Elementary particles for astrophysicists

Graciela Gelmini - UCLA

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Outline I assume that you know QM and some Special Relativity

- The foundation of the SM: Klein Gordon and Dirac equations, Chirality, Antiparticles, Quantum Field Theory
- Gauge symmetry and the development of the gauge theories in the SM: QED, Electroweak, QCD
- The Higgs mechanism
- The Standard Model
- The origin of mass in the SM
- Problems of the SM. Ideas for Beyond the SM physics

The Standard Model is the THEORY of elementary particles and their interactions (excluding gravity). It is embodied in a remarkably concise unified description of electromagnetic, weak and strong interactions. It is a renormalizable relativistic quantum field theory with a gauge symmetry, part of it spontaneously broken by the "Higgs mechanism", and the following particles

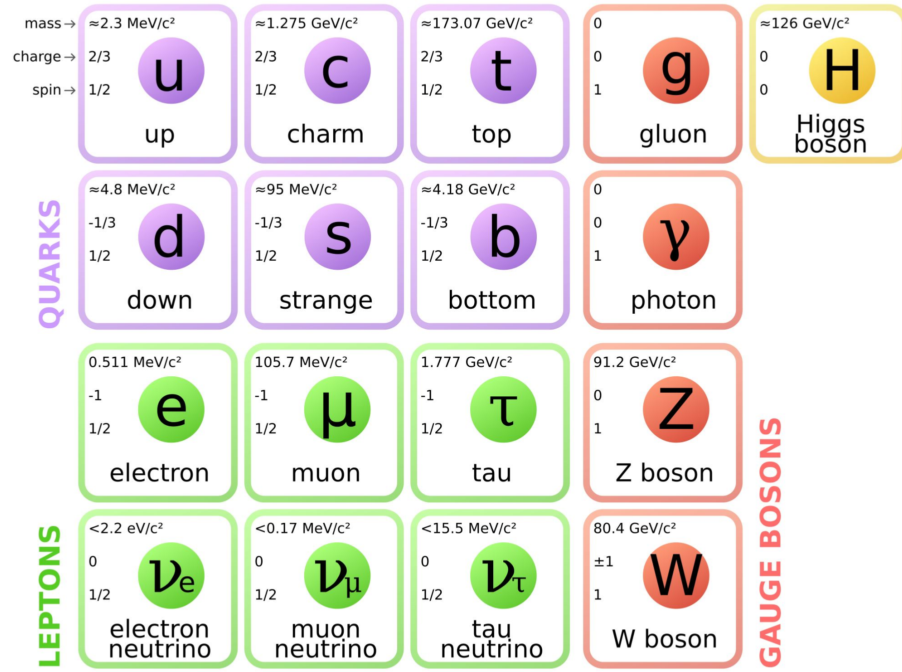
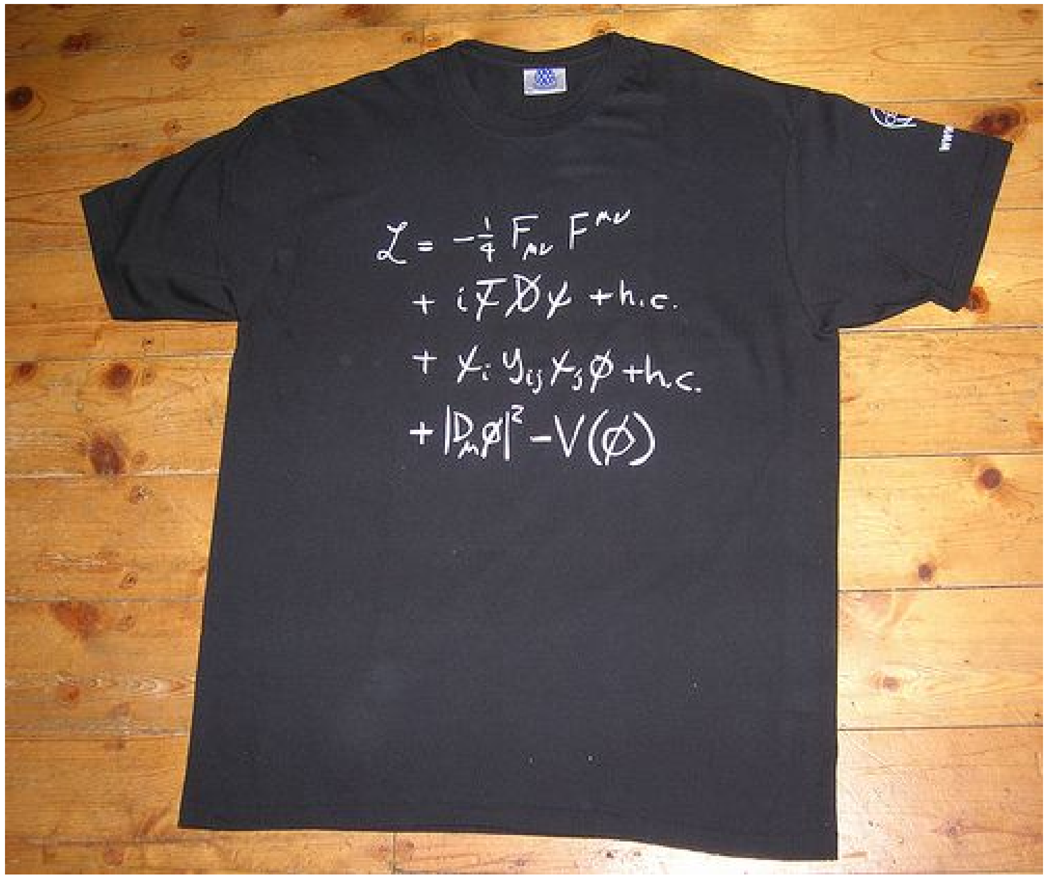


Fig. from wikipidia, Standard Model

The standard model

a unified description concise enough to fit on a t-shirt!



The first interactions- examples of “Unification”

Gravity: described first by Isaac Newton in his Principia, 1669. He introduced also the concept of “force”.

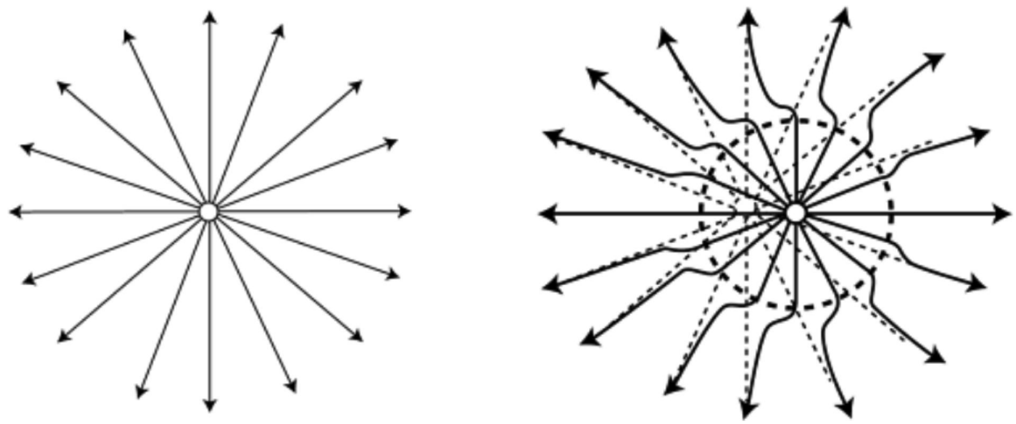
1st example of “unification”: same force responsible for projectile motion (Galileo Galilei) and for the motion of planets (Johanes Kepler)

Electromagnetism: electricity (electron = amber in Greek) and magnetism (Magnesia is a greek city) known at least since VI bc.

Only in 1820 H. C Oersted and in 1831 M. Faraday saw they are related-
M Faraday and J.M. Maxwell:“unification” into electromagnetism

Classical fields

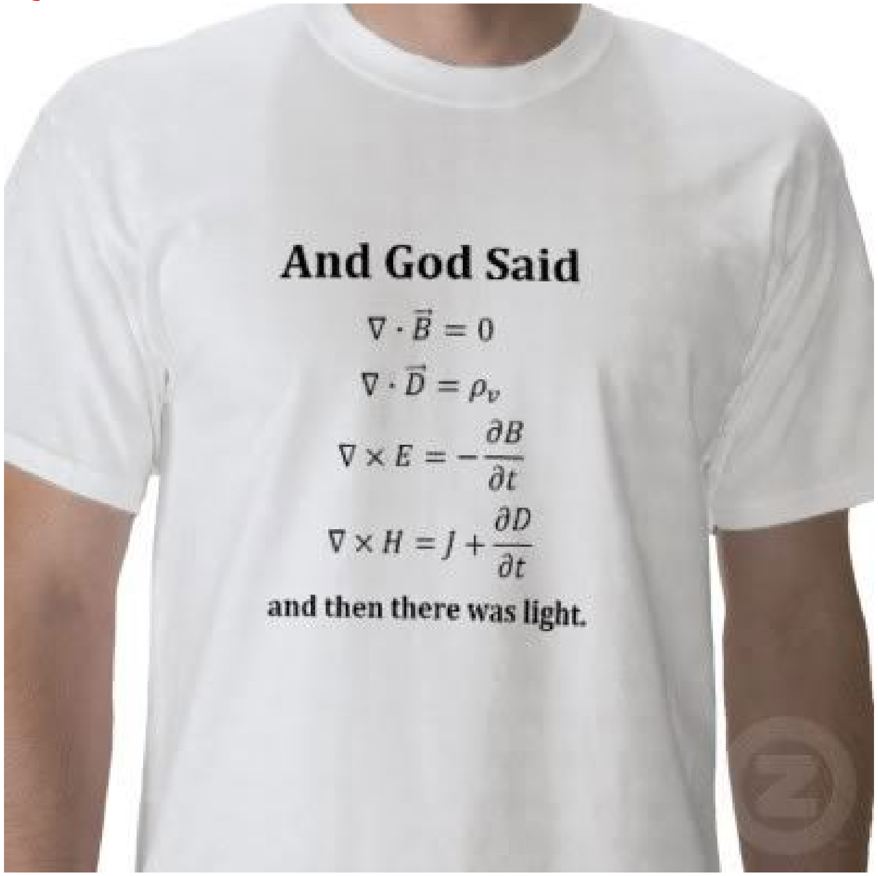
Faraday invented the concept of "field" first visualizing it with "force lines". Conceptual change: interactions are local, signals carried by perturbations in the field (QM "wave-particle duality": signal carried by "exchanged particles")



James Cleck Maxwell, 1865: equations for the electric **E** and magnetic **B** fields; predict electromagnetic waves with only one speed in vacuum c. Lead to

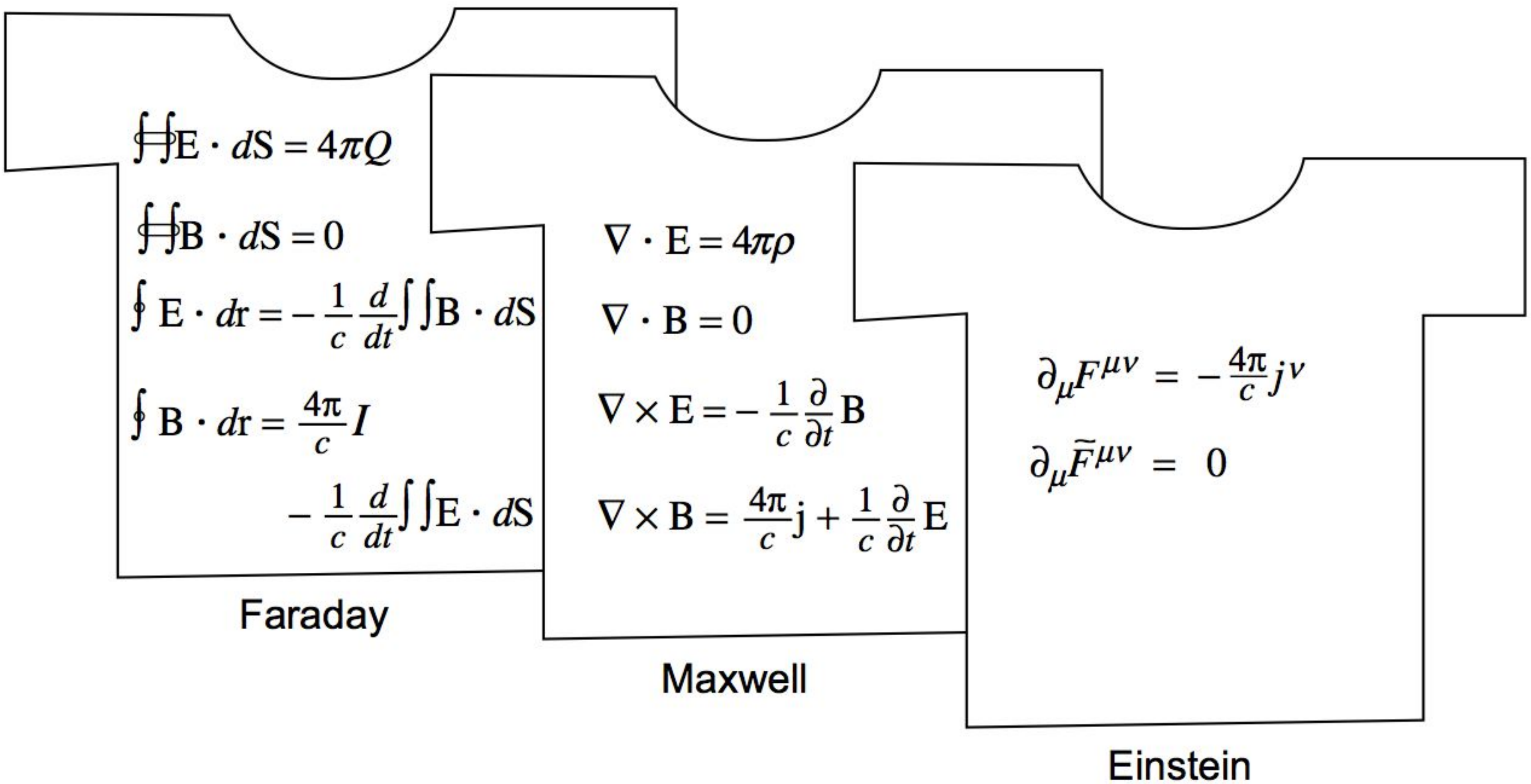
Special Relativity (Einstein 1905): space-time $x^\mu = (ct, \vec{x})$, $p^\mu = (E/c, \vec{p})$ for $\mu = 0, 1, 2, 3$, $E^2 = m^2c^4 + |\vec{p}|^2c^2$, if $m = 0$ $E = |\vec{p}|c$, if $m \neq 0$ $v < c$

Classical Electrodynamics on a t-shirt



A not manifestly covariant God!

Classical Electrodynamics



(from K.Huang: "sophomore, senior and graduate student e.m. t-shirts")

Classical Electrodynamics

units so that $c = \mu_0 = \varepsilon_0 = 1$ (so force between q_1 and q_2 is $F = \frac{q_1 q_2}{4\pi r^2}$) Maxwell's equations are

$$\begin{aligned} \vec{\nabla} \cdot \vec{E} &= \rho \qquad \vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{J} \\ \vec{\nabla} \cdot \vec{B} &= 0 \qquad \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \end{aligned}$$

Potential 4-vector $A^\mu = (\varphi, \vec{A})$: $\vec{B} = \vec{\nabla} \times \vec{A}$ and $\vec{E} = -\vec{\nabla}\varphi - \frac{\partial \vec{A}}{\partial t}$ and the **Electromagnetic Field Strength Tensor** $F^{\mu\nu}$ as

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

Maxwell's eqs. are: $\partial_\mu F^{\mu\nu} = J^\nu$; $\partial_\mu \widetilde{F^{\mu\nu}} = 0$

(the dual tensor $\widetilde{F^{\mu\nu}} \equiv \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$ has $\vec{E} \rightarrow \vec{B}$, $\vec{B} \rightarrow -\vec{E}$ and 4-vector current is $J^\mu = (\rho, \vec{J})$)

Lagrangian In Classical Mechanics, for particles $L \equiv T - V$ where $T = T(q_i, \dot{q}_i)$ and $V = V(q_i)$

Action (a functional) $S \equiv \int_{t_i}^{t_f} dt L(q_i, \dot{q}_i)$ and $0 = \delta S$ implies the
Euler-Lagrange equations (E.L.)

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0$$

For fields $\varphi_i(\vec{x}, t) = \varphi_i(x^\mu)$, $S = \int dt d^3x \mathscr{L} = \int d^4x \mathscr{L}$ and the relativistic generalization of E.L eq. for multiple fields φ_i $(i = 1, \ldots, n)$ is

$$\partial_\mu \left(\frac{\partial \mathscr{L}}{\partial(\partial_\mu \varphi_i)} \right) - \frac{\partial \mathscr{L}}{\partial \varphi_i} = 0$$

Maxwell's eqs are the E.L. eqs. of the Lagrangian density $(\varphi_i \to A_\mu)$

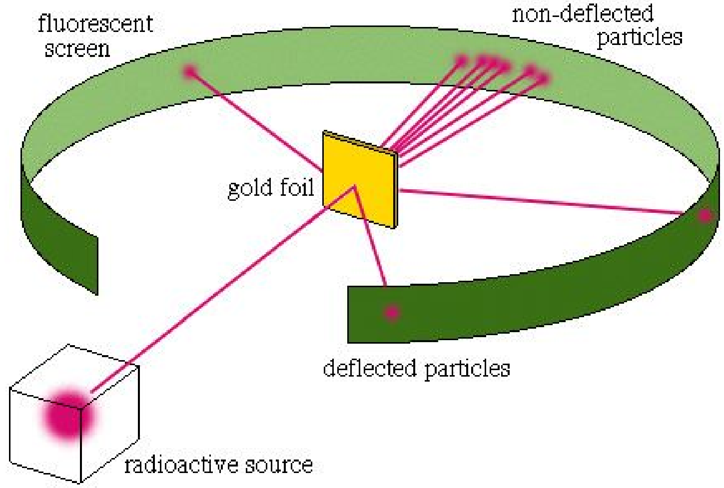
$$\mathscr{L}_{EM} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - J^\mu A_\mu$$

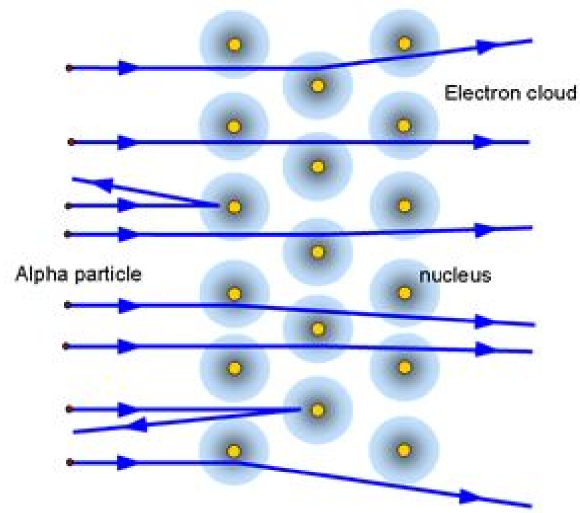
Gauge transformation: if $A^\mu \to A'^\mu = A^\mu + \partial^\mu \chi$ leaves the action invariant

The first particles:

• 1897 J. J. Thomson measured the charge and mass of the particles which constitute "cathodic rays", which he called electrons

• 1911 Ernest Rutherford (diffraction of α particles, He nuclei, by a thin gold foil: most passed undeflected, a few were deflected at large angle)

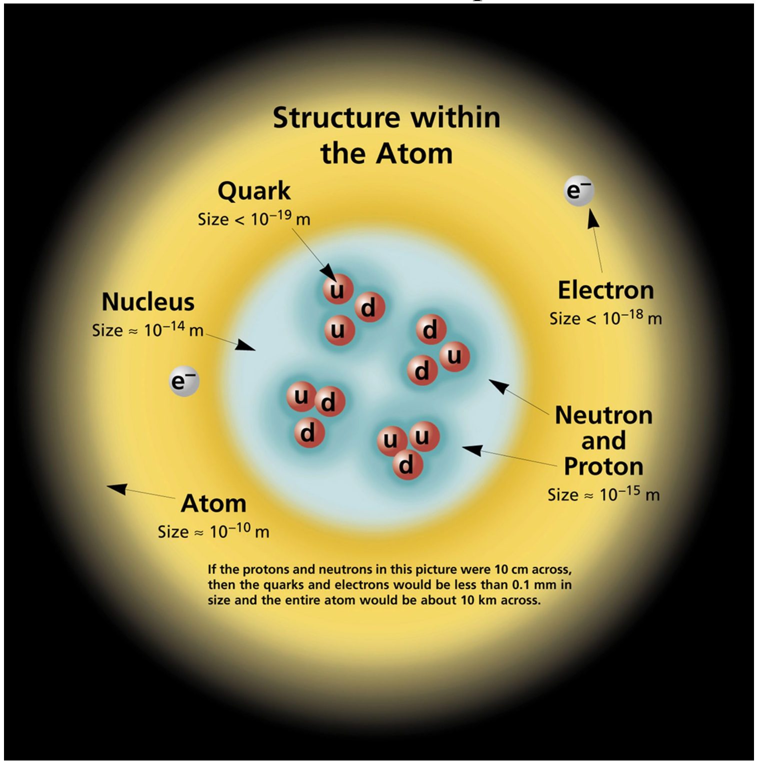




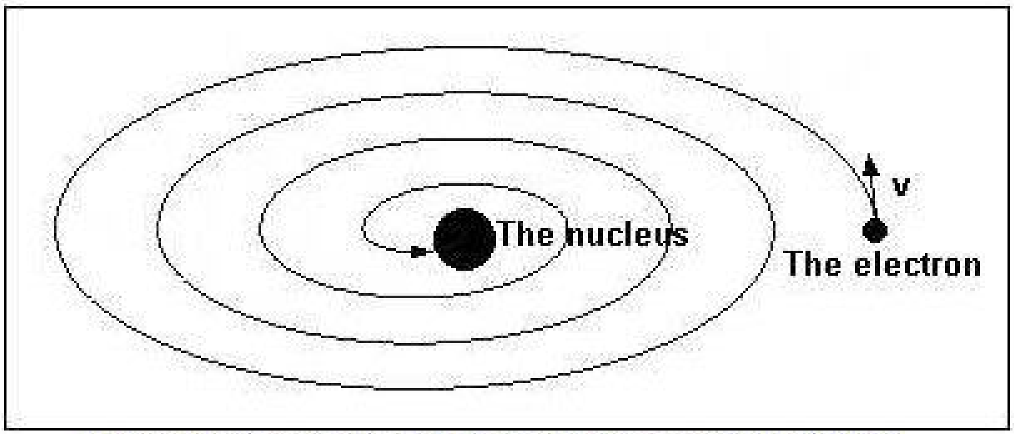
revealed the structure of an atom: positive heavy nucleus, light cloud of electrons.

The first particles:

1911 Ernest Rutherford gave its name to the proton ("protos", first), the H nucleus, and suggested the existence of the neutron (to explain the difference between mass and charge of nuclei); found in 1932 by James Chadwick, $m_n = m_p = 2000 m_e$.



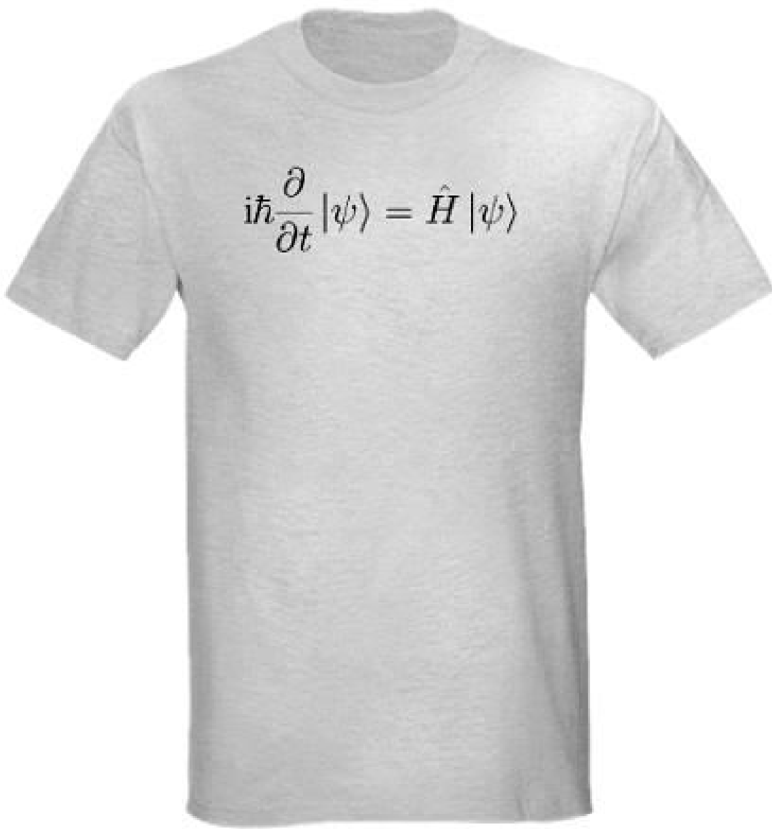
Quantum Mechanics



In the planetary model of atom, the electron should emit energy and spirally fall on the nucleus.

Why electrons do not all fall to the nucleus? Quantum Mechanics
1900 Max Planck, quanta $E = h\nu$
1905 Einstein "light quanta", later called photons (926, Gilbert Lewis);
1913 Niels Bohr, 1925 Werner Heisenberg; 1926 Erwin Schoedinger in 1926 ("antiparticles" in 1932 Carl Anderson discovered the positron)
1926, QM + Relativity = Quantum Field Theory

Quantum Mechanics on a the t-shirt



Non-relativistic QM

Schroedinger's equation (1926) is non-relativistic

For a non-interacting particle,

$$E = \frac{\vec{p}^2}{2m} \quad and \quad E \to H = i\hbar\frac{\partial}{\partial t}, \quad \vec{p} \to -i\hbar\vec{\nabla} \quad so \quad -\frac{\hbar}{2m}\vec{\nabla}^2\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

1) Non-relativistic: second order spatial ($\vec{\nabla}^2$), first order time derivatives
Relativistic choices: both second order (KG) or both first (D)
2) Non-relativistic: $\vec{x}$ is an operator, t a parameter
Relativistic choices: a) promote t to an operator: has irremediable problems, or
b) demote $\vec{x}$ to a parameter and quantize in a new way:
we interpret the particle/field as the operator parameterized by the spacetime coordinates acting on the vacuum itself, $|0\rangle$

This approach, where the quantum mechanical entities are no longer the coordinates acting on the fields, but the fields themselves, is called
Quantum Field Theory (QFT)

Equation quadratic in t and $\vec{x}$: Klein - Gordon (1926)

valid for spin=0 particles

Start with relativistic energy $E^2 = \vec{p}^2 c^2 + m^2 c^4$ and QM operator substitution

$$-\hbar^2 \frac{\partial^2 \varphi}{\partial t^2} = (-\hbar^2 c^2 \vec{\nabla}^2 + m^2 c^4)\varphi \Rightarrow (\partial^0 \partial_0 - \vec{\nabla}^2 + \frac{m^2 c^2}{\hbar^2})\varphi = 0$$

Klein Gordon equation-1926 (in "natural units" $c = \hbar = 1$):

$$(\partial^2 + m^2)\varphi = 0 \qquad \textit{and} \qquad \mathscr{L}_{KG} = \frac{1}{2}\partial^\mu \varphi \partial_\mu \varphi - \frac{1}{2} m^2 \varphi^2$$

Or the Klein Gordon Lagrangian with complex scalar fields φ and $\varphi^\dagger$:

$$\mathscr{L}_{KG} = \partial^\mu \varphi^\dagger \partial_\mu \varphi - m^2 \varphi^\dagger \varphi \equiv |\partial_\mu \varphi|^2 - m^2 |\varphi|^2$$

Notice last line of SM t-shirt, if $\partial \to D$

Dirac s approach: equation linear in t and $\vec{x}$ (1928)

valid for s=1/2 particles

$$\gamma^\mu \partial_\mu \, \psi = [\gamma^0 \partial_0 + \gamma^1 \partial_1 + \gamma^2 \partial_2 + \gamma^3 \partial_3] \, \psi = -im \, \psi$$

such that if operator squared will give the Klein Gordon eq. (relativistic E)

$$\gamma^\mu \partial_\mu \gamma^\nu \partial_\nu \psi = -im(-im\psi) \quad \Rightarrow \quad \left(\gamma^\mu \gamma^\nu \partial_\mu \partial_\nu + m^2\right)\psi = 0$$

$$Requires \quad (\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu) = 2\eta^{\mu\nu} I$$

Thus the γ^μ cannot be numbers! need 4 anti commuting matrices. Dimensions in which properties are possible: 4, 6 , 8 etc. Smallest dimension: 4×4 Dirac matrices. With $\bar{\psi} \equiv \psi^\dagger \gamma^0$ the Lagrangian is

$$\mathscr{L}_D = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \equiv \bar{\psi}(i\not{\partial} - m)\psi$$

Notice 2nd. line of SM t-shirt, if $\partial \to D$ and no mass term.

Left and Right Chirality Weyl spinors 4×4 Dirac matrices- In the "chiral representation"

$$\gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix} \quad \text{and} \quad \gamma^0 = \begin{pmatrix} 0 & \sigma^0 \\ \sigma^0 & 0 \end{pmatrix} \tag{-6}$$

where σ^0 is the 2×2 identity matrix, and σ^i are the Pauli spin matrices.

$With \;\; \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3, \qquad \psi_L \equiv P_L\psi \equiv \frac{1-\gamma^5}{2} = \begin{pmatrix} \chi_L \\ 0 \end{pmatrix}, \qquad \psi_R \equiv P_R\psi \equiv \frac{1+\gamma^5}{2} = \begin{pmatrix} 0 \\ \chi_R \end{pmatrix}$

2-component Weyl spinors χ_L, χ_R , 4-component Dirac spinor $\psi \equiv \begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix}$

In terms of Weyl spinors Dirac's equation becomes

$$i\bar{\sigma}^\mu \partial_\mu \chi_R = +m\chi_L \qquad with \qquad \bar{\sigma}^\mu = (\sigma^0, -\sigma^1, -\sigma^2, -\sigma^3)$$

$$i\sigma^\mu \partial_\mu \chi_L = +m\chi_R \qquad with \qquad \sigma^\mu = (\sigma^0, \sigma^1 \sigma^2, \sigma^3)$$

Using plane wave solutions $\psi_{L(R)} = u_{L(R)}(E, \vec{p}) e^{-iEt + i\vec{p}.\vec{x}}$ for $m = 0$ $(E = |\vec{p}|)$

$$\frac{\vec{\sigma}.\vec{p}}{|\vec{p}|} u_L = -u_L \qquad \frac{\vec{\sigma}.\vec{p}}{|\vec{p}|} u_R = +u_R$$

ONLY IF $m = 0$ chirality coincides with helicity: spin component along $\vec{p}$.
Chirality is a Lorentz invariant property. Helicity is not

Dirac's Lagrangian with L and R fields

With Dirac's fields, with $\bar{\psi} \equiv \psi^\dagger \gamma^0$

$$\mathscr{L}_D = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi = \bar{\psi}_L i\gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R i\gamma^\mu \partial_\mu \psi_R - m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

Or with Weyl fields:

$$\mathscr{L}_D = i\chi_L^\dagger \bar{\sigma}^\mu \partial_\mu \chi_L + i\chi_R^\dagger \sigma^\mu \partial_\mu \chi_R - m(\chi_L^\dagger \chi_R + \chi_R^\dagger \chi_L)$$

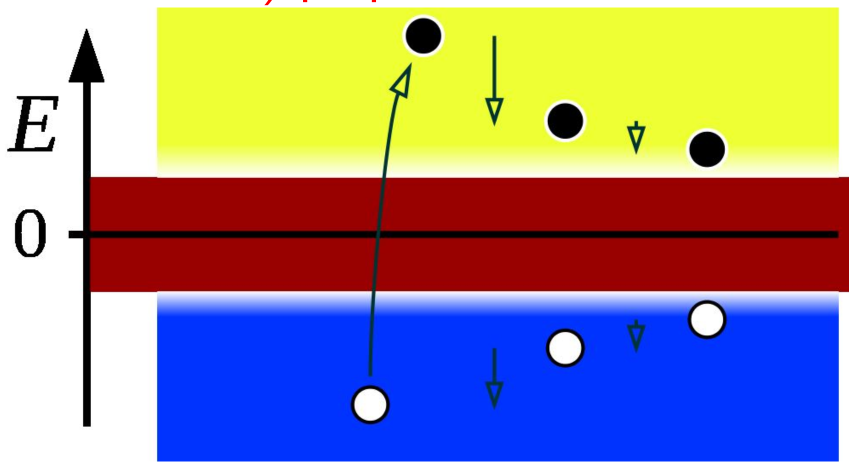
Notice that mass term mixes L and R

P (parity $\vec{x} \to -\vec{x}$) exchanges $L \longleftrightarrow R$ but Charged Weak interactions with only L (violate P maximally) thus mass terms are forbidden
(need the Higgs mechanism to give fermion a mass)

Problem with both relativistic eqs: negative energy states

Eigenvalues $E = \pm\sqrt{m^2c^4 + \bar{p}^2c^2}$ and $E < 0$ means that we don't have a true vacuum, a particle can cascade down forever, giving off an infinite amount of radiation!

Dirac's brilliant (now abandoned) proposal: "Dirac Sea"



Use Pauli Exclusion Principle, infinite number of particles already in the negative energy levels

A hole would be seen as a different particle with same mass but opposite additive changes: **antiparticle**

Antiparticles

The "Dirac-Sea" has major problems: two of them are

1- a supposed theory of single particle requires an infinite number of particles

2 - Particles like photons, or pions, or Klein-Gordon scalars don't obey the Pauli Exclusion Principle, but still have negative energy states

QFT interpretation: an antiparticle is a particle rather than the absence of one- Negative energy: we take out (annihilate) an antiparticle with positive energy

In non-relativistic QM: coordinates are Hermitian operators that act on the state in the Hilbert space representing a particle, e.g. act on the "electron" $|\Psi\rangle$ with the operator $\hat{x}$

In relativistic QM (QFT): the "electron" (parameterized by x) $\Psi(x^\mu)$ acts on the vacuum $|0\rangle$, creating the state $\left(\Psi(x^\mu)|0\rangle\right)$ (vacuum $|0\rangle$, with no particles in it)

E.g. expand the solutions of the Klein Gordon equation in plane waves, $e^{i\vec{k}\cdot\vec{x}\pm i\omega t}$, with $\omega = +\sqrt{\vec{k}^2 + m^2}$ and $\vec{k}$ is the standard wave vector.

$$\varphi(x) = \varphi(x)^\dagger \simeq \int d^3\vec{k}\left[a(\vec{k})e^{ik\cdot x} + a^\star(\vec{k})e^{-ik\cdot x}\right]$$

$k \cdot x = k^\mu x_\mu$- integration variable $\vec{k} \to -\vec{k}$ in the 2nd term of integral to use 4-vector notation.

$a(\vec{k})$ are operators with commutation relations similar to the "raising" and "lowering" operators in a harmonic oscillator (use $a(\vec{k})^\dagger$ for $a^\star(\vec{k})$)

$$[a(\vec{k}), a(\vec{k}^{'})] = 0$$

$$[a^\dagger(\vec{k}), a^\dagger(\vec{k}^{'})] = 0$$

$$[a(\bar{k}), a^\dagger(\vec{k}^{'})] = (2\pi)^3 2\omega\delta^3(\vec{k} - \vec{k}^{'})$$

$a^\dagger(\vec{k})$ *creates* a φ particle with momentum $\vec{k}$ and energy ω,
$a(\vec{k})$ *annihilates* a φ particle with momentum $\vec{k}$ and energy ω.
Normalized on particle state is $|\vec{k}\rangle = \sqrt{2\omega}a^\dagger(\vec{k})|0\rangle$. Probability amplitudes are $\langle\vec{k}_f|\vec{k}_i\rangle$.

For fermions we are forced to use anti-commutation relations: Spin-Statistics

Antiparticles

- All particles have antiparticles (sometimes they coincide)

- When a particle is different than its antiparticle the field is not Hermitian

A_μ: can create or destroy a photon (the photon is its own antiparticle)

ψ_e^-, by convention, can destroy an electron e^- or create a positron e^+
ψ_e^+, by convention, can destroy an electron e^+ or create a positron e^-

- particle and antiparticle have opposite charges

(given that a field always adds the same charges when applied to a state)

Quantum Electrodynamics (QED) to date makes the most accurate experimental predictions ever. Dirac equation plus EM

$$\mathscr{L}_{QED} = \mathscr{L}_D + \mathscr{L}_{EM} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - J^\mu A_\mu$$

If the source of the em field is the electron: $J^\mu = ej^\mu = e\bar{\psi}\gamma^\mu\psi$ (where $\partial_\mu j^\mu = 0$ due to a **global symmetry**- multiplication by an x independent phase)
Using the **"covariant derivative"** $D_\mu \equiv \partial_\mu + ieA_\mu$ this becomes

$$\mathscr{L}_{QED} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \equiv \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Notice 1st and 2nd. line of SM t-shirt, if no mass term (see the 3rd term)

"Gauge Symmetry": Lagrangian invariant under the combined local transformations: $\psi \to e^{i\alpha(x)}\psi$ **and** $A_\mu \to A_\mu - e^{-1}\partial_\mu \alpha(x)$

Internal Symmetry Set of transformations applied on fields at each space-time point (not space-time rotations, translation or boosts) which leave invariant $\mathscr{L}$

- Global transformation is the same everywhere in space-time.
- Local or "gauge" transformation, its parameters depends on spacetime.

Emmy Noether's Theorem (1918): for every transformation of a field $\varphi \to \varphi + \delta\varphi$

there is a current $j^\mu \equiv \frac{\partial \mathscr{L}}{\partial(\partial_\mu \varphi)} \delta\varphi$ and if $\varphi \to \varphi + \delta\varphi$ leaves $\mathscr{L}$ invariant, $\delta\mathscr{L} = 0$,

then the associated current is conserved $\partial_\mu j^\mu = 0 \Rightarrow -\frac{\partial j^0}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$

which emplies that the charge $Q \equiv \int_{all\ space} d^3x\, j^0$ is conserved.

Notice: $\mathscr{L}_D$ is invariant under **global phase transformations** $\psi \to e^{i\alpha}\psi$ thus $j^\mu = \bar{\psi}\gamma^\mu\psi$ and **the total electric change** is conserved! But, to have **local invariance** $\psi \to e^{i\alpha(x)}\psi$, must add the coupling to a "gauge field" A_μ

Nature prefers groups of transformations Lie groups are defined by the commutation relations among its **Generators**

Lie group: transformations parameterized by a set of continuous parameters, α_i for $i = 1, \ldots, n$. Transformations are matrices $D(\alpha_i)$ and for infinitesimal $\delta\alpha_i$

$$D(\delta\alpha_i) = I + i \sum_{i=1}^{n} \delta\alpha_i T^i + \ldots$$

where T_i are constant matrices called the **Generators** of the group, and there is one for each parameter. A finite transformation is $D_n(\alpha_i) = e^{i\alpha_i T_i}$

Gauge Symmetries in the SM

- QED: gauged U(1) symmetry (Abelian $\equiv$ elements commute, just phases)
- Weak int: gauged SU(2) (has 3 generators, Pauli matrices) $\times$ U(1)
- Strong int.: gauged SU(3) (has 8 generators, Gell-Mann matrices)

(SU(2), SU(3) are Non-Abelian, non-commuting matrices)

Gauge Symmetries Guiding principle to write Lagrangians: start with one invariant under a global symmetry and "gauge it", by replacing

$$\partial_\mu \rightarrow D_\mu \equiv \partial_\mu + igA_\mu(x) \qquad with \quad A_\mu(x) = \sum_{i=1}^{n} A^i_\mu(x) T^i$$

for which we need to add one **gauge boson** A^i_μ per generator of the group T^i (which transform $A^i_\mu \rightarrow A'^i_\mu = A^i_\mu + i \sum_{j=1}^n \delta\alpha_j(x) T^{adjoint\ j} A^j_\mu - \frac{1}{g}\partial_\mu(\delta\alpha_j(x))$

• thus mass terms $M^2 A^{\dagger i\mu} A^i_\mu$ are forbidden- **gauge bosons are massless, unless the symmetry is spontaneously broken**)

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and add the "kinetic term" $\mathscr{L}_{Kin,A} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

Abelian gauge symmetry, a U(1), $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ (photon is not self coupled)

Yang-Mills or Non-Abelian gauge symmetries (C.N. Yang and R.L. Mills, 1954)

• $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\left[A_\mu, A_\nu\right]$ thus Yang- Mills gauge bosons,weak bosons and gluons, **are self coupled** due to $F_{\mu\nu} F^{\mu\nu}$

QFT explains (See "QFT" by F. Wilczek- Rev. Mod. Phys 71,S85, 1999)

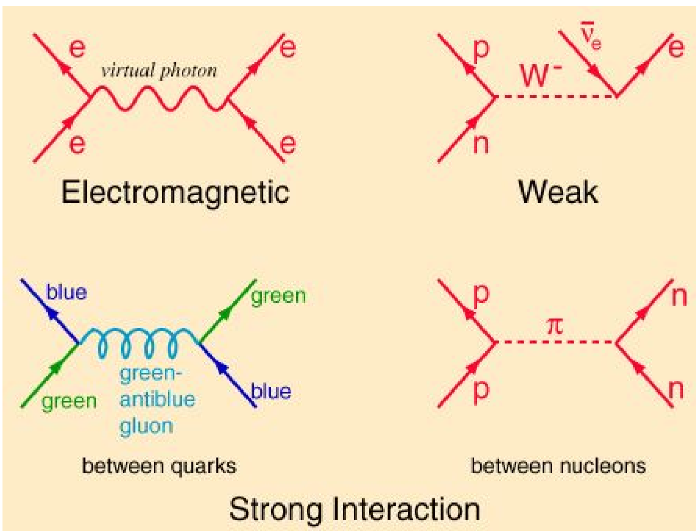
- the existence of different yet indistinguishable copies of elementary particles
- the connection between spin and statistics: integer spin particles are bosons and half-integer spin particles are fermions. The fermion character of electrons explains the stability of matter and the whole periodic table of elements. (In non-relat. QM Pauli uncertainty principle leads to particles being indistinguishable and thus to fermions and bosons- but which particle is which is a phenomenological issue)
- the existence of antiparticles
- the association of forces with particle exchange
- interactions are local: fields multiplied in $\mathscr{L}$ at the same space-time point- "vertices"
- CPT is always a symmetry CPT =product of parity, charge conjugation and time reversal

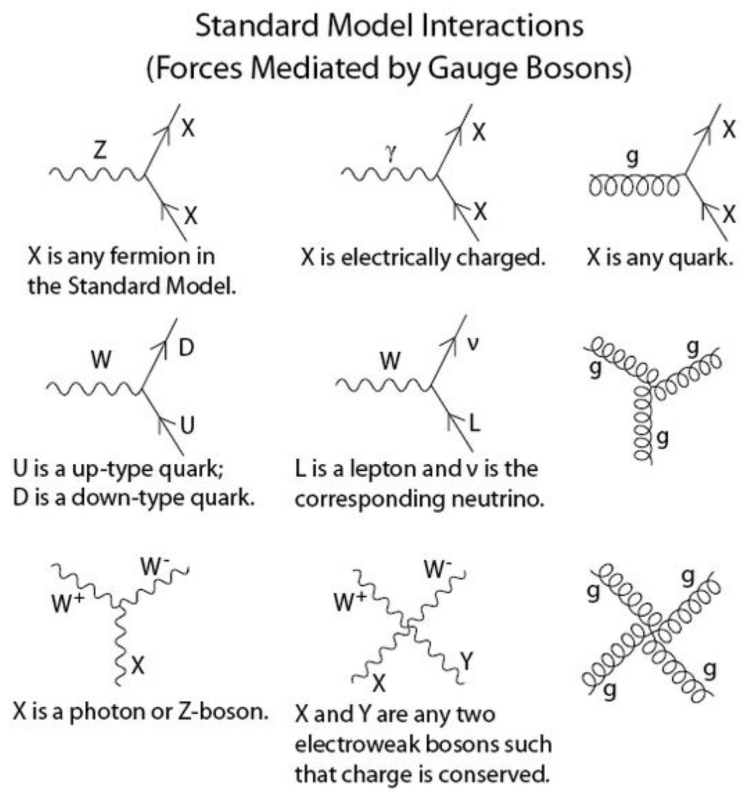
- **From QED onwards: QFT's are not finite but "renormalizable"**. Thus the consistent QFT are limited, requiring only some interactions to be present in the Lagrangian. In particular explicitly broken gauge theories are not renormalizable- exact and spontaneously broken gauge theories are renormalizable.

Renormalization Heroes (K. Huang, "Fundamental Forces of Nature" book)

Fig. 12.8 Richard Feynman (seated, with pen in hand) explains a point at the Shelter Island conference (1947). From left to right, standing: Willis E. Lamb, K. K. Darrow, Victor F. Weisskopf, George E. Uhlenbeck, Robert E. Marshak, Julian S. Schwinger, David Bohm. From left to right, seated: J. Robert Oppenheimer (holding pipe), Abraham Pais, Richard P. Feynman, Herman Feshbach. (Image credit: National Academy of Sciences.)

Feynmann Diagrams useful to represent interactions.
The exchanged particles are "Virtual".
Their existence is due to the
Time-Energy Uncertainty Relation.





Time-Energy Uncertainty Relation The most probable value of the difference ΔE between two measurements of the energy of a system separated by a time interval Δt is

$$\Delta E \simeq (h/2\pi\Delta t)$$

It is of crucial importance: in a quantum system the conservation of energy cannot be verified in an interval Δt with better precision than this ΔE!

While 2 particles are exchanging a third there are 3 particles instead of just two, but the energy has not changed! only possible if the uncertainty in E is $\Delta E \geq M$ which is only possible if the particle M lives a very short time $\Delta t \leq (h/2\pi M)$.

Particles which can only exist due to the uncertainty in the energy are called "virtual" (change completely the concept of "vacuum")

Feynmann Diagrams correspond to “Feynmann rules”

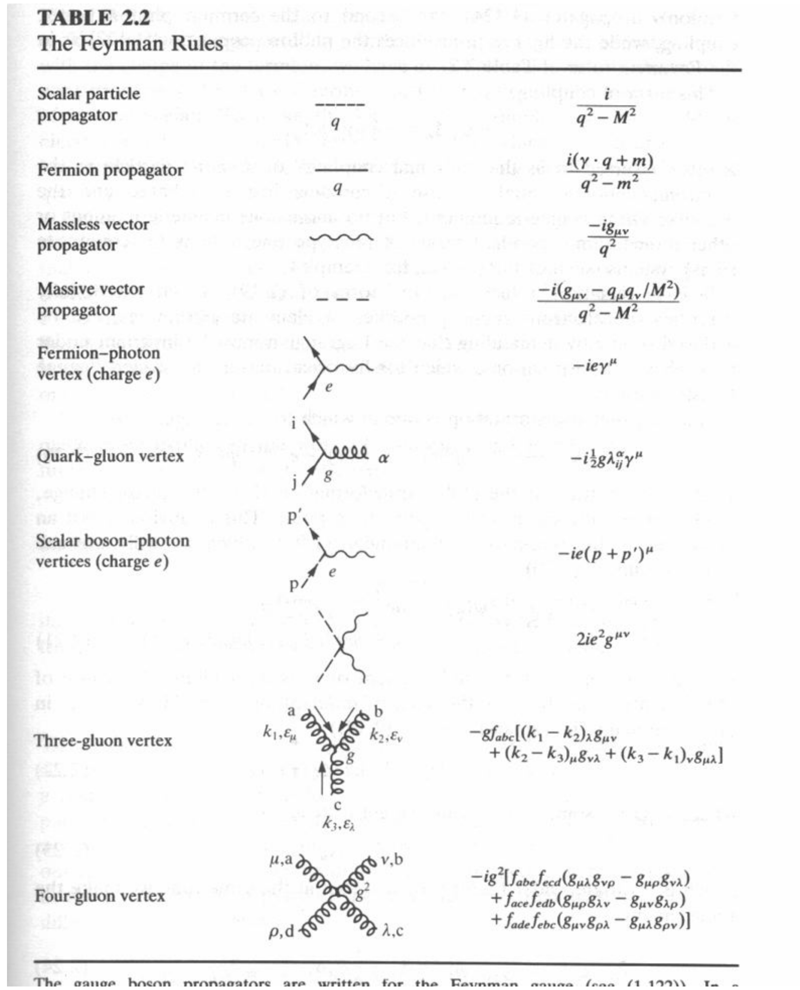


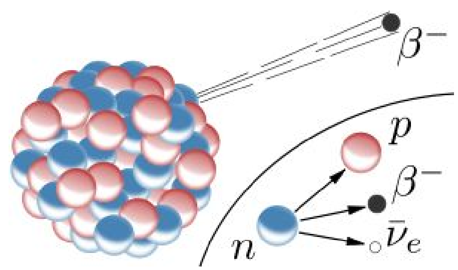
TABLE 2.2
The Feynman Rules

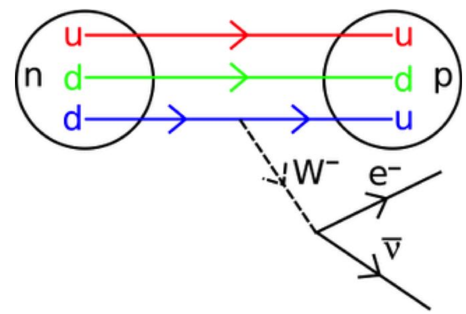
Scalar particle propagator	q	$\frac{i}{q^2 - M^2}$
Fermion propagator	q	$\frac{i(\gamma \cdot q + m)}{q^2 - m^2}$
Massless vector propagator		$\frac{-ig_{\mu\nu}}{q^2}$
Massive vector propagator		$\frac{-i(g_{\mu\nu} - q_\mu q_\nu / M^2)}{q^2 - M^2}$
Fermion–photon vertex (charge e)	e	$-ie\gamma^\mu$
Quark–gluon vertex	i, j, g, α	$-i\frac{1}{2}g\lambda^\alpha_{ij}\gamma^\mu$
Scalar boson–photon vertices (charge e)	p′, p, e	$-ie(p + p')^\mu$
		$2ie^2 g^{\mu\nu}$
Three-gluon vertex	a, b, c, k_1, ε_μ, k_2, ε_ν, k_3, ε_λ, g	$-gf_{abc}[(k_1 - k_2)_\lambda g_{\mu\nu} + (k_2 - k_3)_\mu g_{\nu\lambda} + (k_3 - k_1)_\nu g_{\mu\lambda}]$
Four-gluon vertex	μ,a; ν,b; ρ,d; λ,c; g^2	$-ig^2[f_{abe}f_{ecd}(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}) + f_{ace}f_{edb}(g_{\mu\rho}g_{\lambda\nu} - g_{\mu\nu}g_{\lambda\rho}) + f_{ade}f_{ebc}(g_{\mu\nu}g_{\rho\lambda} - g_{\mu\lambda}g_{\rho\nu})]$

The gauge boson propagators are written for the Feynman gauge (see (1.122)). In a

Weak nuclear interactions responsible for β-radioactivity (Becquerel 1896)

- 1914: J. Chadwick: β spectrum is continuous.
- 1930 Revolutionary proposal of W. Pauli: a new neutral particle with very feeble interactions, the neutrino, so that $n \to p + e^- + \bar{\nu}_e$ (due to a quark weak decay, $u \to d + e^- + \bar{\nu}_e$)





- 1934 Fermi: field theory for β–decay (but it is non-renormalizable)

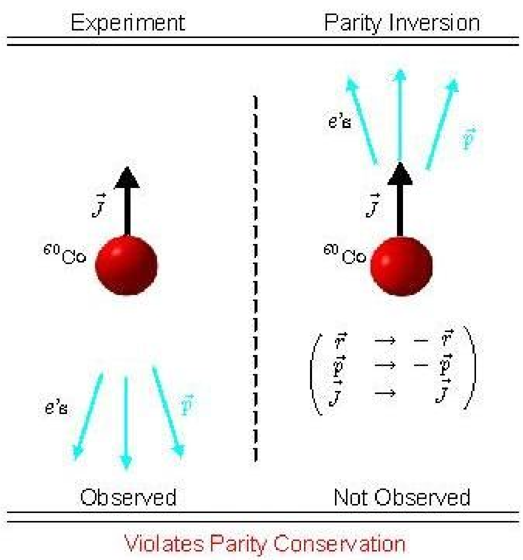
$$\mathscr{L}_{weak} = \frac{G_F}{\sqrt{2}}\left(\bar{\Psi}_p \gamma_\mu \Psi_n\right)\left(\bar{\psi}_e \gamma^\mu \psi_\nu\right) \quad with \quad G_F = \frac{1.03 \times 10^{-5}}{M_p^2}$$

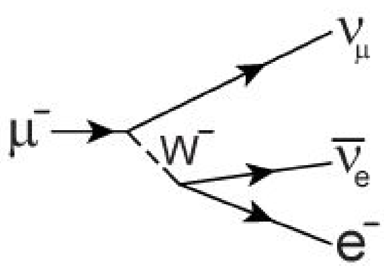
- 1937, 1st. indication of existence of 2nd generation!!! $\mu =$ heavy e !!!

Also $\mu^- \to e^- + (\bar{\nu}_e + \nu_\mu)$ and $\mu^- + p \to \nu_\mu + n$

Charged weak interactions violate parity conservation maximally

- 1956: proposal by T.D. Lee and C. N. Yang (Lederman loughs about it)
- 1957: Madame Chien-Ning Wu (Wu et al) confirms it experimentally after 8 months in ^{60}Co to ^{60}Ni beta-decay (Garwin, Lederman and Weinrich, hurry to see it in $\pi \to \mu \bar{\nu}$ in one weekend)





- 1957: Schwinger, Lee and Yang propose Week Int. mediated by bosons W^+, W^-

$$g_{weak} W_\mu J^{+\,\mu}_{\text{lept-L}} = g_{weak} W_\mu \left[\bar{\Psi}_{e,L}\, \gamma^\mu \Psi_{\nu,L}\right] \quad and \quad \frac{g^2_{weak}}{M^2_W} \simeq G_F$$

If mediator very heavy, both vertices appear joined in one: $\frac{g^2}{M^2_W}\left(\bar{\Psi}_{\nu,L}\gamma_\mu \Psi_{\mu,L}\right)\left(\bar{\Psi}_{e,L}\gamma^\mu \Psi_{\nu,L}\right)$

Glashow, Weinberg, Salam Electroweak model

• 1961 Shelly Glashow adds neutral intermediate weak boson: SU(2)$_{weak}$ x U(1)
W^1, W^2, W^3, B mix to form W^+, W^-, γ, Z^0. Problem: W and Z masses !!!
ψ_L are doublets of SU(2)$_{weak}$, ψ_R are singlets. Thus mass terms, which mix L and R $m\bar{\psi}_L\psi_R$, are forbidden and all quarks and leptons are massless too

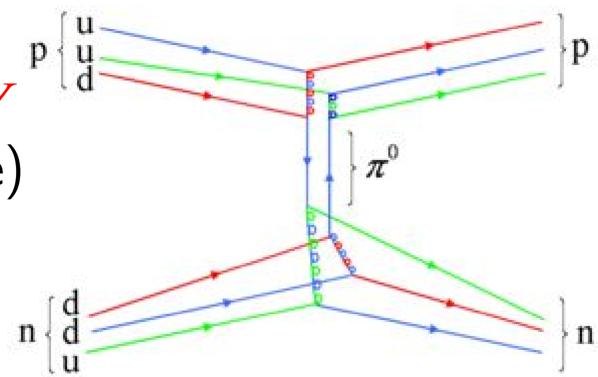
• 1964, "Higgs mechanism". 1967, S. Weinberg ("A model of Leptons") and in 1968 A. Salam introduce the electroweak spontaneous symmetry breaking to give W and Z masses: SU(2)$_{weak}$ x U(1) → U(1)$_{em}$

• 1970 M. Veltman and G. 't Hooft prove Yang-Mills theories are renormalizable.
• 1971 't Hooft proves spontaneously broken YM theories are too.
• 1973 neutral weak current observed at CERN- Still, model only for leptons? (With only three quarks u, d, s, the Z^0 can couple to d y s)
• 1974 quark c found experimentally (no Flavour Changing Neutral Currents): GWS model accepted universally
• 1983 W y Z^0 bosons found at CERN

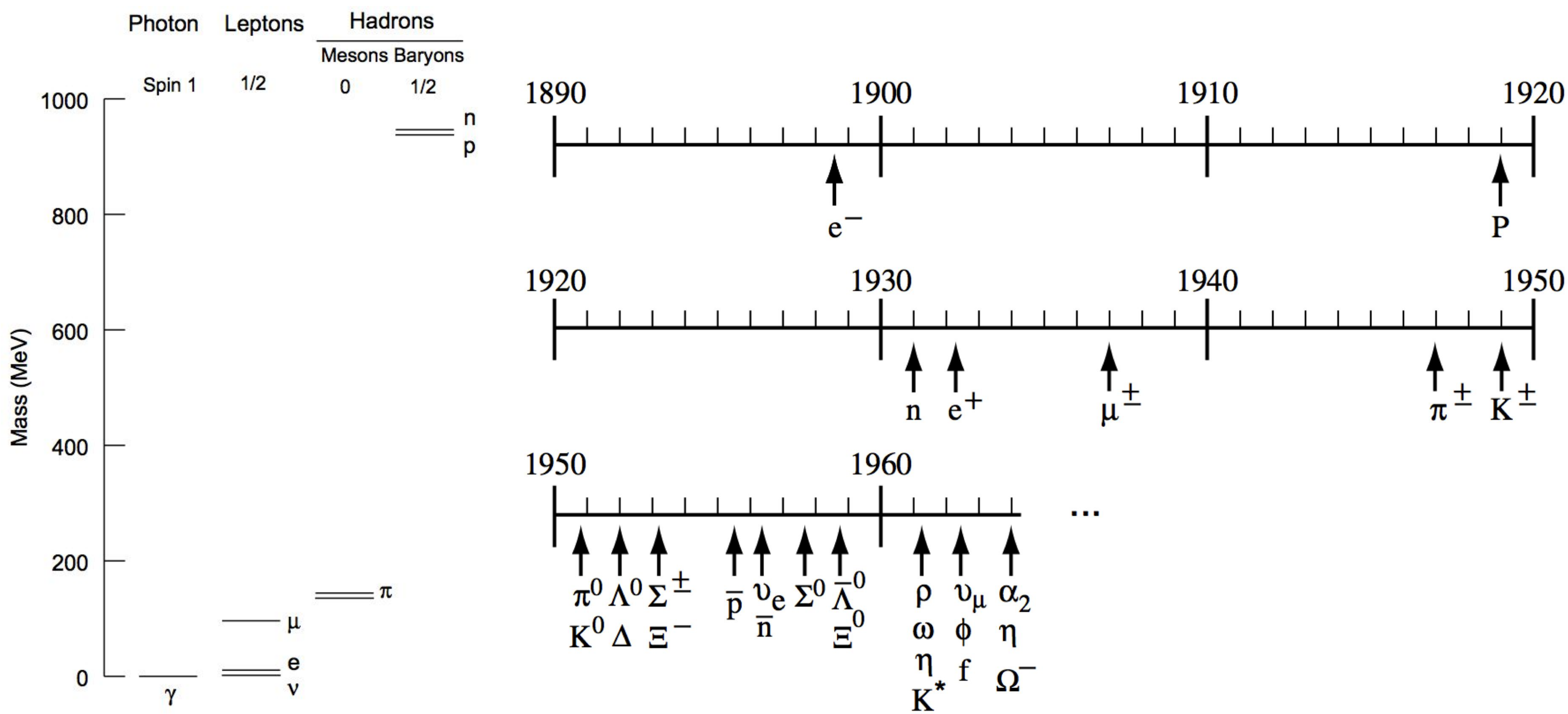
Strong nuclear Interactions keeps p and n bound to the atomic nucleus
Force stronger that the EM repulsion of the equal p charges, acting at short range, only inside nucleus.

The success of Pauli in inventing a particle replicated in
•1935 Hideki Yukawa proposes the pions π, $\mathscr{L}_{Yukawa} \sim \bar{\Psi}_N \varphi_\pi \Psi_N$
(three charges to mediate all possible interactions between p and n)
Range of interaction $\sim$ inverse of mediator mass M, thus $M_\pi \simeq 100$ MeV
Simplest explanation (C.G. Wick in 1938) use time-energy uncertainty relation, need $\Delta E \geq M$ which is only possible if the particle M lives a very short time $\Delta t \leq (h/2\pi M)$ (and range $< c\Delta t$)

• 1947 Cecil Powel discovers the π, $M_\pi = 140 MeV$
corresponds to 2×10^{-13}cm (fraction of nuclear size)

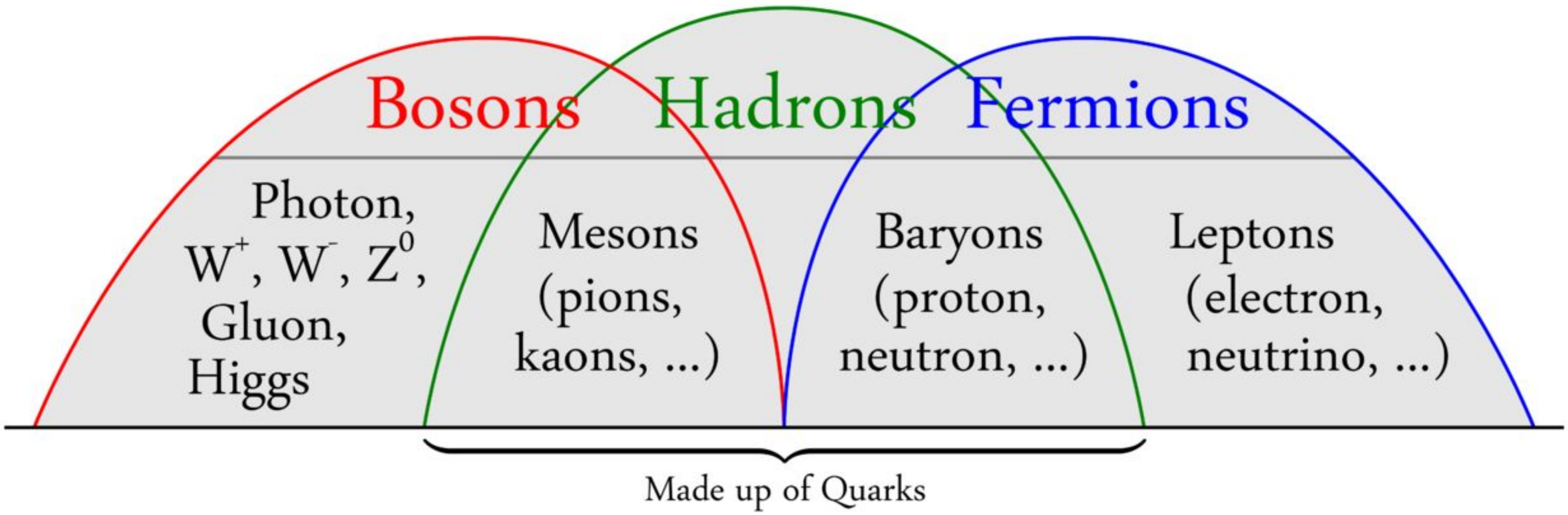


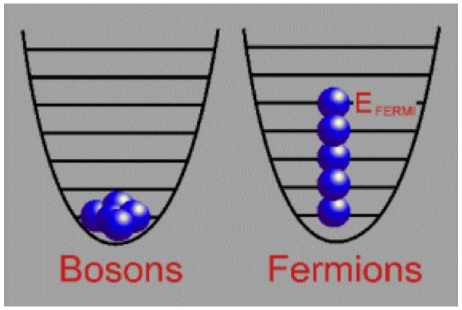
Particles as of 1947 – and later (1950's 1960's explosion of hadrons!)



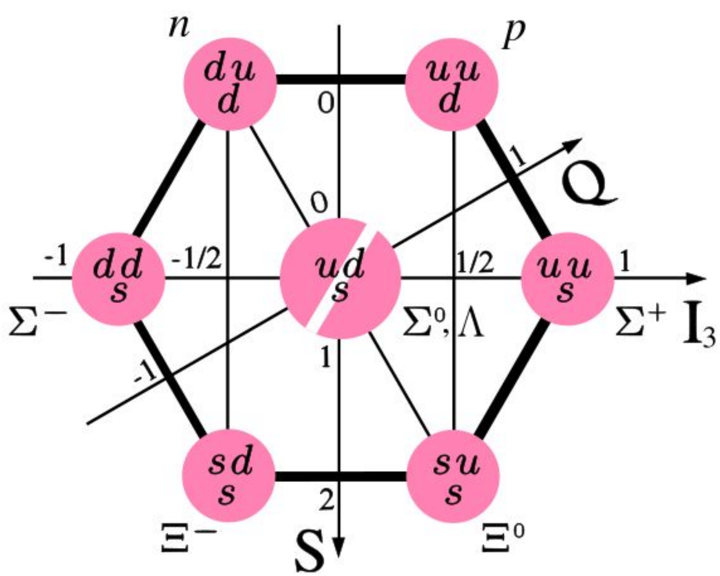
Hadrons (baryons, mesons), Leptons, Bosons, Fermions

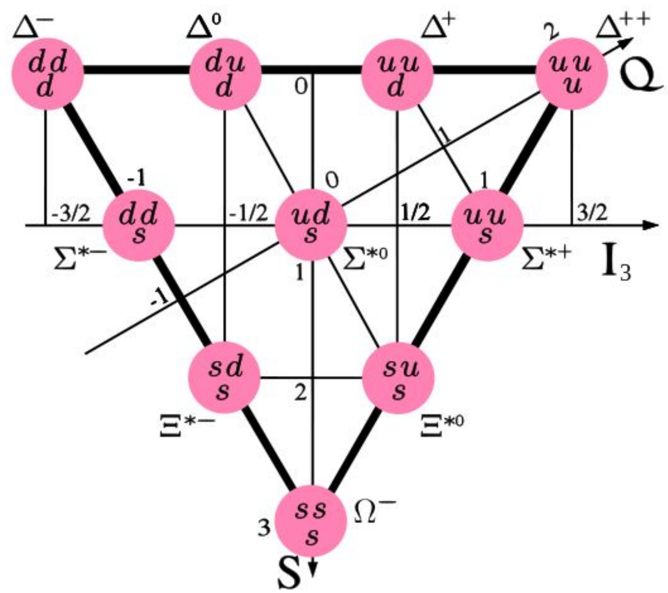
Hadrons (Lev Okun 1962) have strong interactions. Leptons do not.





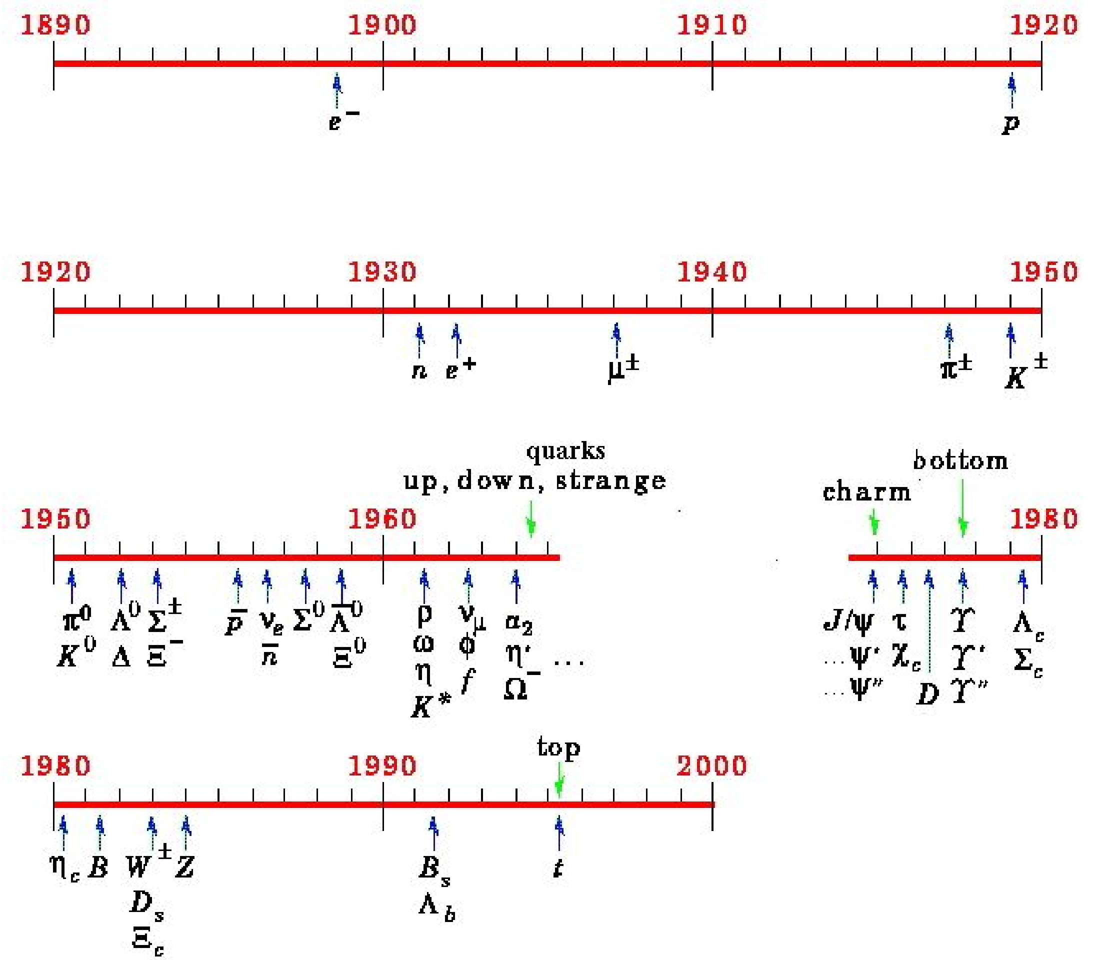
Quarks M. Gell-Mann and G. Zweig (1964) classified all known particles as composed of three constituents: u,d, and s. Baryons as qqq, Mesons as $q\bar{q}$ (electric charges $2/3$, $-1/3$, $2/3$, “baryon number” $1/2$). Found experimentally as free particles inside p and n in 1971. Now three more “heavy” quarks: $c, b,, t$.





New quantum number: color. Three colors carried by quarks, necessary so that Color=0 only in $q\bar{q}$ and qqq (1964, O .Greenberg and also M. Han and Y. Nambu)

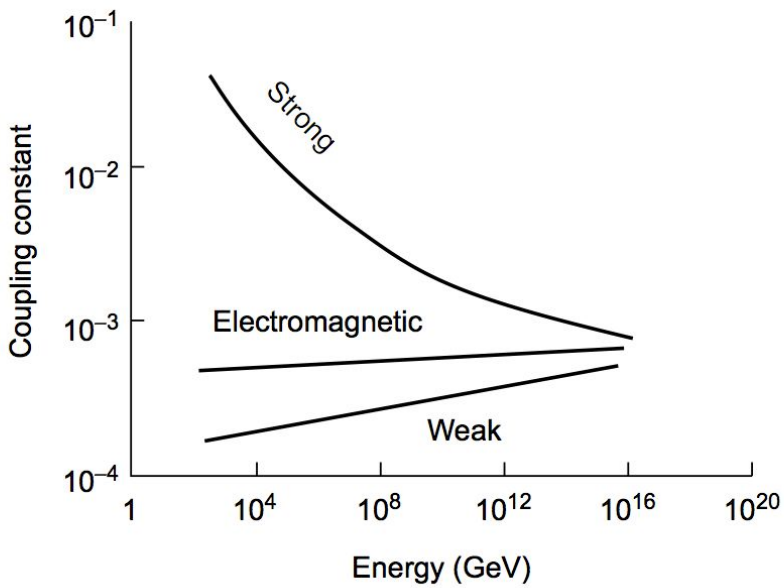
Timeline of the discovery of quarks

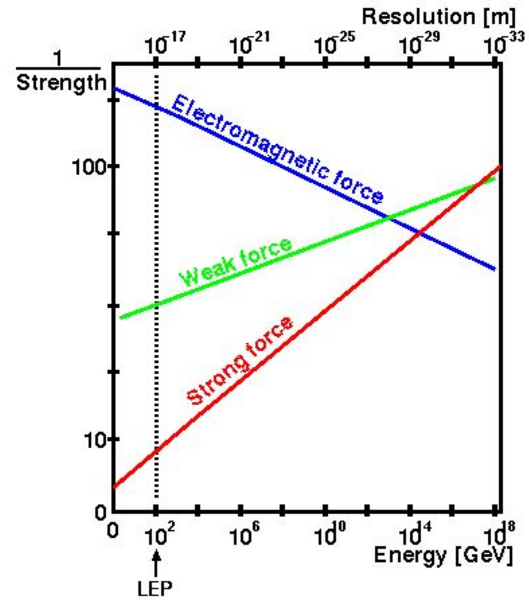


Quantum Chomo Dynamics ("chromos" = color)

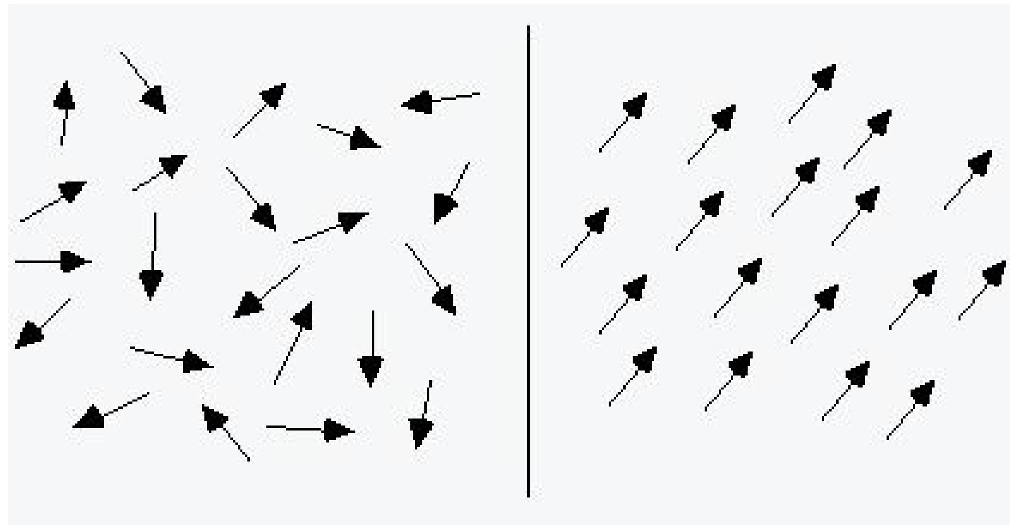
• 1971, Harald Fritzsch y Murray Gell-Mann, proposed to make color the charge of a SU(3)$_{color}$ Yang-Mills theory, with gauge bosons they called "gluons"

• 1973, David Polizer, and also David Gross and Frank Wilczek proved that due to the gluons self interactions QCD could leave quarks free a very short distance and instead confine them within a hadron: "asymptotic freedom".



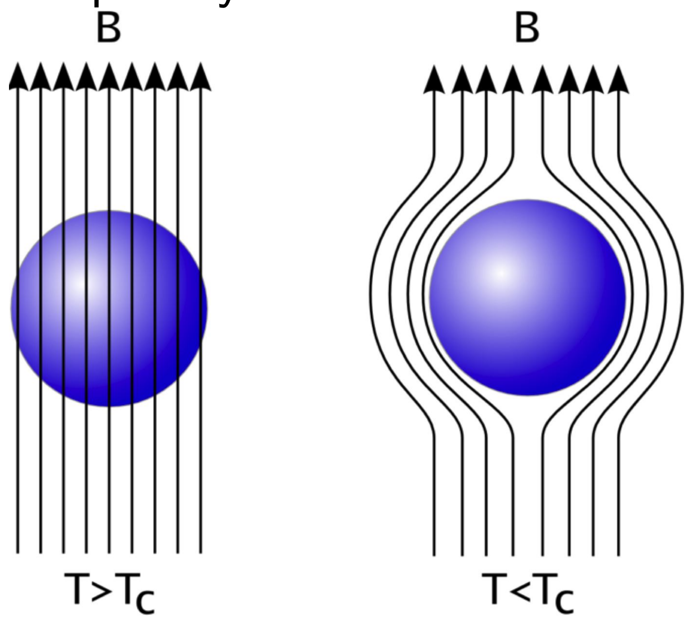


Spontaneous symmetry breaking of a global symmetry examples:
circular table Ferromagnet



When one person chooses the set of glasses to his/her right or left, all the others do the same (breaks left-right symmetry). As T decreases the magnets orient in the same direction at random (breaks rotational symmetry) and there are "spin waves" (whose energy $\rightarrow 0$ as the wavelength $\rightarrow \infty$, this is a Goldstone mode).

Spontaneous symmetry breaking of a gauge symmetry example:
the Meissner effect: when a superconductor forms at $T < T_c$, it expels magnetic fields from the interior, except very near the surface.



This means the photon has a mass $m_\gamma \simeq 1/\lambda$ inside the material, where λ is the penetration depth of $\boldsymbol{B}$. The superconductor forms via a Bose-Einstein condensate of Cooper pairs (Ginzburg & Landau 1950, Bardeen, Cooper & Schrieffer 1957) which breaks spontaneously the gauge symmetry.

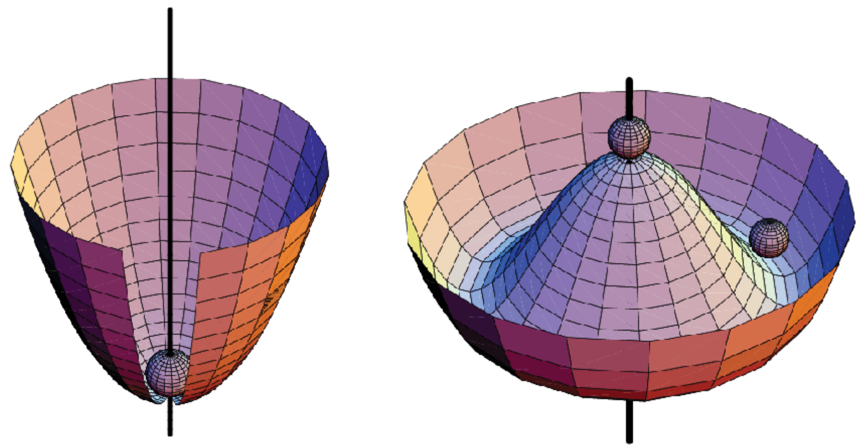
Spontaneous symmetry breaking of a global symmetry Simple field theory model, complex scalar φ with a potential energy $V(\varphi)$

$$\mathscr{L} \;=\; (\partial_\mu\varphi)(\partial^\mu\varphi^*) - V(\varphi) \quad \textit{and} \quad V(\varphi) = M^2\varphi\varphi^* + \lambda(\varphi\varphi^*)^2$$

Invariant under the group $U(1)$ of global transformations $\varphi(x) \to e^{i\theta}\varphi(x)$

$M^2 > 0$: the minimum of V is at $\varphi = 0$ (symmetric solution - left fig.)

$M^2 < 0$: whole circle of minima at the complex φ-plane with radius $v = (-M^2/2\lambda)^{1/2}$. Any point on the circle corresponds to a spontaneous breaking (right fig.). The component of φ along the orbit of minima is massless, it is a Goldstone boson (Nambu 1960, Goldstone 1961)



7

Spontaneous breaking of a gauge symmetry- “Higgs Mechanism”

(1963 P. Anderson, 1964 F. Englert & R. Brout, P. Higgs, G. Guralnik, C. Hagen & T. Kibble)

let us now “gauge the Lagrangian”: $\partial_\mu \to D_\mu \equiv \partial_\mu + ieA_\mu$ and add the FF term

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\varphi|^2 - [M^2\varphi\varphi^* + \lambda(\varphi\varphi^*)^2]$$

Invariant under gauge transformation: $\varphi(x) \to e^{i\theta(x)}\varphi(x) \quad A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\theta(x)$

Choose $M^2 < 0$, so minimum at $v = (-M^2/2\lambda)^{1/2}$

Parameterize the fields as $\varphi(x) = \frac{1}{\sqrt{2}}[v + \rho(x)]e^{i\theta(x)/v}$; $A_\mu(x) = B_\mu(x) - \frac{1}{ev}\partial_\mu\theta(x)$ and replace in the Lagrangian.

ρ is the radial component displaced by v
θ is the Goldstone boson, along the orbit of degenerate minima

Spontaneous breaking of a gauge symmetry- “Higgs Mechanism”

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Parameterize the fields as $\varphi(x) = \frac{1}{\sqrt{2}}[v + \rho(x)]e^{i\theta(x)/v}$; $A_\mu(x) = B_\mu(x) - \frac{1}{ev}\partial_\mu\theta(x)$

and replacing we get (here $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$)

$$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{e^2 v^2}{2} B_\mu^2 + \frac{1}{2}(\partial_\mu \rho)^2 - \frac{1}{2}(2\lambda v^2)\rho^2 - \frac{\lambda}{4}\rho^4 + \frac{1}{2} e^2 B_\mu^2 (2v\rho + \rho^2)$$

Massive gauge boson $M_B = ev$

Massive scalar boson ρ: $m_\rho = \sqrt{2\lambda} v$ Unknown because of self coupling λ

ρ **is the Higgs Boson** in this simple model Couplings: ρBB, $\rho\rho BB$

The Goldstone field θ disappeared, but is part of $B_\mu = A_\mu(x) + \frac{1}{ev}\partial_\mu\theta(x)$

Standard Model is a renormalizable QFT with a gauge symmetry. Any model of this type is constructed following 6 steps

- 1- Choose the gauge symmetry group:
SU(3)$_{color}$ x SU(2)$_{weak}$ x U(1)$_{hyperchage:Y}$

- 2- Choose the "matter" fields and their assignments to representations (i.e. quantum numbers)

3 repeated generations of quarks q and leptons l:
1) u, d, ν_e, e ; 2) c, s, ν_μ, μ ; and 3) t, b, ν_τ, τ

q are triplets of SU(3)$_{color}$ (carry three colors, are the fundamental representation);
l carry no color (are singlets)

q_L and l_L are doublets of SU(2):

$$l^i_L(x) = \frac{1}{2}(1+\gamma_5)\begin{pmatrix} \nu_i(x) \\ \ell^-_i(x) \end{pmatrix}; \quad q^i_L(x) = \frac{1}{2}(1+\gamma_5)\begin{pmatrix} u_i(x) \\ d_i(x) \end{pmatrix}; \ i = 1,2,3$$

$\ell_{iR}(x)$, $u_{i_R}(x)$ and $u_{i_R}(x)$ are singlets and $\nu_{iR}(x)$ is NOT included in SM (to prevent giving neutrinos a mass)

Hypercharge Y$=$ Q $+$ T$_3$
(T$_3$ $= +1/2$ (-1/2) for the upper (lower) members of each doublet)

- 3- Include scalar fields to allow for the Higgs mechanism
To give mass to three vectors bosons, need at least three real components, plus a physical massive spin zero boson. Thus, the minimal choice is an SU(2) doublet. Need the right Y so that

$$\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$$

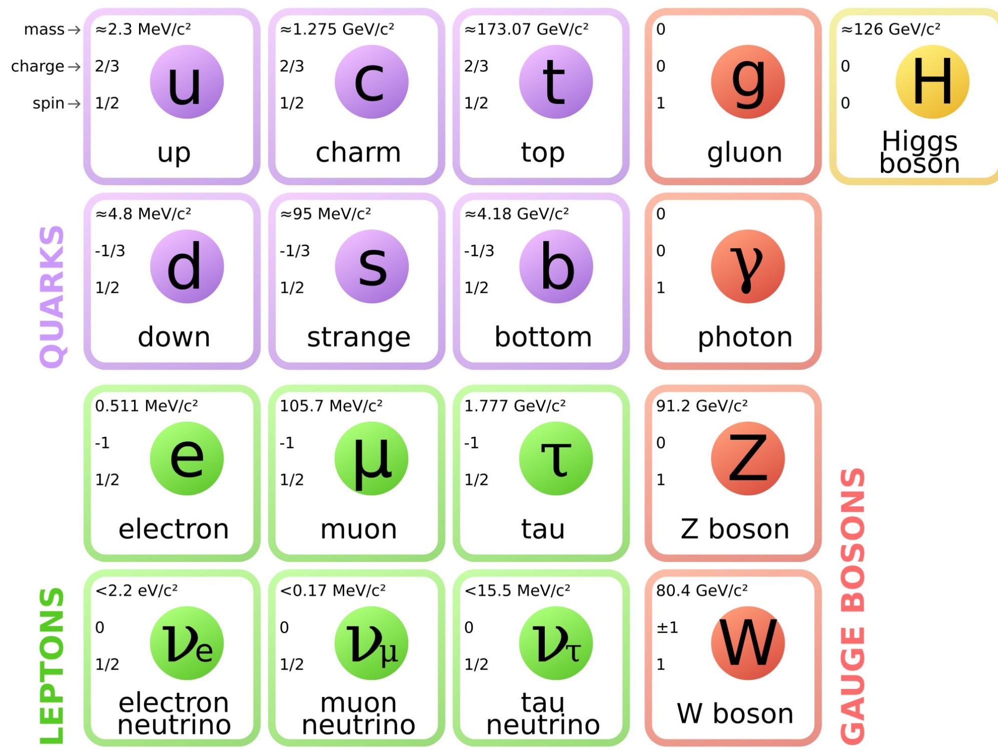
- 4 - Write the most general renormalizable Lagrangian invariant under the gauge group. At this stage gauge invariance is still exact and all gauge vector bosons are massless.

- 5 - Choose parameters of the Higgs potential so that spontaneous symmetry breaking occurs and $\varphi^0 = v/\sqrt{2}$ and $\varphi^+ = 0$ at the minimum.

- 6- Rewrite the Lagrangian in terms of physical fields.
(This last step is actually more complicated when we need to renormalize the theory)

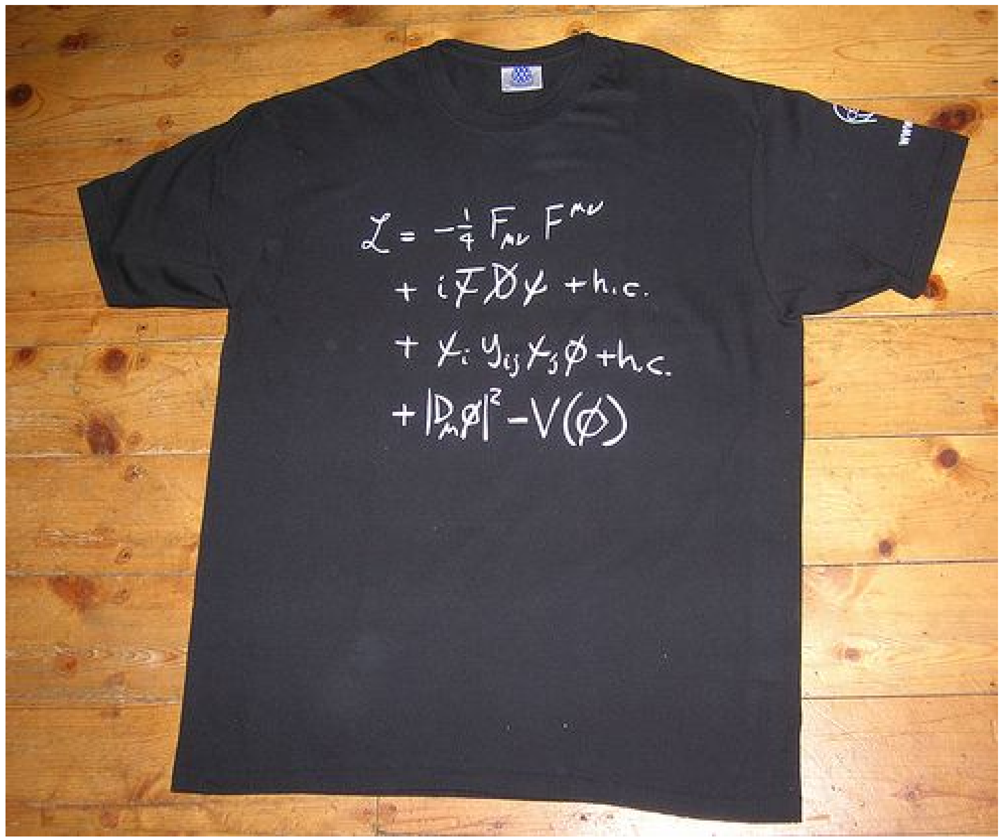
The rest is algebra and all the properties of SM come from these simple choices supplemented several fundamental measurements which allow then to predict the outcome of an enormous array of experiments

The standard model is the THEORY of elementary particles and their interactions (excluding gravity). It is a renormalizable relativistic quantum field theory with a gauge symmetry, part of it spontaneously broken by the "Higgs mechanism", and the following particles



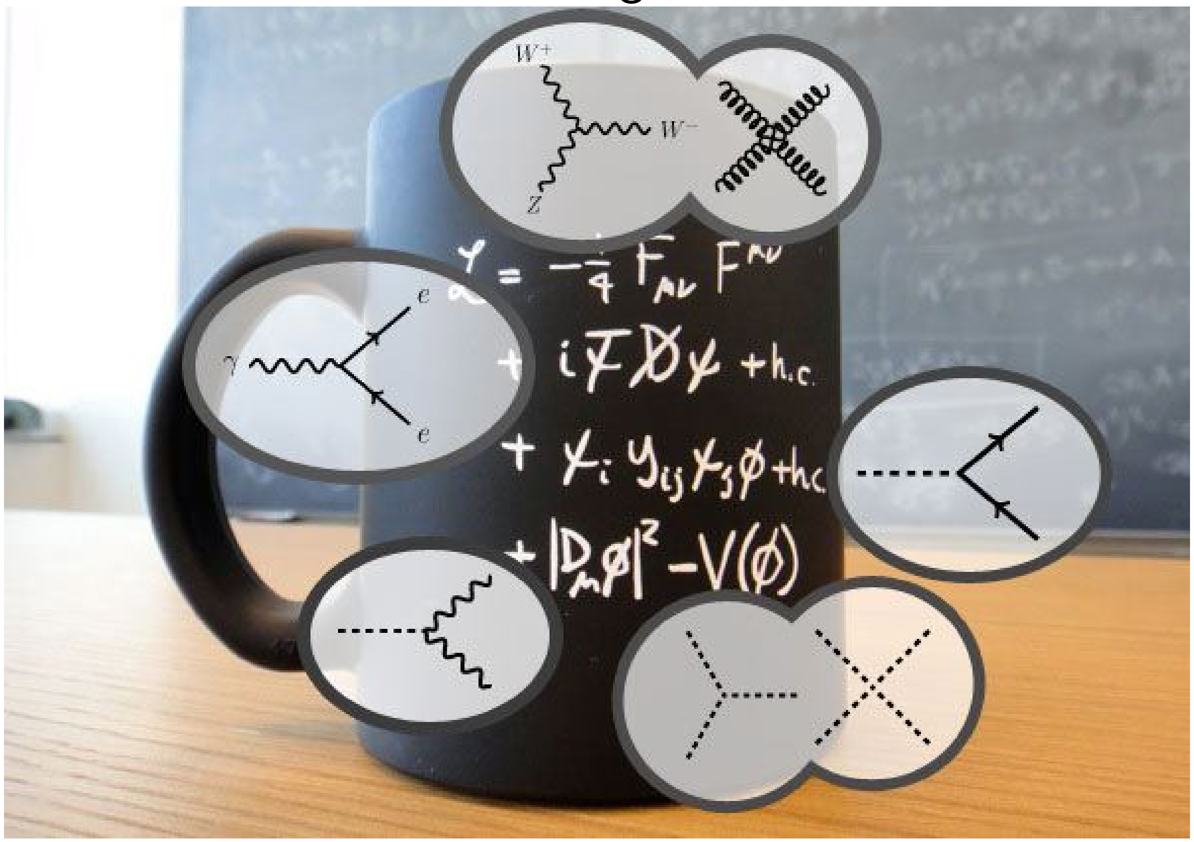
The standard model

a unified description concise enough to fit on a t-shirt!



The standard model

now we can understand the t-shirt or mug



The Origin of Mass in the SM:

See "The origin of mass" F. Wilczek 1206.7114

• Classical Mechanics: mass is a primary, conserved property of matter.

• Special Relativity: energy-momentum is conserved, but mass IS NOT
(and in GR, the source of curvature is energy and momentum, not mass)

• QM: the binding energy (e.g.of a nucleus) changes its "mass", so what is mass?

• **In the SM, there are two different origins of mass,**
1) the "Higgs mechanism" which gives mass to the heavy gauge bosons and to quarks and leptons

2) QCD interactions, which give mass to the hadrons (including p and n, $m_p \simeq m_n \simeq 1$ GeV, but the mass of the u,d quarks inside is O(10 MeV))

Mass terms for the gauge bosons come from the $|D_\mu \Phi|^2$ terms

once we replace $\Phi(x) \rightarrow \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$ we get $M_W \sim M_Z \sim v$.

One boson stays massless, and we identified it with the photon. This is the boson NOT coupled to the Higgs field component which gets a value v (called VEV, Vacuum Expectation Value) at the minimum, i.e. this Higgs component has electric charge zero, it is φ^0

The same terms gives the coupling of the Higgs boson, ρ, with the gauge bosons

Charged quarks and lepton masses The mass term of fermions mix L and R chiralities, but L are doublets and R are singlets, thus mass terms $m\bar{\psi}_L \psi_R$ are doublets and thus not invariant under SU(2) transformations, but we can write "Yukawa couplings", i.e. with the scalar Higgs field Φ, which respect the symmetry of the SM.

Charged quarks and lepton masses

Given by "Yukawa couplings"

$$\mathscr{L}_{Yuk} = y_{ij} \bar{\psi}_{iL} \Phi \, \psi_{jR}$$

We can do this for "up" and "down" quarks and charged leptons, but not for neutrinos, unless we add the ν_R fields to the SM.

Notice that we get mass matrices $y_{ij} v$ which need to be diagonalized to find the masses and mass eigenstates. All the fermion masses and the mixings between mass eigenstates come from the ARBITRARY CONSTANTS y_{ij}, so the SM does NOT predict any of their values!

The coupling of the physical Higgs boson with fermions is proportional to the fermion mass (thus the t coupling dominates).

Neutrino masses

Are strictly zero in the SM because of the absence of the R neutrinos Weyl spinors (which if they exist, are singlets of all SM interactions).

But neutrinos are now known to have mass, at least two of them, much smaller than that of any other fermion. Thus something needs to be added to the SM to account for those, just the ν_R fields, or other scalar bosons. . .

Charged fermions are Dirac fields have four independent components

$$\psi^C_{Dirac} \neq \psi_{Dirac}$$

(C is charge conjugation) particles and antiparticles are different. Neutrinos could be either Dirac or Majorana. A Majorana fermion is its own antiparticle and has only two independent components. In terms of fields

$$\psi^C_{Majorana} = \psi_{Majorana}$$

Mass of atoms is about the mass of all "nucleons" N (i.e. n and p) inside them. The mass of hadrons comes from QCD interactions"the back reaction of color gluon fields resisting accelerated motion of quarks and gluons inside them" and can now be calculated from first principles, with supercomputers.

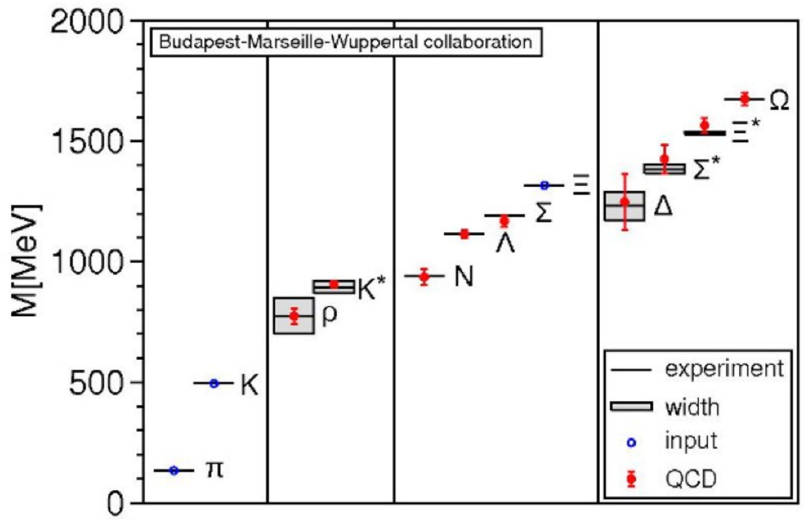
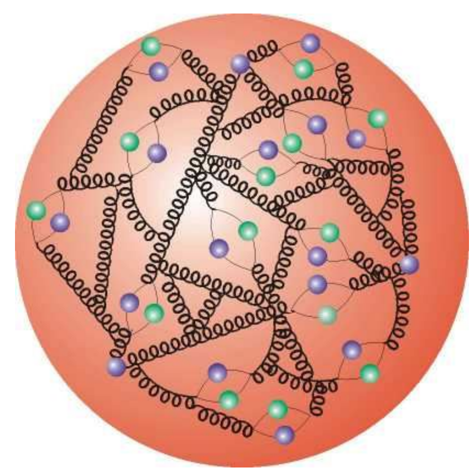


Figure 1. Spectrum of low-lying mesons and baryons, calculated from first principles in quantum chromodynamics, compared to experiment [10]. π and K mesons masses are used to fix the light quark and strange quark masses, respectively, and Ξ to fix the overall scale. "N" denotes nucleon. This result reveals the origin of the bulk of the mass of standard matter.



Problems of the SM

So far the SM has been enormously successful, proven to be right in the 100's of experimental tests (maybe too successful at this point). But we believe it cannot be the last word.

• It does not include gravitational interactions

• Has many (too many?) free parameters: 20 for massless neutrinos + 7(9) for Dirac (Majorana) neutrinos. It does not explain why the electric charge of quarks is exactly related to that of electrons, so that atoms are neutral (in the SM this is an accident). There is no explanation of why there are 3 generations of repeated fermions and of their mass hierarchy.

• There is no explanation of neutrino masses.

• No solution for the "strong CP problem" (due to a term $\theta F_{\mu\nu} \widetilde{F^{\mu\nu}}$ in the QCD Lagrangian -only viable solution so far is to add a global Peccei-Quinn symmetry)

• There are no cold or warm Dark Matter particle candidates (so the bulk of the dark matter cannot be accounted for within the SM)

• There is no explanation of the Dark Energy

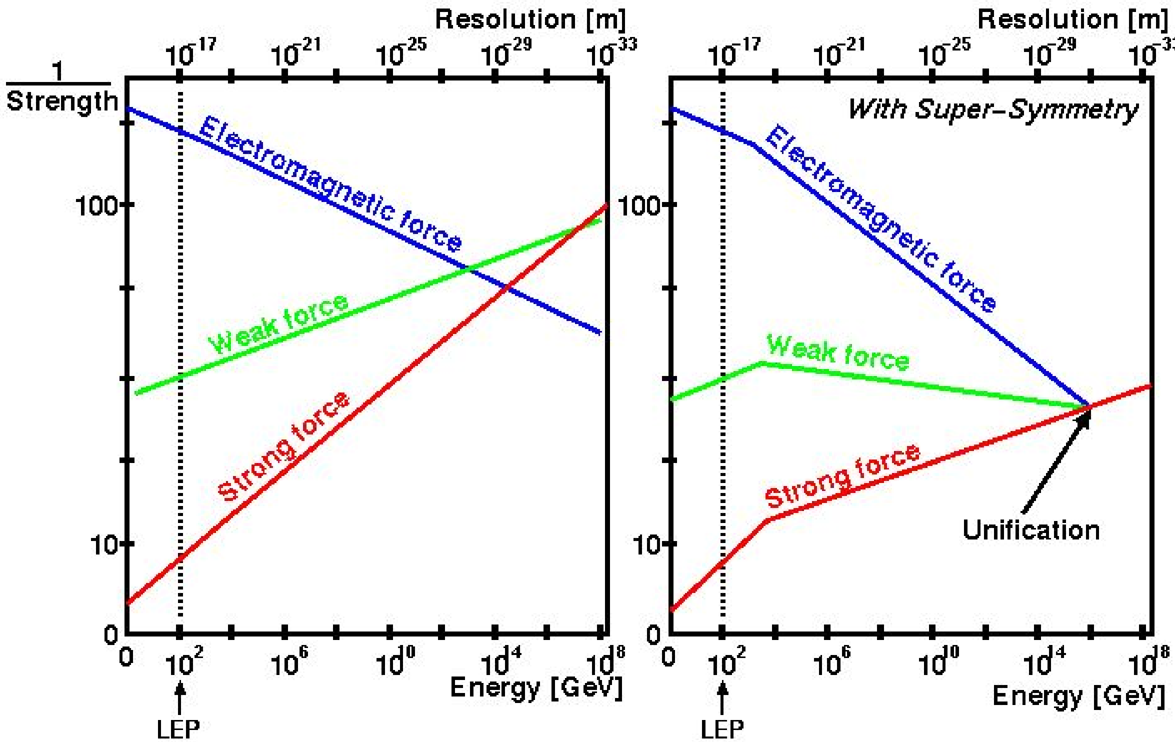
• Problem of stability of the Higgs mass if there is any physical scale Λ where new physics arises. The tree-level (bare) Higgs mass, the one which appears in the Lagrangian we dealt with, receives quadratically-divergent corrections from one loop diagrams, $M_H^2 = (M_H^2)_{bare} + O(\lambda, g^2, h^2)\Lambda^2$, which take the corrected mass to $O(\Lambda)$, much larger than measured

(Solutions: TeV scale supersymmmetry (so far not found by the LHC) where there is cancellation of fermionic and bosonic contributions to the loop, Little Higgs models, where the Higgs is light because it is almost a Goldstone boson... all already constrained by the LHC)

Ideas to go beyond the SM

- More symmetry

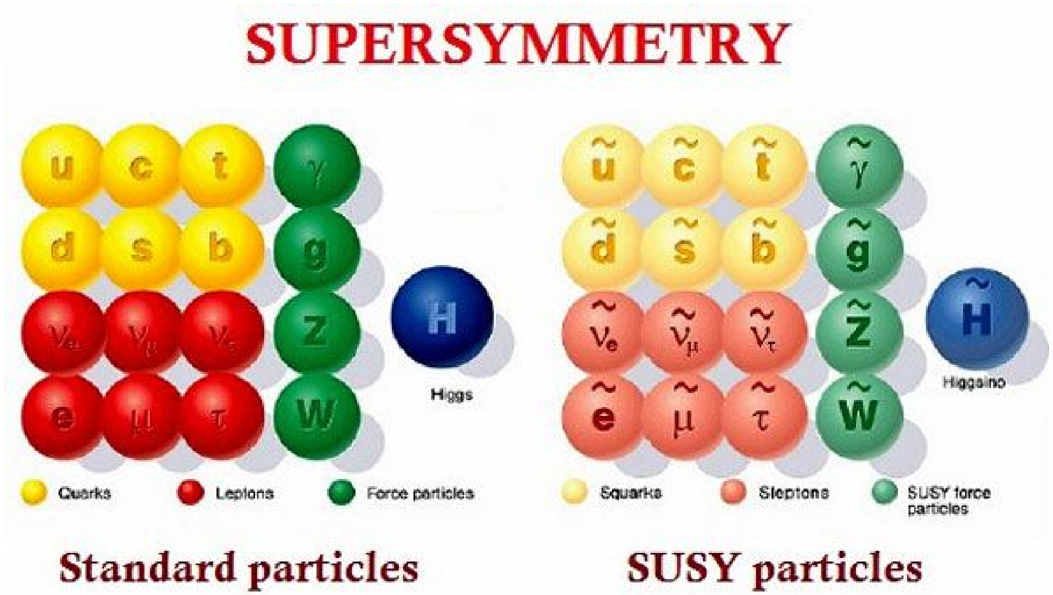
Grand Unified Theories (GUT), unifications of electroweak and strong interactions at high energies?



Ideas to go beyond the SM

- More symmetry

Supersymmetry (SUSY): Symmetry between bosons and fermions (need to duplicate all the particles of the SM, and at least an additional Higgs doublet)!



Ideas to go beyond the SM

• Compositeness Technicolor to bind Technifermions inside quarks and leptons; Little Higgs models, where the Higgs is light because it is almost a Goldstone boson of a complicated sector...

• Extra-dimensions which are inhabited by some of the fields which also inhabit our space-time. Maybe at low energy scale of 100's of TeV

• Physics related to the Dark Matter, usually with complicated "hidden" sectors (talk of Paolo Gondolo)

Let us hope the LHC finds something else besides the Higgs boson soon. Otherwise, our only windows to the physics beyond the SM will be precision measurements of properties of the Higgs boson and flavor physics