Cosmological constant from gravity-mediated interactions in the hadronic vacuum

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"We mathematicians are all a bit crazy…"

Lev Landau

The greatest challenge of Fundamental Physics

Nobel Prize 2011

"…for the discovery
of the accelerating expansion
of the Universe through
observations of distant supernovae"

Energy density of the Universe is dominated by Vacuum component!

Hypothesis about time-independent Λ-term = Dark Energy (74%)

Vacuum in Particle Physics has incredibly wrong energy scale!

Vacuum energy from **Particle Physics**

$$\rho_{vac} \sim M^4$$

$$\rho_{vac} \sim 1\,\mathrm{GeV}^4, \quad \textbf{QCD}$$

$$\sim 10^8\,\mathrm{GeV}^4, \quad \textbf{EW}$$

and more…

Ooops!

Vacuum energy from **Cosmology**

$$\rho_\Lambda \sim \rho_c \sim 10^{-46}\,\mathrm{GeV}^4$$

"…the worst theoretical prediction in the history of physics!" (Hobson 2006)

"Vacuum Catastrophe"

No single established theoretical framework cancels near to *many orders of magnitude*!

Cancellation of QCD component at large distances

NPT topological (instanton) QCD contribution

$$\varepsilon_{vac(top)} = -\frac{9}{32}\langle 0| : \frac{\alpha_s}{\pi} F^a_{ik}(x) F^{ik}_a(x) : |0\rangle + \frac{1}{4}\left(\langle 0| : m_u \bar{u} u : |0\rangle + \langle 0| : m_d \bar{d} d : |0\rangle + \langle 0| : m_s \bar{s} s : |0\rangle\right)$$

$$\simeq -(5 \pm 1) \times 10^9 \ \mathrm{MeV}^4.$$

large negative due to gluons!

Induced due to quantum tunneling of quark and gluon fields between topologically different states

A contribution of a different nature should cancel it, but existing at the same hadron scale ~ 200 MeV!

Two possibilities:

- ***hypothesis about the existence of cosmological Yang-Mills fields in Nature***

  (homogeneous/isotropic gluon field with unbroken symmetry at cosmological scales)

  …no physical arguments against it
  …can be created during Inflation (e.g. Adshead'12, Elizalde'12, Maeda'12)

- ***there are extra (non-topological) contributions to QCD vacuum energy***

  (which cancel the instanton one due to a fine-tuning of QCD parameters)

Possibility I: Cosmological Yang-Mills

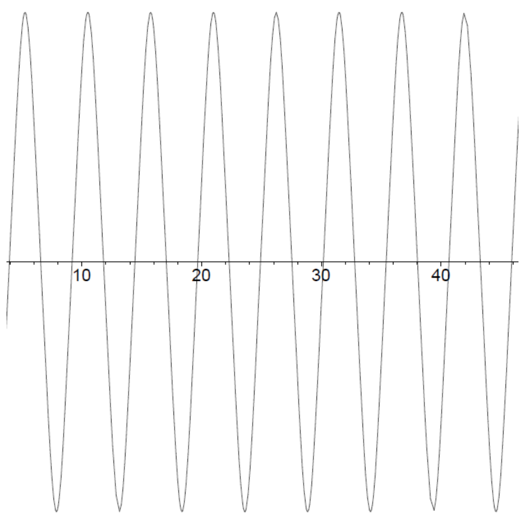
Assumption: **the gluon field with unbroken SU(3) fills up the whole Universe!**

Freedman flat Universe:

$$g_{\mu\nu} = a^2(\eta) g_{\mu\nu(M)}$$
$$dt = a(\eta) d\eta$$

Non-linear solution:

$$A(\eta) \simeq A_0 \cos\left(\frac{6}{5} A_0 \eta\right)$$



In Hamilton gauge and homogeneous/isotropic Universe:

$$A_i^a = \begin{cases} \delta_i^a A(\eta), & i, a = 1, 2, 3 \\ 0, & i = 1, 2, 3; \ a > 3 \end{cases}$$
$$A_0^a = 0$$

Classical EYM fields equations ("radiation" medium):

$$\frac{3}{\varkappa} \frac{a'^2}{a^4} = \frac{3}{2 g_{YM}^2 a^4} \left(A'^2 + A^4\right),$$
$$A'' + 2A^3 = 0.$$

Classical EYM equations are unstable (form non-invariant) w.r.t. radiation corrections!

Lagrangian of the gluonic field taking into account vacuum polarisation in one-loop approximation:

$$L_{YM} = -\frac{1}{4 g_{YM}^2} \frac{F_{\alpha\beta}^a F_a^{\alpha\beta}}{\sqrt{-g}} \left[1 + \frac{\beta}{2} \ln\left(\frac{J}{\Lambda_{QCD}^4}\right)\right] = -\frac{11}{128\pi^2} \frac{F_{\mu\nu}^a F_a^{\mu\nu}}{\sqrt{-g}} \ln\left(\frac{J}{\Lambda_{QCD}^4}\right),$$
$$J = \frac{1}{\xi^4} \frac{|F_{\alpha\beta}^a F_a^{\alpha\beta}|}{\sqrt{-g}}, \qquad \sqrt{-g} = a^4(\eta),$$

e.g. Savvidi et al '79, '81

"Time instantons" cancel "topological instantons"!

Classical EYM fields equations **with vacuum polarisation:**

$$\frac{6}{\varkappa}\frac{a''}{a^3} = \varepsilon - 3p + 4\bar{\Lambda} + T_{\mu}^{\mu,YM}\,, \qquad T_{\mu}^{\mu,YM} = \frac{33}{16\pi^2}\frac{1}{a^4}\left(A'^2 - A^4\right)$$

$$\frac{\partial}{\partial\eta}\left(A'\ln\frac{6e|A'^2 - A^4|}{a^4(\xi\Lambda_{QCD})^4}\right) + 2A^3\ln\frac{6e|A'^2 - A^4|}{a^4(\xi\Lambda_{QCD})^4} = 0.$$

Vacuum energy (w/o matter):

$$\bar{\Lambda} = \Lambda_{inst} + \Lambda_{cosm}$$

instanton QCD vacuum contribution — **observable cosmol. constant**

Exact first integral with POSITIVE constant energy exists!

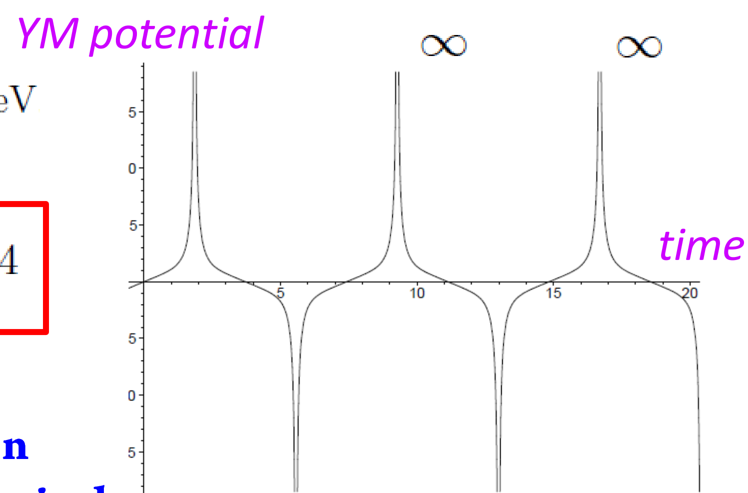
*Simple! System with **minimal energy**!*

$$\boxed{\frac{6e(A'^2 - A^4)}{a^4(\xi\Lambda_{QCD})^4} = 1}$$

Classical solution is not stable w.r.t QCD vacuum fluctuations!

$$T_0^{0,\,tot} = T_0^{0,\,mat} + \bar{\Lambda} + \frac{33}{64\pi^2}\frac{(\xi\Lambda_{QCD})^4}{6e} \qquad \Lambda_{QCD} \simeq 280\,\mathrm{MeV}$$

$$\boxed{\frac{33}{64\pi^2}\frac{(\xi\Lambda_{QCD})^4}{6e} + \Lambda_{inst} = 0 \qquad \Lambda_{inst} \simeq -265^4\,\mathrm{MeV}^4 \qquad \xi \simeq 4}$$



$$\frac{3}{\varkappa}\frac{a'^2}{a^4} = \varepsilon + \Lambda_{cosm}\,,$$

$$A'^2 - A^4 = a^4\frac{(\xi\Lambda_{QCD})^4}{6e}$$

Dynamical cancellation of the quantum-topological QCD contribution!

Possibility II: Topological vs collective contributions

NPT QCD vacuum

Quantum-topological (instanton) fluctuations

instantons/dyons carrying chromomagnetic and chromoelectric charges

$$\varepsilon_{vac(top)} < 0$$

Quantum-wave (hadronic) fluctuations

have quantum numbers of light hadrons

$$m_h \leq l_{g(min)}^{-1}$$

$$\varepsilon_{vac(h)} > 0$$

exist at the same typical space-time scales

$$l_{g(min)} < l_g < l_{g(max)},$$

$$l_{g(min)} \simeq (1500\ \text{MeV})^{-1}, \qquad l_{g(max)} \simeq (500\ \text{MeV})^{-1}$$

Can they mutually cancel each other? In principle, YES!

Taking into account ONLY metastable hadrons

$$B = \{N,\ \Lambda,\ \Sigma,\ \Xi\}$$

$$M = \{\pi,\ K,\ \eta,\ \eta'\}$$

$$\varepsilon_{vac(h)} = \frac{1}{32\pi^2}\left(2\sum_B (2J_B+1)m_B^4 \ln\frac{\mu}{m_B} - \sum_M (2J_M+1)m_M^4 \ln\frac{\mu}{m_M}\right)$$

$$\mu \simeq l_{g(min)}^{-1}$$

$$\boxed{\varepsilon_{vac(top)} + \varepsilon_{vac(h)} = 0 \ \text{ for } \ \mu = 1.22\ \text{GeV}}$$

!!!

Zeldovich-Sakharov-Kardashev scenario

Ya. Zeldovich (1967): $\Lambda \sim G m^6$

Gravitational constant (G)

Characteristic mass scale of elementary particles (m)

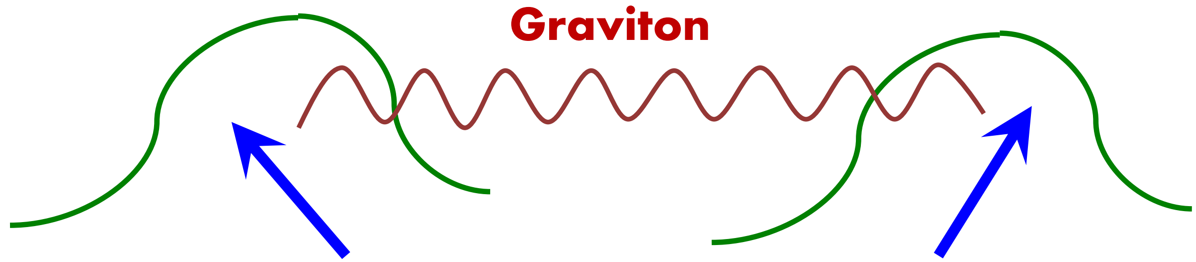
A. Sakharov (1967):

extra terms describing an effect of graviton exchanges between *identical particles* *(bosons occupying the same quantum state) should appear in the* *right hand side of Einstein equations* *(averaged over quantum ensemble)*

…first quantum gravity correction to "bare" vacuum!

Basic idea:

Gravity-induced Λ-term



N. Kardashev (1997):

Only NPT QCD vacuum at lowest (hadron) scales of Particle Physics gives rise to observable Λ-term

All perturbative vacuum components are cancelled???

Quasiclassical (semiquantum) gravity

Action $S = \int L d^4x, \qquad L = -\frac{1}{2\varkappa}\sqrt{-\hat{g}}\hat{g}^{ik}\hat{R}_{ik} + L\left(\hat{g}^{ik}, \chi_A\right)$

Metric operator $\hat{g}^{ik}$

Macroscopic geometry (c-number part) g^{ik}

Quantum graviton field Φ_i^k

$$\langle 0|\Phi_i^k|0\rangle = 0$$

Heisenberg state vector containing info about initial states of all fields exists!

Independent variations over classical and quantum fields:

$$\delta \int L d^4x = -\frac{1}{2}\int d^4x \left(\sqrt{-g}\delta g^{ik}\hat{G}_{ik}\right)_{\Phi_i^k = \mathrm{const}}$$
$$= -\frac{1}{2}\int d^4x \left(\sqrt{-g}\delta\Phi^{ik}\hat{G}_{ik}\right)_{g^{ik} = \mathrm{const}}$$

Averaging over initial states

same operator eqns:

$$\hat{G}_i^k = \frac{1}{2}\left(\delta_l^k\delta_i^m + g^{km}g_{il}\right)\left(\frac{\hat{g}}{g}\right)^{1/2}\hat{E}_m^l = 0,$$
$$\hat{E}_m^l = \frac{1}{\varkappa}\left(\hat{g}^{lp}\hat{R}_{pm} - \frac{1}{2}\delta_m^l\hat{g}^{pq}\hat{R}_{pq}\right) - \hat{g}^{lp}\hat{T}_{pm}\left(\hat{g}^{ik}, \chi_A\right)$$

e.o.m. for macroscopic geometry

$$\langle 0|\hat{G}_i^k|0\rangle = 0$$

e.o.m. for graviton field

$$\hat{G}_i^k - \langle 0|\hat{G}_i^k|0\rangle = 0$$

Metric fluctuations in exponential parameterization

Independent variations fix the exponential parameterization:

$$\sqrt{-\hat{g}}\hat{g}^{ik} = \sqrt{-g}g^{il}(\exp\psi)_l^k = \sqrt{-g}g^{il}\left(\delta_l^k + \psi_l^k + \tfrac{1}{2}\psi_l^m\psi_m^k + \ldots\right) \qquad \psi_i^k = \Phi_i^k - \frac{1}{2}\delta_i^k\Phi$$

up to quadratic terms in graviton field we get:

$$\hat{G}_i^k = \frac{1}{2\varkappa}\left(\psi_{i;l}^{k;l} - \psi_{i;l}^{l;k} - \psi_{l;i}^{k;l} + \delta_i^k\psi_{l;m}^{m;l} + \psi_i^l R_l^k + \psi_l^k R_i^l - \delta_i^k\psi_l^m R_m^l\right) + \frac{1}{\varkappa}\left(R_i^k - \frac{1}{2}\delta_i^k R\right) - \hat{T}_i^k$$

where $$\hat{T}_i^k = \hat{T}_{i(G)}^k + \frac{1}{2}\left(\delta_l^k\delta_i^m + g^{km}g_{il}\right)\left(\frac{\hat{g}}{g}\right)^{1/2}\hat{g}^{lp}\hat{T}_{pm}\left(\hat{g}^{ik},\chi_A\right)$$

with energy-momentum tensor of gravitons

$$\hat{T}_{i(G)}^k = \frac{1}{4\varkappa}\left(\psi_{m;i}^l\psi_l^{m;k} - \frac{1}{2}\psi_{;i}\psi^{;k} - \psi_{i;m}^l\psi_l^{m;k} - \psi_l^{k;m}\psi_{m;i}^l\right) - \frac{1}{8\varkappa}\delta_i^k\left(\psi_{m;n}^l\psi_l^{m;n} - \frac{1}{2}\psi_{;n}\psi^{;n} - 2\psi_{n;m}^l\psi_l^{m;n}\right)$$

$$-\frac{1}{4\varkappa}\left(2\psi_n^l\psi_i^{k;n} - \psi_n^k\psi_i^{l;n} - \psi_i^n\psi^{kl}{}_{;n} + \psi_i^{n;k}\psi_n^l + \psi_{n;i}^k\psi^{nl} + \delta_i^k\left(\psi_m^n\psi_n^l\right)^{;m}\right)_{;l}$$
$$-\frac{1}{4\varkappa}\left(\psi_i^m\psi_n^l R_l^k + \psi_n^k\psi_l^n R_i^l - \delta_i^k\psi_l^n\psi_n^m R_m^l\right) + O(\psi^3).$$

e.o.m. for macroscopic geometry $$\frac{1}{\varkappa}\left(R_i^k - \frac{1}{2}\delta_i^k R\right) = \langle 0|\hat{T}_i^k|0\rangle$$

e.o.m. for graviton field $$\psi_{i;l}^{k;l} - \psi_{i;l}^{l;k} - \psi_{l;i}^{k;l} + \delta_i^k\psi_{l;m}^{m;l} + \psi_i^l R_l^k + \psi_l^k R_i^l - \delta_i^k R_l^m\psi_m^l = 2\varkappa\left(\hat{T}_i^k - \langle 0|\hat{T}_i^k|0\rangle\right)$$

Gluodynamics with vacuum anomaly

Let us now include non-perturbative gluon and quark fields fluctuations into quasiclassical gravity theory!

We need energy-momentum tensor for NPT vacuum fluctuations with conformal anomalies!

Classical conformal symmetry (under rescalings of the background metric and simultaneously fields) is broken by quantum gravity effects!

Basic recipe:

$$\mathcal{A}_i^a = g_s A_i^a$$

$$\mathcal{F}_{ik}^a = \partial_i \mathcal{A}_k^a - \partial_k \mathcal{A}_i^a + f^{abc} \mathcal{A}_i^b \mathcal{A}_k^c$$

stress tensor operator

Chromodynamical coupling has to be considered as an operator depending on operators of quantum fields through RG evolution equation

$$2J \frac{dg_s^2(J)}{dJ} = g_s^2(J)\beta[g_s^2(J)]$$

operator RG equation

$$J = \mathcal{F}_{ik}^a \mathcal{F}_a^{ik}$$

Invariant operator of least dimension

Operator gluodynamics with conformal anomaly:

$$\boxed{L_{eff} = -\frac{1}{4g_s^2(J)} \mathcal{F}_{ik}^a \mathcal{F}_a^{ik}}$$

can now be incorporated into quasiclassical gravity! (after covariant generalization)

$$\begin{cases} \hat{T}_{i(g)}^k = \dfrac{1}{g_s^2(J)} \left(-\mathcal{F}_{il}^a \mathcal{F}_a^{kl} + \dfrac{1}{4} \delta_i^k \mathcal{F}_{ml}^a \mathcal{F}_a^{ml} + \dfrac{\beta[g_s^2(J)]}{2} \mathcal{F}_{il}^a \mathcal{F}_a^{kl} \right) \\ D_k^{ab} \left\{ g_s^{-2}(J) \left(1 - \dfrac{\beta[g_s^2(J)]}{2} \right) \mathcal{F}_b^{ik} \right\} = 0, \\ D_k^{ab} = \delta^{ab} \partial_k - f^{abc} \mathcal{A}_k^c. \end{cases}$$

QCD vacuum energy-momentum tensor

To **one-loop approximation:** $\beta[g_s^2(J)] = -\frac{b g_s^2(J)}{16\pi^2}, \qquad \frac{g_s^2(J)}{4\pi} \equiv \alpha_s(J) = \frac{8\pi}{b\ln(J/\lambda^4)}$

An account for **quarks changes b-factor**! $\quad b = b(3) = 9$

Phenomenology provides with:

correlation length of fluctuations!

$$\langle 0| : \bar{s}s : |0\rangle \simeq \langle 0| : \bar{u}u : |0\rangle = \langle 0| : \bar{d}d : |0\rangle = -\langle 0| : \frac{\alpha_s}{\pi} F^a_{ik} F^{ik}_a : |0\rangle L_g = -(225 \pm 25\ \mathrm{MeV})^3$$

the **same for quantum-wave fluctuations**!

$$L_g \simeq (1500 \pm 300\ \mathrm{MeV})^{-1}$$

summing gluon and quark contributions:

$$\hat{T}^k_{i(QCD)} = \frac{b_{eff}}{32\pi^2}\left(-\mathcal{F}^a_{il}\mathcal{F}^{kl}_a + \frac{1}{4}\delta^k_i \mathcal{F}^a_{ml}\mathcal{F}^{ml}_a\right)\ln\frac{eJ}{\lambda^4} - \delta^k_i \frac{b_{eff}}{128\pi^2}\mathcal{F}^a_{ml}\mathcal{F}^{ml}_a$$

$$b_{eff} = b(3) + 8L_g(m_u + m_d + m_s) \simeq 9.6$$

instead of 11 for pure gluodynamics!

to a good approximation:

$$\ln\frac{eJ}{\lambda^4} = \ln\frac{e\langle 0|J|0\rangle}{\lambda^4} + \cancel{\frac{J - \langle 0|J|0\rangle}{\langle 0|J|0\rangle}} + \ldots \qquad \ln\frac{e\langle 0|J|0\rangle}{\lambda^4} = 4\ln\frac{L_g^{-1}}{\Lambda_{QCD}}$$

coming back to original fields:

Averaged conformal anomaly!

$$\boxed{\hat{T}^k_{i(QCD)} = \frac{b_{eff}\alpha_s}{2\pi}\left(-F^a_{il}F^{kl}_a + \frac{1}{4}\delta^k_i F^a_{ml}F^{ml}_a\right)\ln\frac{L_g^{-1}}{\Lambda_{QCD}} - \delta^k_i \frac{b_{eff}}{32}\langle 0|\frac{\alpha_s}{\pi}F^a_{ml}F^{ml}_a|0\rangle}$$

Λ-term calculation

We start from the Einstein equations for macroscopic geometry:

$$\frac{1}{\varkappa}\left(R_i^k-\frac{1}{2}\delta_i^k R\right)=\langle 0|\hat{T}_i^k|0\rangle \qquad \hat{T}_i^k=\hat{T}_{i(G)}^k+\frac{1}{2}\left(\delta_l^k\delta_i^m+g^{km}g_{il}\right)\left(\frac{\hat{g}}{g}\right)^{1/2}\hat{g}^{lp}\hat{T}_{pm}\left(\hat{g}^{ik},\chi_A\right)$$

Trace: $R+4\varkappa\Lambda=0$

$$\boxed{\Lambda=-\frac{b_{eff}}{32}\langle 0|\frac{\alpha_s}{\pi}\left(\frac{\hat{g}}{g}\right)^{1/2}\hat{g}^{il}\hat{g}^{km}\hat{F}_{ik}^a\hat{F}_{lm}^a|0\rangle+\frac{1}{4}\langle 0|\hat{T}_{(G)}|0\rangle}$$

Stress tensor in Riemann space is found from YM eqs:

$$\left(\delta^{ab}\frac{\partial}{\partial x^k}-g_s f^{abc}\hat{A}_k^c\right)\sqrt{-\hat{g}}\hat{g}^{il}\hat{g}^{km}\hat{F}_{lm}^b=0$$

$$\hat{F}_{ik}^a=F_{ik}^a+\underbrace{\frac{1}{2}\psi F_{ik}^a-\psi_i^l F_{lk}^a-\psi_k^l F_{il}^a}_{\text{induce interactions of YM field with metric fluctuations}}+O(\alpha_s G)$$

Equation for gravitons turns into:

$$\psi_{i,\,l}^{k,\,l}-\psi_{i,\,l}^{l,\,k}-\psi_{l,\,i}^{k,\,l}+\delta_i^k\psi_{l,\,m}^{m,\,l}=\frac{\varkappa b_{eff}\alpha_s}{\pi}\left(-F_{il}^a F_a^{kl}+\frac{1}{4}\delta_i^k F_{ml}^a F_a^{ml}\right)\ln\frac{L_g^{-1}}{\Lambda_{QCD}}$$

After exact cancellation of unperturbed part of EMT tensor we get:

$$\boxed{\Lambda=-\frac{b_{eff}}{16}\ln\frac{L_g^{-1}}{e\Lambda_{QCD}}\langle 0|\frac{\alpha_s}{\pi}F_{il}^a F_a^{kl}\left(\psi_k^i-\frac{1}{4}\delta_k^i\psi\right)|0\rangle}$$

Λ-term calculation

Fock gauge: $$\psi^{k}_{i;\,k} = 0$$

Metric fluctuations are induced by QCD vacuum fluctuations!

Exact solution of graviton equation:

$$\psi^k_i(x) = \varkappa b_{eff} \ln\frac{L_g^{-1}}{\Lambda_{QCD}} \int d^4x' \underbrace{\mathcal{G}(x-x')}_{} \times \overbrace{\left(\frac{\alpha_s}{\pi} F^a_{il}(x') F^{kl}_a(x') - \delta^k_i \frac{\alpha_s}{4\pi} F^a_{ml}(x') F^{ml}_a(x')\right)}$$

Green function: $$\mathcal{G}_{;\,l}^{;\,l} = -\delta(x-x')$$

After explicit calculation of averages, we get

$$\boxed{\Lambda = -\pi G \langle 0| : \frac{\alpha_s}{\pi} F^a_{ik} F^{ik}_a : |0\rangle^2 \times \left(\frac{b_{eff}}{8}\right)^2 \ln\frac{L_g^{-1}}{e\Lambda_{QCD}} \ln\frac{L_g^{-1}}{\Lambda_{QCD}} \int d^4y\, \mathcal{G}(y) D^2(y) = (1 \pm 0.5) \times 10^{-29} \Delta \text{ MeV}^4.}$$

where

$$\langle 0|\frac{\alpha_s}{\pi} F^a_{ik}(x) F^{ik}_a(x')|0\rangle = \langle 0| : \frac{\alpha_s}{\pi} F^a_{ik}(0) F^{ik}_a(0) : |0\rangle D(x-x'),$$

$$\Delta = -\frac{1}{L_g^2} \int d^4y\, \mathcal{G}(y) D^2(y)$$

$$D(x-x') = D_{top}(x-x') - D_h(x-x'), \qquad D(0) = 0.$$

must be established in a dynamical theory of NPT QCD vacuum!

It is expected to be generated by chiral symmetry breaking

In terms of known NPT QCD parameters

$$1/L_{top} \sim 1/L_h \sim 1/L_g,$$

$$|1/L_{top} - 1/L_h| \sim m_u + m_d + m_s$$

$$\boxed{\Delta = k \cdot \frac{(m_u + m_d + m_s)^2 L_g^2}{(2\pi)^4} \sim 3 \cdot 10^{-6}}$$

!!!

Summary

Physically reasonable conditions for ZSK scenario are formulated.

- **Perturbative part of the Physical Vacuum should exactly be compensated at every known energy scales separately as a necessary requirement for the "Theory of Everything" (exact SUSY is the only known way to avoid the major fine tuning of vacuum parameters)**

- **QCD vacuum is a special case: has strongest non-perturbative component (responsible e.g. for color confinement) at lowest Particle Physics energy scale (~200 MeV)**

- **Large negative NPT (topological) QCD vacuum component can be canceled (1) by ground state energy of cosmological YM field, and/or (2) by collective (hadron) quantum-wave fluctuations in the hadron vacuum**

- **A small quantum gravity correction to NPT QCD vacuum fluctuations (graviton exchanges in the vacuum) induces an uncompensated Λ-term. Our estimate based upon phenomenological parameters reproduces the observable value. Uncertainties are due to unknown non-perturbative dynamics of the QCD vacuum.**