Planet Formation

and evolution of young planetary systems

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Overview

Part I: Basic framework

- ***Protoplanetary discs*** and their evolution

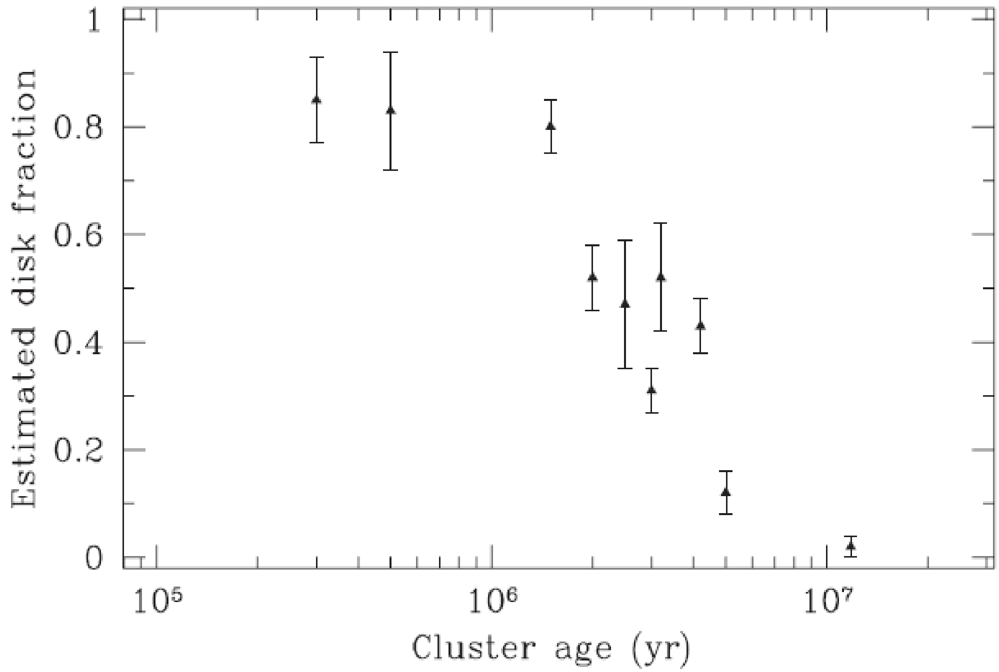
Part II: Planet formation

- Formation of ***planetesimals***
- Formation of ***terrestrial planets*** and ***planet cores***
- Formation of ***gas giants***

Part III: Dynamics of young systems

- Gas- and planetesimal-driven ***planet migration***
- Formation of ***resonant systems*** and the ***"Nice model"***

Protoplanetary discs

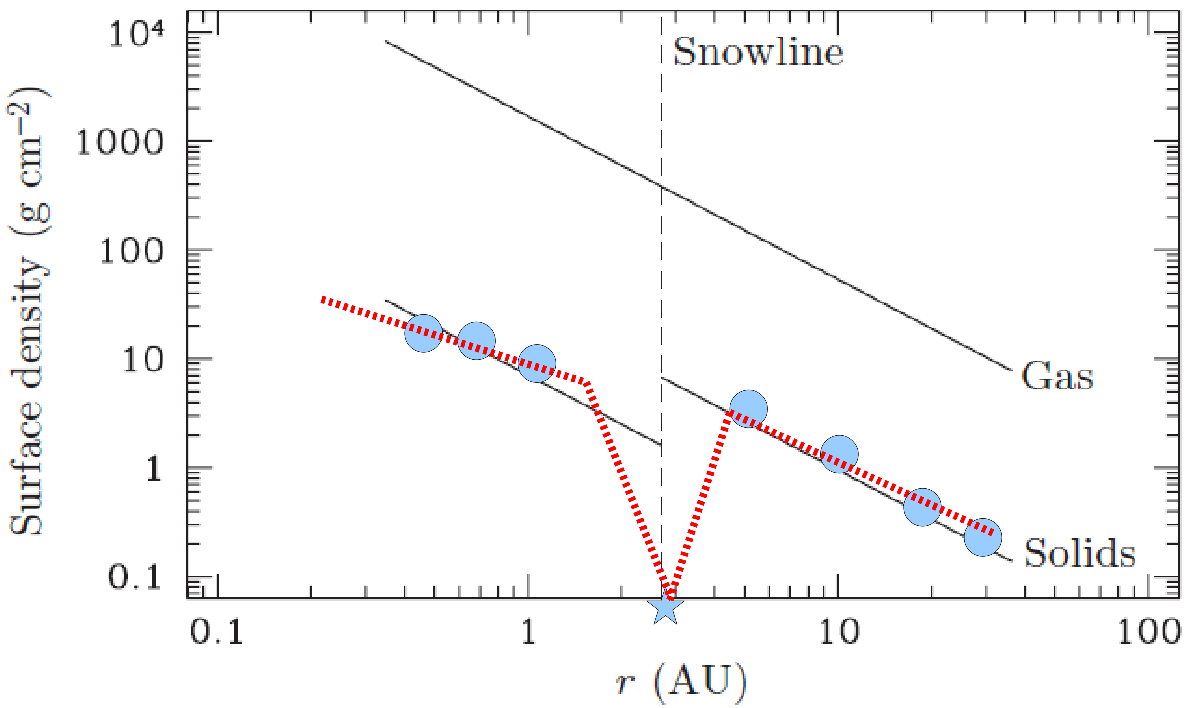


- Observed in YSOs from IR excess (dust) and possibly UV excess (accretion 'hot-spots')
- Estimated lifetimes of a few Myrs
- Mass ~0.01-0.1M_* and extent of ~10s-100s AU
- orbital periods (at 1 AU) are ~10^6 times smaller than the lifetime

→ we can treat discs as "very slowly evolving" structures, roughly in steady state

Disc mass profile: the MMSN model

The Minimum Mass Solar Nebula (MMSN) recipe:
- take the solids in all planets
- augment the mass of H/He to solar composition
- spread in annuli around each orbit



Derived surface density:

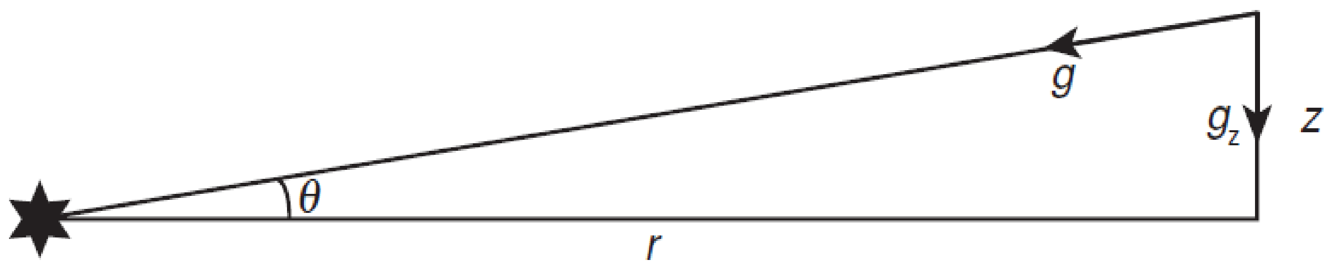
$$\Sigma(r) = \Sigma_0\, r^{-3/2} \quad (\text{g cm}^{-2})$$

Σ_0 = 1700 for gas
Σ_0 = 7 (rocky, a<2.7 AU)
Σ_0 = 30 (ices, a>2.7 AU)

gas/solids ~ 100/1
Total mass ~ $0.01M_*$

Snowline: defined as the distance from the star beyond which water condenses to ice, greatly augmenting the concentration of solids (2.7 AU, for low pressure)

Basic disc physics



Vertical structure: we assume a vertically stable structure with a small scale-height → *h/r*<<1

This means that the vertical component of the star's gravity

$$g_z = g\sin\theta = \frac{GM_*}{(r^2+z^2)}\frac{z}{(r^2+z^2)^{1/2}}$$

is balanced by the vertical pressure gradient $(1/\rho)(\mathrm{d}P/\mathrm{d}z)$

Assuming an ideal gas e.o.f. $P = \rho c_s^2$

$$c_s^2\frac{\mathrm{d}\rho}{\mathrm{d}z} = -\frac{GM_* z}{(r^2+z^2)^{3/2}}\rho \quad \longrightarrow \quad \rho = C\exp\left[\frac{GM_*}{c_s^2(r^2+z^2)^{1/2}}\right]$$

* Problem: Assume z/r <<1 and use the fact that $\Omega = \sqrt{GM_*/r^3}$

To show that $\rho = \rho_0 e^{-z^2/2h^2}$ with $\rho_0 = \frac{1}{\sqrt{2\pi}} \frac{\Sigma}{h}$

Where Σ is the column-averaged surface density and the scale-height is defined by:

$$h \equiv \frac{c_s}{\Omega}$$

We also assume the disc to be *vertically isothermal*, so that

$$c_s^2 = \frac{k_B T}{\mu m_p}$$

For μ=2.3 (mixture of molecular H/He gas) and T~100 K at r =1 AU and h/r=0.02, we obtain c_s~1 km/s

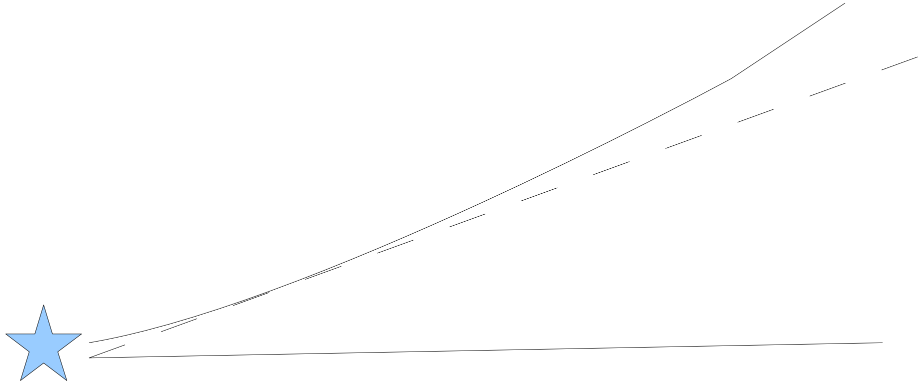
NOTE: we cannot (yet) determine the radial profile of disk quantities!

→ also the shape of the disc depends on $h(r)/r$

If e.g. we assume that the sound speed goes like: $c_s \propto r^{-\beta}$

then $$\frac{h}{r} \propto r^{-\beta+1/2}$$

i.e. for β<1/2 (which means $T(r) \sim r^{-1}$ or shallower) the disc will be ***"flaring"*** (h/r increasing with distance)



Radial disc structure

- The radial profile of (Σ,T) cannot be determined without asking how the disc evolves (angular momentum transport)

- For given (Σ,T) profiles, the gas velocity profile is found by balancing the forces in the radial direction. For a *stationary* axisymmetric flow:

$$\underbrace{\frac{v_{\phi,\mathrm{gas}}^2}{r}}_{\text{centrifugal}} = \underbrace{\frac{GM_*}{r^2}}_{\text{star's gravity}} + \underbrace{\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r}}_{\text{radial pressure gradient}}$$

If e.g. $P = P_0 \left(\frac{r}{r_0}\right)^{-n}$ then: $v_{\phi,\mathrm{gas}} = v_\mathrm{K}\left(1 - n\frac{c_\mathrm{s}^2}{v_\mathrm{K}^2}\right)^{1/2}$

- For $\Sigma \sim 1/r$ and h/r = 0.05=const, we get n=3 and

$$\boxed{\mathbf{v_\varphi \sim 0.996\ v_K}} \longrightarrow \text{slightly “sub-Keplerian” motion}$$

Radial temperature profile

- Important to understand the chemical evolution of solid particles in the disc (condensation of different species)

- Two main heating processes:
  (a) stellar irradiation
  (b) accretional heating by liberation of potential energy of infalling gas at the star's surface

- Simple calculation: what is the accretion rate for which the disc is still "passive" (-a- more important than -b-):

$$\frac{\mathrm{G} M_* \dot{M}}{R_*} = \frac{1}{4} L_*$$

assumes that the disk intercepts a fraction (1/4) of the star's luminocity

- For a Sun-like star it gives $\dot{M} \approx 2 \times 10^{-8}\ M_{\odot}\mathrm{yr}^{-1}$

- Then, assuming black-body re-radiation by the disc, we get:

$$T_{\text{disk}} \propto r^{-3/4}$$

Disc evolution

- Central issue: for the gas to move inwards and accrete on the star it has to ***lose angular momentum!***

  ⇒ the only way to change $\Sigma(r,t)$ in a time-independent potential

Q: by what kind of "friction" ?

Fluid resistance to Keplerian shear ⇒ **Viscosity!**

$$\underset{\text{shear stress}}{\tau} = F/\delta l = \mu \, \underbrace{r \frac{d\Omega}{dr}}_{\text{rate of shear}} = \nu \, \Sigma \, r \frac{d\Omega}{dr}$$

μ = dynamic viscosity = $\nu \, \Sigma$

ν = kinematic viscosity (cm^2/s)

The torque exerted on an annulus by shear stresses is:

$$G = \underbrace{2\pi r}_{\text{circumference}} \cdot \nu \, \Sigma \, r \frac{d\Omega}{dr} \cdot \underset{\text{lever arm}}{r}$$

Evolution of surface density

- Conservation of mass:

$$\frac{\partial}{\partial t}(2\pi r \Delta r \Sigma) = 2\pi r \Sigma(r) v_r(r) - 2\pi(r+\Delta r)\Sigma(r+\Delta r) v_r(r+\Delta r)$$

$$\Rightarrow \quad r\frac{\partial \Sigma}{\partial t} + \frac{\partial}{\partial r}(r\Sigma v_r) = 0$$

- Conservation of angular momentum (see Pringle 1981):

$$r\frac{\partial}{\partial t}(r^2\Omega\Sigma) + \frac{\partial}{\partial r}(r^2\Omega \cdot r\Sigma v_r) = \frac{1}{2\pi}\frac{\partial G}{\partial r}$$

- Combining the above (eliminate v_r) we get the final evolution equation for $\Sigma(r)$:

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{r}\frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial}{\partial r}\left(\nu\Sigma r^{1/2}\right)\right]$$

Problem: derive that!

- The evolution equation $$\frac{\partial \Sigma}{\partial t} = \frac{3}{r}\frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial}{\partial r}\left(\nu \Sigma r^{1/2}\right)\right]$$

using a change of variables, this takes the form of a typical

diffusion equation

$$X \equiv 2r^{1/2},$$
$$f \equiv \frac{3}{2}\Sigma X$$

$$\Rightarrow \quad \boxed{\frac{\partial f}{\partial t} = D\frac{\partial^2 f}{\partial X^2}} \quad \text{where} \quad D = \frac{12\nu}{X^2}$$

the diffusion coefficient defines the characteristic *viscous time-scale of the disc:*

$$\tau_\nu \approx \frac{r^2}{\nu}$$

which is ~ a few Myrs for typical discs

$\Rightarrow$ *the disc evolves diffusively under the effect of viscosity!*

$$\Sigma(r,t) = \Sigma_0 r^{-\beta} \exp(-t/\tau_\nu)$$

Steady-state solution

Take $\frac{\partial}{\partial t} = 0$ in the momentum equation and solve the resulting ODE :

$$2\pi r \Sigma v_r \cdot r^2 \Omega = 2\pi r^3 \nu \Sigma \frac{d\Omega}{dr} + \text{const}$$

Problem: derive that! Then, assuming that the shear stress vanishes on the surface of the star (r=R_s) and taking into account the definition of Ω and the fact that the accretion rate is

$$\dot{M} = -2\pi r \Sigma v_r$$

show that the solution simplifies to:

$$\nu \Sigma = \frac{\dot{M}}{3\pi}\left(1 - \sqrt{\frac{R_s}{r}}\right) \sim \frac{\dot{M}}{3\pi}$$

So, for a constant accretion rate, viscosity and density have steady-state radial profiles such that the *r*-dependence “cancels-out”

The Shakura-Sunyaev *α*-prescritpion

Q1: what is the *nature* of the viscosity?
Q2: how can we *parametrize* it ?

Molecular viscosity turns out to be several orders of magnitude weaker than needed → diffusion times ~ 10^{13} yrs!

Molecular viscosity is so low and the Reynolds number so large that the disc would be highly turbulent under small perturbations!
→ We assume that the disc is *turbulent*, so that turbulent viscosity is the source of angular momentum transport!

* how big is it? By dimensional arguments:
a) the larger turbulence scale should be ~ the scale height, *h*
b) the velocity should be < c_s, otherwise we would have strong dissipation in shocks

→ $$\boxed{\nu = \alpha\, c_s\, h}$$ with α<1 being the dimensionless Shakura-Sunyaev parameter

A complete disc description

$$\nu = \alpha c_s h,$$

$$c_s^2 = \frac{k_B T_c}{\mu m_p}, \quad h = \frac{c_s}{\Omega}$$

$$\rho = \frac{1}{\sqrt{2\pi}}\frac{\Sigma}{h}$$

$$\nu\Sigma = \frac{\dot{M}}{3\pi},$$

$$\sigma T_{\rm disk}^4 = \frac{9}{8}\nu\Sigma\Omega^2$$

$$\tau = \frac{1}{2}\Sigma\kappa_R$$

$$T_c^4 = \frac{3}{4}\tau T_{\rm disk}^4$$

Specifying the opacity of the material in the disk (dust), ***κ***, then you have 8 eqs with 8 unknowns. Giving the accretion rate and *α*, you can find all the disk quantities at any distance *r*.

This is a steady-state, axisymmetric and vertically isothermal (and averaged) disc!

e.g. For $\kappa \sim T_c^{\,2}$ eliminating variables gives: $\Sigma^3 = \frac{64}{81\pi}\frac{\sigma}{\kappa_0}\left(\frac{\mu m_p}{k_B}\right)^2 \alpha^{-2}\dot{M}$

and for $\alpha \sim 0.01$ and $dM/dt \sim 10^{-7}$ we get $\nu \sim 10^{15}$ and $t_\nu \sim 1$ My (at 30 AU)

Ang.Mom transport, and time evolution

What did we ignore so far:
Magnetic fileds, disc self-gravity, ionization, radiative transfer,...

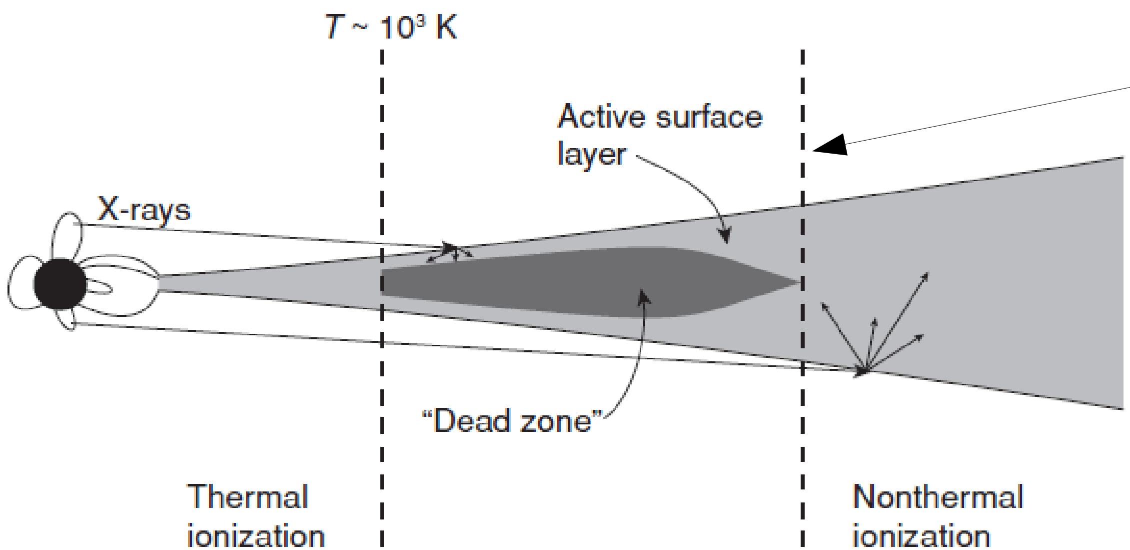
Magnetic field:
- the magnetorotational instability (MRI) is the main source of turbulence.
- magentically driven winds can lead to ang.mom loss!

Ionization:
- partial ionization (by the star's radiation, X-rays, radioactive decay) can mitigate the MRI (by ohmic dissipation, diffusion, etc.)

Self-gravity:
- can give rise to local hydrodynamical turbulence, when the density is high-enough

...

These processes can form a *layered disk*, where mass flows through an active layer that surrounds a dead zone (no MRI)



This creates density and pressure 'bumps'

important for planet formation ?

Disc dispersal:

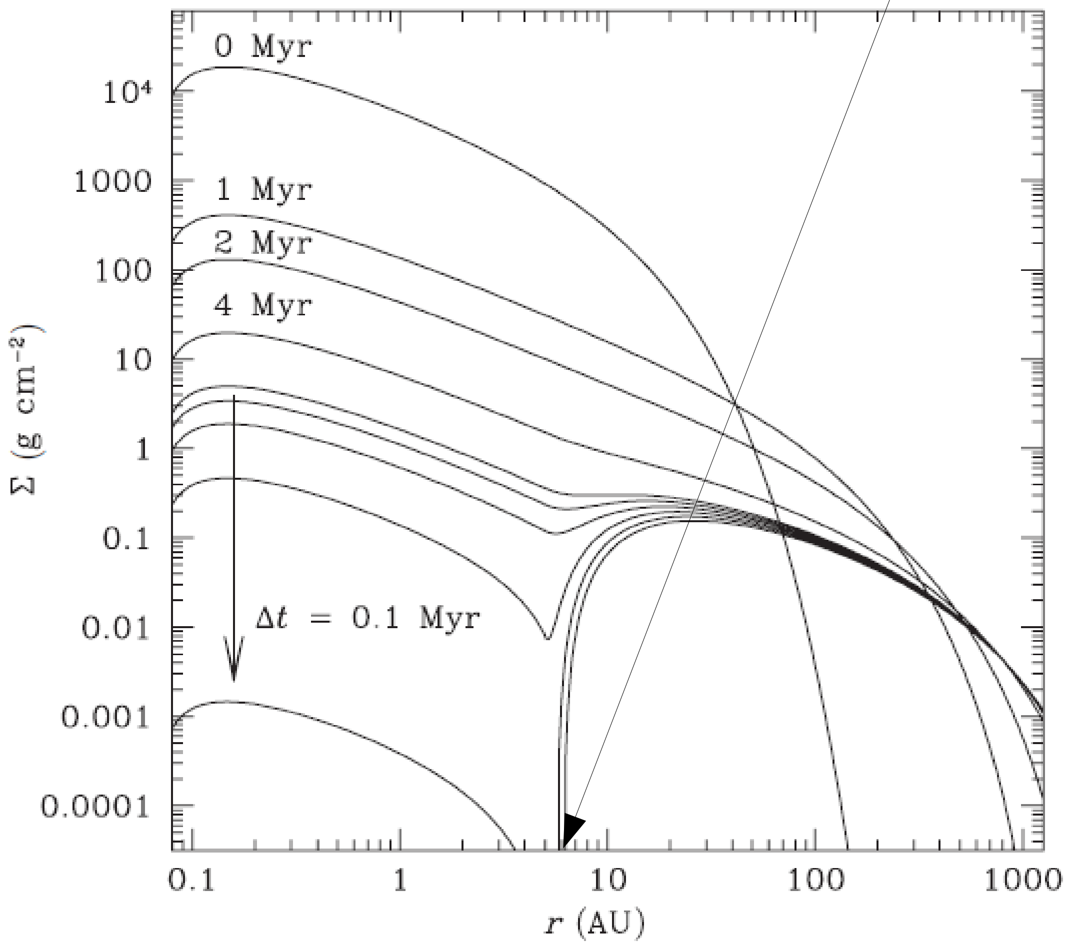
Accretion is a slow, gradual, process and cannot give the abrupt age-profile of discs. Thus, another process needs to act at the same time.

→ ***Photo-evaporation*** (a thermally driven wind)

If we combine viscous evolution with photo-evaporation (mass loss):

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{r}\frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial}{\partial r}\left(\nu \Sigma r^{1/2}\right)\right] + \dot{\Sigma}_{\rm wind}(r,t)$$

Where a characteristic distance, r_g, is set as the limit where the sound speed of the ionized gas equals the orbital velocity



Slow (viscous) diffusion and a slowly driven wind (beyond r_g) act together.

The accretion rate on the star drops with time

When it becomes equal to the rate of mass loss due to the wind:

- mass at $r>r_g$ preferentially lost as wind
- the inner disc decouples and drains rapidly on the star, following its own viscous time-scale

Concluding remarks on discs

- A protoplanetary disc is a thin disc with gas/solid ~ 100/1
- The gas moves in a slightly sub-Keplerian fashion. The disc can be considered as vertically stable and nearly locally isothermal
- Radial force balance gives the steady-state profiles of disc quantities

- We need a source of viscosity in order to have any time evolution of the surface density (ang.mom loss)
- Magnetic turbulence is the main considered source
- In passive discs, most heating comes from the star's radiation and mass accretion rates are low
- Solid dust is the main source of opacity and modifies the vertical temperature profile (central vs. surface)
- The temperature and pressure profiles dictate the snowline location and the condensation of solids

- To disperse in a realistic way we need photoevaporation to couple with viscous evolution
- We have swept under the carpet a lot of processes and difficulties...

Part II: planet formation

There are 3 steps (and 12 orders of magnitude!) to consider:

- *Planetesimal formation*
  solid bodies of ~10 km decoupled from the gas. These are the building blocks of planets. How do they form? How does the disc affect growth from mm- to km-sized bodies? (aerodynamic drag and mutual interactions...)

- *Terrestrial planets (TP) and core formation*
  Mutual interactions (Newtonian gravity and collisions) of planetesimals. Easy physics, but huge number of events (~1 billion large planetesimals to form the Solar System Tps) and different sizes complicate things...

- *Giant planet (GP) formation*
  once a core like the Earth is formed it can accrete a gas envelope. For large-enough masses, the accretion rate increase and the envelope can collapse, giving a giant planet

NOTE a time-scale issue: GPs must form while the disc is still arround.
TPs can form later...

Planetesimal Formation

- *Aerodynamic drag*
  two different regimes

Epstein drag, for $s < \lambda$ $\quad \mathbf{F}_\mathrm{D} = -\frac{4\pi}{3}\rho s^2 v_\mathrm{th}\mathbf{v}$

Stokes drag, for $s > \lambda$ $\quad \mathbf{F}_\mathrm{D} = -\frac{C_\mathrm{D}}{2}\pi s^2 \rho v \mathbf{v}$

s = size of solid particle
λ = mean free path of gas molecules
v = relative velocity and v_{th} = thermal gas velocity
C_D depends on the Reynolds number of the flow (and the shape of particles..)

- For s>9λ/4 they are equal – one can interpolate between the two to make a smooth transition 'law'

- Assume μm-sized dust. The *friction time* is $\quad t_\mathrm{fric} = \frac{m v}{|F_\mathrm{D}|} = \frac{\rho_\mathrm{m}}{\rho}\frac{s}{v_\mathrm{th}}$

this is ~1s ! ⇨ they are strongly coupled to the gas

now, let's see what happens if we add the star's gravity and look into the z-direction...

The solid particles do not feel the vertical force due to the pressure gradient, but feel the drag!

Equating the gravitational force (star) to the Epstein drag, we get the *terminal velocity* and the *settling time*:

$$v_{\rm settle} = \frac{\rho_{\rm m}}{\rho} \frac{s}{v_{\rm th}} \Omega^2 z \qquad t_{\rm settle} = \frac{z}{|v_{\rm settle}|}$$

For typical disc conditions, the settling time is ~10^5 yr

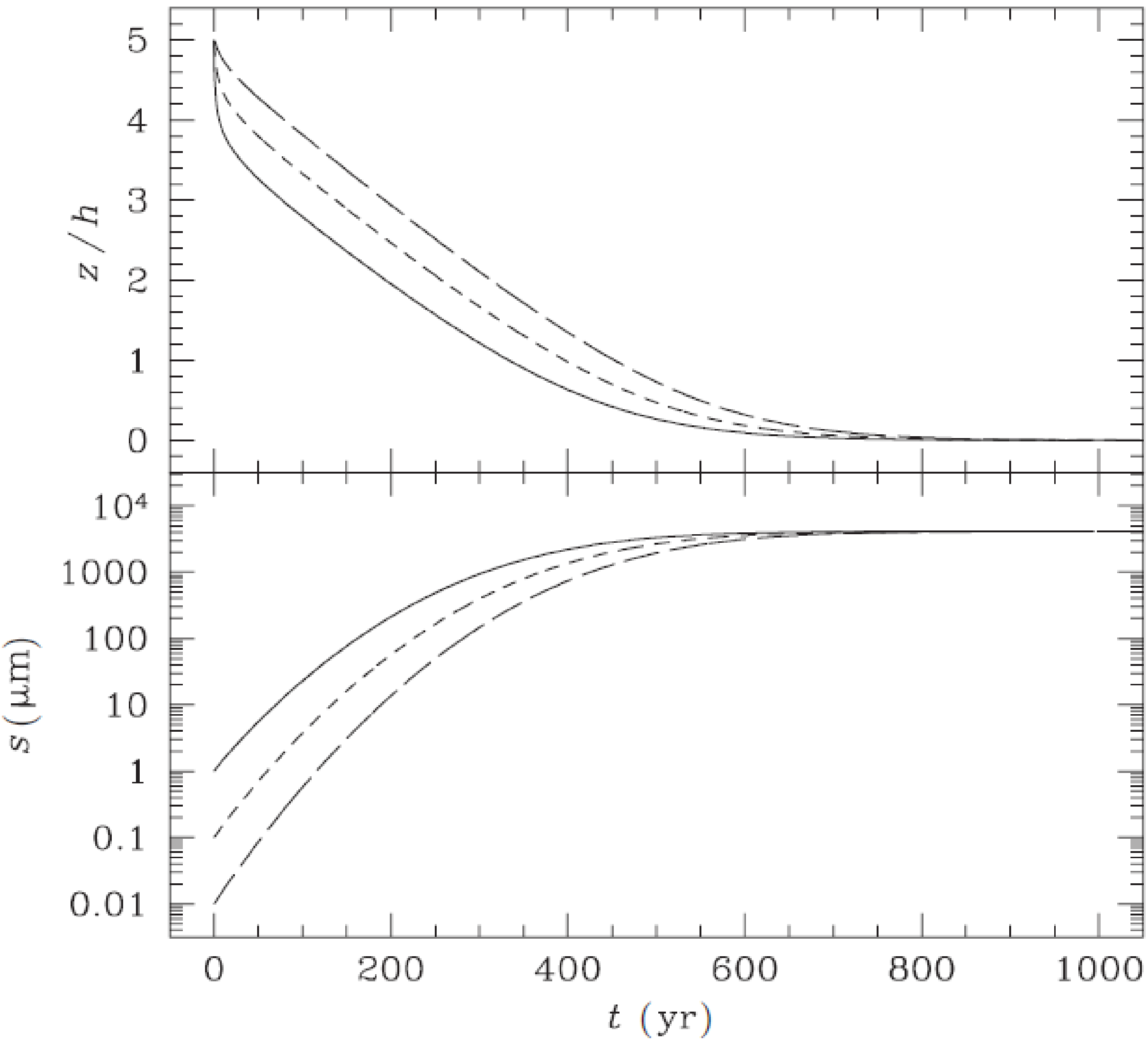
Problem: assuming a vertically isothermal disc, find out how the settling time depends on Σ and h

- *Coagulation*

Assume that, as it falls, the particle collides with smaller ones and all collisions are adhesive $\Rightarrow$ it's mass grows $\Rightarrow$ it settles faster!

Solving for mass growth $$\frac{\mathrm{d}m}{\mathrm{d}t} = \pi s^2 |v_{\mathrm{settle}}| f \rho(z) = \frac{3}{4} \frac{\Omega^2 f}{v_{\mathrm{th}}} z m$$

and z-motion (simultaneously) $$\frac{\mathrm{d}z}{\mathrm{d}t} = -\frac{\rho_{\mathrm{m}}}{\rho} \frac{s}{v_{\mathrm{th}}} \Omega^2 z$$



Dust particles can grow to ~cm size within only 1000 yrs!

Coagulation helps a lot !

This is a simple model (no radial motion, no turbulence, only adhessive collitions...)

Turbulent diffusion inhibits this process and we need growth to occur relatively fast to recover such short time-scales ...

... the problem starts when we consider radial motion ...

Radial drift due to drag

small guys coupled to the gas will move at sub-Keplerian velocity
⇨ they spiral in because the centrifugal force cannot balance gravity...

bigger guys see a 'headwind' due to the sub-Keplerian motion of the gas
⇨ they spiral in because of friction with the gas...

- Either way, there is radial motion inwards!

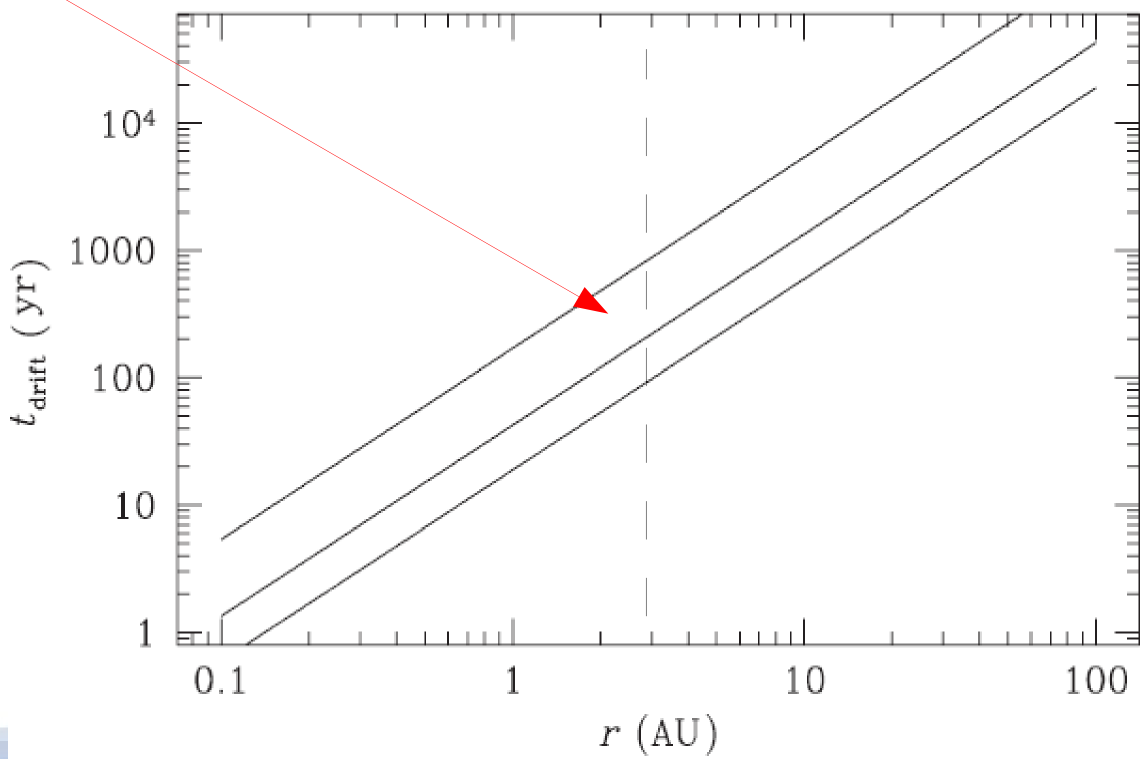
$$\frac{\mathrm{d}v_r}{\mathrm{d}t} = \frac{v_\phi^2}{r} - \Omega_\mathrm{K}^2 r - \frac{1}{t_\mathrm{fric}}\left(v_r - v_{r,\mathrm{gas}}\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(r v_\phi\right) = -\frac{r}{t_\mathrm{fric}}\left(v_\phi - v_{\phi,\mathrm{gas}}\right)$$

Amazingly fast drift for particles with s~10-100 cm !!!

... the m-size barrier...

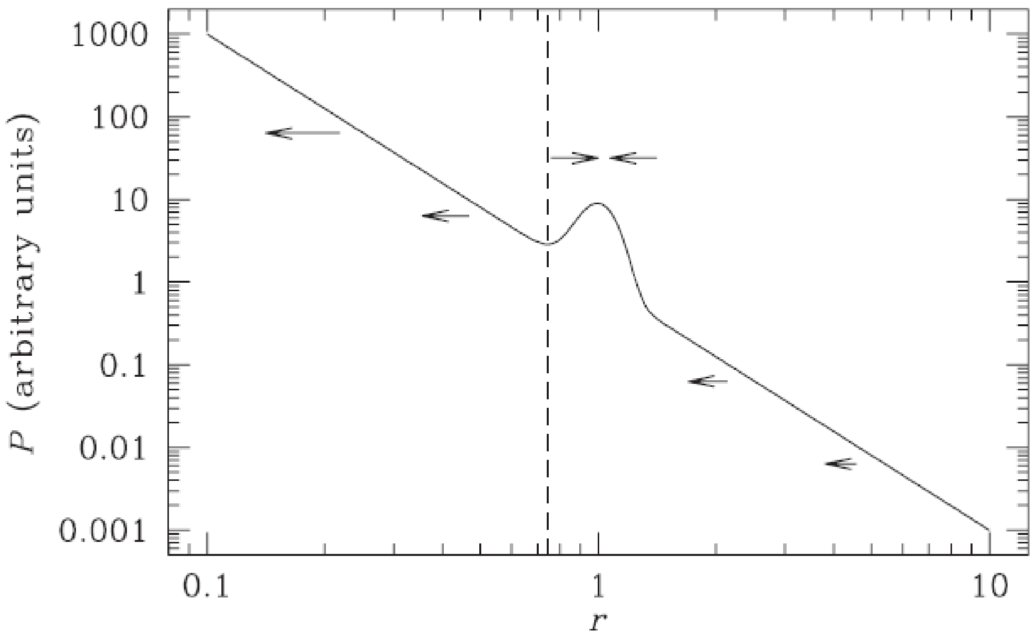
how do cm-particles survive to form planetesimals???



- Coagulation can help also in the radial direction. It can be shown that particles ~1 m can grow faster than they decay to the star. (good!)

- However, collisions can be destructive (f*#k!)

So, is there any hope ???



One solution is that the disc possesses *local pressure maxima*

Since the drift is directed towards the maximum!

Pressure maxima can occur due to turbulence (also possibly eddies)

or at the edges of a dead zone!

another way out is if planetesimal growth is not due to slow coagulation but due to another mechanism that can by-pass a few scales in magnitude...

The gravitational instability mechanism

If a dense-enough (?) layer of particles forms in the midplane:

- it can become gravitationally unstable (Safronov 1969, Goldreich & Ward 1973) and form large planetesimals very fast!

- It can also lead to modifications in the turbulence properties of the disc due to feedback from the solids.

- 'Two-fluid' instabilities can lead to local clumping and collapse

all these processes assume important self-gravity

- We consider a 'fluid' of solid particles that forms a disc of zero thickness (midplane) with surface density Σ and revolves around the star. The question is whether this *can be stable against its own self-gravity* that would tend to collapse it and form bigger objects...

For this 'fluid', the evolution is given by the following equations:

$$\frac{\partial \Sigma}{\partial t} + \nabla \cdot (\Sigma \mathbf{v}) = 0,$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{\nabla p}{\Sigma} - \nabla \Phi$$

$$\nabla^2 \Phi = 4\pi G \rho.$$

we assume that the surface density, Σ, is the sum of the unperturbed density plus small disturbances:

$$\Sigma = \Sigma_0 + \Sigma_1 = \Sigma_0 + \delta\Sigma_j \, e^{i(\mathbf{k}r - \omega t)}$$

Similarly, we expand all quantities and perform linear stability analysis

... The result is ...

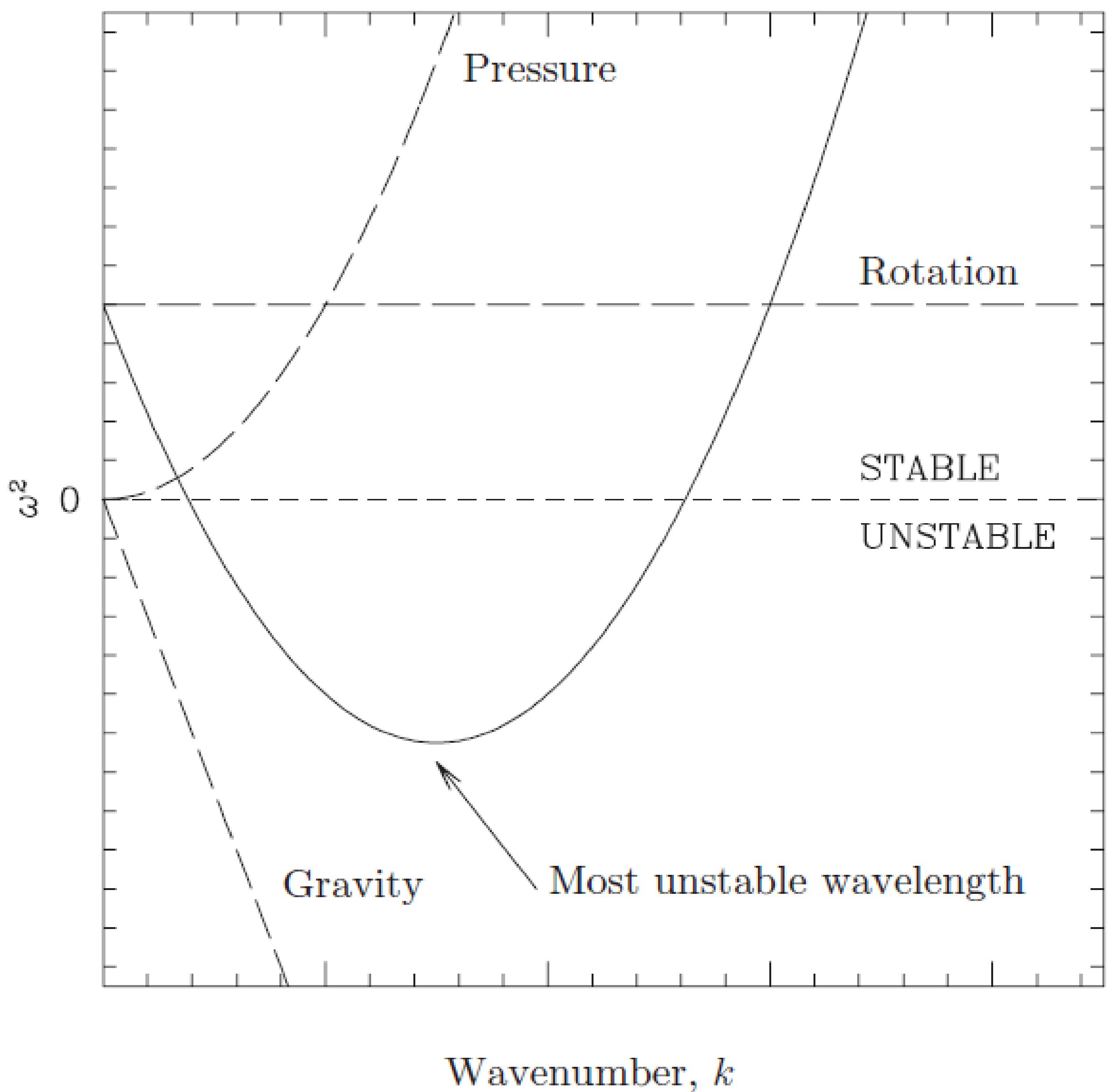
all modes are unstable iff *'Toomre's Q'* satisfies: $Q \equiv \frac{c_s \Omega}{\pi G \Sigma_0} < 1$

* here the 'sound speed' is actually taken equivalent to the particles velocity dispersion

→ Circular ('cold') orbits, massive discs and large distances favor this!

The *dispersion relation* of wave-like perturbations tells you whether a mode (wave-number) is stable or not:

$$\omega^2 = c_s^2 k^2 - 2\pi G \Sigma_0 |k| + 4\Omega^2$$

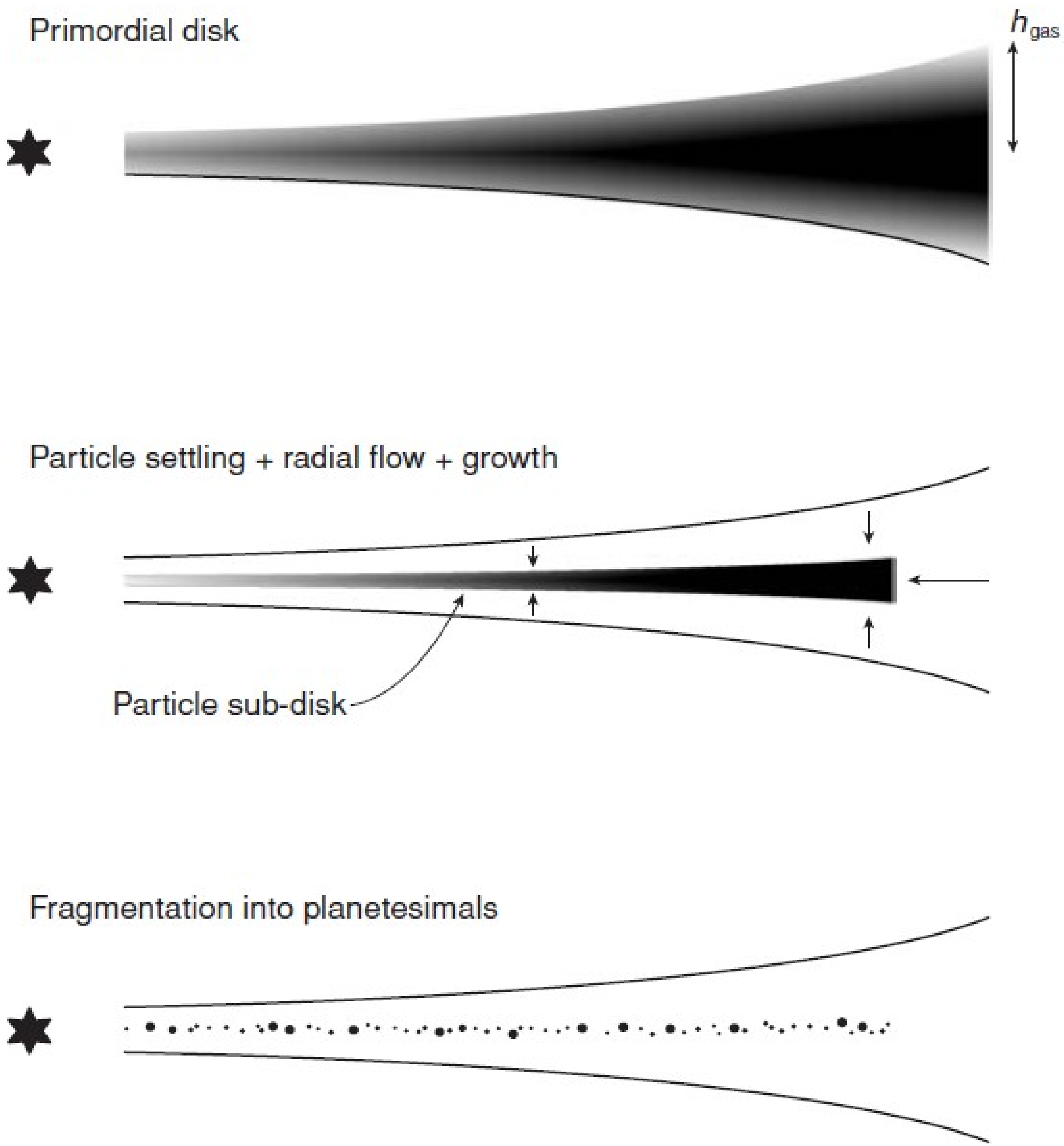


Rotation: stabilizing effect, independent of *k*

Pressure: stabilizing effect, strong for large wavenumbers (quadratic in *k*)

Self-gravity: destabilizing effect (linear in *k*)

The Goldreich-Ward scenario



The formation time is *~yr !*

(assuming that once instability sets in particles collapse on the free-fall time-scale)

Problem:

Use the dispersion relation to find the wave-length of the most unstable mode

and

calculate the size of the 'planetesimal' that forms, considering realistic values for Σ_s, Q=1 and r=5 AU.

For particles well-coupled to the gas, what is the physical meaning of Q ?

Concluding remarks on planetesimal formation

- It seems that growth to ~cm sizes has no problems. Dust settling of μm-sized particles and coaggulation to mm-sizes occurs within 1,000-10,000 yr

- The problem is for cm-m sized particles. Rapid growth is needed to overcome the fast radial drift. Not really sure how we can beat that...

- Possible solutions:
  (a) pressure fluctuations that act as particle 'traps' and
  (b) adhessive collisions even for larger sizes (relative velocities?)

  ... all these have to happen within 10^5 yr ...

- 'Terminal soution': gravitational instability sets-in and the solid particles sub-disc is fragmented to ~km-sized planetesimals

  ... this needs very large local density enhancement ...

Formation of TPs and GP-cores

- We need to go up ~3 orders of magnitude in size, i.e. ~10^9-10^{10} objects to form a few planets of ~1-10 Earth masses

- Gas is not so important – only provides some damping of the velocity dispersion (or, eccentricity, *e*, and relative inclination, *i*)

- The 3 main questions are:

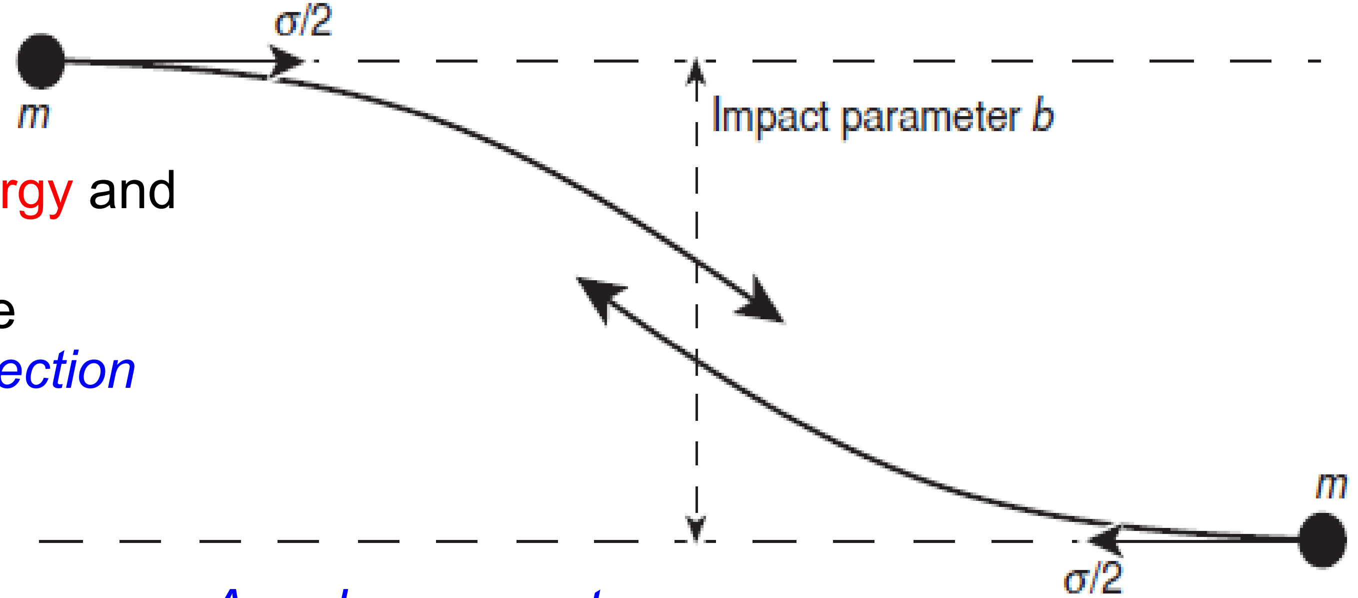
  - what is the mass and velocity (e,i) distribution that is consistent with the gravitational interactions (+ gas...) ?

  - given such a distribution, what is the rate of collisions?

  - how efficient are collisions and what formation times do we get?

Gravitational focusing and collisions

Two body approach:



Ignore the star. Use energy and angular momentum conservation to calculate the gravitational *cross-section*

Energy cons.

$$\frac{1}{4}m\sigma^2 = m v_{\max}^2 - \frac{Gm^2}{R_c}$$

Angular momentum cons.

$$v_{\max} = \frac{1}{2}\frac{b}{R_c}\sigma$$

If the closest approach occurs at the physical radius, R_s, they collide! The cross-section of collisions is πb^2, where b the max impact parameter for collision

$$\Gamma = \pi R_s^2 \underbrace{\left(1 + \frac{v_{\rm esc}^2}{\sigma^2}\right)}_{\textit{gravitational focusing factor}}$$

with $v_{\rm esc}^2 = 4Gm/R_s$ the *escape velocity* from the point of contact

Three-body approach:

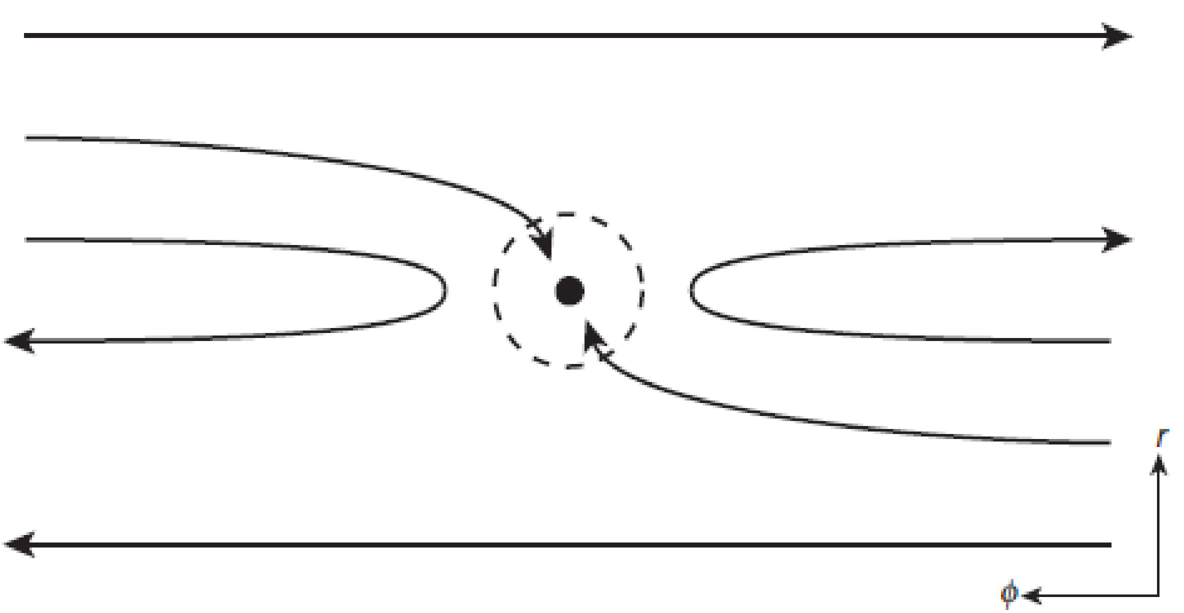
we need to compare the gravity that a particle feels when approaching a proto-planet, against the 'tidal field' of the star

We can equate the angular velocities: $\sqrt{GM_p/r^3} \sim \sqrt{GM_*/a^3}$

This gives a characteristic radius of influence for the proto-planet

$\Rightarrow r_H \sim \left(\frac{M_p}{M_*}\right)^{1/3} a$ *Hill radius*, and $v_H \sim \sqrt{\frac{GM_p}{r_H}}$ the *Hill velocity*

For $r<r_H$, the growing proto-planet dominates the dynamics



The dynamics are controled by the ratio of

$$\sigma = (e^2 + i^2)^{1/2} v_K$$

to v_H:

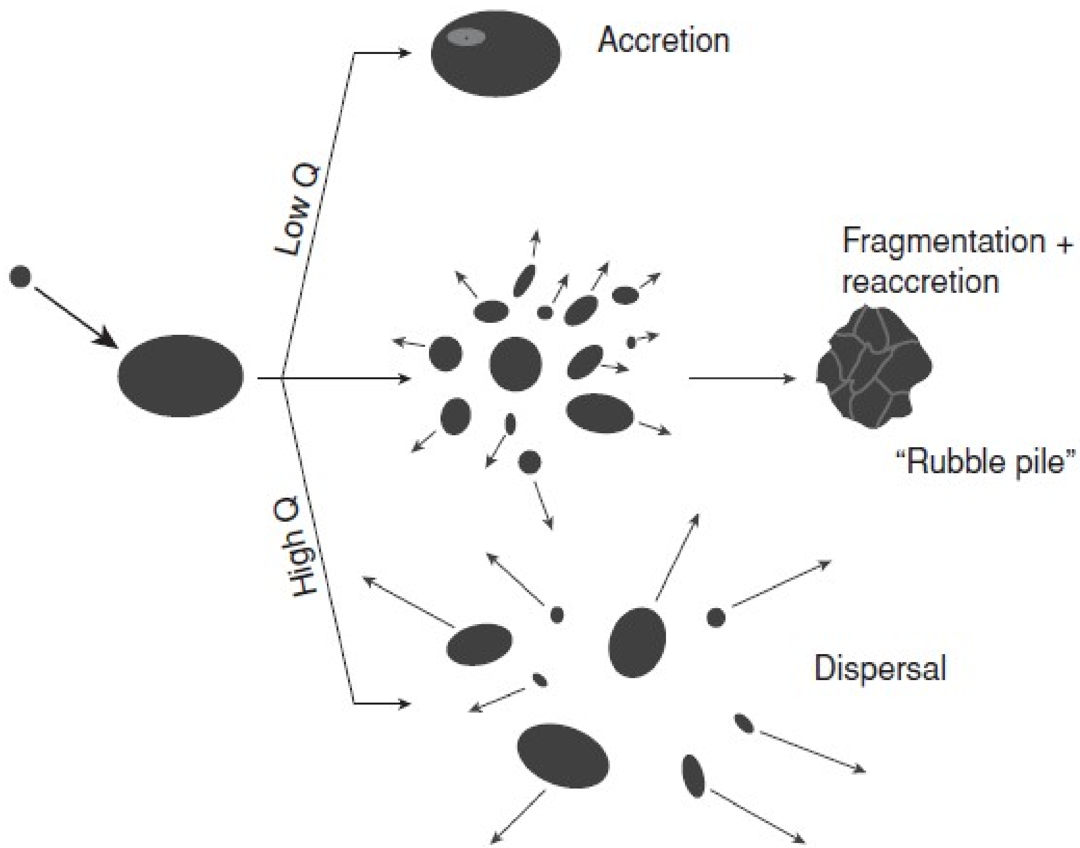
dispersion ($\sigma > v_H$) vs

shear ($\sigma < v_H$) dominated

Accretion vs Disruption

The result of a collision depends on the *specific impact energy* $Q \equiv \frac{mv^2}{2M}$

where *m* the impactor and *M* the target mass (v = relative velocity). There are three possible outcomes, depending on the value of Q vs Q_S (shattering) and Q_D (disruption; $Q_D > Q_S$)



These quantities depend upon

material strength (decreases with size) and

gravitational binding energy (increases with size)

for planetesimals hitting proto-planets and small relative velocities, we can assume efficient accretion

Approximate statistical treatment of planet growth

Take a relatively larger object -- *proto-planet* -- (M, R_s, v_{esc}) embedded in a 'sea' of planetesimals with Σ_p, σ, $h_p = \sigma/\Omega$ and $\rho_{SW} = \Sigma_p/(2h_p)$. In the *dispersion dominated regime*:

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \rho_{\mathrm{sw}} \sigma \pi R_s^2 \left(1 + \frac{v_{\mathrm{esc}}^2}{\sigma^2}\right)$$

[density **x** relative velocity **x** cross-section]

which gives:

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \frac{\sqrt{3}}{2} \Sigma_{\mathrm{p}} \Omega \pi R_s^2 \left(1 + \frac{v_{\mathrm{esc}}^2}{\sigma^2}\right)$$

Problem: show that if $v_{esc} \sim \sigma$ (e.g the planet is still small) and growth does not really affect the global disc properties, the planet's radius grows linearly with time

For an icy object at Jupiter's orbit in a MMSN, the rate is **extremely slow!**
It would take 10 My to form a 100-km body!!!

Fast growth occurs only when gravitational focusing becomes strong (i.e. 'cold' disc and large M_p)

Growth is slower at large distances!

Problem: show that for $v_{esc} >> \sigma$ the mass growth of a planet is given by

$$\frac{dM}{dt} = \frac{\sqrt{3}\pi G \Sigma_p \Omega}{\sigma^2} M R_s = k M^{4/3}$$

Under the same assumption as before, show that this gives....

Runnaway growth!

$$M(t) = \frac{1}{(M_0^{-1/3} - k't)^3}$$

This assumes that the planet has not yet grown so much that it dominates the velocity dispersion of planetesimals → *σ remains small*

Can runnaway growth be sustained?

When σ is still small, particles have ~circular orbits

→ a finite supply of particles within the planet's *feeding zone* can be accreted. Once these are consumed, ***growth has to slow down!***

NOTE: as the planet grows, it increases σ → ***growth will also slow down!***

N-body effects and velocity dispersion

- The main effects produced by the gravitational interaction between planetesimals (and proto-planets) are:

  - Viscous stirring:
  weak (distant) encounters have a cumulative effect of exiting the mean eccentricitiy and inclination in the disc. The only important process if all bodies are of similar size

  - Dynamical friction:
  when a mass spectrum exists, the system tries to reach equipartition of energy by taking energy from the big guys and giving it to the small ones. This gives a mass-dependent velocity dispersion

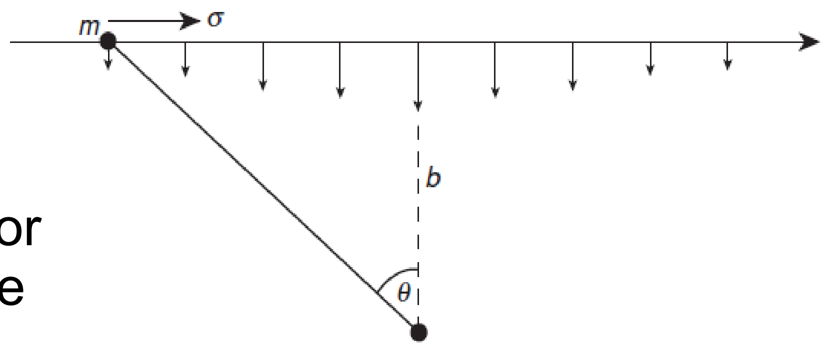
- we will ingore:

  - Aerodynamic drag:
  exerted by the residual gas disc. This is very weak (for large bodies) but still provides some damping of e and i.

  - Collisional damping: inellastic collisions (also shattering) between growing proto-planets also dissipates energy and damps e and i.

Viscous stirring

Assume a two-body encounter at impact parameter b and velocity (at ∞) σ



Impulse approx: the maximum deflection is for $d \sim b$, where the particle feels a velocity impulse

$\delta v = F\ \delta t / m = Gm/b^2$ for a time interval $\delta t = 2b/\sigma$ ⇒ $\delta v = 2Gm/(b\sigma)$

and $$\delta E = 2G^2 \frac{m^3}{b^2} \sigma^2$$

The rate of encounters with impact parameters in (b, b+db):

$\Gamma = 2\pi b\ db\ n_{sw}\ \sigma$ where n_{sw} the number density of planetesimals

Summing up for all possible encounters, we get:

$$\frac{dE}{dt} = \frac{4\pi G^2 m^3 n_{sw}}{\sigma} \int_{b_{min}}^{b_{max}} \frac{db}{b} = \frac{4\pi G^2 m^3 n_{sw}}{\sigma} \ln \Lambda$$

Coulomb logarithm
(depends on system's size)

For a planetesimal disc, we have:

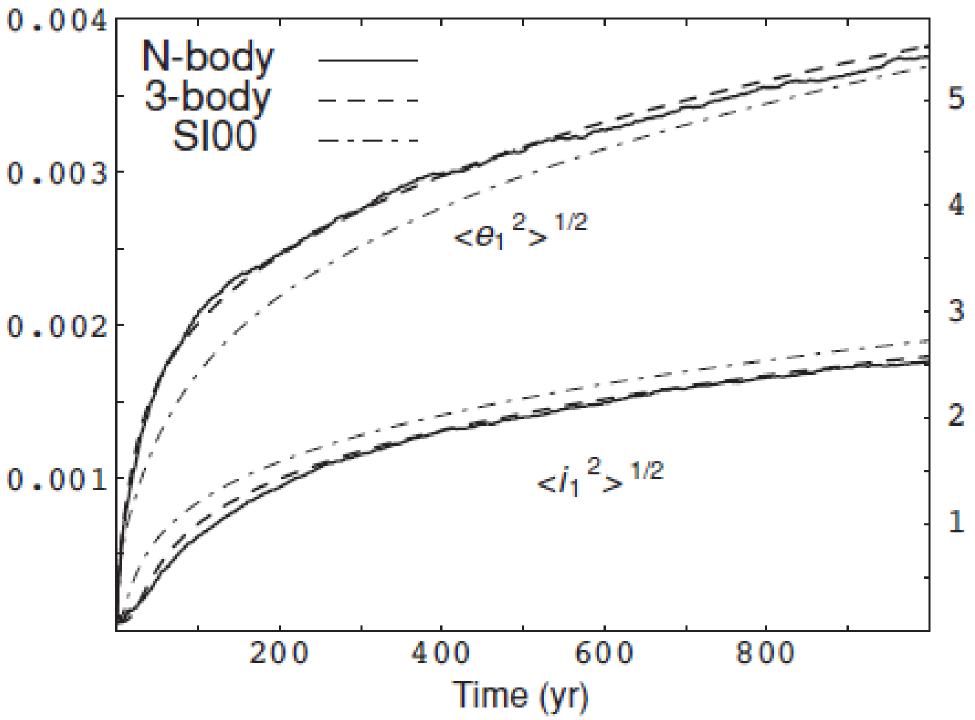
$$\frac{d\sigma}{dt} = \frac{2\pi G^2 m \Sigma_{sw} \Omega \ln \Lambda}{\sigma^3}$$

and 'integrating' we get:

Problem: how ???

$$\sigma(t) \propto t^{1/4}$$

For the TP region, the *dynamical heating* time-scale is ~ 10,000 yr only!



numerical simulations (single-species disc) confirm this estimate

NOTE: the equlibirum distribution has

$$e \simeq 2i$$

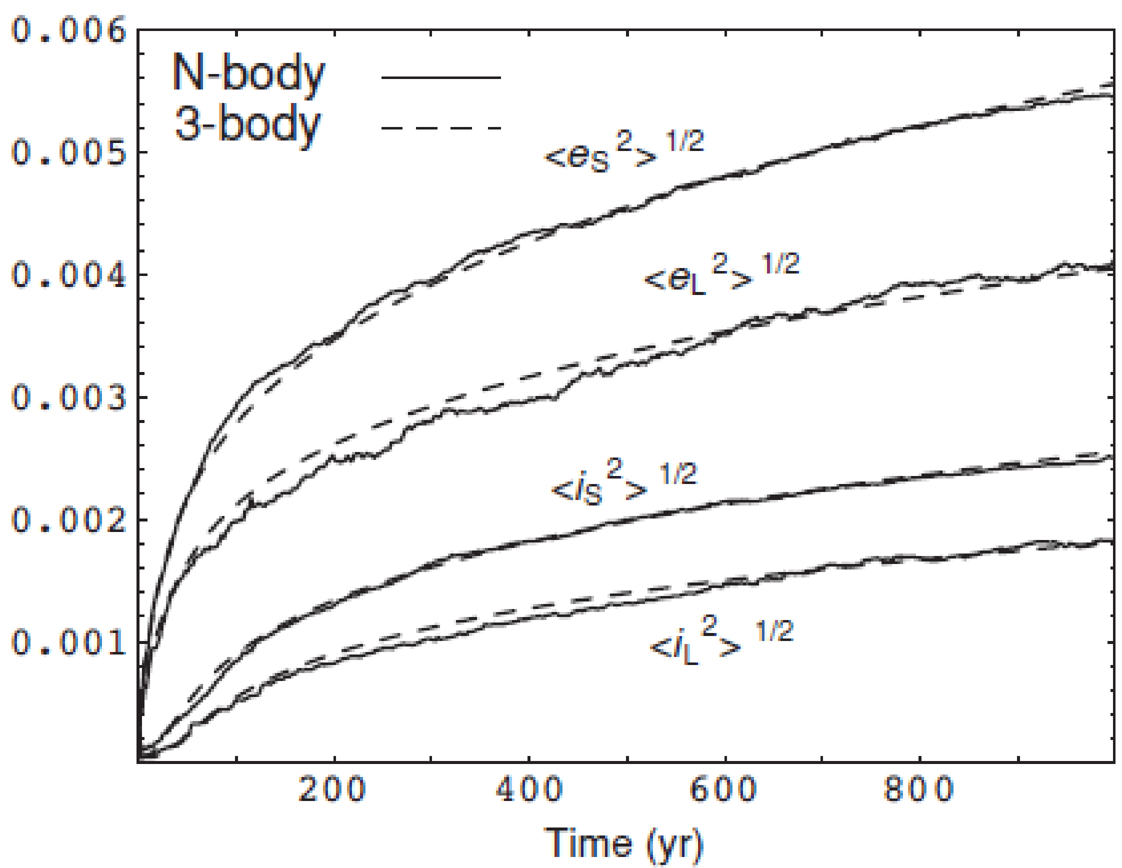
(Rayleigh distr.)

Dynamical friction

Basically the same as before. However, if two different *species* are considered, 'large' (M) and 'small' (m), the system tries to reach equipartition in energy

$$\frac{1}{2}m\sigma_{\rm m}^2 = \frac{1}{2}M\sigma_{\rm M}^2$$

as small guys receive bigger velocity kicks



also confirmed numerically...

Lower random velocities for the big guys means lower *relative velocities* between the planets and the planetesimals than between planetesimals

→ more gravitational focusing

→ enhanced runnaway growth!

Isolation mass

- This is the limiting mass that the planet can reach by runnaway growth, as the planet depletes its feeding zone (and increases σ)

  → it becomes *isolated* from the planetesimal disc

- This mass is given by: $M_{\rm iso} = \frac{8}{\sqrt{3}}\pi^{3/2}C^{3/2}M_*^{-1/2}\Sigma_{\rm p}^{3/2}a^3$

ıif we assume that the feeding zone is $\Delta a \sim C\, r_H$, since only particles near the Hill radius can be deflected to encounter the planet. The isolation mass is then reached when the planet's mass becomes equal to the mass of planetesimals in the original disc

Problem: derive that!

- For the TP region this gives ~0.1 Earth masses, while for the Jupiter region it gives ~10 Earth masses

... this is close to the estimated mass of Jupiter's solid core...

Final formation stages of TPs

- Formation of TPs is a 3-step procedure:

 - *Runnaway growth:* no large bodies initially, random velocities set by viscous stirring among planetesimals (and damping). Strong gravitational focusing (initially cold disc) *and dynamical friction* results to *runnaway* growth of a small fraction of bodies

 - *Oligarchic growth:* runnaway stops when a few big guys grow so much that they stir-up planetesimals more severely than planetesimals do and have largely depleted their feeding zones. This limits the gravitational cross-section but these *oligarchs* continue to grow (more slowly)

 * These steps are very rapid. For our solar system, ~1,000 *planetary embryos* with masses ~ 0.01-0.1 form within 0.1-1 My

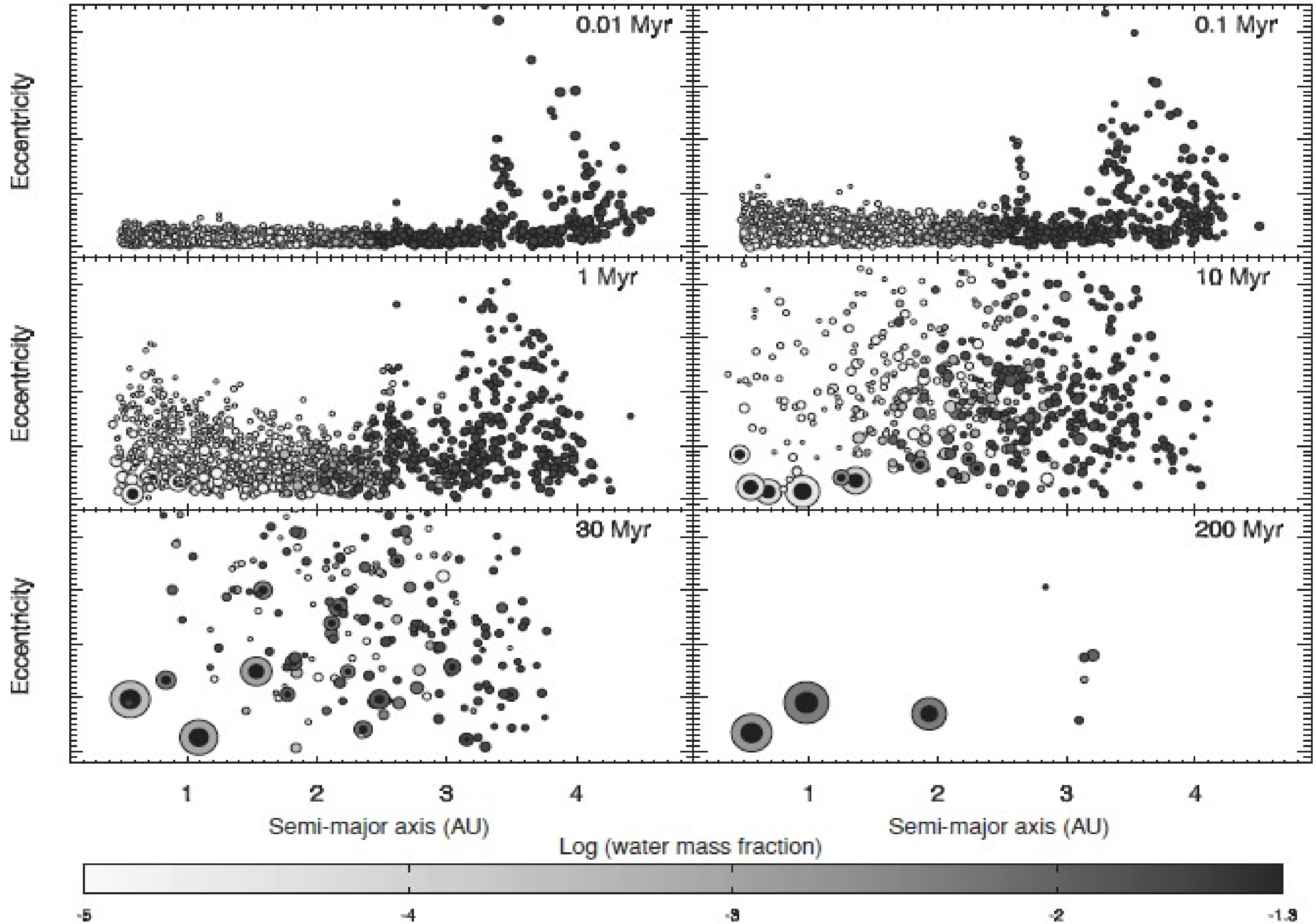
 - *Final assembly stage:* the embryos have depleted the disc significantly and dynamical friction can no longer keep their random velocities small. They start interacting violently, *colliding with each other* and scattering away small guys.

- ADD: if this phase starts after a ~My, then Jupiter has laready formed
  more violent evolution

- This final stage can take as long as 100 My to complete!

- One can assume different compositions in order to estimate e.g. the concentration of water on the final planets

- Simulations ***in general*** agree with the number, masses and orbits of the TPs



Concluding remarks on TP formation

- Mass and random velocity distribution controls the process of formation in the first two stages, i.e. until *planetary embryos* form

- Key factors: *gravitational focusing*, controled by the *velocity dispersion* (i.e viscous stirring and dynamical friction)

- Runnaway growth: larger bodies form faster than small ones. They can reach a limiting *isolation mass*, by depleting their *feeding* zones.

- Oligarchic growth sets in when *big-enough* guys dominate the stir-up of planetesimals. Accretion slows down

- *Planetary embryos* become *unstable* when dynamical friction can no longer keep their relative velocities small. They start hitting each other and (slowly) reach TP masses...

* *Collision efficiency (accretion vs. disruption) should be taken into account at every step along with gas drag (first steps) ...*

Formation of Gas giants

- *The main problem:* form Jupiter-sized planets before the gas disc disperses (not easy!!!)

- First, their *cores* must form within <1 My (... assuming we can beat the planetesimal formation problem ... and the embryos migration problem...)

- Then, the gas has to collapse on the core quickly enough...

... these are the problems in the core-accretion model ...

- These can be by-passed if the gas-analogue to the Goldreich-Ward mechanism (gravitational instability and collapse) can occur

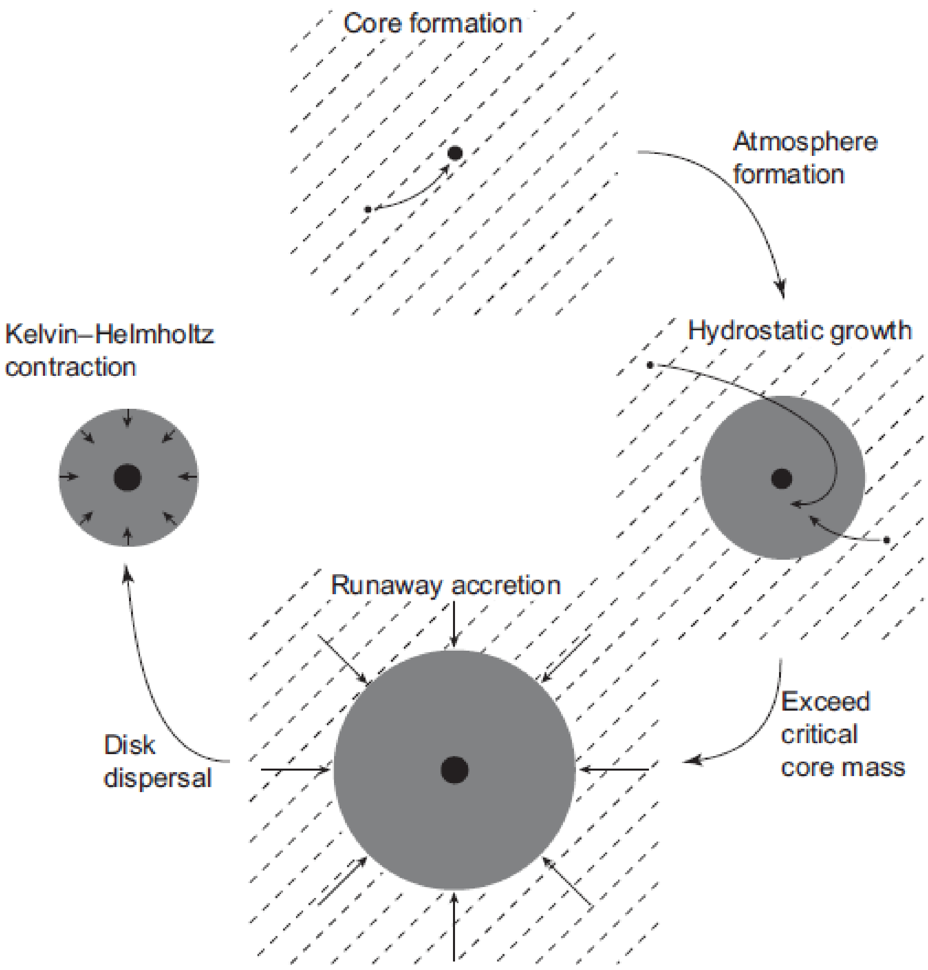
... here, the gas needs to *cool very fast* in order for the disc to *fragment*

there are good aspects and problems in both models...

- For low-mass (MMSN) discs core-accretion seams more plausible

Core-accretion model

- GP formation is a *4-stages* process:



- core formation:
a solid core becomes massive enough to retain an atmosphere

- hydrostatic growth:
the system is initially in hyrdostatic equilibrium. Energy exchange results in slow growth of both core and envelope up to a critical mass

- runnaway accretion:
the system is massive enough to accrete (very fast) all available gas around

- end phase:
no more gas is available around and the planet cools...

Some estimates:

- To retain an atmosphere, the gas sound speed has to be smaller than the escape velocity. This gives

$$M_{\rm p} > \left(\frac{3}{32\pi}\right)^{1/2} \left(\frac{h}{r}\right)^{3} \frac{M_*^{3/2}}{\rho_{\rm m}^{1/2} a^{3/2}}$$

Problem: how?

... but this is a tiny number ...

- Let's do better... start with a core-envelope system in hydrostatic equilibrium, where we assume the envelope mass, $M_{\rm env}$, to be a small fraction, ε, of the total mass, M_P:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM_{\rm p}}{r^2}\rho$$

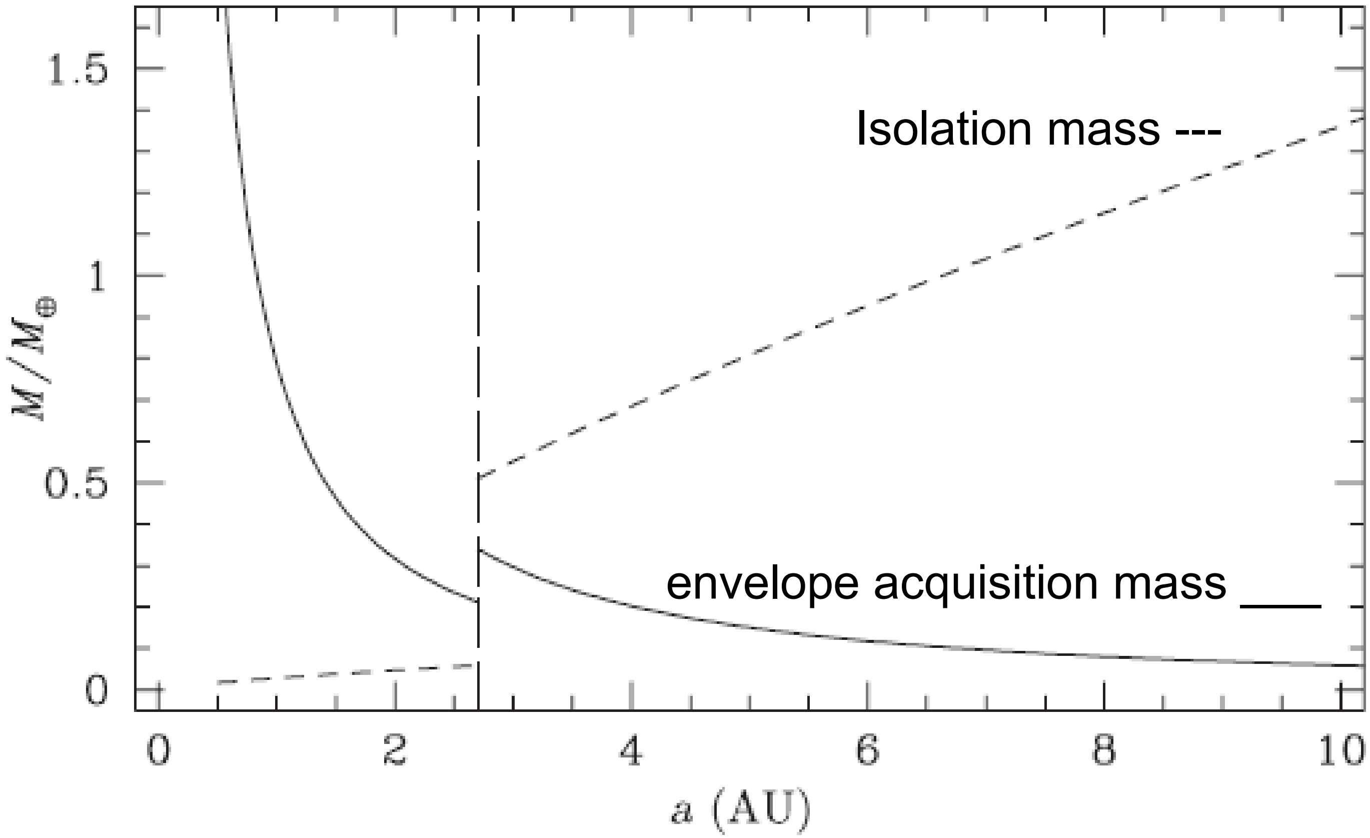
Now, assume a vertically isothermal disc of ideal gas, integrate and set as ρ_0 to be the density at the distance r where sound speed = escape velocity. Finally, assume that most of the envelope mass is close to the surface of the core (is that OK?). Then, $M_{\rm env} = \varepsilon M_P$ gives:

$$M_{\rm p} \gtrsim \left(\frac{3}{4\pi\rho_{\rm m}}\right)^{1/2} \left(\frac{c_{\rm s}^2}{G}\right)^{3/2} \left[\ln\left(\frac{\epsilon\rho_{\rm m}}{\rho_0}\right)\right]^{3/2}$$

Problem: do it!

- These mass estimates give ~ 0.2 Earth masses for the Jupiter region (in a MMSN disc) and ~1 Earth mass in the TP region!

- If we calculate the *isolation mass* of a solid core (in a MMSN) and the planet mass that can attain a significant envelope, we get:



➡ TPs could not have acquired gas envelopes, in contrast to the GPs in the solar system!

Evolution of the core-envelope structure

- We need the full set of evolution equations:

$$\left.\begin{aligned}\frac{\mathrm{d}M}{\mathrm{d}r} &= 4\pi r^2 \rho \\ \frac{\mathrm{d}P}{\mathrm{d}r} &= -\frac{GM}{r^2}\rho\end{aligned}\right\} \text{ mass and momentum conservation}$$

$$P = \frac{k_B}{\mu m_p}\rho T$$

equation of state (ideal gas)

We also need to specify how temperature evolves, i.e. how the envelope cools. If we assume to be trhough radiation (ignore convection):

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3\kappa_R \rho}{16\sigma T^3}\frac{L}{4\pi r^2}$$

and that the luminosity is due to the energy of planetesimals falling on the core:

$$L \simeq \frac{GM_{\mathrm{core}}\dot{M}_{\mathrm{core}}}{R_s} \propto M_{\mathrm{core}}^{2/3}\dot{M}_{\mathrm{core}}$$

We try to calculate how much the mass of the core can grow, while keeping the envelope in hyrdostatic equilibrium!

$$\left.\begin{aligned} \frac{\mathrm{d}P}{\mathrm{d}r} &= -\frac{\mathrm{G}M}{r^2}\rho \\ \frac{\mathrm{d}T}{\mathrm{d}r} &= -\frac{3\kappa_\mathrm{R}\rho}{16\sigma T^3}\frac{L}{4\pi r^2} \end{aligned}\right\} \xrightarrow{\text{eliminate } \rho} \frac{\mathrm{d}T}{\mathrm{d}P} = \frac{3\kappa_\mathrm{R} L}{64\pi\sigma \mathrm{G} M T^3} \xrightarrow[M(r)=M_P)]{\text{integrate } (L=const,}$$

$$T^4 \simeq \frac{3}{16\pi}\frac{\kappa_\mathrm{R} L}{\sigma \mathrm{G} M_\mathrm{p}} P$$

Now, use the e.o.s and the eq. for dT/dr, to get

$$T \simeq \left(\frac{\mu m_\mathrm{p}}{\mathrm{k_B}}\right)\frac{\mathrm{G}M_\mathrm{p}}{4r} \quad \text{and} \quad \rho \simeq \frac{64\pi\sigma}{3\kappa_\mathrm{R} L}\left(\frac{\mu m_\mathrm{p} \mathrm{G} M_\mathrm{p}}{4\mathrm{k_B}}\right)^4 \frac{1}{r^3}$$

and find the envelope mass:

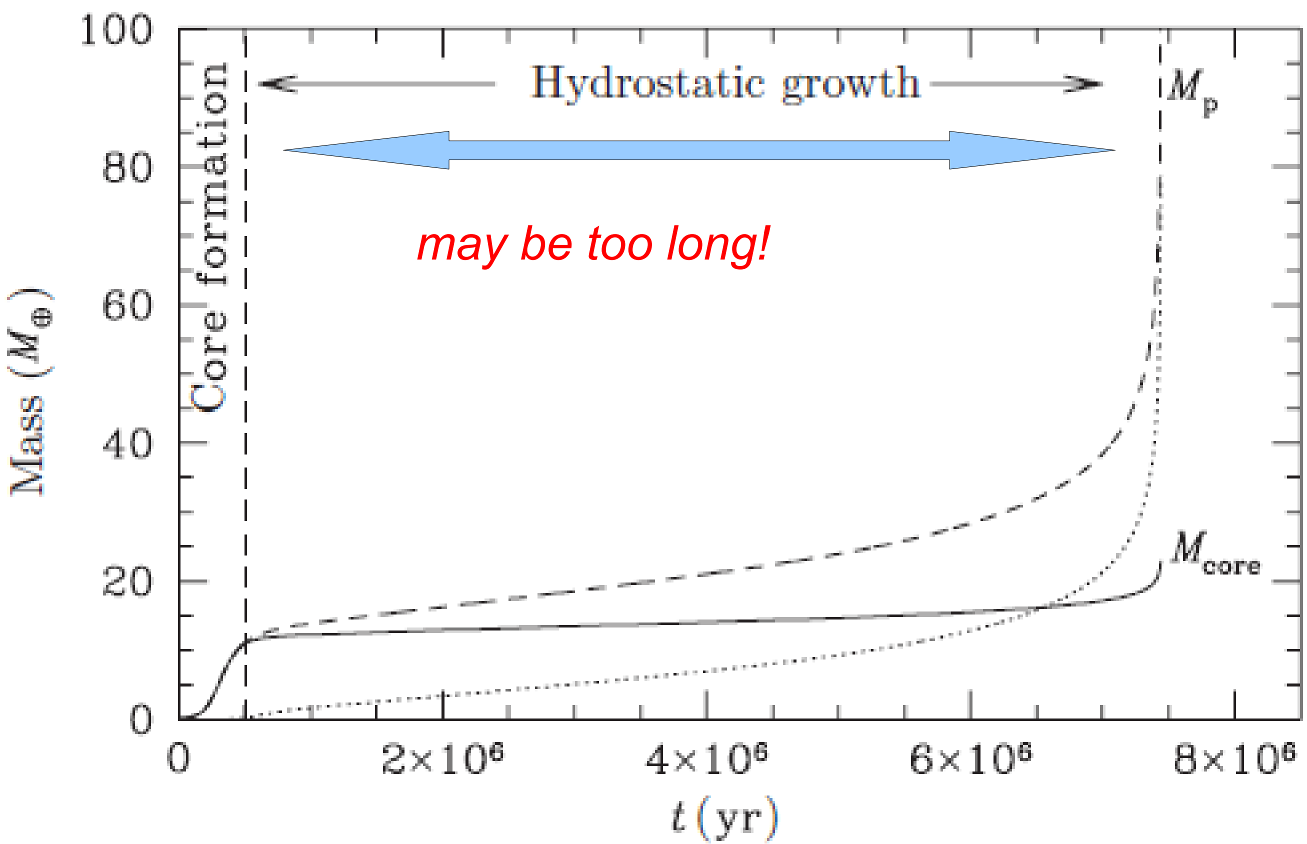
$$M_\mathrm{env} = \int_{R_\mathrm{s}}^{r_\mathrm{out}} 4\pi r^2 \rho(r)\mathrm{d}r = \frac{256\pi^2\sigma}{3\kappa_\mathrm{R} L}\left(\frac{\mu m_\mathrm{p} \mathrm{G} M_\mathrm{p}}{4\mathrm{k_B}}\right)^4 \ln\left(\frac{r_\mathrm{out}}{R_\mathrm{s}}\right)$$

taking into account that $M_\mathrm{core} = M_\mathrm{p} - M_\mathrm{env}$ and using the expression for the luminosity, we get ....

$$M_{\rm core} = M_{\rm p} - \left(\frac{C}{\kappa_{\rm R}\dot{M}_{\rm core}}\right)\frac{M_{\rm p}^4}{M_{\rm core}^{2/3}}$$

This equation has *no solution*, beyond a *critical core mass* (~10-20 Earth masses at *r*~5 AU in a MMSN)

→ ***hydrostatic equilibrium cannot be maintained!***



Collapse!

- Gas accretion is no longer demand-limited but *supply-limited*!
- The planet grows very fast!
- growth stops when all gas in the neighborhood is accreted (or the disc disperses)

The gravitational-instability model

- Similar reasoning to the planetesimal-formation Goldreich-Ward mechanism, i.e the local density is too high for the disc to be stable against its own self-gravity:

$$Q \equiv \frac{c_s \Omega}{\pi G \Sigma} < Q_{\rm crit}$$

local instability criterion global

$$\frac{M_{\rm disk}}{M_*} \gtrsim \frac{h}{r}$$

- This requires massive discs (~0.1 M_*) that may be present in very early phases of star formation. For the Sun, it gives Σ ~10 x MMSN !!!

- If indeed the disc fragments, the wave-length of the most unstable mode

$\lambda \sim 2c_s^2/(G\Sigma)$ suggests formation of planets $M_p \sim \pi\lambda^2\Sigma \approx 8\ M_J$

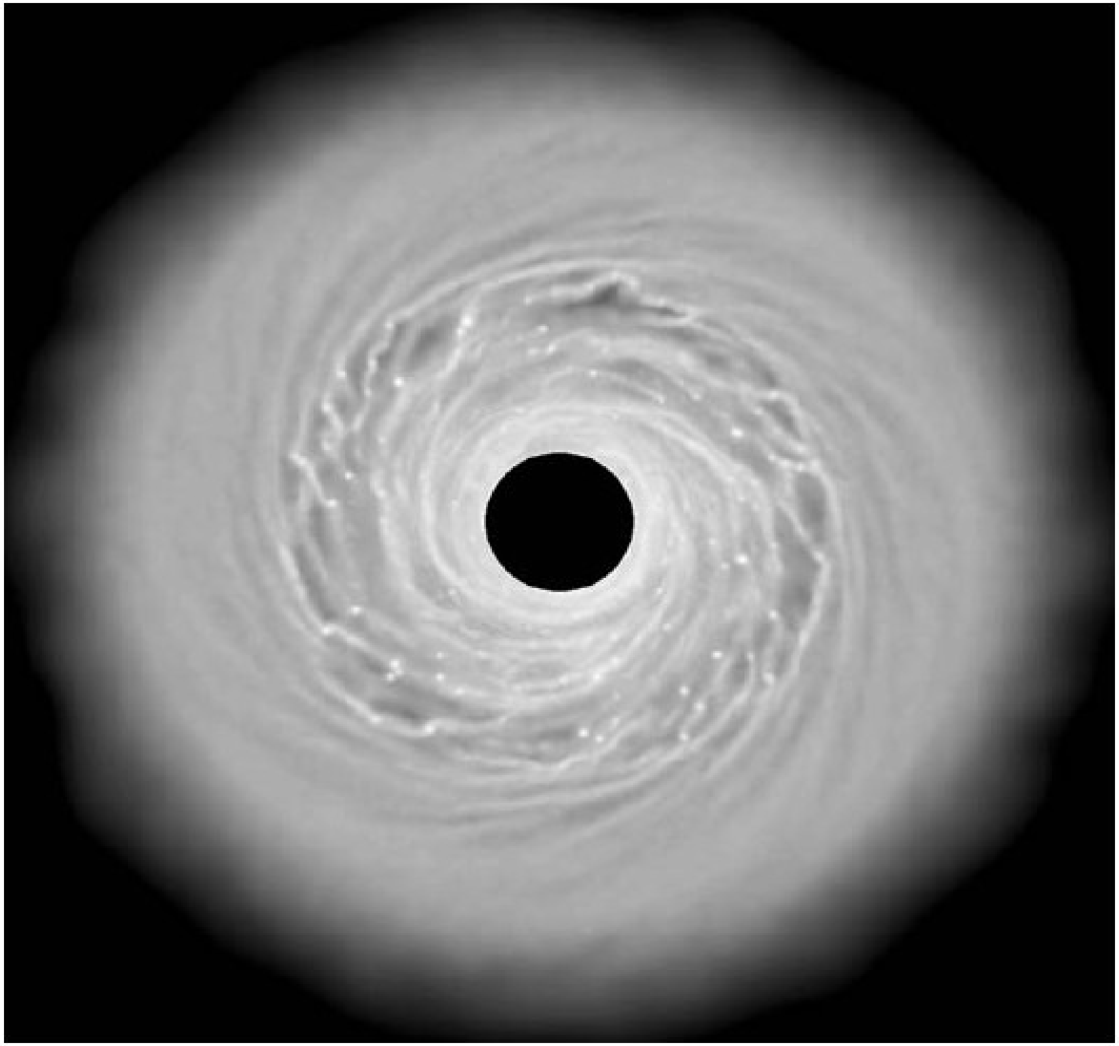
- This is probably *too much* even for extra-solar planets (or *brown dwarfs*)

- More refined calculations are needed to see if indeed the disc can fragment easily and what range of masses it can produce

- Unstable modes can be non-axisymmetric,leading to the formation of spiral waves, angular momentum transport and increase of accretion energy

- this energy can heat-up the disc and kill the instability

→ *no fragmentation!*



Fragmentation occurs for *short-enough* cooling time-scales

These are ~ $1/\Omega$ ~ *orbital period!*

Typical disc models suggest that this cannot happen at least at small orbital radii (~30 AU)

Finer estimates suggest that discs with

$$\Sigma_{\rm crit} \simeq 5 \times 10^3 \left(\frac{r}{5\ {\rm AU}}\right)^{-2}$$

can do it (~$0.5 M_*$ within 30 AU)

Concluding remarks on GP formation

- Two basic models: *core-formation* and *gravitational instability*

- The core-formation model is probably more realistic for *less massive*, passive discs (like the MMSN)

- It can explain why we don't have *gaseous TPs*

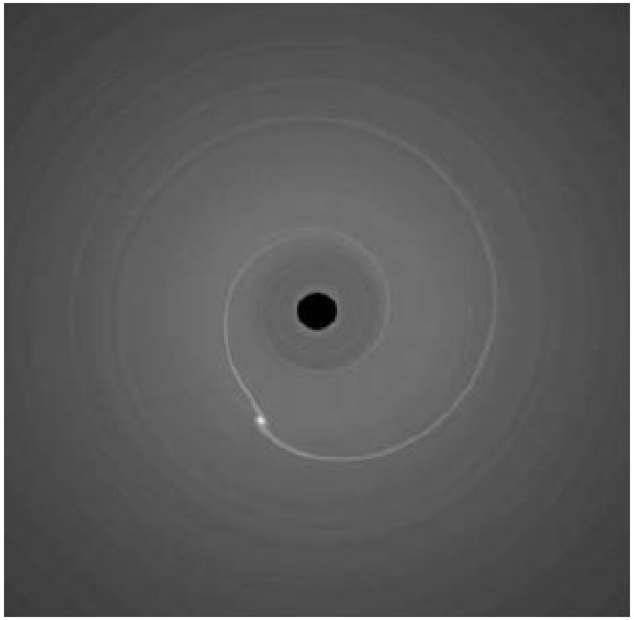
- There are *time-scale issues*: the core needs to form fast (~1 My) and the hydrostatic growth phase cannot take much longer...

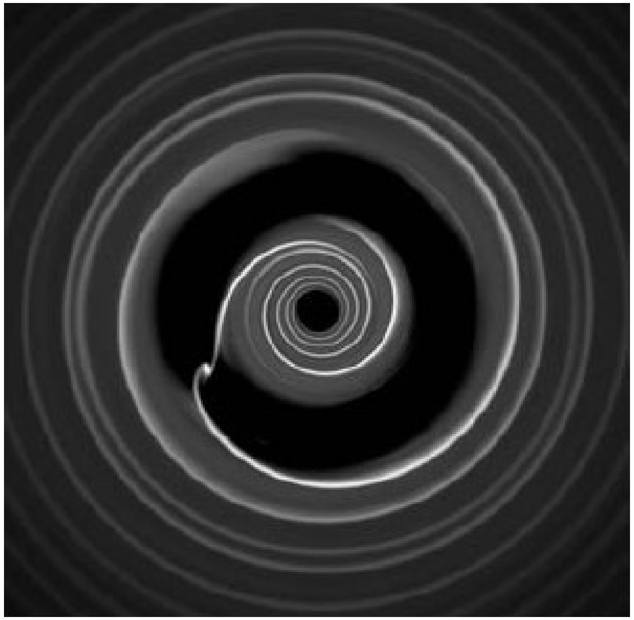
In any case, everything has to be done within a few My!

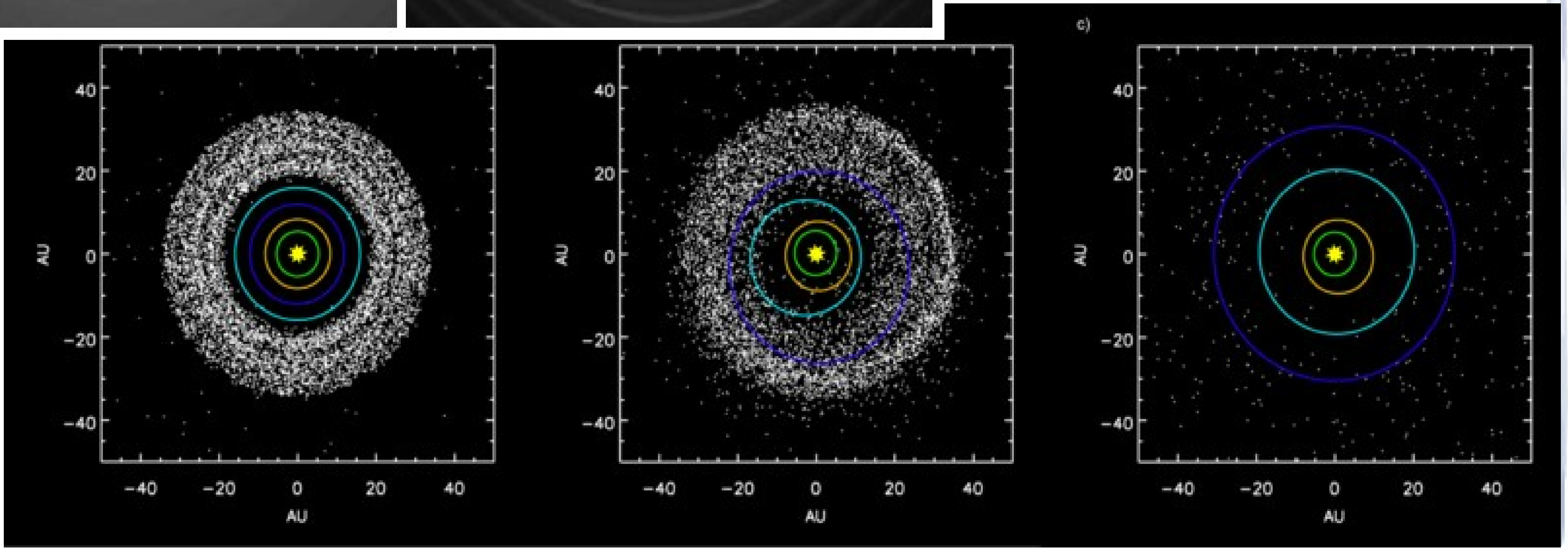
- For *multiple-planet systems*, we need to understand the balance between core-growth and gas-accretion and their competetion

- We did not discuss the possibility of *core mixing* (and erosion), *convection-dominated* envelopes and how these lead to different possible results in terms of internal structure

Dynamics of young planetary systems







- Assume 'planets' have formed. The main interactions to consider are:

  - planet-disc interactions:
  exchange of angular momentum with the remnant disc (gas, or solid planetesimals) leads to radial migration of the planets

  - planet-planet interactions:
  distant interactions are small, quasi-periodic perturbations that become important only when a resonance is established (either in mean motion or in secular precession)

- Combined: we can have capture into a stable resonant configuration, or resonance-crossing that can de-stabilize a system

- In an unstable system, planet loss (by gravitational scattering) can occur!

- Instability may also be suppressed by dynamical friction exerted by the debris disc

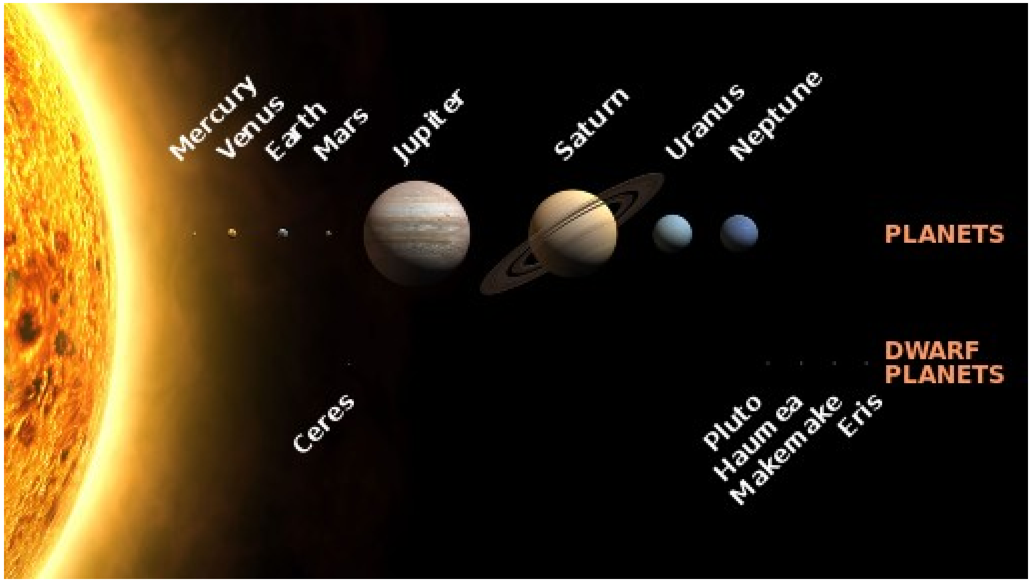
→ a 'new' stable configuration is reached

The Solar System

Contains:

- Sun (M_*=1.989x10^{33} g)
  - 73% H, 25% He, 2% other (Z=0.02)
- Planets
  - mass = 0.13%
  - angular momentum J_J/J_*~100
- Dwarf planets (Pluto, Eris,...), minor planets (asteroids), comets, etc. (total mass ~0.1 M_E)

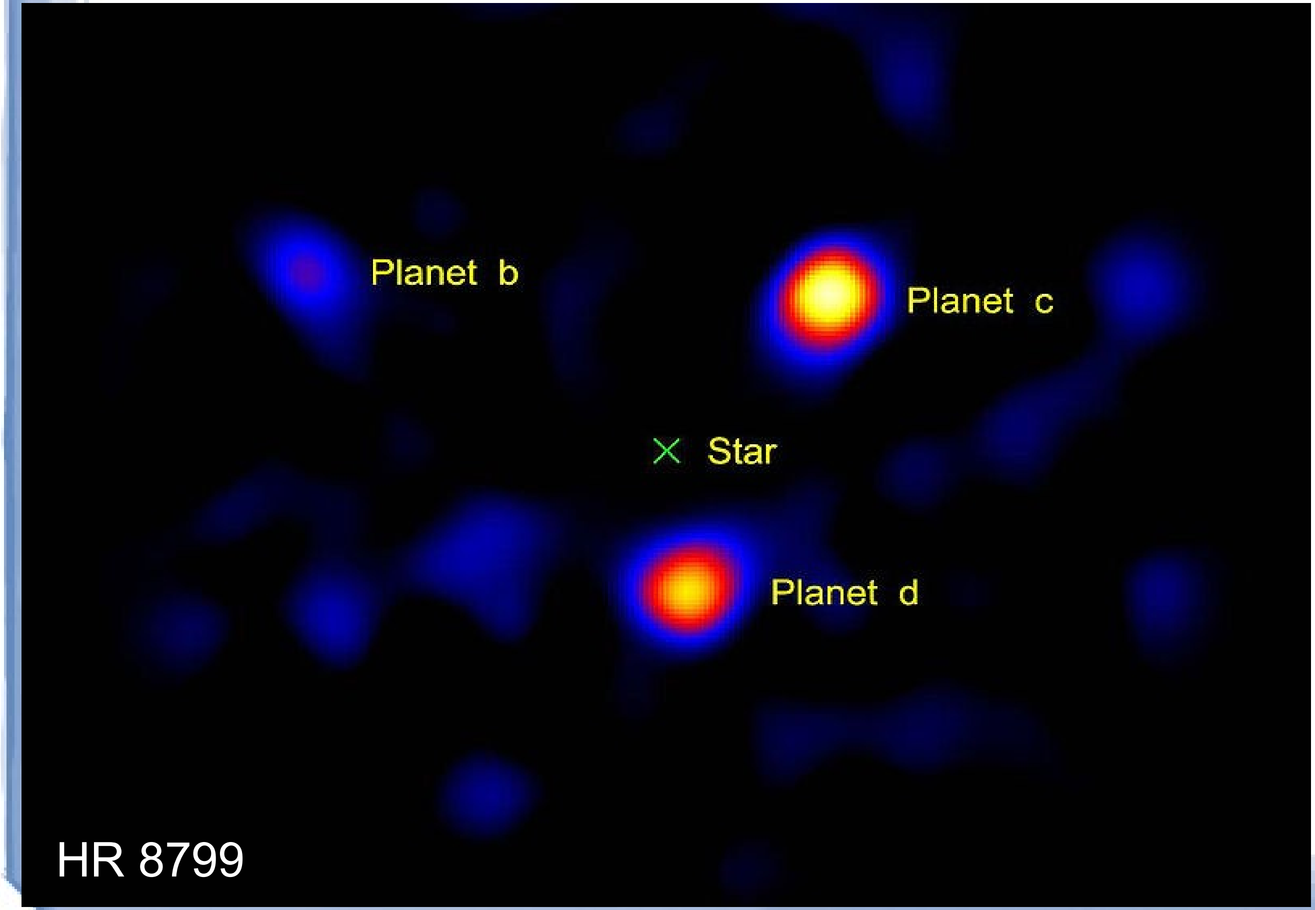
	a(AU)	e	i(deg)	M_p(g)	R_p(cm)
Mercury	0.3871	0.2056	7.00	3.302×10^{26}	2.440×10^{8}
Venus	0.7233	0.0068	3.39	4.869×10^{27}	6.052×10^{8}
Earth	1.000	0.0167	0.00	5.974×10^{27}	6.378×10^{8}
Mars	1.524	0.0934	1.85	6.419×10^{26}	3.396×10^{8}
Jupiter	5.203	0.0484	1.30	1.899×10^{30}	7.149×10^{9}
Saturn	9.537	0.0539	2.49	5.685×10^{29}	6.027×10^{9}
Uranus	19.19	0.0473	0.77	8.681×10^{28}	2.556×10^{9}
Neptune	30.07	0.0086	1.77	1.024×10^{29}	2.476×10^{9}



Extrasolar planets

First detection: (Wolcszczan & Frail 1992) planets around pulsar PSR1257+12
First around a solar-type star: (Mayor & Queloz, 1995) 51-Peg

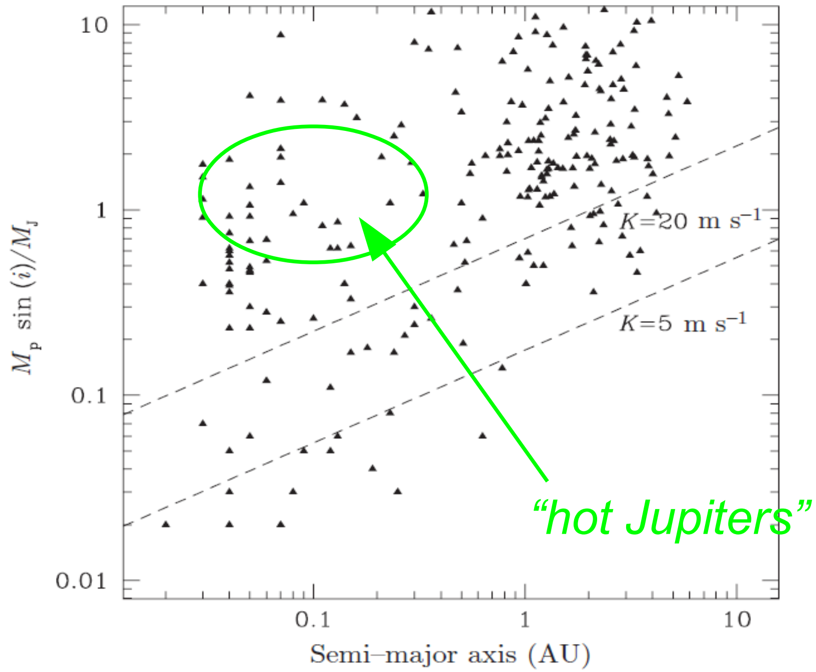
Now (July 2012): **777** planets, **623** systems (**105** multiple)

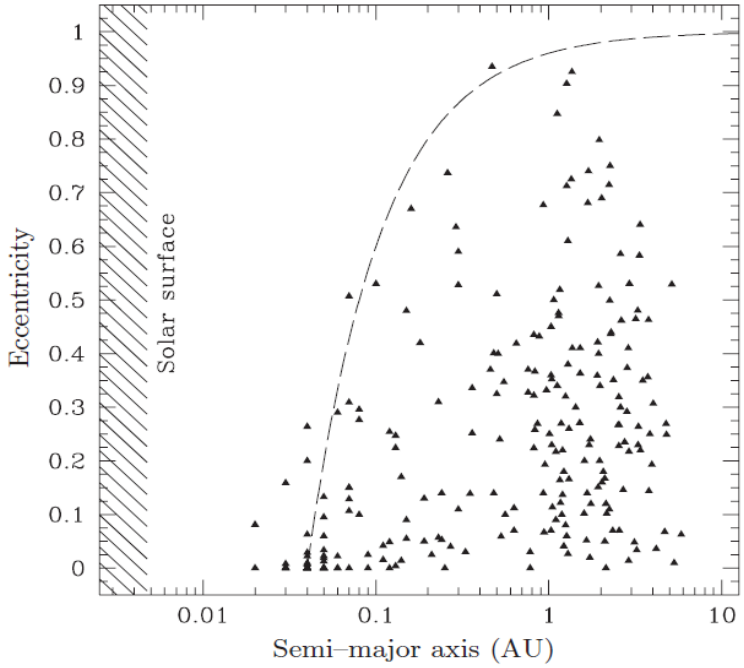


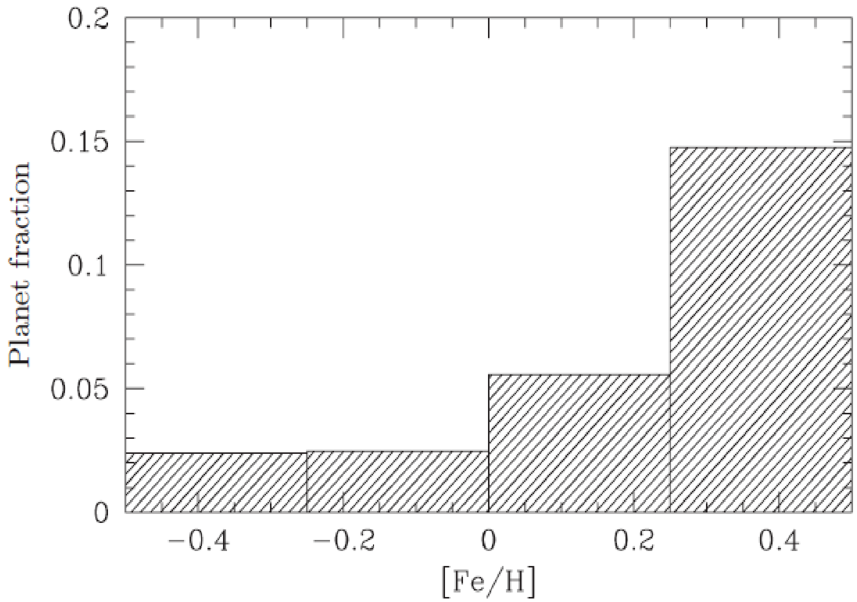
Observational techniques:

- direct imaging
- radial velocity
- astrometry
- transits
- microlensing

Statistics of planetary systems







A huge variety of systems !

- Massive planets, close to the star (obs. biases)
- Very eccentric orbits (dynamics)
- Planet frequency increases with Z

Gas-driven migration

- Exchange of angular momentum between the planet and gas parcells. Use the *impulse approximation*, now looking into the *radial motion*:

$$|\delta v_\perp| = \frac{2GM_\mathrm{p}}{b\Delta v} \quad \Rightarrow \quad \delta v_\parallel \simeq \frac{1}{2\Delta v}\left(\frac{2GM_\mathrm{p}}{b\Delta v}\right)^2$$

$$\Delta v^2 = |\delta v_\perp|^2 + (\Delta v - \delta v_\parallel)^2$$

and a change in specific ang.mom.

$$\boxed{\Delta j = \frac{2G^2 M_\mathrm{p}^2 a}{b^2 \Delta v^3}}$$

- Sign of angular momentum change:
  - Gas *exterior* to the planet's orbit *moves slower* and is 'overtaken' by the planet, its angular mometum is *increased* (moves outwards) and is *repelled away* from the planet
  - Gas *interior* to the planet's orbit *moves faster* and 'overtakes' the planet, its angular mometum is *decreased* (moves inwards) and is *again repelled away* from the planet

$\Rightarrow$ *External disc* tries to push the planet *inwards* (*decreasing* J_p)
Internal disc tries to push the planet *outwards* (*increasing* J_p)

- The end result depends on the disc characteristics (net torque)

- Problem: take a ring of material of width db, compute its mass, dm, and the time interval, Δt, needed for all the ring material to encounter the planet. Then, sum-up to find the total torque exerted on the planet by the disc:

$$\frac{\mathrm{d}J}{\mathrm{d}t} = -\frac{8}{27}\frac{\mathrm{G}^2 M_\mathrm{p}^2 a \Sigma}{\Omega_\mathrm{p}^2 b_\mathrm{min}^3}$$

- Applied to an Earth-mass core at ~5 AU this gives inwards migration at a characteristic *decay time-scale* of ~1 My !

$\Rightarrow$ another problem for core formation!!!

- For larger masses it's even worse, since da/dt ~ M_P

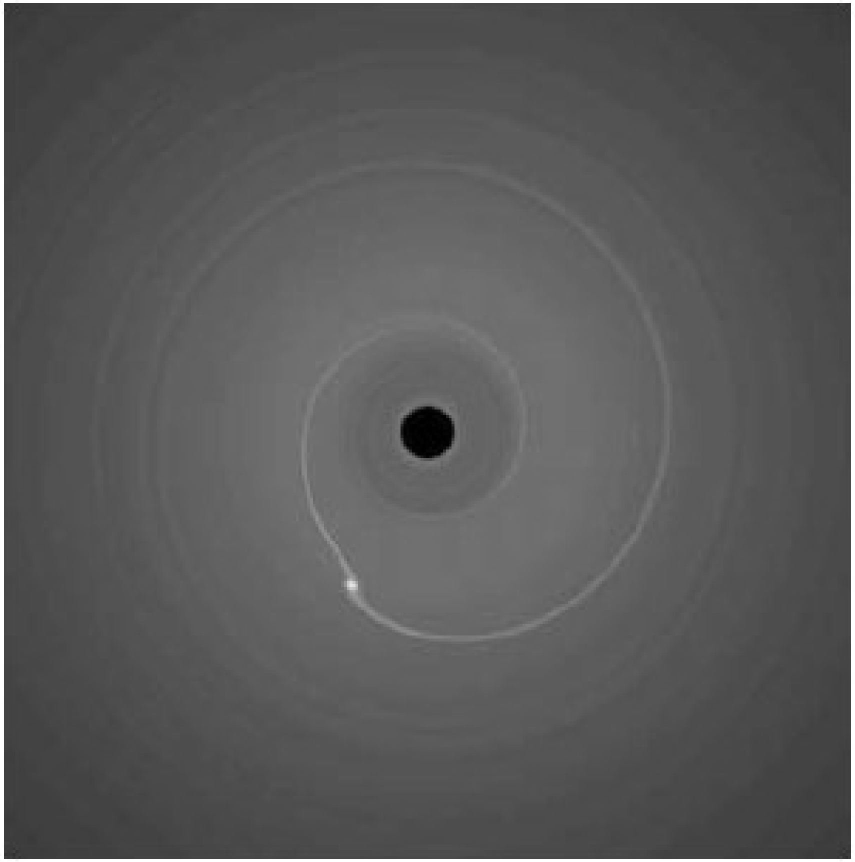
- Finer calculations (using linearization) show that the change in angular momentumis just the sum of the torque felt by gas elements in *orbital resonance* with the planet:

$$m[\Omega(r) - \Omega_\mathrm{p}] = \pm\kappa(r) \Rightarrow r_\mathrm{L} = \left(1 \pm \frac{1}{m}\right)^{2/3} a$$

since, at these locations, *standing waves* are excited, while at all other r's the quasi-periodic nature of the pertutbation gives phase-mixing

- There are *two types* of migration, depending on the mass of the planet and the characteristics of the disc:

- *Type I migration:*
 a low-mass planet, weak interaction (linear regime) resulting in a nearly unperturbed disc structure.



This implies that gas is *always* present in resonances

The *viscous* redistribution of angular momentum overcomes the gravitational torque by the planet

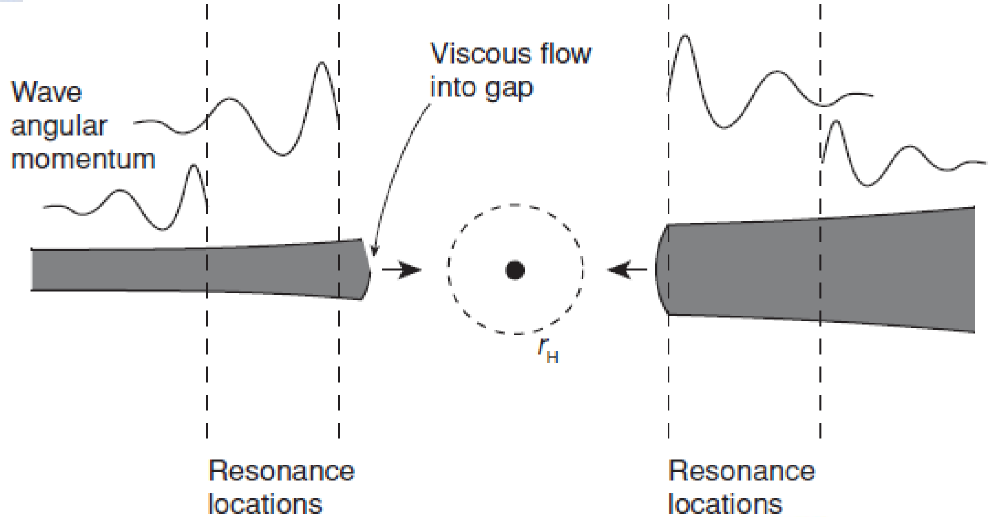
The planet remains fully *embedded* in the gas and moves (inwards) on the previously defined scale

For a $\Sigma(r) \propto r^{-\alpha}$ profile:

$$\Gamma_{\rm total} = -(1.36 + 0.54\alpha)\left(\frac{M_{\rm p}}{M_*}\right)^2 \left(\frac{h}{r}\right)^{-2} \Sigma a^4 \Omega_{\rm K}^2$$

- *Type II migration:*
 a massive planet, strongly interacting with the gas, repels gas very efficiently and *opens a gap* in the disc around its orbit!

- Most resonances (which accumulate near the planet) are *severely depleted* and so the torque that the planet feels from the disc drops!



The smallest possible gap has size *h* (scale hight). The gas tries to fill this gap on the viscous time-scale

$$t_{\rm close} = \frac{h^2}{\nu}$$

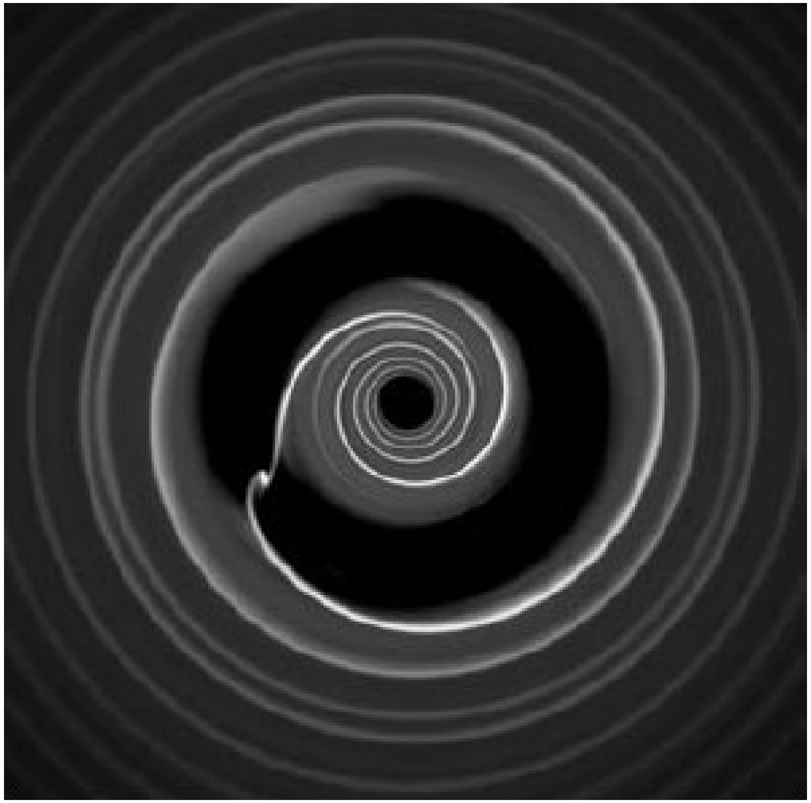
while the planet tries to empty it on a time-scale

$$t_{\rm open} = \frac{\Delta J}{|{\rm d}J/{\rm d}t|}$$

where $\Delta J = 2\pi a h \Sigma \cdot \left.\frac{{\rm d}l}{{\rm d}r}\right|_a \cdot h$ is the total angular momentum content

The minimum mass ratio $q \equiv M_{\rm p}/M_*$ needed for gap-opening is:

$$q_{\rm crit} \simeq \left(\frac{27\pi}{8}\right)^{1/2} \left(\frac{h}{r}\right)^{5/2} \alpha^{1/2}$$



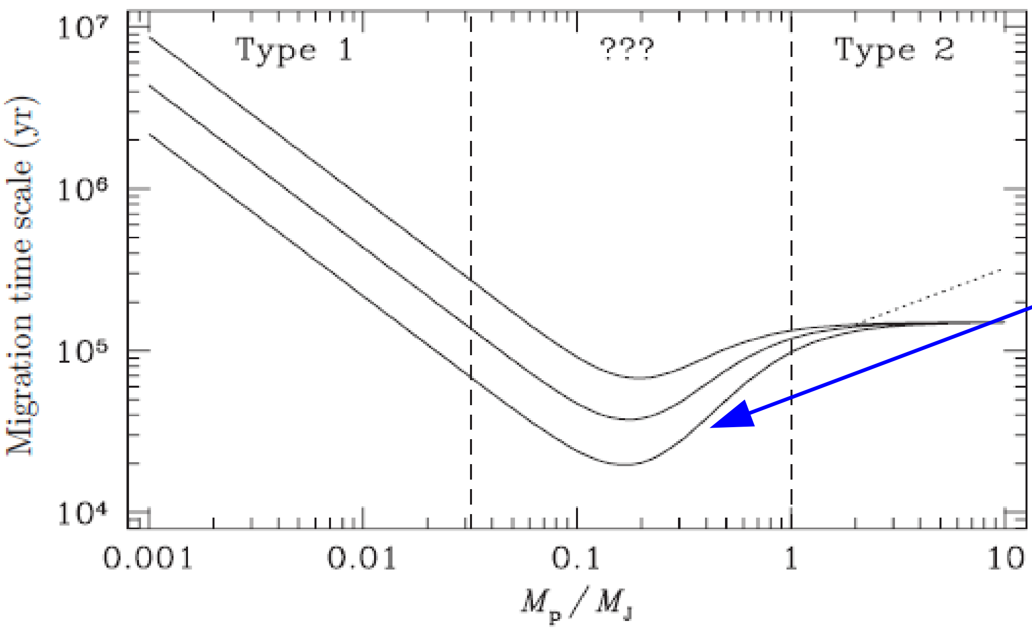
For a 'normal' MMSN disc, we get:

q~2 x 10^{-4} , i.e. Saturn's size

For larger masses, *the planet opens a clear gap* which cannot be replenished by viscous diffusion.

The planet-disc system is 'locked' in this configuration and the planet *has to follow the evolution of the disc*, i.e it moves inwards on the *viscous* time-scale:

$$v_{\rm nominal} = -\frac{3\nu}{2r} = -\frac{3}{2}\alpha\left(\frac{h}{r}\right)^2 v_{\rm K}$$



The intermediate parameters region is essentially accessible only by numerical experiments

Type II migration is considered as the most viable explanation for *hot Jupiters*

Planetesimal-driven migration

- Apart from the gas, the planets interact with the *remnant disc of planetesimals.*

- After the dissipation of the gas, the *debris disc* becomes very important. This dics will be primarily localted *outside* the orbit of the last giant planet

- In the solar system, we estimate the total mass beyond Neptune to have been

$$M_{\rm disk} = 4\pi\,\Sigma_0\left(r_{\rm out}^{1/2} - r_{\rm in}^{1/2}\right) \simeq 40\ M_\oplus$$

which is *~100-1,000 times greater* than nowdays KB. Moreover, we need this large mass to form objects the size of Pluto

- This mass must have been there and somehow *got eliminated by dynamical interaction* with the planets

⇨ the planets must have *migrated outwards* (on average)

⇨ their *initial orbits were closer* to the Sun than now (also needed because of large formation time-scale...)

- Assume one planet (M_P, a) moving *interior* to a disc (Σ_P, m) and scatters particles such that they end-up to *lower a* (i.e removes ang.mom.). The zone that the planet can significantly perturb has ~ $\Delta r \approx \left(\frac{M_\mathrm{p}}{3M_*}\right)^{1/3} a$ and contains mass ~ $\Delta m = 2\pi a \Sigma_\mathrm{p} \Delta r$. Each orbit has *specific angular momentum* $l = \sqrt{\mathrm{G}M_* r}$

- If all particles are scattered inwards, the total loss of ang.mom is:

$$\Delta J \approx \Delta m \left.\frac{\mathrm{d}l}{\mathrm{d}r}\right|_a \Delta r.$$

- This is *gained by the planet* who moves by: $\Delta a \approx \frac{2\pi a \Sigma_\mathrm{p} \Delta r^2}{M_\mathrm{p}}$

for this to be comparable to Δ*r* the planet has to satisfy: $M_\mathrm{p} \lesssim 2\pi a \Sigma_\mathrm{p} \Delta r.$

- The total rate of change of the planet's semi-major axis is given by:

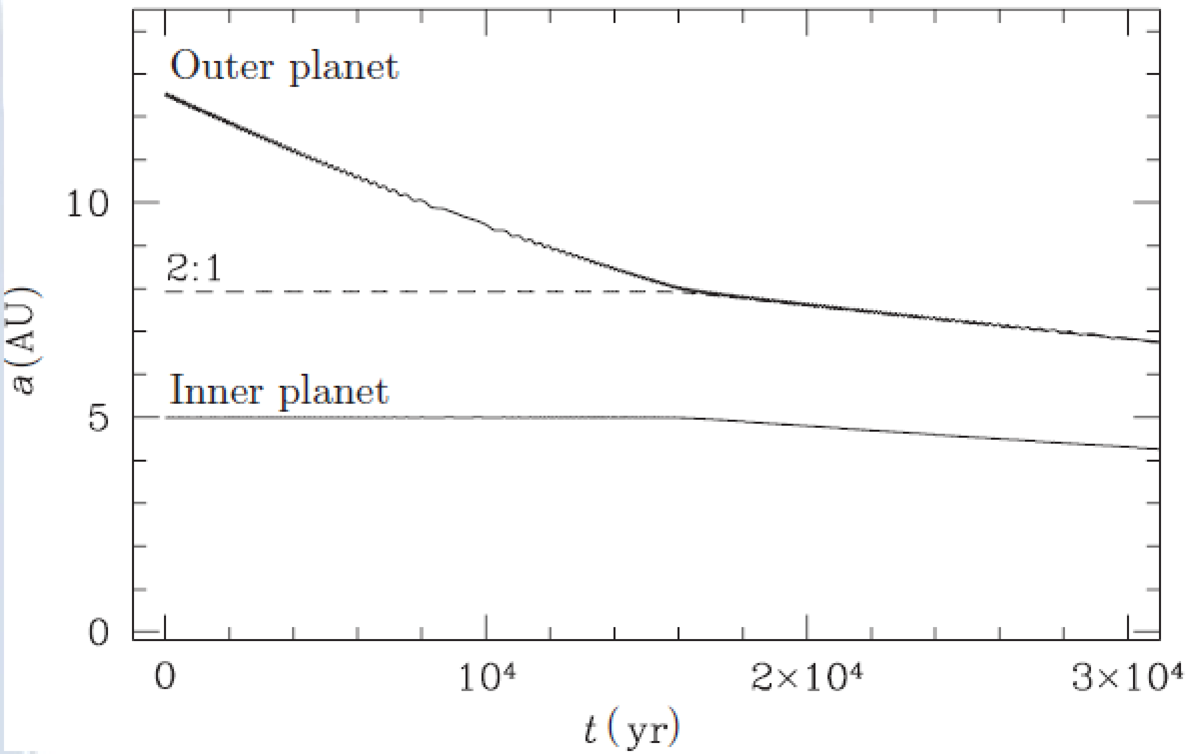
$$\boxed{\frac{\mathrm{d}a}{\mathrm{d}t} \sim \frac{a}{P} \frac{\pi a^2 \Sigma_\mathrm{p}}{M_*}}$$

given that the time it takes for all particles to be deflected is $\Delta t \sim \frac{2}{3} \frac{a}{\Delta r} P$

- More *massive* planets will also move, but *at a slower rate*. The planet will *'stall'* if Σ_P drops such that *'fresh'* mass within Δ*r* *decreases constantly*

Formation of resonant systems

Numerical simulations of giant planets evolving under the effects of Type II migration show the possibility to have *resonant capture**



The resonance is characterized by *near-preservation of the period ratio* and *libration* of the *resonant arguments:*

$$\theta_1 = \lambda_{\rm in} - 2\lambda_{\rm out} + \varpi_{\rm in},$$

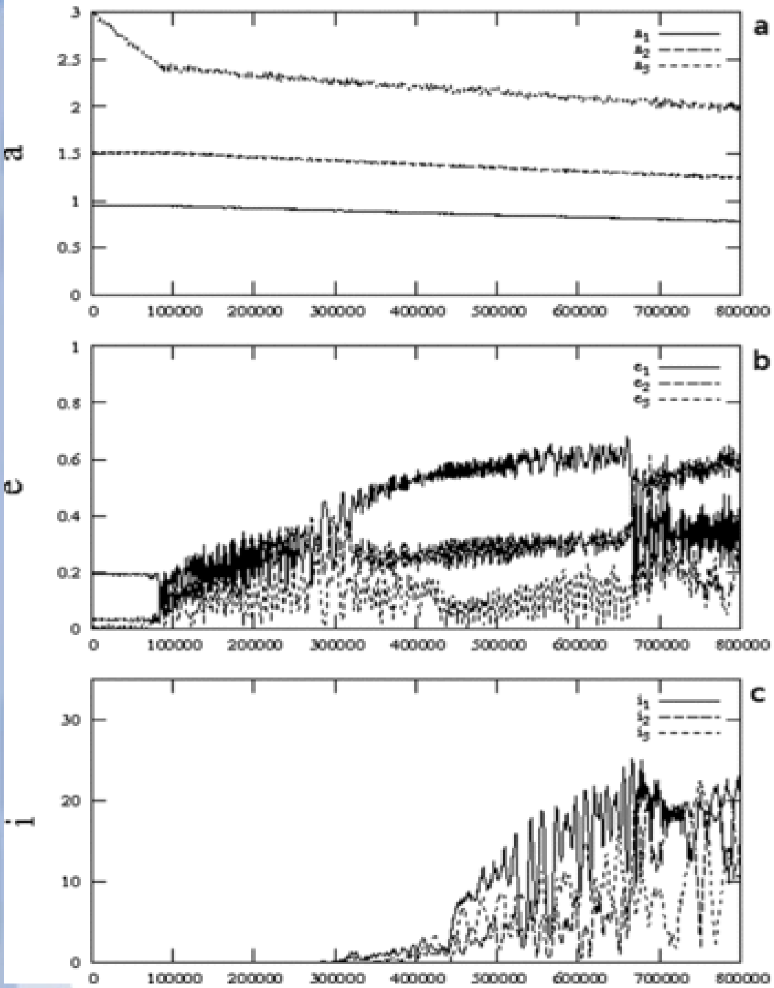
$$\theta_2 = \lambda_{\rm in} - 2\lambda_{\rm out} + \varpi_{\rm out}$$

As the disc tries to push the planets inwards, the resonance tries to preserve the ang.mom related to radial *epicyclic* motion

As a result, the planets *eccentricities* go up. If the gas in the disc manages to *damp* the eccentricities efficiently, a final, stable *resonant system* occurs.

If not, the orbits will start *crossing each other* and the system may *dissolve!*

* requires *converging* orbits!

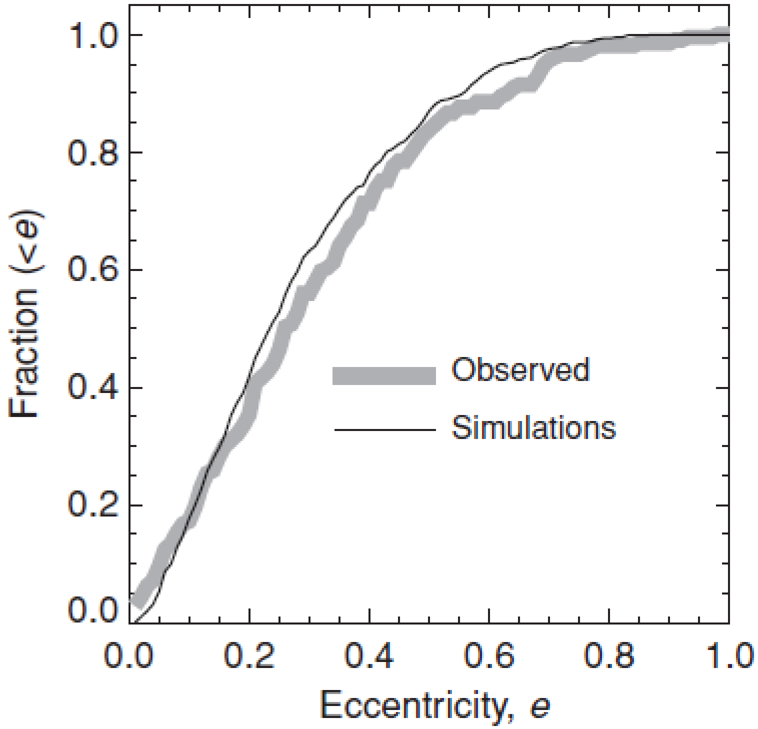


Numerical simulations show that:

- *multi-resonant systems* of moderately eccentric and inclined planet orbits can also be formed

- if the gas surface *density drops relatively fast*, these systems may be disrupted, leading to *planet-planet scattering and planet loss*.

In 3-planet simulations, the eccentricity distribution of *surviving planets* matches *the one of EPSs*.



Resonance-crossing and the *'Nice model'*

- If the planets move inwards by gas-driven migration *without getting captured* e.g. in the 2:1 resonance (e.g. Saturn's mass has the highest migration rate and can 'jump' over the resonance), then

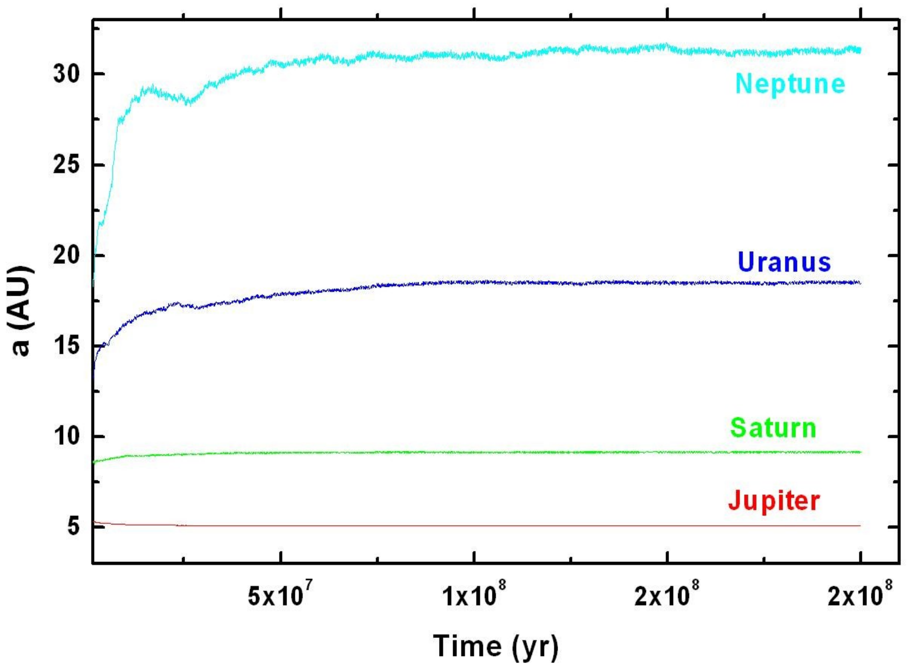
... when the gas dissipates, the planetary system will have to disrupt the massive planetesimals disc → *planet-driven migration*

- In a *many-planets system*, the outermost planets deflects particles into the sphere of influence of the next one, and so on... → a *particle 'chain'* forces all planets to migrate

Actually Jupiter moves slightly inwards, as it ejects all particles on hyperbolic oribts

It can explain the semi-major axes of the planets and the capture of Pluto in a 3:2 resonance with Neptune

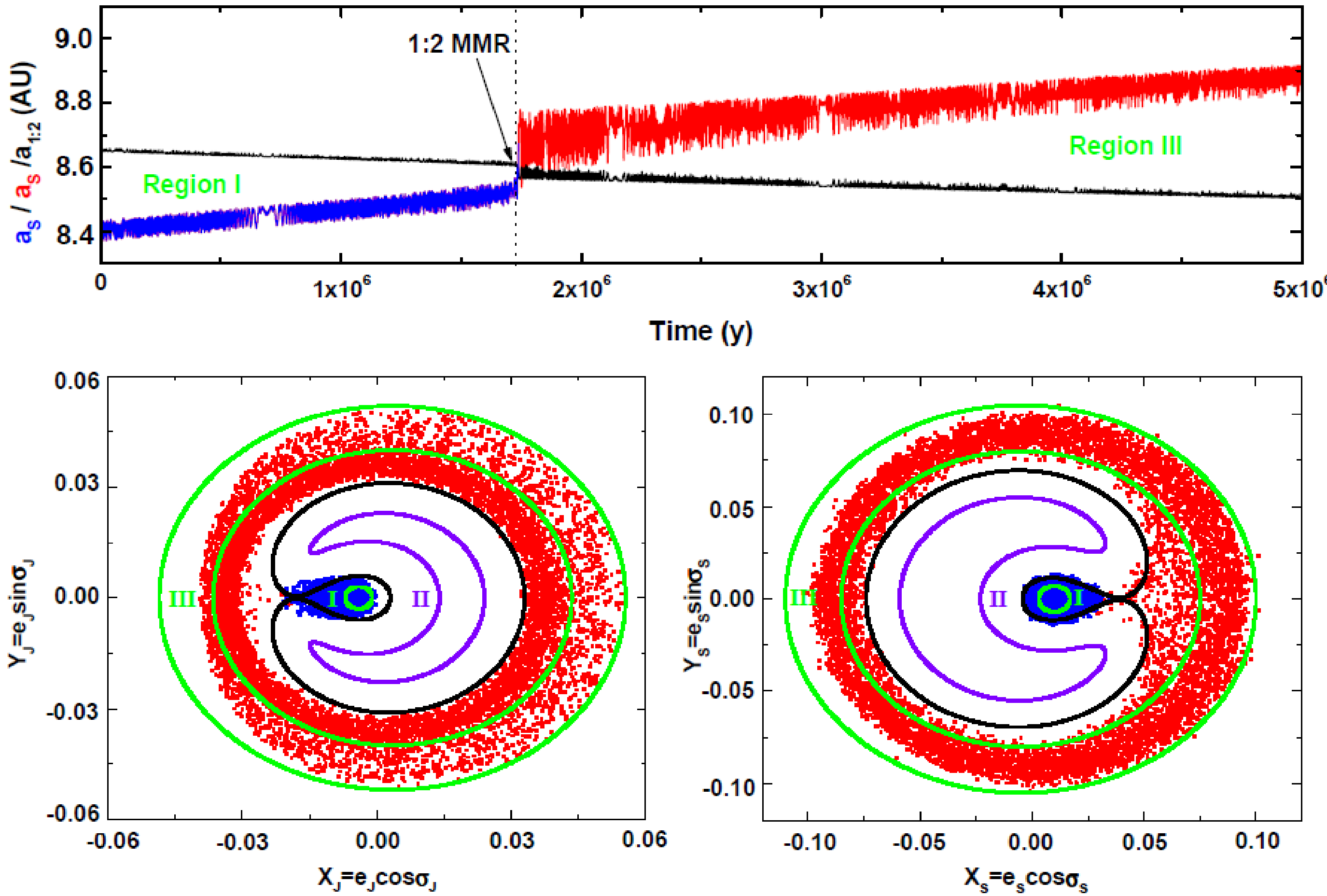
It cannot explain the other orbital elements (e.i) neither how the core of Neptune got to be so big (~20 Earth masses) out there

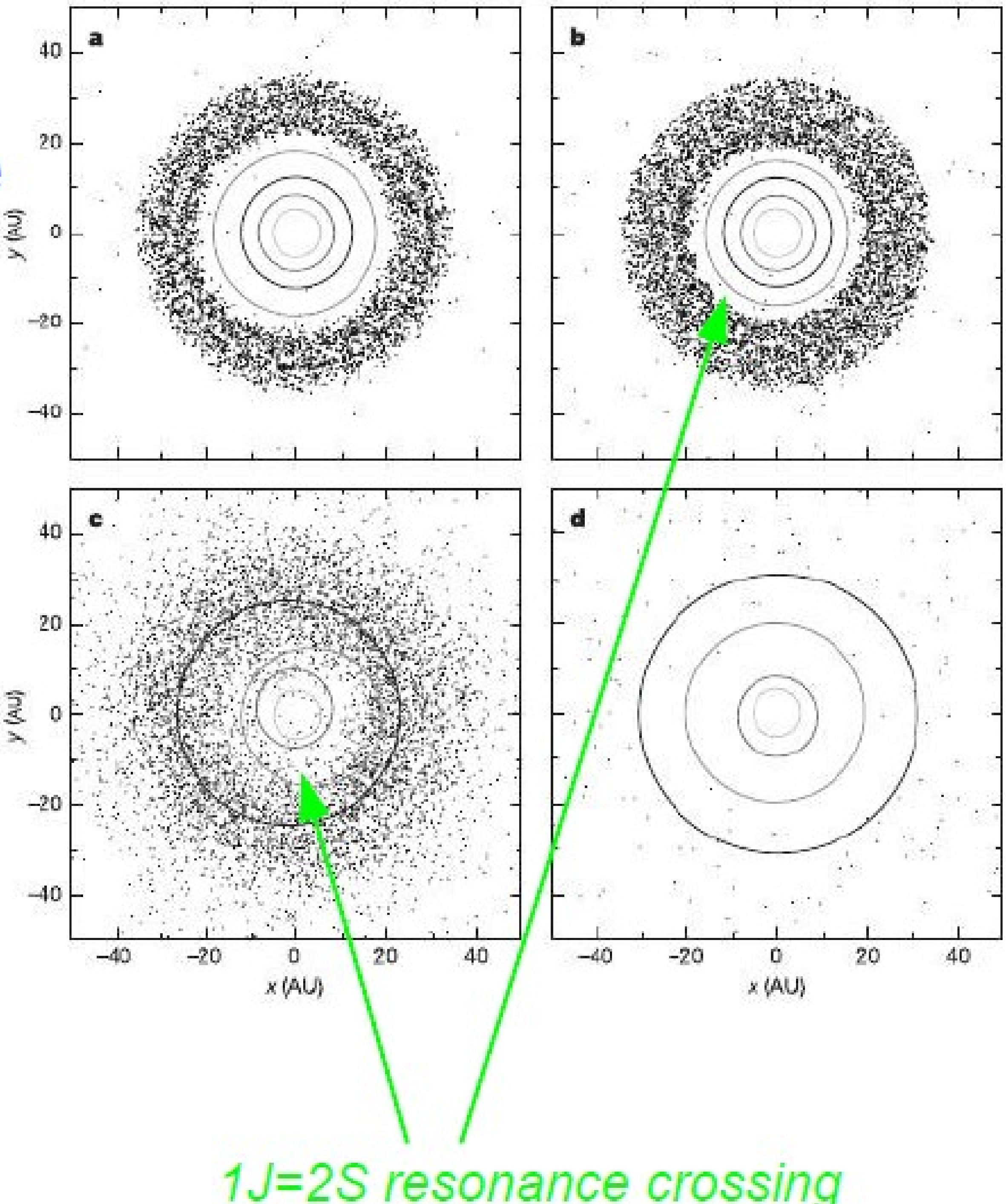


The 'Nice model'

- Assume Saturn starts off interior to the 1:2 resonance with Jupiter. As they move *on diverging* orbits, the resonance is approached but *capture is not possible*:

→ *Resonance crossing* that increases the eccentricities of the planets!





This destabilizes the whole system!

Destroying the particles disc...

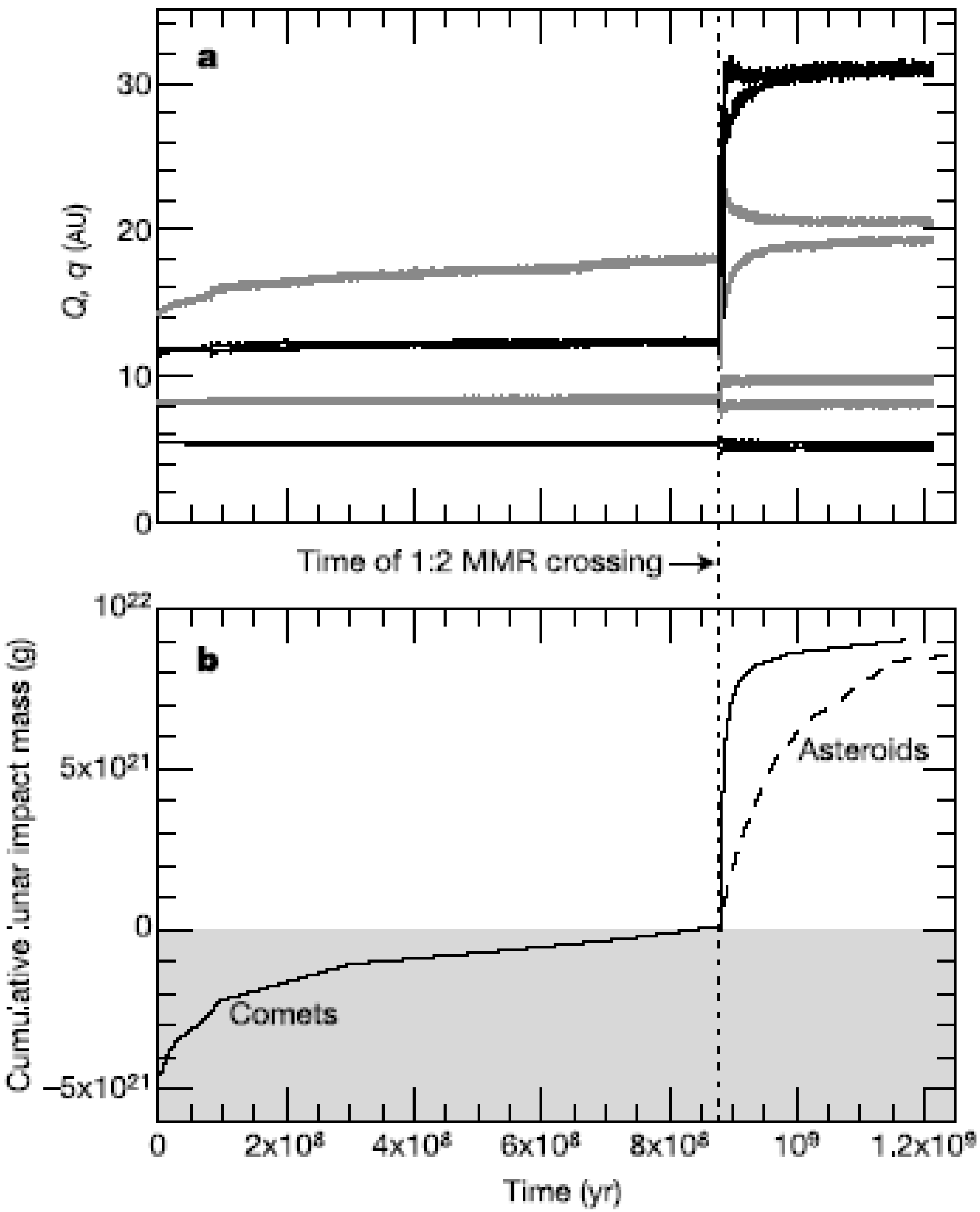
Which acts as an 'amortiseur' (dynamical friction) and *tries to circularize* the planetary orbits

Eventually the system *relaxes* in a new, stable configuration and the disc has been almost completely depleted...

Leaving behind a 'relic' that we now call *Kuiper Belt*...

- what do the *orbits* look like at the end?

- where did this *mass* go?



- The model kills (more than) 2 birds in one stone....
- The orbits of the planets are very close to the observed ones in *all 3 elements (a,e,i)*
- Some of the mass flowing towards the TP region *hits the Moon and the Earth*
- This bombardment has all the characteristics of the so-called *Late Heavy Bombardment*

  (*mass*, *duration*, and *time-delay*)
- Also explain the mass and *orbital distribution* in the asteroid belt and the Kuiper Belt ...

Concluding remarks on the dynamics of young systems

- Migration is crucial for understanding the *observed variety* of EPSs
- Type I (gas-driven) migration poses problems (agian..) or understanding the fast *formation of GP cores*
- Type II (gas-driven) migration can explain the existence of *hot Jupiters*. In a multi-planet system it can lead to the *formation of resonant systems*

  → the stable ones should be observed, while unstable ones should have lead to *planet loss* and *excitation* of the remaining ones

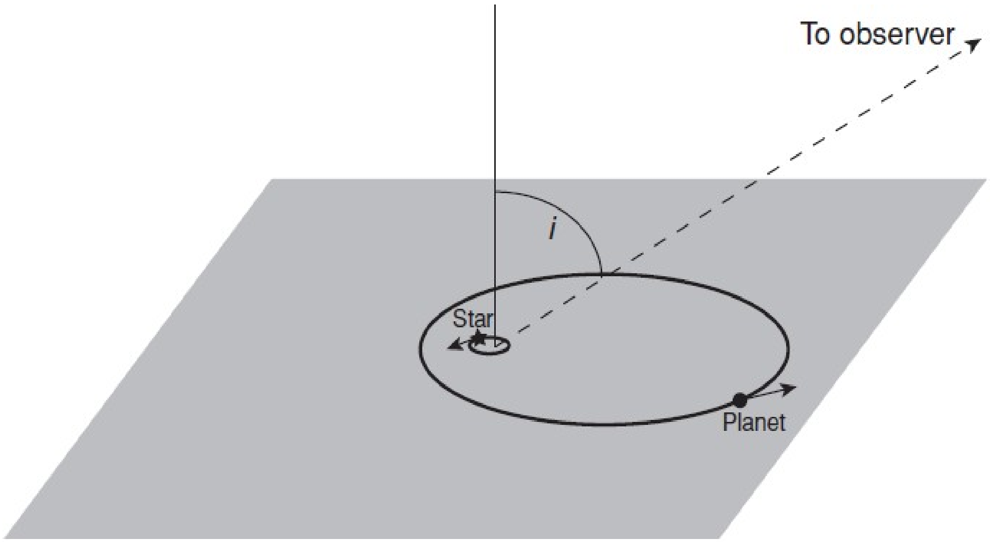
  → this can explain the *eccentricity distribution* of observed EPSs
- In the solar system, Type-II could have lead to a *non-resonant* system, with the planets being closer to the Sun
- Planetesimal driven migration – in conjunction with *resonance crossing* – probably shaped our system, through a *temporary instability*
- This may have been important to other systems, possessing *relatively massive debris discs*

Some notes on observations and parameter estimation for EPSs*

- Direct imaging is very difficult (involves obscuring the star, being able to measure the light from the planet and being able to spatially resolve the signals)

- Problem: given that the magnitude of an object is proportional to the log of its brightness ($m = -2.5 \log(l) + C$) calculate the amount of starlight that an Earth-like planet intercepts and re-radiates (with albedo A) to find how many magnitudes fainter it is.
  If the system is 10 pc away, how big a telescope would we need in order to spatially resolve the two signals?

Radial velocity and orbital parameters estimation



We observe the system at an unknown angle, *i*.

Circular orbit: $v_{\mathrm{K}} = \sqrt{\frac{\mathrm{G}M_*}{a}}$.

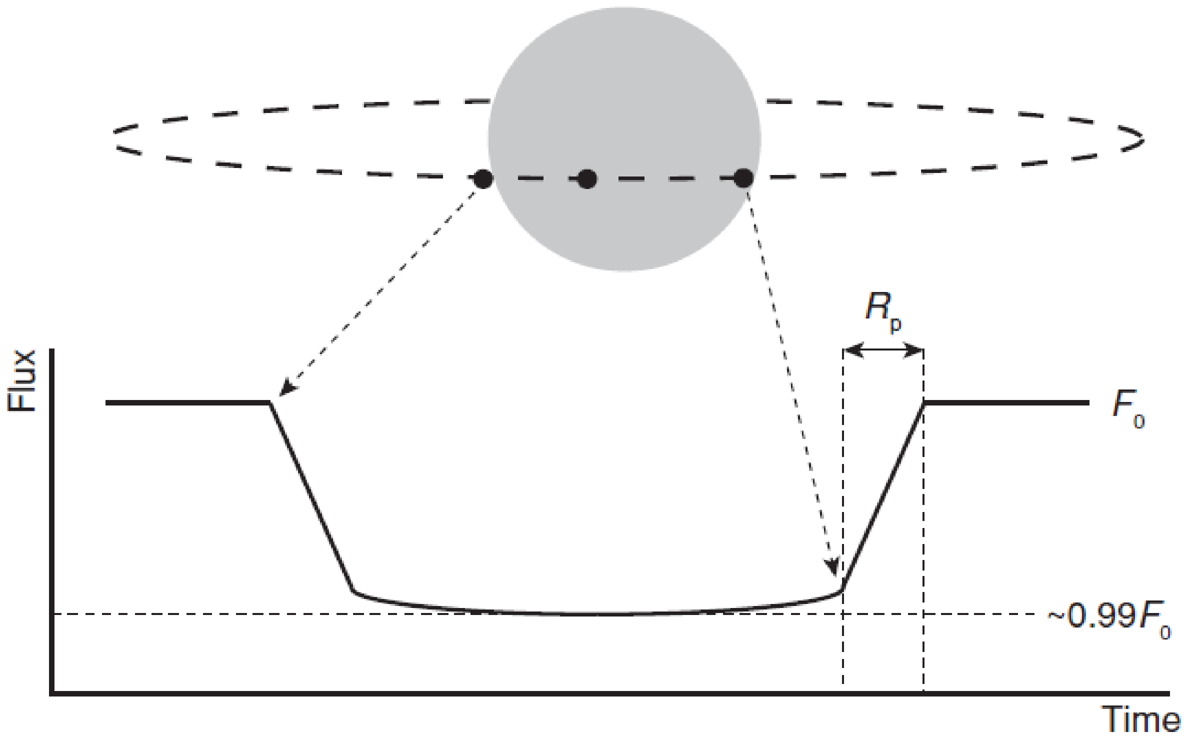
c.of mass: $M_* v_* = M_{\mathrm{p}} v_{\mathrm{K}}$

Directly observable: $P = 2\pi\sqrt{\frac{a^3}{\mathrm{G}M_*}}$, $K = v_* \sin i = \left(\frac{M_{\mathrm{p}}}{M_*}\right)\sqrt{\frac{\mathrm{G}M_*}{a}} \sin i$

→ Having the star's mass we can compute the mass (lower limit), $M_P \sin i$, and orbital radius, *a*, of the planet

- Similarly for **eccentric** orbits, one obtains the eccentricity, *e*, and pericenter longitude, *ω*, given several measurements of *K*.

Transit method



- Gives the radius of the planet and the orbital period
  Not very easy, the inclination has to satisfy:
  $\cos i \leq (R_* + R_p)/a$
- Variations in transit timing and duration reveals additional planets