Magnetic field induced ***quasi-helical liquid state*** in a ***disordered*** 1D electron system with ***spin-orbit interaction***

Anders Ström, Bernd Braunecker,
George I. Japaridze

University of Gothenburg, Sweden
Universidad Autónoma de Madrid, Spain
Ilia State University, Georgia

Phys. Rev. B 87, 075151 (2013)

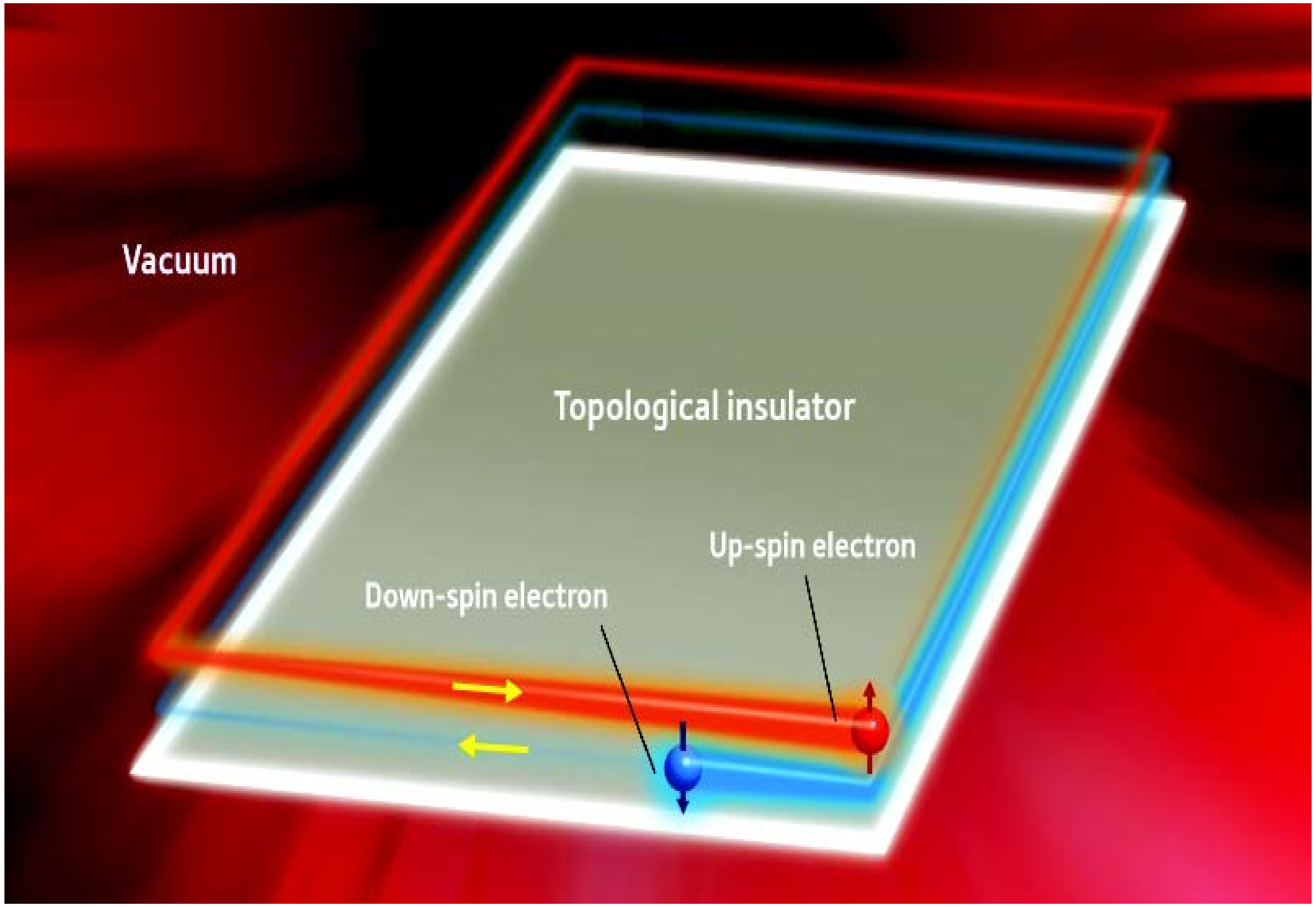
Workshop “Quantum Engineering of States and Devices“
Nordita 15.08.14

What is a helical liquid (HL)?

Unconventional State of matter with spin-momentum locking

Helical modes: where do they occur?

topological insulators (edges):	Pankratov, Pakhomov & Volkov, SSC 61, 93 (1987) Hasan and Kane, RMP 82, 3045 (2010)
semiconducting nanowires: in a magnetic field	Streda and Seba, PRL 90, 256601 (2003) Braunecker, Japaridze, Klinovaja, DL, PRB 82, 045127 (2010) Kloeffel, Trif, and DL, arXiv:1107.4870
electrons holes (SiGe)	Braunecker, Simon, DL, PRL 102, 116403 (2009)
carbon nanotubes:	Klinovaja, Schmidt, Braunecker, DL, PRL 106 156809 (2011), and PRB 84, 085452 (2011)

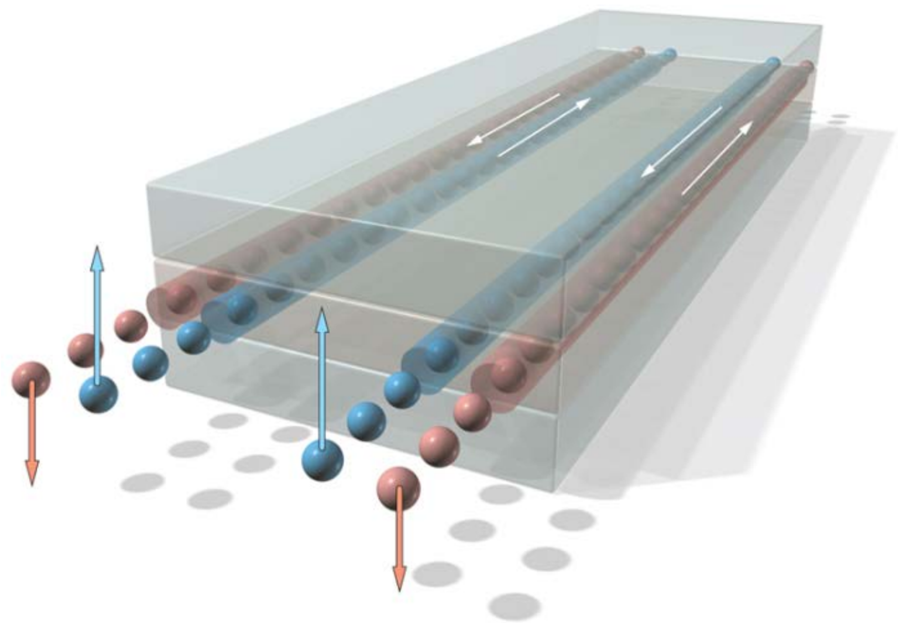


L. Fu, C. L. Kane and E. J. Mele PRL 98, 106803 (2007).
J. E. Moore and L. Balents, PRB 75, 121306(R) (2007).
S.-C. Zhang, Nature Physics 1, 6 (2008).

Helical conductors

Opposite spins move in opposite directions

Helical modes: where do they occur?



Picture: http://www.wissenschaft.de

Edges of 2D topological insulators

König & al.: J Phys Soc Jap. 77, 031007 (-08)
Hasan & Kane: Rev Mod Phys 82, 3045 (-10)

Quasi--1D superconductors

Potter and Lee, PRL 105, 227003 (2010)

Sato, Loss, Tserkovnyak PRL, 105, 226401 (-10)

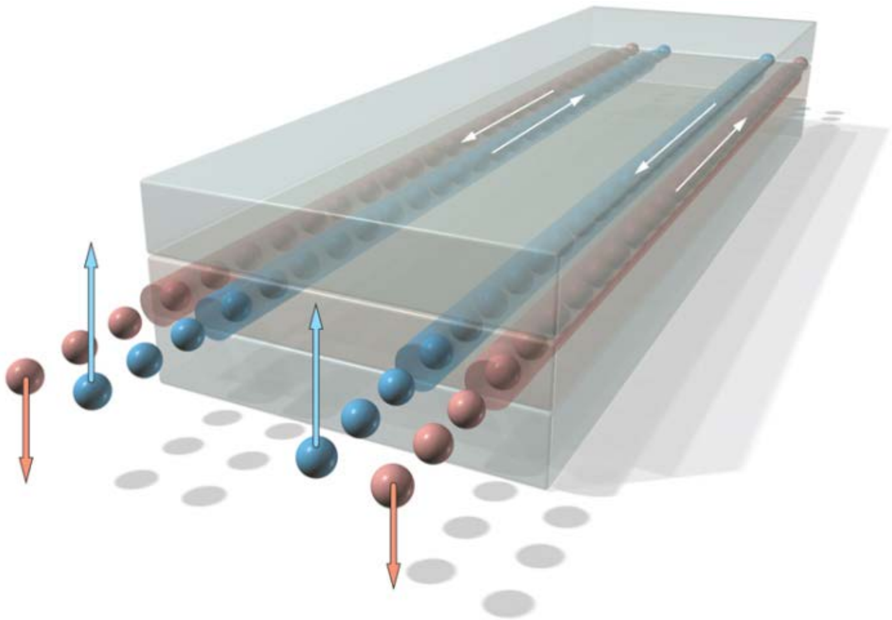
Carbon Nanotubes

Klinovaja et. al., PRL 106 156809 (2011)

Helical conductors

Opposite spins move in opposite directions

(Quasi)-Helical modes: where do they occur?



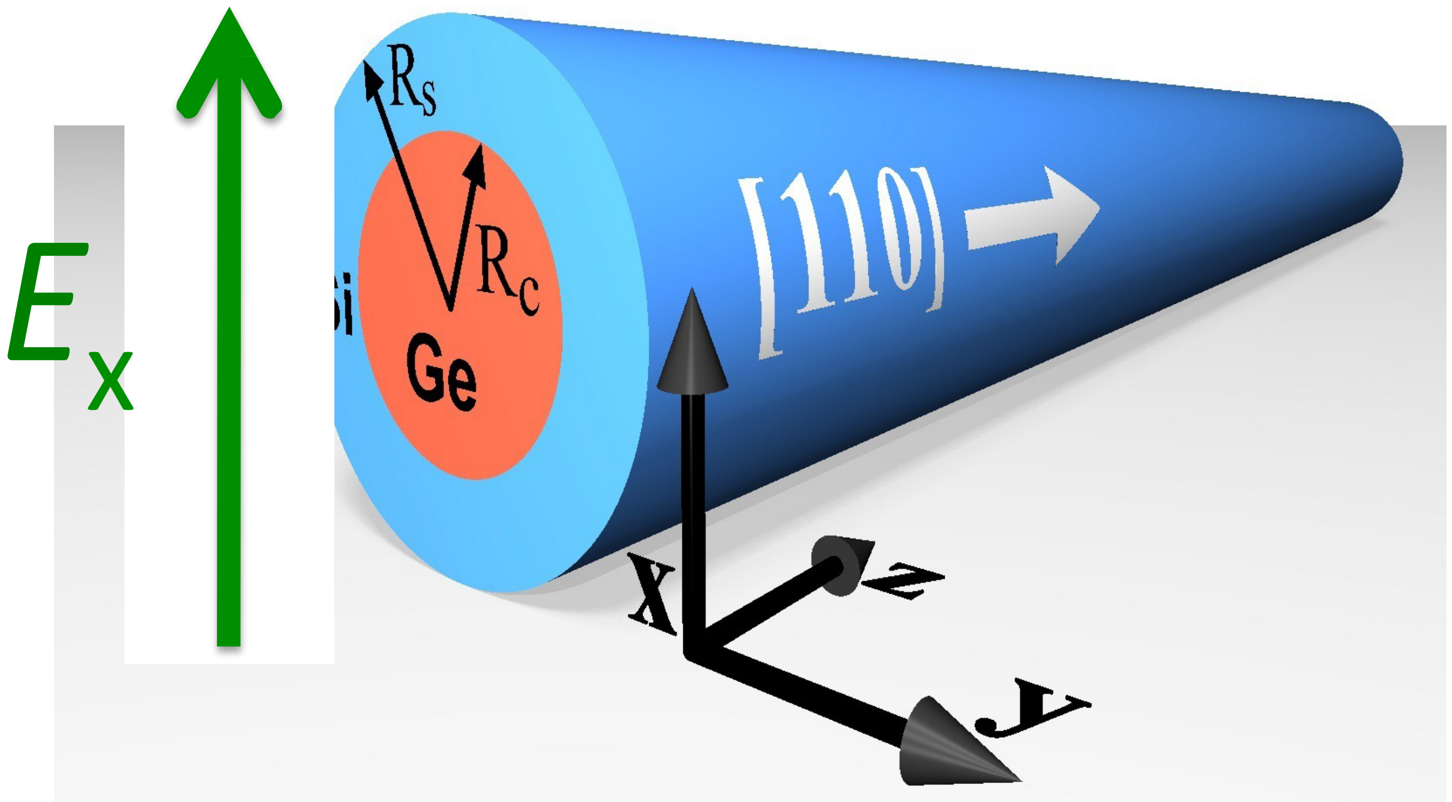
Picture: http://www.wissenschaft.de

1D semiconductor wires with strong SOI in external magnetic field

Streda & Seba: PRL **90**, 256601 (-03)
Braunecker, GIJ, Klinovaya and Loss: PRB **82**, 045127 (-10)
Braunecker, Bena, Simon PRB 85, 035136 (2012)
Quay et. al. Nature Phys. 6, 336, ('10)
Transport experiments

Ge/Si Core/Shell Nanowires

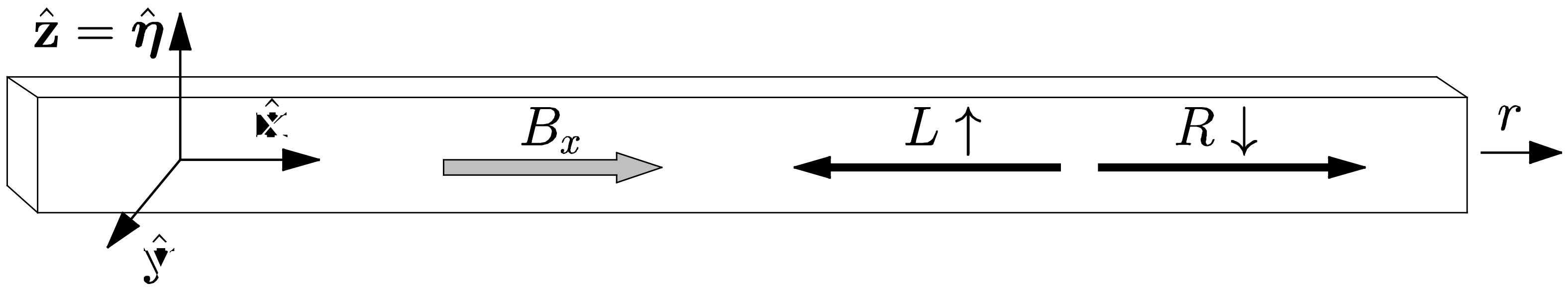
Xiang *et al.* Nature (2006); Hu *et al.*, Nat. Nano (2007); Hu *et al., preprint (2011)*

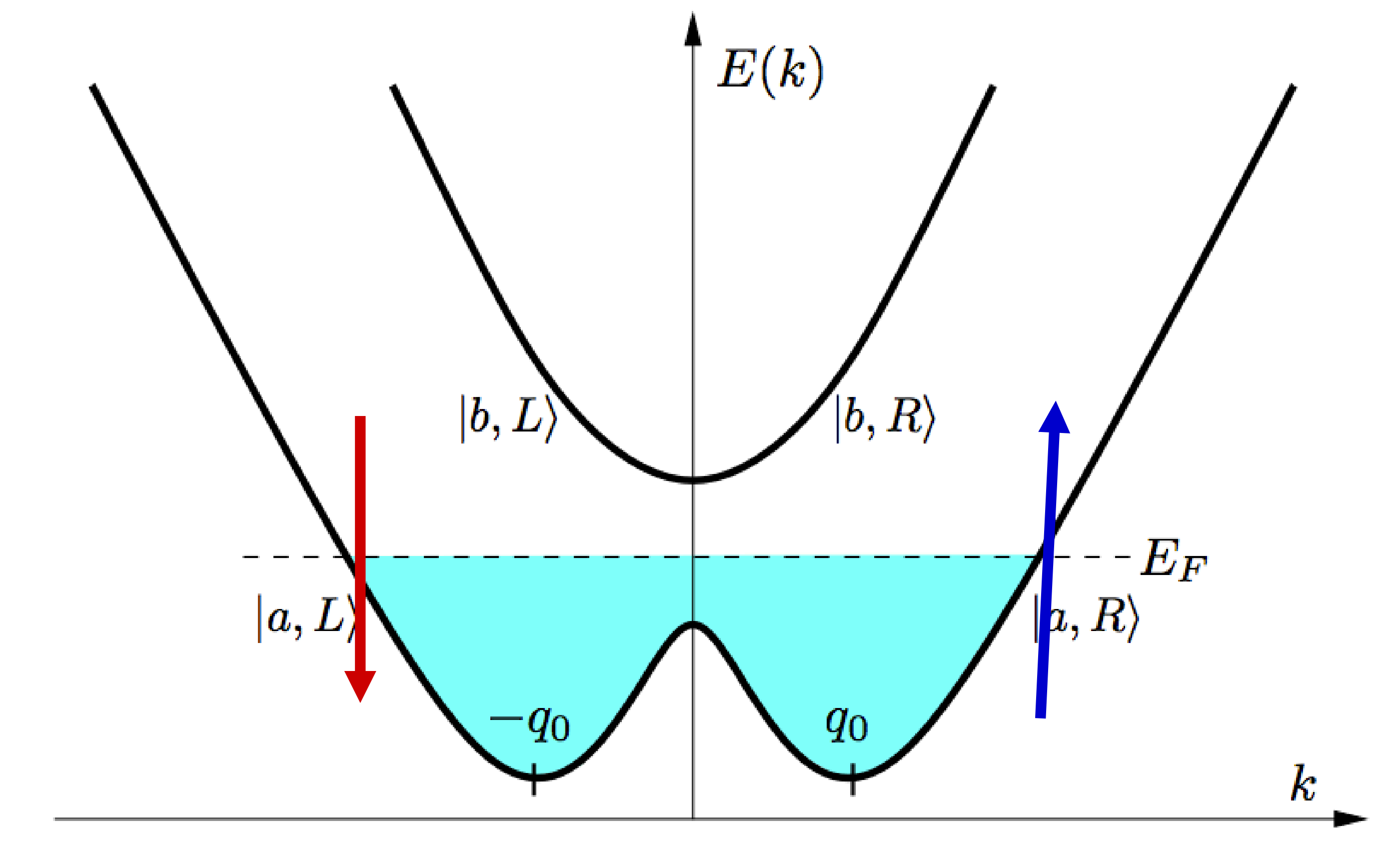


Nanowire grown along [110]

Large Ge/Si valence band offset of ~ 0.5 eV, narrow interfaces
→ **replace with hard wall at core radius $R_c \equiv R$**

Lauhon *et al.*, Nature (2002), Lu *et al.*, PNAS USA (2005)





Start with spinful LL

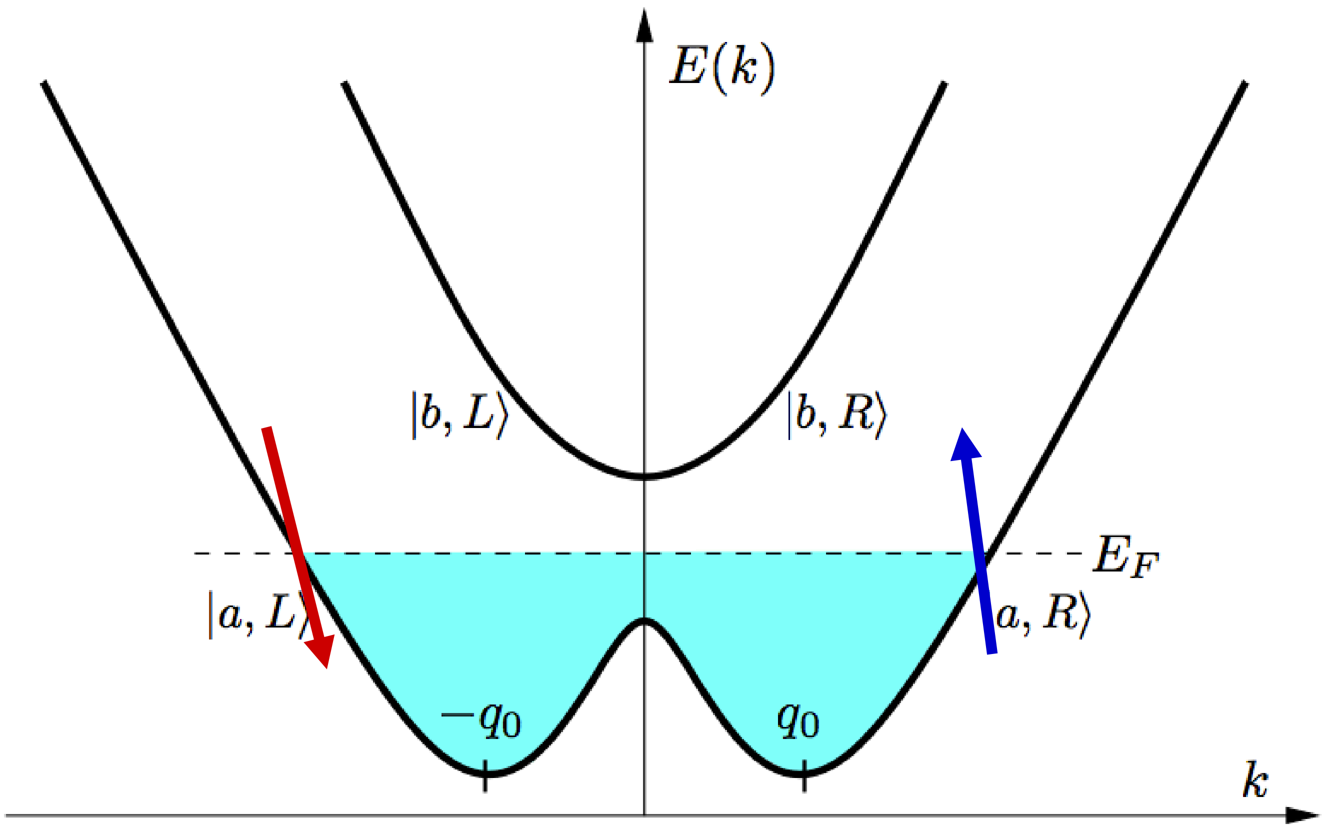
Add Rashba SOI

Add perpendicular magnetic field H_x

Gap needs to be larger than thermal energy:

$$1\text{K} \sim 0.086 \text{ meV}, 1\text{T} \sim 0.3 \text{ meV}$$

Band structures



Spins look anti-parallell for opposite velocities. Helical?

Not quite, the magnetic field H_x will twist them somewhat!

Let’s calculate the spin overlaps!

Band structures

Spin Overlaps

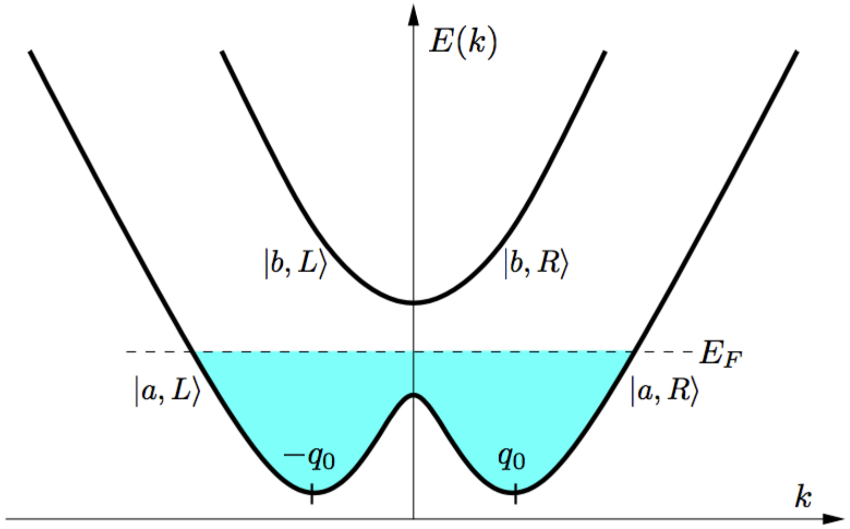
$$|aL\rangle = -\beta(k)\,|\uparrow\rangle + \alpha(k)\,|\downarrow\rangle$$
$$|aR\rangle = -\alpha(k)\,|\uparrow\rangle + \beta(k)\,|\downarrow\rangle$$
$$|bL\rangle = \alpha(k)\,|\uparrow\rangle + \beta(k)\,|\downarrow\rangle$$
$$|bR\rangle = \beta(k)\,|\uparrow\rangle + \alpha(k)\,|\downarrow\rangle\,,$$

Limits:

$$H_x = 0 \Rightarrow \alpha(k) = 1 \ \&\ \beta(k) = 0$$
$$\eta_R = 0 \Rightarrow \alpha(k) = \beta(k) = 1/\sqrt{2}$$

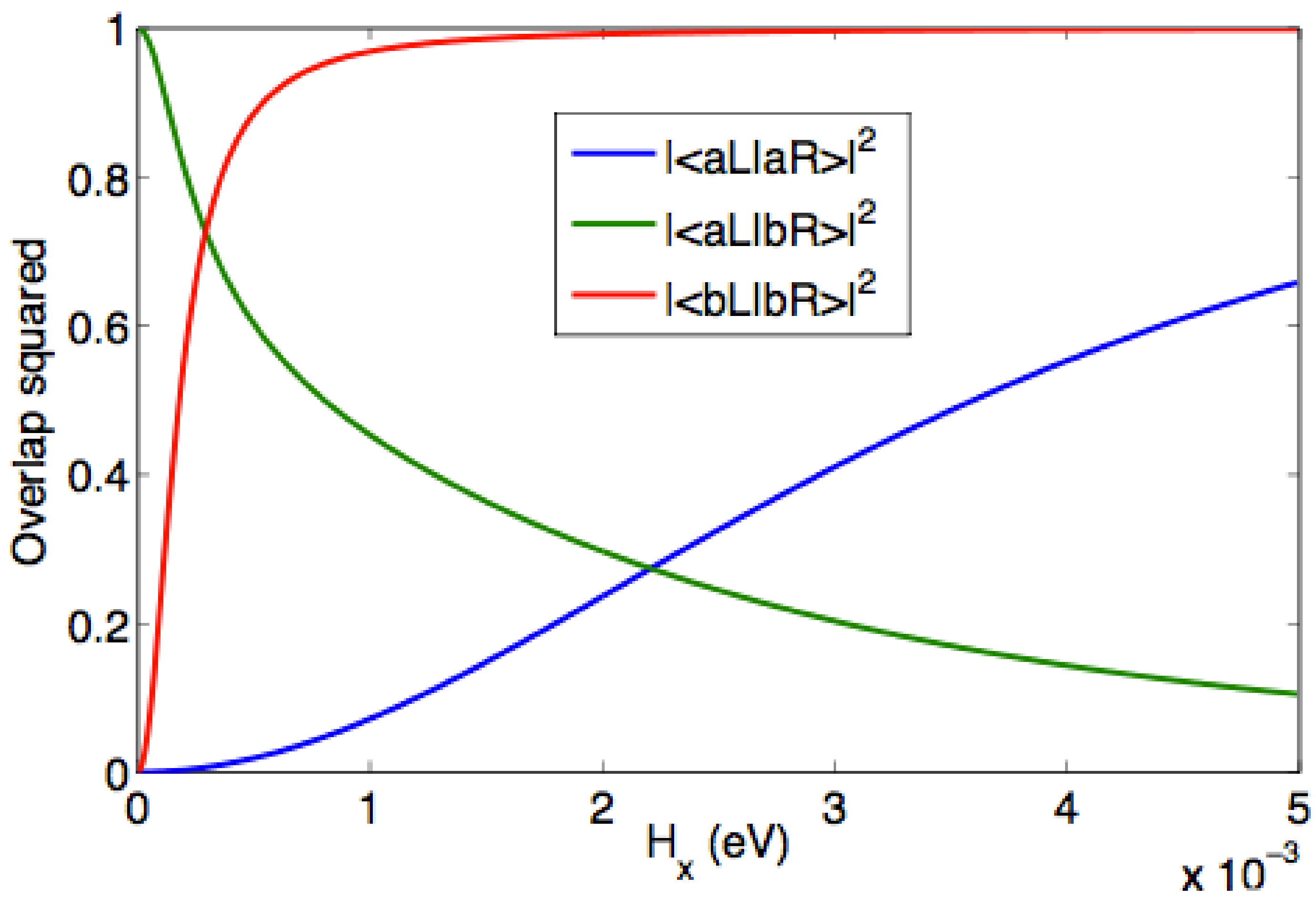
$$\alpha(k) = \sqrt{\frac{\sqrt{H_x^2 + (2\eta_R \sin k)^2} + 2\eta_R \sin|k|}{2\sqrt{H_x^2 + (2\eta_R \sin k)^2}}}$$

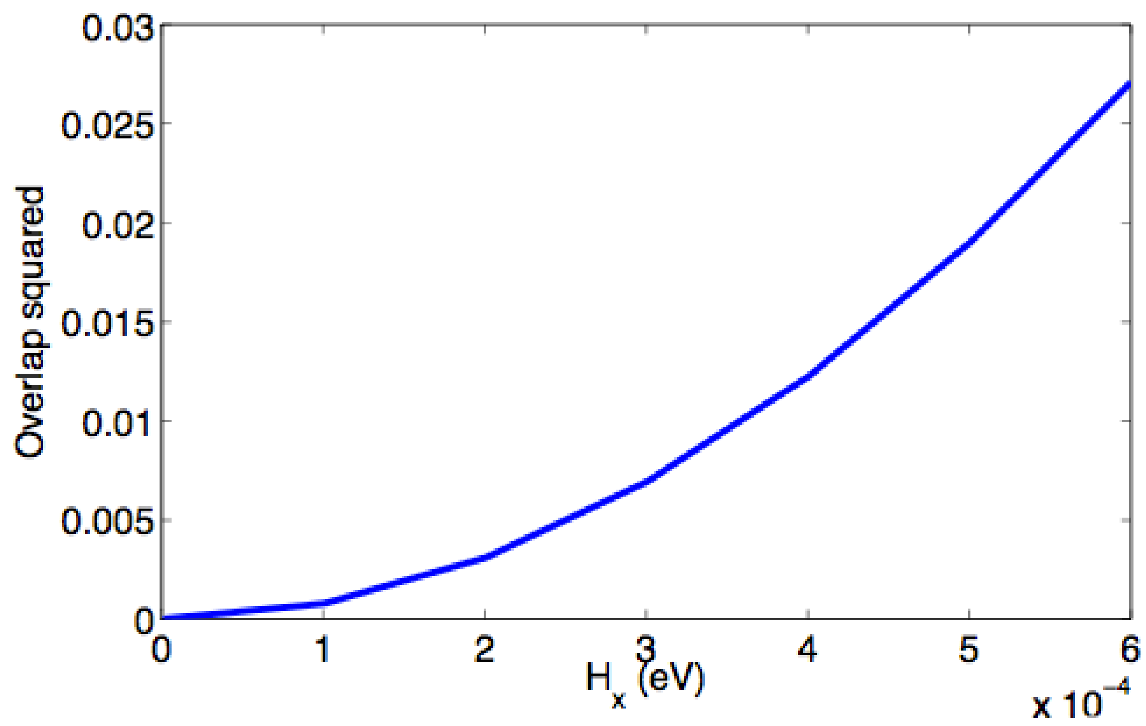
$$\beta(k) = \sqrt{\frac{\sqrt{H_x^2 + (2\eta_R \sin k)^2} - 2\eta_R \sin|k|}{2\sqrt{H_x^2 + (2\eta_R \sin k)^2}}}$$



Band structures

Spin Overlaps





Detail for aa overlap. $0.3\ \mathrm{meV}$ (ca $1\ \mathrm{T}$) yields overlap squared of about 0.01.

Disorder

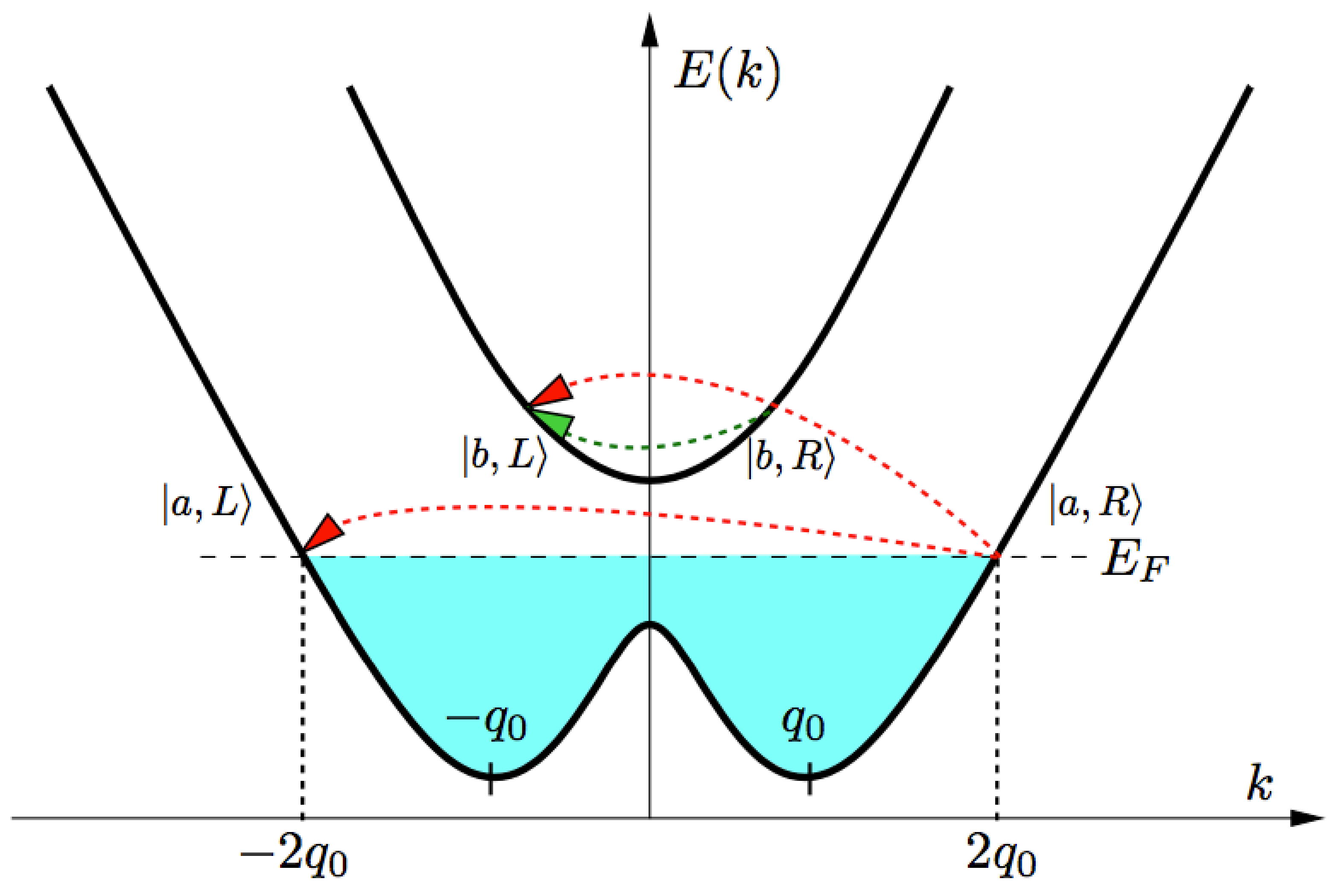
Backscattering against randomly distributed impurities, may cause **localisation.**

We deal with this by bosonising the theory and introducing the disorder strength D_{ij}
(see **Giamarchi's *Quantum Physics in One Dimension***)

D_{ij} for one backscattering channel:
$D_{ij} = n_{\mathrm{imp}} V_{ij}^2$ and $V_{ij} \propto |\langle iL|jR\rangle|^2$
$$\Rightarrow D_{ij} \propto |\langle iL|jR\rangle|^4$$

Disorder

Channels for backscattering off impurities:



Disorder Localization RG approach

Start with high-E cutoff

Integrate out everything

between Λ/b and Λ Low-E eff. theory

$\Rightarrow$

Vary b from 1 and up, see how eff. theory responds.

This is done on the bosonised theory.

Disorder is treated with the *replica method.*

$\rightarrow$ Scaling equations!

Disorder

RG

Scaling equations!

For full Luttinger liquid with spin:

$$\frac{\partial K_\rho}{\partial l} = -\frac{u_\rho}{2u_\sigma} K_\rho^2 \tilde{D}_{ij}(l)$$

$$\frac{\partial K_\sigma}{\partial l} = -\left(\frac{\tilde{D}_{ij}(l)}{2} + \frac{y_{ij}^2(l)}{2}\right) K_\sigma^2$$

$$\frac{\partial y}{\partial l} = (2 - 2K_\sigma)y - \tilde{D}_{ij}(l)$$

$$\frac{\partial \tilde{D}_{ij}}{\partial l} = (3 - K_\rho - K_\sigma - y)\tilde{D}_{ij}$$

$$\frac{\partial u_\rho}{\partial l} = -\frac{u_\rho^2}{2u_\sigma} K_\rho \tilde{D}_{ij}(l)$$

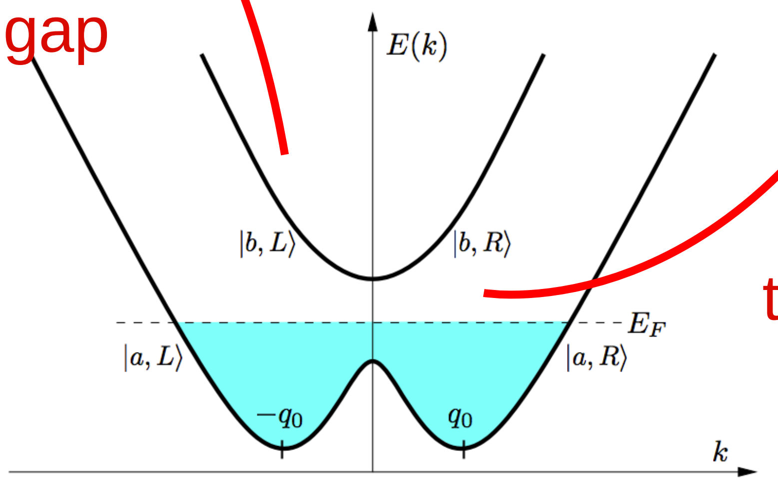
$$\frac{\partial u_\sigma}{\partial l} = -\frac{u_\sigma K_\sigma}{2} \tilde{D}_{ij}(l),$$

For spinless or helical Luttinger liquid:

$$\frac{\partial K}{\partial l} = -\frac{K^2}{2} \tilde{D}_{ij}(l)$$

$$\frac{\partial \tilde{D}_b}{\partial l} = (3 - 2K)\tilde{D}_{ij}$$

$$\frac{\partial u}{\partial l} = -\frac{uK}{2} \tilde{D}_{ij}(l)$$



RG

Method: **Two step RG.**

1. Start at high energies and renormalise the LL coupling constants:

$$K_\rho, K_\sigma, y, \tilde{D}_b, u_\rho \text{ and } u_\sigma.$$

2. When gap is reached, transform to helical coupling constants,

$$K, u \text{ and } \tilde{D}_b.$$

$$K = \sqrt{\frac{K_\rho(K_\rho K_\sigma u_\rho + u_\sigma)}{K_\sigma(K_\rho K_\sigma u_\sigma + u_\rho)}} \qquad u = \frac{1}{2}\sqrt{u_\rho^2 + u_\sigma^2 + u_\rho u_\sigma \left(K_\rho K_\sigma + \frac{1}{K_\rho K_\sigma}\right)}$$

Disorder

Localisation length

We need to know the length scale where
the system is an insulator - **localisation length**, ξ_{loc}.

Define $l = \ln b$

All lengths scale as $x \to x e^{-l}$

Define l^*: $\tilde{D}_\xi(l^*) \sim 1$

There rescaled loc. length is the size of lattice constant:

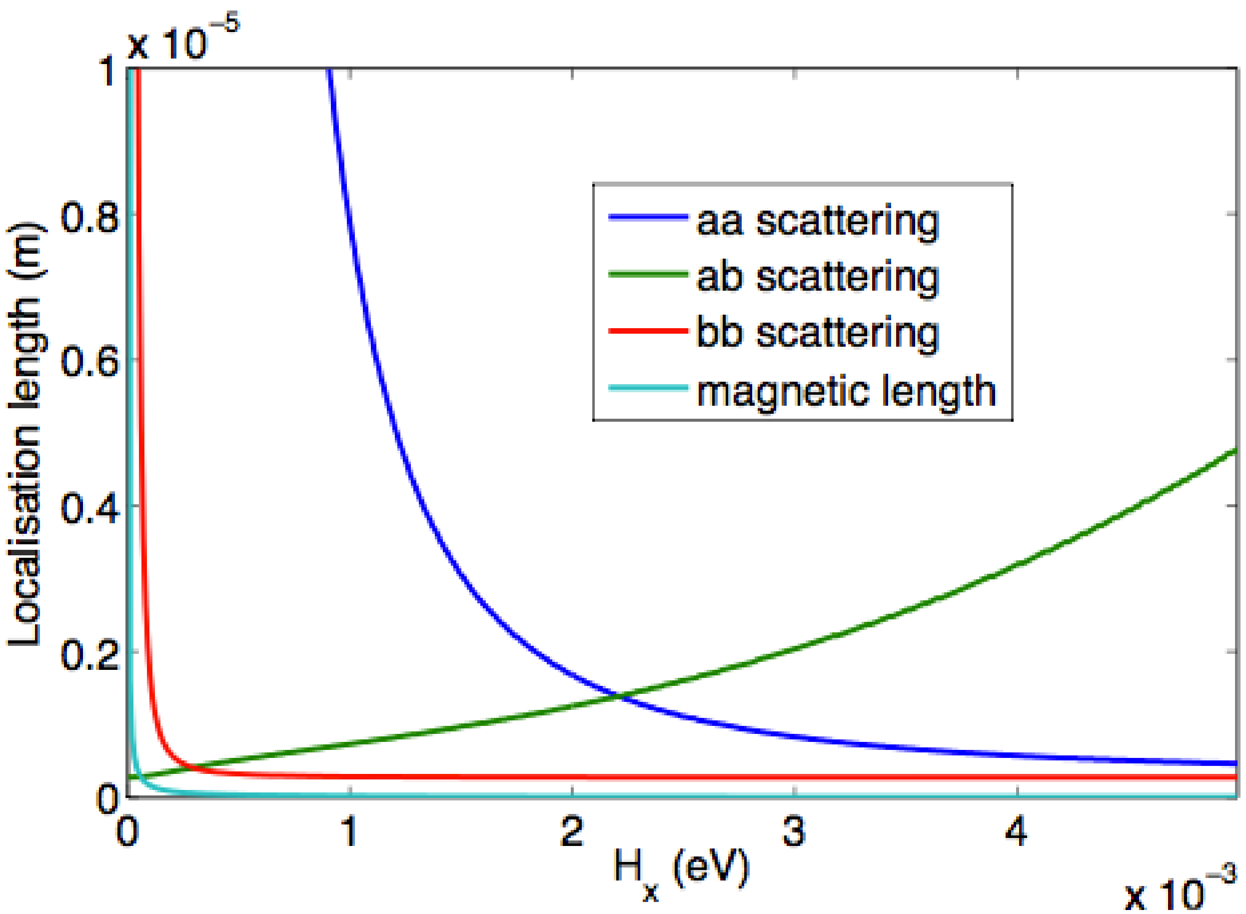
$\xi_{\rm loc.} e^{-l^*} = \kappa$

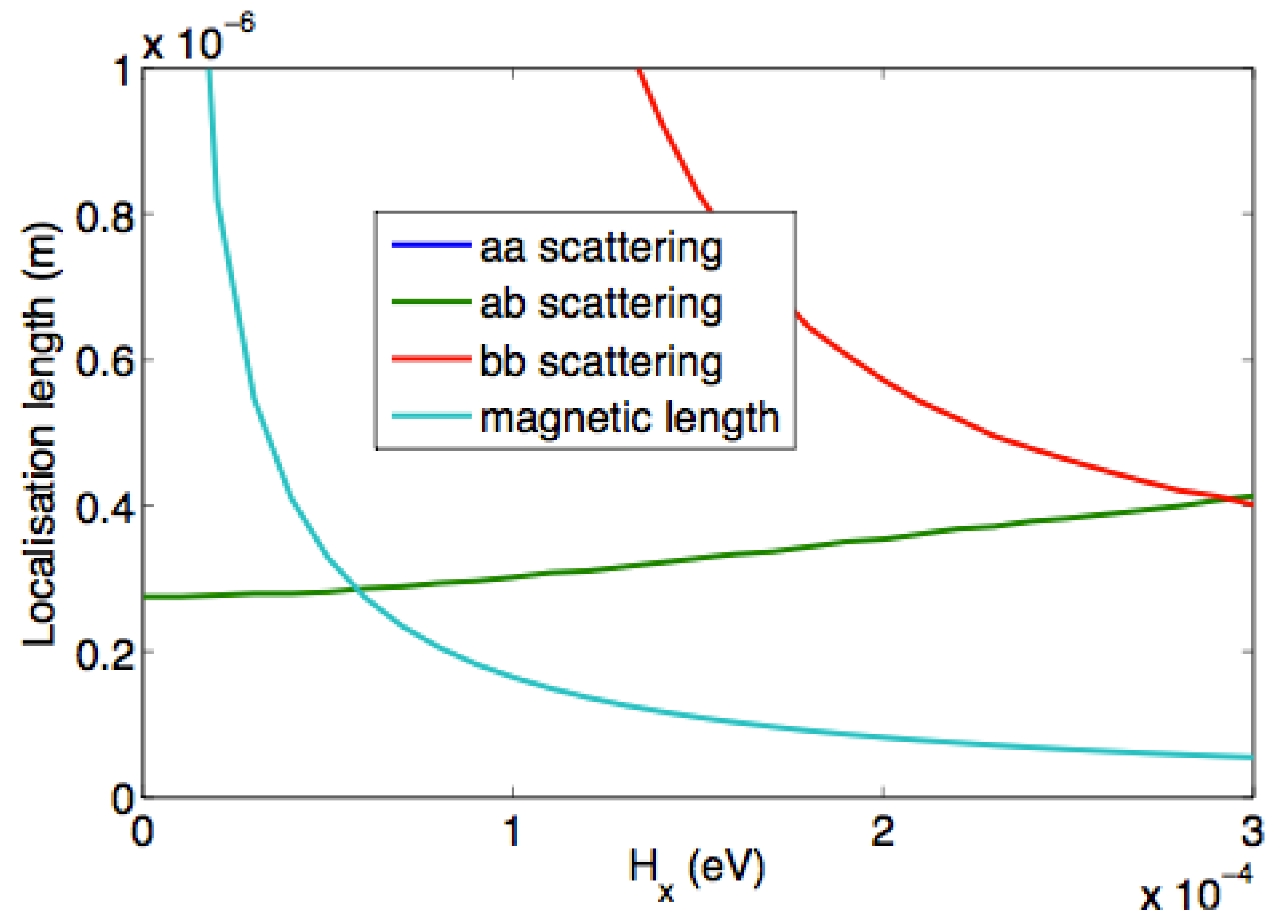
Thus: $\xi_{\rm loc.} = \kappa e^{l^*}$

Disorder

Localisation length

Can have loc. from ab disorder, if gap is smaller than ca 6×10^{-5} eV, almost 1 K (or 0.2 T.)



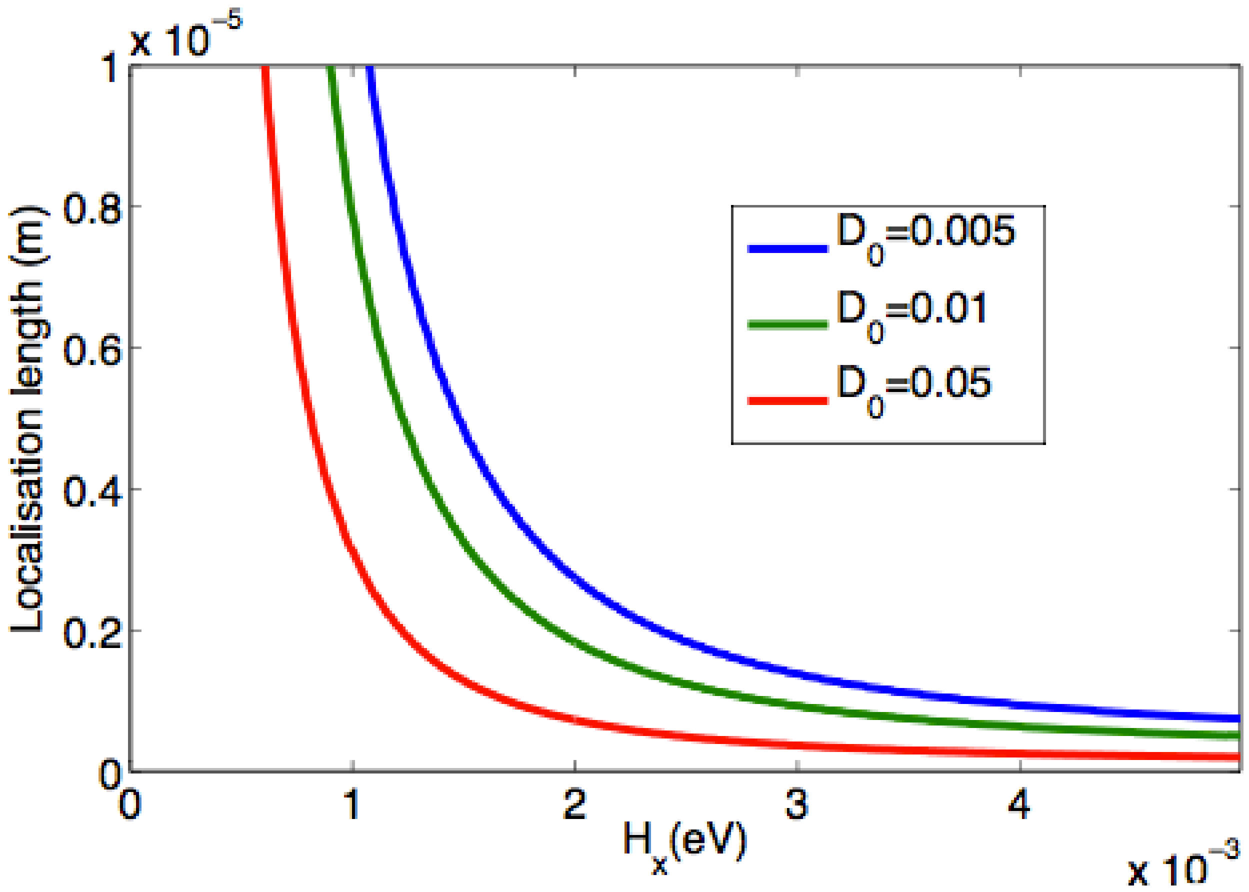


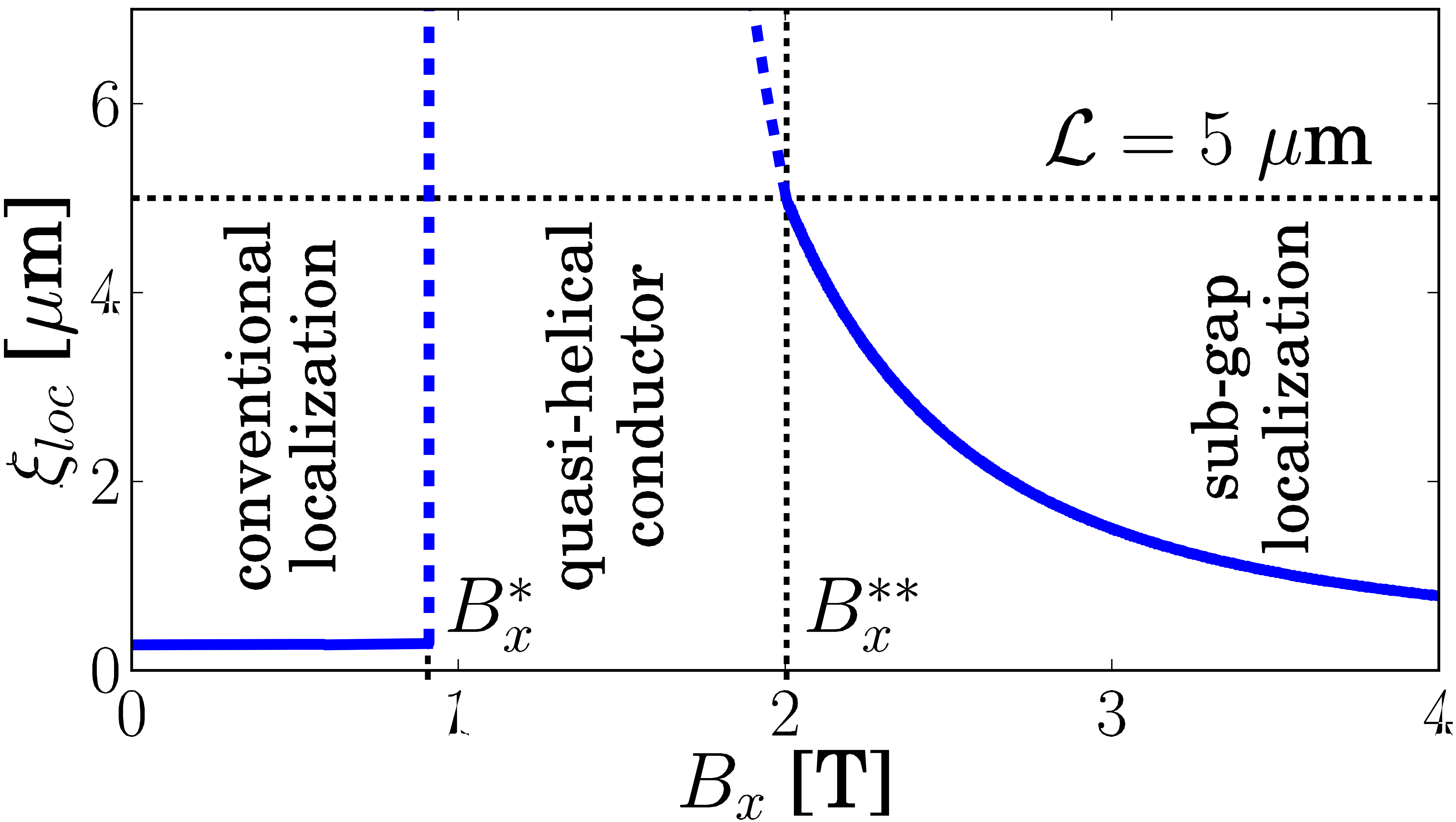
Below the gap, we only need to consider disordered aa backscattering.

Disorder

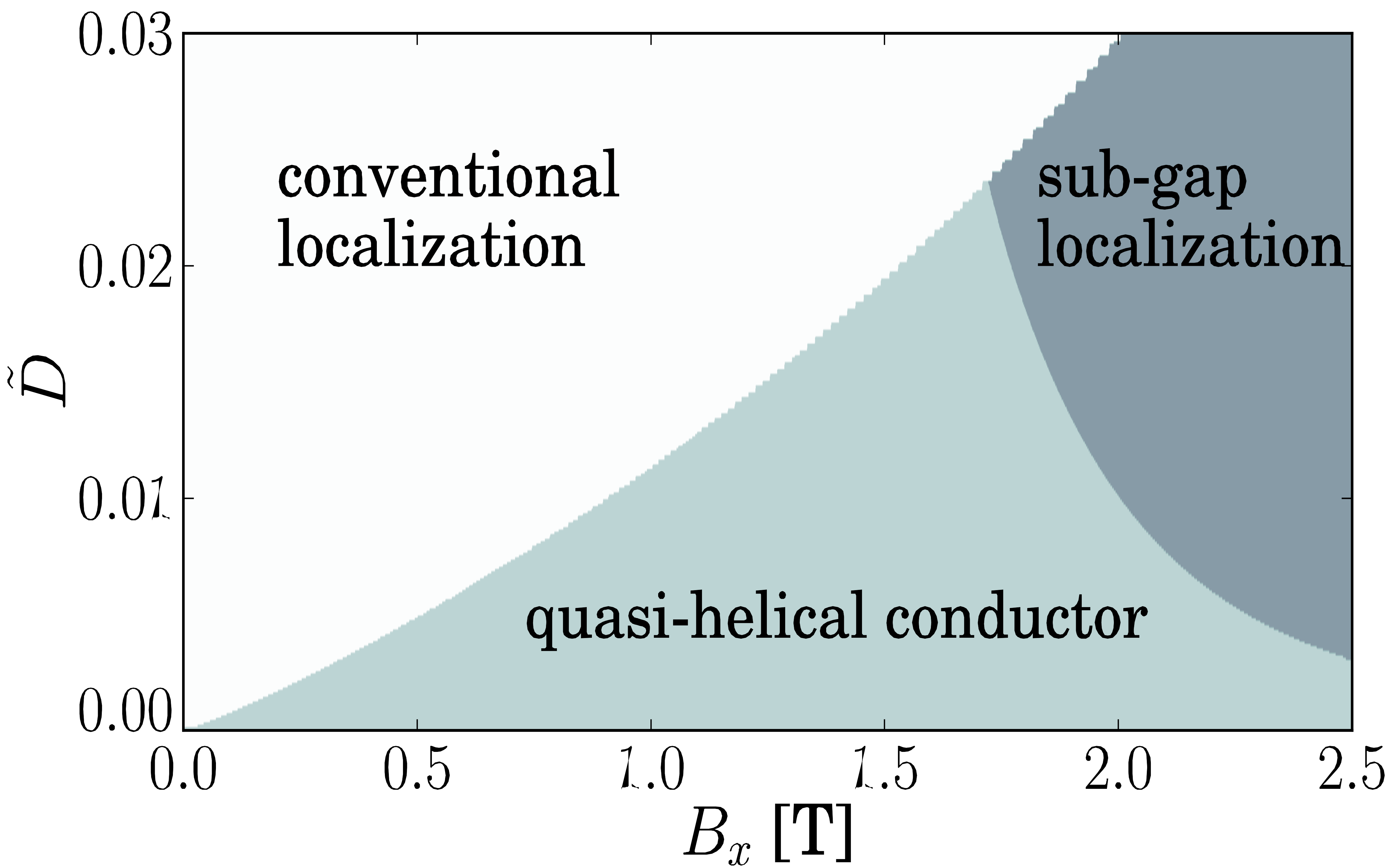
Localisation length

Now, in the magnetic gap,
only spinless (semi-helical) liquid.





$\mathcal{L} = 5\ \mu\mathbf{m}$
$\xi_{loc}\ [\mu\mathbf{m}]$
B_x [T]
conventional localization
quasi-helical conductor
sub-gap localization
B_x^*
B_x^{**}
0
2
4
6
0
1
2
3
4



0.03
0.02
0.01
0.00
0.0
0.5
1.0
1.5
2.0
2.5
$\tilde{D}$
B_x [T]
conventional localization
sub-gap localization
quasi-helical conductor

Summary

Calculated spin overlaps show that the liquid is ”quite helical” for ”moderate” magnetic fields.

($|\langle a|\, a\rangle|^2 = 0.01$) for $H_x = 1\mathrm{T}$.

Localisation can occur by two different disorder-induced scattering:

1. Interband (ab) backscattering off impurities, is relevant for $H_x < 0.06\ \mathrm{meV}$.

2. Intraband (aa) backscattering off impurities, relevant for $H_x \gtrsim 1\ \mathrm{meV}$.

Phys. Rev. B 87, 075151 (2013)

Thanks for support!

NORDITA

Shota
Rustaveli
National
Science
Foundation

STCU

SCIENCE & TECHNOLOGY
CENTER IN UKRAINE

Thank you.