Minimal Flavour Violation in Two Higgs doublet Models

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on-going collaboration with

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Two Higgs doublet models (2HDM)

Several Motivations

- New Sources of CP violation

  SM cannot account for BAU

- Possibility of having spontaneous CP violation

  EW sym breaking and $\not{CP}$ same footing

  T. D. Lee 1973 ; Kobayashi and Maskawa 1973

- Strong CP problem, Peccei-Quinn

- Supersymmetry

LHC important rôle

Neutral currents have played an important rôle in the construction and experimental tests of unified gauge theories

EPS Prize in 2009 to Gargamelle, CERN

In the Standard Model Flavour Changing Neutral Currents (FCNC) are forbidden at tree level

- in the gauge sector, ie no Z FCNC
- in the scalar sector, ie no H FCNC

- Models with two or more Higgs doublets potentially large HFCNC

strict limits on FCNC processes!

Proposed solutions, case of Multi-Higgs models

without HFCNC

NFC — Weinberg, Glashow (1977)
Paschos (1977)

Aligned two-Higgs-doublet model
Pich, Tuzon (2009)

with HFCNC

existence of suppression factors in HFCNC
Antaramian, Hall, Rasin (1992)
Hall, Weinberg (1993)
Joshipura, Rindani (1991)

- first models of this type with no ad-hoc assumptions
suppression by small elements of V_{CKM} : BGL models
Branco, Grimus, Lavoura (1996)

Minimal Flavour Violation

Models With NFC

single scalar doublet coupling to each type of f_R

The softly broken Z_2 symmetric 2HDM potential

CP conserving type I and type II

recent work Barroso, Ferreira, Haber, Ivanov, Santos, Sher, Silva

Cheon, Kang (2012); Grinstein, Uttayarat (2013); Eberhardt, Nierste, Wiebusch (2013)

2HDM type II Yukawa With CP violation

Basso, Lipniacka, Mahmoudi, Moretti, Osland, Pruna, Purmohammadi (2012) + (2013)

Notation

Yukawa interactions

$$\mathcal{L}_Y = -\,\overline{Q}_L^{0}\,\Gamma_1\,\Phi_1\,d_R^{0} - \overline{Q}_L^{0}\,\Gamma_2\,\Phi_2\,d_R^{0} - \overline{Q}_L^{0}\,\Delta_1\,\tilde{\Phi}_1\,u_R^{0} - \overline{Q}_L^{0}\,\Delta_2\,\tilde{\Phi}_2\,u_R^{0} + h.c.$$

$$\tilde{\Phi}_i = -i\,\tau_2\,\Phi_i^{*}$$

Quark mass matrices

$$M_d = \frac{1}{\sqrt{2}}\left(v_1\,\Gamma_1 + v_2\,e^{i\alpha}\,\Gamma_2\right)\;;\; M_u = \frac{1}{\sqrt{2}}\left(v_1\,\Delta_1 + v_2\,e^{-i\alpha}\,\Delta_2\right)$$

Diagonalized by

$$U_{d_L}^{\dagger}\,M_d\,U_{d_R} = D_d \equiv \mathrm{diag}\,(m_d, m_s, m_b)$$

$$U_{u_L}^{\dagger}\,M_u\,U_{u_R} = D_u \equiv \mathrm{diag}\,(m_u, m_c, m_t)$$

Leptonic Sector

$$-\overline{L_L^0}\, \Pi_1\, \phi_1\, l_R^0 - \overline{L_L^0}\, \Pi_2\, \phi_2\, l_R^0 + h.c.$$

$$\left(-\overline{L_L^0}\, \Sigma_1\, \tilde{\phi}_1\, \nu_R^0 - \overline{L_L^0}\, \Sigma_2\, \tilde{\phi}_2\, \nu_R^0 + h.c \right)$$

$$\left(\frac{1}{2} \nu_R^{0T}\, C^{-1}\, M_R\, \nu_R^0 + h.c \right)$$

Expansion around the vev's

$$\phi_j = \begin{pmatrix} \phi_j^+ \\ \frac{e^{i\alpha_j}}{\sqrt{2}}(v_j + \rho_j + i\eta_j) \end{pmatrix}, \quad j = 1,2$$

we perform the following transformation

$$\begin{pmatrix} H^0 \\ R \end{pmatrix} = U \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}; \quad \begin{pmatrix} G^0 \\ I \end{pmatrix} = U \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}; \quad \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = U \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}$$

$$U = \frac{1}{v}\begin{pmatrix} v_1 e^{-i\alpha_1} & v_2 e^{-i\alpha_2} \\ -v_2 e^{-i\alpha_1} & v_1 e^{-i\alpha_2} \end{pmatrix}; \quad v = \sqrt{v_1^2 + v_2^2} = \left(\sqrt{2}\, G_F\right)^{-1/2} \simeq 246\,\text{GeV}$$

- U singles out

H^0 with couplings to quarks proportional to mass matrices

G^0 neutral pseudo-Goldstone boson

G^+ charged pseudo-Goldstone boson

Physical neutral Higgs fields are combinations of H^0, R and I

Neutral and charged Higgs interactions for the quark sector

$$\mathcal{L}_Y(\text{quark, Higgs}) = -\bar{d}_L^0 \frac{1}{v}\left(M_d H^0 + N_d^0 R + i N_d^0 I\right) d_R^0 +$$

$$+ \bar{u}_L^0 \frac{1}{v}\left[M_u H^0 + N_u^0 R + i N_u^0 I\right) u_R^0 -$$

$$- \frac{\sqrt{2} H^+}{v}\left(\bar{u}_L^0 N_d^0 d_R^0 - u_R^0 N_u^{0\dagger} d_L^0\right) + h.c.$$

$$N_d^0 = \frac{1}{\sqrt{2}}\left(v_2 \Gamma_1 - v_1 e^{i\alpha}\Gamma_2\right), \quad N_u^0 = \frac{1}{\sqrt{2}}\left(v_2 \Delta_1 - v_1 e^{-i\alpha}\Delta_2\right)$$

Flavour structure of quark sector of 2HDM characterized by

M_d, M_u, N_d^0, N_u^0

likewise leptonic sector, Dirac neutrinos

M_e, M_ν, N_e^0, N_ν^0

Yukawa couplings in terms of quark mass eigenstates for H^+, H^0, R, I

$$\mathcal{L}_Y = \dots \sqrt{2}\,\frac{H^+}{v}\,\bar{u}\left(-V N_d \gamma_R + N_u^\dagger V \gamma_L\right) d \; + \text{h.c.} \; -$$

$$- \frac{H^0}{v}\left(\bar{u} D_u u + \bar{d} D_d d\right) -$$

$$- \frac{R}{v}\left[\bar{u}\left(N_u \gamma_R + N_u^\dagger \gamma_L\right) u + \bar{d}\left(N_d \gamma_R + N_d^\dagger \gamma_L\right) d\right] +$$

$$+ i\,\frac{I}{v}\left[\bar{u}\left(N_u \gamma_R - N_u^\dagger \gamma_L\right) u - \bar{d}\left(N_d \gamma_R - N_d^\dagger \gamma_L\right) d\right]$$

$\gamma_L = (1-\gamma_5)/2$; $\gamma_R = (1+\gamma_5)/2$ $\qquad V \equiv V_{CKM}$

Flavour changing neutral currents controlled by:

$$N_d = \frac{1}{\sqrt{2}} U_{d_L}^{\dagger} \left(v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2 \right) U_{d_R}$$

$$N_u = \frac{1}{\sqrt{2}} U_{u_L}^{\dagger} \left(v_2 \Delta_1 - v_1 e^{-i\alpha} \Delta_2 \right) U_{u_R}$$

For generic two Higgs doublet models
N_u, N_d non-diagonal arbitrary

For definiteness rewrite N_d :

$$N_d = \frac{v_2}{v_1} D_d - \frac{v_2}{\sqrt{2}} \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_{d_L}^{\dagger} e^{i\alpha} \Gamma_2 U_{d_R}$$

conserves flavour

leads to FCNC

The flavour structure of Yukawa couplings is not constrained by gauge invariance

All flavour changing transitions in SM are mediated by charged weak currents with flavour mixing controlled by V_{CKM}

MFV essentially requires flavour and CP violation linked to known structures of Yukawa couplings

[all new flavour changing transitions are controlled by the CKM matrix]

About Minimal Flavour Violation

Buras, Gambino, Gorbahn, Jager, Silvestrini (2001)

D'Ambrosio, Giudice, Isidori, Strumia (2002)

leptonic sector

Cirigliano, Grinstein, Isidori, Wise (2005)

$G_F = U(3)^5$ largest symmetry of the gauge sector

flavour violation completely determined by Yukawa couplings

Our framework

- multi-Higgs models
- no Natural Flavour Conservation
- must obey above condition (one of the defining ingredients of MFV framework)

In order to obtain a structure for Γ_i, Δ_i such that FCNC at tree level strength completely controlled V_{CKM}

Branco, Grimus, Lavoura imposed symmetry

$$Q^0_{Lj} \to \exp(iz) Q^0_{Lj} \; ; \; u^0_{Rj} \to \exp(2iz) u^0_{Rj} \; ; \; \phi_2 \to \exp(iz)\phi_2 \; , \; z \neq 0, \pi$$

$$\Gamma_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix} ; \; \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix} ; \; \Delta_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \; \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$$

$j = 3$

Both Higgs have non-zero Yukawa couplings in the up and down sector

Special WB chosen by the symmetry

FCNC in down sector

if instead of $u^0_{Rj} \to \exp(2iz) u^0_{Rj}$ impose $d^0_{Rj} \to \exp(2iz) d^0_{Rj}$

then FCNC in up sector

Six different BGL models

$$(N_d)_{rs} = \frac{v_2}{v_1}(D_d)_{rs} - \left(\frac{v_2}{v_1}+\frac{v_1}{v_2}\right)\underbrace{(V_{CKM}^{\dagger})_{r3}(V_{CKM})_{3s}}_{\text{MFV}}(D_d)_{ss}$$

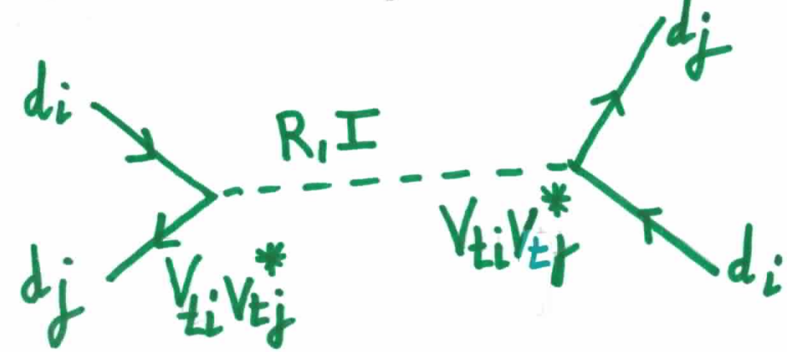
$$j = 3$$

$$N_u = -\frac{v_1}{v_2}\,\mathrm{diag}(0,0,m_t) + \frac{v_2}{v_1}\,\mathrm{diag}(m_u,m_c,0)$$

FCNC only in the down sector
suppression by the 3rd row of V_{CKM}
dependence on V_{CKM} and $\tan\beta$ only

Strong and Natural suppression of the most constrained processes

e.g. $|V_{td}V_{ts}^{*}|^2 \sim \lambda^{10}$



What is the necessary condition for N_d^0, N_u^0 to be of MFV type?

Should be functions of M_d, M_u no other flavour dependence

Furthermore, N_d^0, N_u^0 should transform appropriately under WB

$$Q_L^0 \rightarrow W_L Q_L^0 \quad , \quad d_R^0 \rightarrow W_R^d d_R^0 \quad , \quad u_R^0 \rightarrow W_R^u u_R^0$$

$$M_d \rightarrow W_L^\dagger M_d W_R^d \quad , \quad M_u \rightarrow W_L^\dagger M_u W_R^u$$

N_d^0, N_u^0 transform as M_d, M_u

$$N_d^0 \propto M_d \;;\; (M_d M_d^\dagger) M_d \;;\; (M_u M_u^\dagger) M_d$$

$$Y_d \;;\; (Y_d Y_d^\dagger) Y_d \;;\; (Y_u Y_u^\dagger) Y_d \quad \text{Yukawa}$$

see previous references

What is particular about BGL models in MFV context?

$$M_d M_d^\dagger \equiv H_d \quad ; \quad U_{d_L}^\dagger M_d U_{d_R} = D_d \quad ; \quad U_{d_L}^\dagger H_d U_{d_L} = D_d^2$$

$$D_d^2 = \mathrm{diag}(m_d^2, m_s^2, m_b^2) = m_d^2 \underbrace{\begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \end{pmatrix}}_{P_1} + m_s^2 \underbrace{\begin{pmatrix} 0 & & \\ & 1 & \\ & & 0 \end{pmatrix}}_{P_2} + m_b^2 \underbrace{\begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix}}_{P_3}$$

$$D_d^2 = \sum_i m_{d_i}^2 P_i$$

$$H_d = U_{d_L} D_d^2 U_{d_L}^\dagger = \sum_i m_{d_i}^2 U_{d_L} P_i U_{d_L}^\dagger = \sum_i m_{d_i}^2 P_i^{d_L}$$

$U_{d_L} P_i U_{d_L}^\dagger$ rather than $Y_d Y_d^\dagger$ are the minimal building blocks to be used in the expansion of N_d^0, N_u^0 conforming to the MFV requirement

Botella, Nebot, Vives 2004

It is convenient to write H_d, H_u in terms of projection operators

Botella, Nebot, Vives 2004

$$H_d = \sum_i m_{d_i}^2 P_i^{dL} \; ; \quad P_i^{dL} = U_{dL} P_i U_{dL}^\dagger \; ; \quad (P_i)_{jk} = \delta_{ij}\delta_{ik} \qquad u \leftrightarrow d$$

MFV expansion for N_d^0 and N_u^0

$$N_d^0 = \lambda_1 M_d + \lambda_{2i} U_{dL} P_i U_{dL}^\dagger M_d + \lambda_{3i} U_{uL} P_i U_{uL}^\dagger M_d + \ldots$$

$$N_u^0 = \tau_1 M_u + \tau_{2i} U_{uL} P_i U_{uL}^\dagger M_u + \tau_{3i} U_{dL} P_i U_{dL}^\dagger M_u + \ldots$$

In green terms that do not lead to FCNC

In red terms that lead to FCNC

In the quark eigenstate basis

$$N_d = \lambda_1 D_d + \lambda_{2i} P_i D_d + \lambda_{3i} (V_{CKM})^\dagger P_i V_{CKM} D_d + \ldots$$

$$N_u = \tau_1 D_u + \tau_{2i} P_i D_u + \tau_{3i} V_{CKM} P_i (V_{CKM})^\dagger D_u + \ldots$$

At this stage λ and τ coefficients appear as free parameters, MFV

Need for additional symmetries in order to constrain these coeff.

WB covariant definition for BGL models

$$N_d^0 = \frac{v_2}{v_1} M_d - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_j^{\gamma} M_d$$

$$N_u^0 = \frac{v_2}{v_1} M_u - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_j^{\gamma} M_u$$

together with

$$\mathcal{P}_j^{\gamma} \Gamma_2 = \Gamma_2 \quad , \quad \mathcal{P}_j^{\gamma} \Gamma_1 = 0$$

$$\mathcal{P}_j^{\gamma} \Delta_2 = \Delta_2 \quad , \quad \mathcal{P}_j^{\gamma} \Delta_1 = 0$$

γ stands for u (up) or d (down)

$\mathcal{P}_j^{\gamma}$ are projection operators — Botella, Nebot, Vives 2004

$$\mathcal{P}_j^{u} = U_{uL} P_j U_{uL}^{\dagger} \qquad \mathcal{P}_j^{d} = U_{dL} P_j U_{dL}^{\dagger}$$

$$(P_j)_{lk} = \delta_{jl} \delta_{jk}$$

e.g. $P_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

BGL is the only implementation of models where Higgs FCNC are a function of V_{CKM} only (together with v_1, v_2) which are based on an Abelian symmetry obeying the sufficient conditions of having M_u block diagonal together with the existence of a matrix P such that

$$P\Gamma_2 = \Gamma_2 \quad ; \quad P\Gamma_1 = 0$$

Ferreira, Silva arXiv: 1012287

Alternative MFV implementations in 2HDM

Dery, Efrati, Hiller, Hochberg, Nir (2013)

$$Y^U = \frac{\sqrt{2}\, M^U}{v}, \quad Y^D = \frac{\sqrt{2}\, M^D}{v}, \quad Y^E = \frac{\sqrt{2}\, M^E}{v} \; ; \; Y_S^F, \quad S = h, H, A$$

e.g. leptonic sector $G^{\ell}_{global} = SU(3)_L \times SU(3)_E$

Definition leptonic MFV, only one spurion breaks G^{ℓ}_{global}

$$\hat{Y} \sim (3, \bar{3})$$

In the most general case, each Yukawa matrix Y_1, Y_2 is a power series in this spurion

$$Y_i = \left[a_i + b_i \hat{Y}\hat{Y}^\dagger + c_i (\hat{Y}\hat{Y}^\dagger)^2 + \cdots \right] \hat{Y} \qquad i = 1, 2$$

For each sector $F = U, D, E$ there are two Yukawa matrices $Y^F_{1,2}$

- Is there a loss of generality when we choose as basic spurion one over the other?
- Can we choose the mass matrices $(\sqrt{2}/v)\, M^F$ to play the role of spurions?

Flavour structure (quark sector)

M_d, M_u, N_d^0, N_u^0

Freedom of choice of WB

Zero textures are WB dependent

Symmetries are only apparent in particular WB

WB transformations do not change the physics

Symmetries have physical implications

Above four matrices encode breaking of flavour symmetry present in gauge sector

large redundancy of parameters

WB invariants are very useful to study flavour

see talk by G.C. Branco

How to recognize a BGL model when written in arbitrary WB

Necessary and sufficent conditions for BGL

$\Delta_1^\dagger \Delta_2 = 0$; $\Delta_1 \Delta_2^\dagger = 0$; $\Gamma_1^\dagger \Delta_2 = 0$; $\Gamma_2^\dagger \Delta_1 = 0$

Higgs mediated FCNC in the down sector

Imply existence of WB where these matrices can be cast in the form given before

The leptonic sector

Required for completeness

- study of experimental implications
- study of stability under RGE

Models with two Higgs doublets with FCNC

- controlled by V_{CKM} in the quark sector
- controlled by V_{PMNS} in the leptonic sector

Case of Dirac neutrinos, straightforward

Same flavour structure
Six different BGL-type models

Scalar Potential

The softly broken Z_2 symmetric 2HDM potential

$$V(\Phi_1,\Phi_2) = m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 - \left(m_{12}^2 \phi_1^\dagger \phi_2 + h.c.\right) + \frac{1}{2}\lambda_1 \left(\phi_1^\dagger \phi_1\right)^2 + \frac{1}{2}\lambda_2 \left(\phi_2^\dagger \phi_2\right)^2$$
$$+ \lambda_3 \left(\phi_1^\dagger \phi_1\right)\left(\phi_2^\dagger \phi_2\right) + \lambda_4 \left(\phi_1^\dagger \phi_2\right)\left(\phi_2^\dagger \phi_1\right) + \frac{1}{2}\left[\lambda_5 \left(\phi_1^\dagger \phi_2\right)^2 + h.c.\right]$$

$$\phi_1 \to \phi_1 \,, \quad \phi_2 \to -\phi_2$$

in our case $\phi_1 \to \phi_1$, $\phi_2 \to e^{iz} \phi_2$, $z \neq 0, \pi$ no λ_5 term

V does not violate CP neither explicitly nor spontaneously

7 free parameters : m_h, m_H, m_A, $m_{H^\pm}$, $v = \sqrt{v_1^2 + v_2^2}$, $\tan\beta$, α (H^0, R)

soft symmetry breaking prevents ungauged accidental continuous symmetry

Analysis of implications, 36 BGL models

$m_h = 126$ GeV

v fixed

no mixing α (good approximation)

results in terms of regions in
$m_{H^\pm}$ versus $\tan\beta$ plane

oblique parameters require (S, T)

$$m_{H^\pm} \sim m_H \sim m_A$$

m_h, m_H, m_A, $m_{H^\pm}$, $v = \sqrt{v_1^2 + v_2^2}$, $\tan\beta$, α
(R) (I)

Analysis of implications, 36 BGL models

Decays mediated by charged currents

i) pure leptonic of type $l_i \to l_j \bar{\nu}_j \nu_i$

FCNC if present always negligible

$m_{H^\pm}$ < LEP bound ~ 80GeV   irrelevant

ii) leptonic decays of pseudoscalar mesons $M \to l\nu$

eg $B^+ \to \tau^+\nu$ $D_s^+ \to \mu^+\nu$ $D_s^+ \to \tau^+\nu$

helicity suppressed in SM ; new physics contributions more relevant in the case of heavy pseudoscalar mesons, dependence $m_M^2/m_{H^\pm}^2$

iii) semileptonic processes of the form $l \to M\nu$ eg $\tau^- \to \pi^-\nu$

iv) semileptonic decays of pseudoscalar mesons $M \to M' l\nu$

eg $B \to D\tau\nu$, $B \to D^*\tau\nu$

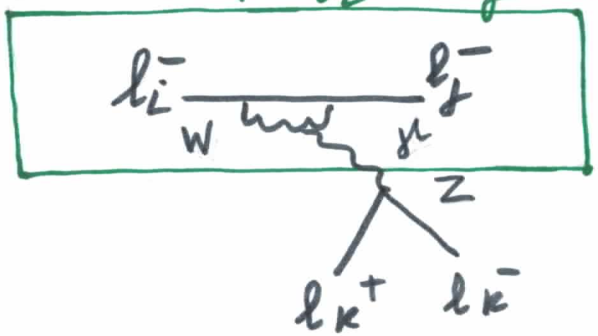
Analysis of implications, 36 BGL models

Flavour changing neutral currents at tree level

i) $l_i^- \to l_j^- l_k^- l_l^+$

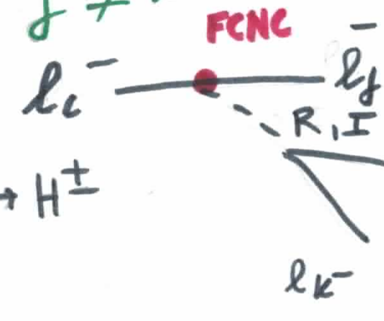
a) $l_i^- \to l_j^- l_k^- l_k^+$ $\quad j = k$ or $j \neq k$

SM



BGL

($\mu \to e\gamma$) $\quad W^\pm \to H^\pm$



tree level diagrams are irrelevant in this case, mass suppression

b) $l_i^- \to l_j^- l_j^- l_k^+$ $\quad k \neq j$

boxes $W^\pm, H^\pm$ SM diagram above with R, I

tree level with two FCNC vertices

very suppressed

ii) decays of pseudoscalar mesons into charged leptons

$B^0 \to \mu^+\mu^-$ $\quad B_s \to \mu^+\mu^-$ $\quad$ SM helicity suppression

iii) neutral meson mixing

SM loop level, BGL models there are tree level cont. which might lead to stringent constraints $\quad j=3$ FCNC up quark, FCNC ν sector

Analysis of implications, 36 BGL models

Loop induced processes

i) radiative leptonic decays of the form $\ell_1 \to \ell_2 \gamma$, $\mu \to e\gamma$

ii) $b \to s\gamma$ no trivial translation, important

iii) $Z \to b\bar{b}$ very powerful constraint

$B \to \tau\nu$, $B \to D\tau\nu$, $B \to D^*\tau\nu$

eliminates the region where are improved

Oblique Parameters and Direct searches

S, U in 2HDM tend to be small corrections

T receives corrections can be sizable

Grimus, Lavoura, Ogreid, Osland (2007)

$m_{H^\pm}$, m_H, m_A not very different

Results in $m_{H^\pm}$, $\tan\beta$ plane

Conclusions

LHC results may bring surprises for the Higgs sector, e.g. discovery of charged Higgs

There are new mechanisms beyond NFC to obtain strong suppression of FCNC as required by experiment

BGL-type models are very interesting candidates for New Physics

Minimal Flavour Violation with Majorana neutrinos

Low energy effective theory and stability

$$\mathcal{L}_{\text{Majorana}} = \frac{1}{2} \nu_L^{0T} C^{-1} m_\nu \nu_L^0 + h.c.$$

generated from effective dimension five operator

$$\mathcal{O} = \sum_{i,j=1}^{2} \sum_{\alpha,\beta=e,\mu,\tau} \sum_{a,b,c,d=1}^{2} \left(L_{L\alpha a}^T \kappa_{\alpha\beta}^{(ij)} C^{-1} L_{L\beta c} \right) \left(\varepsilon^{ab} \phi_{ib} \right) \left(\varepsilon^{cd} \phi_{jd} \right)$$

$$\mathcal{L}_{Y_\ell} = - \overline{L_L^0}\, \Pi_1 \phi_1 \ell_R^0 - \overline{L_L^0}\, \Pi_2 \phi_2 \ell_R^0 + h.c.$$

Π_1, Π_2, κ^{11}, κ^{12}, κ^{21}, κ^{22} $\quad (\kappa^{(ij)})$

$L_{Lj}^0 \rightarrow \exp(i\alpha) L_{Lj}^0$, $\phi_2 \rightarrow \exp(i\alpha)\, \phi_2$

$\alpha = \pi/2$, Z_4 symmetry

Imposing this Z_4 symmetry implies: $(j=3)$

$$k^{(12)} = k^{(21)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$k^{(11)} = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad k^{(22)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$$

$\alpha = \pi/2$ ensures $k^{(22)}_{33} \neq 0$

$$\frac{1}{2} m_\nu = \frac{1}{2} v_1^2 \, k^{(11)} + \frac{1}{2} v_2^2 e^{2i\theta} k^{(22)}$$

$$\Pi_1 = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{bmatrix}, \quad \Pi_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{bmatrix}$$

Higgs FCNC in the charged sector

Stability: $k^{(12)} = k^{(21)} = 0$ $\qquad k^{(11)} P_3^\nu = 0 \qquad k^{(22)} P_3^\nu = k^{(22)}$

$P_3^\nu \, \Pi_1 = 0 \qquad P_3^\nu \, \Pi_2 = \Pi_2$

stable under renormalization

W. Grimus, L. Lavoura 2005

Seesaw framework

$$\mathcal{L}_Y + \text{mass} = -\overline{L}^0_L \,\Pi_1\, \phi_1\, \ell^0_R - \overline{L}^0_L\, \Pi_2\, \phi_2\, \ell^0_R -$$
$$- \overline{L}^0_L\, \Sigma_1\, \tilde{\phi}_1\, \nu^0_R - \overline{L}^0_L\, \Sigma_2\, \tilde{\phi}_2\, \nu^0_R +$$
$$+ \frac{1}{2}\, \nu_R^{0\,T}\, C^{-1}\, M_R\, \nu^0_R + h.c.$$

$$m_\ell = \frac{1}{\sqrt{2}}\left(v_1 \Pi_1 + v_2 e^{i\theta}\Pi_2\right), \quad m_D = \frac{1}{\sqrt{2}}\left(v_1 \Sigma_1 + v_2 e^{-i\theta}\Sigma_2\right)$$

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}}\, W^+_\mu\, \overline{\ell}^0_L\, \gamma^\mu\, \nu^0_L + h.c$$

$$\mathcal{L}_Y(\text{neutral, lepton}) = -\overline{\ell}^0_L \frac{1}{v}\left[m_\ell H^0 + N^0_\ell R + i N^0_\ell I\right]\ell^0_R -$$
$$- \overline{\nu}^0_L \frac{1}{v}\left[m_D H^0 + N^0_\nu R + i N^0_\nu I\right]\nu^0_R + h.c.$$

$$N^0_\ell = \frac{v_2}{\sqrt{2}}\Pi_1 - \frac{v_1}{\sqrt{2}} e^{i\theta}\Pi_2$$
$$N^0_\nu = \frac{v_2}{\sqrt{2}}\Sigma_1 - \frac{v_1}{\sqrt{2}} e^{-i\theta}\Sigma_2$$

$$\mathcal{L}_{mass} = -\bar{\ell}_L^0 m_\ell \ell_R^0 + \frac{1}{2}\left(\nu_L^{0T}, (\nu_R^0)^{cT}\right) C^{-1} \mathcal{M}^* \begin{pmatrix} \nu_L^0 \\ (\nu_R^0)^c \end{pmatrix} + hc$$

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \qquad (\psi_L)^c = C\gamma_0^T (\psi_L)^*$$

BGL type example , Z_4 symmetry

$L_{L3}^0 \to \exp(i\alpha) L_{L3}^0$, $\nu_{R3}^0 \to \exp(i2\alpha)\nu_{R3}^0$, $\phi_2 \to \exp(i\alpha)\phi_2$

$\alpha = \frac{\pi}{2}$

$$\Pi_1 \begin{pmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{pmatrix}, \quad \Pi_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{pmatrix}$$

$$\Sigma_1 = \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Sigma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{pmatrix}, \quad M_R = \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{pmatrix}$$

New feature m_{ν_i} from $m_{eff} = -m_D \frac{1}{M_R} m_D^T$ $\quad M_{33} \neq 0$

Three light neutrinos ν_i, plus heavy neutrinos N_j

light-light, light-heavy, heavy-heavy couplings

H^0, R, I couplings

$U^\dagger m_{eff} U^* = d$, $m_D \frac{1}{D} m_D^T = -U d U^T$ (WB M_D diag)

$m_D = i U \sqrt{d}\, O^c \sqrt{D}$ Casas and Ibarra, 2001

$$(N_\ell)_{ij} = \frac{v_2}{v_1}(D_\ell)_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right)(U_\nu^\dagger)_{i3}(U_\nu)_{3j}(D_\ell)_{jj}$$

light-light neutral couplings : diag, d

light-heavy neutral couplings : sensitive to O^c, d, D

heavy-heavy neutral couplings: diag, sensitive to O^c, d, D

H^+ couplings

$$\frac{\sqrt{2}\, H^+}{v}\left(\bar{\nu}_L^0 N_\ell^0 \ell_R - \bar{\nu}_R^0 N_\nu^{0\dagger} \ell_L^0\right) + h.c.$$