

F flavour in the Higgs ERA

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Some of the Open Questions in Flavour Physics.

- Why is there "Flavour"?
i.e. What is the Origin of Family replication
- How to understand the observed spectrum
of Fermion Masses and Mixing?
Why is Leptonic Mixing Large, in
contrast to Small Quark Mixing?
- Why is there Flavour Alignment in
the Quark Sector?

The Question of Alignment

Within the SM, consider a set of Yukawa couplings γ_u, γ_d leading to hierarchical quark masses and small mixing. By small mixing we mean that there is a Weak Basis (WB) where :

$$M_u = \text{diag.} (m_u, m_c, m_t); \quad M_d \approx \text{diagonal}$$

The absolute ordering of quark mass eigenstates has no physical meaning. But the relative ordering, does have physical meaning. Since in the SM, y_u, y_d are entirely independent it is as likely that, in the basis where $M_u = \text{diag.}(m_u, m_c, m_t)$, one has $M_d \approx \text{diag}(m_d, m_s, m_b)$ corresponding to Alignment as to have $M_d \approx (m_b, m_d, m_s)$ implying Misalignment. Obviously, even assuming "small mixing" the probability of Alignment is only $1/6$.

CP Violation is closely related to the Question of Flavour

An open question :

- Is CP broken explicitly through complex Yukawa couplings like in the SM
 - or (T.D.Lee)
- Spontaneously : Lagrangian is invariant under CP but the vacuum violates CP?

At present, there is solid experimental evidence for a complex V_{CKM} .

Does this imply complex Yukawa couplings?

NO!!

There are realistic models of spontaneous CP violation where the vacuum phase generates a complex V_{CKM} .

Can one have geometrical CP violation?

For definiteness, let us consider an extension of the SM where n $SU(2) \times U(1)$ scalar doublets are introduced. In order to include the possible existence of "family" symmetries of the Lagrangian under which the scalar doublets transform non-trivially, one has to consider the most general CP transformation which leaves invariant the kinetic energy terms of the scalar doublets :

$$CP \phi_i (CP)^+ = \sum_{j=1}^n U_{ij} \phi_j^*$$

Let us assume that the vacuum is CP invariant, meaning that:

$$CP|0\rangle = |0\rangle$$

One can then derive, the following relation:

$$\sum_{j=1}^n U_{ij} \langle 0 | \Phi_j | 0 \rangle^* = \langle 0 | \phi_i | 0 \rangle$$

If the vacuum is such that none of the symmetries allowed by the Lagrangian satisfy the above equation, then this means that the vacuum is not CP invariant and we say that CP is spontaneously broken.

There are various examples of multi-Higgs models with "family" symmetries, where in a certain region of parameter space, the minimum of the scalar potential corresponds to fixed values of the vacuum phases, which do not depend on the specific values of the parameters of the potential.

$$Z_2 \text{ symmetry} \rightarrow \langle \phi_1^0 \rangle = v; \langle \phi_2^0 \rangle = v e^{i\pi/2}$$

$$S_3 \text{ symmetry} \rightarrow \langle \phi_1^0 \rangle = v; \langle \phi_2^0 \rangle = v e^{i2\pi/3}; \langle \phi_3^0 \rangle = v e^{i4\pi/3}$$

It has been shown by G.B, Girard, J.M., Grimus, W
that , contrary to naïve expectation
all these vacua are CP conserving .

However, an example was found, based
on the group $\Delta(27)$ leading to genuine
CP and T geometrical valuation.

Recently, interesting other examples have
been found :

I. M. Varzilas, D. E. Costa, P. Lester Phys.Lett (2012)

I. P. Ivanov, L.avoura, Eur. Phys. (2013)

Realistic model ?

8 120

Generation of the Baryon Asymmetry 149

of the Universe (BAU)

The ingredients to dynamically generate **BAU** from an initial state with zero B. A. , were formulated by Sakharov(1967)

- (i) Baryon number violation
- (ii) C and CP Violation
- (iii) Departure from thermal equilibrium

All these ingredients exist in the SM,
but it has been established that in
the SM, one cannot generate the
observed BAU:

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.20 \pm .15) \times 10^{-10}$$

n_B , $n_{\bar{B}}$, n_γ number densities of baryons,
anti baryons and photons at present time.

Reasons why the SM cannot generate sufficient BAU:

(i) CP violation in the SM is too small

$$\frac{\text{tr} [H_u, H_d]^3}{T_{ew}^{12}} \approx 10^{-20}$$

(ii) Successful baryogenesis requires a strongly first order phase transition which would require a light Higgs mass

$$m_H \leq 70 \text{ GeV}$$

The various manifestations of CP violation :

- CP violation in the quark sector
- CP violation in the lepton sector
- CP violation needed to generate BAU

A Common Origin?

- What about the Strong CP problem?

The need to suppress Flavour-Changing - neutral - currents (FCNC) led to the following two Dogmas :

- No Z -mediated FCNC at tree level
- No Scalar-mediated FCNC at tree level
 - { Glashow, Weinberg
Paschos

Can one violate these two dogmas in reasonable extensions of the SM? Yes!

"Reasonable" means that FCNC should be naturally suppressed, without fine-tuning.

- In the gauge sector, the Dogma can be violated through the introduction of a $Q = 1/3$ and/or $Q = 2/3$ vector-like quark.

Naturally small violations of 3×3 unitarity of V^{CKM}

Z -mediated, Naturally suppressed FCNC at tree level

Yukawa interactions in the General two Higgs doublet Model

$$-\mathcal{L}_Y = \bar{Q}_L^0 \Gamma_1 \phi_1 d_R^0 + \bar{Q}_L^0 \Gamma_2 \phi_2 d_R^0 + \bar{Q}^0 \Delta_1 \tilde{\phi}_1 u_R^0 + \bar{Q}_L^0 \Delta_2 \tilde{\phi}_2 u_R^0 + h.c.$$

Quark mass matrices :

$$M_d = \frac{1}{\sqrt{2}} (v_1 \Gamma_1 + v_2 e^{i\alpha} \Gamma_2), \quad M_u = \frac{1}{\sqrt{2}} (v_1 \Delta_1 + v_2 e^{-i\alpha} \Delta_2)$$

Diagonalized by :

$$(U_d)_L^\dagger M_d (U_d)_R = D_d = \text{diag} (m_d, m_s, m_b)$$

$$(U_u)_L^\dagger M_u (U_u)_R = D_u = \text{diag} (m_u, m_c, m_t)$$

Expanding around the vacuum:

$$\langle \phi_j^0 \rangle = e^{i\alpha_j} \frac{1}{\sqrt{2}} (v_j + \rho_j + i\eta_j) \quad j = 1, 2$$

It is useful to introduce new fields:

$$\begin{bmatrix} H^0 \\ R \end{bmatrix} = O \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix}; \begin{bmatrix} G^0 \\ I \end{bmatrix} = O \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}; \begin{bmatrix} G^+ \\ H^+ \end{bmatrix} = O \begin{bmatrix} \phi_1^+ \\ \phi_2^+ \end{bmatrix}$$

$$O = \frac{1}{v} \begin{bmatrix} v_1 & v_2 \\ v_2 & -v_1 \end{bmatrix}; \quad v = \sqrt{v_1^2 + v_2^2} \approx 246 \text{ GeV}$$

$H^0 \rightarrow$ has couplings to quarks proportional to

$G^0 \rightarrow$ neutral pseudo Goldstone boson mass matrices

$H^\pm \rightarrow$ charged pseudo-Goldstone bosons

Neutral and charged Higgs interactions in the quark sector

$$\begin{aligned}
 -\mathcal{L}_Y = & \bar{d}_L^\circ \frac{1}{v} (M_d H^\circ + N_d^\circ R + i N_d^\circ I) d_R^\circ + \\
 & + \bar{u}_L^\circ \frac{1}{v} (M_u H^\circ + N_u^\circ R + i N_u^\circ I) u_R^\circ + \\
 & + \frac{\sqrt{2}}{v} H^+ (\bar{u}_L^\circ N_d^\circ d_R^\circ - \bar{u}_R^\circ N_u^\circ d_L^\circ) + h.c.
 \end{aligned}$$

$$N_d^\circ = \frac{1}{\sqrt{2}} (v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2); \quad N_u^\circ = \frac{1}{\sqrt{2}} (v_2^{A_1} v_1 \bar{e}^{-i\alpha} \Gamma_2).$$

In the quark mass eigenstate basis N_d, N_u
are not flavour diagonal.

Physical neutral Higgs are combinations of H°, R, I

Yukawa Couplings in terms of quark mass eigenstates :

$$\begin{aligned}
 \mathcal{L}_Y = & \frac{\sqrt{2} H^+}{v} \bar{u} \left[-V N_d \gamma_R + N_u^+ V \gamma_L \right] d + h.c. - \\
 & - \frac{H^0}{v} \left[\bar{u} D_u u + \bar{d} D_d d \right] - \\
 & - \frac{R}{v} \left[\bar{u} (N_u \gamma_R + N_u^+ \gamma_L) u + \bar{d} (N_d \gamma_R + N_d^+ \gamma_L) d \right] \\
 & + \frac{I}{v} \left[\bar{u} (N_u \gamma_R - N_u^+ \gamma_L) u - \bar{d} (N_d \gamma_R - N_d^+ \gamma_L) d \right]
 \end{aligned}$$

$$\gamma_L = \frac{1}{2}(1 - \gamma_5) \quad ; \quad \gamma_R = \frac{1}{2}(1 + \gamma_5) ; \quad V \text{ is the}$$

CKM matrix

Flavour changing neutral currents are controlled by :

$$N_d = \frac{1}{\sqrt{2}} U_{dL}^+ (v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2) U_{dR}$$

$$N_u = \frac{1}{\sqrt{2}} U_{uL}^+ (v_2 \Delta_1 - v_1 e^{-i\alpha} \Delta_2) U_{uR}$$

For generic two Higgs doublet models

N_u, N_d are non-diagonal, arbitrary matrices.



too large Higgs mediated Flavour changing neutral currents, unless a suppression mechanism is introduced

G.C.B, Grimus, W and Lavoura, L (BGL) have shown that it is possible to find a symmetry which when imposed on a 2-Higgs doublet extension of the SM, leads to a structure of the Yukawa couplings such that there are FCNC at tree level, with strength completely controlled by V_{CKM} , i.e., N_d , N_u only depend on V_{CKM} and on $\frac{v_1}{v_2}$.

Possible choice of the symmetry S :

$$Q_L^{\circ} \rightarrow \exp(i\alpha) Q_L^{\circ} \quad ; \quad U_R^{\circ} \rightarrow \exp(i2\alpha) U_R^{\circ}$$

$$\phi_2 \rightarrow \exp(i\alpha) \phi_2, \text{ where } \alpha \neq 0, \pi$$

all other fields transform trivially under S .
 This leads to the Yukawa couplings:

$$\Gamma_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}$$

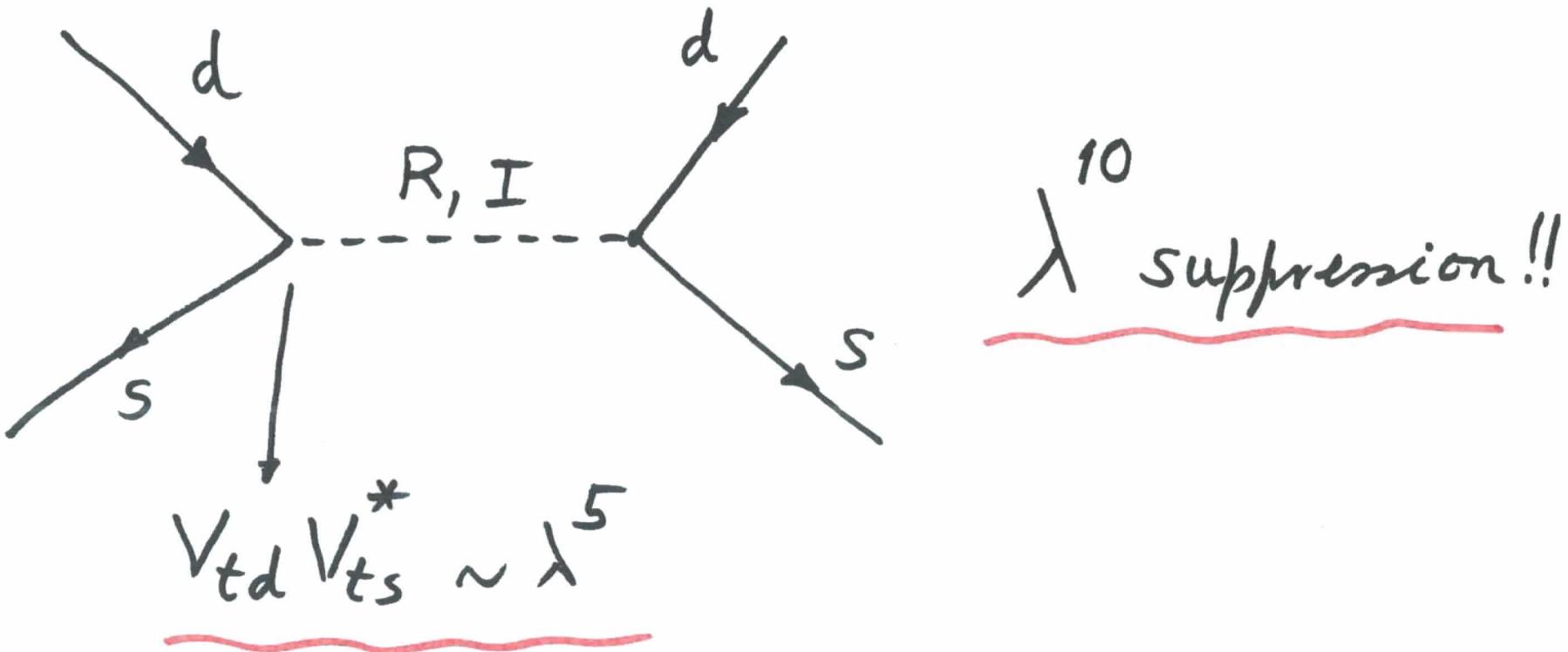
These Yukawa structures lead to :

$$(N_d)_{ij} = \frac{v_2}{v_1} (D_d)_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) [V_{CKM}]_{3j} (D_d)_{jj}$$

$$(N_u) = -\frac{v_1}{v_2} \text{diag.}(0, 0, m_t) + \frac{v_2}{v_1} \text{diag.}(m_u, m_c, 0)$$

In this particular **BGL** model there are **FCNC** only in the down sector, but there is a strong, natural suppression of the most "dangerous" processes

$K^0 - \bar{K}^0$ mixing



There are 6 different BGL models
in the quark sector

An important Question :

Are there any other models, based on abelian symmetries, which lead to FCNC at tree level, completely controlled by V_{CKM} , without further parameters?

Intuitive Answer : Yes

Correct Answer : No!

P. Ferreira and J. Silva (Phys. Rev. D83(2011))
have classified all possible implementations
of an **Abelian symmetry** in 2-Higgs doublet
models, imposing the request of having
non-vanishing quark masses and not block
diagonal $VCKM$.

Their conclusion : **BGL** models are unique

Open question : Non-abelian symmetries?

Other Approaches to Multi-Higgs Models

• Minimal Flavour Violation

G. D'Ambrosio, G.F. Giudici, G. Isidori, A. Strumia

A. Buras, P. Gambino, M. Gorbahn, S. Jagner, (2002)

L. Silvestrini (2001)

A. Dery, A. Efaati, G. Hiller, Y. Hochberg, Y. Nir

(2013)

• Alignment

A. Pich, P. Tuzon (2009)

In the quark mass eigenstate basis flavour mixing can be parametrized by various unitary matrices which arise from the misalignment in flavour space between pairs of various Hermitian flavour matrices.

This is entirely analogous to what one encounters in the SM

$V_{CKM} \rightarrow$ arises from the misalignment between $H_u \equiv M_u M_u^\dagger$ and $H_d \equiv M_d M_d^\dagger$

Weak-basis invariants are a useful to study Flavour.

Example :

$$\text{tr} [H_u, H_d]^3 \xrightarrow{\text{J. Bernabeu, G.C.B., M. Gronau}} \\ \times (m_c^2 - m_u^2) (m_t^2 - m_c^2) (m_t^2 - m_u^2) \times \\ \times (m_s^2 - m_d^2) (m_b^2 - m_s^2) (m_b^2 - m_d^2) \times$$

$\propto \det [H_u, H_d]$

for 3 generations

(C. Jarlskog)

Im Q

Q \rightarrow invariant quartet of
VCKM

One may also study invariants under Higg basis transformations

G.C.B., M.N. Rebelo, Silva-Marcos ;

H. Haber, F. Gunion
S. Davidson, H. Haber

We have seen that under a WB transformation :

$$d_L^o = W_L d_L^{o'} ; \quad d_R^o = W_R^d d_R^{o'}$$

$$u_L^o = W_L u_L^{o'} ; \quad u_R^o = W_R^u u_R^{o'}$$

Under a WB transformation the flavour matrices transform as :

$$M_d \rightarrow M'_d = W_L^+ M_d W_R^d ; \quad M_u \rightarrow M'_u = W_L^+ M_u W_R^u$$

$$N_d^o \rightarrow N_d^{o'} = W_L^+ N_d^o W_R^d ; \quad N_u^o \rightarrow N_u^{o'} = W_L^+ N_u^o W_R^u$$

In view of the freedom of choice of WB, it is useful to express the physical content of M_d, M_u, N_d^o, N_u^o in terms of WB invariants.

Examples of Invariants :

$$I_1 \equiv \text{tr} (M_d N_d^{\circ +}) = m_d (N_d^*)_{11} + m_s (N_d^*)_{22} + m_b (N_d^*)_{33}$$

where N_d is N_d° in the basis where the down quark mass matrix is diagonal. This invariant is not sensitive to Higgs mediated FCNC but $\text{Im } I_1$ is specially important, since it probes the phases of $(N_d)_{jj}$ which contribute to the electric dipole moment of down quarks.

Analogous comments apply to $\text{tr} (M_u N_u^{\circ +})$

3L

Consider now a WB invariant which is sensitive
to the off-diagonal elements of N_d :

27

$$I_2 \equiv \text{tr} \left[M_d N_d^{\circ}{}^+, M_d M_d^+ \right]^2 = -2 m_d m_s (m_s^2 - m_d^2) (N_d^*)_{12} (N_d^*)_{21} \\ - 2 m_d m_b (m_b^2 - m_d^2) (N_d^*)_{13} (N_d^*)_{31} - 2 m_s m_b (m_b^2 - m_s^2) (N_d^*)_{23} (N_d^*)_{32}$$

CP-odd WB invariant :

$$I_2^{CP} \equiv \text{tr} \left[H_u, H_{N_d^{\circ}} \right]^3 = 6 i \Delta_{u_L} \Delta_{N_d} \xrightarrow{(m_u^2 - m_c^2)} \dots \xrightarrow{\text{analogous}} \text{Im } Q_2$$

$Q_2 \rightarrow$ is a rephasing invariant quartet of V_L

$$V_L \equiv U_u^+ (U_{N_d^{\circ}})_L$$

³³ These invariants can be very useful to study the CP properties (as well as FCNC) in specific models. For example in BGL models :

$$\text{Im} \operatorname{tr}(M_d N_d^{\circ+}) = 0$$

In BGL models the lowest order invariant sensitive to CP violation is :

$$I_g^{\text{CP}} = \operatorname{tr} [M_d N_d^{\circ+} M_d M_d^\dagger M_u M_u^\dagger M_d M_d^\dagger]$$

This may be relevant for Baryogenesis.

See M. N. Rebelo talk

Can one have a model where
gauge mediated FCNC exist at
tree level, but are naturally suppressed?

Answer : Yes !!

A Minimal Model

Consider an extension of the SM,
where the following new fields are introduced:

- A vectorial quark D° , with both D_L° and D_R° are $SU(2)_L$ singlets with charge $Q = -1/3$ (or $Q = 2/3$)
- 3 right-handed neutrinos ν_{Rj}°
- A neutral complex singlet S

- Since we want to have Spontaneous CP violation, we impose CP invariance at the Lagrangian level: All couplings real.
- Introduce a Z_4 symmetry on the Lagrangian, under which :

$$\gamma_e^\circ \rightarrow i \gamma_\ell^\circ ; e_{Rj}^\circ \rightarrow i e_{Rj}^\circ ; \nu_{Rj}^\circ \rightarrow i \nu_{Rj}^\circ$$

$$D^\circ \rightarrow - D^\circ ; S \rightarrow - S$$

The Z_4 symmetry is crucial to obtain a solution of the Strong CP problem and Leptogenesis

Scalar Potential

The Scalar potential contains various terms which do not have phase dependence, but there are terms with phase dependence.

$$V_{\text{phase}} = [\mu^2 + \lambda_1 S^* S + \lambda_2 \phi^+ \phi] (S^2 + S^{*2}) + \\ + \lambda_3 (S^4 + S^{*4})$$

There is a range of the parameters of the Higgs potential, where the minimum is at:

$$\langle \phi \rangle = \frac{v}{\sqrt{2}} \quad ; \quad \langle S \rangle = \frac{V}{\sqrt{2}} e^{i\theta}$$

Most general $SU(2)_L \times U(1) \times SU(3)_C^{\text{c}} \times \mathbb{Z}_4$
40
 invariant Yukawa couplings in the quark
 sector :

$$\mathcal{L}_Y = -(\bar{u}^\circ \bar{d}^\circ)_{L_i} [g_{ij} d_R^o + h_{ij} \tilde{\phi} u_R^o] - \bar{M} (\bar{D}_L^o D_R^o) \\ - (f_i S + f' S^*) \bar{D}^o d_R^o + \text{h.c.}$$

Quark mass matrix for down-type quarks:

$$\begin{pmatrix} \bar{d}_{1L} & \bar{d}_{2L} & \bar{d}_{3L} & \bar{D}_L^o \end{pmatrix} \begin{pmatrix} \overset{3 \times 3, \text{ real}}{\begin{matrix} \rho \\ m_d \\ \vdots \end{matrix}} & 0 \\ \vdots & \ddots \\ M_1 & M_2 & M_3 & \bar{M} \end{pmatrix} \begin{pmatrix} d_{1R}^o \\ d_{2R}^o \\ d_{3R}^o \\ D_R^o \end{pmatrix}$$

"zero"
 due to
 \mathbb{Z}_4
 symmetry

$$M_j = f_j V e^{i\theta} + f'_j V e^{-i\theta}$$

$$\mathcal{M} = \begin{bmatrix} m_d & \vdots & 0 \\ \vdots & \ddots & \vdots \\ \bar{M}, \bar{M}_2, \bar{M}_3, & \ddots & \bar{M} \end{bmatrix};$$

$$U_L^+ (\mathcal{M} \mathcal{M}^+) U_L = \text{diag.}(d^2, D^2)$$

$$U = \begin{bmatrix} K & R \\ S & T \end{bmatrix}; \text{ One can easily derive: } \xrightarrow{\text{Complex}}$$

$$K^{-1} \left[m_d m_d^T - \frac{m_d M^T M m_d^T}{(M M^T + \bar{M}^2)} \right] K = d^2$$

A remarkable feature of the Model :

The phase Θ arising from $\langle S \rangle$, generates a non-trivial CKM phase, provided $|M_j|$ and \bar{M} are of the same order of magnitude. (This is "natural")

$$K^{-1} m_{\text{eff}}^+ m_{\text{eff}}^- K = \text{diag. } (m_d^2, m_s^2, m_b^2)$$

$$m_{\text{eff}}^+ m_{\text{eff}}^- = m_d m_d^\dagger - \frac{m_d M^\dagger M m_d^\dagger}{M M^\dagger + \bar{M}^2}$$

$$M_j = (f_j V e^{i\Theta} + f_j' V e^{-i\Theta})$$

41
 Naturally small deviations of 3×3 unitarity

143
 Naturally small Flavour-Changing Neutral Currents

For definiteness, consider the case of one isosinglet $Q = -\frac{1}{3}$
 3×3 CKM quark

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} (\bar{u} \bar{c} \bar{t})_L \gamma^\mu \begin{bmatrix} \overset{\uparrow}{K} R \\ K^+ R^+ \end{bmatrix} \begin{bmatrix} d \\ s \\ b \\ D \end{bmatrix} W_\mu^+$$

$$\begin{aligned} \mathcal{L}_Z = & -\frac{g}{2 \cos \theta_W} \left\{ (\bar{u} \bar{c} \bar{t})_L \gamma^\mu \begin{bmatrix} u \\ c \\ t \end{bmatrix}_L - [\bar{d} \bar{s} \bar{b} \bar{D}] \begin{bmatrix} K^+ k & K^+ R \\ R^+ k & R^+ R \end{bmatrix} \gamma^\mu \begin{bmatrix} d \\ s \\ b \\ D \end{bmatrix} \right. \\ & \left. - \sin^2 \theta_W J_\mu^\mu \right\} Z_\mu \end{aligned}$$

Why deviations of 3×3 unitarity are naturally small :

$$U_L^\dagger M M^+ U_L = \text{diag.}(m_d^2, m_s^2, m_b^2, M_D^2)$$

$$U_L = \begin{bmatrix} K & R \\ S & T \end{bmatrix} ; \quad K^+ K + S^+ S = 1$$

$$\text{but } S \approx -\frac{M m_d^2 K}{M^2} \rightarrow O(m/M) ;$$

$K^+ K = 1 - O(m^2/M^2)$. Note that there is nothing strange about violations of 3×3 unitarity.

The PMNS matrix is not unitary in the framework of seesaw mechanism, type I.

Some comments :

- Nothing "strange" in having deviations of 3×3 unitarity of V_{CKM} . The PMNS matrix in the leptonic sector, in the context of type-one seesaw, is not 3×3 unitary.
- Vector-like quarks provide the simplest model with Spontaneous CP violation, with a complex V_{CKM} , in agreement with experiment
- Provide a framework to have a Common Origin of all CP violations.
- Potential Solution of Strong CP problem without Axions

Conclusions

- We are entering the Higgs Era in Particle Physics.
- A crucial question is : Will Higgs particle(s) play an important rôle in Flavour?
- My guess : Yes, it will not be just Vanilla Higgs.
- It may take some time until Higgs Flavour be tested experimentally. We have to be patient and remember Neutrinos
- We can safely violate the two Flavour Dogmas.