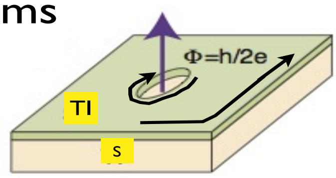
Majorana bound state in hybrid HTc systems



A.Tagliacozzo (Universita' di Napoli "Federico II")

3D Topological Insulators Bi_2Te_3, Bi_2Se_3: boundary states and superconductive proximity

P.Lucignano, *G.Campagnano, V.Parente, D.Giuliano, F.Tafuri*
F. von Oppen (FU), F.Guinea(CSIC), A.Mezzacapo(Bilbao), F.Lombardi(Chalmers)

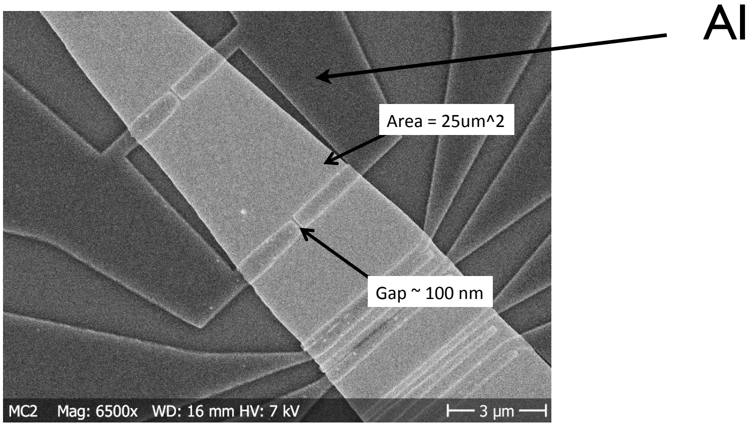
thanks to P. Breuwer (FU),

Nordita 3.9.2014

Proximity with an s-wave Superconductor

SQUID on a Bi2Te3 Flake

Chalmers

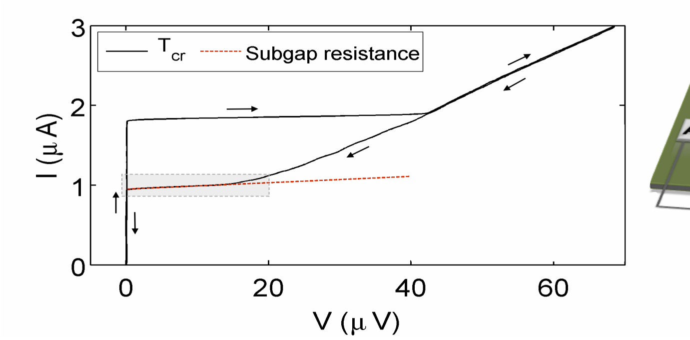


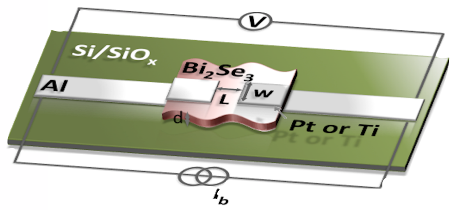
F.Lombardi

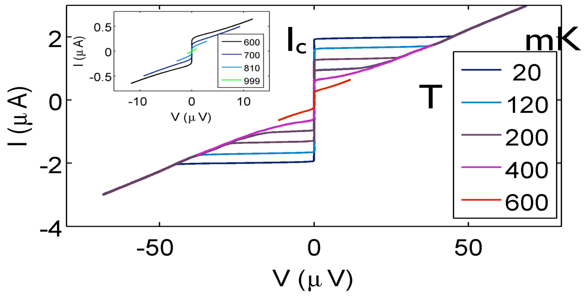
Proximization with an s-wave Superconductor (Al)

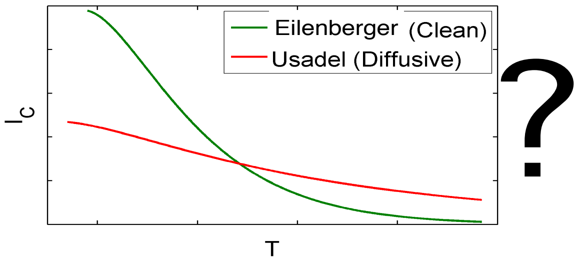
Al/Bi_2Te_3/Al

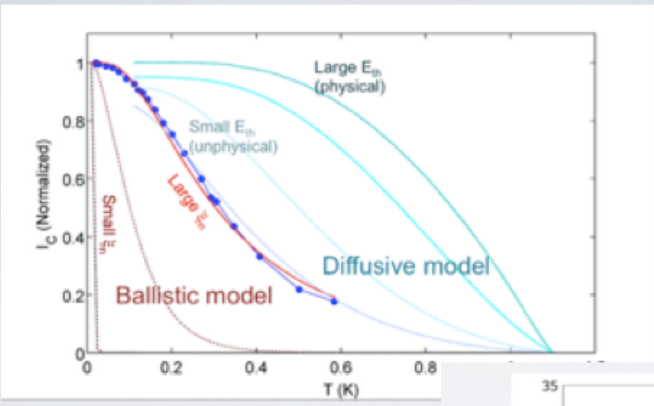
Chalmers





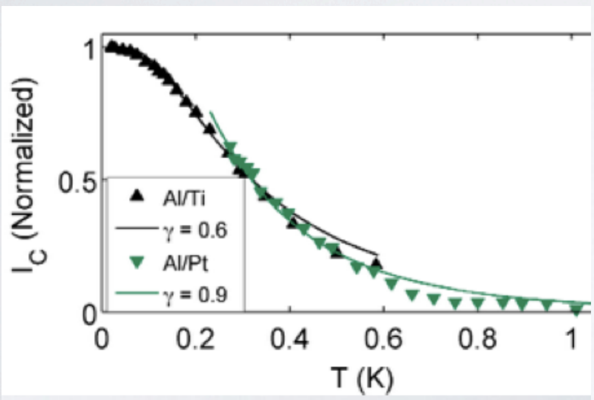


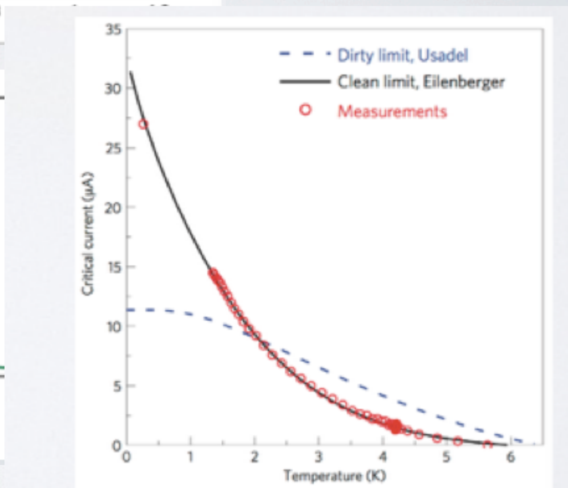




$$\xi_n^B = \frac{\hbar v_F}{E_{Th}}, \quad (\xi_n < L)$$

$$\xi_n^D = \sqrt{\frac{\hbar D}{E_{Th}}}, \quad (\xi_n < L)$$





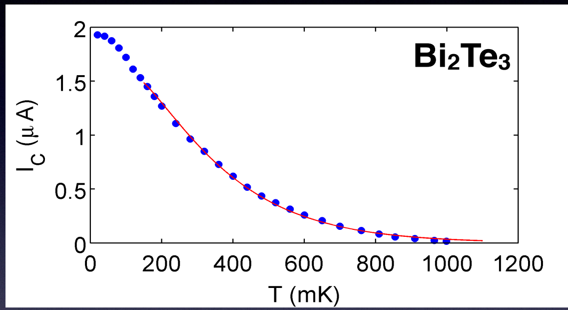
In agreement with Nb/Bi2Te3/Nb samples
Veldhrost et al., Nature Materials (2012)

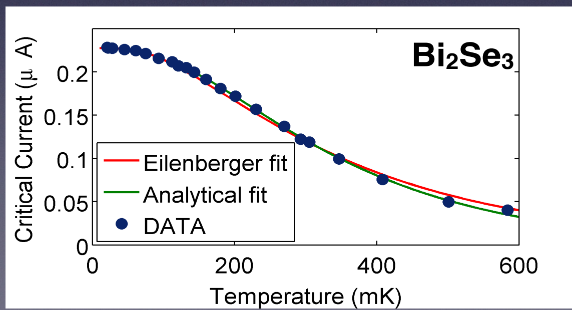
Ballistic Transport Regime

- Bi_2Te_3
  - E_{th} = 96 μeV -> ξ_n = 0.7 μm
  - L = 200 nm
- Bi_2Se_3
  - E_{th} = 100 μeV -> ξ_n = 3 μm
  - L = 300 nm

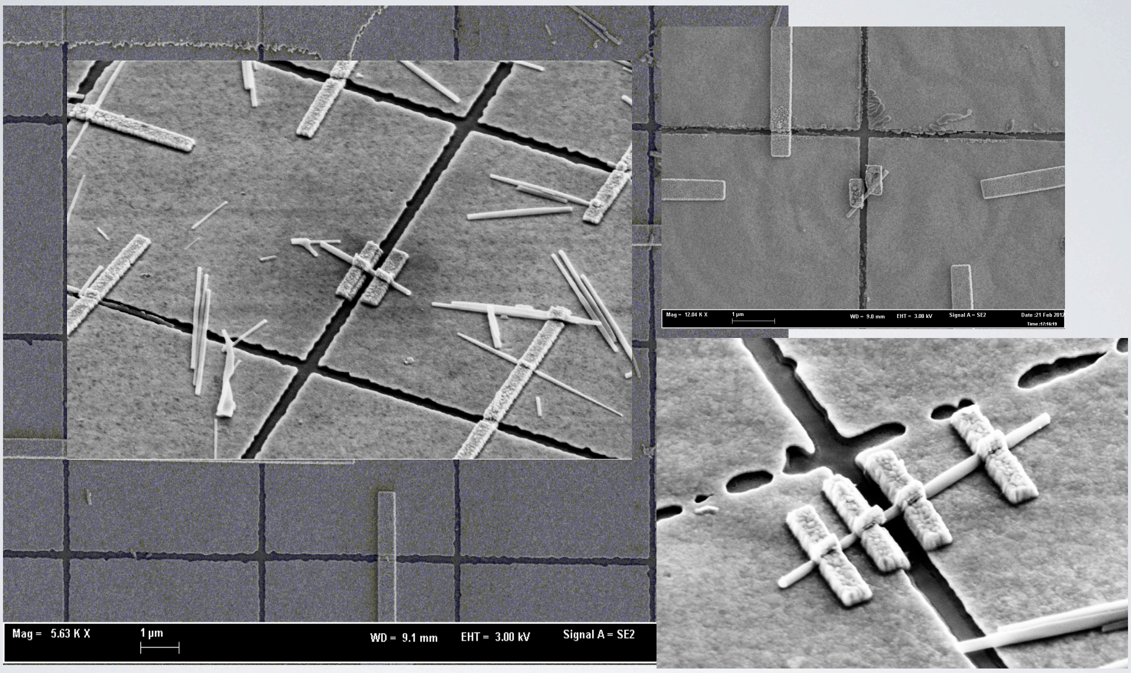
★ **Ballistic proximity regime through the topological edge states**

(L. Galletti et al., PRB89)



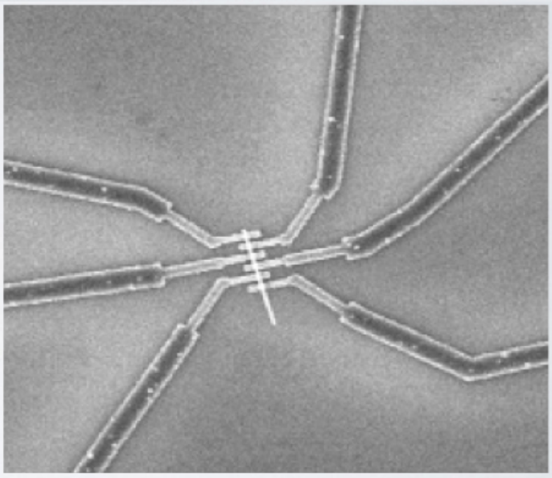


InAs wires on top of YBCO



F. Tafuri Napoli-HTS Josephson junctions group CNR-SPIN & SUN
D. Montemurro, L. Sorba, F. Beltram NEST, Istituto Nanoscienze-CNR and Scuola Normale Superiore Pisa
S. Roddaro CNR-IOM Trieste

InAs wire with YBCO contacts



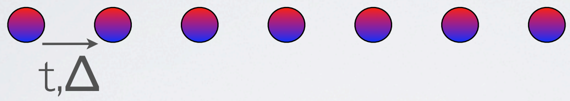
Napoli-HTS Josephson junctions group @ CNR-SPIN & SUN with SNS Pisa and CNR-NEST

Content

- **INTRO:** Josephson junction in Kitaev's chain
- π-junction and YBCO tricrystal geometry
- long wavelength 3D-TI model and helicity of boundary states
- s-wave superconductive proximity of a TI flake
- π-loop: d-wave pairing proximity and Majorana Bound state

Kitaev's Model

$$H - \mu N = \sum_i t(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) - \mu c_i^\dagger c_i + \Delta(c_i c_{i+1} + c_{i+1}^\dagger c_i^\dagger)$$



$$H - \mu N = \frac{1}{2}\sum_k \begin{pmatrix} c_k^\dagger & c_k \end{pmatrix} H_{BdG} \begin{pmatrix} c_k^\dagger \\ c_k \end{pmatrix}$$

$$H_{BdG} = \tau_z(2t\cos k - \mu) + \tau_x \Delta \sin k = \vec{d}(k)\cdot\vec{\tau}$$

One-dimensional p-wave spinless superconductor

A. Kitaev Annals of Physics 2003

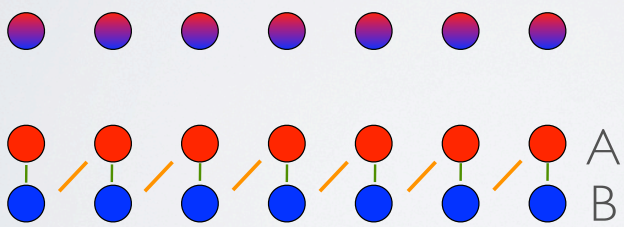
Mapping of spinless Dirac Fermions into Majorana Fermions

$$c_i = \gamma_{Bi} + i\gamma_{Ai} \qquad \{\gamma_i, \gamma_j\} = 2\delta_{ij}$$

$$\mu c_i^\dagger c_i = 2i\mu\gamma_{Bi}\gamma_{Ai} \qquad \gamma_i^2 = 1$$

$$\Delta(c_i c_{i+1} + h.c.) = 2i\Delta(\gamma_{Bi}\gamma_{Ai+1} + \gamma_{Ai}\gamma_{Bi+1})$$

$$t(c_i^\dagger c_{i+1} + h.c.) = 2it(\gamma_{Bi}\gamma_{Ai+1} - \gamma_{Ai}\gamma_{Bi+1})$$



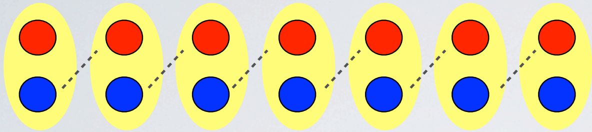
N-site fermion chain

2N-site Majorana chain

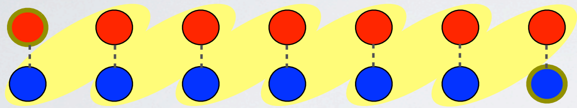
t and Δ couple different site MFs

μ acts within the same site

$t = \Delta$

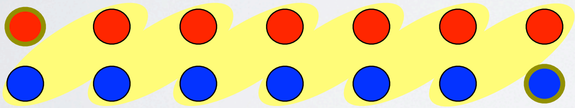


$\mu > 2t$ Intra-site interact.



$\mu < 2t$ Inter-site interact.

Limiting case $\mu=0$, $2t>0$



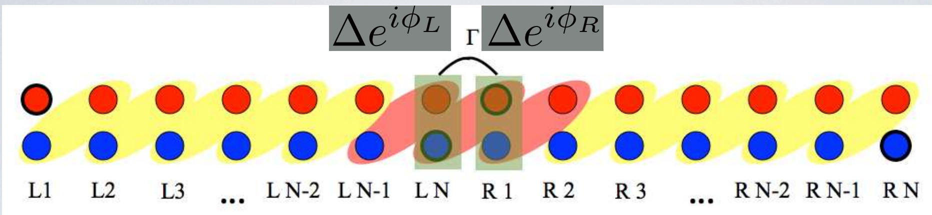
$\mu=0$ Unpaired Majorana end states

$$f = \gamma_{A1} + i\gamma_{BN}$$

$|0\rangle$ $\quad f|0\rangle = 0 \quad f^\dagger|0\rangle = |1\rangle$

$H|0\rangle = H|1\rangle$ Ground state degeneracy (fermion parity)

Josephson conduction in Kitaev chains ($\mu=0$, $t=\Delta$)



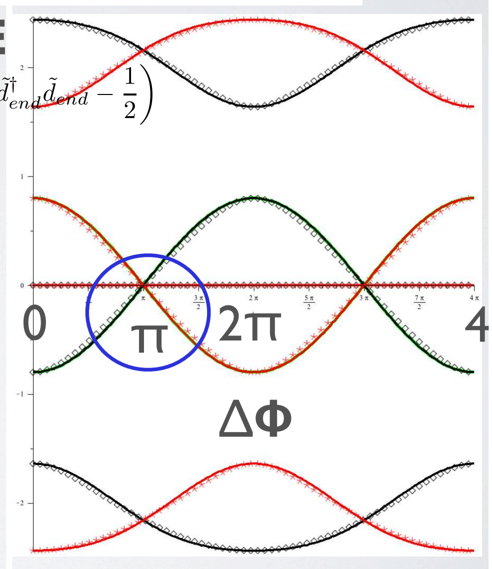
$$H_{eff} = E_A\left(\tilde{d}_A^\dagger \tilde{d}_A - \frac{1}{2}\right) + E_B\left(\tilde{d}_B^\dagger \tilde{d}_B - \frac{1}{2}\right) + E_E\left(\tilde{d}_{end}^\dagger \tilde{d}_{end} - \frac{1}{2}\right)$$

$$d_{end} = \frac{1}{2}(\gamma_{A1}^R + i\gamma_{BN}^L)\,,$$

$$H_\Gamma = -\Gamma\, e^{i\Delta\phi/2}\, c_{LN}^\dagger c_{R1} + h.c.$$

$$I \propto \sin(\Delta\phi/2)\langle d_{end}^\dagger d_{end} - 1/2\rangle$$

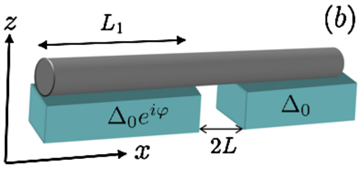
zero energy bound state !



Jason Alicea, Yuval Oreg, Gil Refael, Felix von Oppen & Matthew P.A. Fisher Nature Physics 2011

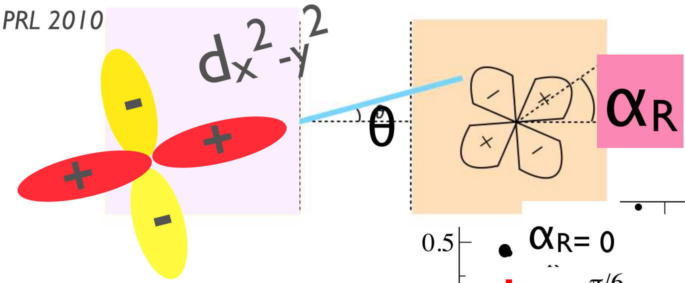
d-wave superconductors: switching from 0- to π-junction

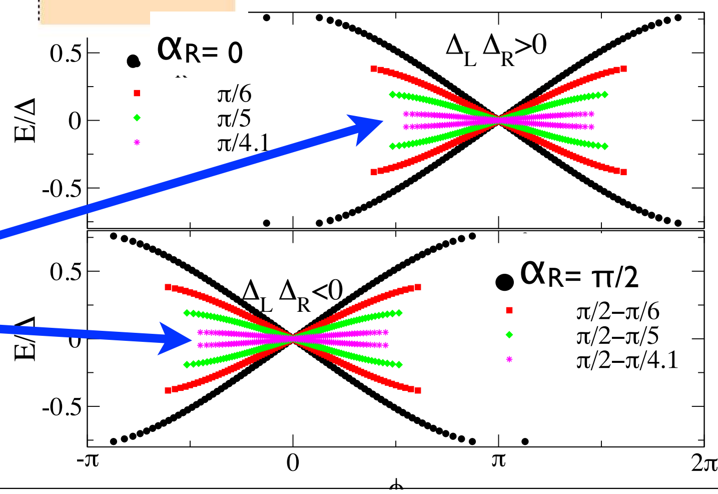
e.g. InAs wire



Oreg et al. PRL 2010
Sau et al. PRL 2010

MBS switches from Φ=π to Φ=0



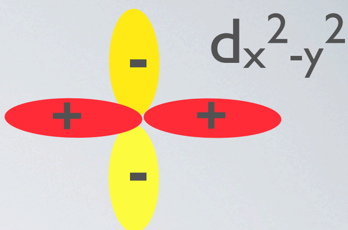


Zero modes of the theory is a BMF !

(θ=0)

P.Lucignano, et al PRB 2012

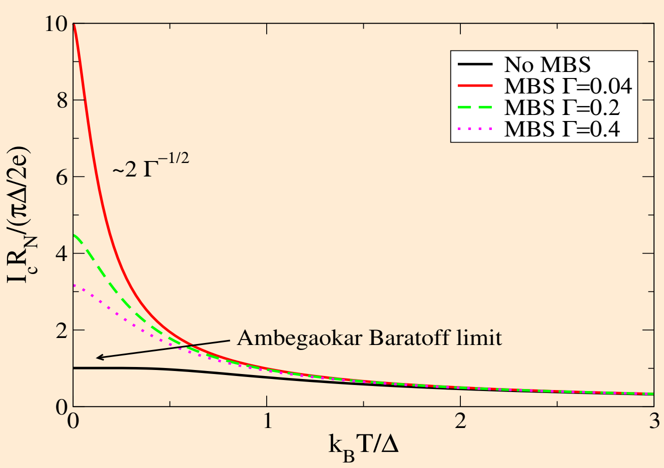
HIGH Tc JOSEPHSON JUNCTION



$$H_S = H_0 + \int_{-\infty}^{\infty} dx \left(\Delta(x)\psi_\uparrow^\dagger(x)\psi_\downarrow^\dagger(x) + h.c. \right)$$

$$H_0 = \int_{-\infty}^{\infty} dx \psi_\alpha^\dagger(x) \left(\left(-\frac{\partial_x^2}{2m^*} - \mu \right) I_2 + i\sigma_y \partial_x + B_x \sigma_x \right)_{\alpha\beta} \psi_\beta(x)$$

$$\Delta(x) = \begin{cases} \Delta_L & for\ \ x < -L_N/2\,, \\ 0 & for\ \ -L_N/2 \le x \le L_N/2\,, \\ \Delta_R & for\ \ x > L_N/2\,, \end{cases}$$



$$\begin{aligned} \Delta_L &= \Delta_0 cos(2(\theta - \alpha_L))\,, \\ \Delta_R &= \Delta_0 cos(2(\theta - \alpha_R)) \exp(-i\varphi)\,. \end{aligned}$$

$$I_c R_N = \begin{cases} \frac{\pi\sqrt{\Delta_L \Delta_R}}{e\sqrt{\Gamma}} & WITH\ \ MBS \\ \frac{\pi\sqrt{\Delta_L \Delta_R}}{2e} & NO\ \ MBS \end{cases}$$

Γ is the barrier transparency

P. Lucignano, et al ,PRB 2012

$$Z \sim \left(1 + \frac{\pi\Gamma}{\Delta}\right)^{-1} \qquad \tilde{\Delta} = \Delta(1 - Z) \qquad \begin{array}{c} \tilde{H}_R = H_R Z \\ \tilde{B} = BZ \end{array}$$

Good proximity: Z~0 — Maximum induced Gap but minimum B!

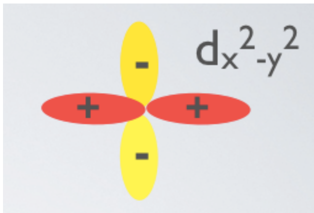
Bad proximity: Z~1 — Minimum induced Gap but maximum B!

We have to optimize Z!

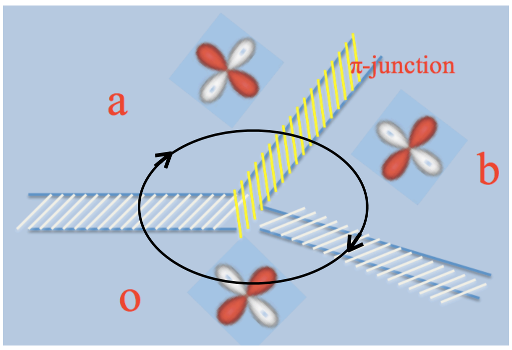
For α=0.2–0.8 K, even after carefully optimizing B , γ , and Δ_0 , only a *p*-wave pairing gap on the order of 0.1–0.4 K is achievable (with low Tc superconductors). Such small excitation gaps would require operating at temperatures **T<< 0.1 K** in order to avoid thermal excitations! A.C. Potter and P.A. Lee PRB & PRL 2011

detour:

YBCO tricrystal junctions



$$U = -E_J\left(\cos\theta_a - 1\right) - E_J\left(\cos\theta_b - 1\right) + E_J\left(\cos(\theta_b - \theta_a) + 1\right)$$



ground state:

$$\Theta_o - \Theta_b = -\theta_b = \frac{\pi}{3}, \quad \Theta_a - \Theta_o = \theta_a = \frac{\pi}{3},$$

$$\Theta_b - \Theta_a = \theta_b - \theta_a + \frac{2\pi}{\phi_o}\int_a^b A \cdot d\ell$$

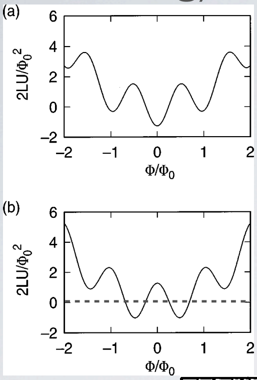
requires half a flux quantum

$$\Phi = \oint A \cdot d\ell = \int_a^b A \cdot d\ell = \frac{\phi_o}{2}$$

$$\phi_o = \frac{hc}{2e}$$

YBCO: TRICRYSTAL GEOMETRY:

free energy

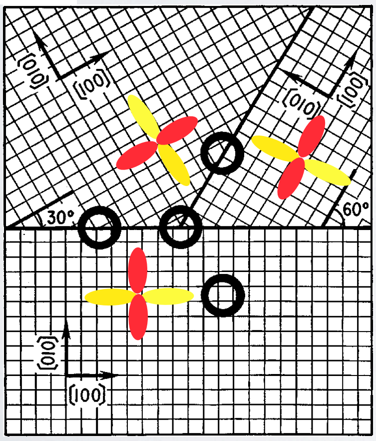


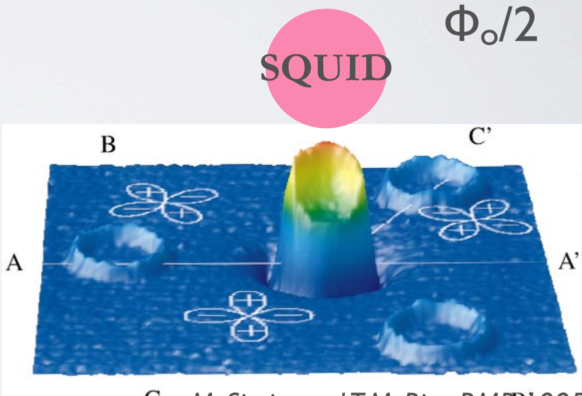
0-loop

$$U(\Phi,\Phi_a)=\frac{\Phi_0^2}{2L}\left\{\left(\frac{\Phi-\Phi_a}{\Phi_0}\right)^2-\left(\frac{L|I_c|}{\pi\Phi_0}\right)\times\cos\left(\frac{2\pi}{\Phi_0}\Phi+\varphi\right)\right\},$$

π-loop : Spontaneous half integer flux

$\Phi_o/2$





M. Sigrist and T.M. Rice RMP 1995
C.C. Tsuei and J.R. Kirtley RMP 2000

Two bands model Hamiltonian for TI

- $Bi_x\,Sb_{1-x}$
- layered, stoichiometric crystals $Sb_2\,Te_3$, $Sb_2\,Se_3$, $Bi_2\,Te_3$ e $Bi_2\,Se_3$

Zhang et al. 2009

minimal model in the continuum limit:

$$\mathcal{H}_0[\vec{r}]=\hbar v_F\begin{pmatrix}-\mathcal{M}-\mu & i\partial_z & 0 & i(\partial_x-i\partial_y)\\ i\partial_z & \mathcal{M}-\mu & i(\partial_x-i\partial_y) & 0\\ 0 & i(\partial_x+i\partial_y) & -\mathcal{M}-\mu & -i\partial_z\\ i(\partial_x+i\partial_y) & 0 & -i\partial_z & \mathcal{M}-\mu\end{pmatrix}$$

$$(\psi_{g\uparrow}\,,\psi_{u\uparrow}\,,\psi_{g\downarrow}\,,\psi_{u\downarrow}\,)$$

Bulk bands dispersion $E=\pm\sqrt{M^2+k^2+k_z^2}$ at the $\bar{\Gamma}$ point

$$\Theta\mathcal{H}_0(\vec{k})\Theta^{-1}=\mathcal{H}_0(-\vec{k})$$ TR invariant

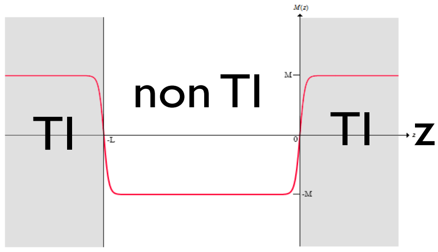
$$\Xi\mathcal{H}_0(\vec{k})\Xi^{-1}=-\mathcal{H}_0(-\vec{k})$$ p-h symmetric (μ=0)

$$\Gamma\mathcal{H}_0(\vec{k})\Gamma^{-1}=-\mathcal{H}_0(-\vec{k})$$ conserves helicity (μ=0) $\Gamma=i\,\Theta\Xi$

$$\Theta=\mathcal{I}\otimes i\sigma_y\,\mathcal{K},\quad \Xi=-i\tau_y\otimes\sigma_x\,\mathcal{K},\quad \Gamma=\tau_y\otimes\sigma_z$$

boundary states

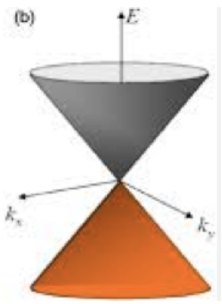
At the boundary with a trivial insulator, M changes sign:



$$k = \sqrt{k_x^2 + k_y^2} = k_+ k_-$$
$$k_\pm = k_x \pm i\, k_y$$

$$|surf, +; k_\parallel\rangle = \begin{pmatrix} M+k \\ k_- + iM \\ M+k \\ k_+ - iM \end{pmatrix}\Bigg|_{M\sim 0} \quad e^{-\int^z dz\, M(z)}\, e^{ik_\parallel \cdot r_\parallel}, \quad E = +k$$

$$|surf, -; k_\parallel\rangle = \Xi\; |surf, +; -k_\parallel\rangle = \begin{pmatrix} k_- + iM \\ M+k \\ k_+ - iM \\ M+k \end{pmatrix}\Bigg|_{M\sim 0} \quad e^{\int^z dz\, M(z)}\, e^{ik_\parallel \cdot r_\parallel}, \quad E = -k$$

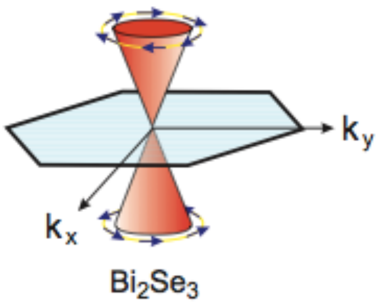


surface state decays as inside TI

$$\boxed{e^{-|M z|/\hbar v_F}}$$

Restricting H onto the L,R helical space:

$$\boxed{\Pi_L = \frac{(\mathbf{1} - \Gamma)}{2} \qquad \Pi_R = \frac{(\mathbf{1} + \Gamma)}{2}}$$



a reduced H for the surface is obtained

$$\mathcal{H}_0\, \Pi_L = \begin{pmatrix} 0 & (\sigma_x + i\,\sigma_z)\, k_- \\ (\sigma_x - i\,\sigma_z)\, k_+ & 0 \end{pmatrix}$$

$$-\Xi \mathcal{H}_0 \Xi\, \Pi_R = \begin{pmatrix} 0 & (\sigma_x + i\,\sigma_z)\, k_- \\ (\sigma_x - i\,\sigma_z)\, k_+ & 0 \end{pmatrix}$$

$$\boxed{E = \pm k}$$

becomes 2X2 with the internal structure $\psi_{a\sigma} = \psi_{g\sigma} - i\,(-1)^\sigma\, \psi_{u\sigma}$

$$H_{red} = \begin{pmatrix} 0 & -i\, k_- \\ i\, k_+ & 0 \end{pmatrix} \equiv -\vec{\sigma} \times \vec{k} \cdot \hat{z}$$

Rashba-like

s-wave proximity 2D boundary TI

start from the Dirac chiral boundary states.

Fu & Kane 08

spin is not an independent degree of freedom: $\sigma \cdot p = cnst$

basis is $[\psi_{k\uparrow}, \psi_{k\downarrow}, \psi^\dagger_{-k\downarrow}, -\psi^\dagger_{-k\uparrow}]$

$$H_{TI} = v \begin{pmatrix} -\mu & p_- & \Delta & 0 \\ p_+ & -\mu & 0 & \Delta \\ \Delta & 0 & \mu & -p_- \\ 0 & \Delta & -p_+ & \mu \end{pmatrix}$$

eigenvalues are: $\pm\sqrt{(\pm v|p| - \mu)^2 + \Delta^2}$

in a spinless fermion picture:

$$c_{\vec{k}} \equiv \begin{bmatrix} c_{\vec{k}\uparrow} \\ c_{\vec{k}\downarrow} \end{bmatrix} \to \frac{1}{\sqrt{2}} \begin{bmatrix} e^{-i\theta_k/2} \\ e^{i\theta_k/2} \end{bmatrix} ; \quad -c_{-\vec{k}}{}^\dagger \equiv \begin{bmatrix} c_{-\vec{k}\downarrow}{}^\dagger \\ c_{-\vec{k}\uparrow}{}^\dagger \end{bmatrix} \to \frac{1}{\sqrt{2}} \begin{bmatrix} e^{-i\theta_k/2} \\ -e^{i\theta_k/2} \end{bmatrix} .$$

a $\Delta\,(p_x - ip_y)$ pairing appears

$$H^{new} = \frac{1}{2} \begin{pmatrix} c_{\vec{k}}{}^\dagger & -c_{-\vec{k}} \end{pmatrix} \begin{pmatrix} [vk - \mu] & \Delta e^{-i\theta_k} \\ \Delta e^{i\theta_k} & -[vk - \mu] \end{pmatrix} \begin{pmatrix} c_{\vec{k}} \\ -c_{-\vec{k}}{}^\dagger \end{pmatrix}$$

effective $p_x - i\,p_y$

Read & Green 2000, Fu & Kane 2008

Edge states are Majorana chiral states!

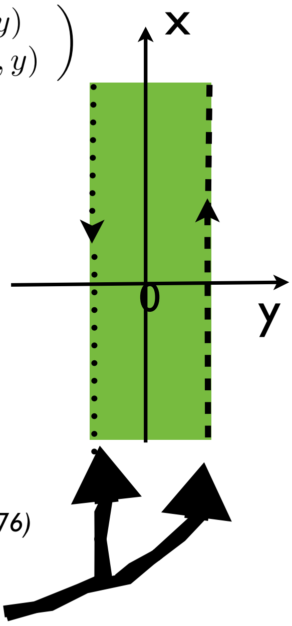
x,y in units of ξ

$$H = \begin{pmatrix} \mathcal{H}_0 & \hat{\Delta} \\ \hat{\Delta}^\dagger & -\mathcal{H}_0^T \end{pmatrix}$$

BdG
in 2D real space

$$h_0(x,y) - \mu \to \tilde{\mu}(y)$$

$$\begin{pmatrix} [h_0(x,y) - \mu] & \Delta\, i\,(\partial_x - i\partial_y) \\ \Delta\, i\,(\partial_x + i\partial_y) & -[h_0(x,y) - \mu] \end{pmatrix} \begin{pmatrix} c_\epsilon(x,y) \\ -\,c^\dagger_\epsilon(x,y) \end{pmatrix} = \epsilon \begin{pmatrix} c_\epsilon(x,y) \\ -\,c^\dagger_\epsilon(x,y) \end{pmatrix}$$



$$H = -\sigma_z\left(\tilde{\mu}(y) - \Delta\sigma_x\partial_y\right) + i\Delta\sigma_x\partial_x$$

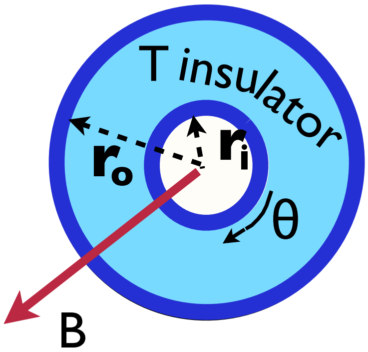
$$\gamma_k(\vec{r}) \propto \begin{pmatrix} c_{\epsilon_\pm}(\vec{r}) \\ -\,c^\dagger_{\epsilon_\pm}(\vec{r}) \end{pmatrix} = e^{\sigma_x \int_0^y dy'\,\tilde{\mu}(y')/\Delta} \begin{pmatrix} 1 \\ \mp 1 \end{pmatrix} \psi_k(x)$$

$$\Delta\partial_x\psi_k(x) \to \hbar v_F \partial_x\, \psi_k(x) = \epsilon_k\, \psi_k(x)$$

Jackiw and Rebbi (1976)

ε=0 Majorana state

$$c_{\epsilon=0}(\vec{r}) + c^\dagger_{\epsilon=0}(\vec{r}) \to e^{-\int_0^y dy'\,\tilde{\mu}(y')/\Delta}\, \psi_0(x)$$



However, in a circular geometry
antiperiodic boundary conditions

$$\Delta(\underline{r}) = i\,\hbar v_\Delta\, e^{-i\varphi(\underline{r})/2} e^{i\theta} \left(\partial_r - i\frac{\partial_\theta}{r}\right)\, e^{-i\varphi(\underline{r})/2}$$

to make H separable, gauge transform H

$$\psi \to \psi\, e^{-i\theta/2}$$

a zero energy solution with periodic boundary conditions only if

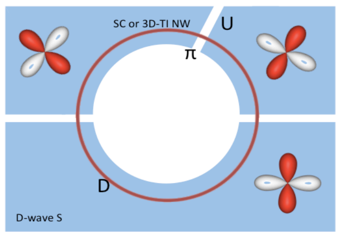
φ(θ)= (2n+1)θ : odd # of trapped flux quanta !

$$\gamma_i\,(\underline{r}) \propto e^{i\sigma_z\varphi(\underline{r})/2}\, e^{-i\sigma_z\theta/2}\, e^{\frac{\sigma_x}{\hbar v_\Delta}\int_{r_i}^{r}\tilde{\mu}(r')dr'} \begin{bmatrix} 1 \\ \pm 1 \end{bmatrix} \psi_{r_i}(\theta)$$

The MF $\psi_{r_i}(\theta)$ is eigenstate of: $H'' = \pm i\,\dfrac{\hbar v_\Delta}{r_i}\,\dfrac{\partial}{\partial\theta}$

Fu Kane...Grosfeld and Stern...

Kitaev π-loop



$$H_U = -\Gamma\,\left(c^\dagger_{\ell 1} c_{rN} + h.c.\right)$$

$$H_D = -u\,t\,\left(c^\dagger_{\ell N} c_{r1} + h.c.\right)$$

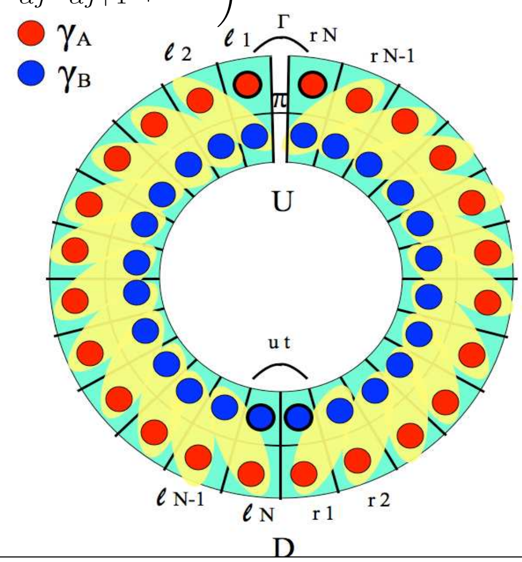
$$\alpha = \ell, r$$

$$H_\alpha = \sum_{j=1}^{N-1}\left(-\frac{t}{2}e^{ig_{\alpha j}}c^\dagger_{\alpha j}c_{\alpha j+1} + \frac{\Delta}{2}\,e^{i(\phi_{\alpha j}+\phi_{\alpha j+1})/2}\,c_{\alpha j}\,c_{\alpha j+1} + h.c.\right)$$

$$g_{\alpha j} = -\frac{e}{\hbar c}\int_{\alpha j}^{\alpha j+1}\vec{A}\cdot d\vec{l},$$

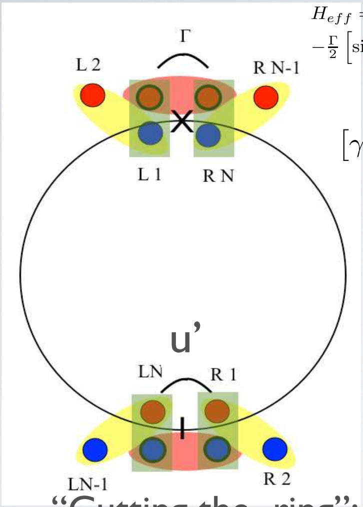
$$\phi_{\alpha j} = \theta_{\alpha j} - \frac{2e}{\hbar c}\int_{\ell 1}^{\alpha j}\vec{A}\cdot d\vec{l},$$

$$\varphi = \phi_{\ell 1} - \phi_{rN} = \frac{2e}{\hbar c}\oint\vec{A}\cdot d\vec{\ell} \quad = \pi$$



P.Lucignano et al. PRB(2013)

$$H_{eff} = \frac{i}{2\sqrt{2}}\, tu \left[f_{end}\left(f_B - f_A^\dagger\right) - h.c.\right] + t\left[d_A^\dagger d_B + h.c.\right]$$
$$-\frac{\Gamma}{2}\left[\sin\frac{\varphi}{2}\left(2d_{end}^\dagger d_{end} - 1\right) - i\sqrt{2}\cos\frac{\varphi}{2}\left(d_{end}(d_B + d_A^\dagger) - h.c.\right) + \sin\frac{\varphi}{2}\left(d_B d_A + d_A^\dagger d_B^\dagger + d_A d_A^\dagger - d_B d_B^\dagger\right)\right]$$



Projecting onto the lowest lying bands

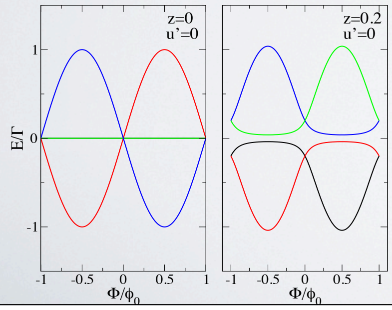
$$\left[\gamma^\ell_{A1},\ \gamma^r_{AN},\ \gamma^\ell_{BN},\ \gamma^r_{B1}\right]$$

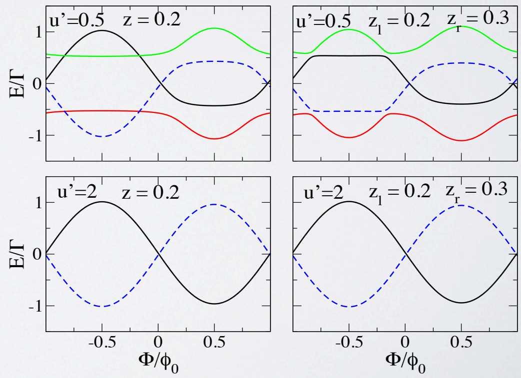
$$H_M = i\Gamma \begin{pmatrix} 0 & \sin\frac{\varphi}{2} & z_\ell & 0 \\ -\sin\frac{\varphi}{2} & 0 & 0 & z_r \\ -z_\ell & 0 & 0 & u' \\ 0 & -z_r & -u' & 0 \end{pmatrix}$$

$$E_\pm \sim \Gamma\sin\left(\frac{\varphi}{2}\right) \quad \langle 2n_{end} - 1\rangle_{\mathcal{P}} = \pm\Gamma\sin\left(\frac{\varphi}{2}\right)$$

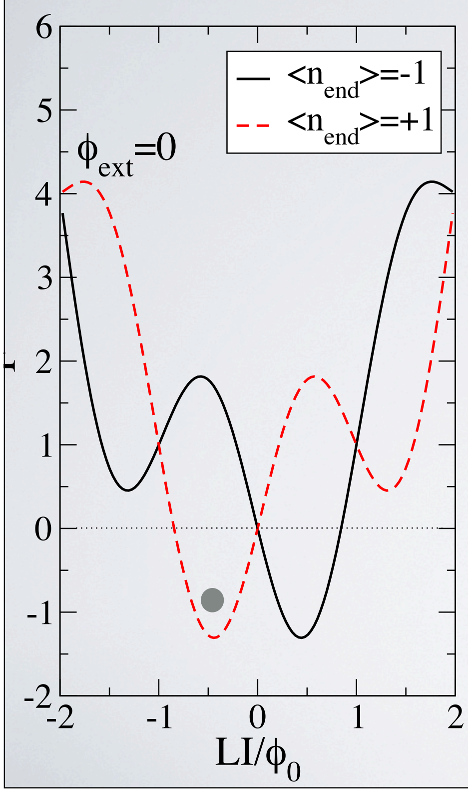
Closed loop structure: the topology forces the zero energy crossing!

"Cutting the ring": the BMS splits

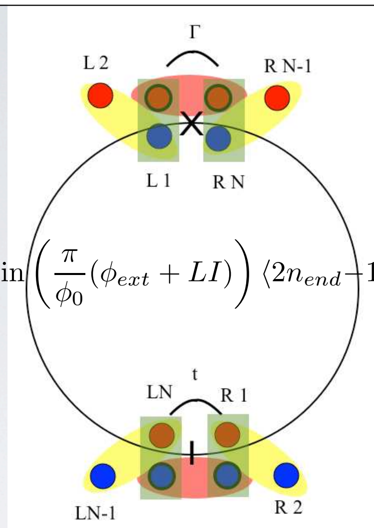




Free energy

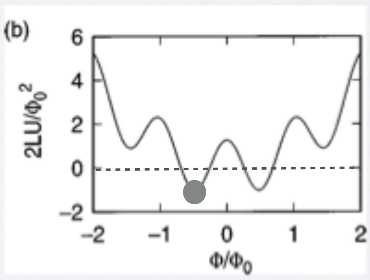


$$F_{\mathcal{P}}\left(I,\phi_{ext}\right) = \frac{1}{2}LI^2 - \frac{\phi_0 I_c}{2\pi}\sin\left(\frac{\pi}{\phi_0}(\phi_{ext} + LI)\right)\langle 2n_{end} - 1\rangle_{\mathcal{P}}$$



minima at $\pm\Phi_0/2$ for the two Fermion parities.

compare with the trivial case:



this one is not sensitive to charge poisoning !

Summary

3D TI in the long wavelength continuum model:

- TI provides one helical Dirac boundary state at each surface
  - s-wave proximity gives spinless helical particles with effective p-wave pairing
  - HTc superconductors with TI offer new functionalities:

frustrated ring : $\lim_{t\to\infty} \langle \Phi \rangle = \mathbf{Tr}\left\{\hat{\rho}\,\Phi\right\} \sim 0$