Bell-paired states inducing volume law for Entanglement Entropy in fermionic lattices

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Entanglement in many-body systems

- Quantum correlations characterizing quantum states
- Characterization of quantum phase transitions
- Possibility to detect novel quantum phases, without local order (topological phases)
- Scaling of the entropy. Thermal/classical states: extensive entropy (no entanglement). What about GS properties of a quantum system?
- Black holes, holographic principle
- Characterizes the efficiency of algorithms such as DMRG

Entanglement Entropy (EE) (Von Neuman Entropy)

- Measure of the entanglement of a <u>pure state</u>
- Bipartition into two subsystems: A and $\bar{A}$

$$\rho_A = Tr_{\bar{A}} \rho$$

$$S_A = -Tr_A(\rho_A \ln \rho_A)$$

Area law

- area law for EE: typically the EE grows as the boundary of the subsystem (proven only for gapped ground states in 1D)
- If dim A=d

$$S_A \sim L^{d-1}$$

Free fermions

Logarithmic deviation to the area law $S_A \sim L^{d-1} \log L$

Entanglement entropy for free fermions

$$H = -\sum_{I,J} c_I^\dagger t_{IJ} c_J$$

EE in terms of correlation matrix

$$S_A = -\sum_{\gamma=1}^{L} \left[(1 - C_\gamma)\ln(1 - C_\gamma) + C_\gamma \ln C_\gamma\right]$$

C_γ eigenvalues of the correlation matrix

$$C_{ij} = \langle \Psi | c_i^\dagger c_j | \Psi \rangle.$$

translational invariance: plane wave solutions

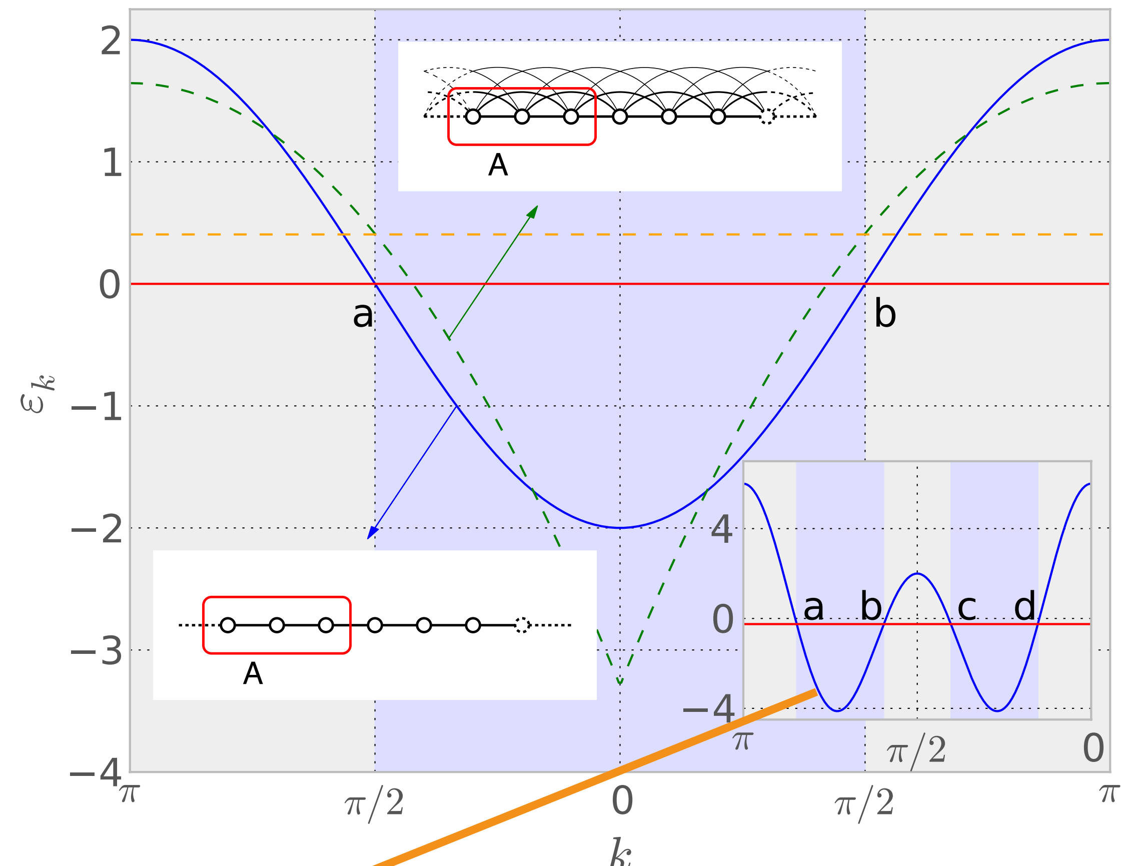
$$\psi_k(J) = \frac{1}{\sqrt{N_S}} e^{ikJ} \qquad C_{ij} = \int_{E<E_F} \frac{dk}{2\pi} e^{ik(i-j)}$$

Non-local power law hopping

- Free fermions with PBC
- Long range hopping: can it determine a violation of the area law beyond the logarithmic correction?

$$t_{I,J} = \begin{cases} 0 & I = J, \\ \frac{t}{|I-J|_p^\alpha} & I \neq J, \end{cases}$$

$$|I - J|_p = \min(|I - J|, N_S - |I - J|)$$



- EE does not change varying α : still area law
- Does not depend on the spectrum but on the topology of the Fermi surface

- Solid line: nearest neighbors
- dashed line $\alpha = 2$
- Inset: next-nearest neighbors model

$$C_{ij} = \int_{k_a}^{k_d} \frac{dk}{2\pi} e^{ik(i-j)} - \int_{k_b}^{k_c} \frac{dk}{2\pi} e^{ik(i-j)}$$

Fully connected case $\alpha = 0$

$$H = -\frac{t}{N_S}\sum_{I\neq J} c_I^\dagger c_J,$$

$$\left\langle c_I^\dagger c_J \right\rangle = \begin{cases} f \;\; for \quad I = J \\ b \;\; for \quad I \neq J \end{cases}$$

$N_S \to \infty$

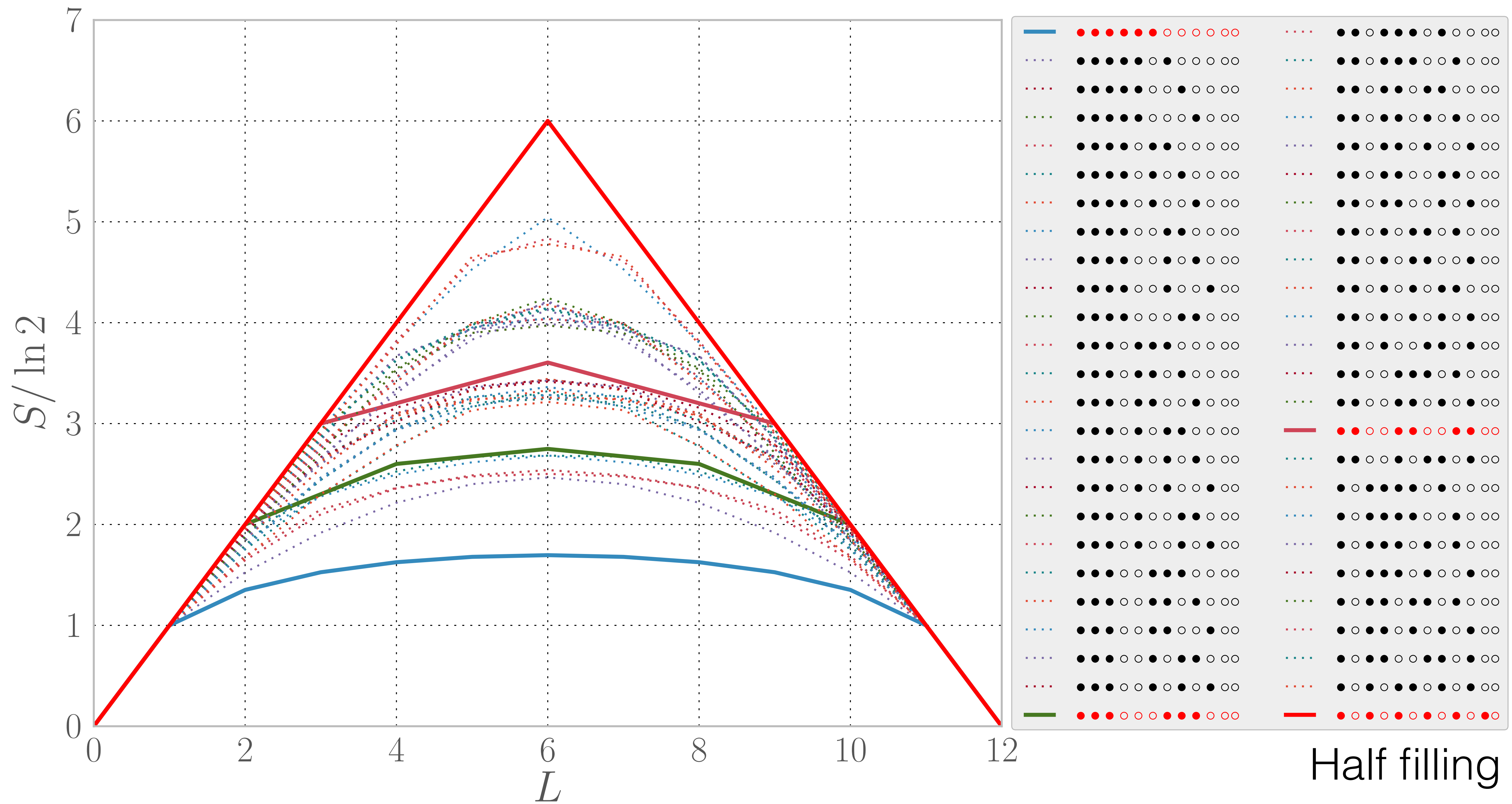
$$S_A \approx -L\left[(1-f)\ln(1-f) + f\ln(f)\right]$$

$$I\left(A:\bar{A}\right) = S(A) + S\left(\bar{A}\right) - S\left(A,\bar{A}\right) = S_A + S_{\bar{A}} - S_T$$

$N_S \to \infty$ $\quad I \simeq 0$

- (Ns-1)-fold degenerate single particle excite state
- Many-body GS degenerate
- Fermi surface not well defined
- The EE scales linearly (but, due to the degeneracy, is not a measurement of entanglement)
- The Mutual Information could be considered instead
- No entanglement: reduction to a classical case

Let's try all the possible Fermi surfaces



Half filling

$N_S = 12$

- Maximal EE for alternating filling of the wave vectors
- Periodicity in k-space: piecewise linear behavior

States with maximal EE

- Single particle vector space $\mathcal{V} = \mathcal{A} \oplus \bar{\mathcal{A}}$
- Basis in the complementary subsystems $\alpha_j \in \mathcal{A},\ \bar{\alpha_j} \in \bar{\mathcal{A}}$

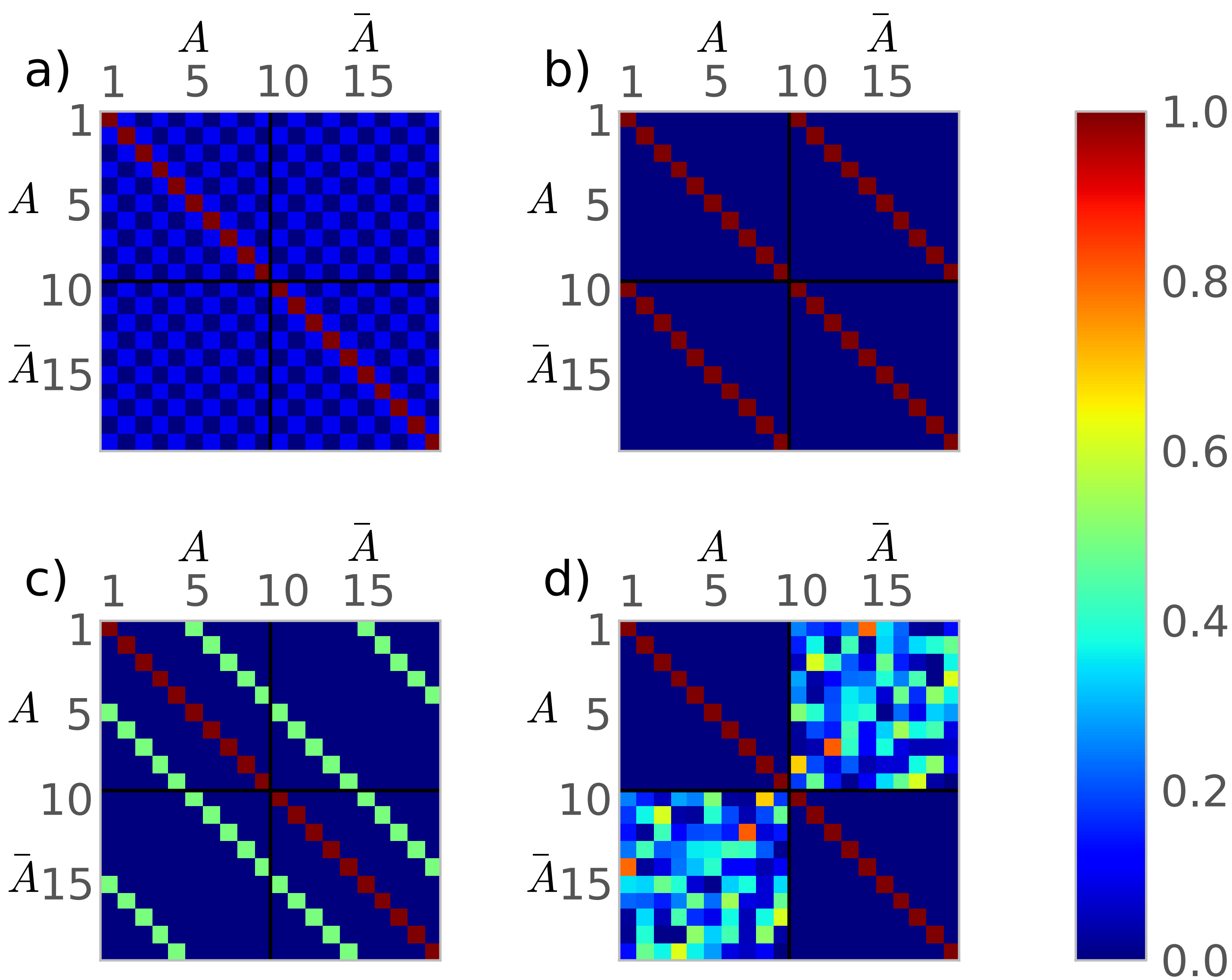
$$\beta_1 = \frac{1}{\sqrt{2}}(\alpha_1 + \bar{\alpha}_1), \beta_2 = \frac{1}{\sqrt{2}}(\alpha_2 + \bar{\alpha}_2), \ldots, \beta_{\dim \mathcal{A}} = \frac{1}{\sqrt{2}}(\alpha_{\dim \mathcal{A}} + \bar{\alpha}_{\dim \mathcal{A}})$$

- single particle occupied states (not necessarily wave vectors) constructed in this way maximize the EE for half filling
- Also all the Rényi entropies
- Bell-paired states

zig-zag structure in the momentum space

$$\langle J|\alpha_k\rangle = \begin{cases} \frac{1}{\sqrt{N_S/2}} e^{2\pi i n_k J/(N_S/2)} & \text{for } J \le N_S/2 \\ 0 & \text{for } J > N_S/2 \end{cases}$$

$$\langle J|\bar{\alpha}_k\rangle = \begin{cases} 0 & \text{for } J \le N_S/2 \\ \pm\frac{1}{\sqrt{N_S/2}} e^{2\pi i n_k J/(N_S/2)} & \text{for } J > N_S/2, \end{cases}$$



$$\mathcal{C}_{I,J} = \langle c_I^\dagger c_J + c_J^\dagger c_I\rangle$$

A. contiguous occupation of k
B. zigzag state
C. two filled and two empty momenta alternates
D. Bell-paired state with random connection between the complementary subsystems

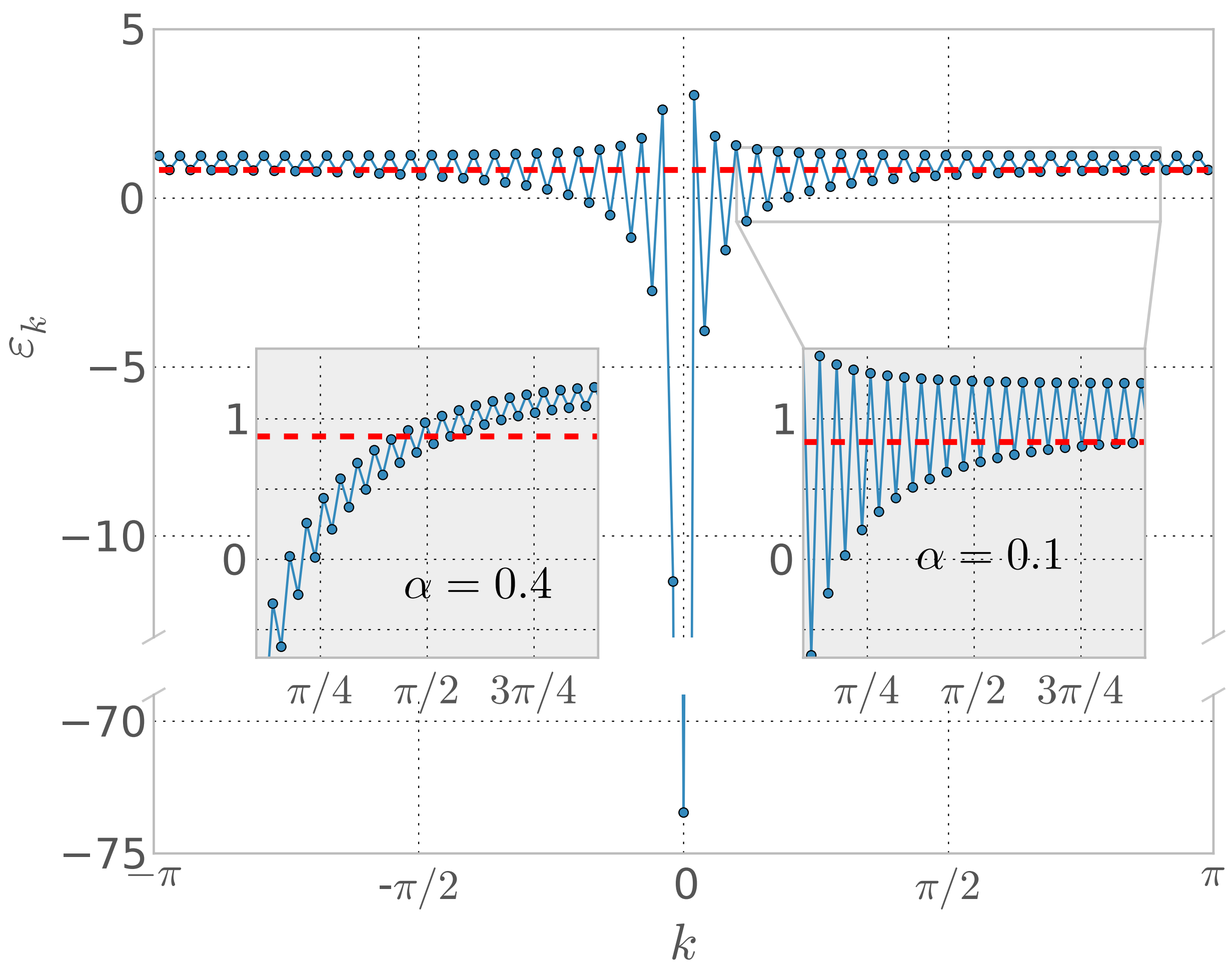
Examples of models violating the area law

A model can be constructed for the GS to have the structure of the Bell-paired states.

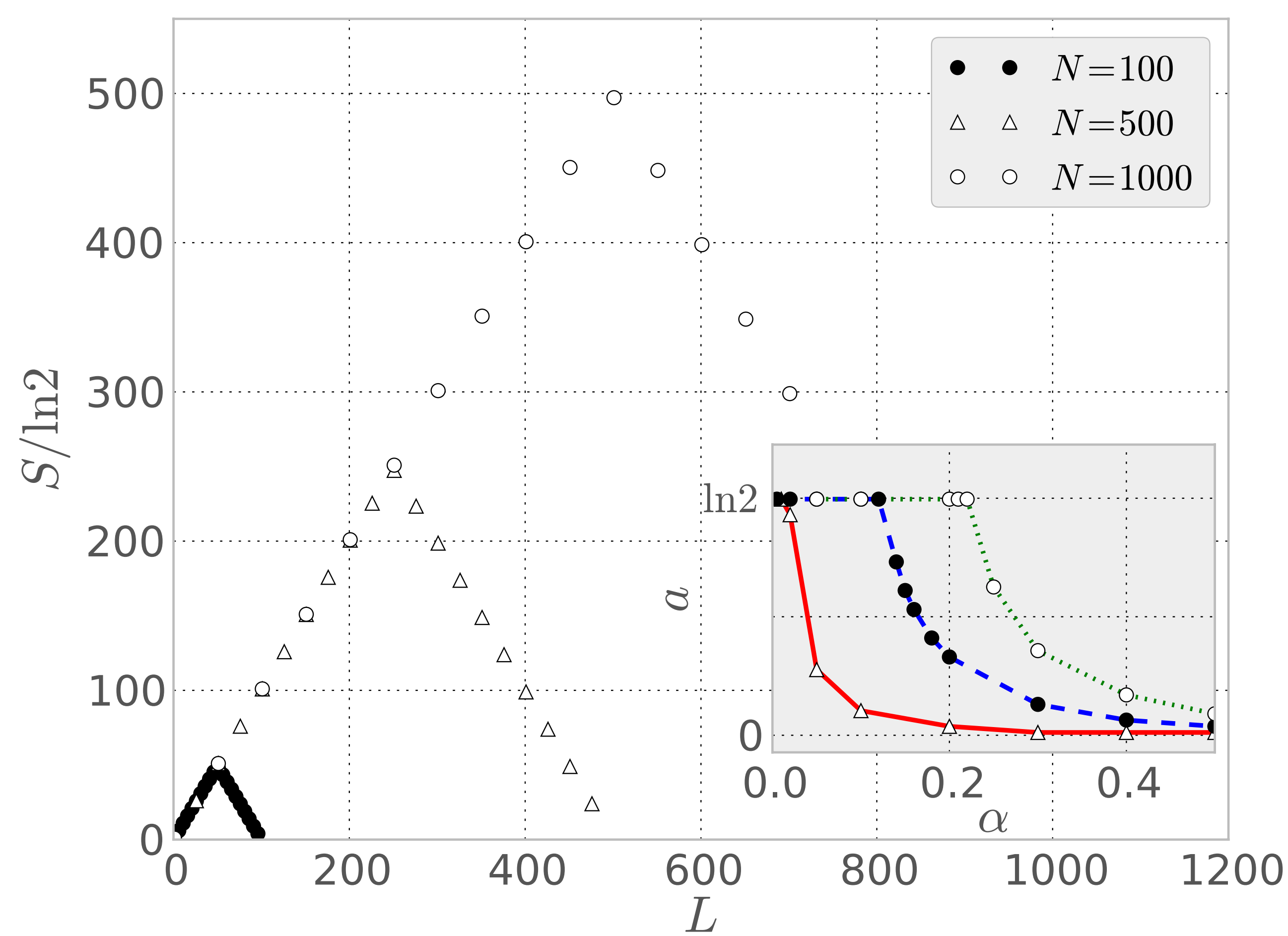
$$t_{I,J} = \begin{cases} t & for\,|I-J|_p = \frac{N_S}{2} \\ 0 & otherwise, \end{cases}$$

Long-Range with a Magnetic flux

$$t_{I,J} = \frac{t \cdot e^{i\phi d_{I,J}}}{|I-J|_p^{\alpha}} \qquad\qquad \phi = \frac{2\pi}{N_S}\Phi$$



N=100 $\Phi = 0.1$



$\Phi = 0.001$ $\Phi = 0.1$ $\Phi = 0.3$

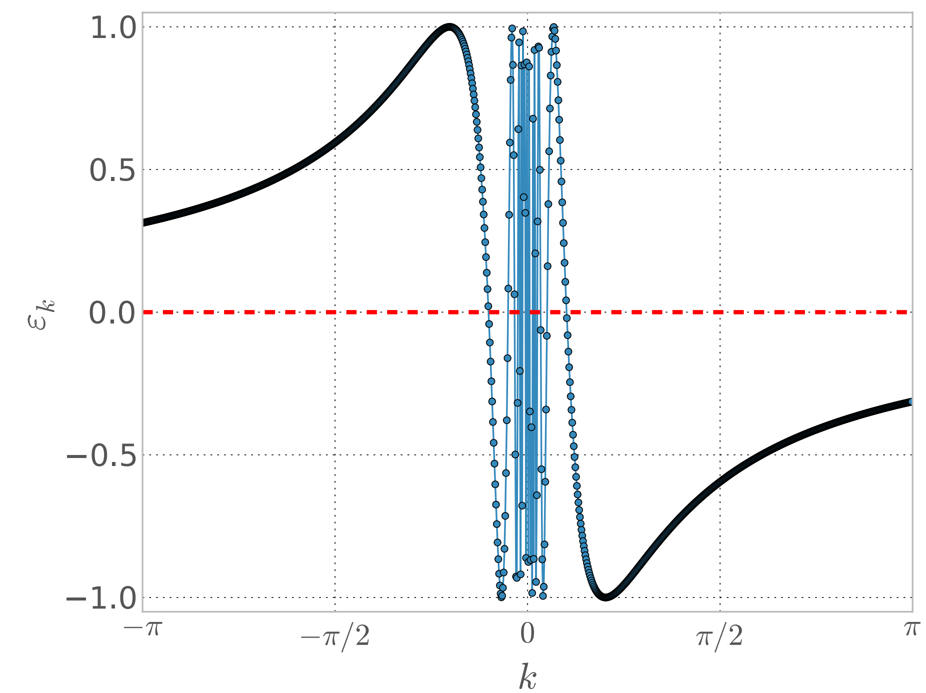
fitting function: S=a L

Fermi surface with an accumulation point

$$\varepsilon_k = -t \cdot \sin\left(\frac{1}{k^\alpha}\right),$$

Fermi surface:

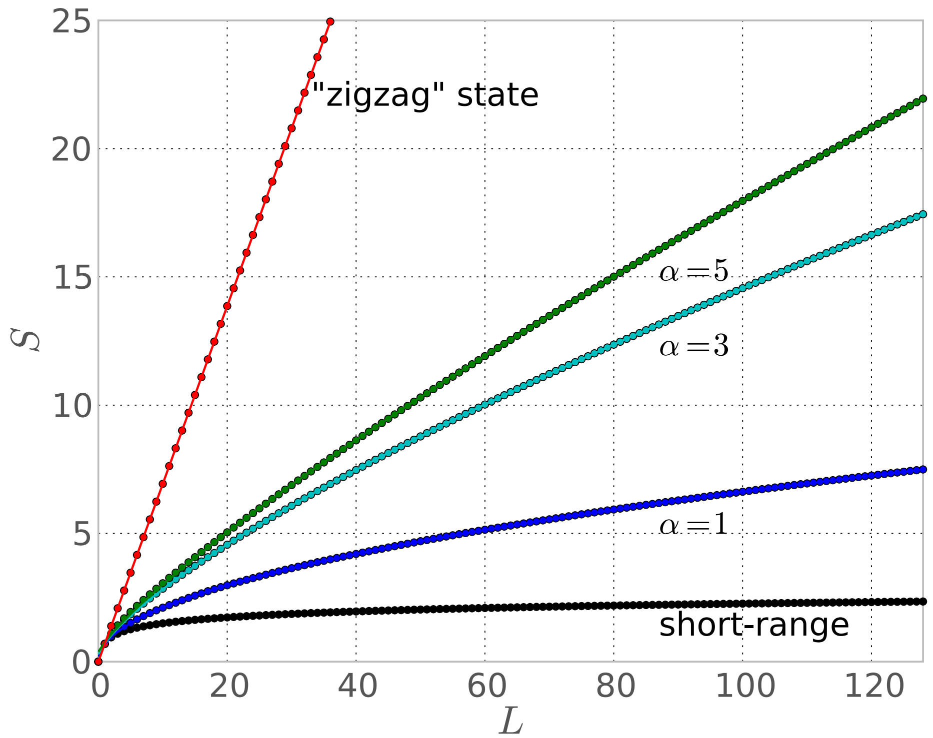
$$\{\pm\frac{1}{\pi}, \pm\frac{1}{\pi 2^\alpha} \pm \frac{1}{\pi 3^\alpha} \dots\}$$

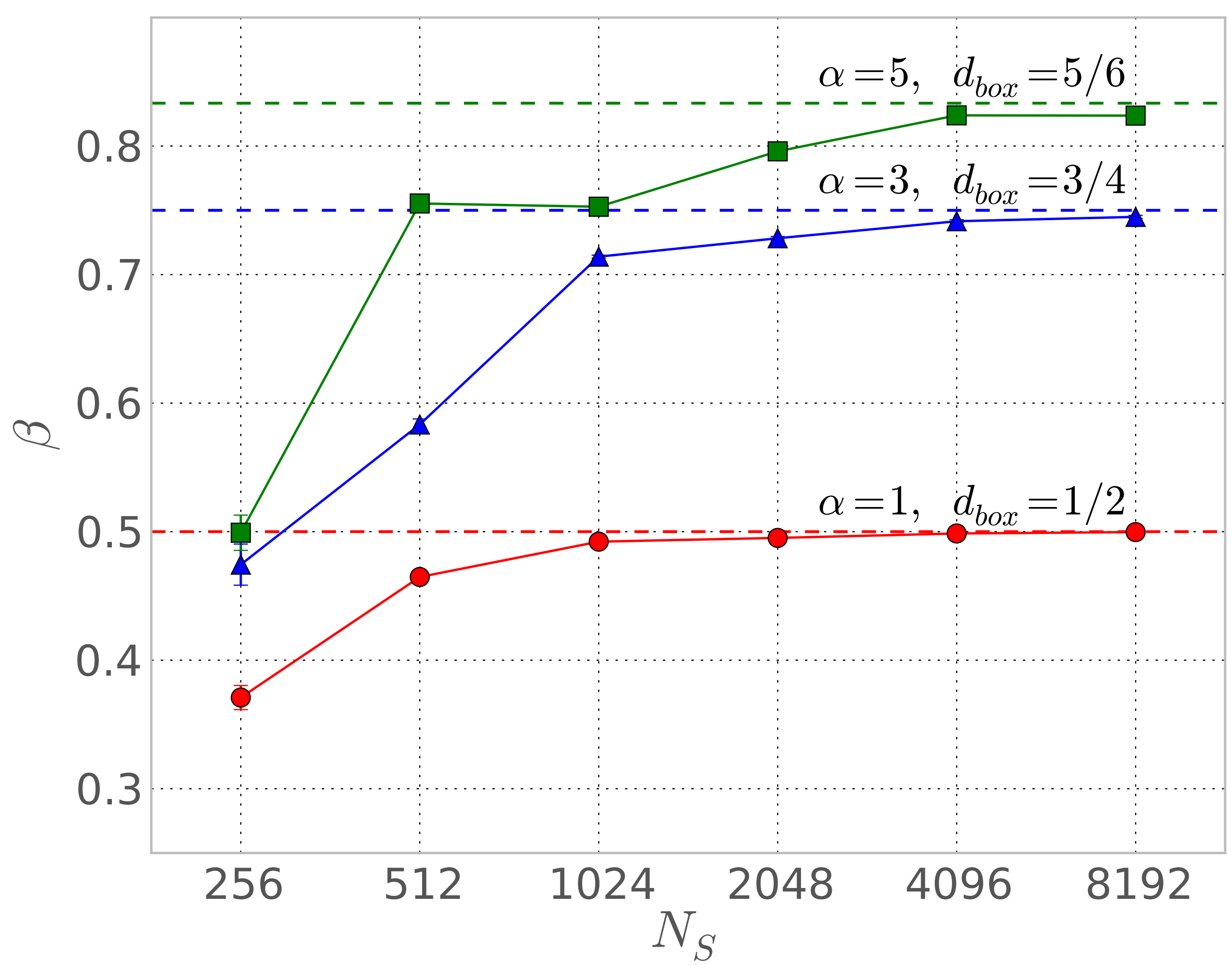


Set with box counting dimension: $d_{box} = \frac{\alpha}{\alpha+1}$

Fit function:

$$S = a + bL^\beta$$





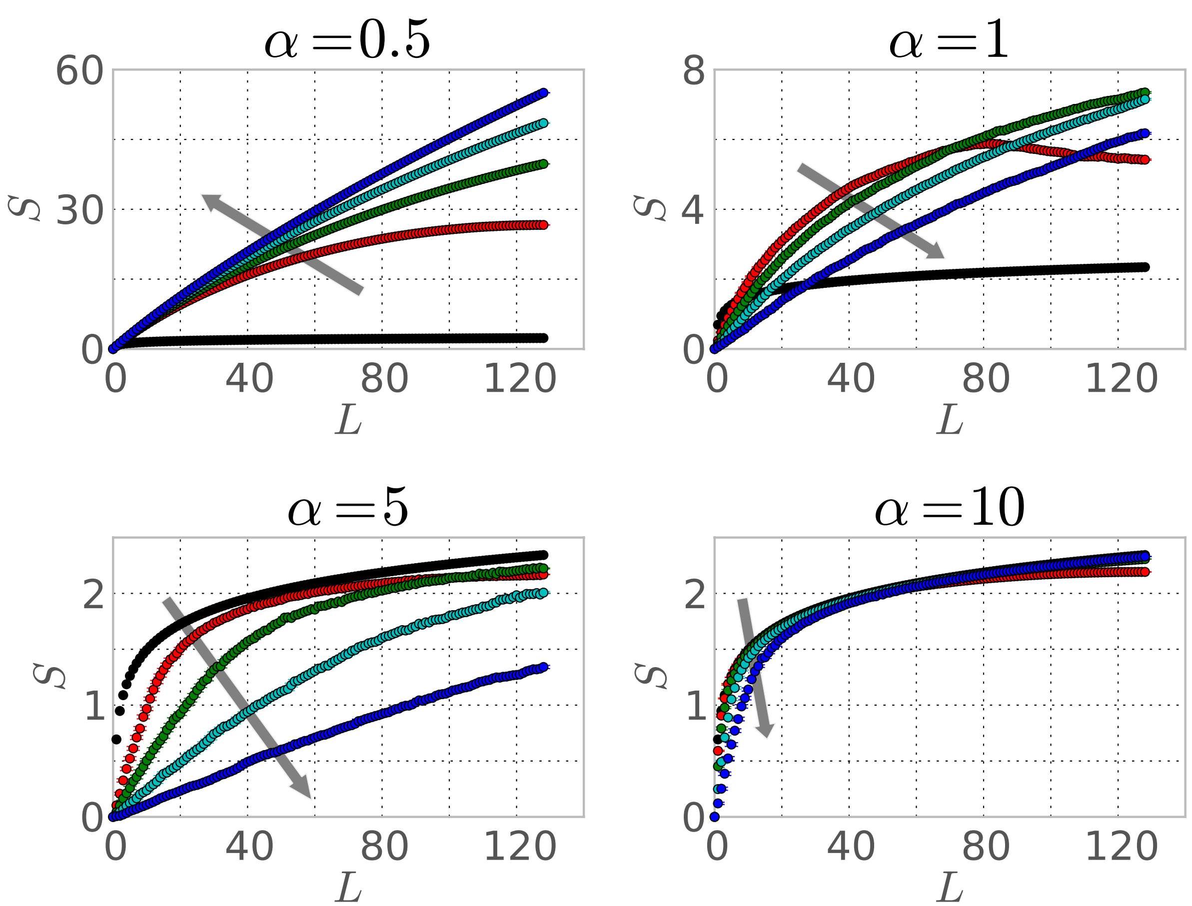
L=1…128. Short range: S=a+b ln L

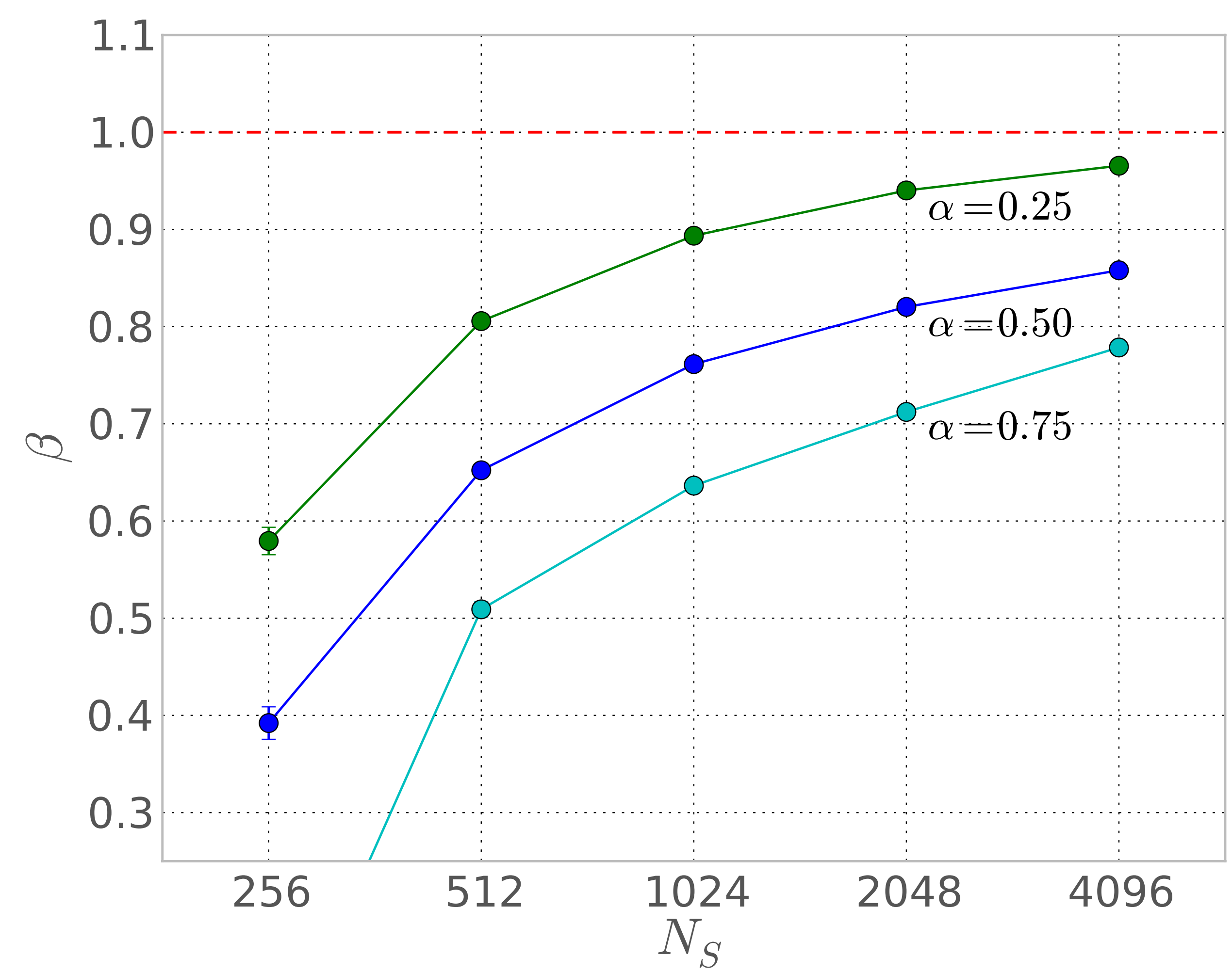
Random Long-Range hopping

$$t_{I,J} = \frac{t \cdot \eta_{I,J}}{|I-J|_p^\alpha}$$

$$\eta_{I,J} = \pm 1$$

- Breaking of translational symmetry
- Logarithm behavior for $\alpha \gg 1$
- Linear behavior for $\alpha \ll 1$





Conclusions

- Violation of the area law has been investigated for free-fermions
- Long range hopping is not a sufficient condition. A more complex structure of the spectrum and of the Fermi surface is needed and/or a breaking of the translational invariance.
- Explicit construction of the states maximizing the EE in terms of Bell pairs.
- Explicit examples: magnetic phase; FS with an accumulation point
- Preliminary results for a disordered lattice