Brian Muenzenmeyer

Dissipative preparation of squeezed states with ultracold atomic gases

GW & Mäkelä, Phys. Rev. A **85**, 023604 (2012)

Caballar *et al*., Phys. Rev. A **89**, 013620 (2014)

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Quantum state engineering using dissipation

What is dissipation?

Dissipation (*Longman Advanced American Dictionary*)

1. the process of making something disappear or scatter
2. the act of wasting money, time, energy etc.
3. the enjoyment of physical pleasures that are harmful to your health

"Diligence and Dissipation" by Northcote

Dissipation: caused by coupling with environment

Dissipation in cold atom gases

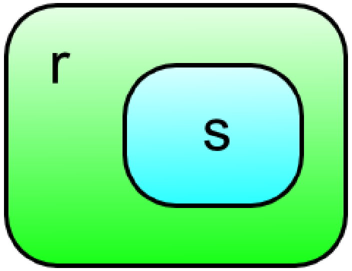
Dissipation: caused by coupling with environment

- Particle losses (1-, 2-, & 3-body losses)
- Incoherent scattering of trap lasers etc.

Usually, "dissipation" ≈ "decoherence"

But, "dissipation" ≠ "decoherence"

Quantum sys. coupled to a reservoir.



(total) = (system) + (reservoir)

ρ_{sr} : total density operator (system + reservoir)

$\rho \equiv \rho_s = \mathrm{Tr}_r[\rho_{sr}]$: system density operator

What we need is the system density operator ρ.

Master equation

Master equation: EOM for the system density op. within Born-Markov approx.

$$\frac{d\rho}{dt} = -i[H,\rho] + \frac{\gamma}{2}(2c\rho c^\dagger - c^\dagger c\rho - \rho c^\dagger c)$$

Dissipation

c : jump op.

γ : dissipation rate

Born-Markov approx.

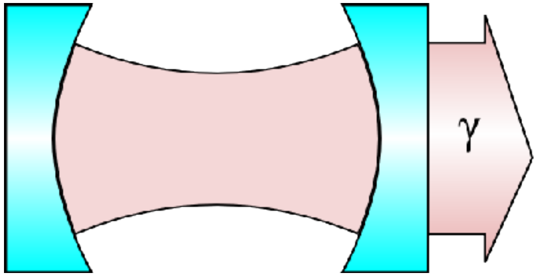
- Weak system-bath coupling (Born)
  coupling term << H or reservoir
- Short correlation time of the reservoir (Markov)
  << dynamical timescale of sys
  Reservoir has no memory.

Poor man's derivation of master eq. (1)

ex. Photons in a lossy cavity

γ: loss rate

a: annihilation op. of photons



Prob. of a photon escaping from cavity in δt.

$$\Delta P = \gamma \langle \Psi | a^\dagger a | \Psi \rangle \delta t$$

$$|\Psi\rangle \xrightarrow{\delta t} \begin{cases} |\Psi_{\mathrm{emit}}\rangle & \Delta P \\ |\Psi_{\mathrm{no\ emit}}\rangle & 1 - \Delta P \end{cases}$$

$$|\Psi_{\mathrm{emit}}\rangle = \frac{a|\Psi\rangle}{\langle \Psi | a^\dagger a | \Psi \rangle^{1/2}} = (\gamma \delta t / \Delta P)^{1/2} a |\Psi\rangle$$

$$|\Psi_{\mathrm{no\ emit}}\rangle = \frac{e^{-iH_{\mathrm{eff}}\delta t}|\Psi\rangle}{\langle \Psi | e^{iH^\dagger_{\mathrm{eff}}\delta t} e^{-iH_{\mathrm{eff}}\delta t} | \Psi \rangle^{1/2}} \simeq \frac{(1 - iH\delta t - \frac{\gamma}{2}\delta t a^\dagger a)|\Psi\rangle}{(1 - \Delta P)^{1/2}}$$

$$H_{\mathrm{eff}} = H - i\frac{\gamma}{2} a^\dagger a$$

Measurement of "no emission". ➡ non-Hermitian

Poor man's derivation of master eq. (2)

Density operator at t + δt

$$\rho(t+\delta t) = \Delta P |\Psi_{\rm emit}\rangle\langle\Psi_{\rm emit}| + (1-\Delta P)|\Psi_{\rm no\ emit}\rangle\langle\Psi_{\rm no\ emit}|$$

$$\simeq |\Psi\rangle\langle\Psi| - i\delta t\ [H, |\Psi\rangle\langle\Psi|]$$

$$+\frac{\gamma}{2}\delta t\,(2a|\Psi\rangle\langle\Psi|a^\dagger - a^\dagger a|\Psi\rangle\langle\Psi| - |\Psi\rangle\langle\Psi|a^\dagger a)$$

Since $|\Psi\rangle\langle\Psi| = \rho(t)$

$$\frac{d\rho}{dt} = -i[H,\rho] + \frac{\gamma}{2}(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)$$

Quantum jump by emission

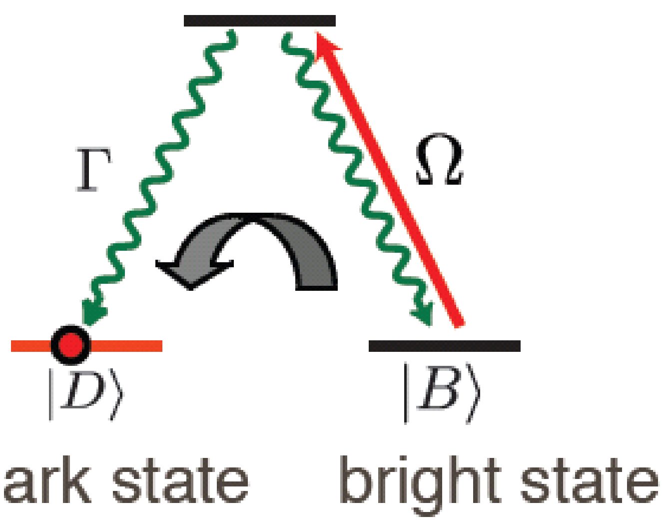
Damping by non-unitary evolution.

Jump operator = *a*

State preparation using dissipation (1)

Usually, "dissipation" ≈ "decoherence"

But, "dissipation" ≠ "decoherence"



(presentation by P. Zoller)

Analogy to optical pumping

$$\rho(t) \xrightarrow[t \to \infty]{} |D\rangle\langle D|$$

Source of Γ: Dissipation in cold atom gases

- By spontaneous emission of photons
  ⇐ coupling with vac. st.
- By emission of Bogoliubov excitations
  ⇐ coupling with background BEC

Dissipation can be used for preparation of pure states.

[Agarwal (1988), Aspect *et al*. (1988), Kasevich & Chu (1992), Diehl *et al*. (2008), Kraus *et al*. (2008)]

State preparation using dissipation (2)

Kraus *et al*. PRA **78**, 042307 (2008).

Master eq.

$$\frac{d\rho}{dt} = \mathcal{L}(\rho) = -i[H,\rho] + \sum_i \frac{\gamma_i}{2}(2c_i\rho c_i^\dagger - c_i^\dagger c_i \rho - \rho c_i^\dagger c_i)$$

1. $H|\Psi\rangle = E|\Psi\rangle$
2. $\forall i \quad c_i|\Psi\rangle = 0$: dark st.
3. Uniqueness
   $|\Psi\rangle$ is the only st.;
   $\mathcal{L}(|\Psi\rangle\langle\Psi|) = 0$

target st.

$$\rho \xrightarrow[t\to\infty]{} |\Psi\rangle\langle\Psi|$$

Task: Construct a parent Liouvillian

Condition 3. → c_i must be non-Hermitian.

$\because$ If c_i is Hermitian → $\dot{\rho} = -i[H,\rho] + \sum_i \frac{\gamma_i}{2}[[c_i,\rho],c_i]$

$\therefore \ \rho \propto I$ is also a steady st.; $\mathcal{L}(I) = 0$

State preparation using dissipation (3)

State preparation using dissipation!

Advantages:

- Works for any initial state.

[Kraus *et al*. PRA **78**, 042307 (2008)]

- Self-driven: no need of active control.

"Just wait."

- Engineered dissipation can overwrite unwanted one.

Dissipation-induced coherence

Example: Pumped single-mode field with loss

[G. S. Agarwal, J. Opt. Soc. Am. B, **5**, 1940 (1988)]

Pumping Hamiltonian:

$$H = \frac{g}{2}(a + a^\dagger)$$

Master eq.:

$$\frac{d\rho}{dt} = -i[H, \rho] + \frac{\gamma}{2}(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)$$

$$= \frac{\gamma}{2}(2c\rho c^\dagger - c^\dagger c\rho - \rho c^\dagger c) \equiv \mathcal{D}[c]\rho$$

with $c \equiv a + ig/\gamma$

$\frac{d\rho}{dt} = \mathcal{D}[c]\rho = 0$ if & only if $(a + ig/\gamma)\rho = 0 = \rho(a + ig/\gamma)^\dagger$

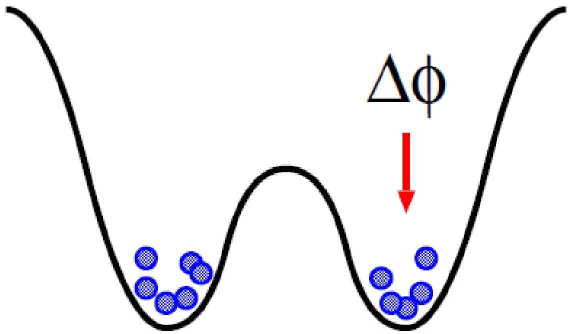
→ $\rho = |-ig/\gamma\rangle\langle -ig/\gamma|$: coherent st.

"Dissipation-induced coherence"

Matter-wave interf. and 2-mode sys.

Matter-wave interferometer : interferometer with cold atoms

Ex. cold Bose gases in a double-well pot.



Release from the trap.

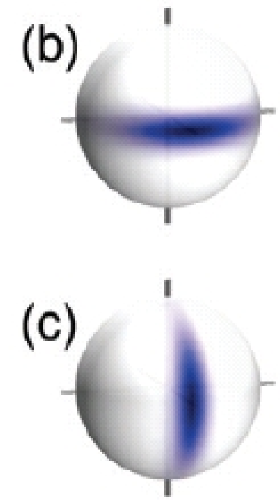
Expand & interfere

Measure the force field, etc.
from the interference pattern.

Squeezed st. in matter-wave interf.

Uncertainty relation $\Delta X_1 \Delta X_2 \gtrsim \hbar$

Squeezing: decreasing uncertainty of one variable
(in expense of increasing the other's)

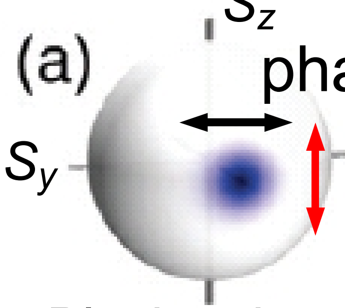


Grond *et al.* (2010)

Two-mode case

Number squeezing $\xi_N \equiv \langle \Delta S_z^2 \rangle^{1/2} / \sqrt{N/2} < 1$

Phase squeezing $\xi_{\text{phase}} \equiv \langle \Delta S_y^2 \rangle^{1/2} / \sqrt{N/2} < 1$



Bloch sphere

$$S_x = (a_1^\dagger a_2 + a_2^\dagger a_1)/2$$

$$S_y = (a_1^\dagger a_2 - a_2^\dagger a_1)/2i$$

$$S_z = (a_1^\dagger a_1 - a_2^\dagger a_2)/2$$

Phase sq. st.: High visibility of interf. pattern.
Good for readout.

(Number sq. st.: Small phase diffusion.
Longer phase accum. time.)

Bottom lines

GW & Mäkelä, PRA **85**, 023604 (2012)

Caballar *et al*., Phys. Rev. A **89**, 013620 (2014)

Dissipative preparation scheme of
phase- & number-squeezed st. using cold atom gases

- Proposal of the physical setup

- Resulting master eq. leads to a pure squeezed st. in the ideal limit.

- Squeezing develops on a rapid time scale $\propto 1/N$ due to the bosonic enhancement.

Two-mode Bose squeezing using dissipation

Squeezing jump operator

Coherent st.: $|\phi\rangle \propto (e^{i\phi/2}a_1^\dagger + e^{-i\phi/2}a_2^\dagger)^N|0\rangle$

Squeezing jump op. [GW & Mäkelä, PRA **85**, 023604 (2012)]

$$c = (a_1^\dagger + a_2^\dagger)(a_1 - a_2) + \epsilon(a_1^\dagger - a_2^\dagger)(a_1 + a_2) \quad (-1 < \epsilon < 1)$$

$|\phi = 0\rangle \propto (a_1^\dagger + a_2^\dagger)^N|0\rangle$
is a dark st.
$\because \ [a_1 - a_2, a_1^\dagger + a_2^\dagger] = 0$

$|\phi = \pi\rangle \propto (a_1^\dagger - a_2^\dagger)^N|0\rangle$
is a dark st.
$\because \ [a_1 + a_2, a_1^\dagger - a_2^\dagger] = 0$

$$c = 2(1+\epsilon)S_z - 2i(1-\epsilon)S_y$$

when ε =1, $c = 4S_z \propto \Delta N$

Dark st. is a Fock st. with ΔN=0.

2-mode squeezing

Coherent st. analysis (valid for *N*>>1)

$$\langle S_x^2 \rangle \simeq \langle S_x \rangle^2 \quad \text{with} \quad \langle S_x \rangle \simeq N/2 + O(N^0)$$

$$\frac{d}{dt}\langle S_{y,z}^2 \rangle = \mathrm{Tr}\left[\dot{\rho} S_{y,z}^2\right]$$

master eq.

$$\Rightarrow \frac{d}{dt}\langle S_{y,z}^2 \rangle \simeq -4N\gamma(1-\epsilon^2)\langle S_{y,z}^2 \rangle + N^2\gamma(1 \pm \epsilon)^2$$

In the steady state

Number squeezing param.: $$\xi_N = \frac{\langle S_z^2 \rangle^{1/2}}{\sqrt{N/2}} \simeq \sqrt{\frac{1-\epsilon}{1+\epsilon}}$$

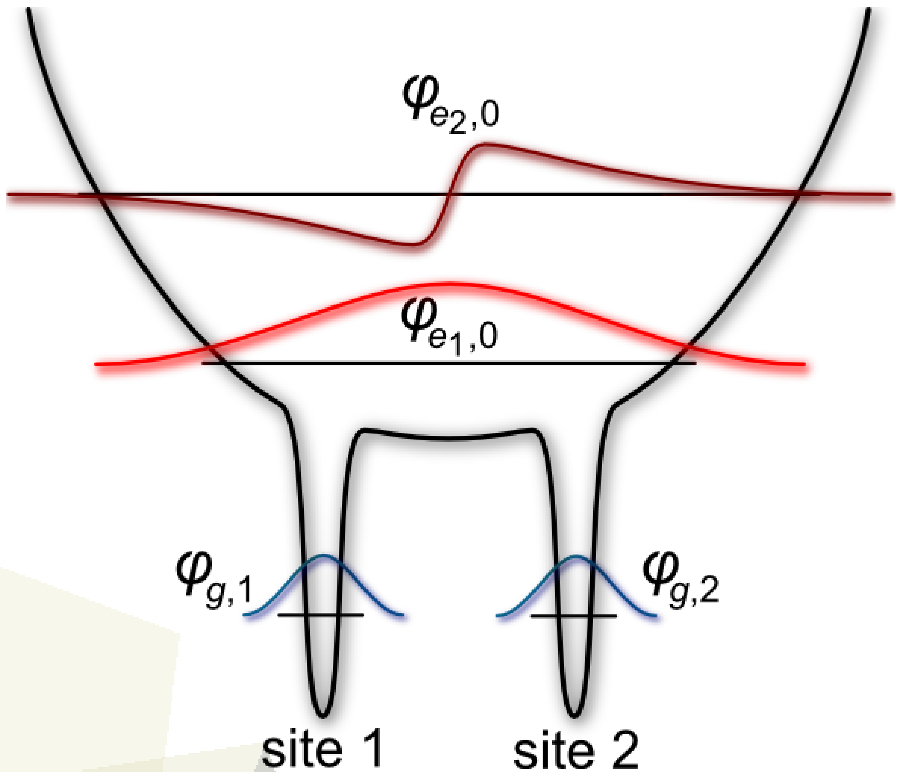
Phase squeezing param.: $$\xi_{\text{phase}} = \frac{\langle S_y^2 \rangle^{1/2}}{\sqrt{N/2}} \simeq \sqrt{\frac{1+\epsilon}{1-\epsilon}}$$

ε > 0 ⇒ number sq. & phase anti-sq.

ε < 0 ⇒ phase sq. & number anti-sq.

Setup

- System: *a*-atoms in "spherical cow" trap



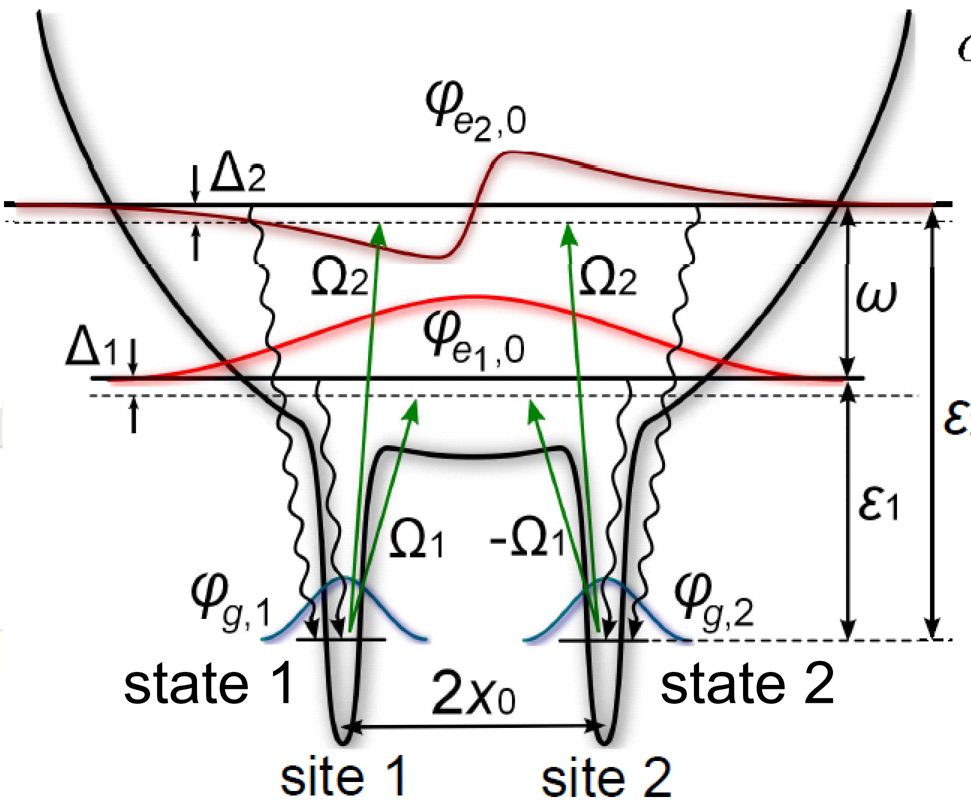
- Environment: background BEC of *b*-atoms

Reservoir of superfluid phonons

Setup: trapped atoms + background BEC

system — reservoir of s.f. phonons

- States 1& 2 are Raman coupled to two excited states with even & odd parity.
- Atoms in the excited states decay into the states 1 & 2 through s.f. phonon emission.



$$c = (a_1^\dagger + a_2^\dagger)(a_1 - a_2) + \epsilon(a_1^\dagger - a_2^\dagger)(a_1 + a_2)$$

F_1 F_2

→ F_2

→ F_1

$$\epsilon = \frac{\Omega_2}{\Omega_1}\frac{\Delta_1}{\Delta_2}\frac{\phi_{e_2,0}(x_0)}{\phi_{e_1,0}(x_0)}$$

Ω_n : effective Rabi freq.

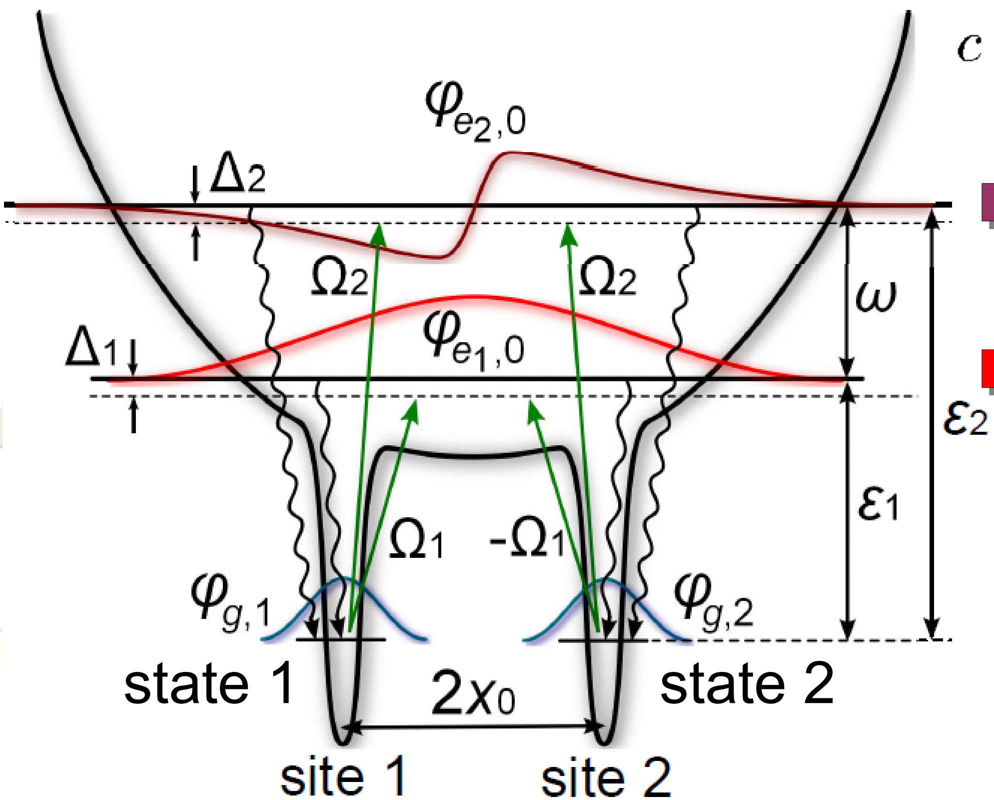
Δ_n : detuning

$|\Omega_n/\Delta_n| \ll 1$: far-detuned

→ Two excited states are adiabatically eliminated.

Conditions

- For the coherence btwn site 1 & 2: $k_n x_0 \ll 1$
- For the coherence btwn F_1 & F_2: $|k_2 - k_1| \ll k_1, k_2$
  $(\omega \ll \epsilon_1, \epsilon_2)$



$$c = \underbrace{(a_1^\dagger + a_2^\dagger)(a_1 - a_2)}_{F_1} + \epsilon \underbrace{(a_1^\dagger - a_2^\dagger)(a_1 + a_2)}_{F_2}$$

→ F_2

→ F_1

$$\epsilon = \frac{\Omega_2}{\Omega_1} \frac{\Delta_1}{\Delta_2} \frac{\phi_{e_2,0}(x_0)}{\phi_{e_1,0}(x_0)}$$

Ω_n : effective Rabi freq.

Δ_n : detuning

$|\Omega_n / \Delta_n| \ll 1$: far-detuned

→ Two excited states are adiabatically eliminated.

Hamiltonian

$$H = \underline{H_a} + \underline{H_b} + \underline{H_{ab}}$$

trapped atoms (a-atoms)	Bogoliubov excit. (b-atoms)	coupling btwn. a-atoms & Bogoliubov excit.

$$H_{ab} \simeq \sum_{\mathbf{k}\neq 0} g_k A^\dagger_{\mathbf{k}} b_{\mathbf{k}} + \mathrm{h.c.} \qquad g_k \equiv \frac{2\pi a_{ab}}{\mu}\sqrt{\rho_b} S_k^{1/2}$$

$$A^\dagger_{\mathbf{k}} \propto \sum_{\underbrace{n,n'}_{\text{level}},\underbrace{i,i'}_{\text{site}}} \mathcal{A}^{(n,n')}_{k_x;i,i'} a^\dagger_{n,i} a_{n',i'}$$

transition of a-atoms
$\phi_{n',i'} \to \phi_{n,i}$

$$\mathcal{A}^{(n,n')}_{k_x;i,i'} \equiv \int dx\, e^{ik_x x} \phi^*_{n,i}(x) \phi_{n',i'}(x)$$

g. st. $\phi_{g,i} \sim \sqrt{N}$ $\gg$ excit. st. $\phi_{e_n,0} \sim 1$

Keep $e_n \to g$ and neglect $e_2 \to e_1$.

Born-Markov approx.

$$\dot\rho(t) = -\int_0^\infty dt' \,\mathrm{Tr}_R \left[H_{ab}(t), [H_{ab}(t-t'), \rho(t) \otimes R] \right]$$

Master equations

$$\mathcal{L}\rho \simeq \frac{\gamma}{2}\sum_{n,n'=1,2} C_{n,n'}\left(e^{-i(\epsilon_n-\epsilon'_n)t}[F_n\rho, F^\dagger_{n'}] + \mathrm{h.c.}\right)$$

$$F_1 \equiv F_2^\dagger \equiv c_+^\dagger c_-$$

$$C_{11}=1,\ C_{22}=\eta\epsilon^2,\ C_{12}=\epsilon,\ C_{21}=\eta\epsilon$$

$$\eta(k_1,k_2)\equiv k_2\gamma_2/k_1\gamma_1 \qquad \gamma_n \propto k_n|g_{k_n}|^2/v(k_n)$$

- Ideal limit: $\omega = \epsilon_2 - \epsilon_1 \to 0$

$$k_2 \to k_1 \quad \& \quad \gamma_2 \to \gamma_1 \quad \Rightarrow \quad \eta \to 1$$

$$\mathcal{L}\rho \simeq \frac{\gamma}{2}(2c\rho c^\dagger - c^\dagger c\rho - \rho c^\dagger c)$$

with $c = F_1 + \epsilon F_2$

Master eq. with squeezing jump op.!

$$\mathcal{L}\rho \simeq \frac{\gamma}{2} \sum_{n,n'=1,2} C_{n,n'} \left(e^{-i(\epsilon_n - \epsilon'_n)t} [F_n \rho, F^\dagger_{n'}] + \text{h.c.} \right)$$

$$F_1 \equiv F_2^\dagger \equiv c_+^\dagger c_-$$

$$C_{11} = 1,\ C_{22} = \eta\epsilon^2,\ C_{12} = \epsilon,\ C_{21} = \eta\epsilon$$

$$\eta(k_1, k_2) \equiv k_2\gamma_2/k_1\gamma_1 \qquad \gamma_n \propto k_n |g_{k_n}|^2 / v(k_n)$$

- Large *ω* limit: $\omega = \epsilon_2 - \epsilon_1 \to \infty$

  rapidly osc. factor $\exp[\pm i(\epsilon_2 - \epsilon_1)t] \to 0$

$$\mathcal{L}\rho \simeq \frac{\gamma}{2} \left[(2F_1\rho F_1^\dagger - \{F_1^\dagger F_1, \rho\}) + \eta\epsilon^2 (2F_2\rho F_2^\dagger - \{F_2^\dagger F_2, \rho\}) \right]$$

Two processes of F_1 & F_2 contribute incoherently.

Time scales

EOM of $\langle S^2{}_{y,x} \rangle$ by coherent st. approx.

$$\frac{d}{dt}\langle S^2_{y,z}\rangle \simeq -4N\gamma(1-\epsilon^2)\langle S^2_{y,z}\rangle + N^2\gamma(1\pm\epsilon)^2$$

- Time scale for squeezing: $$\tau_\gamma \equiv \frac{1}{4N(1-\epsilon^2)}$$

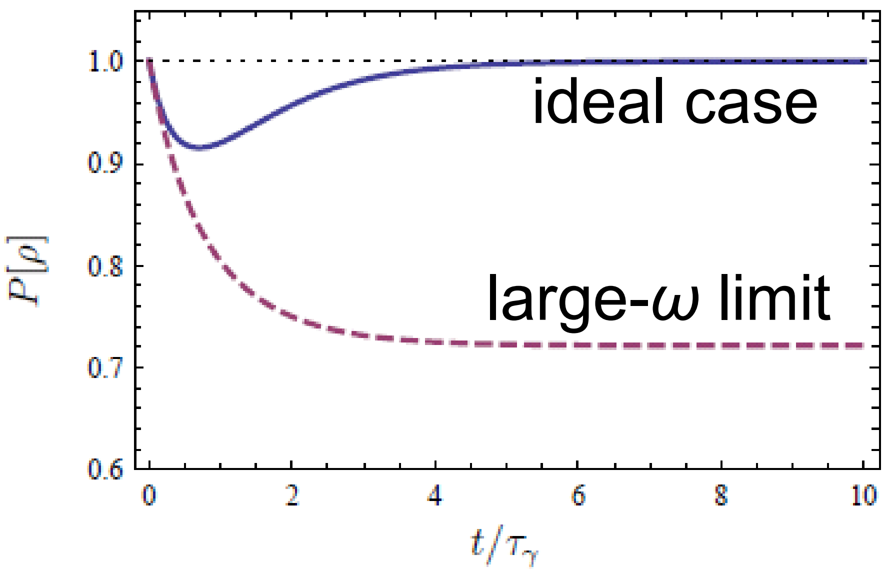
  Short time scale $\tau_\gamma \propto 1/N$ ⬅ Bose enhancement

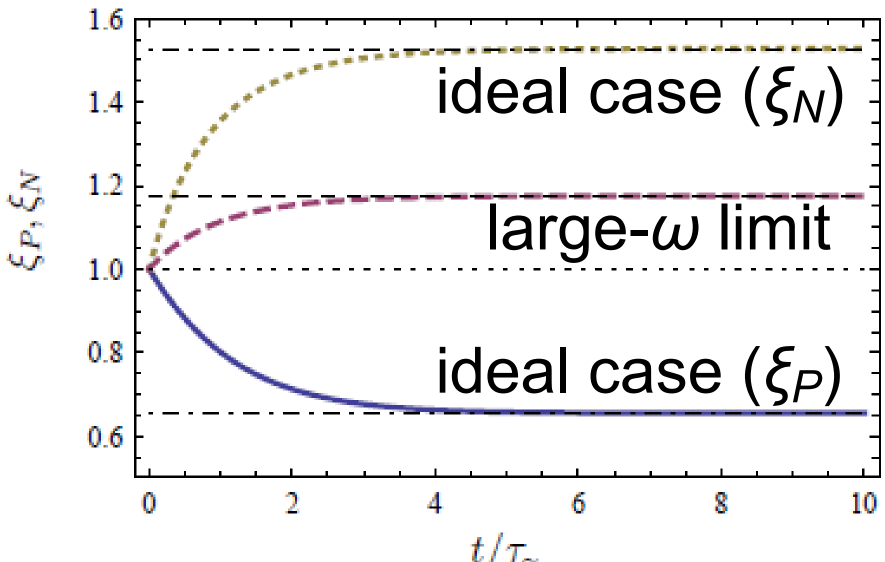
- Dephasing time scale: $T_\omega \equiv 2\pi/\omega = 2\pi/(\epsilon_2 - \epsilon_1)$

Squeezed st. is generated at $\tau_\gamma < t < T_\omega$

Time evolution for ideal & large-ω cases

N = 100, ε = -0.4, η = 1





Purity

$$P[\rho] \equiv \frac{1}{N}\left\{(N+1)\mathrm{Tr}[\rho^2]-1\right\}$$

pure st.: $P[\rho]$ = 1

max. mixed st.: $P[\rho]$ = 0

ideal case ➡ pure steady st.

Squeezing

Ideal case

$$\xi_P^{ss} \simeq \sqrt{\frac{1+\epsilon}{1-\epsilon}}\,, \quad \xi_N^{ss} \simeq \sqrt{\frac{1-\epsilon}{1+\epsilon}}$$

$\varepsilon < 0 \Rightarrow$ phase sq. ($\xi_P < 0$)

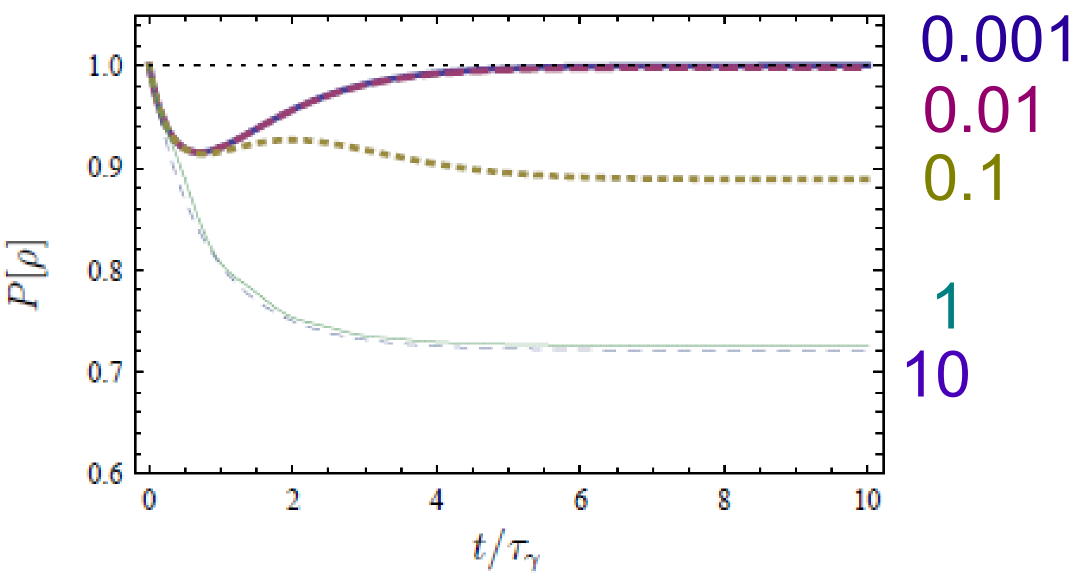
$\varepsilon > 0 \Rightarrow$ number sq. ($\xi_N < 0$)

Large-ω limit

$$\xi_P^{ss,\mathrm{inco}} = \xi_N^{ss,\mathrm{inco}} = \sqrt{\frac{1+\eta\epsilon^2}{1-\eta\epsilon^2}} > 1$$

Time evolution for intermediate cases

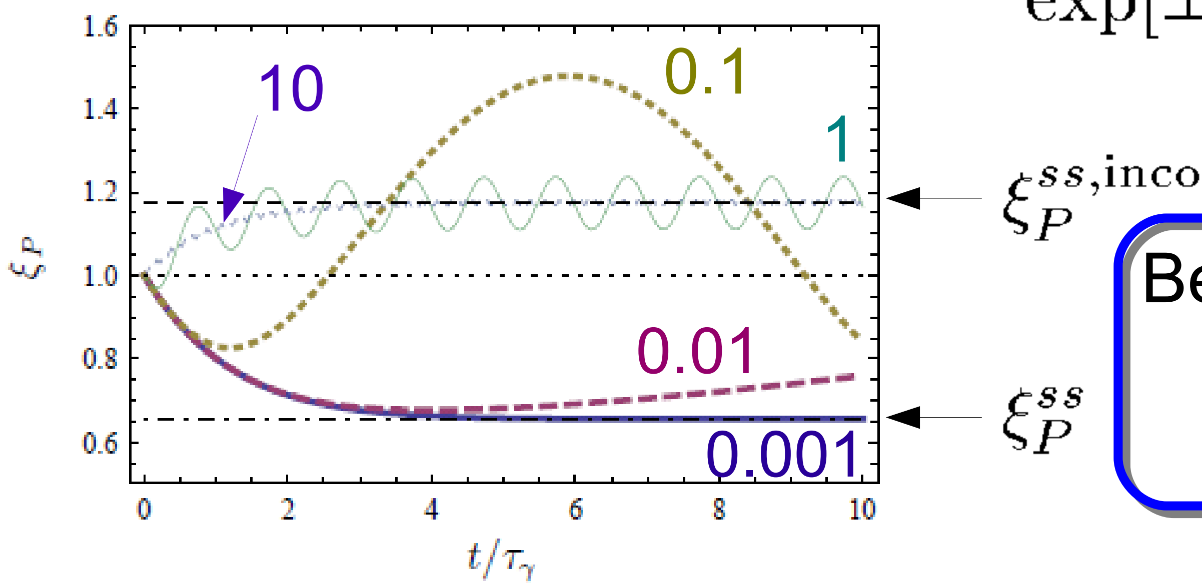
$N = 100$, $\varepsilon = -0.4$, $\eta = 1$ τ_γ / T_ω



Change from nearly ideal to large-ω regimes

$\tau_\gamma / T_\omega \uparrow$ ➡ purity ↓

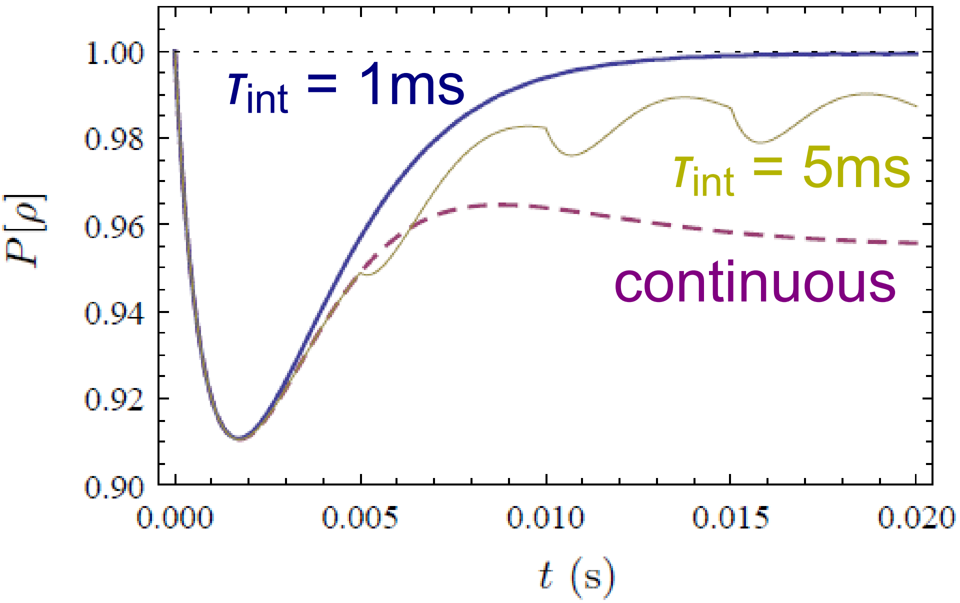
$\exp[\pm i\omega t]$ ➡ oscillation of ξ

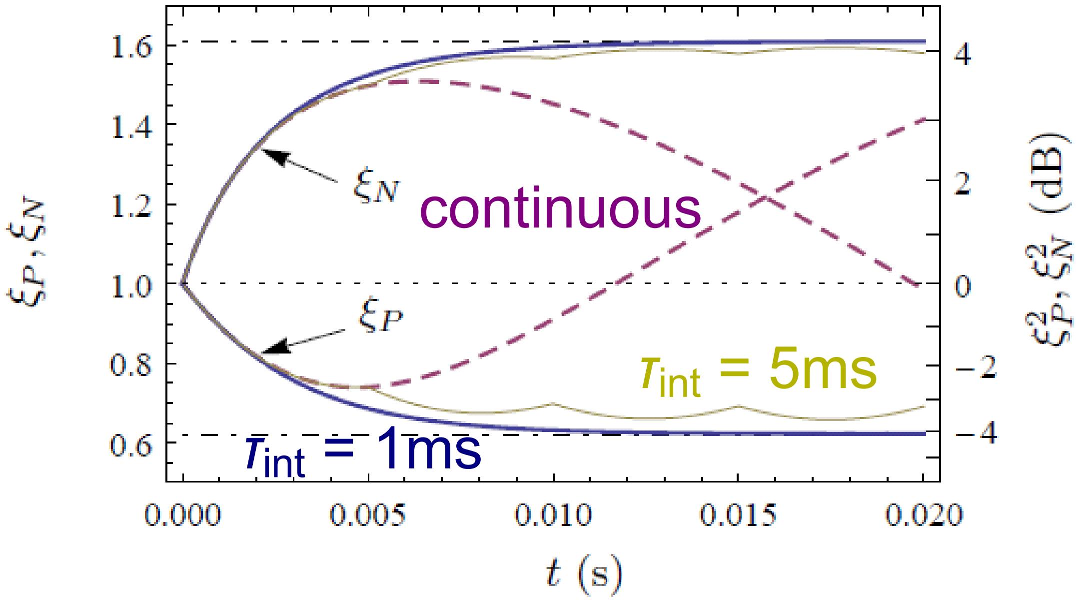


Behaviors of P & ξ resemble incoherent cases when $\tau_\gamma / T_\omega \gtrsim 1$

Suppression of dephasing

Parameters: $\epsilon = -0.40$ $T_\omega = 0.053$ s $\tau_\gamma / T_\omega = 0.049$





"Stroboscopic" method (cf. Polzik *et al*.)

Switching driving lasers off and on

continuous evolution ➡ stroboscopic elements with $\tau_{\rm int}$

Dephasing can be suppressed!

Summary & conclusion

GW & Mäkelä, PRA **85**, 023604 (2012)
Caballar, Diehl, Mäkelä, Oberthaler & GW, PRA **89**, 013620 (2014)

Dissipative preparation scheme of
phase- & number-squeezed st. using cold atom gases

- Proposal of the squeezing jump op. & its physical setup

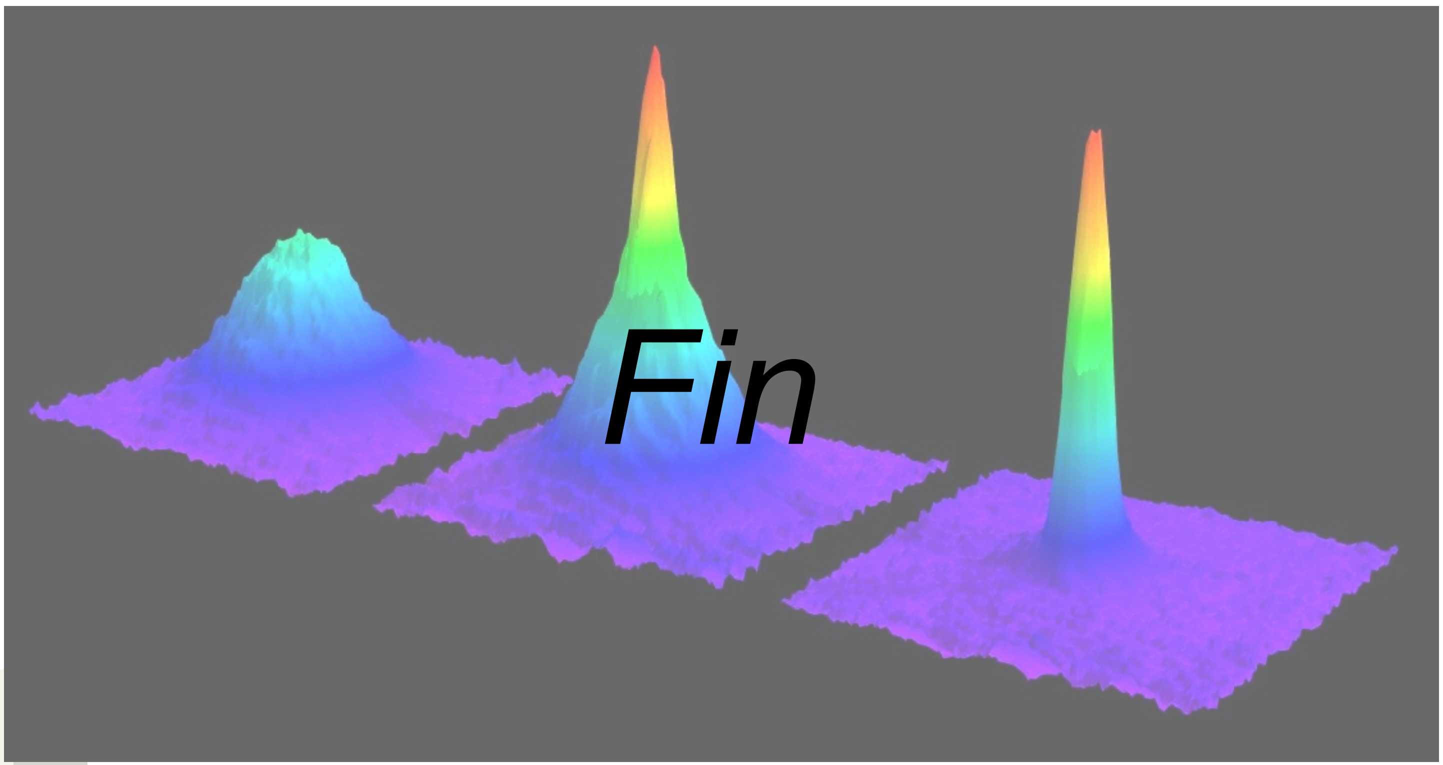
  Atoms in a double well immersed in a b.g. BEC acting as a reservoir.

- Master eq. is derived starting from microscopic physics.

- Squeezing develops on a rapid time scale $\propto 1/N$ due to the bosonic enhancement.

- Dephasing can be avoided by switching the driving on and off, which leads to robust steady squeezed st.

- Ingredients for experimental implementation using Rb atoms.



Fin

Thank you for your attention.

System-reservoir coupling: Atoms & Bogoliubov excit.

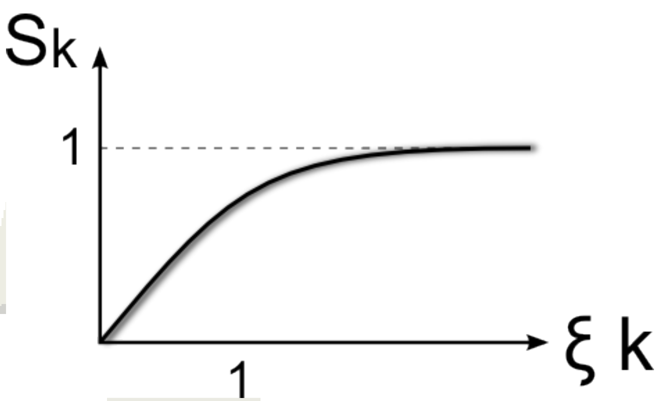
$$\hat{H}_{ab} = \frac{1}{2}\frac{4\pi a_{ab}}{\mu}\int d^3r\ \hat{\psi}_a^\dagger \hat{\psi}_a \hat{\psi}_b^\dagger \hat{\psi}_b \simeq \sum_{\mathbf{k}\neq 0} g_k (\hat{A}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{k}} + \mathrm{h.c})$$

$\hat{\psi}_a$: trapped atoms

$\hat{\psi}_b = \sqrt{\rho_b} + \delta\hat{\psi}_b(\mathbf{r})$: background s.f. atoms

$$\delta\hat{\psi}_b(\mathbf{r}) = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}}(u_{\mathbf{k}}\hat{b}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} + v_{\mathbf{k}}\hat{b}_{\mathbf{k}}^\dagger e^{-i\mathbf{k}\cdot\mathbf{r}})$$

$\hat{b}_{\mathbf{k}}$: Bogoliubov excitations



$$g_k \propto S_k^{1/2} = \sqrt{\frac{k^2}{2m_b E_{\mathbf{k}}}}$$: static structure factor of BEC

Master equation

$$\mathcal{L}\rho \simeq \frac{\gamma}{2} \sum_{n,n'=1,2} C_{n,n'} \left(e^{-i(\epsilon_n - \epsilon'_n)t} [F_n \rho, F^\dagger_{n'}] + \text{h.c.} \right)$$

$$F_1 \equiv F_2^\dagger \equiv c_+^\dagger c_- \qquad c_+ \equiv a_1 + a_2, \quad c_- \equiv a_1 - a_2$$

$$C_{11} = 1,\ C_{22} = \eta\epsilon^2,\ C_{12} = \epsilon,\ C_{21} = \eta\epsilon$$

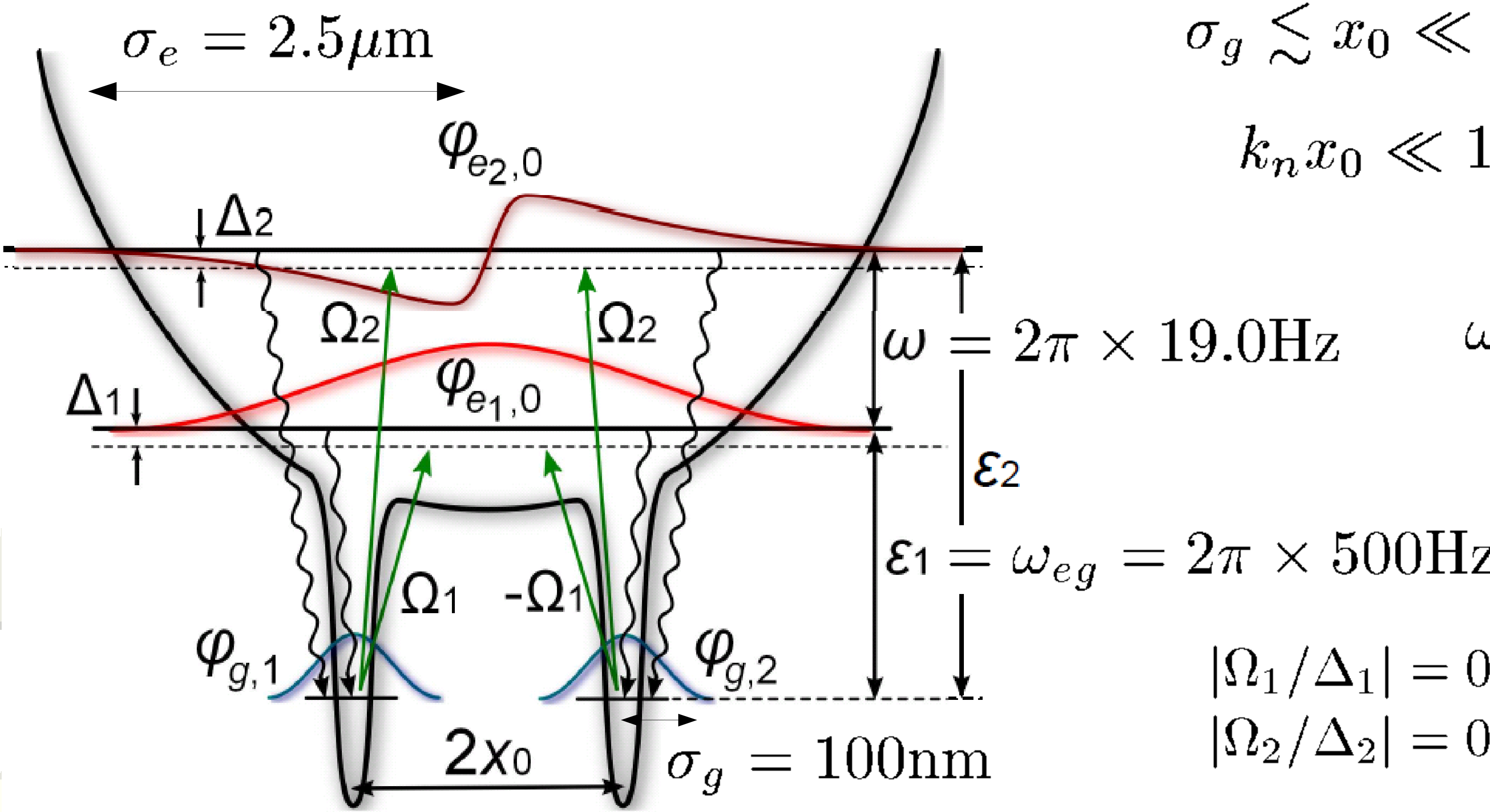
$$\eta \equiv k_2\gamma_2/k_1\gamma_1 \qquad \gamma_n \propto k_n |g_{k_n}|^2/v(k_n)$$

$$\gamma \propto k_1 \sigma_g \gamma_1 \phi_{e_1}^2(x_0)(\Omega_1/\Delta_1)^2$$

Towards implementation with cold atoms (1)

Trapped atoms (a-atoms): Rb85 (set a_{aa}=0), N=10^5

Background atoms (b-atoms): Rb87



$\omega_{\rm well} = 2\pi \times 11.9\mathrm{kHz}$

$x_0 = 350\mathrm{nm}$

$\sigma_g \lesssim x_0 \ll \sigma_e$

$k_n x_0 \ll 1$

$\omega \ll \omega_{eg}$

$|\Omega_1/\Delta_1| = 0.075 \ll 1$

$|\Omega_2/\Delta_2| = 0.15 \ll 1$

Towards implementation with cold atoms (1)

Trapped atoms (a-atoms): Rb85 (set a_{aa}=0), N=10^5

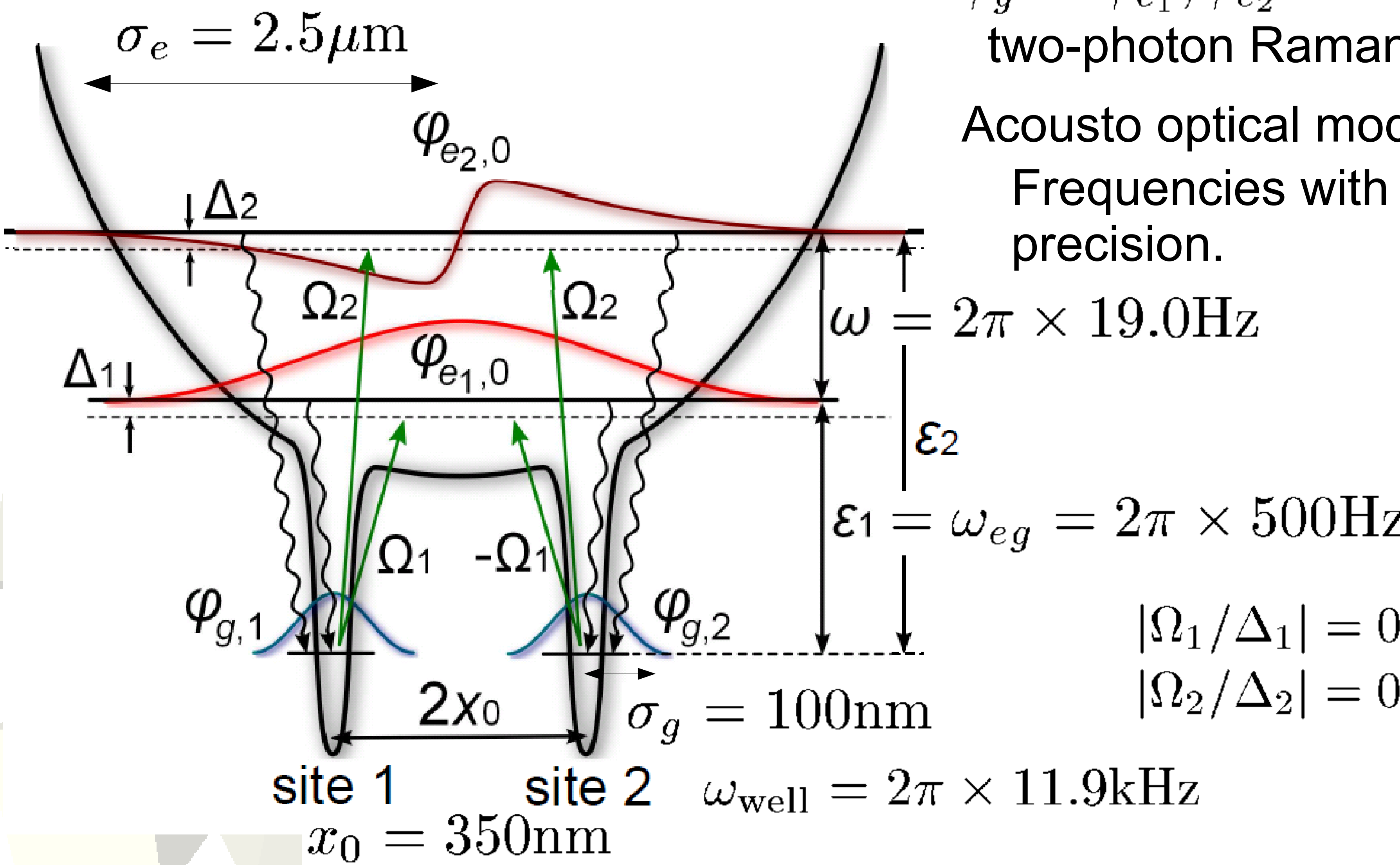
Background atoms (b-atoms): Rb87

$\phi_g \rightarrow \phi_{e_1}, \phi_{e_2}$

two-photon Raman coupling

Acousto optical modulators

Frequencies with > few Hz precision.

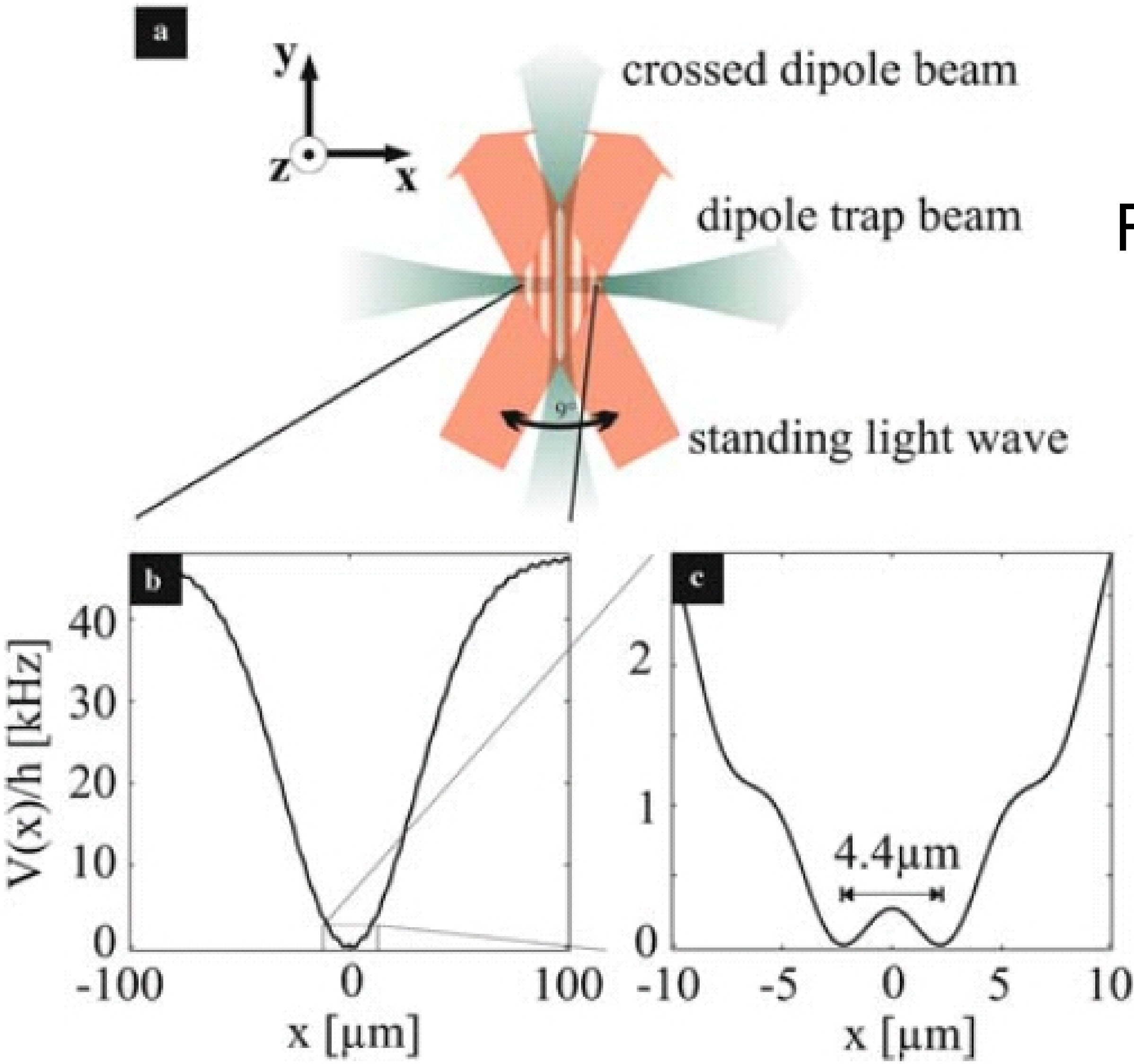


$$|\Omega_1/\Delta_1| = 0.075 \ll 1$$

$$|\Omega_2/\Delta_2| = 0.15 \ll 1$$

Realization of double-well potential

Gati *et al.*, Appl. Phys. B 82, 207 (2006)



g. st. extension (osc. length)

$$\sigma = \sqrt{\hbar/m\omega}$$

For Rb:

σ ~100 nm ➡ ω = 20-30 kHz

(feasible)

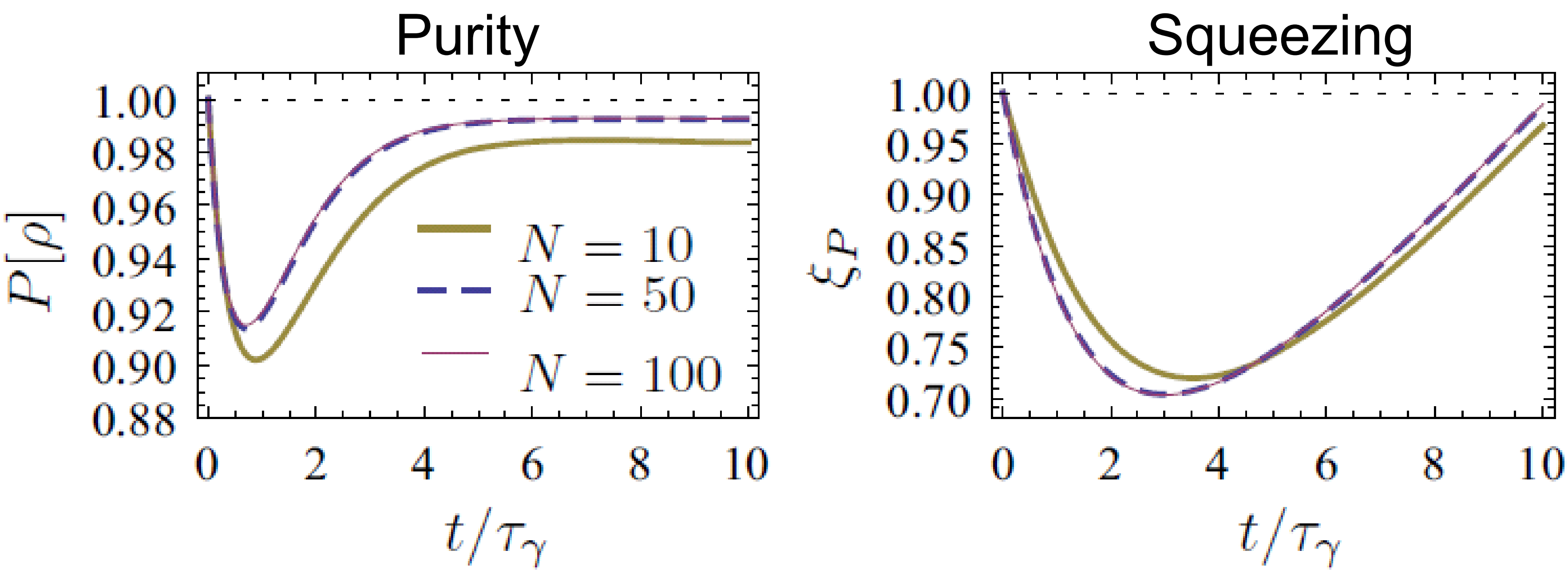
Difficult part:

interwell dist. $2\,x_0$ = 700 nm

interference patter by
2 beams of λ=1.4 μm,
but challenging.

N dependence for fixed τ_γ / T_ω

$$\tau_\gamma / T_\omega = 0.02, \quad \epsilon = -0.4, \quad \eta = 1$$



N dependence saturates already at *N*~100!

In numerical calculations, we can use smaller *N*~100 for the same value of τ_γ / T_ω .

(i.e., adjust γ to get the same τ_γ / T_ω)