = N_c \sum_q e_q^2 (1 + \frac{\alpha_s}{\pi} \frac{\Lambda^4_{\rm QCD}}{s^2} + \cdots)$$

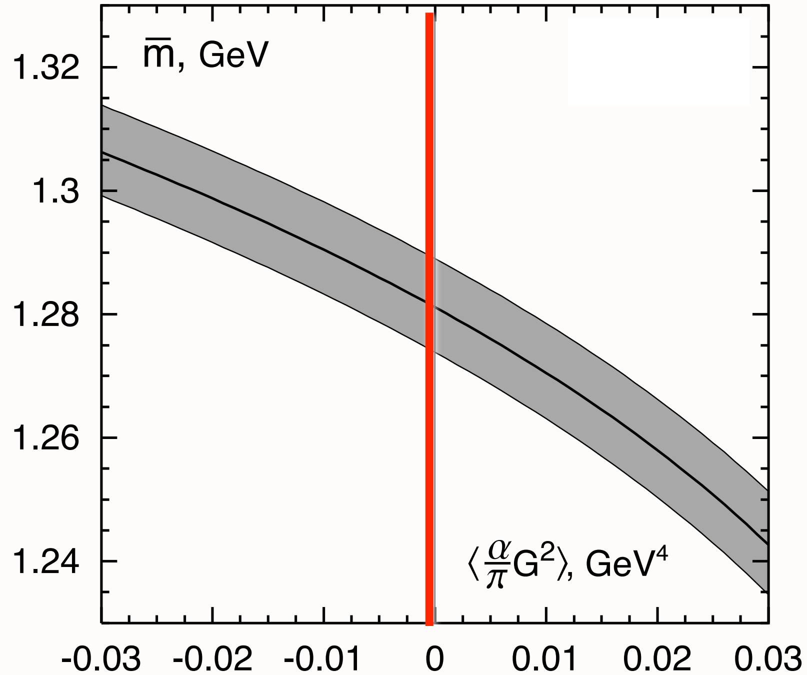
Determinations of the vacuum Gluon Condensate

$$< 0|\frac{\alpha_s}{\pi}G^2|0 > [\mathrm{GeV}^4]$$

$$-0.005 \pm 0.003 \text{ from } \tau \text{ decay.}$$ Davier et al.

$$+0.006 \pm 0.012 \text{ from } \tau \text{ decay.}$$ Geshkenbein, Ioffe, Zyablyuk

$$+0.009 \pm 0.007 \text{ from charmonium sum rules}$$ Ioffe, Zyablyuk

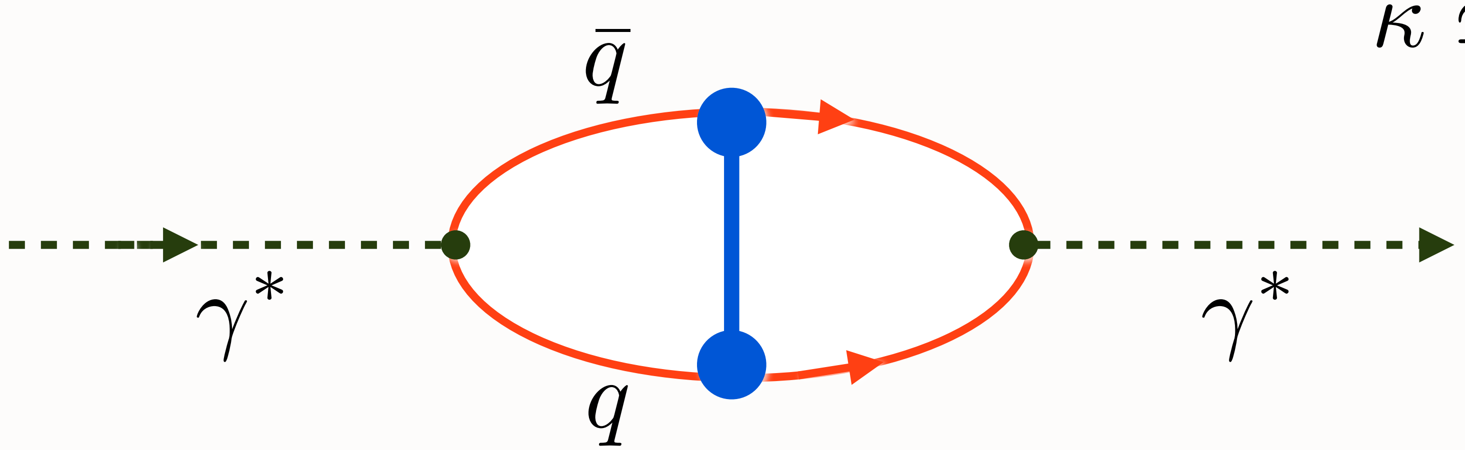


Consistent with zero vacuum condensate

Effective Confinement potential from soft-wall AdS/QCD gives Regge Spectroscopy plus higher-twist correction to current propagator

$$M^2 = 4\kappa^2(n + L + S/2)$$ light-quark meson spectra

$\kappa \simeq 0.5$ GeV



$$R_{e^+e^-}(s) = N_c \sum_q e_q^2 (1 + \mathcal{O}\frac{\kappa^4}{s^2} + \cdots)$$

mimics dimension-4 gluon condensate $< 0|\frac{\alpha_s}{\pi} G^{\mu\nu}(0) G_{\mu\nu}(0)|0 >$ *in*

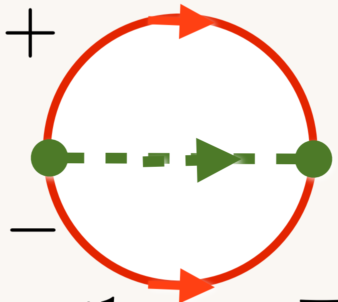
$e^+e^- \to X$, τ decay, $Q\bar{Q}$ phenomenology

Quark and Gluon condensates reside within hadrons, not vacuum

Casher and Susskind　　Maris, Roberts, Tandy　　Shrock and sjb

- **Bound-State Dyson Schwinger Equations**
- **AdS/QCD**
- **Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict**

Instant Form Vacuum in QED



- Loop diagrams of all orders contribute; Frame-Dependent
- Huge vacuum energy? $\Omega_\Lambda \sim 10^{120}$
- $\frac{E}{V} = \int \frac{d^3k}{2(2\pi)^3}\sqrt{\vec{k}^2 + m^2}$ Cutoff quad divergent at Planck scale?
- Why not use :Normal order: prescription?
- Divide S-matrix by disconnected vacuum diagrams
- Contrast: Light-Front Vacuum empty since plus momenta are positive and conserved: $k^+ = k^0 + k^3 > 0$
  Ω_Λ=ZERO!

Goal: An analytic first approximation to QCD

- As Simple as Schrödinger Theory in Atomic Physics
- Relativistic, Frame-Independent, Color-Confining
- QCD Coupling at all scales
- Hadron Spectroscopy
- Light-Front Wavefunctions
- Form Factors, Hadronic Observables, Constituent Counting Rules
- Insight into QCD Condensates
- Systematically improvable

String Theory

Goal: First Approximant to QCD

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

Counting rules for Hard Exclusive Scattering
Regge Trajectories

AdS/QCD

Conformal behavior at short distances
+ Confinement at large distance

QCD at the Amplitude Level

Semi-Classical QCD / Wave Equations

Holography

Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

Features of AdS/QCD LF Holography

- Based on Conformal Scaling of Infrared QCD Fixed Point
- Conformal template: Use isometries of AdS_5
- Interpolating operator of hadrons based on twist, superfield dimensions
- Finite Nc = 3: Baryons built on 3 quarks -- Large Nc limit not required
- Break Conformal symmetry with dilaton
- Dilaton introduces confinement -- positive exponent for spacelike observables
- Origin of Linear and HO potentials: Stochastic arguments (Glazek); General 'classical' potential for Dirac Equation (Hoyer)
- Effective Charge from AdS/QCD at all scales
- Conformal Dimensional Counting Rules for Hard Exclusive Processes
- Use CRF (LF Constituent Rest Frame) to reconstruct 3D Image of Hadrons (Glazek, de Teramond, sjb)

Use AdS/CFT orthonormal Light Front Wavefunctions as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximation
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations
- **Hamiltonian light-front field theory within an AdS/QCD basis.**
  J.P. Vary, H. Honkanen, Jun Li, P. Maris, A. Harindranath,
  G.F. de Teramond, P. Sternberg, E.G. Ng, C. Yang, sjb

Basis functions

- HO basis for transverse momentum states:

$$\Phi_{n,m}(p^{\perp}) = \Phi_{n,m}(\rho,\phi) = \sqrt{2\pi}\frac{1}{b}\sqrt{\frac{2n!}{(|m|+n)!}}e^{im\phi}\rho^{|m|}e^{-\rho^2/2}L_n^{|m|}(\rho^2),$$

with

$$\rho = \frac{|p^{\perp}|}{b}, \quad b = \sqrt{\mathbf{M_0\Omega}}$$

- Discretize longitudinal momentum:

$$\psi_k(x^-) = \frac{1}{\sqrt{2L}}\,\mathrm{e}^{i\,\frac{\pi}{L}k\,x^-},$$

$$k = \begin{cases} k = 1, 2, 3, \ldots \text{(periodic boundary condition for bosons)}, \\ \\ k = \frac{1}{2}, \frac{3}{2}, \ldots \text{(antiperiodic boundary condition for fermions)} \end{cases}$$

- Full 3-D:

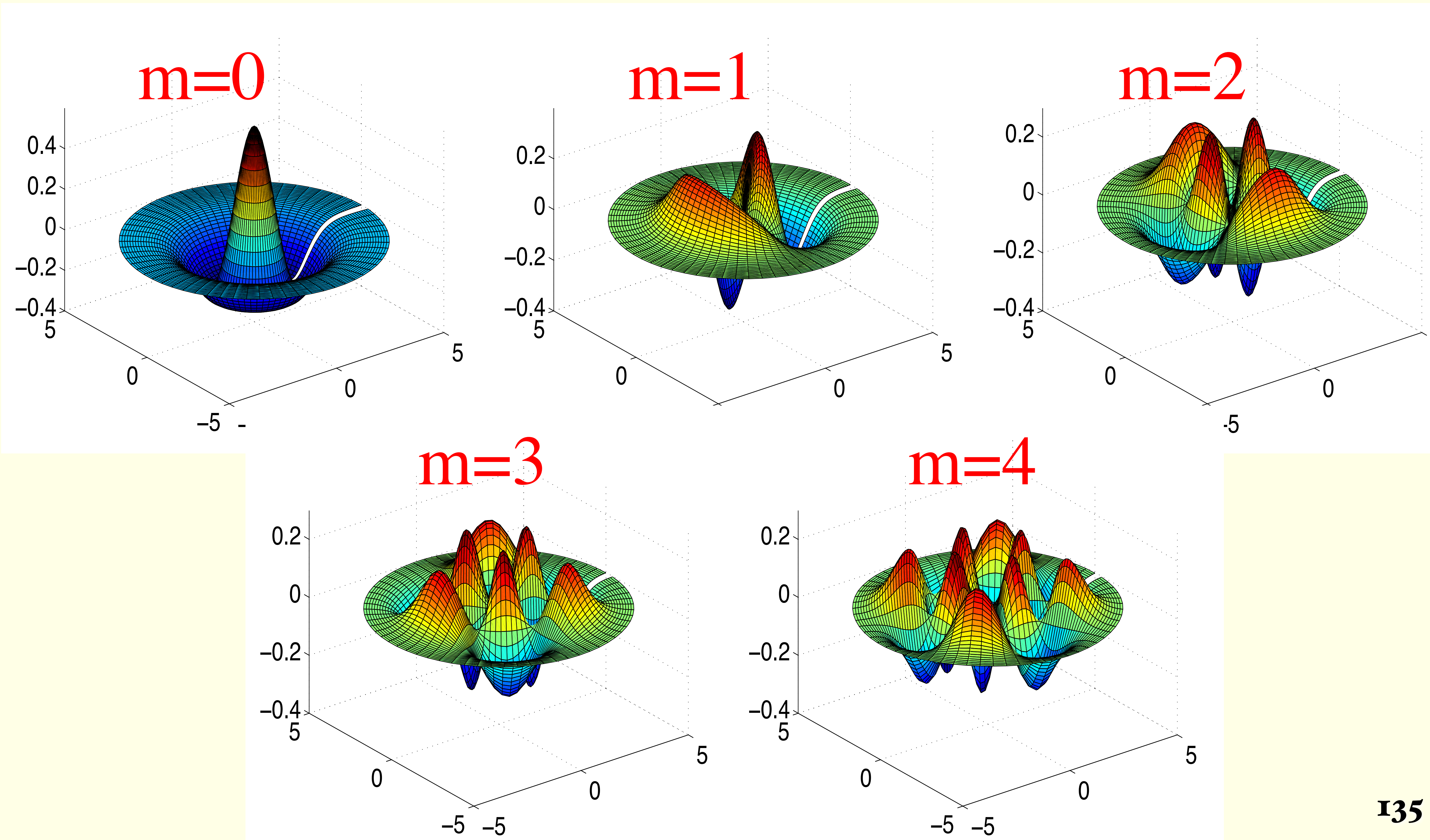
$$\Psi_{k,n,m}(x^-,\rho,\phi) = \psi_k(x^-)\Phi_{n,m}(\rho,\phi). \tag{1}$$

- 2-D harmonic trap with the basis function scale

Heli Honkanen, Jun Li, Pieter Maris, James Vary (Iowa State University)
Stan Brodsky (SLAC National Accelerator Laboratory, Stanford University)
Avaroth Harindranath (Saha Institute of Nuclear Physics, 1/AF, Bidhannagar, Kolkata, India)

Set of transverse 2D HO modes for $n = 1$

J.P. Vary, H. Honkanen, Jun Li, P. Maris, S.J. Brodsky, A. Harindranath, G.F. de Teramond, P. Sternberg, E.G. Ng, C. Yang, PRC



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- Effective Charge from AdS/QCD at all scales
- Conformal Dimensional Counting Rules for Hard Exclusive Processes

Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrodinger equation
- Massless pion (m_q =0)
- Regge Trajectories: universal slope in n and L
- Valid for all integer J & S.
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- Large Nc limit not required
- Add quark masses to LF kinetic energy
- Systematically improvable -- diagonalize H_{LF} on AdS basis

Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum.**

Proton spin
carried by quark angular momentum!

- **Massless Pion**
- **Hadron Eigenstates have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$J^z = +1/2 :< L^z >= 1/2, < S_q^z = 0 >$$

- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

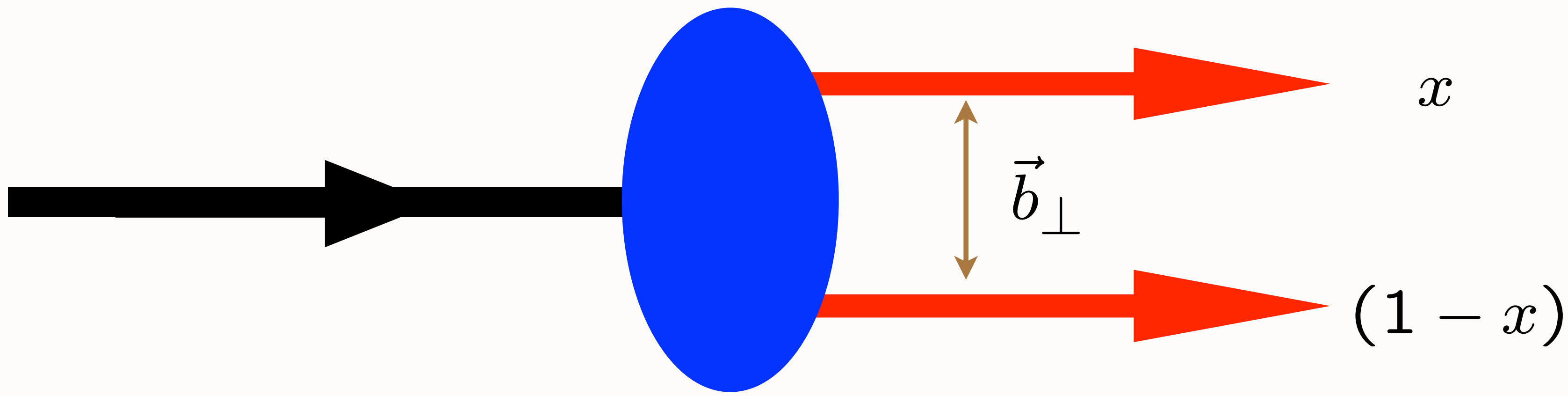
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{4\zeta^2} + U(\zeta, S, L)\right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2 .$$



$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L+S-1)$$

soft wall
confining potential:

G. de Teramond, sjb

Light-Front Schrödinger Equation

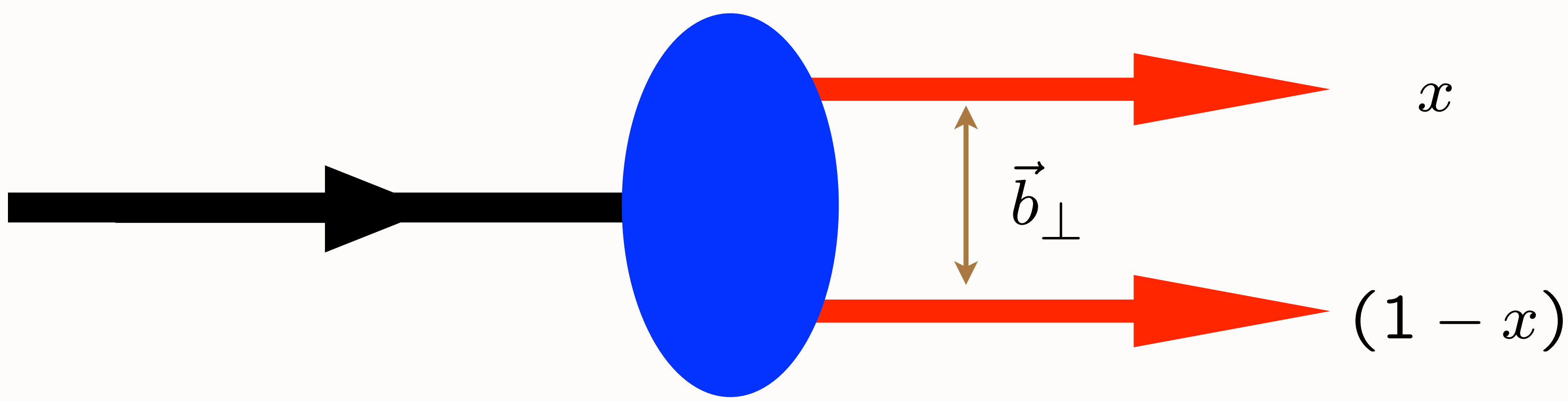
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

$$\left[-\frac{d^2}{d\zeta^2}+\frac{4L^2-1}{4\zeta^2}+U(\zeta^2,J,L,M^2)\right]\Psi_{J,L}(\zeta^2)=M^2\Psi_{J,L}(\zeta^2)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2 .$$

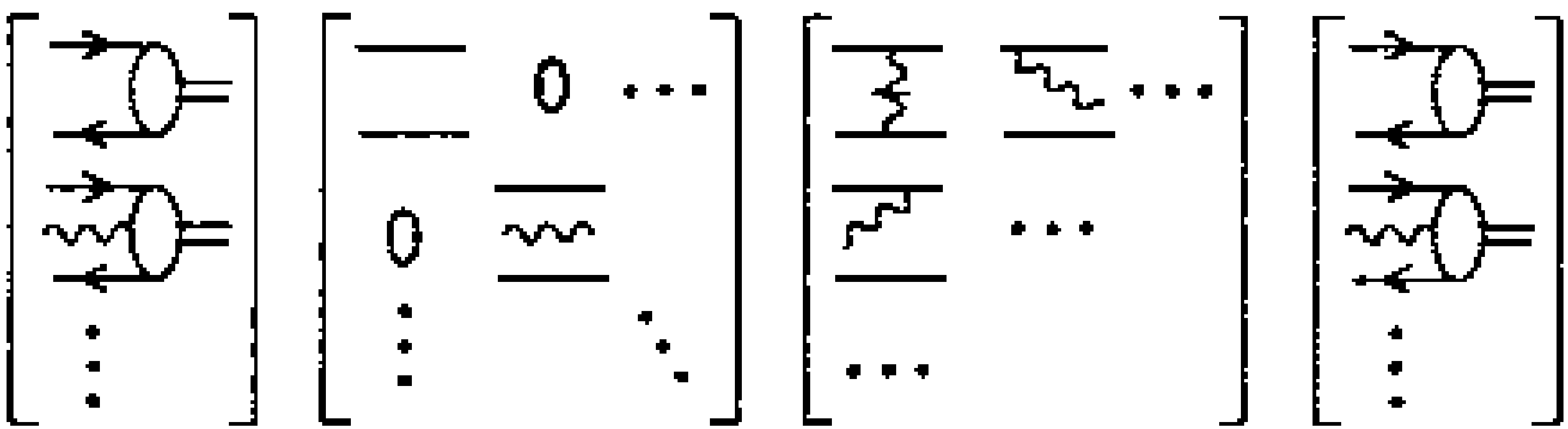


where the potential $U(\zeta^2, J, L, M^2)$ represents the contributions from higher Fock states. It is also the kernel for the forward scattering amplitude $q\bar{q} \to q\bar{q}$ at $s = M^2$. It has only "proper" contributions; i.e. it has no $q\bar{q}$ intermediate state. The potential can be constructed systematically using LF time-ordered perturbation theory. Thus the exact QCD theory has the identical form as the AdS theory, but with the quantum field-theoretic corrections due to the higher Fock states giving a general form for the potential. This provides a novel way to solve nonperturbative QCD.

LIGHT-FRONT SCHRODINGER EQUATION

Direct connection to QCD Lagrangian

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^{2} + m_i^2}{x_i}\right)\begin{bmatrix}\psi_{q\bar{q}/\pi}\\ \psi_{q\bar{q}g/\pi}\\ \vdots\end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\,V\,|q\bar{q}\rangle & \langle q\bar{q}|\,V\,|q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\,V\,|q\bar{q}\rangle & \langle q\bar{q}g|\,V\,|q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}\begin{bmatrix}\psi_{q\bar{q}/\pi}\\ \psi_{q\bar{q}g/\pi}\\ \vdots\end{bmatrix}$$



$A^+ = 0$

G.P. Lepage, sjb

Systematically eliminate non-valence Fock states;
project to a single radial variable

Fock vacuum $|0\rangle$ eigenstate of the full Hamiltonian

$$
\begin{aligned}
P^- \;=\;& \frac{1}{2}\int dx_+ d^2x_\perp \left(\overline{\Psi}\gamma^+ \frac{\bar{m}^2+(i\nabla_\perp)^2}{i\partial^+}\Psi + A^\mu_a (i\nabla_\perp)^2 A^a_\mu\right) \quad \text{free} \\
+\;& g\int dx_+ d^2x_\perp J^\mu_a A^a_\mu \quad \text{vertex interaction} \\
+\;& \frac{g^2}{4}\int dx_+ d^2x_\perp B^{\mu\nu}_a B^a_{\mu\nu} \quad 4-\text{point gluon} \\
+\;& \frac{g^2}{2}\int dx_+ d^2x_\perp J^+_a \frac{1}{(i\partial^+)^2} J^+_a \quad \text{instantaneous gluon interaction} \\
+\;& \frac{g^2}{2}\int dx_+ d^2x_\perp \overline{\Psi}\gamma^\mu T^a A^a_\mu \frac{\gamma^+}{i\partial^+}\left(\gamma^\nu T^b A^b_\nu \Psi\right), \quad \text{instantaneous fermion interaction}
\end{aligned}
$$

where

$$
J^\mu_a = \bar{\Psi}\gamma^\mu T^a \Psi \chi^\mu_a + f^{abc}\partial^\mu A^\nu_b A_\nu .
$$

Goals

- Test QCD to maximum precision
- High precision determination of $\alpha_s(Q^2)$ at all scales
- Relate observable to observable --no scheme or scale ambiguity
- Eliminate renormalization scale ambiguity in a scheme-independent manner
- Relate renormalization schemes without ambiguity
- Maximize sensitivity to new physics at the colliders

Need to set multiple renormalization scales -- Lensing, DGLAP, ERBL Evolution ...

$$\text{Choose renormalization scheme; e.g. } \alpha_s^R(\mu_R^{\text{init}})$$

$\downarrow$

$$\text{Choose } \mu_R^{init}\text{; arbitrary initial renormalization scale}$$

$\downarrow$

$$\text{Identify } \{\beta_i^R\} - \text{terms using } n_f - \text{terms}$$

$$\text{through the PMC} - \text{BLM correspondence principle}$$

$\downarrow$

$$\text{Shift scale of } \alpha_s \text{ to } \mu_R^{\text{PMC}} \text{ to eliminate } \{\beta_i^R\} - \text{terms}$$

$\downarrow$

$$\text{Conformal Series}$$

$$\text{Result is independent of } \mu_R^{\text{init}} \text{ and scheme at fixed order}$$

PMC/BLM

No renormalization scale ambiguity

Result is independent of Renormalization scheme and initial scale

Apply to Evolution kernels, hard subprocesses

Eliminates unnecessary systematic uncertainty

Principle of Maximum Conformality

Xing-Gang Wu
Leonardo di Giustino, SJB

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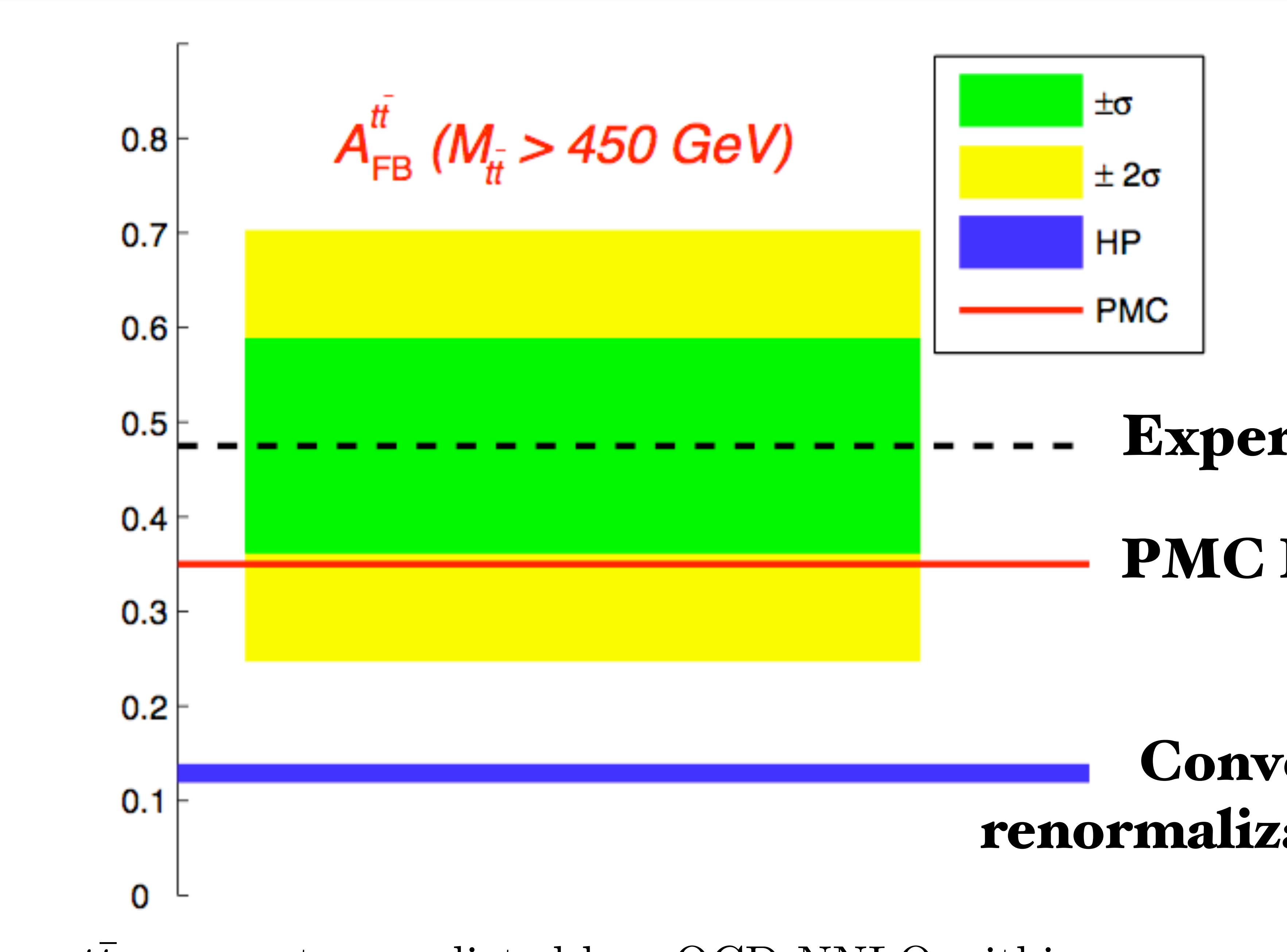


Experimental asymmetry

PMC Prediction

Conventional: guess renormalization scale and range

$t\bar{t}$ asymmetry predicted by pQCD NNLO within
1 σ of CDF/D0 measurements using PMC/BLM scale setting

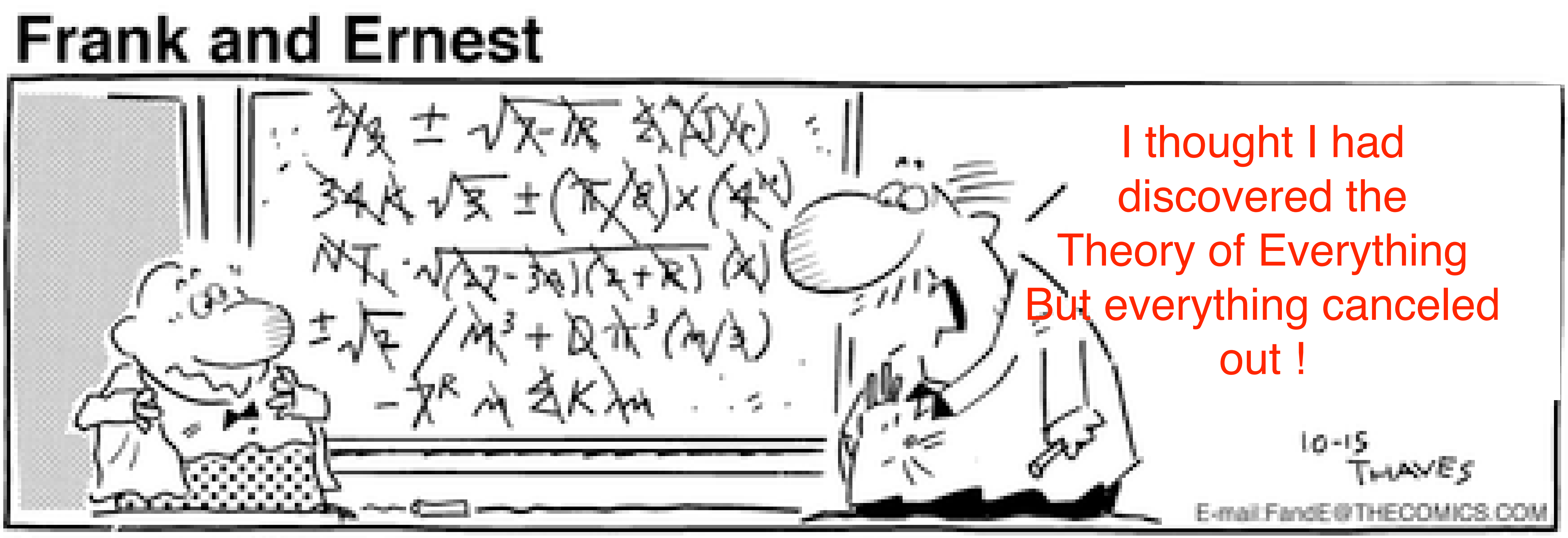
Eliminating the Renormalization Scale Ambiguity for Top-Pair Production Using the Principle of Maximum Conformality

Xing-Gang Wu
SJB

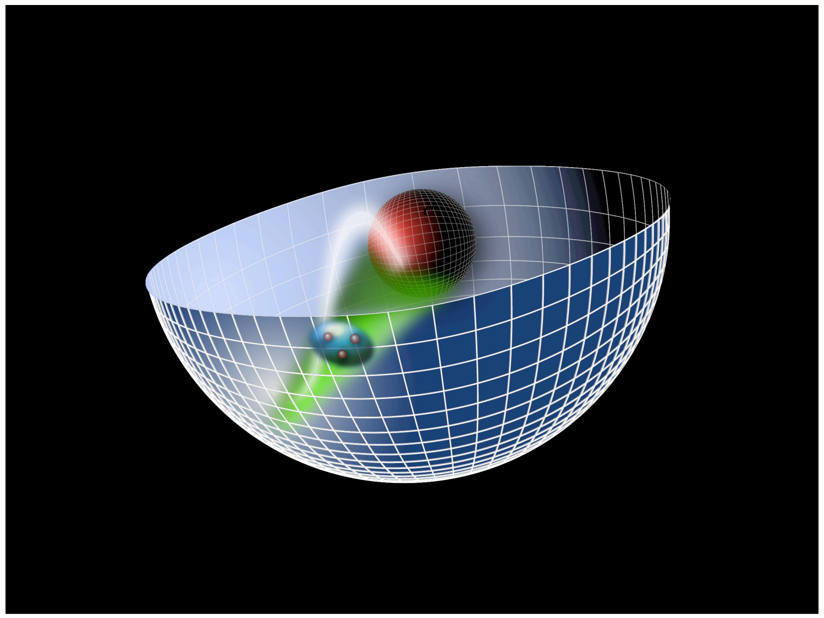
SCIENCE VOL 265 15 SEPTEMBER 1995

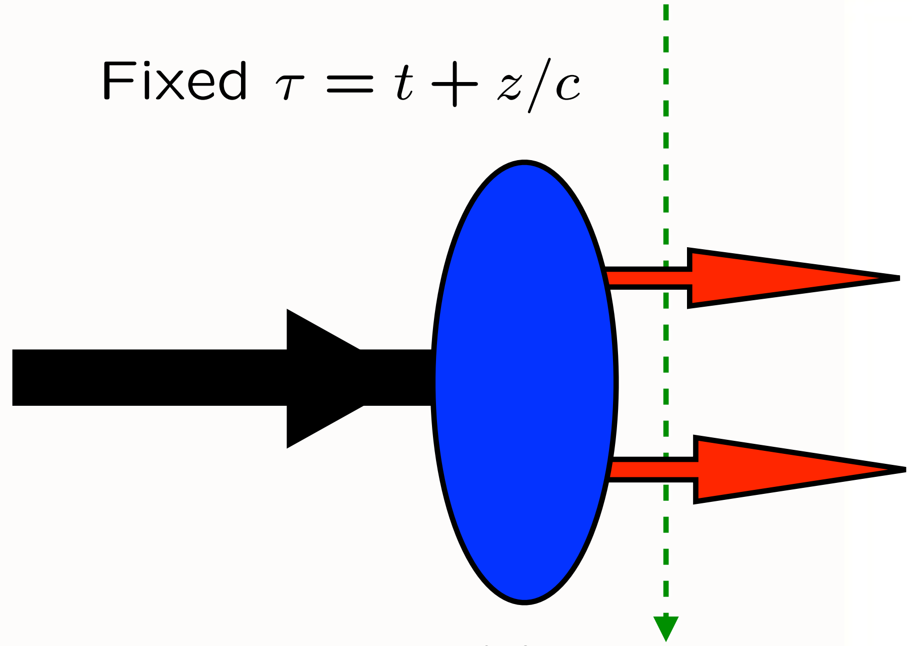
A Theory of Everything Takes Place

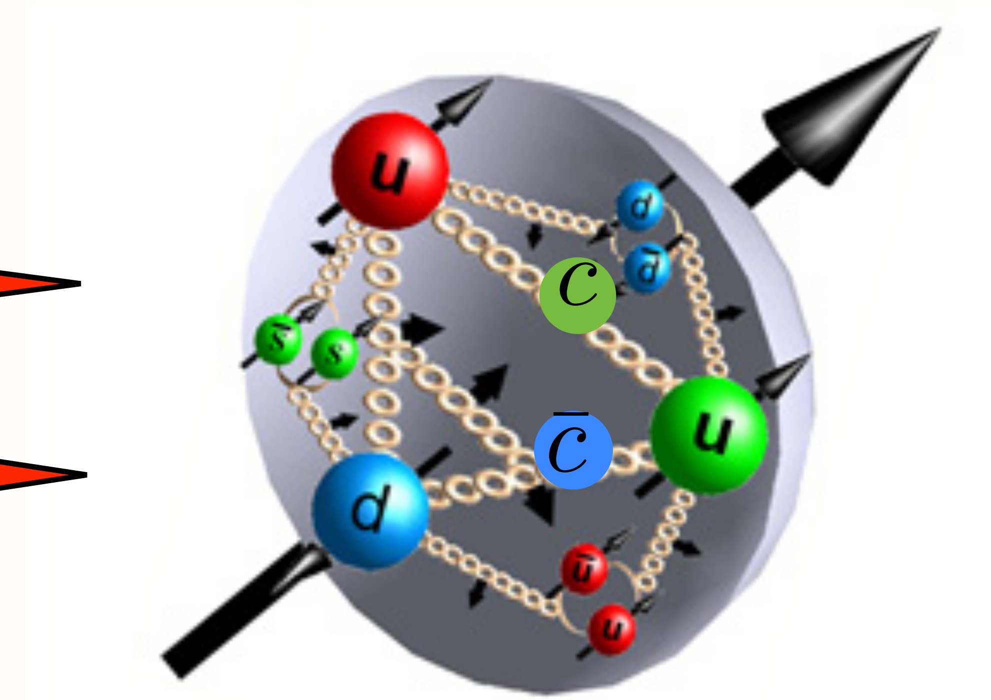
String theorists have broken an impasse and may be on their way to converting this mathematical structure -- physicists' best hope for unifying gravity and quantum theory -- into a single coherent theory.

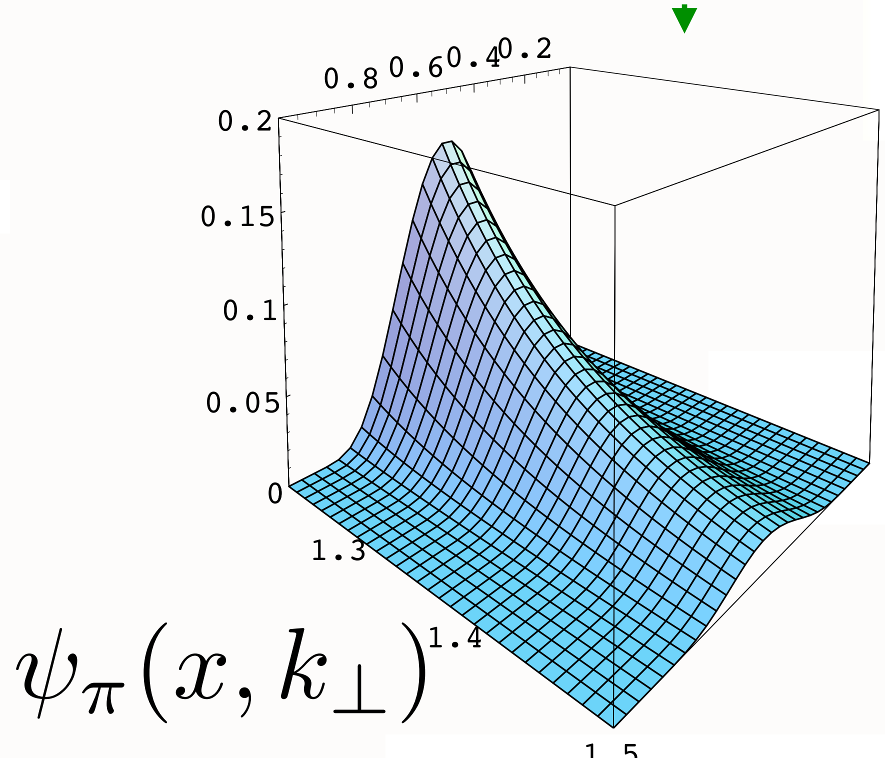


AdS/QCD and Light-Front Holography: New Perspectives for QCD and the Light-Front Vacuum









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Conference on the Origin of Mass 2012

11-17 June 2012