Sponsored by:
NORDITA
Vetenskapsrådet
STINT
The Swedish Foundation for International Cooperation in Research and Higher Education

SYNTHETIC HELICAL LIQUID IN A QUANTUM WIRE

Mariana Malard
University of Brasilia
mmalard@unb.br

George I. Japaridze
Andronikashvili Institute of Physics

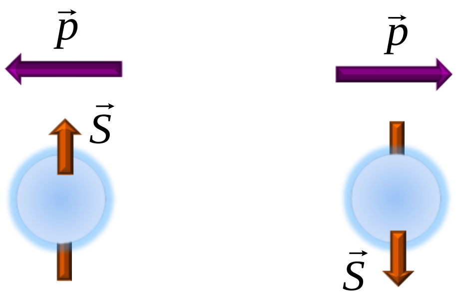
Henrik Johannesson
University of Gothenburg

Outline

- Helical liquid basics
- An alternative route for realization of a helical liquid
- Model
- Effective theory and results
- Estimates
- Conclusions

What is a helical liquid?

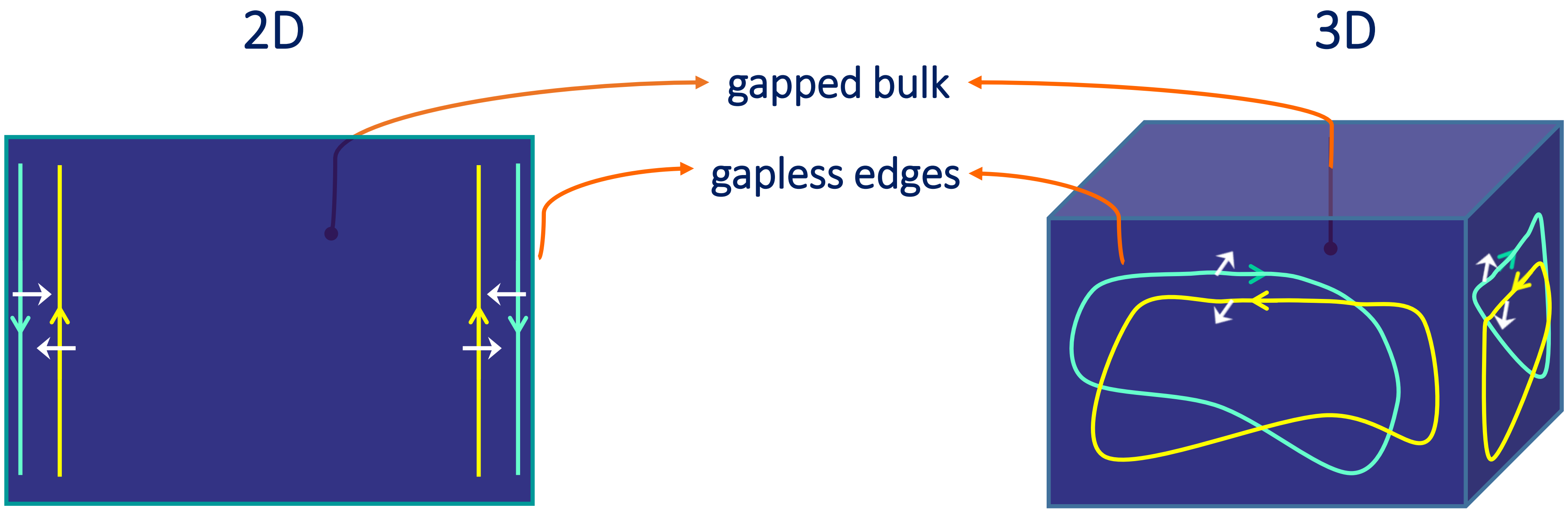
phase of matter with spin-momentum locking



time reversal symmetric

What is interesting about helical liquids?

- Edge states of topological insulators

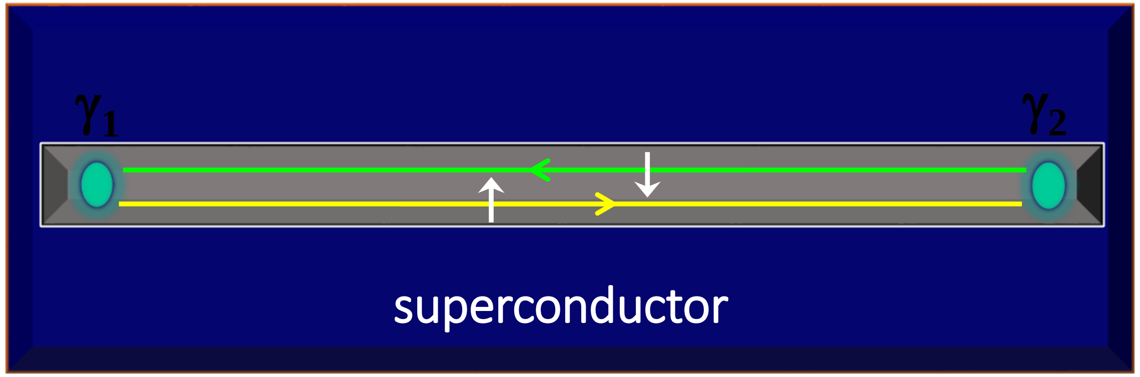


What is interesting about helical liquids?

- Ballistic conduction protected by time reversal symmetry

What is interesting about helical liquids?

- Can host Majorana bound states



Theoretical predictions of helical phases

PRL 95, 226801 (2005) PHYSICAL REVIEW LETTERS week ending 25 NOVEMBER 2005

Quantum Spin Hall Effect in Graphene

weak spin-orbit

C. L. Kane and E. J. Mele
Dept. of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA
(Received 29 November 2004; published 23 November 2005)

We study the effects of spin orbit interactions on the low energy electronic structure of a single plane of graphene. We find that in an experimentally accessible low temperature regime the symmetry allowed spin orbit potential converts graphene from an ideal two-dimensional semimetallic state to a quantum spin Hall insulator. This novel electronic state of matter is gapped in the bulk and supports the transport of spin and charge in gapless edge states that propagate at the sample boundaries. The edge states are nonchiral, but they are insensitive to disorder because their directionality is correlated with spin. The spin and charge conductances in these edge states are calculated and the effects of temperature, chemical potential, Rashba coupling, disorder, and symmetry breaking fields are discussed.

DOI: 10.1103/PhysRevLett.95.226801 PACS numbers: 73.43.−f, 72.25.Hg, 73.61.Wp, 85.75.−d

Theoretical predictions of helical phases

Quantum Spin Hall Effect and Topological Phase Transition in HgTe Quantum Wells

strong spin-orbit

B. Andrei Bernevig,[1,2] Taylor L. Hughes,[1] Shou-Cheng Zhang[1*]

We show that the quantum spin Hall (QSH) effect, a state of matter with topological properties distinct from those of conventional insulators, can be realized in mercury telluride–cadmium telluride semiconductor quantum wells. When the thickness of the quantum well is varied, the electronic state changes from a normal to an "inverted" type at a critical thickness d_c. We show that this transition is a topological quantum phase transition between a conventional insulating phase and a phase exhibiting the QSH effect with a single pair of helical edge states. We also discuss methods for experimental detection of the QSH effect.

SCIENCE VOL 314 15 DECEMBER 2006

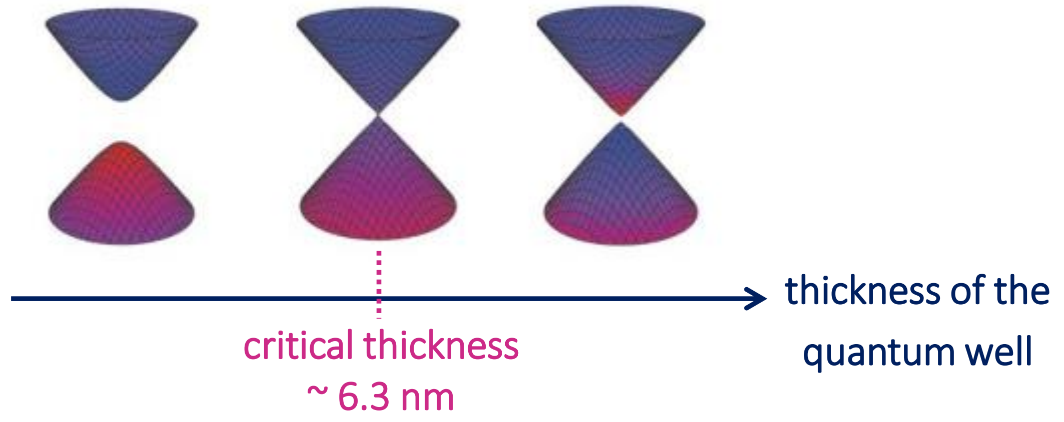
Theoretical predictions of helical phases

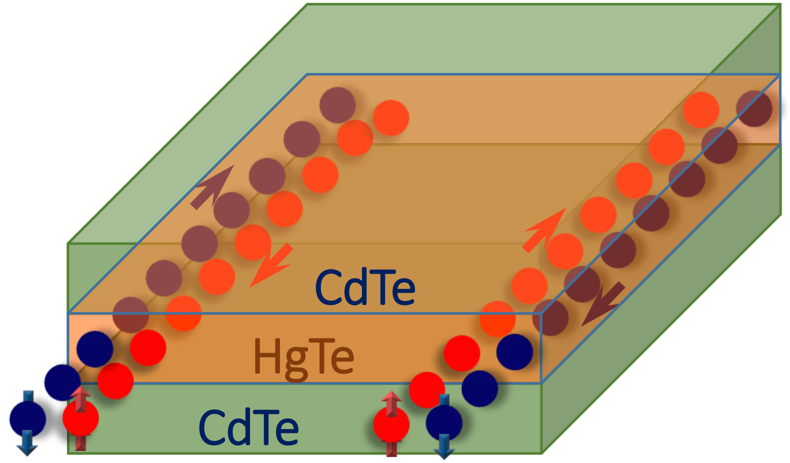
Quantum Spin Hall Effect and Topological Phase Transition in HgTe Quantum Wells

B. Andrei Bernevig,[1,2] Taylor L. Hughes,[1] Shou-Cheng Zhang[1*]

We show that the quantum spin Hall (QSH) effect, a state of matter with topological properties distinct from those of conventional insulators, can be realized in mercury telluride–cadmium telluride semiconductor quantum wells. When the thickness of the quantum well is varied, the electronic state changes from a normal to an "inverted" type at a critical thickness d_c. We show that this transition is a topological quantum phase transition between a conventional insulating phase and a phase exhibiting the QSH effect with a single pair of helical edge states. We also discuss methods for experimental detection of the QSH effect.

SCIENCE VOL 314 15 DECEMBER 2006



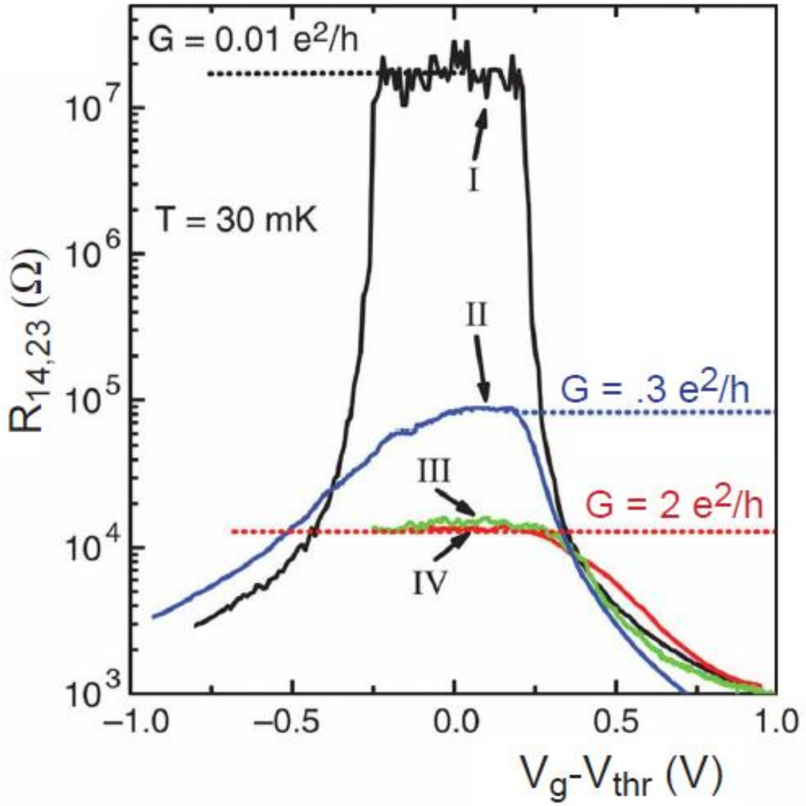


Experimental realizations of 1D helical liquids

HgTe quantum well

M. König, S. Wiedmann, C. Brüne, A. Roth, H. Buhmann, L. W. Molenkamp, X.-L. Qi, and S.-C. Zhang, Science **318**, 766 (2007).

A. Roth, C. Brune, H. Buhmann, L. W. Molenkamp, J. Maciejko, X.-L. Qi, and S.-C. Zhang, Science **325**, 294 (2009).



Experimental realizations of 1D helical liquids

More recently: InAs/GaSb quantum well

I. Knez, R.R. Du and G. Sullivan, Phys. Rev. Lett. **107**, 136603 (2011).

I. Knez, R.R. Du and G. Sullivan, Phys. Rev. Lett. **109**, 186603 (2012).

K. Suzuki, Y. Harada, K. Onomitsu and K. Muraki, Phys. Rev. B **87**, 235311 (2013).

Experimental realizations of 1D helical liquids

2D topological insulators

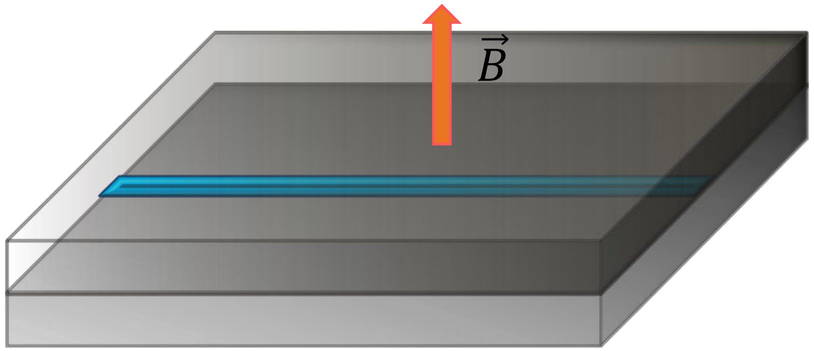
$\Downarrow$

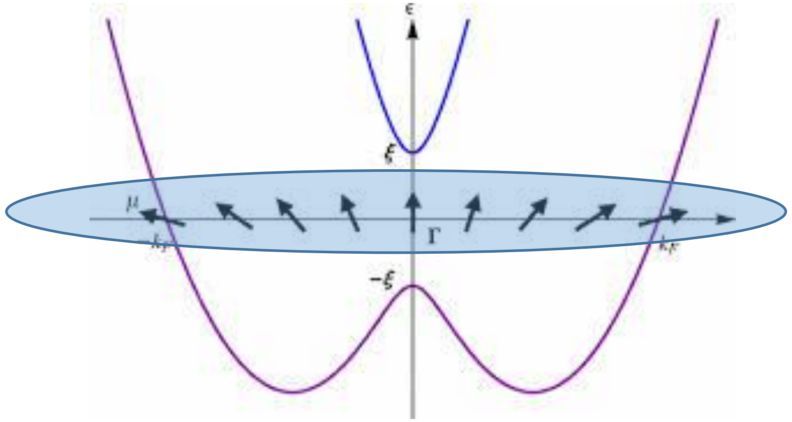
holographic 1D helical liquids

Is it possible to engineer an 1D helical liquid in an 1D system?

Experimental realizations of 1D helical liquids

quantum wire
in a semiconductor quantum well with strong spin-orbit
+
transverse magnetic field



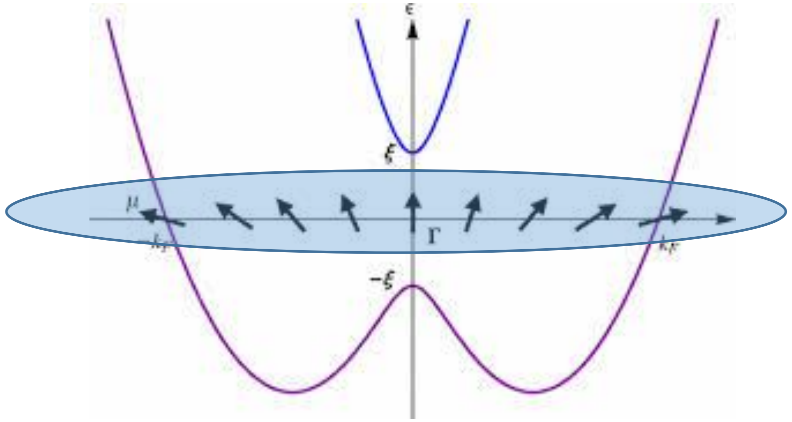


P. Středa and P. Šeba, Phys. Rev. Lett. **90**, 256601 (2003).

Experimental realizations of 1D helical liquids

quantum wire
in a semiconductor quantum well with strong spin-orbit
+
transverse magnetic field

- quasi-helical
- breaks time reversal symmetry
- Conduction will not be ballistic



P. Středa and P. Šeba, Phys. Rev. Lett. **90**, 256601 (2003).

Can one do better?

RAPID COMMUNICATIONS

PHYSICAL REVIEW B **89**, 201403(R) (2014)

Synthetic helical liquid in a quantum wire

George I. Japaridze,[1,2] Henrik Johannesson,[3] and Mariana Malard[4]

[1]*Andronikashvili Institute of Physics, Tamarashvili 6, 0177 Tbilisi, Georgia*

[2]*Ilia State University, Cholokasvili Avenue 3-5, 0162 Tbilisi, Georgia*

[3]*Department of Physics, University of Gothenburg, SE 412 96 Gothenburg, Sweden*

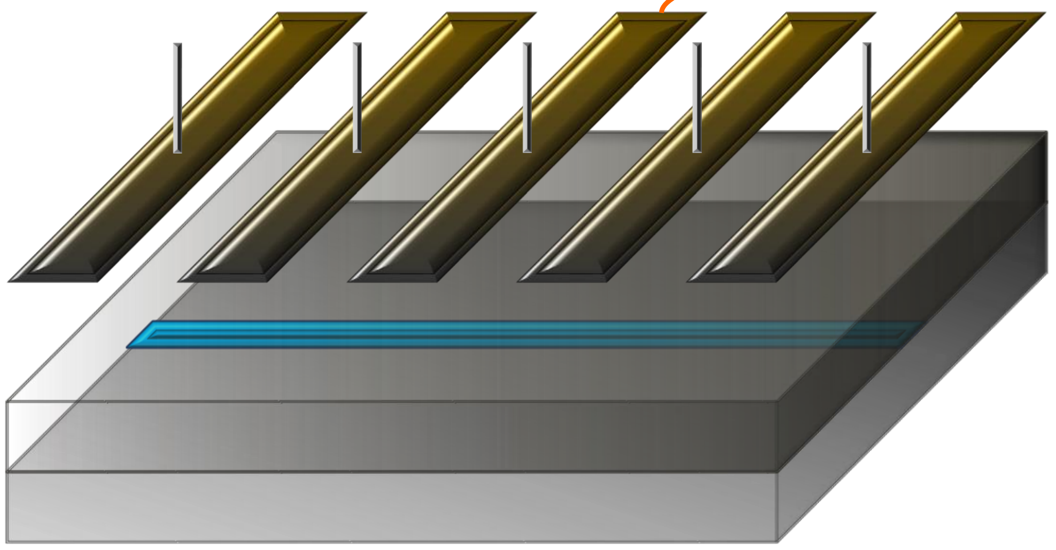
[4]*Faculdade UnB Planaltina, University of Brasilia, 73300-000 Planaltina-DF, Brazil*

(Received 14 November 2013; revised manuscript received 17 April 2014; published 15 May 2014)

We show that the combination of a Dresselhaus interaction and a spatially periodic Rashba interaction leads to the formation of a helical liquid in a quantum wire when the electron-electron interaction is weakly screened. The effect is sustained by a helicity-dependent effective band gap which depends on the size of the Dresselhaus and Rashba spin-orbit couplings. We propose a design for a semiconductor device in which the helical liquid can be realized and probed experimentally.

DOI: 10.1103/PhysRevB.89.201403 PACS number(s): 71.30.+h, 71.70.Ej, 85.35.Be

Idea: replace the magnetic field by electric fields!



modulated Rashba spin-orbit interaction

+ Dresselhaus spin-orbit interaction

+ electron-electron interactions

Advantages of an electric field over a magnetic field:

- Preserves time reversal symmetry and ballistic transport.
- Gives you a true-helical liquid, not a quasi-one.
- Can be easily generated and applied locally.

hopping and chemical potential

$$H_0 = -t \sum_{n,\alpha} c^\dagger_{n,\alpha} c_{n+1,\alpha} + \frac{\mu}{2} \sum_{n,\alpha} c^\dagger_{n,\alpha} c_{n,\alpha} + \text{h.c.}$$

+

uniform Rashba and Dresselhaus interactions

$$H_{\text{DR}} = -i \sum_{n,\alpha,\beta} c^\dagger_{n,\alpha} \left[\gamma_D\, \sigma^x_{\alpha\beta} + \gamma_R\, \sigma^y_{\alpha\beta}\right] c_{n+1,\beta} + \text{h.c.}$$

+

modulated Rashba interaction

$$H^{\text{mod}}_{\text{R}} = -i\gamma'_{\text{R}} \sum_{n,\alpha,\beta} \cos(Qna) c^\dagger_{n,\alpha} \sigma^y_{\alpha\beta} c_{n+1,\beta} + \text{h.c.}$$

+

modulated chemical potential

$$H^{\text{mod}}_{\text{cp}} = \frac{\mu'}{2} \sum_{n,\alpha} \cos(Qna) c^\dagger_{n,\alpha} c_{n,\alpha} + \text{h.c.}$$

+

e-e interaction

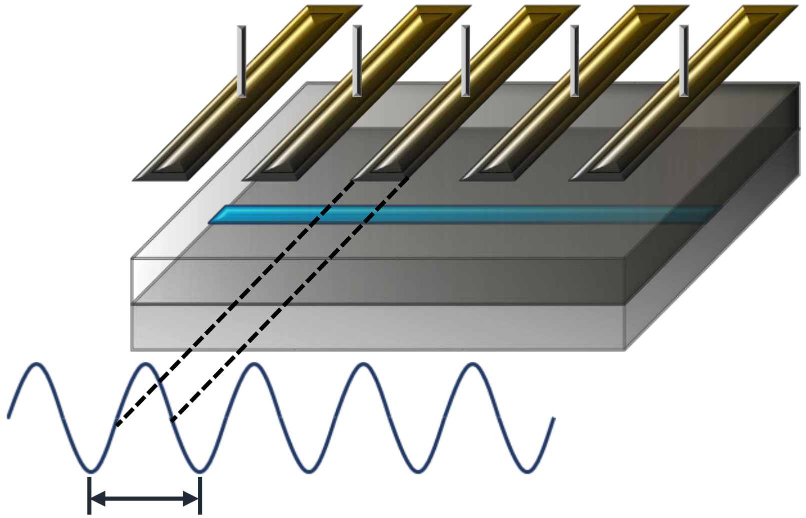
$$H_{e\text{-}e} = \sum_{n,n';\alpha,\beta} V(n-n') c^\dagger_{n,\alpha} c^\dagger_{n',\beta} c_{n',\beta} c_{n,\alpha}$$

modulated Rashba interaction

$$H_{\mathrm{R}}^{\mathrm{mod}} = -i\gamma_{\mathrm{R}}' \sum_{n,\alpha,\beta} \cos(Qna) c_{n,\alpha}^{\dagger} \sigma_{\alpha\beta}^{y} c_{n+1,\beta} + \mathrm{h.c.}$$

modulated chemical potential

$$H_{\mathrm{cp}}^{\mathrm{mod}} = \frac{\mu'}{2} \sum_{n,\alpha} \cos(Qna) c_{n,\alpha}^{\dagger} c_{n,\alpha} + \mathrm{h.c.}$$



$$\boxed{Q = 2(k_F + q_0)}$$

$$k_F = \pi\nu/a$$

$$\nu = N_e/2N$$

$$q_0 a = \arctan\left(\sqrt{(\tilde{t}/t)^2 - 1}\right)$$

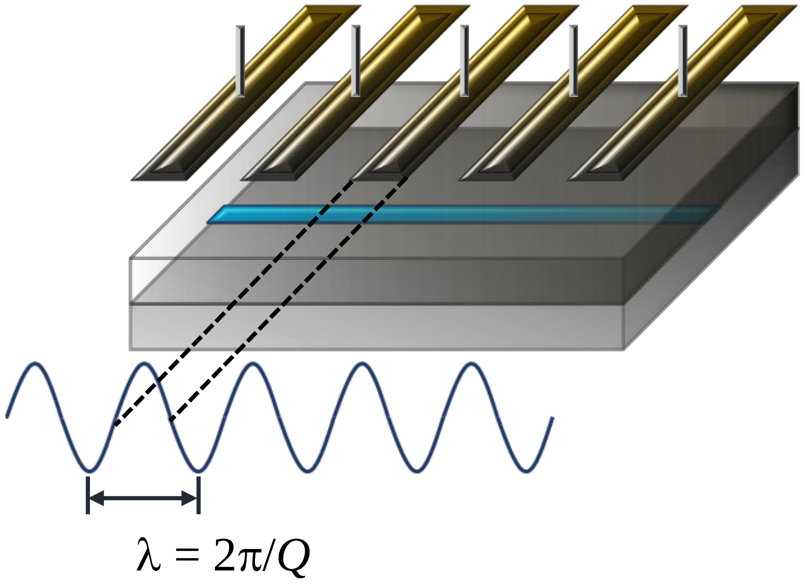
$$\tilde{t} = \sqrt{t^2 + \gamma_R^2 + \gamma_D^2}$$

modulated Rashba interaction

$$H_{\mathrm{R}}^{\mathrm{mod}} = -i\gamma'_{\mathrm{R}} \sum_{n,\alpha,\beta} \cos(Qna) c^{\dagger}_{n,\alpha} \sigma^{y}_{\alpha\beta} c_{n+1,\beta} + \mathrm{h.c.}$$

rapidly oscillating
vanish upon integration
in the continuum limit

$$H_{\mathrm{cp}}^{\mathrm{mod}} = \frac{\mu'}{2} \sum_{n,\alpha} \cos(Qna) c^{\dagger}_{n,\alpha} c_{n,\alpha} + \mathrm{h.c.}$$



$$\boxed{Q = 2(k_F + q_0)}$$

$$k_F = \pi\nu/a$$

$$\nu = N_e/2N$$

$$q_0 a = \arctan\left(\sqrt{(\tilde{t}/t)^2 - 1}\right)$$

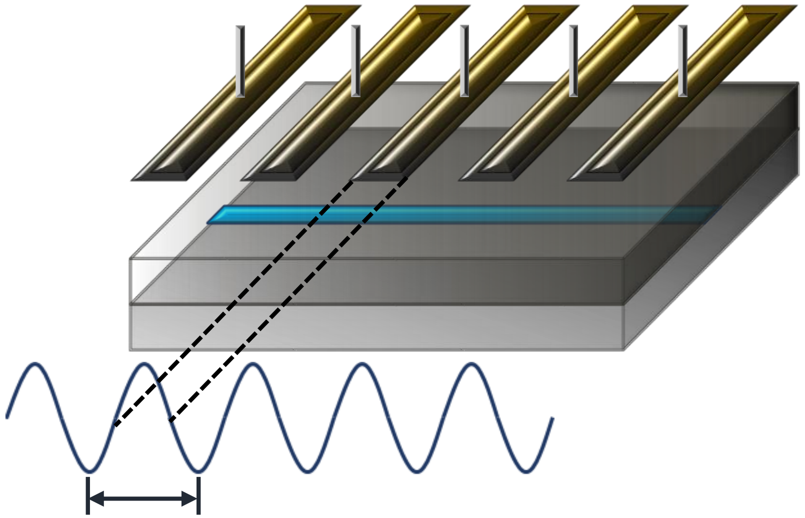
$$\tilde{t} = \sqrt{t^2 + \gamma_R^2 + \gamma_D^2}$$

modulated Rashba interaction

$$H_{\mathrm{R}}^{\mathrm{mod}} = -i\gamma_{\mathrm{R}}' \sum_{n,\alpha,\beta} \cos(Qna) c_{n,\alpha}^{\dagger} \sigma_{\alpha\beta}^{y} c_{n+1,\beta} + \mathrm{h.c.}$$

Has an interesting effect!

$$H_{\mathrm{cp}}^{\mathrm{mod}} = \frac{\mu'}{2} \sum_{n,\alpha} \cos(Qna) c_{n,\alpha}^{\dagger} c_{n,\alpha} + \mathrm{h.c.}$$



$\lambda = 2\pi/Q$

$$\boxed{Q = 2k_F}$$

$$k_F = \pi\nu/a$$

$$\nu = N_e/2N$$

$$q_0 a = \arctan\left(\sqrt{(\tilde{t}/t)^2 - 1}\right)$$

$$\tilde{t} = \sqrt{t^2 + \gamma_R^2 + \gamma_D^2}$$

modulated Rashba interaction

$$H_{\mathrm{R}}^{\mathrm{mod}} = -i\gamma_{\mathrm{R}}' \sum_{n,\alpha,\beta} \cos(Qna) c_{n,\alpha}^{\dagger} \sigma_{\alpha\beta}^{y} c_{n+1,\beta} + \mathrm{h.c.}$$

Has an interesting effect!

$$H_{\mathrm{cp}}^{\mathrm{mod}} = \frac{\mu'}{2} \sum_{n,\alpha} \cos(Qna) c_{n,\alpha}^{\dagger} c_{n,\alpha} + \mathrm{h.c.}$$

PHYSICAL REVIEW B **84**, 075466 (2011)

Modulated Rashba interaction in a quantum wire: Spin and charge dynamics

Mariana Malard
Faculdade UnB Planaltina, Universidade de Brasilia, 73300-000 Planaltina-DF, Brazil

Inna Grusha and G. I. Japaridze
Andronikashvili Institute of Physics, Tamarashvili 6, 0177 Tbilisi, Georgia and College of Engineering, Ilia State University, Cholokasvili Ave. 3-5, 0162 Tbilisi, Georgia

Henrik Johannesson
Department of Physics, University of Gothenburg, SE-412 96 Gothenburg, Sweden

(Received 19 May 2011; revised manuscript received 29 June 2011; published 12 August 2011)

modulated Rashba interaction

+

uniform Rashba and Dresselhaus

+

e-e interactions

$$H_{\mathrm{R}}^{\mathrm{mod}} = -i\gamma'_{\mathrm{R}} \sum_{n,\alpha,\beta} \cos(Qna) c^{\dagger}_{n,\alpha} \sigma^{y}_{\alpha\beta} c_{n+1,\beta} + \mathrm{h.c.}$$

$$\boxed{Q = 2(k_F + q_0)}$$

$$k_F = \pi\nu/a$$

$$\nu = N_e/2N$$

$$q_0 a = \arctan\left(\sqrt{(\tilde{t}/t)^2 - 1}\right)$$

$$\tilde{t} = \sqrt{t^2 + \gamma_R^2 + \gamma_D^2}$$

Gap out a doublet of Fermi points,
leaving behind a helical liquid in the other doublet.

Switch to a basis that diagonalizes $H_0 + H_{DR}$ in spin space.

↓

Take the low energy and continuum limit.

↓

Bosonize.

↓

Integrate out quadratic modes.

↓

Perform perturbative RG.

$$Q = 2(k_F + q_0)$$

$$H_0 + H_{DR} + H_R^{mod} + H_{e-e} = \int dx\, \mathcal{H}[\underbrace{\theta_1, \phi_1}_{\text{branch 1}}; \underbrace{\theta_2, \phi_2}_{\text{branch 2}}]$$

$$\mathcal{H} = \mathcal{H}^{(1)} + \mathcal{H}^{(2)} + \mathcal{H}^{(12)}$$

sine-Gordon potential

relevant when **K** < ½

Gaps out the 2nd branch

$$\mathcal{H}^{(i)} = \mathcal{H}_0^{(i)} - \delta_{i2} \frac{\lambda^2}{4\pi a^2 v_F} \cos(\sqrt{16\pi K}\,\phi_2)$$

mass parameter (spin-orbit interactions)

Luttinger liquid parameter (e-e interactions)

helical Dirac hamiltonian

$$\mathcal{H}_0^{(i)} = u[(\partial_x \theta_i)^2 + (\partial_x \phi_i)^2]$$

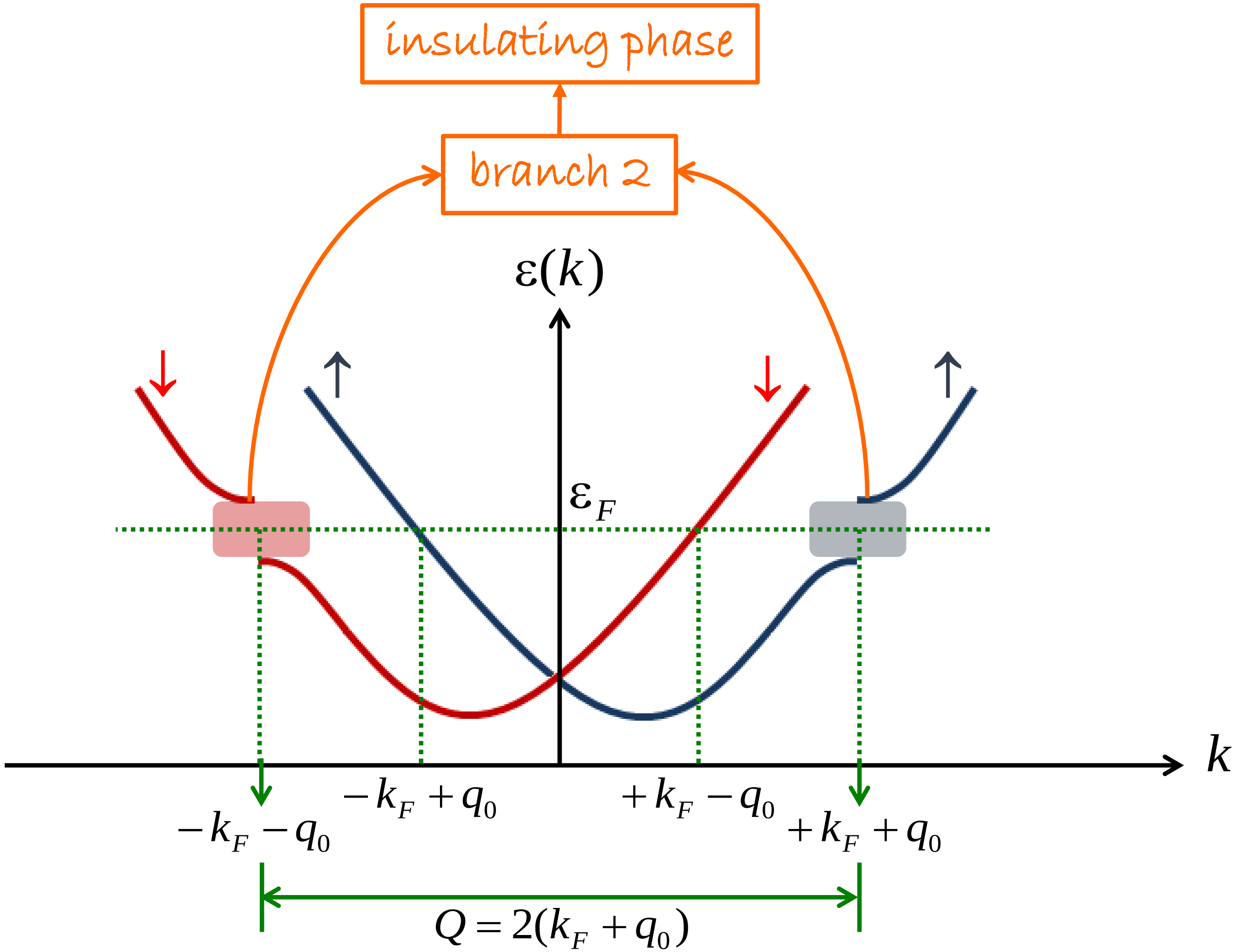
dressed Fermi velocity (spin-orbit interactions and e-e interactions)

supressed when **K** < ½

$$\mathcal{H}^{(12)} = \frac{g_2 K}{\pi} \partial_x \phi_1 \partial_x \phi_2$$

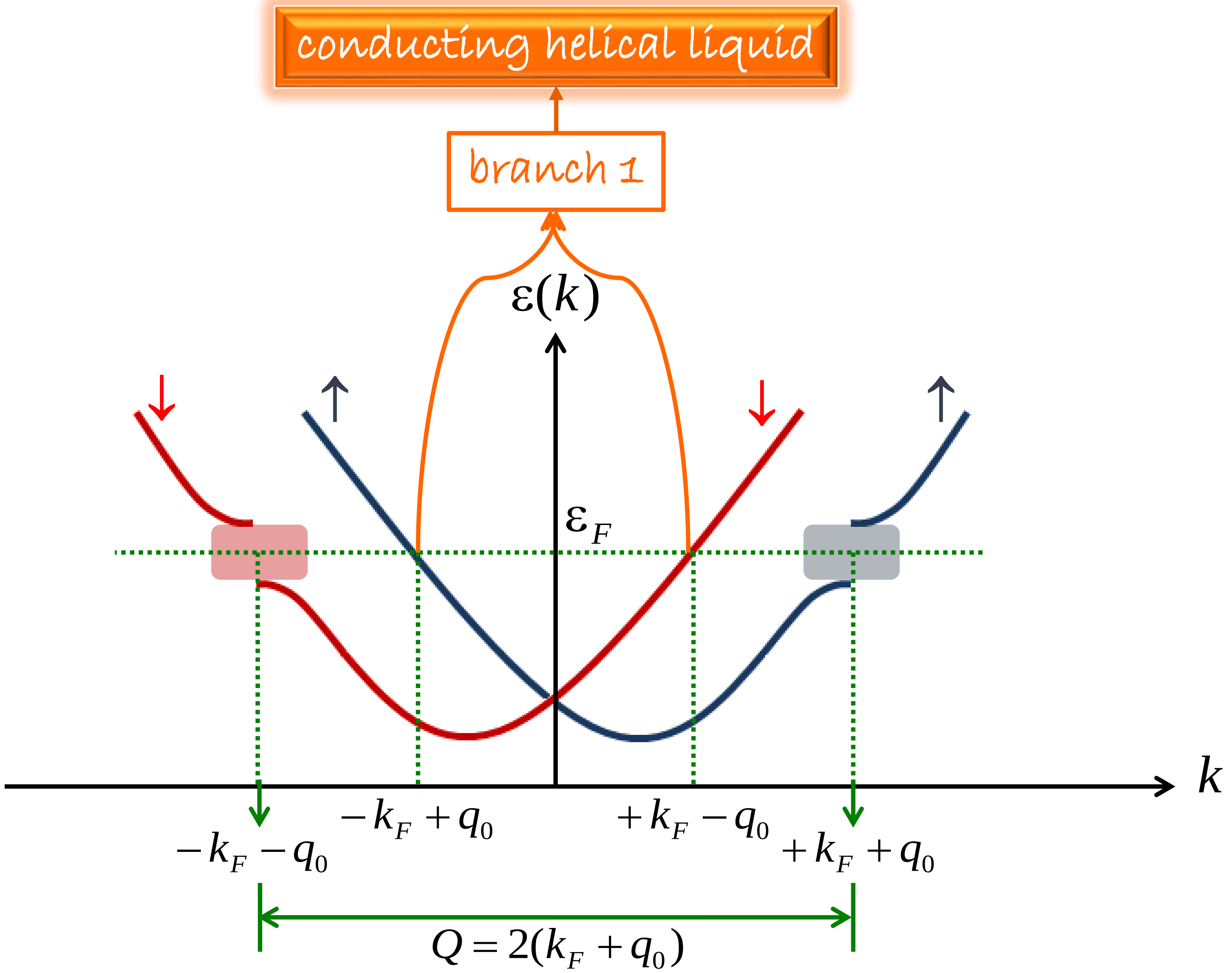
$$\boxed{Q = 2(k_F + q_0)}$$

$$\boxed{K < \frac{1}{2}}$$



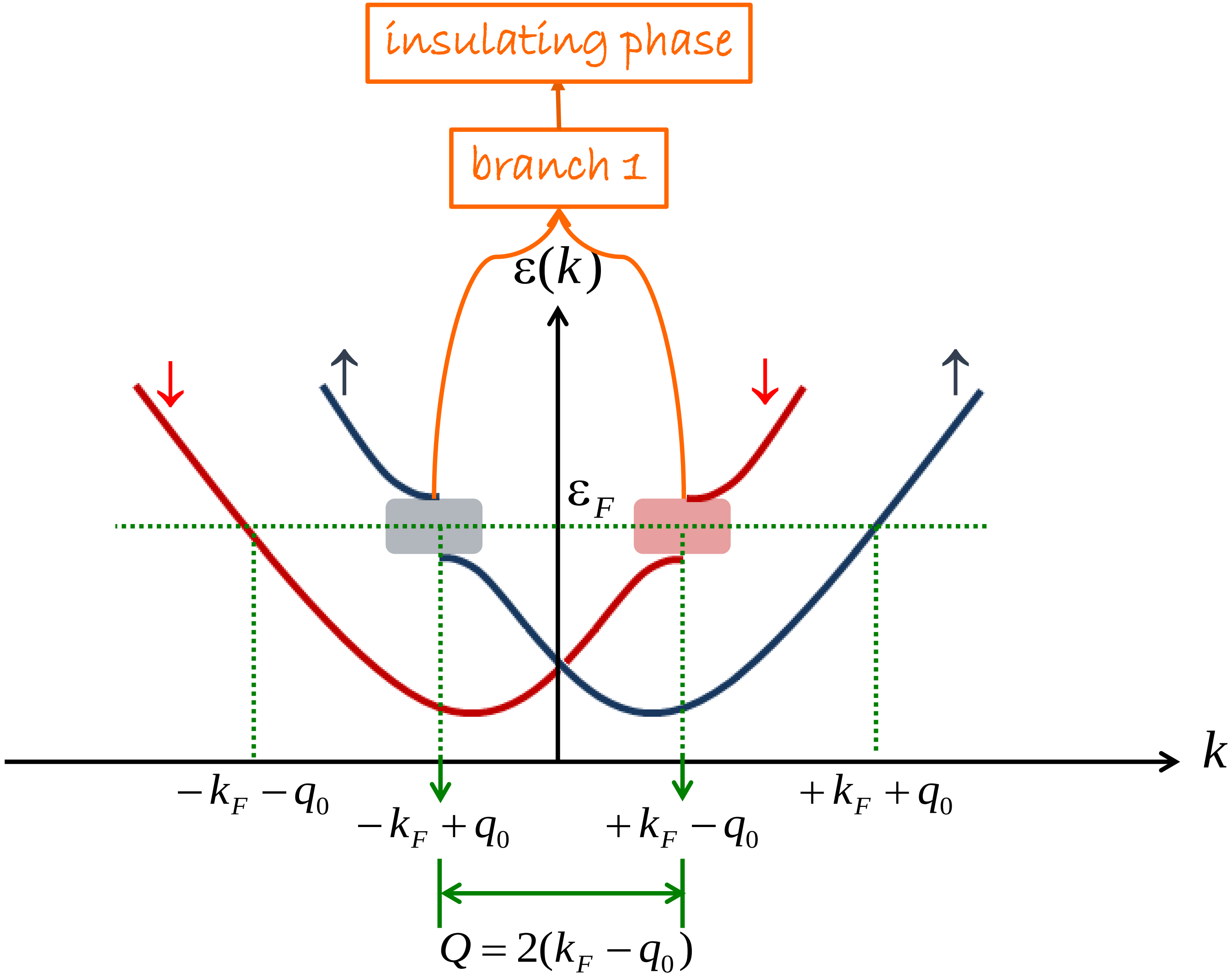
$$\boxed{Q = 2(k_F + q_0)}$$

$$\boxed{K < \frac{1}{2}}$$



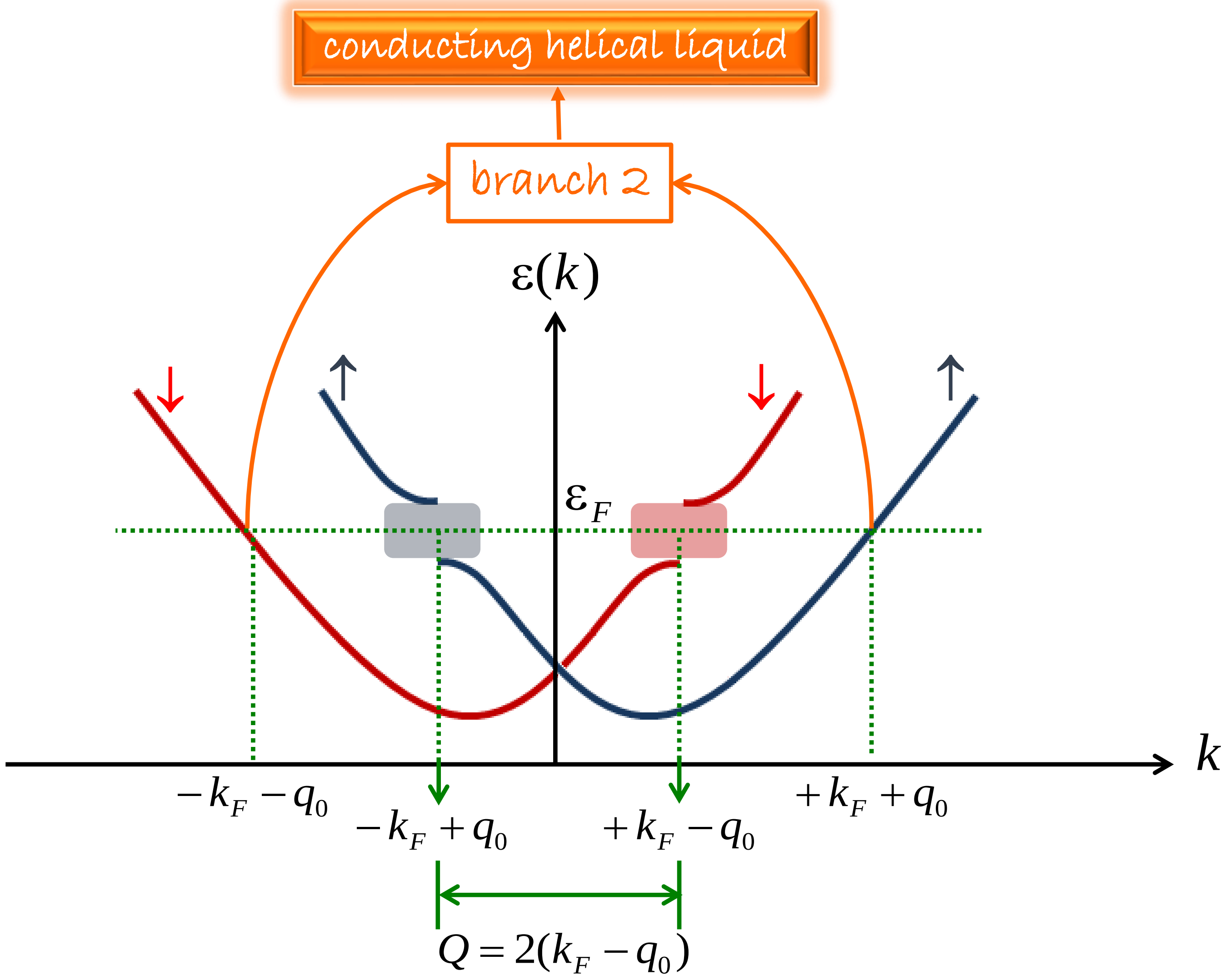
$$\boxed{Q = 2(k_F - q_0)}$$

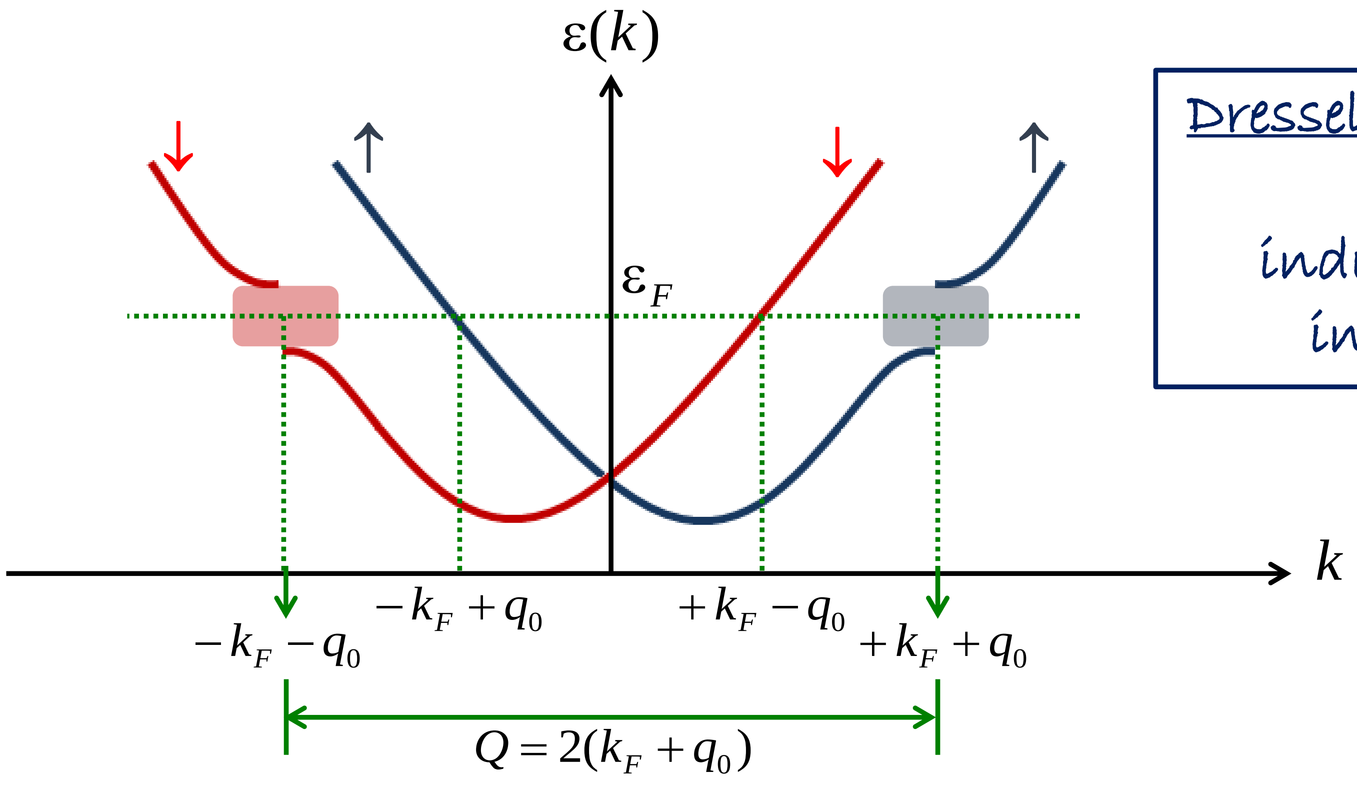
$$\boxed{K < \frac{1}{2}}$$



$$\boxed{Q = 2(k_F - q_0)}$$

$$\boxed{K < \frac{1}{2}}$$





Q-modulated Rashba spin-orbit interaction

primary source of the pinning potential

Dresselhaus spin-orbit interaction

induces spin-flipping terms in the pinning potential

trap designers

e-e interaction

makes the potential relevant

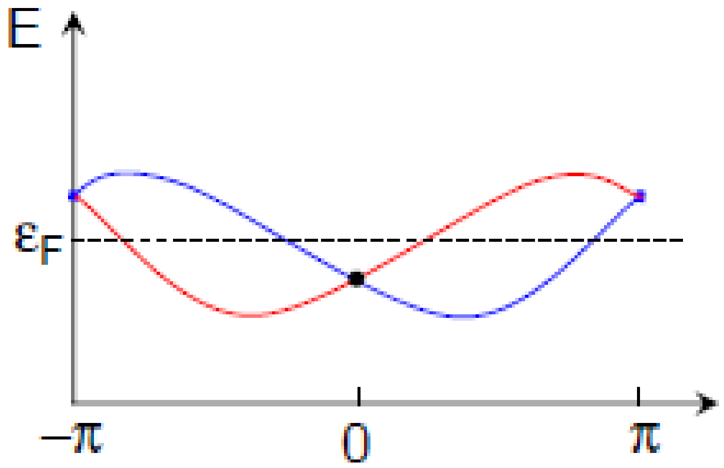
→ trap empowerer

Are you aware of Kramers theorem, Dr. Malard?

Time-reversal symmetry

Kramers theorem

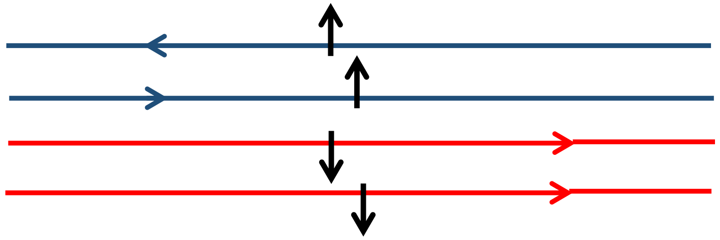
Rules out helicity in a 1D band structure



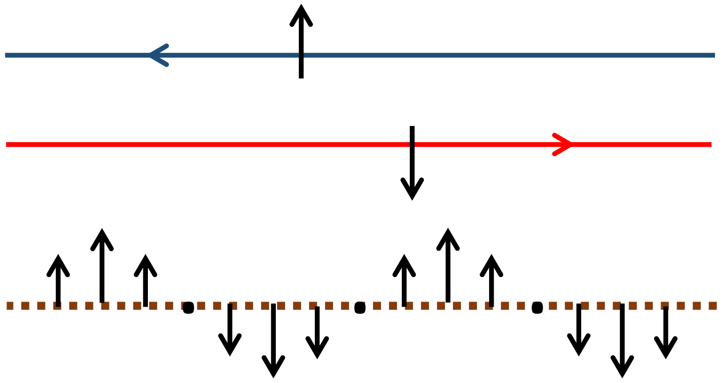
What's the trick then?

Breakdown of time reversal symmetry, as it must be.

- But only in the insulating branch, which spontaneously develops a spin density wave.
- The helical liquid keeps its inherent time-reversibility!



Luttinger liquid with spin-orbit interactions and e-e interactions



ballistic HL protected by TRS

TRS breaking insulating channel


Fair enough...
But will it work in the lab?

Top-2 practical requirements:

- Sufficiently strong e-e interactions so that $K < 1/2$

- large enough gap to block thermal excitations

Case study: quantum wire patterned in an InAs quantum well

- Sufficiently strong e-e interactions so that K < ½

$$K < 1/2$$

$$\Downarrow$$

with $\boxed{\varepsilon_r} \approx 12$

relative permittivity of the dopant and capping layers

$\boxed{d} > 1\,\mu\mathrm{m}$

distance between the back gate and the wire

Case study: quantum wire patterned in an InAs quantum well

- large enough gap to block thermal excitations

$$\Delta \approx 0.3\,\text{meV}$$
$$\Downarrow$$
$$\Delta > k_B T \quad \text{for} \quad T < 3.5\,\text{K}$$

- We have unveiled a scheme for fabricating a synthetic HL in an interacting quantum wire using electric fields only and exploiting an interplay between the Dresselhaus and a modulated Rashba spin-orbit interactions.

- This synthetic HL is of a different type than existing varieties being neither holographic nor quasi-helical.

- Its realization is well within present-day experimental capabilities.

- The simplicity and robustness of our proposed setup makes it a handy testing system for exploring the physics of 1D HLs.

Find the full work:

- PRB 89, 201403(R) (2014)
- arXiv: 1311.4716v1

Thank you!

Tack!