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# Quantum quenches, Entanglement and the transverse field Ising chain from excited states

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mainly based on

LB, Kormos, Calabrese 1401.7250; Kormos, LB, Calabrese, 1406.5070

# Outline of the talk

I. Introduction to non-equilibrium many-body quantum physics

II. State of the art: Relaxation, Light-Cone effect, Entanglement entropy

III. Extension to a quench in the transverse field Ising chain with a new ingredient: initial excited states

IV. Numerical & exact results for the long-time dynamics

V. Conclusions & Outlooks

# I. Introduction

Crucial to non-equilibrium physics is the concept of thermalization

$$\langle \hat{A}(t \to \infty) \rangle = \mathrm{Tr}(\hat{A} \rho_{\mu-\mathrm{can}})$$

Nowadays, we have many numerical and experimental evidences supporting thermalization in some quantum systems...

**[Rigol et al '09, Cassidy et al '11, Eisert et al '11, Trotzky et al. '12, Pozsgay '14... ]**

...but why should a quantum system thermalize?

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## Long-Standing Questions

[Von Neumann '29; Birkhoff '30]

- Does an isolated quantum system reach a stationary state starting from an arbitrary initial state?
- If so, is there a way to economically describe the stationary state?
- Can we describe it according to a statistical ensemble, i.e. by maximising an entropy functional under some constraints?
- How do correlation functions and observables depend on time?

# Out of equilibrium quantum physics: timely subject...

Long-standing
Theoretical questions

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Experimentally highly
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Cold atoms, optical lattices,
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1D models:
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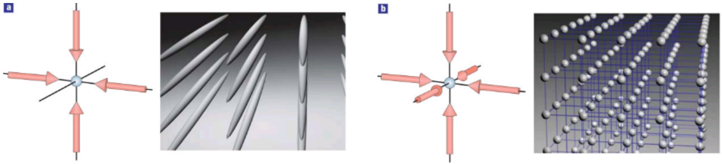
Fruitful comparison between non equilibrium theories based on simple models and carefully designed experiment with tunable parameters

# The benefit of ultracold atoms

Optical lattices allow to tune dimensionality, interactions and also statistics.




Observation of quantum coherent dynamics:

Ultracold atoms allow to avoid dissipation and decoherence on time scales of order milliseconds to seconds (ps for "usual" systems!), long enough to study collective behaviour and (thermal) equilibrium.

# How to drive a system out-of-equilibrium?

- Pumping energy or particles into the system (open systems)
- Acting with a driving field
- Using time-dependent Hamiltonians

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It consists in changing suddenly a parameter of the system and then let it evolve unitarily.

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study of non equilibrium dynamics of closed and isolated interacting quantum systems

# The Quench paradigm

- prepare a many-body quantum system in an eigenstate $|\psi_0\rangle$ of a pre-quenched hamiltonian $H$
- from $t = 0$ let it evolve unitarily with a different post-quenched time-independent hamiltonian $H'$

$$|\psi(t)\rangle = e^{-iH't}|\psi_0\rangle, \qquad [H, H'] \neq 0$$

Initial state is NOT an eigenstate
nor a finite superposition of eigenstates of H'

Evolution from an out-of-equilibrium state $|\psi_0\rangle$

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Example: Interaction Quench

$$H(\lambda) \xrightarrow[\text{Quench}]{t=0} H(\lambda')$$

$t > 0$, evolution under Quantum Mechanics from a pure state of an isolated system

# II. State-of-the-art:

# 1. Relaxation

Can the whole system attain stationary behaviour?

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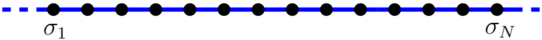
Can the whole system attain stationary behaviour?

Initial pure state + unitary evolution → it will be in a pure state $\forall t$

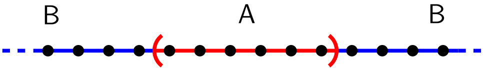
Global observables (i.e. the whole system) can never relax

As an example, a spin-chain




$\langle\psi(t)|\sigma_1 \cdots \sigma_N|\psi(t)\rangle$: persistent oscillations, quantum recurrence

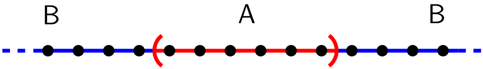
What about local observables?




First taking B infinite, then $t \to \infty$ a finite subsystem A can relax!

## Only local observables relax!

Physical picture: B acts like a "thermal" bath on A
No time averaging involved!

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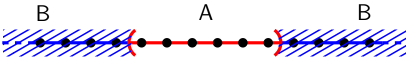
Density matrix:

$\rho_{\mathsf{A}\cup\mathsf{B}}(t) = e^{-iH't}|\psi_0\rangle\langle\psi_0|e^{iH't}$

Reduced Density Matrix of A:

$\rho_{\mathsf{A}}(t) \equiv \mathrm{Tr}_{\mathsf{B}}\left[\rho_{\mathsf{A}\cup\mathsf{B}}(t)\right]$




$$\lim_{t\to\infty}\lim_{N\to\infty}\rho_{\mathsf{A}}(t) = \lim_{N\to\infty}\mathrm{Tr}_{\mathsf{B}}\left[\rho^{\mathrm{mixed}}_{\mathsf{A}\cup\mathsf{B}}\right]$$

- $\rho_{\mathsf{A}}$ stationary and allows for an ensemble description (mixed state)
- determines all local correlation functions

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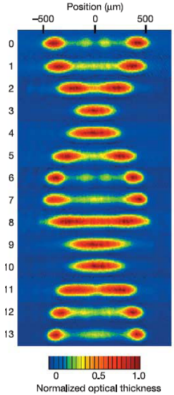
## "Quantum Newton Cradle" [Kinoshita et al '06]

40-250 Rb atoms (hard core bosons) in 1D optical trap, prepared at $t = 0$ in a superposition of states with opposite momentum




- in $D = 2, 3$, after few collisions, the system thermalizes
- in $D = 1$, after thousands of collisions, it does not thermalize

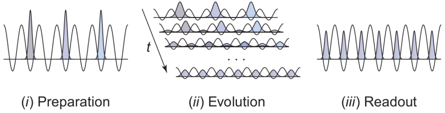
The experiment shows that dimensionality and integrability play a central role in non equilibrium physics.

| Integrable 1D systems do not seem to thermalize |
| --- |

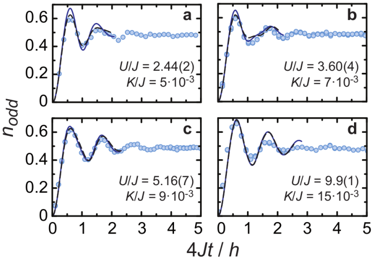
# Probing relaxation in Bose-Hubbard system [Trotsky et al '12]

$$H = \sum_j \left[ -J(a_j^\dagger a_{j+1} + h.c.) + \frac{U}{2} n_j(n_j - 1) + K n_j j^2 \right]$$

Preparation




Results



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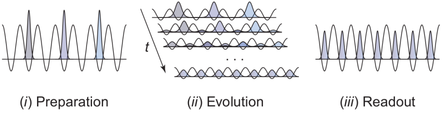


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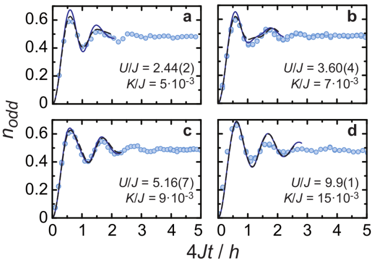


Thermalization: relaxation to a universal value independently of $J$, $U$, $K$

- Initial details of the non integrable system do not matter for the asymptotic behaviour
- Values compatible with the system globally being in a maximum entropy state (with constrained energy and # of particles)

Non Integrable systems thermalize!

$$\lim_{t\to\infty}\lim_{N\to\infty}\rho_{\mathsf{A}}(t)=\mathrm{Tr}_{\mathsf{B}}\left[\rho^{\mathrm{mixed}}_{\mathsf{A}\cup\mathsf{B}}\right]$$

Which is the statistical ensemble for $\rho^{\mathrm{mixed}}_{\mathsf{A}\cup\mathsf{B}}$?

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Non Integrable Systems

$$\rho^{\mathrm{Gibbs}}_{\mathsf{A}\cup\mathsf{B}} = \frac{e^{-H/T_{\mathrm{eff}}}}{Z_{\mathrm{Gibbs}}}$$

Thermal ensemble
only one integral of motion $E$
few info on the whole Initial state

**[Deutsch '91; Srednicki '95]**

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**Integrable Systems**

$$\rho_{\mathsf{A}\cup\mathsf{B}}^{\mathrm{GGE}} = \frac{e^{-\sum_m \beta_m I_m}}{Z_{\mathrm{GGE}}}$$

Non thermal ensemble
complete set of local commuting integrals of motions $I_m$
$I_m = \sum_{j=1}^{N} O_{j,j+1,\cdots,j+m}$,
$\mathcal{O}(m)$-support, m finite
full info on the whole Initial state

[Rigol et al '07; Eisert; Cramer...]

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- based on many theoretical, experimental and numerical outcomes
  [Rigol, Muramatsu, Olshanii; Cazalilla; Calabrese, Cardy; Fioretto, Mussardo; Caux, Mossel...]
- not quite the end of the story [De Nardis et al '14, Kormos et al '14, Andrei et al '14]

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Main test: exact solution of the full dynamics (free theories, TFIC, XY...)

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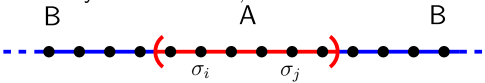


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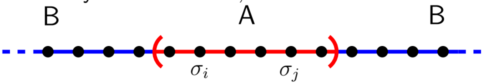


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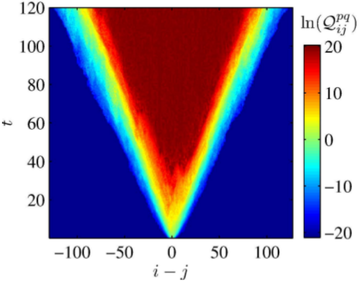
In non relativistic quantum systems with finite-range interactions and a finite local Hilbert space: $\exists$ finite group velocity $v_{max}$, with exponentially small effects outside an effective light cone




**[Cheneau et al '12]**

Is this a general feature? YES $\to$ Lieb-Robinson Bound!

## Causality at equilibrium:

Lieb-Robinson bound **[Lieb, Robinson '72]**

*In local Hamiltonians with a finite local Hilbert space:*

$$||[\mathcal{O}_A(0,x_1),\mathcal{O}_B(t,x_2)]|| \leq \mathcal{C}_{AB}\ e^{(v_{max}|t|-|x_1-x_2|)/\xi}$$

- $\xi$ is connected to the finite range of interactions
- $v_{max}$: maximal group velocity
- independent of the state of the system

Correlations are exponentially suppressed outside the light cone.

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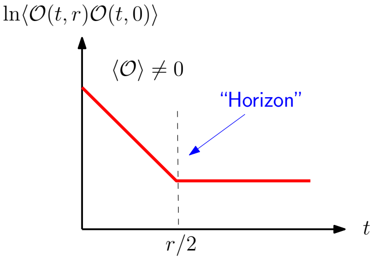
## Causality out of equilibrium:

In a global quench, at equal time, it implies that

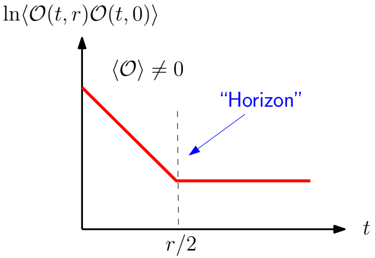
$$\langle\psi(t)|\mathcal{O}_A(x_1)\mathcal{O}_B(x_2)|\psi(t)\rangle \le \mathcal{C}_{AB}e^{(\mathbf{2}v_{max}|t|-|x_1-x_2|)/\xi}$$

**[Hastings et al '06]**




Equal time two point function for fixed separation r

- exponential decay in time for $t \lesssim r/2$
- saturation to t-independent values for $t \gtrsim r/2$

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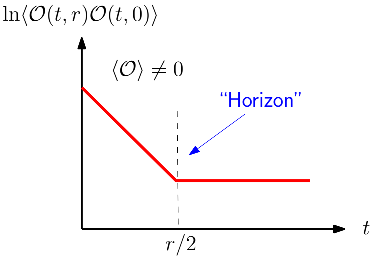
Physical Interpretation

[Calabrese, Cardy '07]

$E_{\psi_0} \gg E_{\mathsf{GS}}$, $|\psi_0\rangle$ acts as a source of excitations

- quasi-particle emitted on scales $E_{\psi_0}^{-1}$ are entangled
- they move classically with light-cone trajectories and spread
- for $t \lesssim r/2$ causally disconnected regions
- after a transient $t \gtrsim r/2$ observables freeze-out

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Horizon effect predicts freeze-out of n ($>$2)-point functions

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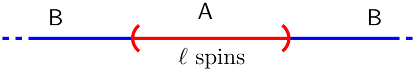
$$S_{\mathsf{A}} = -\mathrm{Tr}[\rho_{\mathsf{A}} \ln \rho_{\mathsf{A}}]$$




Entanglement entropy is a measure of how much a configuration of the subsystem A depends on one of B.

- product state: $|\psi\rangle = |\phi\rangle_A \otimes |\phi\rangle_B$: $S_A = 0$
- maximally entangled state: $|\psi\rangle = \frac{1}{\sqrt{D}} \sum_l |\phi_l\rangle_A \otimes |\phi_l\rangle_B$,
  $S = \log(D)$, $D$: dimension of Hilbert space for substyems A and B

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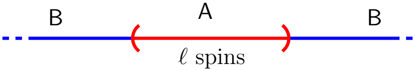


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Example: take a qubit in a singlet state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B), \qquad \rho_A = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

Entanglement in a quantum coherent system is responsible for appearance of entropy, hence for thermalization process!

# III. What we did

So far, the focus has been put on initial states that are ground states of local hamiltonians

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## Objective

Study the time evolution of local observables after a quench
[1 & 2-point functions, entanglement entropy ...]

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In the Transverse field Ising chain
[solvable but non-trivial as free theories]

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## Objective

Study the time evolution of local observables after a quench
[1 & 2-point functions, entanglement entropy ...]

Starting from an initial excited state
Let's discuss first this point

In the Transverse field Ising chain
[solvable but non-trivial as free theories]

# Why should we focus on excited states?

- Radically different behaviour of entanglement entropy for excited states:

ground states:

  - massive non degenerate GS: $S_{GS} \simeq \partial l$ [Bombelli '88;Srendicki '93]
  - critical conformal theories: $S_{GS} \simeq \frac{c}{3}\log(l) + c_1'$ [Calabrese Cardy]

higly-excited states (# excitations $\simeq$ N)

$$S_{\text{exc}} \simeq l + \mathcal{O}(\log l)$$ [Alba, Fagotti, Calabrese, '09; Sierra, ...]

insensitive to the criticality of the ground states

- Look for universal behaviour
- Room for new effects

# Quenched Transverse field Ising chain

$$H(h) = -\frac{1}{2}\sum_{j=1}^{N}\left[\sigma_j^x\sigma_{j+1}^x + h\sigma_j^z\right] \quad + \quad \text{PBC}$$

$\langle 0|\sigma_j^x|0\rangle \neq 0$    $h_c = 1$    $\langle 0|\sigma_j^x|0\rangle = 0$    $h$

$|0\rangle$: ground state of $H(h)$

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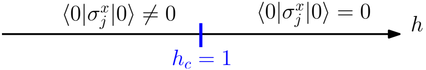


$|0\rangle$: ground state of $H(h)$

From interacting spins $\sigma_i$ to free spinless fermions $b_k$

$$H(h) = \sum_k \epsilon_h(k)\left(b_k^\dagger b_k - \frac{1}{2}\right) \qquad \epsilon_h^2(k) = 1 + h^2 - 2h\cos\frac{2\pi k}{N}$$

# Quenched Transverse field Ising chain

$$H(h) = -\frac{1}{2}\sum_{j=1}^{N}\left[\sigma_j^x\sigma_{j+1}^x + h\sigma_j^z\right] \quad + \quad \text{PBC}$$

$\langle 0|\sigma_j^x|0\rangle \neq 0$ for $h < h_c$, $\langle 0|\sigma_j^x|0\rangle = 0$ for $h > h_c$, with $h_c = 1$ (axis: $h$)

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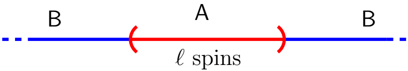
Interaction quench $h \to h'$

Initial state: $\quad |\psi_0\rangle = |m_k\rangle \equiv \prod_k (b_k^\dagger)^{m_k}|0\rangle$

- excited state of pre-quenched hamiltonian $H(h)$
- $Z_2$-invariant: $\langle\psi_0|\sigma_j^x|\psi_0\rangle = 0$
- $m_k$: fermionic initial occupation number of k-mode

# IV. Our results

# Local relaxation in the TFIC from excited states




"A" is a block of $\ell$ contiguous spins

$$\rho_A(t) = \mathrm{Tr}_B\left(|\psi_0(t)\rangle\langle\psi_0(t)|\right) \qquad |\psi_0(t)\rangle = e^{-iH(h')t}|\psi_0\rangle$$

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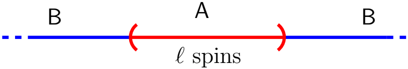


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**Result: GGE works even for excited states!**

$$\rho_{\mathrm{GGE},A} = \rho_A(\infty)$$

Idea:

Free systems $\to$ Wick's thm $\to$ just need to prove it for propagators!

- exactly solvable dynamics
- ensemble averages $\qquad \rho_{\mathrm{GGE,A}} = \dfrac{e^{-\sum_k \lambda_k n_k}}{Z}$

$n_k$: post-quench conserved fermionic occupation number operators

# Local conserved charges from excited states

$$\langle I_n^+ \rangle = \int_{-\pi}^{+\pi} \frac{dk}{4\pi} \cos(nk)\epsilon_k \left[1 + m_k^S \cos\Delta_k\right] \qquad m_k^S \equiv m_{-k} + m_k - 1$$

$$\langle I_n^- \rangle = -\int_{-\pi}^{+\pi} \frac{dk}{4\pi} \sin[(n+1)k] m_k^A \qquad m_k^A \equiv m_{-k} - m_k$$

Two classes of IS

- $m_k^A = 0$: Only $\langle I_n^+ \rangle \neq 0$ (GS belongs to this class!)
- $m_k^A \neq 0$: Both $\langle I_n^+ \rangle$ and $\langle I_n^- \rangle \neq 0$

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Result: Doubling of non zero VEVs local conserved charges wrt ground state

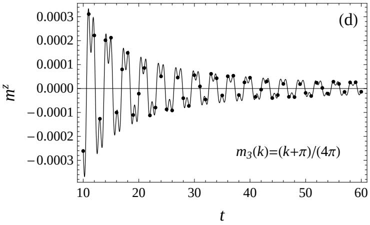
Does the increased number of conserved charges in $m_k^A$
alter the asymptotic time dependence of correlations?

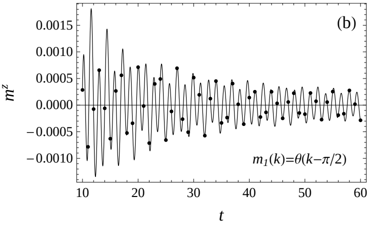
# Transverse magnetization

$$m^z(t) = \underbrace{\int_{-\pi}^{\pi} \frac{dk}{4\pi} e^{i\theta_k} m_k^S \cos\Delta_k}_{\text{stationary part}} - i \underbrace{\int_{-\pi}^{\pi} \frac{dk}{4\pi} e^{i\theta_k} m_k^S \sin\Delta_k \cos(2\epsilon_k t)}_{\text{time}-\text{dependent}}$$

Asymptotic behaviour: stationary phase approximation

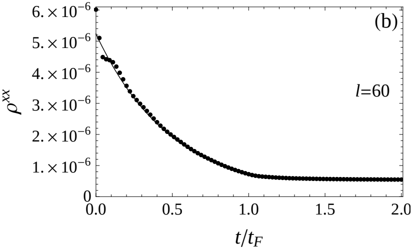
| $m(k)$ analytic | $m(k)$ non-analytic |
|---|---|
| $m^z(t) \simeq t^{-\frac{3}{2}} + \mathcal{O}(t^{-\frac{2n+1}{2}})$ | $m^z(t) \simeq t^{-1} + \mathcal{O}(t^{-\frac{2n+1}{2}})$ |
| AS GROUND STATE | NOVELTY! |



# Longitudinal spin-spin function

$$\rho^{xx}(\ell, t) \equiv \langle \Psi_0(t)|\sigma_n^x \sigma_{\ell+n}^x|\Psi_0(t)\rangle$$




$$m(k) = \frac{k^2}{(2\pi)^2}$$

$$\ell = 60$$

$$h = 1/3, \quad h' = 2/3$$

$$t_F = \ell/(2v_{\max})$$

$$v_{\max} = \min[h, 1]$$

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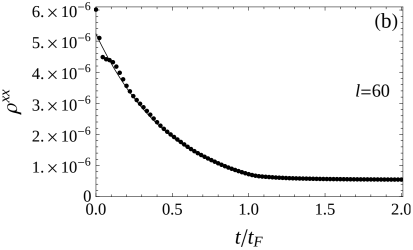


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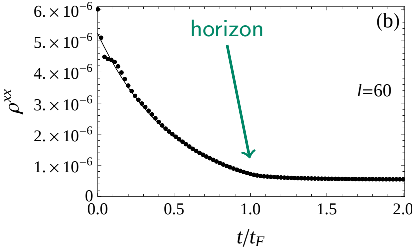


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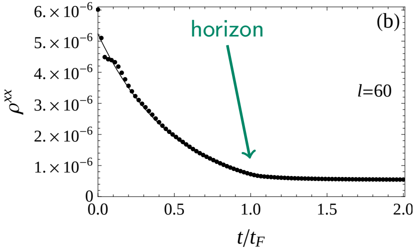


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Results

- Emergent light-cone spreading of correlations (as for GS)
- Common behaviour $\forall m_k$ analyzed (double-stepfunction, linear, quadratic)...

...EXCEPT ONE!

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Different behaviour for different $\ell$!

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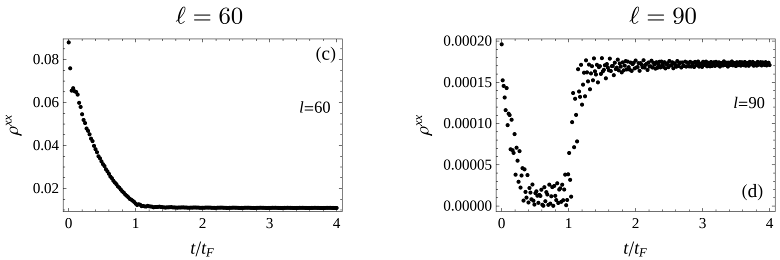


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- Is it related to $\boxed{\langle I_n^- \rangle \neq 0}$?
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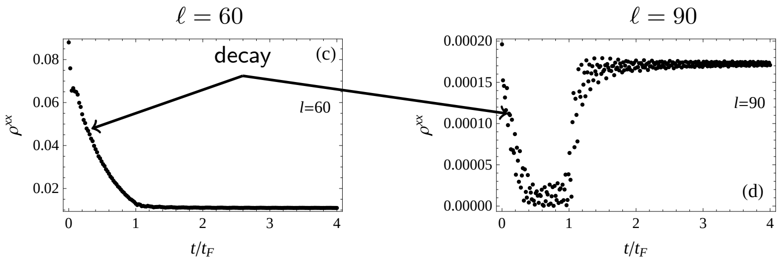


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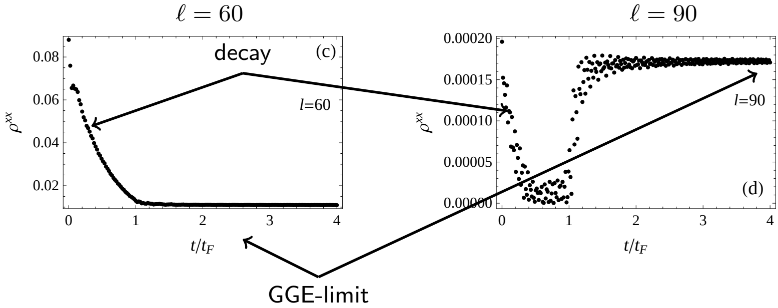


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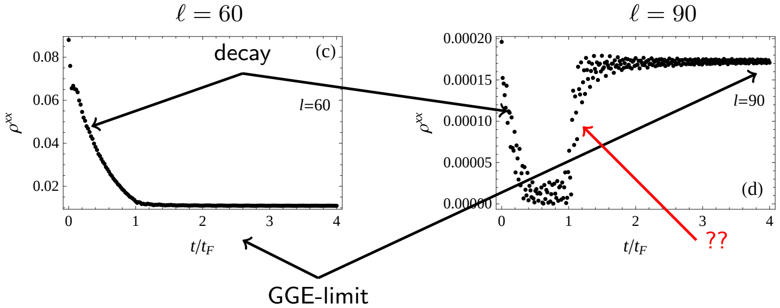


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# Analytical full time evolution of $\rho^{xx}(\ell, t)$

- Focus on quenches within the ferromagnetic phase $h, h_0 < 1$
- Method: multi-dimensional stationary phase **[Fagotti, Essler, Calabrese '08]**
- Extension only to $m_k^A = 0$

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- Method: multi-dimensional stationary phase [Fagotti, Essler, Calabrese '08]
- Extension only to $m_k^A = 0$

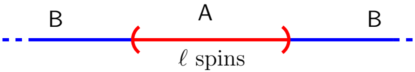
$$\rho^{xx}_{m_k}(\ell,t) \simeq C_{m_k} \overbrace{\exp\left[\ell \int_{-\pi}^{\pi} \frac{dk}{2\pi}\left(1 - 2|\epsilon'_k|\frac{t}{\ell}\right)\ln(|m_k^S|)\theta(\ell - 2|\epsilon'_k|t)\right]}^{\text{typical of excited states}}$$
$$\times \exp\left[2t \int_{-\pi}^{\pi} \frac{dk}{2\pi}|\epsilon'_k|\ln[|\cos\Delta_k m_k^S|]\theta(\ell - 2|\epsilon'_k|t)\right]$$
$$\times \exp\left[\ell \int_{-\pi}^{\pi} \frac{dk}{2\pi}\ln[|\cos\Delta_k m_k^S|]\theta(2|\epsilon'_k|t - \ell)\right]$$

Universal properties:

- $t \ll t_F$, evolution in $t$ does not depend on $m_k^S$ (first two lines)
- $t \gg t_F$, constant in time (third line)
- At fixed time, exponential decreasing with $\ell$

# Entanglement Entropy

$S_{\mathsf{A}} = -\mathrm{Tr}[\rho_{\mathsf{A}} \ln \rho_{\mathsf{A}}]$



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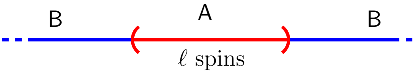


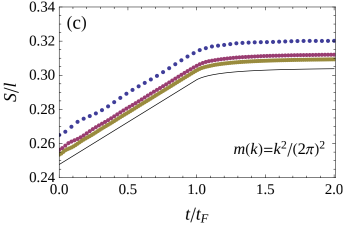


- Light-cone behaviour
- Dependence on $m_k^S$
- $S_\ell/\ell \neq 0$ at $t = 0$ due to excitations

# V. Conclusions and Outlook

We have considered quenches from excited states

Validity of GGE

Horizon effect for $S_\ell$ and $\rho_\ell^{xx}$

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*Thank you for your attention*