

Higgs Boson self-coupling measurements @ the LHC

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aims:

- provide summary of ‘recipe’ of the electroweak sector of the Standard Model.
- provide overview of status of Higgs Boson self-coupling measurements.*
- consider the prospects for the future.

* concentrate on the SM: I will only briefly mention BSM.

electroweak cooking

- ingredients:

$SU(2) \times U(1)$ gauge symmetry

+ complex doublet scalar, ϕ

+ potential for ϕ : $\mathcal{V}(\phi^\dagger \phi)$



electroweak cooking, steps



- choose a minimum in a particular direction, maintaining U(1) invariance \hookrightarrow symmetry breaking.

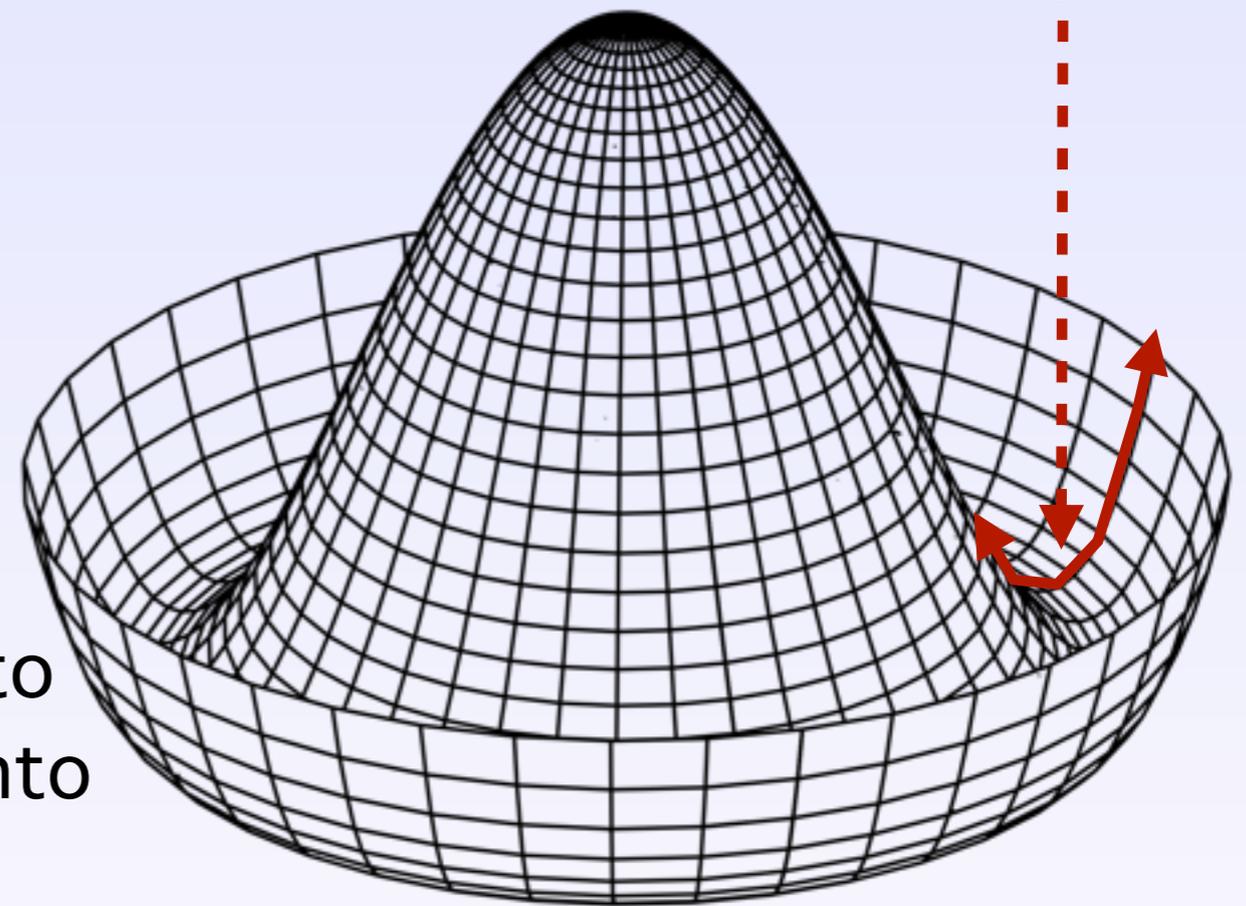
$$\phi_{\min.} \propto (0, v)$$

- consider fluctuations of the scalar field about the minimum:

$$\phi \propto (0, v + H)$$

- make a gauge transformation to absorb the Goldstone modes into the gauge bosons.

- Recipe makes massive W, Z, massless photons and Higgs boson (H). Topped with QCD and served with fermions to complete the SM.



Higgs potential

- focus on the resulting potential for the scalar field H:

$$\mathcal{V} = \frac{1}{2} M_H^2 H^2 + \lambda v H^3 + \frac{\tilde{\lambda}}{4} H^4.$$

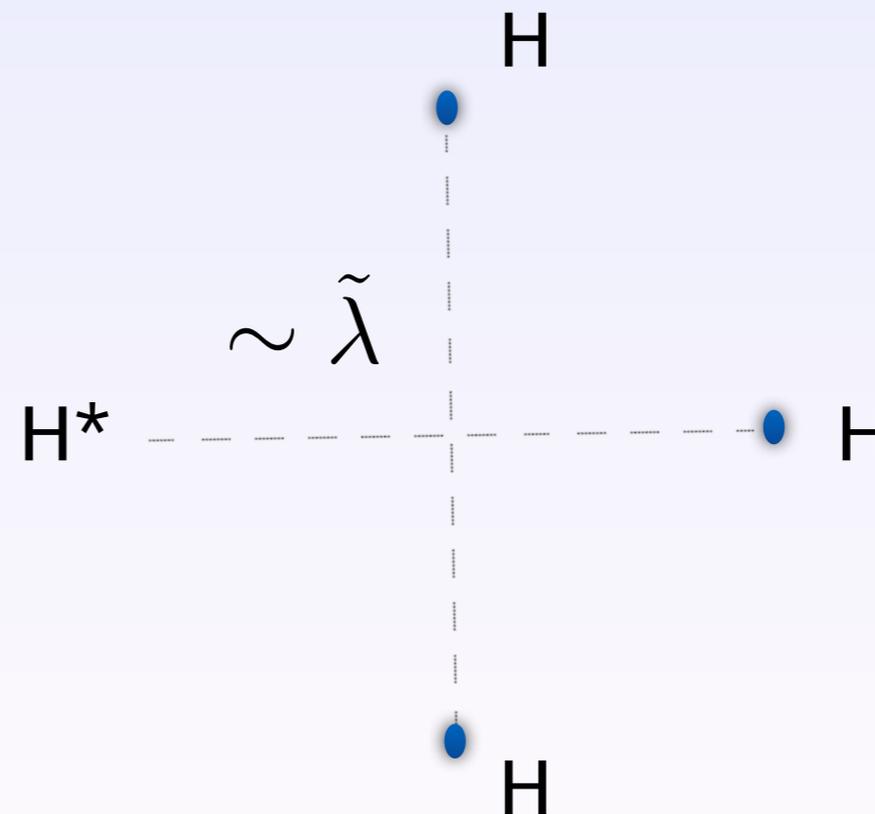
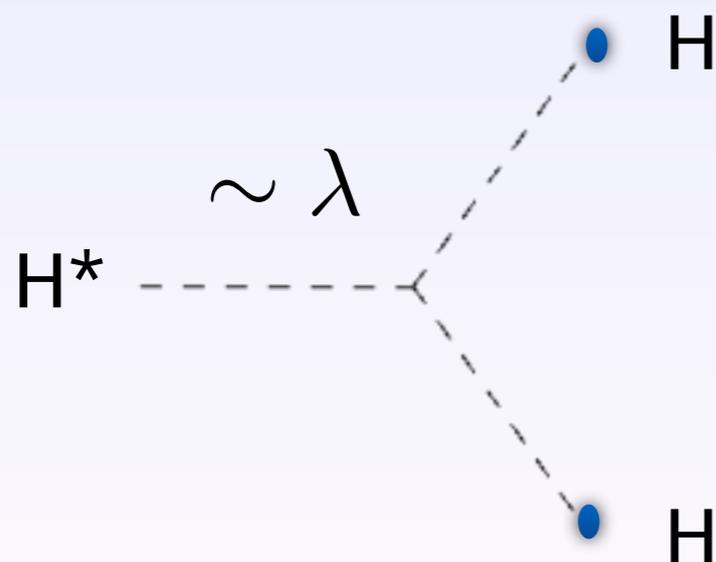
- M_H , the Higgs mass, has been measured in July 2012 ~ 125 GeV.
- v , the vacuum expectation value, from the 4-fermion interaction at low energies ~ 246 GeV.
- two ‘unknowns’:
 - λ , the triple self-coupling
 - $\tilde{\lambda}$, the quartic self-couplingnot measured directly

why should we care?

- standard model prediction: $\tilde{\lambda} = \lambda = \frac{M_H^2}{2v^2} \simeq 0.13$.
- here: regard SM as an **effective theory**, with the self-couplings as per se **free** parameters.
- direct confirmation of the standard model relation probes new physics effects, e.g.
 - extended Higgs sectors.
 - heavy particles running in loops.
 - + other higher-dimensional operators.
- + probes the general consistency of the standard model.

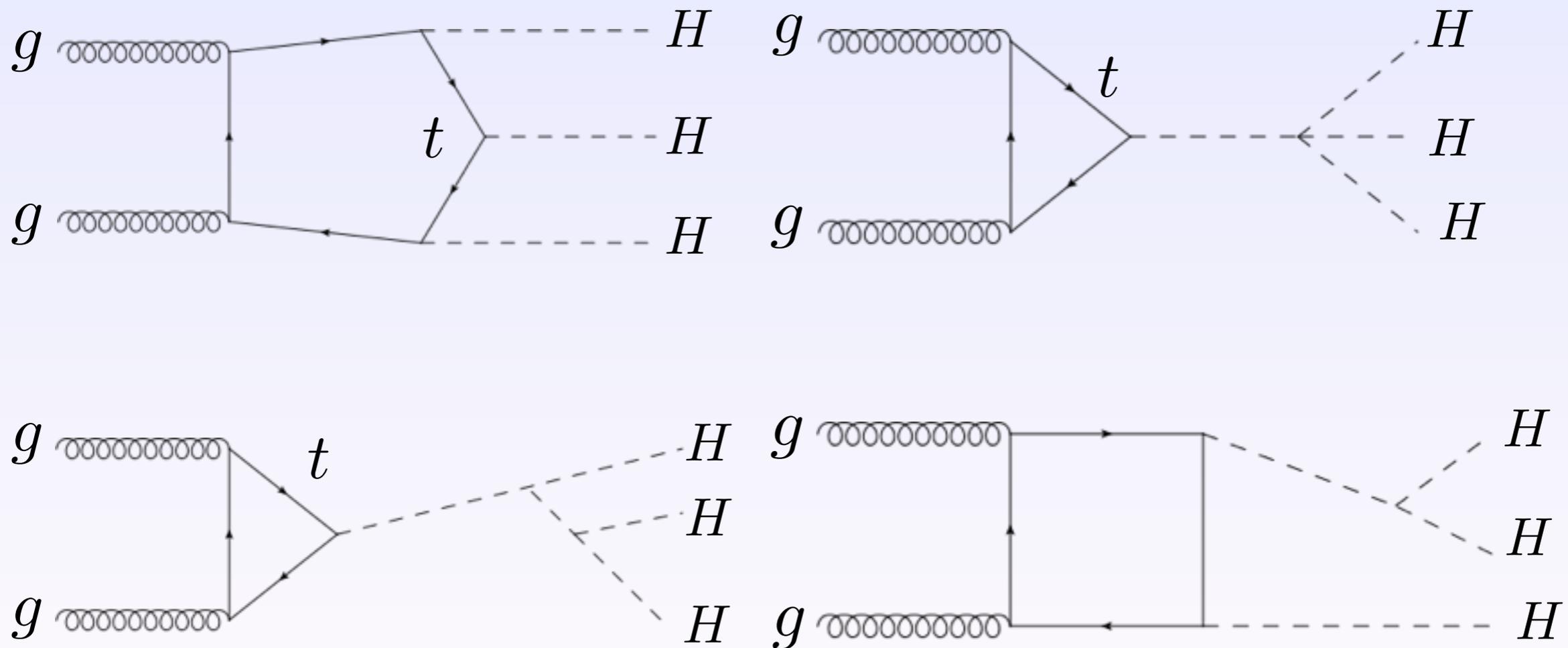
probing Higgs boson self-interactions

- determination of λ and $\tilde{\lambda}$: through multi-Higgs boson production @ colliders:
- triple coupling, λ , \rightarrow Higgs pair production.
- quartic coupling, $\tilde{\lambda}$, \rightarrow Higgs triple production.



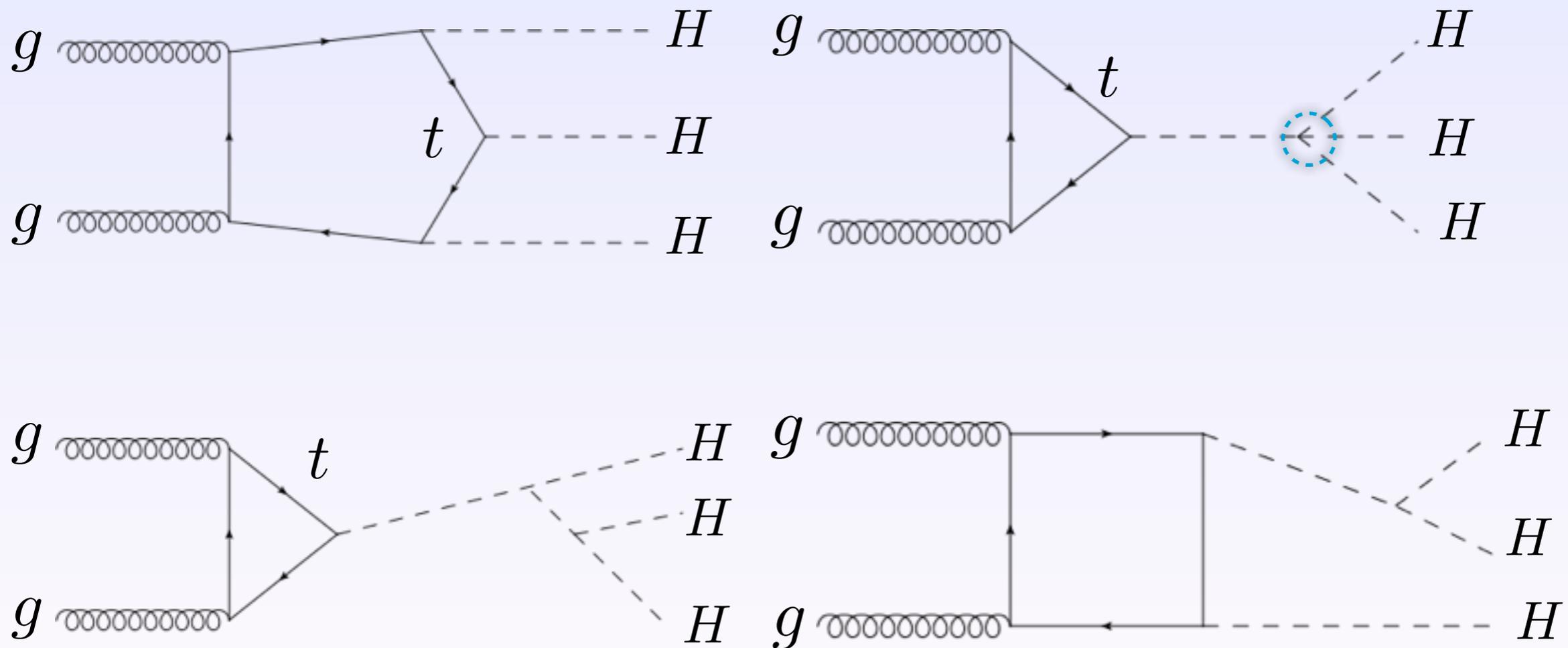
quartic Higgs coupling (I)

- consider triple Higgs boson production at hadron colliders,
- contributing diagrams: $gg \rightarrow HHH$



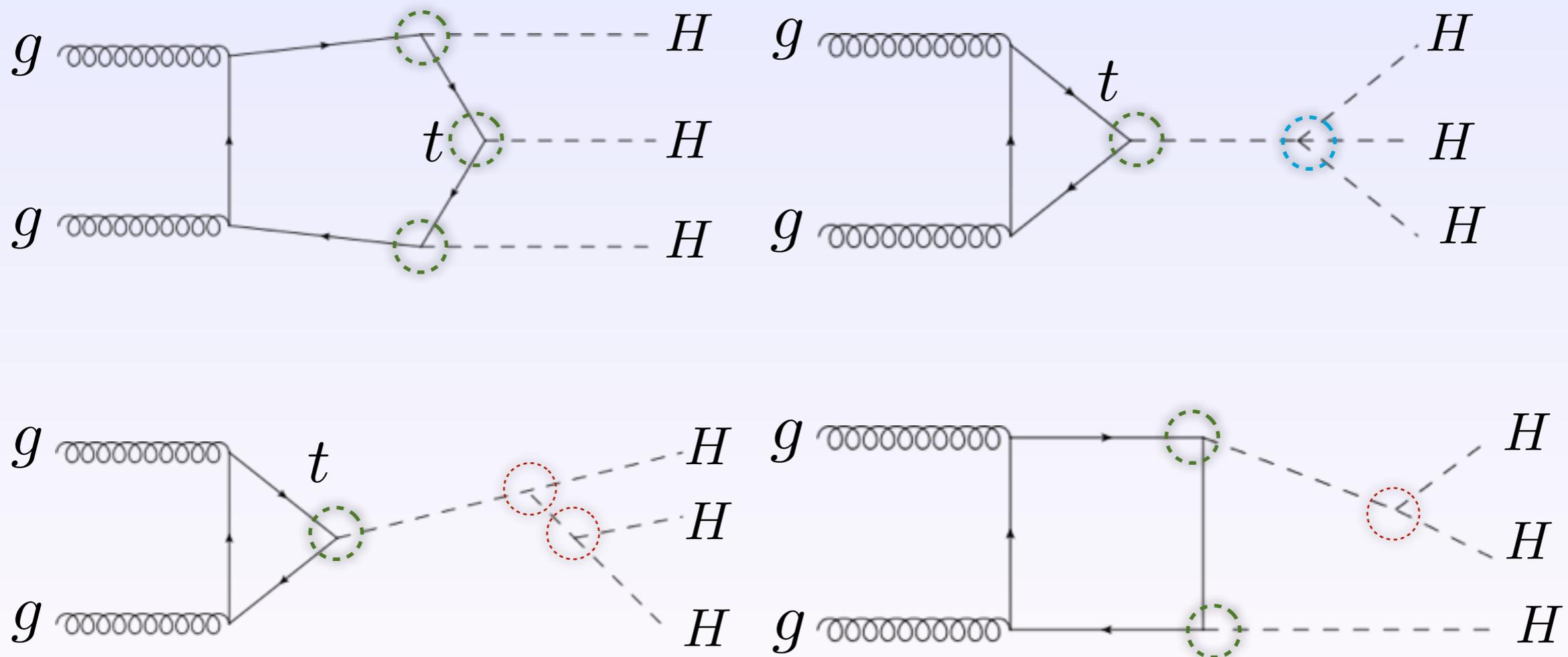
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quartic Higgs coupling (I)

- consider triple Higgs boson production at hadron colliders,
- contributing diagrams: $gg \rightarrow HHH$



quartic Higgs coupling (II)

- total HHH production @ LHC:

$$\sigma_{HHH}(\text{LHC@14 TeV}) \simeq 0.04 \text{ fb}$$

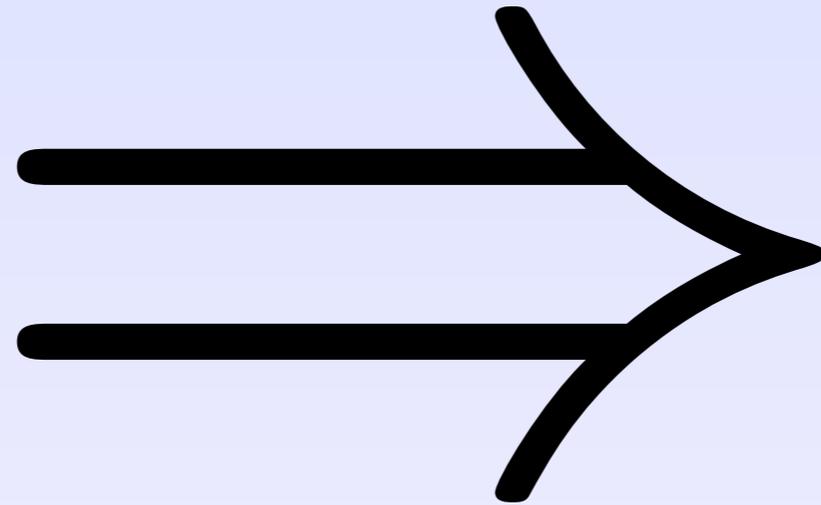
- even for V^n LHC:

$$\sigma_{HHH}(V^n\text{LHC@200 TeV}) \simeq 10 \text{ fb}$$

- AND: must know the triple coupling and top Yukawa well.

\Rightarrow extremely difficult to measure this coupling @ LHC or even any future hadron collider. [Plehn, Rauch, hep-ph/0507321]

(compare to: $\sigma_{gg \rightarrow H}(\text{LHC@14 TeV}) \simeq 50 \times 10^3 \text{ fb}$)

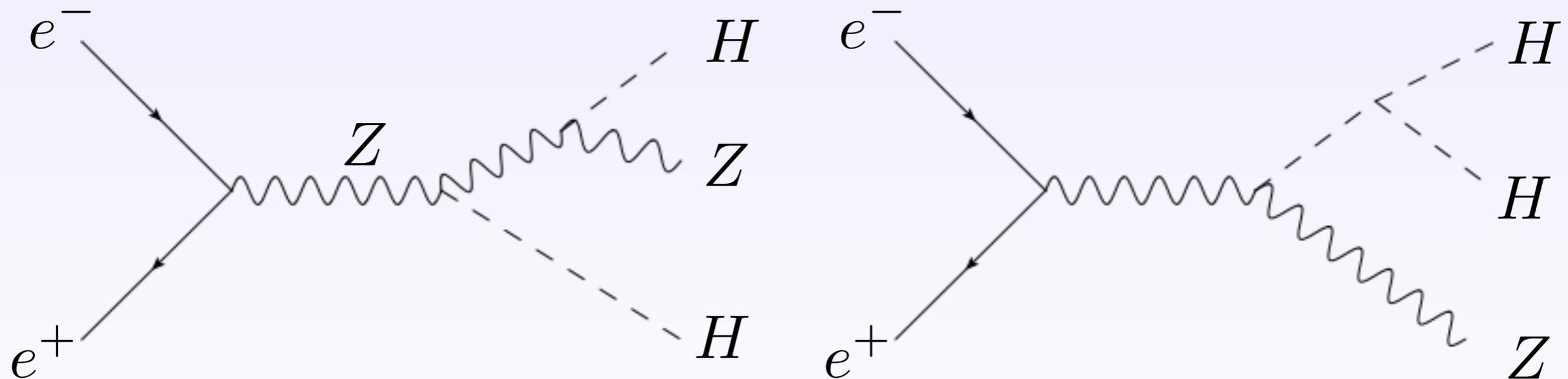


**focus on triple Higgs coupling
(through HH production)**

triple coupl. @ lin. colliders (I)

- at a linear collider, a few studies exist,
- based on processes such as:

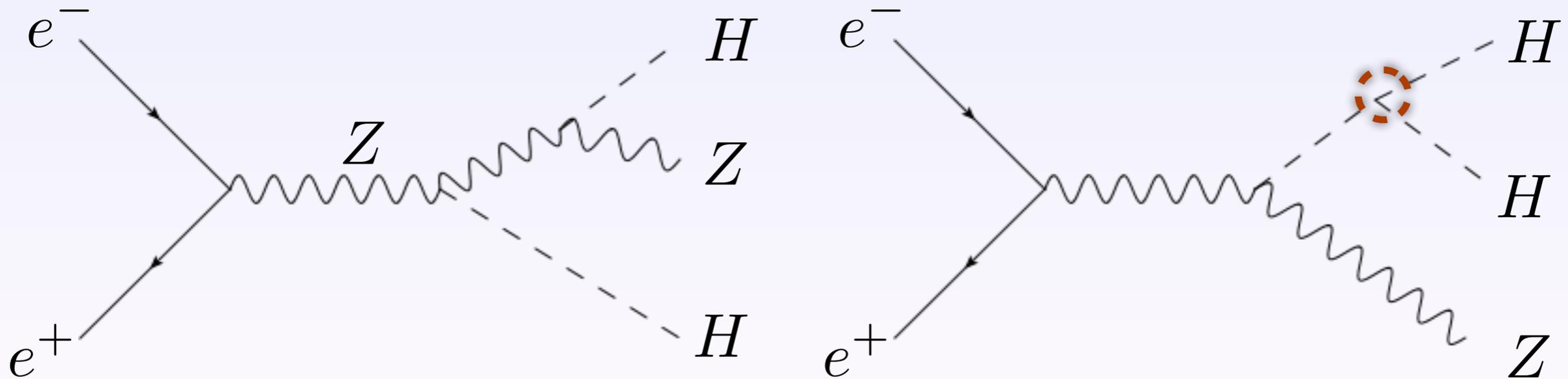
$$e^+e^- \rightarrow ZHH$$



triple coupl. @ lin. colliders (I)

- at a linear collider, a few studies exist,
- based on processes such as:

$$e^+e^- \rightarrow ZHH$$





triple coupl. @ lin. colliders (II)

- e.g. ILC [1306.6352] or TESLA TDR [hep-ph/0106315]:

$$e^+e^- \rightarrow ZHH \quad (\text{and both } H \rightarrow b\bar{b})$$

with:

$$\sigma(\sqrt{S} = 500 \text{ GeV}) \simeq 0.15 \text{ fb for: } M_H \simeq 125 \text{ GeV}$$

TESLA TDR (2001): cross section with $\sim 20\%$ error,

and λ with accuracy $\sim 20\%$: at 1000 fb^{-1} .

ILC TDR (2013): cross section with $\sim 27\%$ error,

and λ with accuracy $\sim 44\%$: at 2000 fb^{-1} .

ILC discrepancy:
'mis-clustering of
color-singlet groups'



'A new jet clustering
algorithm is now
being developed.'

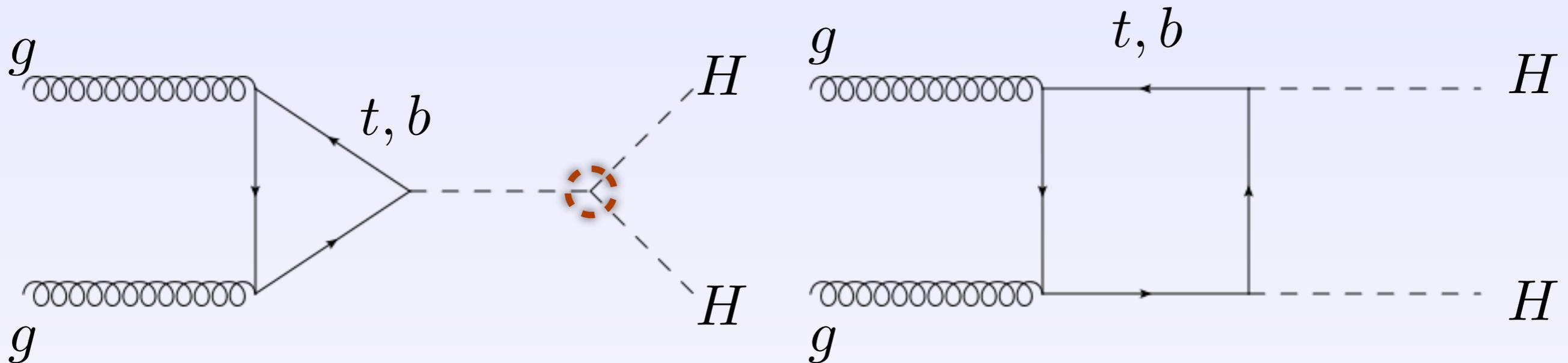


what about HH @ LHC (14 TeV)?

**first: dissection of the
production cross section.**

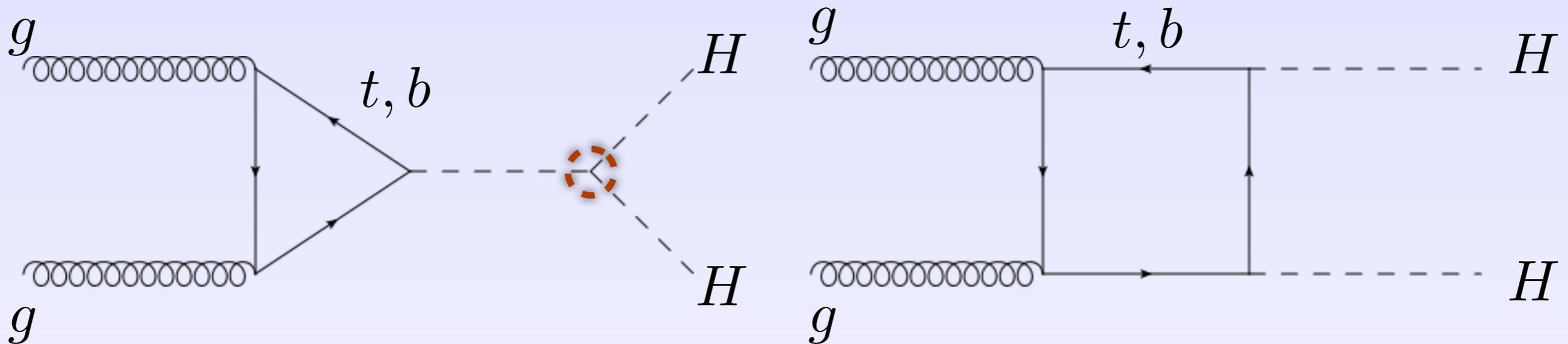
(LO) HH production @ LHC

- focus on dominant initial state: gluon-gluon fusion.
- leading order, two diagrams:



- effective theory (infinite top mass) insufficient: $Q^2 \gtrsim M_{\text{top}}^2$.
- loop calculation necessary to reproduce kinematical properties.

(LO) HH production @ LHC



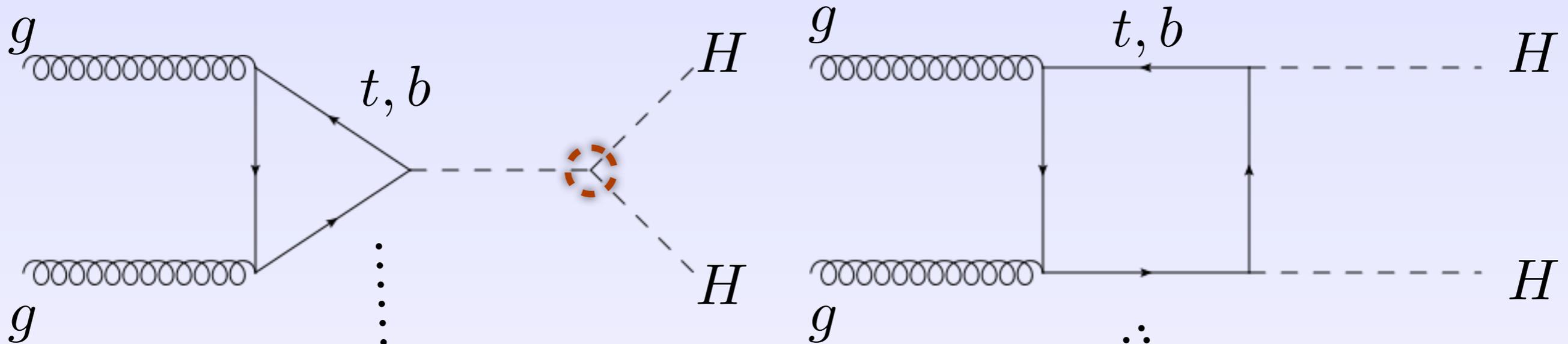
box and triangle topologies, Lorentz structures corresponding to spin-0 and spin-2 gg configurations.

$$\sigma_{HH}^{LO} = \left| \sum_q (\lambda y_q C_{q,\text{tri}}^{(\text{spin}-0)} + y_q^2 C_{q,\text{box}}^{(\text{spin}-0)}) \right|^2 + \left| \sum_q y_q^2 C_{q,\text{box}}^{(\text{spin}-2)} \right|^2$$

(sum over quarks $q = t, b$)

(couplings normalized to SM)

(LO) HH production @ LHC



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HH production @ LHC

- NLO calculation is only available in the infinite top mass limit. [Dawson, Dittmaier, Spira, [hep-ph/9805244]].
- K-factor in this limit ~ 2 .
- recently, corrections in inverse powers of the top quark mass have been computed, found to be significant. [Grigo, Hoff, Melnikov, Steinhauser, [1305.7340]].
- interesting fact: gg box and triangle contributions exactly cancel at the partonic threshold at LO. This causes a large enhancement of the K-factor in the threshold region.



HH production @ LHC

- using HPAIR (M. Spira), fits: Florian Goertz, AP, Li Lin Yang, and José Zurita [1301.3492]

$$\sigma_{HH}^{\text{LO}} [\text{fb}] = 5.22\lambda^2 y_t^2 - 25.1\lambda y_t^3 + 37.3y_t^4$$

$$\sigma_{HH}^{\text{NLO}} [\text{fb}] = 9.66\lambda^2 y_t^2 - 46.9\lambda y_t^3 + 70.1y_t^4$$

(couplings
normalized to
SM)

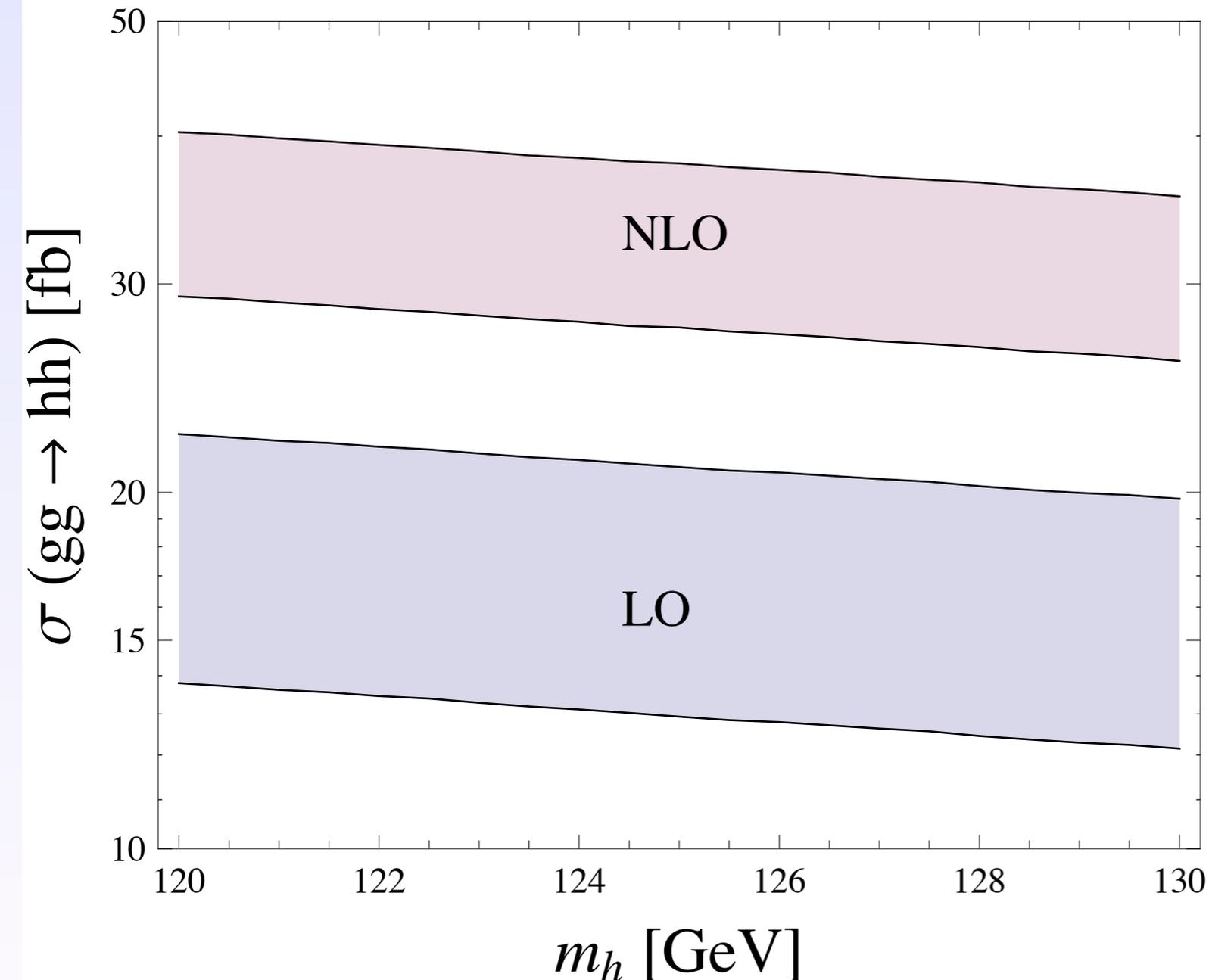
neglecting bottom quark contributions:
O(1%) at total cross section

- negative interference term between triangle and box.
- [interesting: a symmetry point exists at $\lambda \sim 2.5 y_t$ (NLO)].



HH production @ LHC

$$\sqrt{s} = 14 \text{ TeV}, m_{hh}/2 < \mu_F = \mu_R < 2 m_{hh}$$

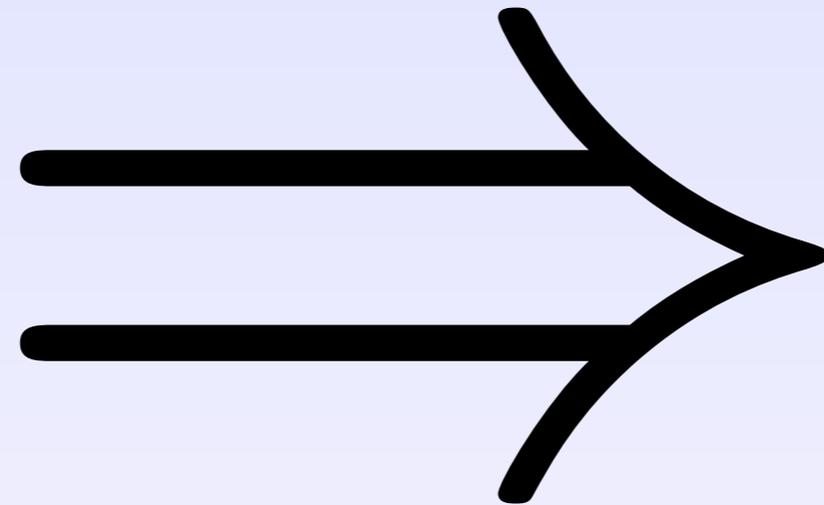


$$\sigma_{(M_H=125 \text{ GeV})}^{NLO} = 32.3^{+5.6}_{-4.7} \text{ fb}$$

AP, Li Lin Yang, and José Zurita
[arXiv:1209.1489]

(using HPAIR)

(NLO calculation exists only
in the heavy top limit)

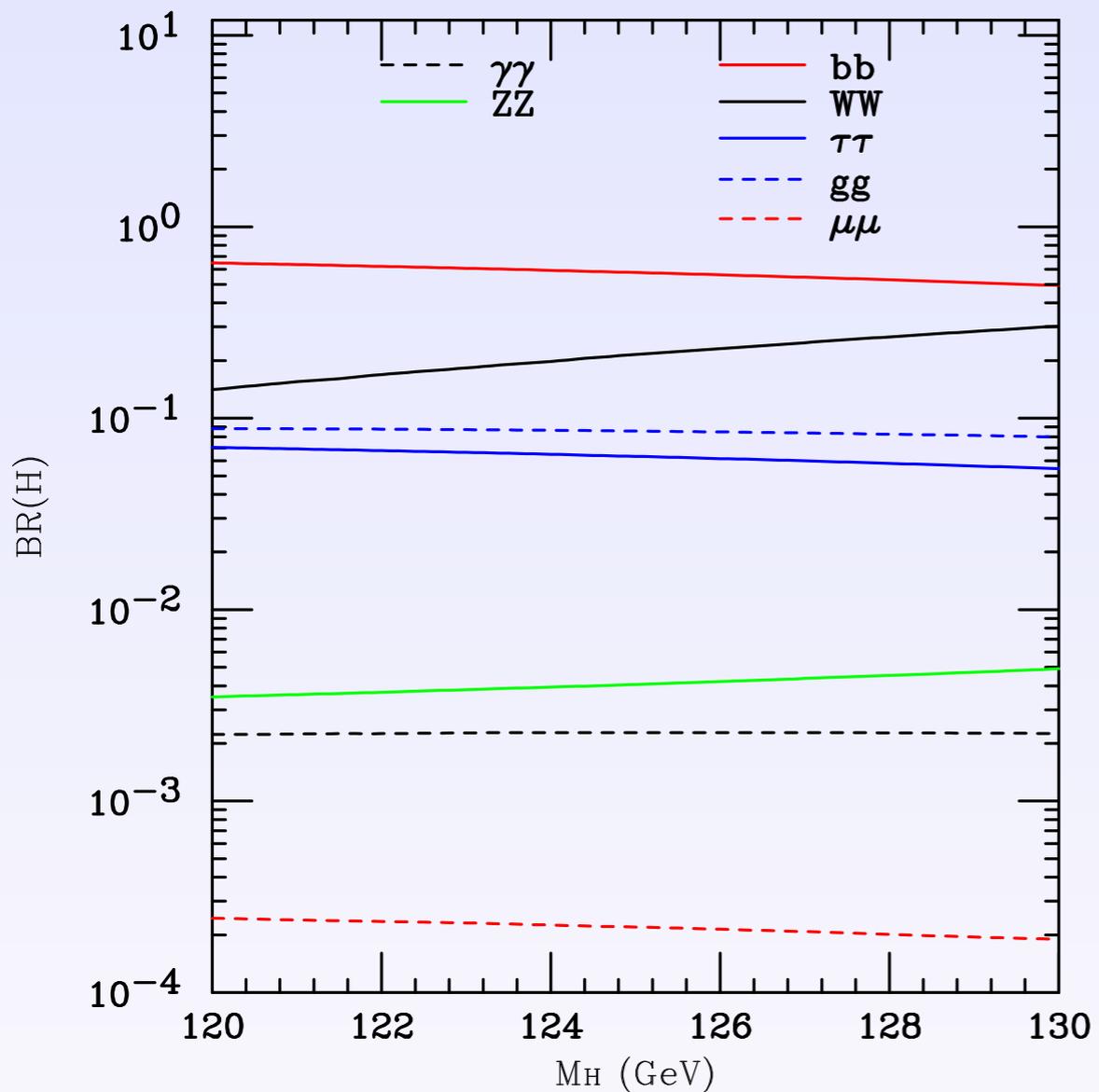


the decay channels

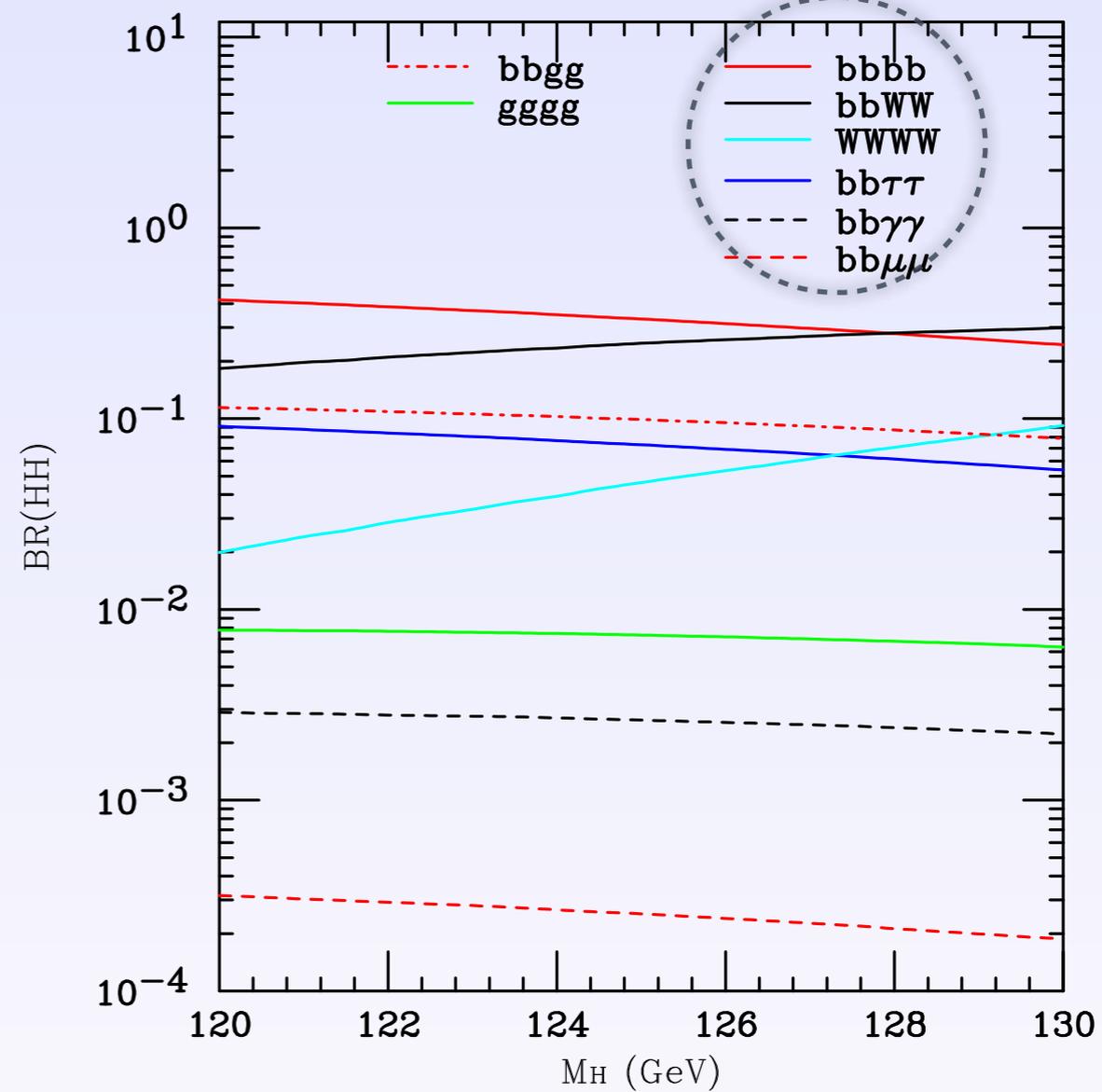
branching ratios

2

==



single production



pair production



branching ratios ($M_H = 125 \text{ GeV}$)

pair production

$$BR[b\bar{b}b\bar{b}] = 33.3\%$$

$$BR[b\bar{b}WW] = 24.8\%$$

$$BR[b\bar{b}\tau\tau] = 7.29\%$$

$$BR[WWWW] = 4.62\%$$

$$BR[WW\tau\tau] = 2.71\%$$

$$BR[\tau\tau\tau\tau] = 0.399\%$$

$$BR[b\bar{b}ZZ] = 0.305\%$$

$$BR[b\bar{b}\gamma\gamma] = 0.263\%$$

$$BR[b\bar{b}Z\gamma] = 0.178\%$$

$$BR[b\bar{b}\mu\mu] = 0.025\%$$

note: each 1% corresponds
to ~ 100 events per 300
 fb^{-1} of luminosity.



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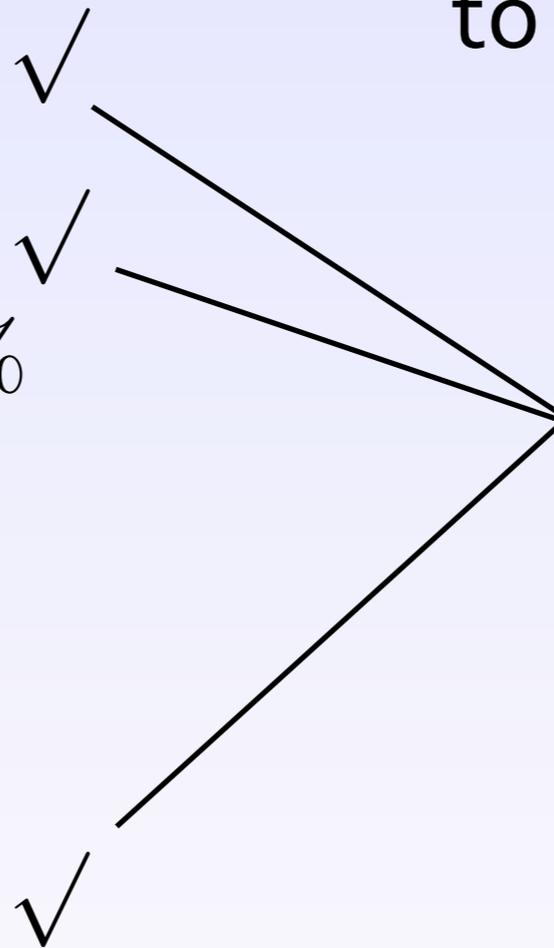
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shown to be potentially viable (in the SM)





establishment of HH process

- first step towards constraining the triple coupling: establishing (i.e. discovering) the HH process @ LHC.
- need large integrated luminosity: 600 fb^{-1} (certain) and 3000 fb^{-1} (possible).
- first: look at the channels that are considered to be 'viable'.



$$HH \rightarrow b\bar{b}\tau\tau$$

Dolan, Englert, Spannowsky, [1206.5001], Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira [1212.5581].

- BR = 7.29%, cross section $\sim 2.4\text{fb}$ (~ 700 events @ 300fb^{-1}).
- reconstruction of τ leptons experimentally delicate.
- backgrounds relatively low: electroweak and top decays with taus in the final states.
- Higgses naturally boosted: use a fat jet: sub-structure of the two b-quark system: like in Higgs+vector boson.
[Butterworth, Davison, Rubin, Salam, 0802.2470]
- results promising given a high τ -tagging efficiency (80%), b-tagging assumed 70%, low fake rates.
- $S \sim 50$ versus $B = 100$ at 600fb^{-1} ($\sim 5\sigma$).



$$HH \rightarrow b\bar{b}\gamma\gamma$$

Baur, Plehn, Rainwater, [hep-ph/031005], Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira [1212.5581].

- BR = 0.263%, cross section = 0.09 fb, (~ 27 events @ 300 fb^{-1}).
- low rate but ‘clean’. backgrounds generally low and mostly coming from reducible backgrounds due to mis-identification of b-jets or photons (jet-to- γ).
- $S \sim 30$ versus $B \sim 60$ at 3000 fb^{-1} ($\sim 4\sigma$).



$$HH \rightarrow b\bar{b}WW$$

Dolan, Englert, Spannowsky, [1206.5001], Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira [1212.5581], AP, Li Lin Yang, and José Zurita [arXiv:1209.1489]

- BR = 24.8%, cross section = 8.0 fb, (~ 2400 events @ 300 fb^{-1}).
- high rate, can have leptons + missing energy in the final state.
- but: huge backgrounds from top-anti-top production.
- with one leptonic W and one hadronic W was shown to be viable using jet sub-structure techniques. [AP, Li Lin Yang, and José Zurita [arXiv:1209.1489]]
- S = 11 versus B = 7 at 600 fb^{-1} ($\sim 4\sigma$).



more HH channels?

- $\underline{b\bar{b}b\bar{b}}$: highest BR, but fully hadronic (triggering?) and huge QCD backgrounds ($\sigma \sim 10.8$ fb).
- $\underline{b\bar{b}\mu\bar{\mu}}$: small initial cross section, essentially found to be impossible ($\sigma \sim 0.008$ fb). [Baur, Plehn, Rainwater [hep-ph/0304015]].
- \underline{WWWW} : good for high-mass Higgs. for low mass seems to be hard due to BR of Ws ($\sigma \sim 1.5$ fb).
- $\underline{\tau\tau\tau\tau}$: low rate and τ -tagging ($\sigma \sim 0.13$ fb).
- $\underline{WW\tau\tau}$: τ -tagging, W BRs ($\sigma \sim 0.86$ fb)
- $\underline{b\bar{b}Z\gamma}$, $\underline{b\bar{b}ZZ}$: low rates and BR for Zs ($\sigma < 0.1$ fb).

other production modes?

- several associated production modes exist:

cross section@14 TeV

$$qq \rightarrow qqHH \quad \sim 1.8 \text{ fb}$$

$$qq \rightarrow WHH \quad \sim 0.4 \text{ fb}$$

$$qq \rightarrow ZHH \quad \sim 0.3 \text{ fb}$$

Baglio, Djouadi, Gröber,
Mühlleitner, Quevillon, Spira
[1212.5581]

- note that: behaviour w.r.t. λ is different for each channel.
- with $HH \rightarrow b\bar{b}b\bar{b}$, could be looked into with sub-structure techniques, but initial cross section low.



summary of HH 'searches':

- 3 channels shown to be potentially viable.
- these should be looked at by experimentalists.
- some others could be investigated again.

Now: what about measuring λ ?



first: how well do we need to measure λ ?

- recent study estimates that λ will likely need to be measured to better than 20% to see a deviation from the SM expectation. [Gupta, Rzehak, Wells [1305.6397]]
- (considers new mixed-in singlets, MSSM Higgses and composite models.)
- other studies of HH in relation to BSM:
 - in composite Higgs models, e.g. [Gillioz, Gröber, Grojean, Mühlleitner, Salvioni, 1206.7120],
 - in warped extra-dimensional models, e.g. [Gouzevitch, Oliveira, Rojo, Rosenfeld, Salam, Sanz, 1303.6636],
 - Higgs portal/MSSM/pseudo-Nambu-Goldstone boson, e.g. [Dolan, Englert, Spannowsky, 1210.8166],
 - + many more...



how well can we measure λ ?

- large theoretical uncertainties: scale, PDF, α_s & effective theory (EFT) uncertainty: $\sim 30-40\%$. [Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira [1212.5581]].
- note: cross section has been shown to be larger than NLO EFT:
- including top mass corrections: $\sim 10\%$ increase. [Grigo, Hoff, Melnikov, Steinhauser, [1305.7340]]. Moreover, the scale dependence reduced ($\sim 5\%$).
- in the effective theory at NNLO: $\sim 20\%$ increase. [de Florian, Mazzitelli, [1305.5206]]



how well can we measure λ ?

- older studies considered analysis of shapes of distributions. [e.g. Baur, Plehn, Rainwater [hep-ph/0310056]].
- shapes may not be so well predicted at the moment: use rates instead. [Goertz, AP, Yang, Zurita [arXiv:1301.3492]].

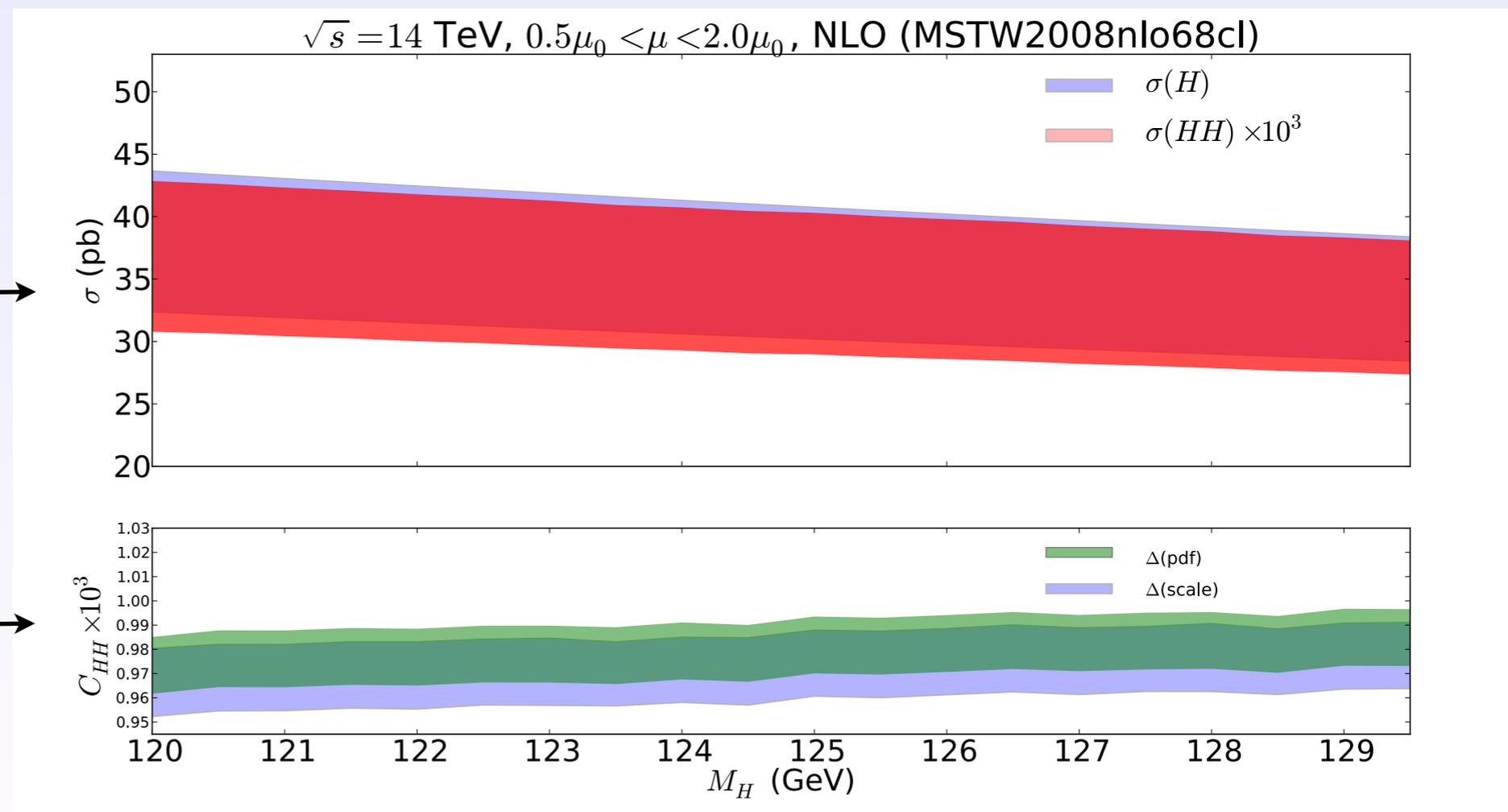
how well can we measure λ ?

- claim: single H and double H production possess similar topologies and hence QCD corrections could be also similar.
- can use ratios of cross section of HH to single H to cancel out part of the theoretical uncertainties.

$\sigma_{HH} \times 1000$

σ_H

$C_{HH} = \sigma_{HH} / \sigma_H$





how well can we measure λ ?

- using the three channels shown to be potentially viable, at 3000 fb^{-1} , LHC@14 TeV:

$$\begin{array}{lll} HH \rightarrow b\bar{b}\tau\tau & \Rightarrow & \lambda = 1.00^{+0.40}_{-0.31} \\ HH \rightarrow b\bar{b}\gamma\gamma & \Rightarrow & \lambda = 1.00^{+0.87}_{-0.52} \\ HH \rightarrow b\bar{b}WW & \Rightarrow & \lambda = 1.00^{+0.46}_{-0.35} \end{array} \quad \begin{array}{l} \diagup \\ \diagdown \end{array} \begin{array}{l} \text{times} \\ \text{the SM} \\ \text{value} \end{array}$$

[Goertz, AP, Yang, Zurita [arXiv:1301.3492]]

- “naively” combining: $\sim +30\%$, $\sim -20\%$ error.
- with this method, to get down to e.g. $O(10\%)$, we would need an extra 3-4 channels with an error of $\sim 40\%-50\%$ each.



outlook: theoretical improvements

- a better NLO calculation could be performed. hard: two loops with two mass scales (H and top mass).
- then this could be matched with the parton shower (MC@NLO/POWHEG) to provide better description of kinematical distributions.
- until then perhaps resummed calculations?
- or an improved Monte Carlo with the real emission matrix elements?
- with an improvement on the kinematical description, shape analyses could improve the limits from each channel.



outlook: 'other' improvements

- use 'improved' boosted jet techniques to push the channels further: e.g. Shower deconstruction or Q-jets.
[Soper, Spannowsky, [1102.3480, 1211.3140], Ellis, Hornig, Roy, Krohn, Schwartz [1201.1914]].
- other channels could be made viable.
- further 'technological' improvements in jet sub-structure techniques?
- long-term (& expensive!): an LHC energy upgrade will allow for more events to 'play with'. e.g. $\sigma(\text{HH}@33\text{TeV}) \sim 210 \text{ fb}$.



summary (I)

- with the discovery of the Higgs boson, an important next step is to determine the form of its potential.
- this can be done via measurement of the self-couplings.
- the triple Higgs coupling, λ , can be probed via Higgs pair production (HH).
- total cross section is low and the HH process is challenging.

summary (II)

- several investigations performed so far,
- but more work needs to be done:
- **theoretically:** improving description of the kinematics and the total cross section,
- **in phenomenology:** re-examine channels, search new,
- **experimentally:** to assess the viability of the promising channels.



special thanks

- special thanks to my collaborators:
Florian Goertz, José Zurita, Li Lin
Yang.



special thanks

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Florian Goertz, José Zurita, Li Lin
Yang.
- ...and thanks for your attention!

auxiliary slides



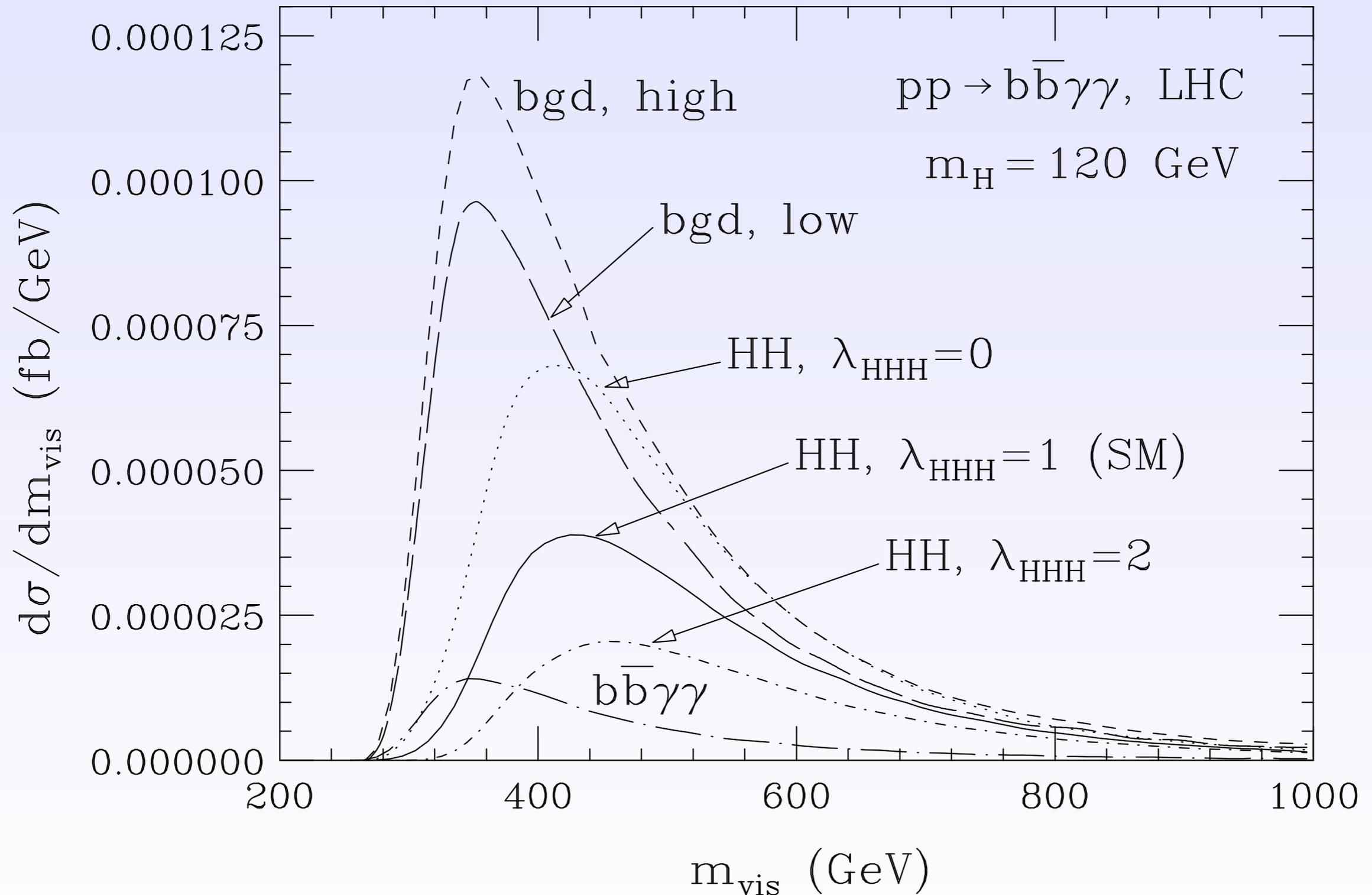
**how do we (actually) measure the
triple coupling λ ?**

using differential distributions



- (as seen in: Baur, Plehn, Rainwater [hep-ph/0310056])
- perform the analysis, e.g. for $b\bar{b}\gamma\gamma$.
- construct a differential distribution for signal and background using Monte Carlo.
- compare to Monte Carlo events to get expected bounds on the self-coupling.

using differential distributions (an example from Baur, Plehn, Rainwater):





using rates (i.e. cross sections)

- differential distributions for both signal and background may not be very well modeled.
- we can use the total rate predictions for signal and background instead.
- BUT: these can be dominated by large systematic uncertainties, originating either from:
 - unknown higher-order corrections,
 - parton density function uncertainties,
 - experimental errors,
 - + more.



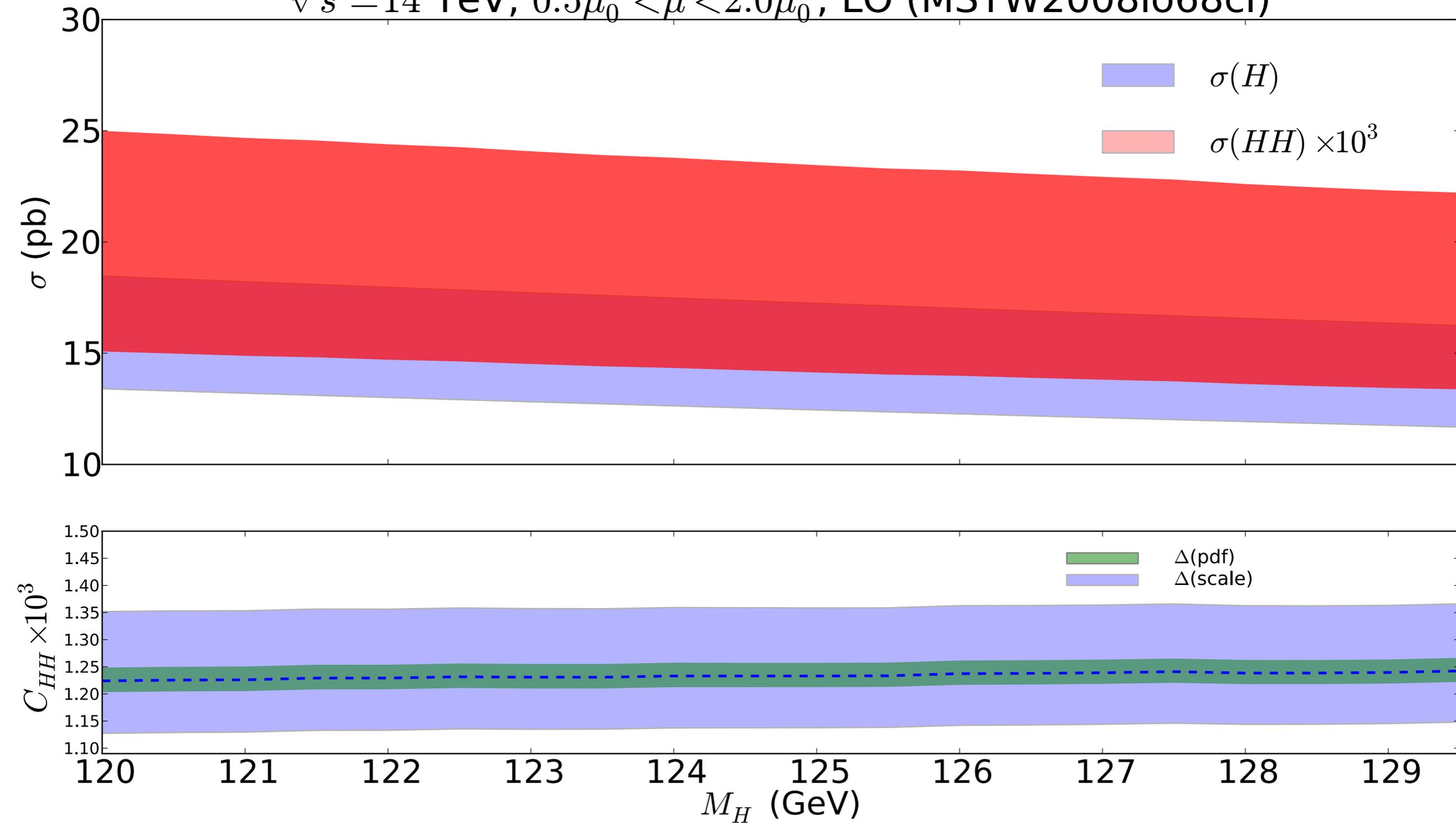
using ratios of cross sections

- consider:
$$C_{HH} = \frac{\sigma(gg \rightarrow HH)}{\sigma(gg \rightarrow H)},$$
- single Higgs production may possess similar higher-order QCD corrections to Higgs pair production.
- these may cancel out in the ratio, leading to a more stable prediction.
- moreover, experimental systematic uncertainties may cancel out, e.g. the luminosity uncertainty.
- we can check the degree to which extent the scale and pdf uncertainties cancel out.



leading order

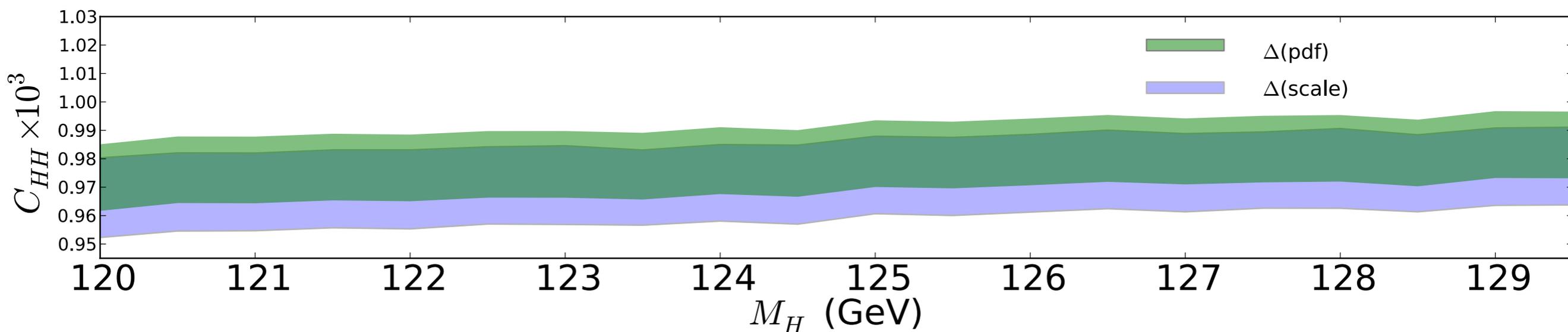
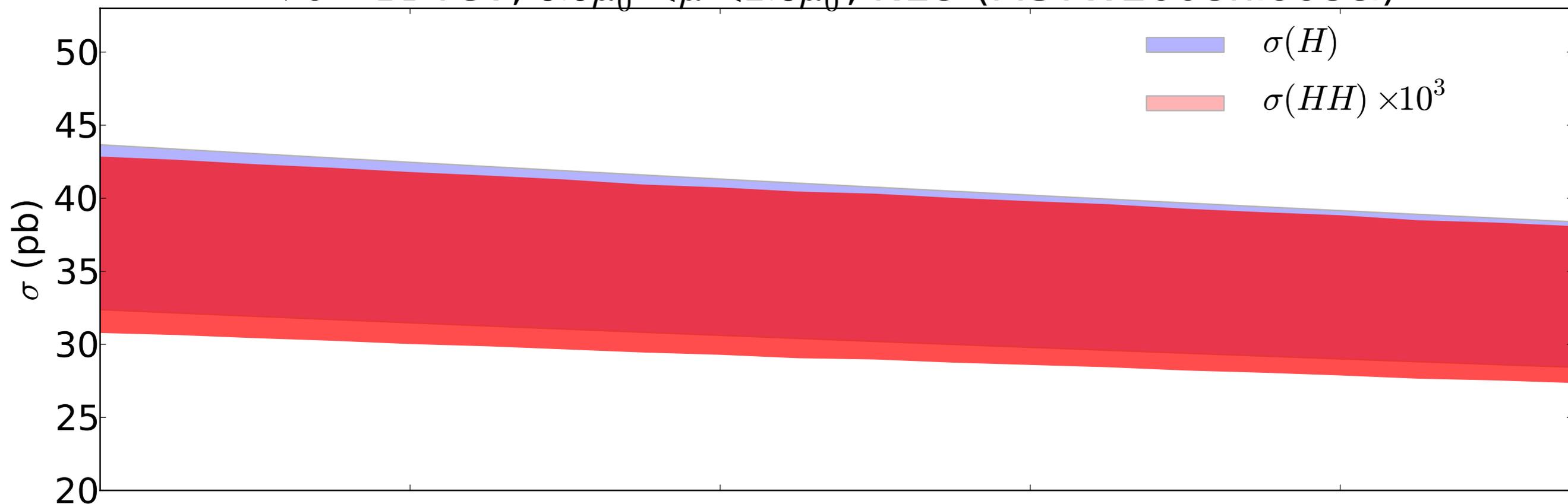
$\sqrt{s} = 14 \text{ TeV}, 0.5\mu_0 < \mu < 2.0\mu_0, \text{ LO (MSTW2008lo68cl)}$





next-to-leading order

$\sqrt{s} = 14 \text{ TeV}, 0.5\mu_0 < \mu < 2.0\mu_0, \text{ NLO (MSTW2008nlo68cl)}$





comments on ratio

- assuming that the scale uncertainties are correlated is a reasonable assumption.
- ratio goes from ~ 1.25 to ~ 1.0 from LO to NLO even though the K-factor is ~ 2 .
- a total theoretical uncertainty of $\sim 5\%$ is not unreasonable for the ratio, as opposed to $\sim 20\%$ for the cross section itself.
- we used the ratio, along with conservative expected experimental uncertainties to construct expected exclusion regions.



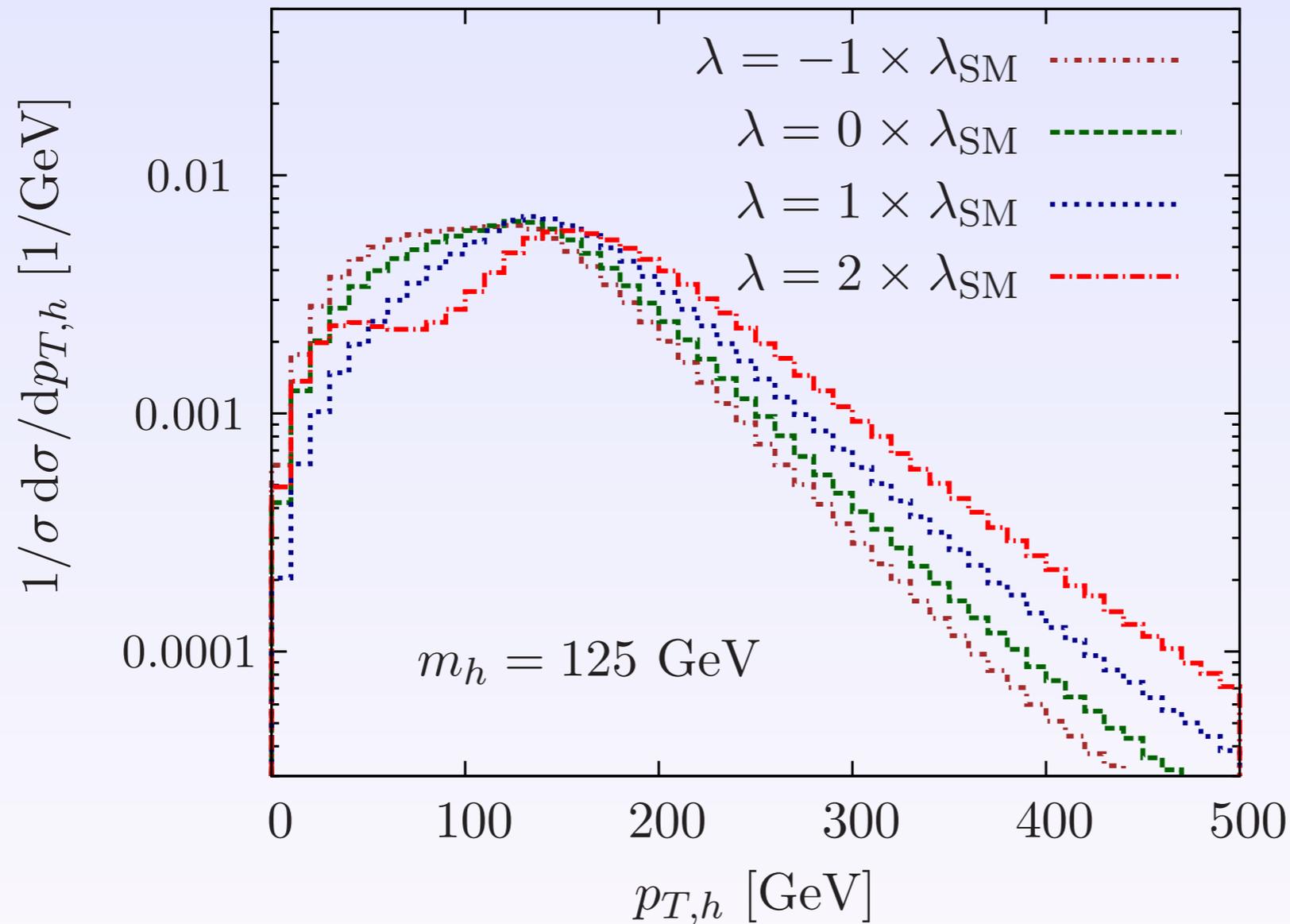
H+V, BDRS Analysis

[Butterworth, Davison, Rubin, Salam, 0802.2470]

- “BDRS” analysis:
 - Higgs decays to two b-quarks.
 - Cambridge/Aachen jet algorithm, $R=1.2$, get “fat jets”.
 - apply a “mass-drop” condition on a hard jet:
 - picks up the decay of a massive particle, e.g. $H \rightarrow b\bar{b}$
 - “filter” the jet: re-apply the jet algorithm with a smaller R , on the “fat” jet constituents, take three hardest “sub-jets”.
 - ask for the two hardest “sub-jets” to contain b-tags.
 - “filtering” reduces the effective area of the “Higgs”-jet,
 - hence reduces pollution from Underlying Event.

BDRS analysis on H+H

- the Higgs bosons in HH are naturally boosted:



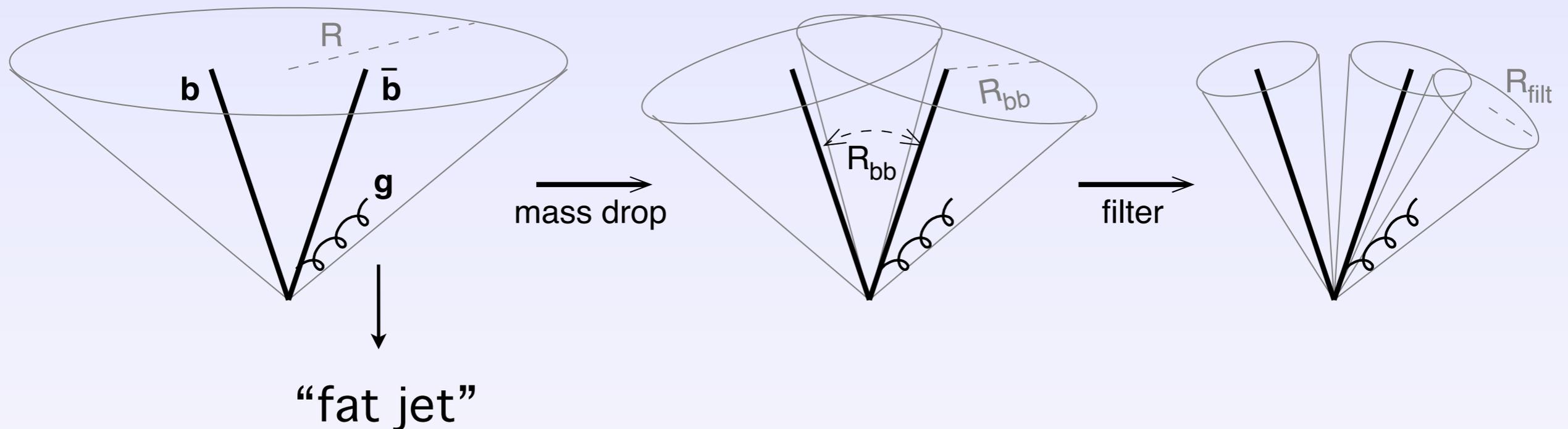
[Dolan, Englert, Spannowsky, 1206.5001]

- + other arguments of BDRS technique apply.

H+V

- “BDRS” analysis, pictorially:

[Butterworth, Davison, Rubin, Salam, 0802.2470]



- HV: yields good sensitivity (4.5σ) @ 14 TeV @ 30 fb^{-1} .
- perhaps an improvement of previous HH results can be also achieved!

electroweak Lagrangian (I)

- ingredients of the 'recipe':



+ (...)



electroweak Lagrangian (I)

- ingredients of the ‘recipe’:

an $SU(2) \times U(1)$ gauge symmetry

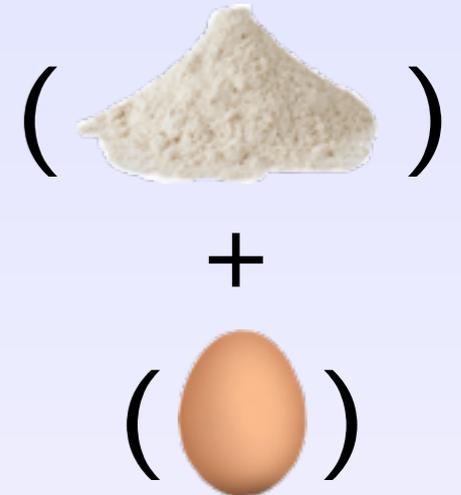
+ a complex doublet scalar, ϕ .

electroweak Lagrangian (I)

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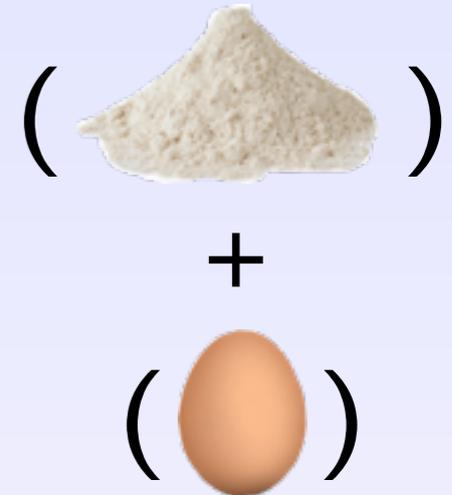


electroweak Lagrangian (I)

- ingredients of the ‘recipe’:

an $SU(2) \times U(1)$ gauge symmetry

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- start by writing (i.e. Higgs boson Lagrangian):

$$\mathcal{L} = (D^\mu \phi)(D_\mu \phi) - \mathcal{V}(\phi^\dagger \phi)$$

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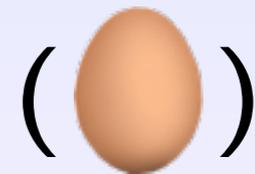
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$$\mathcal{L} = (D^\mu \phi)(D_\mu \phi) - \mathcal{V}(\phi^\dagger \phi)$$

the covariant derivative:

$$D^\mu = \partial^\mu + ig_2 (T \cdot W^\mu) + iY g_1 B^\mu$$

SU(2) coupl.

SU(2) gens.

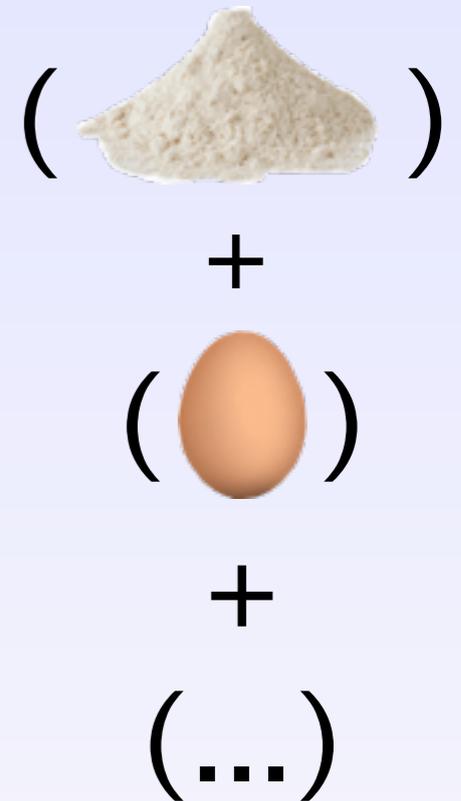
U(1) coupl.

electroweak Lagrangian (II)

- with potential:

$$\mathcal{V}(\phi^\dagger \phi) = \lambda(\phi^\dagger \phi)^2 + \mu^2 \phi^\dagger \phi,$$

$$(\lambda > 0, \mu^2 < 0)$$



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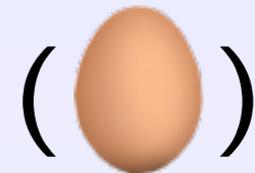
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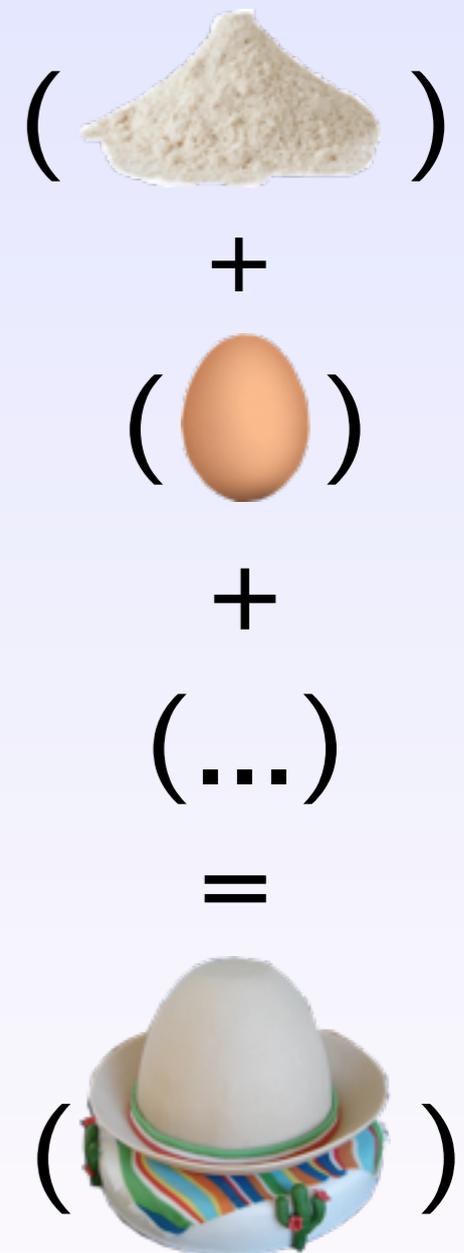
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electroweak Lagrangian (III)

- further steps:
 - choose minimum in particular direction:

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad (\text{implies: residual } U(1) \text{ invariance})$$

- consider fluctuations of scalar field about that minimum,
- and make a gauge transformation to absorb the Goldstone modes into the gauge bosons.



electroweak Lagrangian (IV)

- hence, after symmetry breaking, the Higgs + SU(2)xU(1) Lagrangian becomes:

$$\mathcal{L} = \frac{1}{2} \partial_\mu H \partial^\mu H - \mathcal{V}(H; \lambda, v) + \frac{(v + H)^2}{8} \begin{pmatrix} 0 & 1 \end{pmatrix} (2g_2 T \cdot W_\mu + g_1 B_\mu) \times (2g_2 T \cdot W^\mu + g_1 B^\mu) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

fluct. about min.
 $\phi \propto (0, v + H)$

(recall: μ , λ and v are related and hence only 2/3 are independent.)

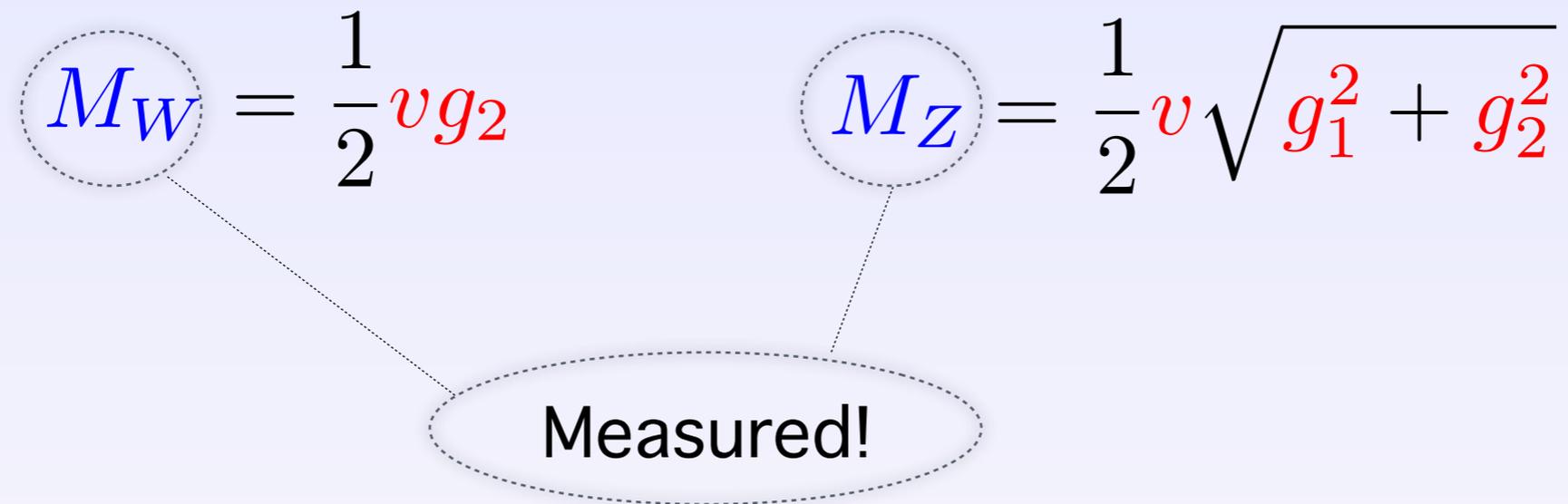
↪ 'Free' parameters: v, g_1, g_2, λ

'fixing' free params. (I)

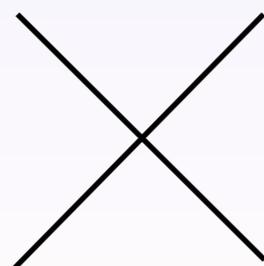
- diagonalize the quadratic terms in vector boson fields,
- and deduce the masses of Z and W bosons:

$$M_W = \frac{1}{2} v g_2$$
$$M_Z = \frac{1}{2} v \sqrt{g_1^2 + g_2^2}$$

Measured!



- 4-fermion interaction at low energies can fix the Fermi constant:


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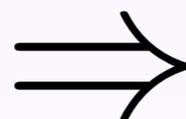
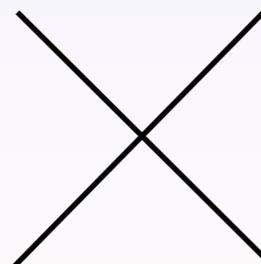
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WARNING: Leading Order!

- 4-fermion interaction at low energies can fix the Fermi constant:



$$\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}$$

'fixing' free params. (II)

- until very recently, only had 3 out of 4 constraining equations...
- ...in July 2012, we obtained the fourth:

$$M_H = \sqrt{2\lambda}v$$

Measured!

$\hookrightarrow \sim 125 \text{ GeV}$