Transport Models for Galactic Cosmic Rays
Results of a Markov-Chain-Monte-Carlo Study

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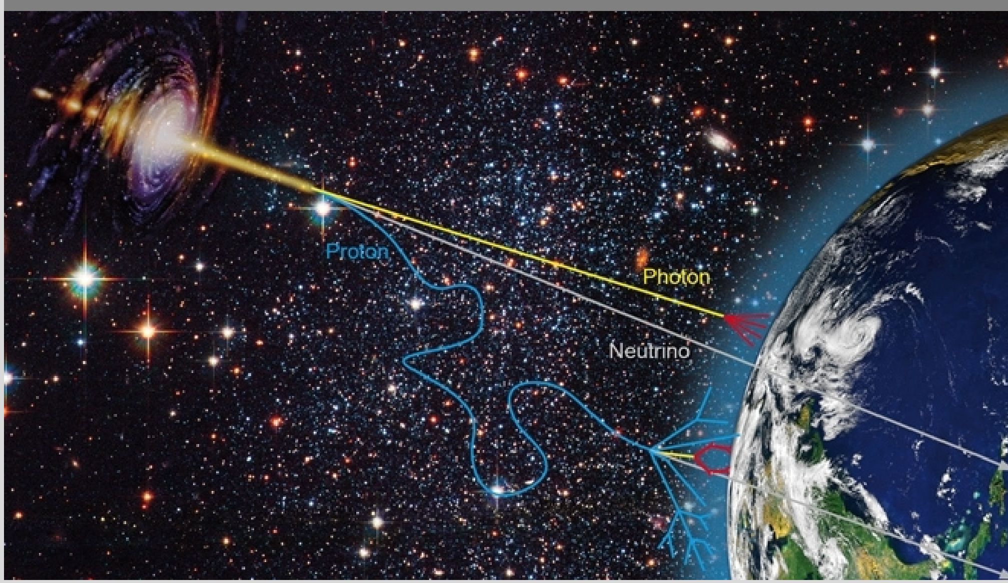
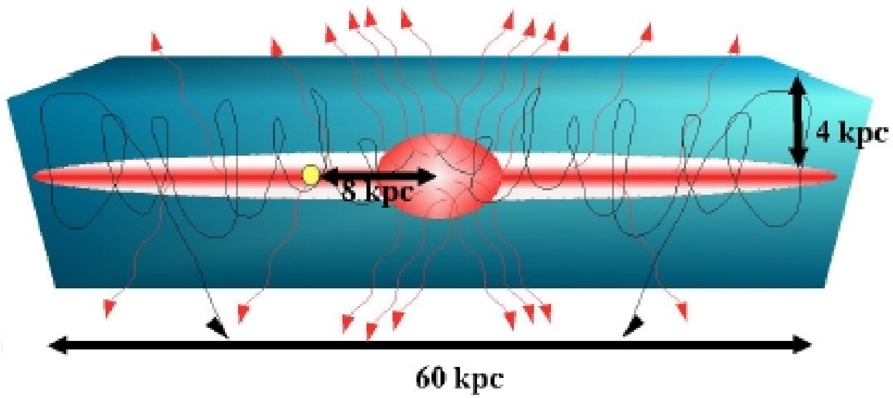


Transport models

- Aim: Reproduction of the locally measured spectra of cosmic rays
- Numerical solution of the transport equation (e.g. GalProp, Dragon)

$$\frac{\partial \Psi}{\partial t} = q(\vec{r}, p) + \vec{\nabla} \cdot (D_{xx} \vec{\nabla} \Psi - \vec{V} \Psi) + \frac{\partial}{\partial p} p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} \Psi - \frac{\partial}{\partial p} [\dot{p} \Psi - \frac{p}{3} (\vec{\nabla} \cdot \vec{V}) \Psi] - \frac{1}{\tau_f} \Psi - \frac{1}{\tau_r} \Psi$$



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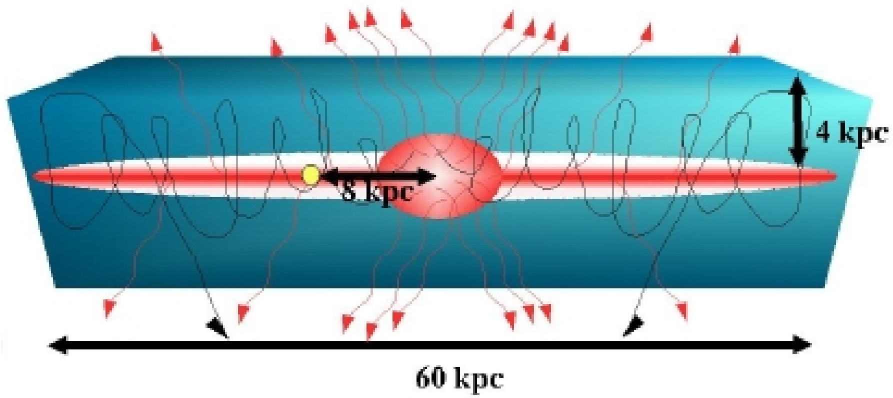
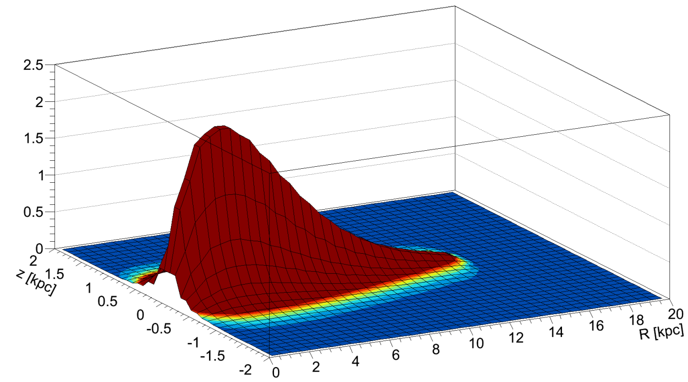


- *Source distribution*



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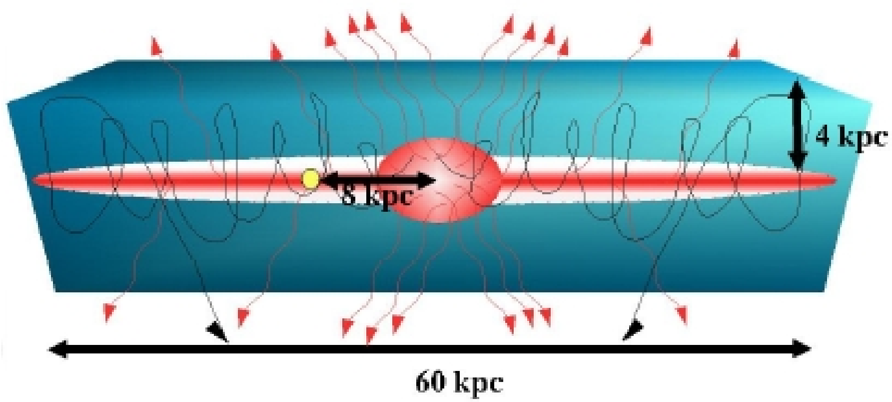
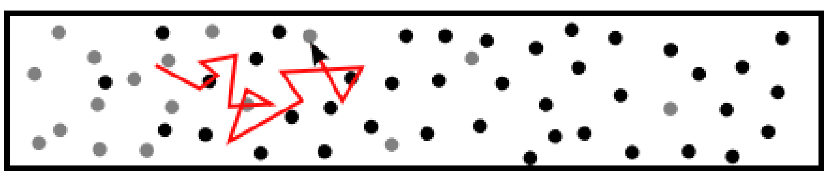


- *Source distribution*
- *Diffusion*



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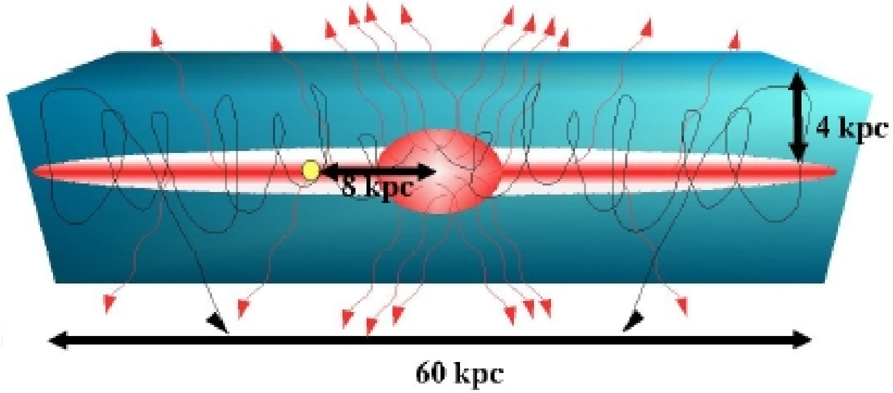
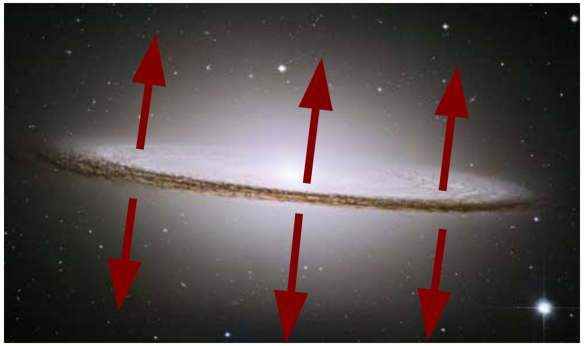


- *Source distribution*
- *Diffusion*
- *Convection (galactic winds)*



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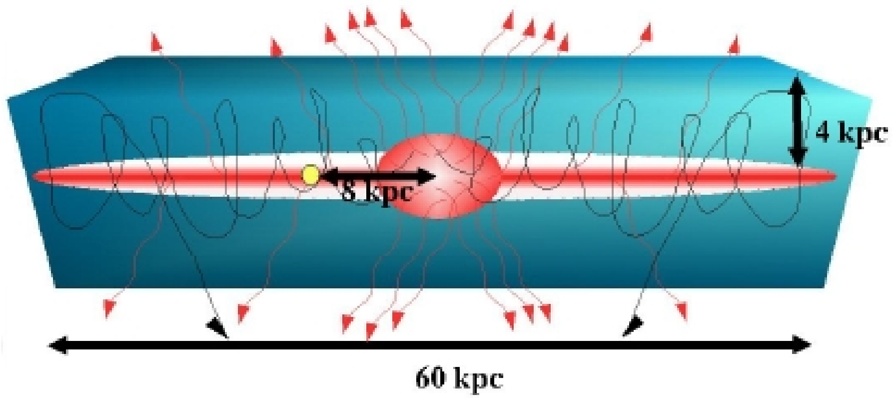
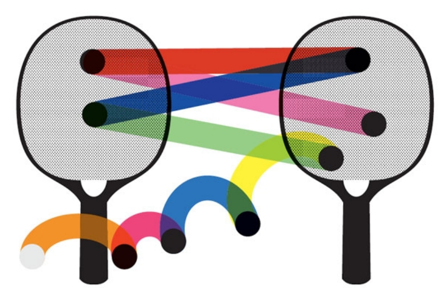


- *Source distribution*
- *Diffusion*
- *Convection (galactic winds)*
- *Diffusive reacceleration*



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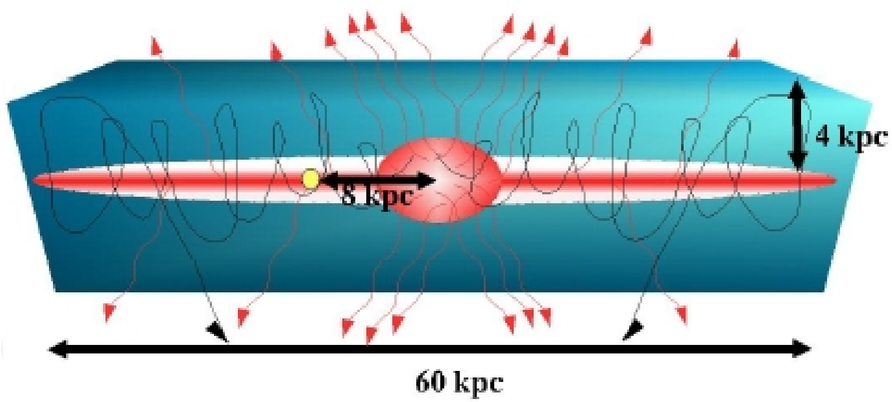
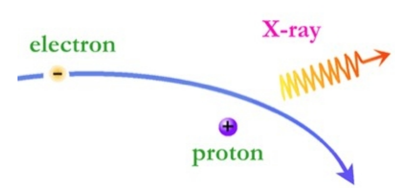


- *Source distribution*
- *Diffusion*
- *Convection (galactic winds)*
- *Diffusive reacceleration*
- *Momentum losses* | *particle losses*



Bremsstrahlung

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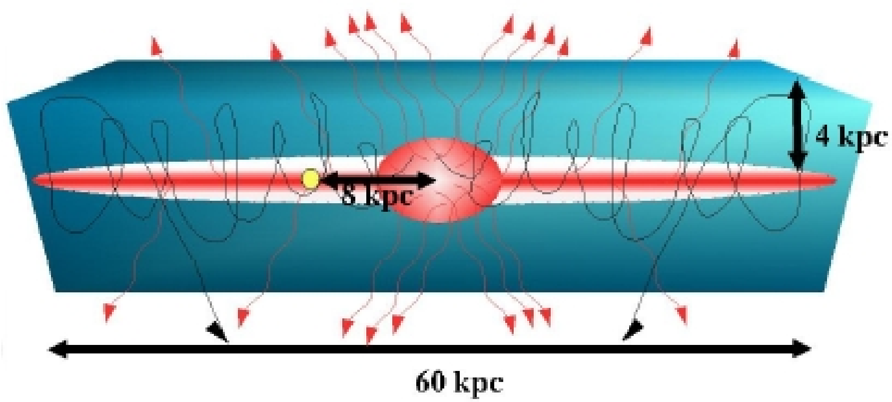

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→ **Many unknown parameters! (16)**

Fitness of model by comparing results with experimental data: $p, \bar{p}, \frac{\bar{p}}{p}, \frac{B}{C}, \frac{Be^{10}}{Be^{9}}$

Why use MCMC and not a simple minimizing algorithm?

→ not feasible for such high dimensional problems!
Evaluation of a single model can take minutes to hours

→ We do **NOT** want to find a best fit model
Instead, want to explore wide ranges of parameter space (exotic models?)
Examine the full potential of these kind of models

→ Use MCMC in order to **sample** parameter space in an efficient way

Markov-Chain-Monte-Carlo

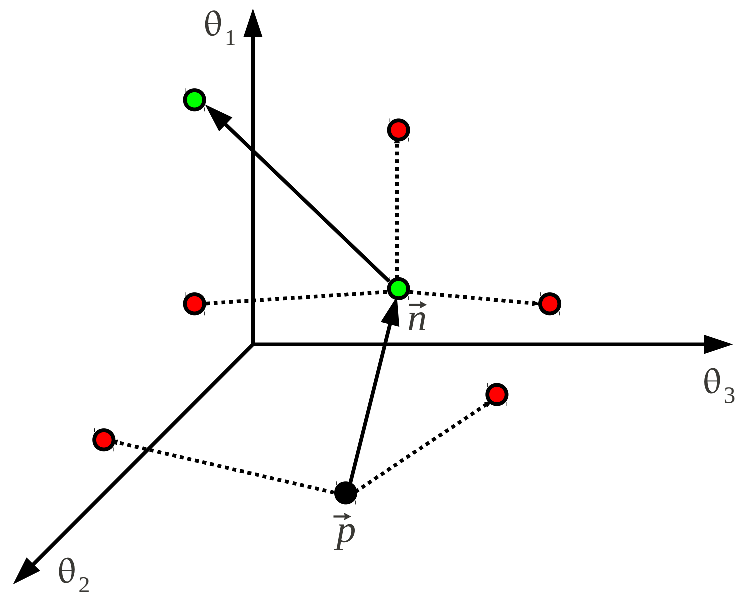
Decider

- Metropolis-Hastings-Algorithm

  (Metropolis, N., Rosenbluth, A.W. , Rosenbluth , M.N. Teller, A.H., 1953)

Acceptance criterion by comparing 2 models

$$\alpha \propto \frac{F(n)}{F(p)}$$



Density of chain points ~ Fitness!

Markov-Chain-Monte-Carlo

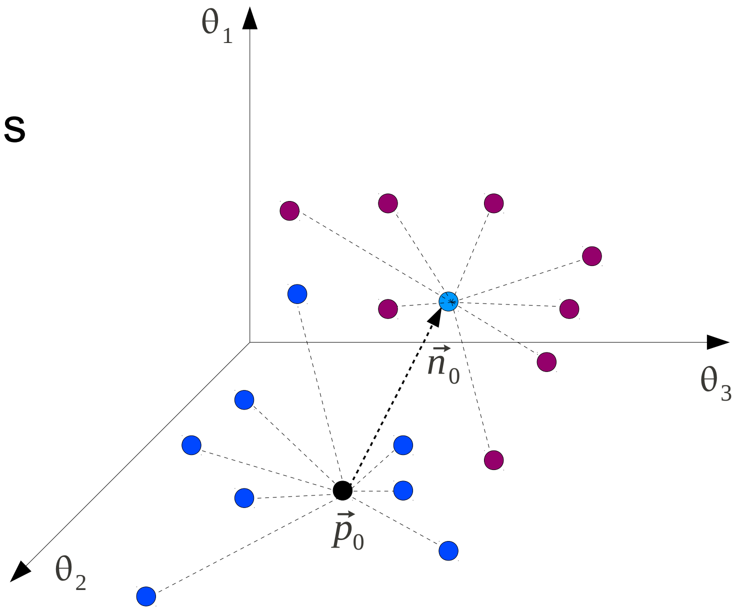
Decider

- Multiple-Try-Metropolis-Algorithm (MTM)
  (Liu, J. S., Liang, F. and Wong, W. H., 2000)

Acceptance criterion by comparing 2 **sets** of models

$$\alpha = min\left(1, \frac{F(p_1)+...+F(p_k)}{F(n_1)+...+F(n_{k-1})+F(p_0)}\right)$$

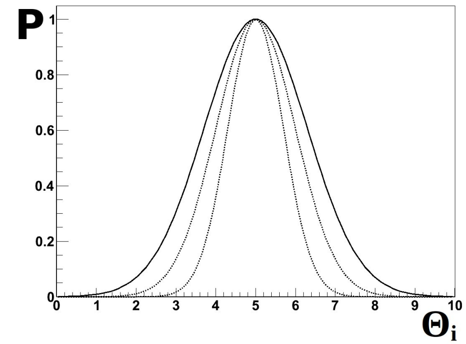
→ Use much more information of parameter space
→ Allows evaluation of several models in parallel



Karlsruher Institut für Technologie

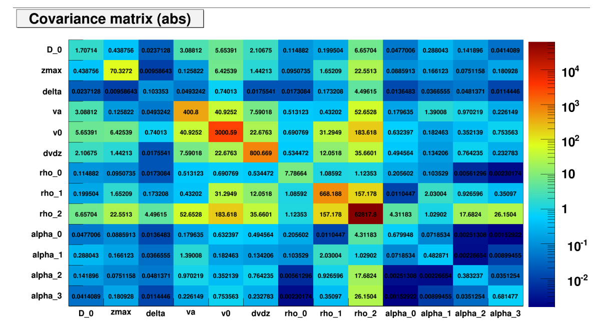
Markov-Chain-Monte-Carlo

Proposal functions (A. Putze et al. 1001.0551)



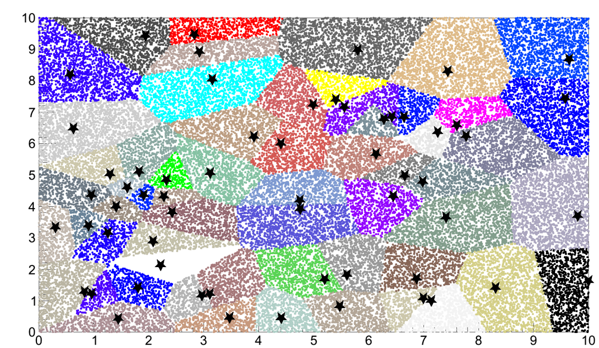
I. **Gaussian function** in every dimension

- Step size (σ) must be given
- Most probable point = current point



II. Multivariate Normal Distribution

- build empirical covariance matrix based on chain points of I.
- Correlations/step sizes are given



III. Next Neighbour (NN) Search

- Propose points according to the binned fitness distribution of all known points

Results

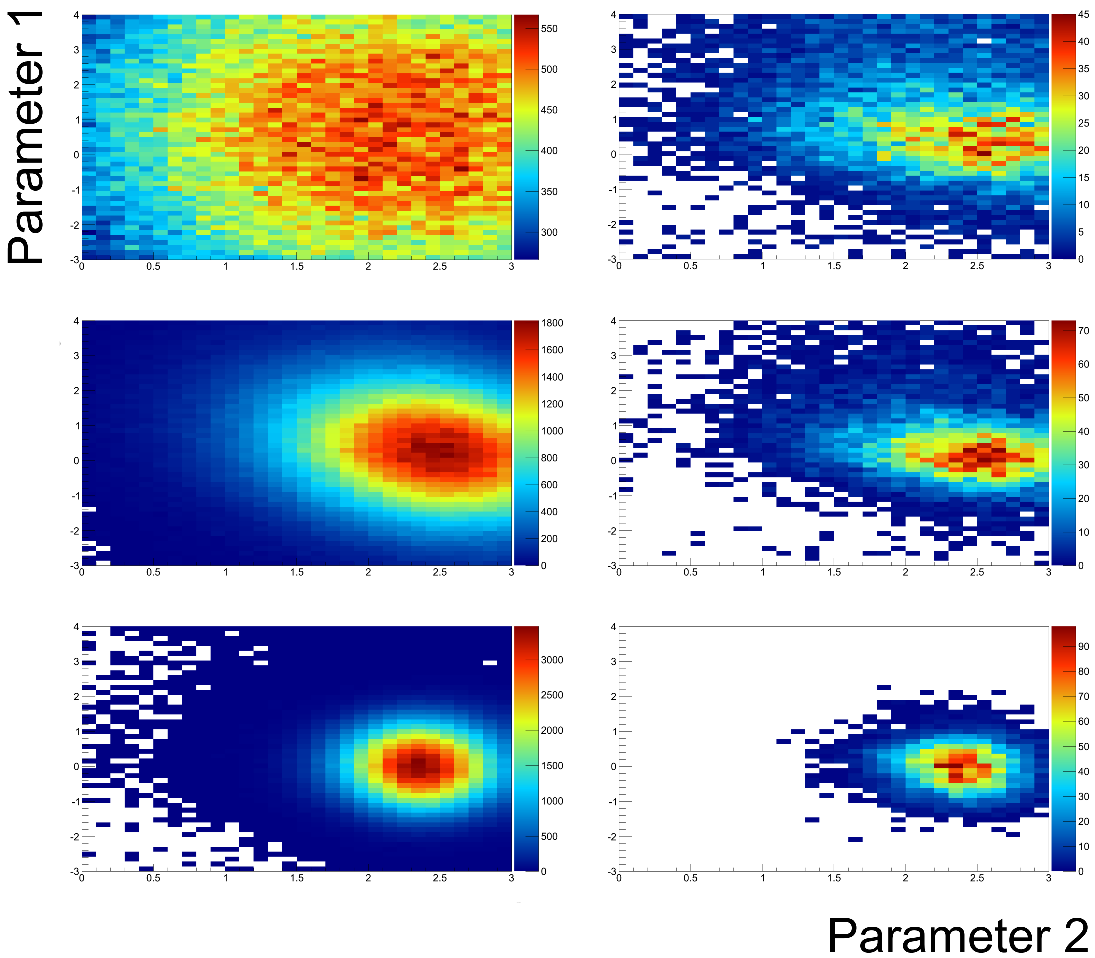
Efficiency

Proposed points **chain points**

I. Gaussian function

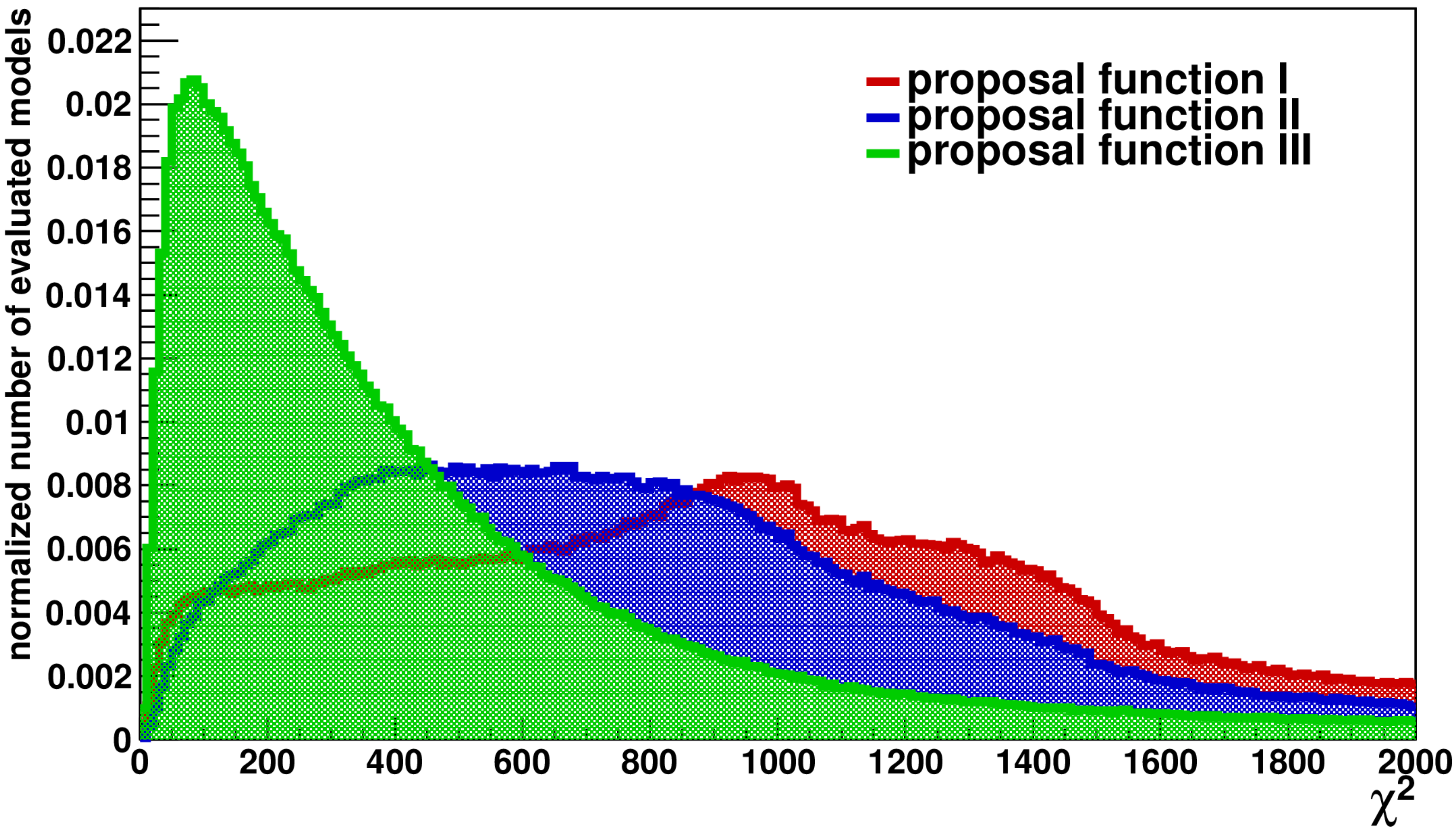
II. Multiv. Norm. Distr.

III. NN Search



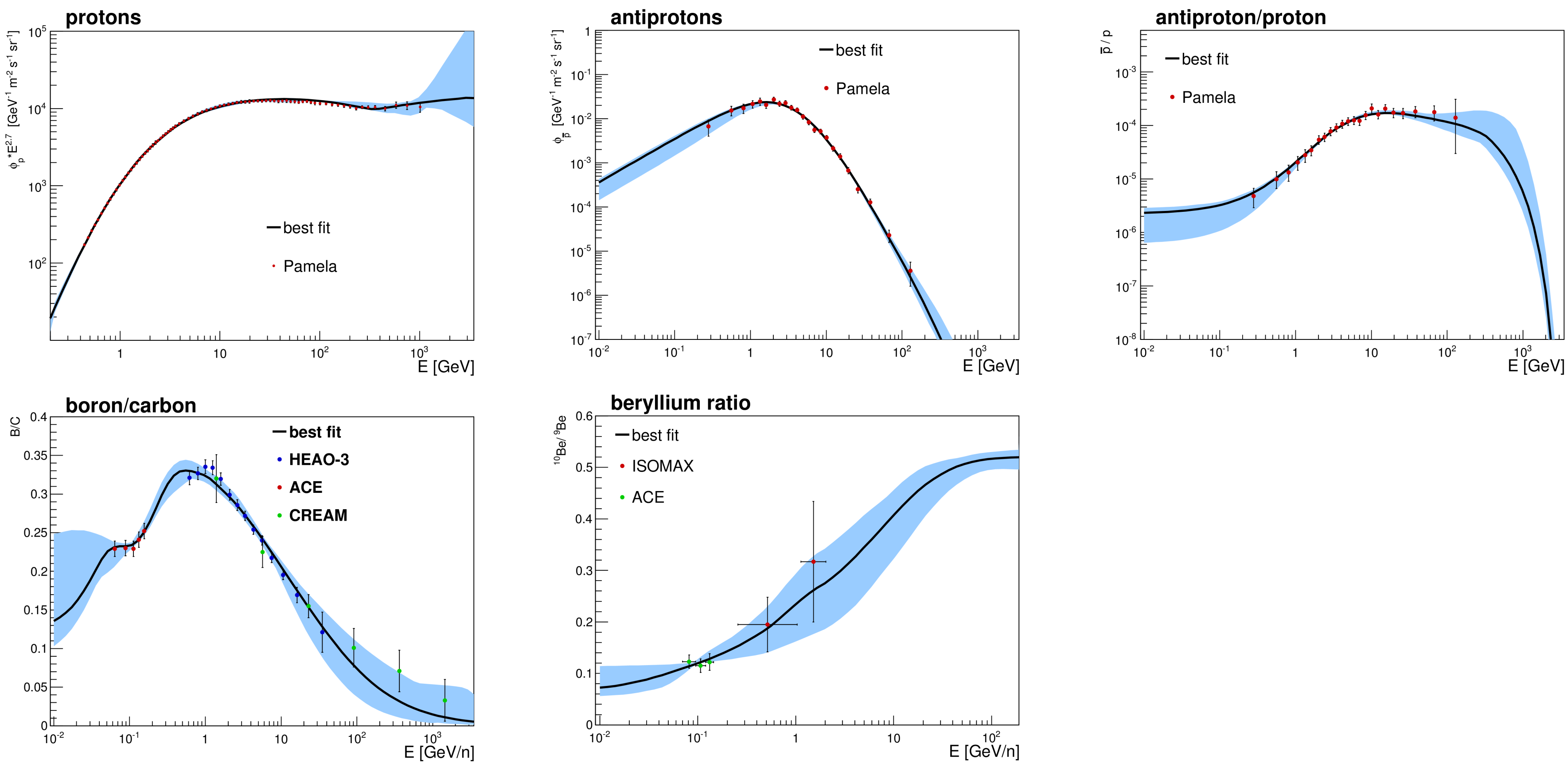
Results

Efficiency

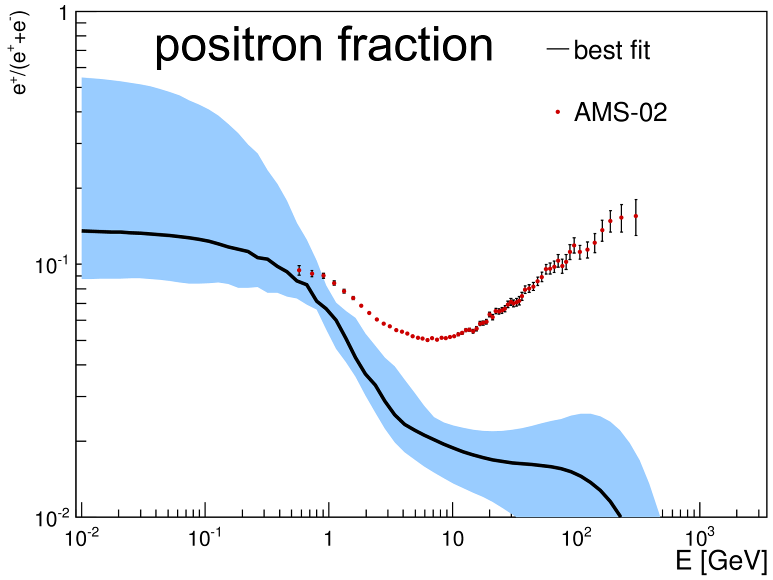


Results

→ more than 10 million models have been evaluated
> 1 year with 500 markov chains, running simultaneously



What about leptons?

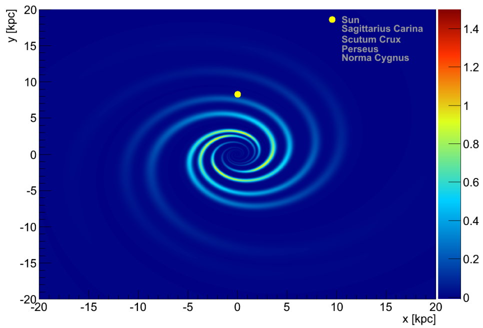


Assume local transport = global transport

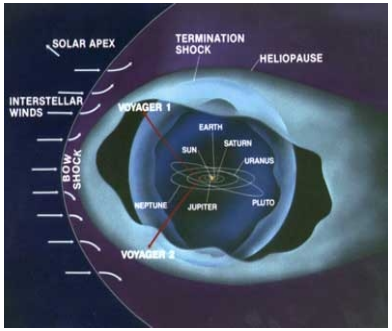
Positron fraction is impossible to mimic with a pure secondary production of positrons

Most favored sources:
- Pulsars
- Dark Matter annihilation

→ large energy losses due to synch. radiation and inv. compton scattering
→ short propagation lengths of O(500pc)
→ *local transport,* highly affected by the local environment



Spiral arm structure of the Milky Way



The local bubble

Separate study, *see Gebauer, SK, Weinreuter ICRC13*