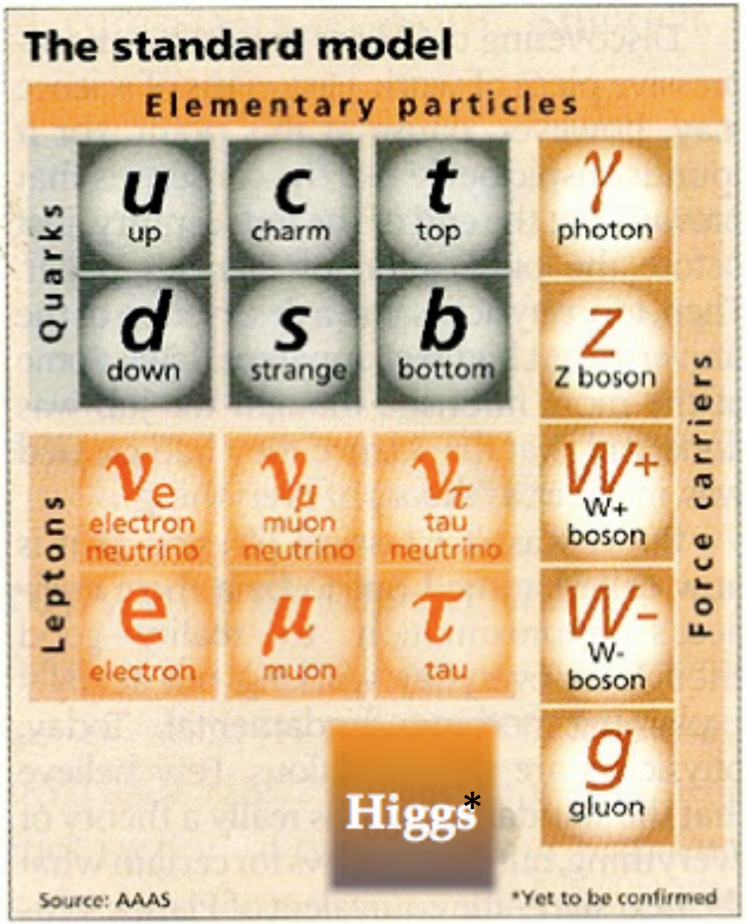
Recap: The Standard Model particles and forces

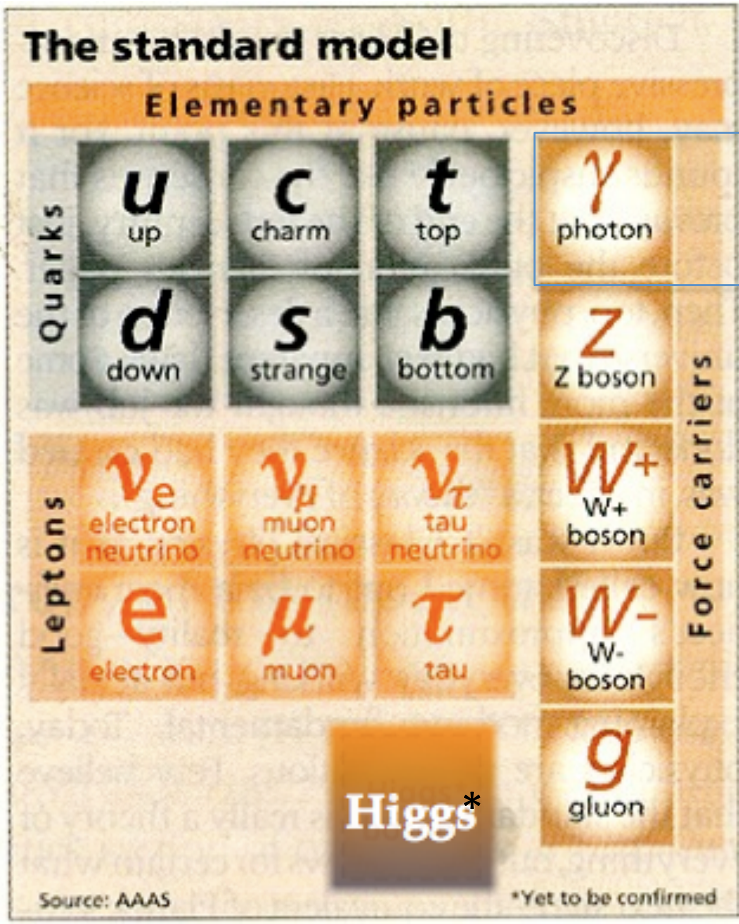


4 known Fundamental Interactions

U(1)	Electromagnetic interaction
SU(2)	Weak interaction
SU(3)	Strong interactions

Gravity $G_{\rm N} m_P^2 \sim 10^{-36}$

Recap: Listed the SM particles and forces



4 known Fundamental Interactions

U(1)	Electromagnetic interaction
SU(2)	Weak interaction
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Derived *scalar quantum electrodynamics :*
i.e. we derived *the masslessness* and *interactions* of photons (*spin-1 U(1) gauge boson*) with matter(scalar field for simplicity) in QFT from gauge symmetry!

The SM, the Higgs and beyond

- Lecture 3 - Goldstone model, Abelian Higgs model,
  the Higgs Mechanism of the Standard Model

We began by considering the Klein-Gordon Lagrangian

$$\mathrm{L} = \underbrace{\left(\partial_\mu \phi(x)\right)^\dagger \partial^\mu \phi(x)}_{T} - \underbrace{m^2 \phi(x)^\dagger \phi(x)}_{V}$$

Free (only quadratic terms in ϕ)
massive $m^2 \neq 0$
Complex scalar field

$$\phi(x) = \phi_1(x) + i\phi_2(x)$$

invariant under $\phi(x) \rightarrow \phi'(x) = e^{i\alpha}\phi(x)$

In terms of the real components

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \phi_1' \\ \phi_2' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

Note, global rotation, nothing to do with space-time transformations
(argument x is unchanged)

Lets generalize the potential by considering the theory

$$L = \partial_\mu \phi^\dagger \partial^\mu \phi - \frac{1}{2}\mu^2 \mid \phi \mid^2 - \frac{1}{2}\lambda^2 \mid \phi \mid^4$$

T V

This is now the *Goldstone Model* – similar to our example of *spontaneous symmetry breaking* in first lecture but now ϕ is complex:

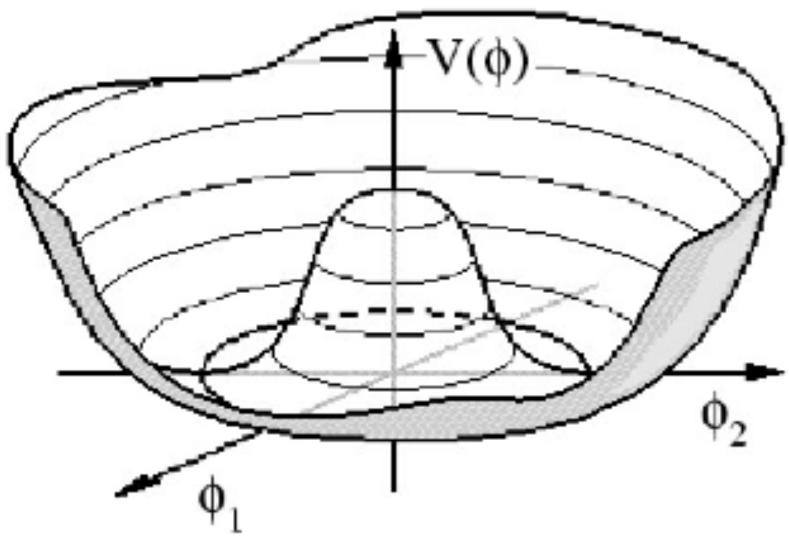
Still invariant under

$$\phi(x) \rightarrow \phi'(x) = e^{i\alpha}\phi(x)$$

Now we have a circle of minima given by

$$|\phi| = \sqrt{\phi_1^2 + \phi_2^2} = \frac{\mu}{\lambda} \equiv v$$

$$\mu^2 < 0$$



Goldstone model

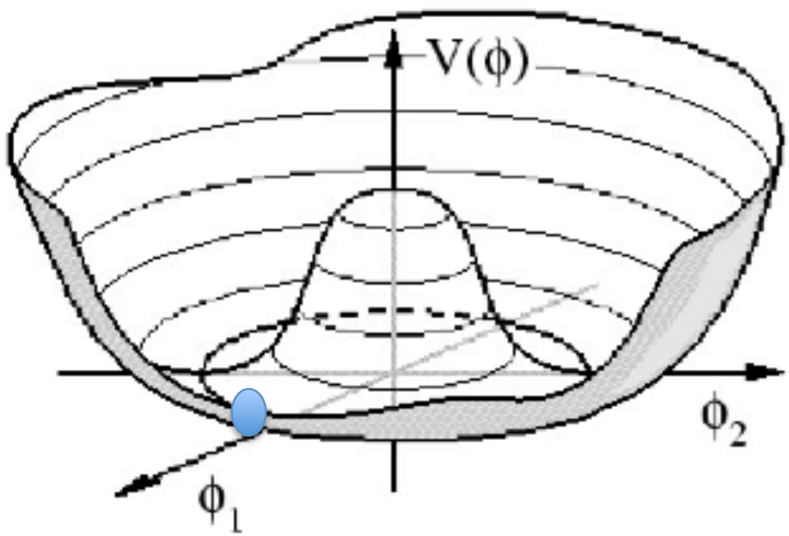
$$L = \partial_\mu \phi^\dagger \partial^\mu \phi - \frac{1}{2}\mu^2 \mid \phi \mid^2 - \frac{1}{2}\lambda^2 \mid \phi \mid^4$$

T V

Now lets choose one minima to do pertubation theory around

$$\phi_1 = v + \chi_1, \quad \phi_2 = 0 + \chi_2$$

$$\mu^2 < 0$$



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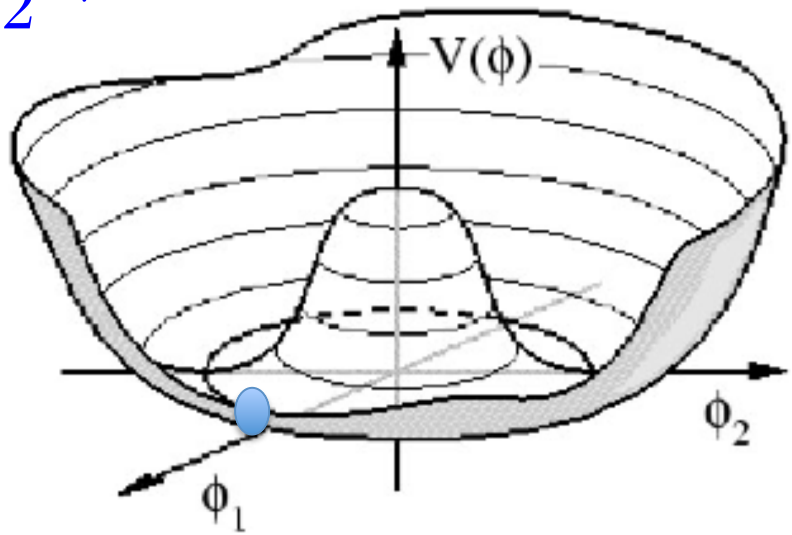
Now vacuum no longer invariant invariant under rotations – how does L look in these Field observables?

$$L = (\frac{1}{2}\partial_\mu \chi_1 \, \partial^\mu \chi_1 - \frac{1}{2}\mu^2 \chi_1^2) + \frac{1}{2}\partial_\mu \chi_2 \, \partial^\mu \chi_2 + \ldots \qquad \mu^2 < 0$$

What particles is it describing?



Goldstone model

$$L = \partial_\mu \phi^\dagger \partial^\mu \phi - \frac{1}{2}\mu^2 \mid \phi \mid^2 - \frac{1}{2}\lambda^2 \mid \phi \mid^4$$

$\underbrace{\partial_\mu \phi^\dagger \partial^\mu \phi}_{T}$ $\qquad$ $\underbrace{\mid \phi \mid^2}_{V}$

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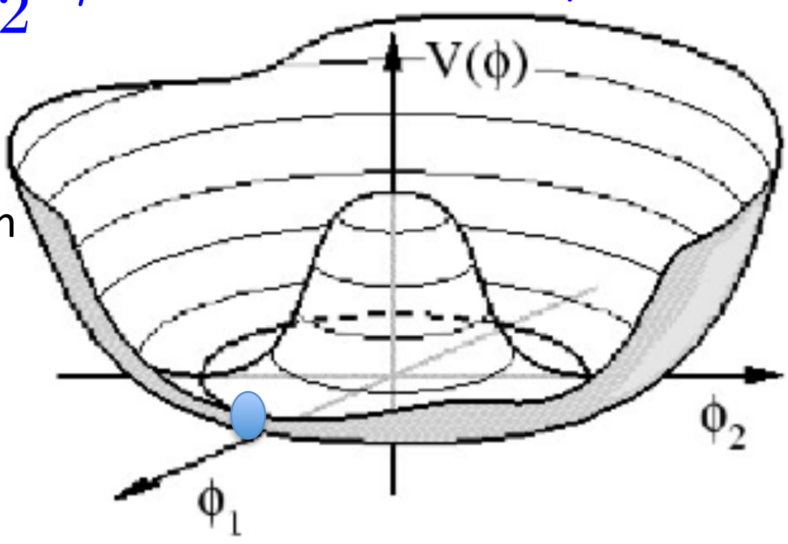
What particles is it describing?

We get a *massless, spin-0 Goldstone Boson* from *spontaneous breaking* of *a global* symmetry



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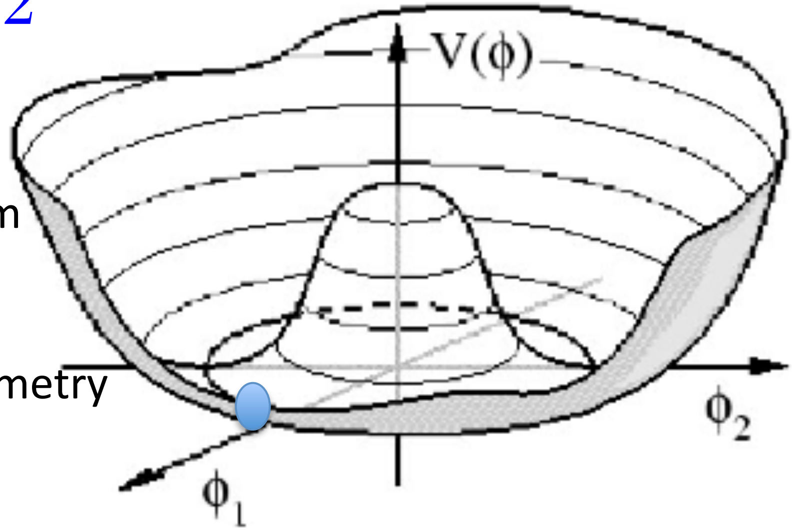
We get a *massless, spin-0 Goldstone Boson* from *spontaneous breaking* of *a global* symmetry

We got a *massless spin-1* field from *gauge* symmetry



Gauging the Klein-Gordon model

$$\phi(x) \to e^{ie\alpha(x)}\phi(x)$$

$$D_\mu\phi \to e^{i\alpha(x)e}D_\mu\phi$$

$$A_\mu \to A_\mu + \partial_\mu\alpha$$

Recall from last lecture that we gauged the free (Klein-Gordon) theory by :

$$\partial_\mu\phi \to D_\mu\phi = \partial_\mu\phi - ieA_\mu\phi$$

$$\mathrm{L} = \left(D_\mu\phi(x)\right)^\dagger D^\mu\phi(x) - m^2\phi(x)^\dagger\phi(x)$$

is invariant under local U(1)

Note : $\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu$ is equivalent to $p^\mu \to p^\mu + eA^\mu$

universal coupling of electromagnetism *follows* from local gauge invariance

i.e. $\mathrm{L} = \mathrm{L}^{\mathrm{KG}} = \left(\partial_\mu\phi(x)\right)^\dagger \partial^\mu\phi(x) - m^2\phi(x)^\dagger\phi(x) - j_\mu^{KG}A^\mu + O(e^2)$

$$j_\mu^{KG} = -ie\left(\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*\right)$$

Abelian Higgs model

Can do exactly the same for the Goldstone model
(potential explicitly gauge invariant):

$$\phi(x) \rightarrow e^{ie\alpha(x)}\phi(x)$$

$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha$$

$$D_\mu \phi \rightarrow e^{ie\alpha(x)} D_\mu \phi$$

$$L = [(\partial_\mu + ieA_\mu)\phi]^\dagger \, [(\partial^\mu + ieA^\mu)\phi] \;-\; \frac{1}{2}\mu^2 \mid \phi \mid^2 \;-\; \frac{1}{2}\lambda^2 \mid \phi \mid^4$$

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

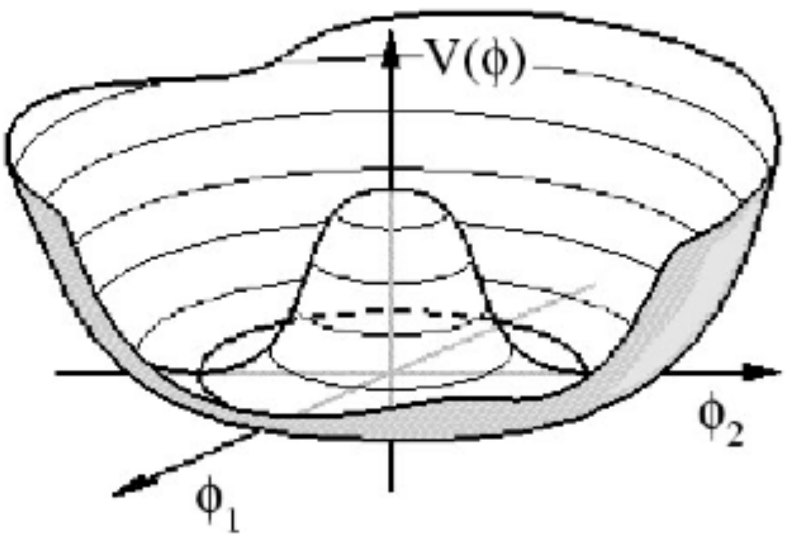
is invariant under local U(1) phase rotations

This time lets describe the theory in 'polar field coordinates'

$$\mu^2 < 0$$

$$\phi = \frac{1}{\sqrt{2}}(v + \rho(x)) \; e^{i\theta(x)/v}$$

What are gauge transformation in terms of these fields?



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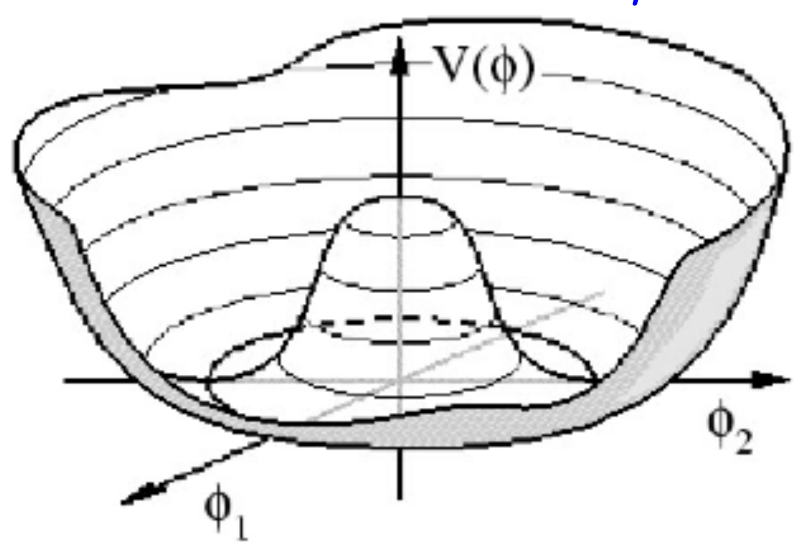

Gauge transformation in terms of these fields

$$\rho(x) \to \rho(x)$$
$$\theta(x) \to \theta(x) - e\,v\,\alpha(x)$$

Abelian Higgs model

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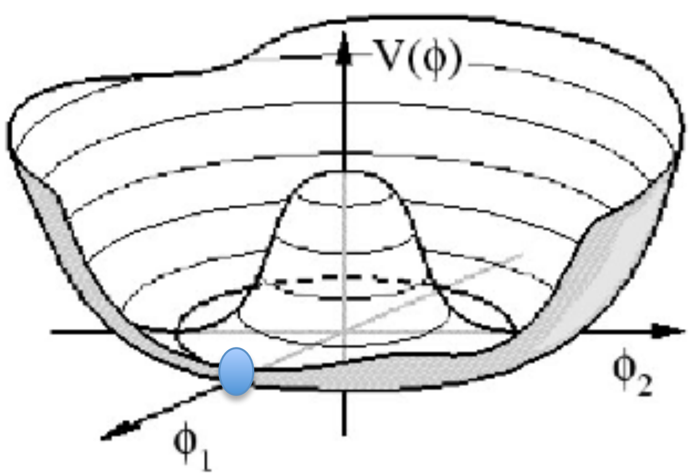
Can do exactly the same for the
Goldstone model (potential explicitly invariant):

We can again tranform to same minimum (φ_2=0), corresponding to Θ=0 by choosing

$$\alpha(x) = \frac{\theta(x)}{ev} \; : \; \phi \to \phi' = \frac{1}{\sqrt{2}}(v + \rho(x)) \quad A_\mu \to A'_\mu = A_\mu + \frac{1}{ev}\partial_\mu \theta(x)$$



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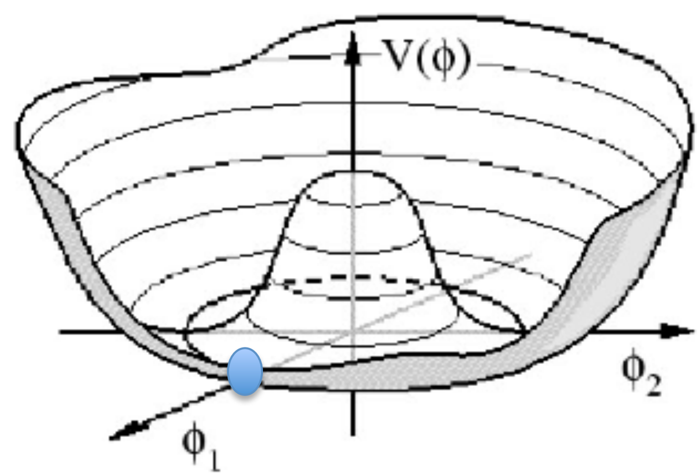
But this adds a 'contribution' to the gauge field – whats the Lagrangian in these fields

$$L(\phi', A'_\mu) = (\frac{1}{2}\partial_\mu \rho\, \partial^\mu \rho \;-\; \frac{1}{2}\mu^2\rho^2) \;-\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} \;+\; \frac{1}{2}e^2v^2A'_\mu A'^\mu \;+ \ldots$$



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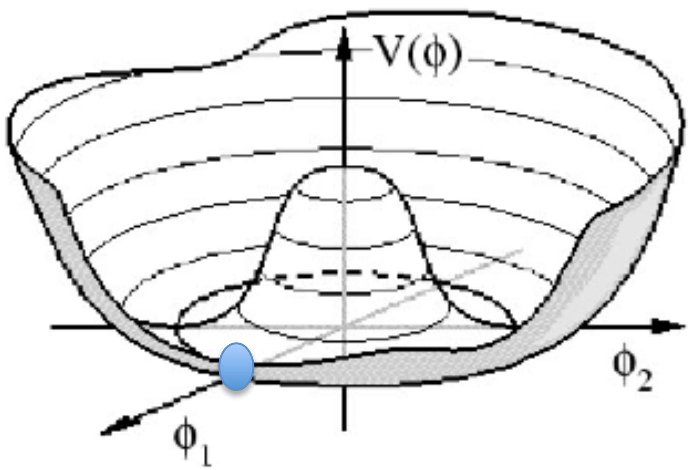
What is this Lagrangian describing?



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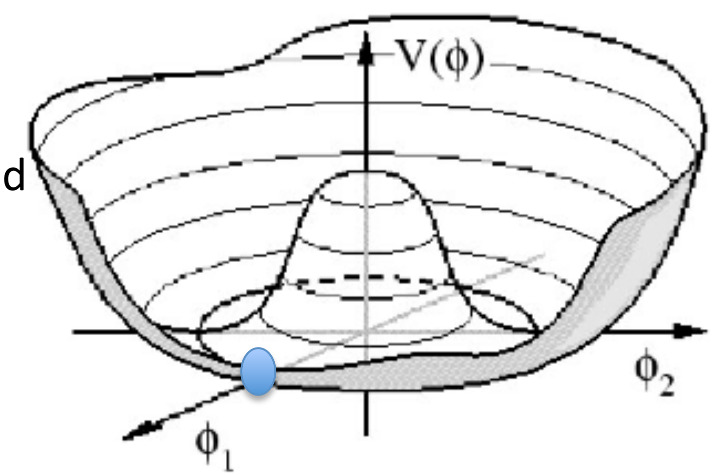
The gauge field has become massive!
The Goldstone boson has been *absorbed* and
become the longitudinal mode of the *massive* gauge field



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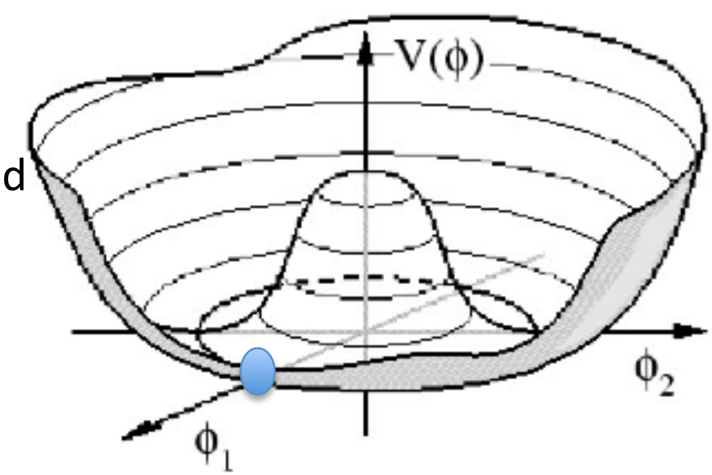
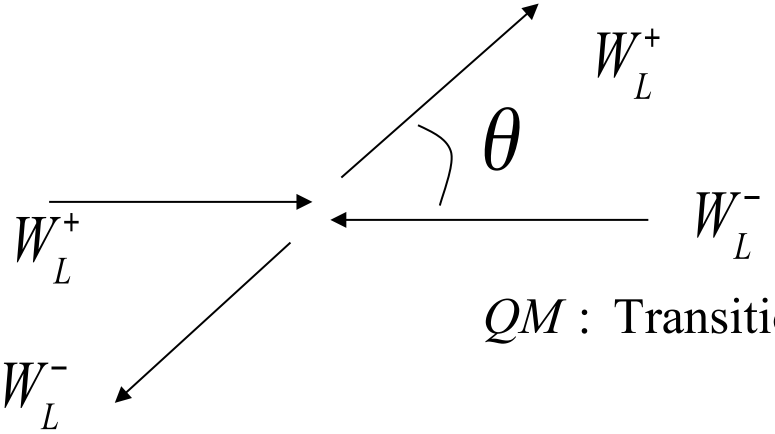
Become the longitudinal mode of the *massive* gauge field

ρ is the accompanying scalar field – U(1) Higgs boson
includes interactions like

$$v\rho A'_\mu A'_\mu$$



Recall Scattering of massive W-bosons



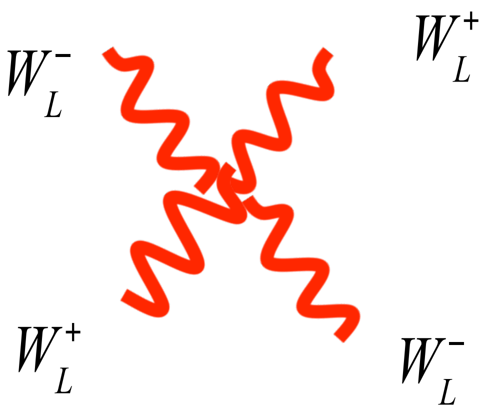
$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 E_{CM}^2}|M|^2$$

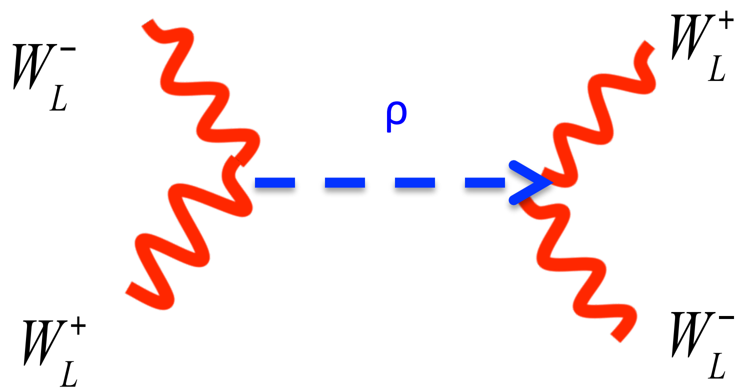
QM : Transition amplitude <final state$|H_I|$initial state>

$$M \propto < W^+W^- \,|\, H_I \,|\, W^+W^- >$$

$$M(W_L^+W_L^- \rightarrow W_L^+W_L) \sim \quad const!$$

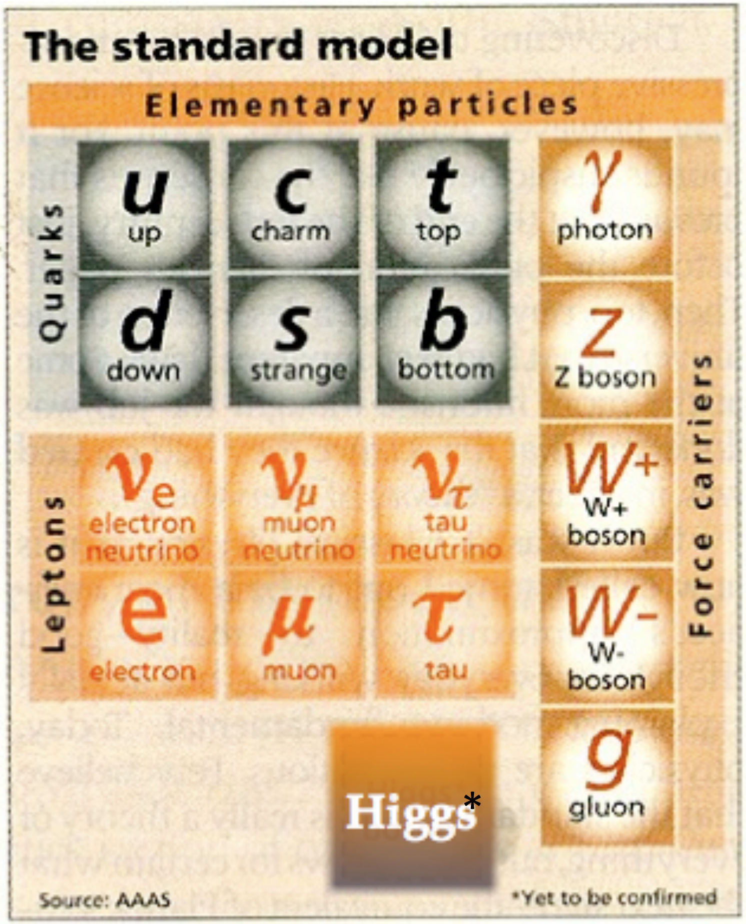
Feynman diagram





The Higgs state *unitarizes* the scattering process of massive gauge bosons

SM Higgs mechanism



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Of course the photon of the SM is *massless*! it is the W and Z bosons which are massive
So how many gauge fields and how many Goldstone Bosons do we need?

Extension to non-Abelian symmetry

Yang-Mills (+Shaw)

$(The\ Standard\ Model\quad SU(3)\otimes SU(2)\otimes U(1))$

$SU(2)$ local gauge invariance

$$\Phi = \begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix} \qquad \begin{aligned}\Phi_1 &= \phi_1 + i\phi_2\\ \Phi_2 &= \phi_3 + i\phi_4\end{aligned}$$

2 complex
4 real scalars

$$\Phi \to \Phi' = e^{ig_2\vec{\alpha}(x)\frac{\vec{\sigma}}{2}}\Phi$$

$$\sigma_1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \quad \sigma_2 = \begin{pmatrix}0 & i\\ -i & 0\end{pmatrix} \quad \sigma_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$D_\mu\Phi \to D_\mu\Phi' = e^{ig_2\vec{\alpha}(x)\frac{\vec{\sigma}}{2}}D_\mu\Phi \qquad \mathrm{D}_\mu = \partial_\mu + ig_2\frac{\sigma_i}{2}W_\mu^i$$

Need 3 gauge bosons

W^+, W^-, W^3

where $W_{\mu,i} \to W_{\mu,i} - \partial_\mu\alpha_i - g_2\varepsilon_{ijk}\alpha_j W_{\mu,k}$

$$L_H = D_\mu\Phi^\dagger D^\mu\Phi - V(\Phi)$$

$$\left(\left[\frac{\sigma_i}{2}, \frac{\sigma_j}{2}\right] = i\varepsilon_{ijk}\frac{\sigma_k}{2}\right)$$

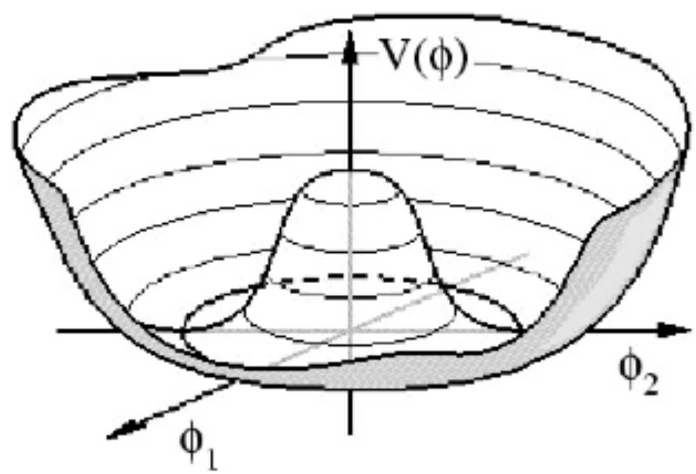
SM Higgs mechanism – first consider global symmetries

$$L = \partial_\mu \Phi^\dagger \partial^\mu \Phi - \frac{1}{2}\mu^2 \mid \Phi \mid^2 - \frac{1}{2}\lambda^2 \mid \Phi \mid^4$$

Now have an SO(4) symmetry before gauging

$$\mid \Phi \mid = \sqrt{\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2} \qquad \mid \Phi \mid_{\min} = \frac{\mu}{\lambda} \equiv v$$

How many broken symmetry direction at the minimum, i.e. how many masless Goldstone Bosons?



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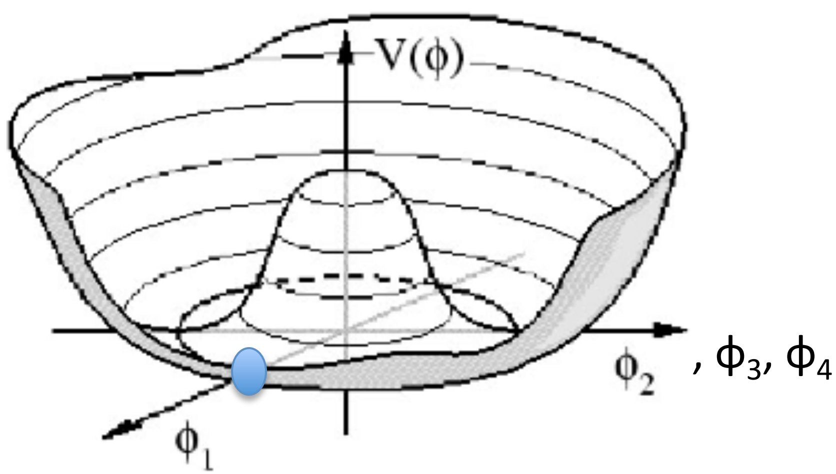

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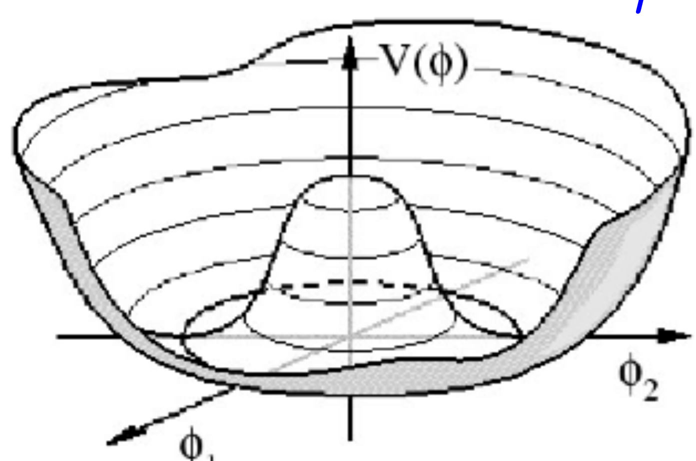
3 broken symmetry direction at the minimum, i.e. 3 Goldstone Bosons?

Now *gauge Φ* under the SU(2)~SO(3) symmetry

$$L = D_\mu \Phi^\dagger D^\mu \Phi - \frac{1}{2}\mu^2 \mid \Phi \mid^2 - \frac{1}{2}\lambda^2 \mid \Phi \mid^4$$

$$\mu^2 < 0$$

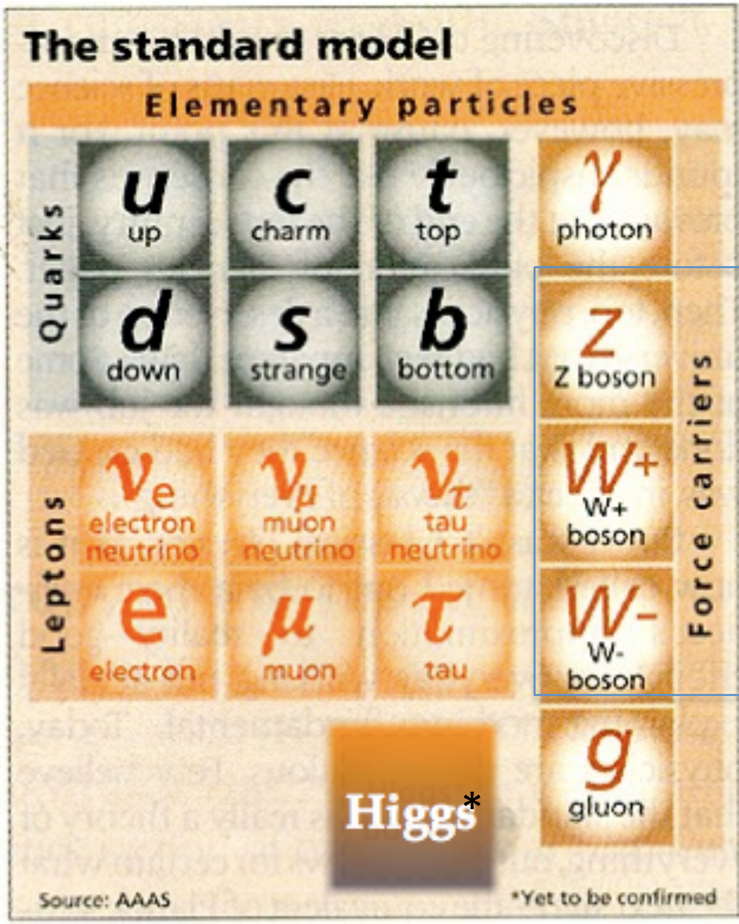
$$\Phi(x) = \frac{1}{\sqrt{2}} e^{i\vec{\theta}(x)\frac{\vec{\sigma}}{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$



, ϕ_3, ϕ_4

If you write out L you now see
You have *3 massive spin-1 particles*,
0 *Goldstone bosons* anymore
1 spin-0 massive *Higgs*

SM Higgs mechanism



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This is the structure Nature ordered – The SM Higgs *can* do the job, but does it?