Improving Measurement Precision with Weak Measurements

Yaron Kedem

Quantum Engineering of States and Devices
04/09/2014

Outline

- Overview of weak measurements
- Signal to noise ratio in weak measurements
- Using noise to increase the signal to noise ratio

If time permits

- Time and frequency as a meter for weak measurements
- New derivation for imaginary weak values

Weak Measurements

- Three-step procedure:
  1. System preparation (preselection): $|\psi\rangle$
  2. Weak coupling to a meter: $H = g(t)PC$
  3. Postselection on the system: $\langle\phi|$
- The final state of the meter is given by the weak value:

$$\boxed{C_w = \frac{\langle\phi|C|\psi\rangle}{\langle\phi|\psi\rangle}}$$

- For example:

$$\delta Q = g\mathrm{Re}C_w$$

$$\delta P = 2g\mathrm{Var}(P)\mathrm{Im}C_w$$

The Von Neumann Scheme

System $\left| C = ? \right\rangle$

$$H = k\delta\left(t - t_0\right)\hat{C}\hat{P}$$

The Von Neumann Scheme

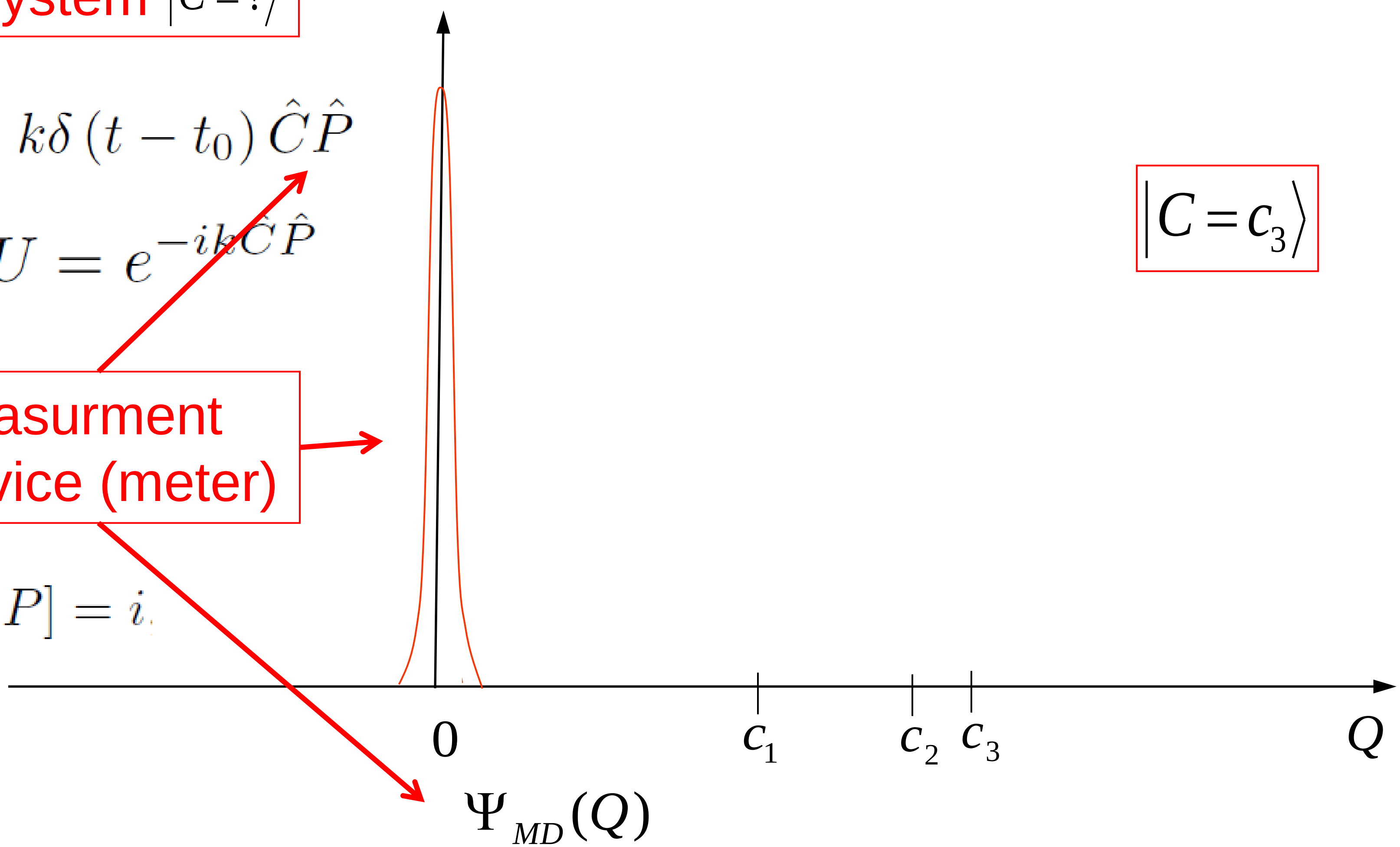
System $|C=?\rangle$

$$H = k\delta\left(t - t_0\right)\hat{C}\hat{P}$$

$$\Rightarrow U = e^{-ik\hat{C}\hat{P}}$$

Measurment Device (meter)

$$[Q, P] = i$$

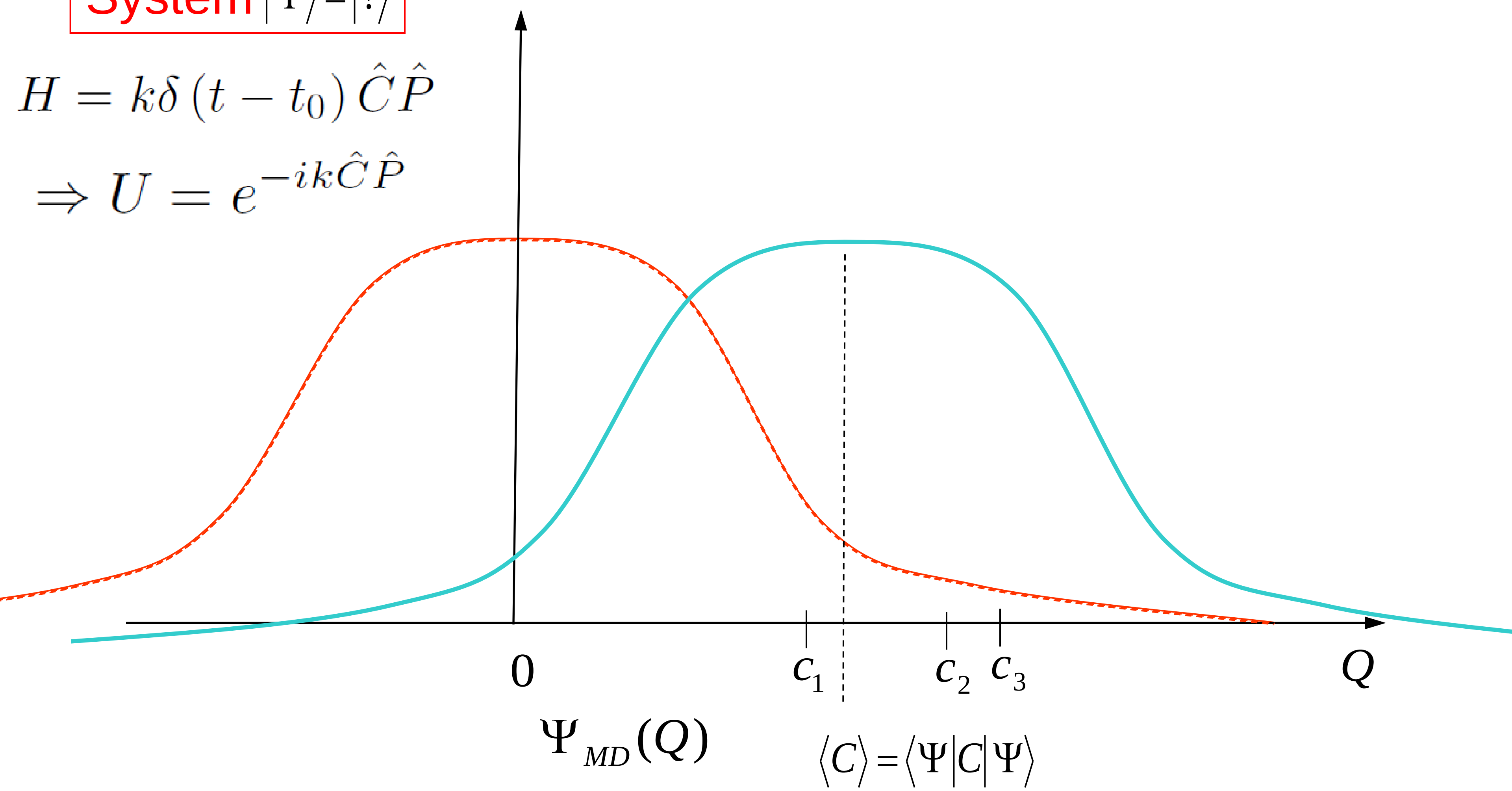


The Von Neumann Scheme

System $|\Psi\rangle = |?\rangle$

$$H = k\delta\left(t - t_0\right)\hat{C}\hat{P}$$

$$\Rightarrow U = e^{-ik\hat{C}\hat{P}}$$



Weak Measurements

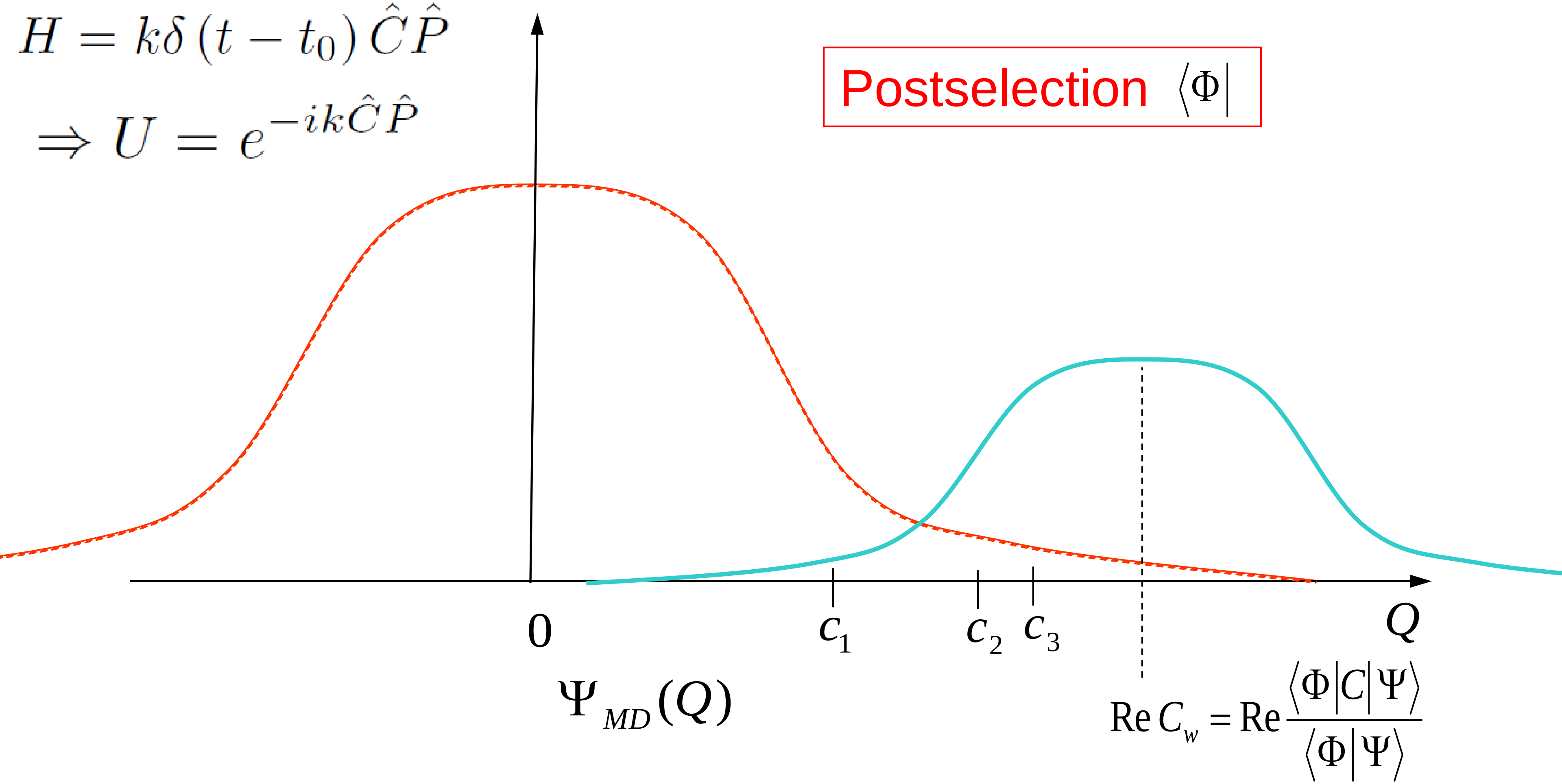
System $|\Psi\rangle$

small ΔQ $\rightarrow$ large ΔP $\rightarrow$ large H $\rightarrow$ “strong”
large ΔQ $\rightarrow$ small ΔP $\rightarrow$ small H $\rightarrow$ “weak”

$$H = k\delta\left(t - t_0\right)\hat{C}\hat{P}$$

$$\Rightarrow U = e^{-ik\hat{C}\hat{P}}$$

Postselection $\langle\Phi|$



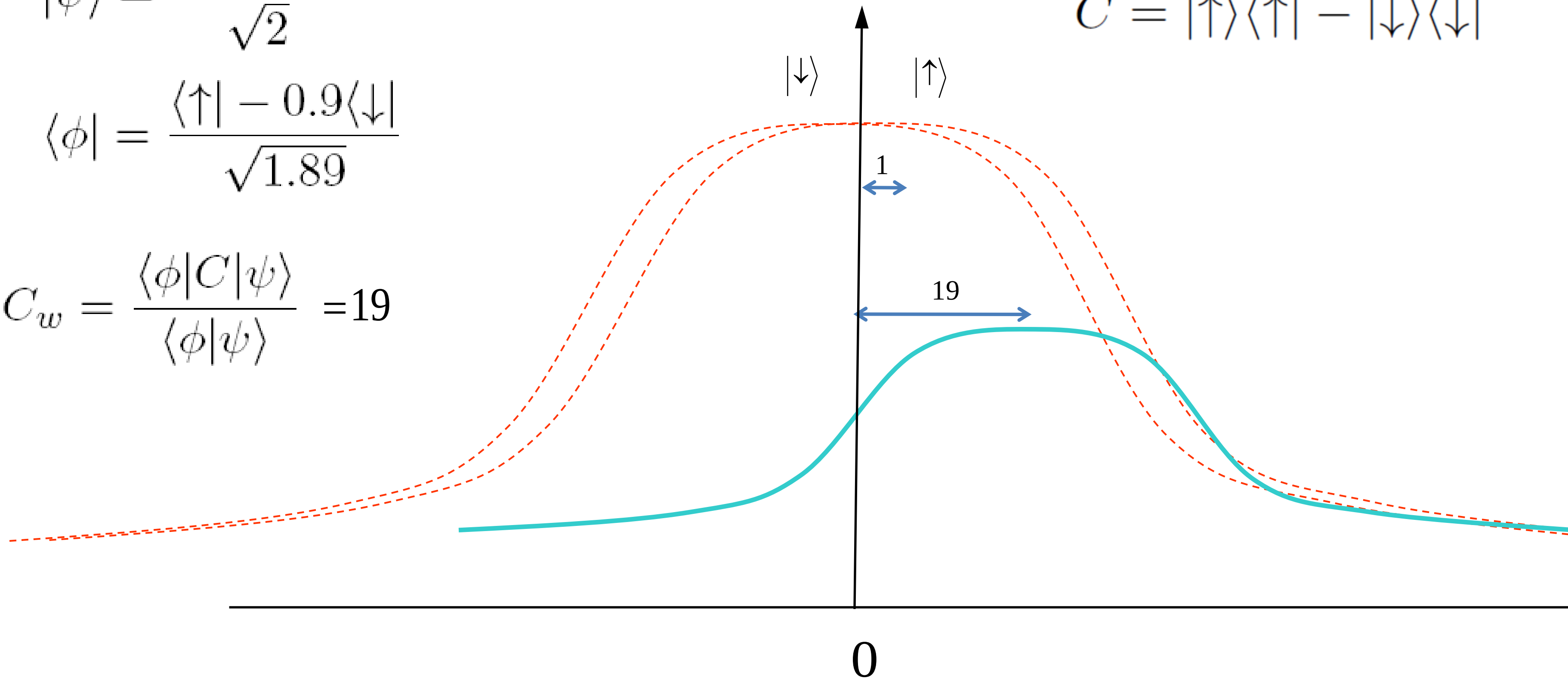
How does it happen?

$$|\psi\rangle = \frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}$$

$$\langle\phi| = \frac{\langle\uparrow| - 0.9\langle\downarrow|}{\sqrt{1.89}}$$

$$C_w = \frac{\langle\phi|C|\psi\rangle}{\langle\phi|\psi\rangle} = 19$$

$$C = |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|$$



Slightly more rigorous derivation

$$|\psi\rangle\,|\Psi_{in}(Q)\rangle$$

The meter's final state is its initial state after it was operated on by the evolution operator:

$$\langle\phi|e^{-ik\hat{C}\hat{P}}|\psi\rangle \simeq \langle\phi|\psi\rangle e^{-ikC_w\hat{P}}$$

For example, a Gaussian meter:

$$\boxed{\delta Q = g\mathrm{Re}C_w}$$

$$\Psi_{in}(Q) \sim e^{-\frac{Q^2}{2\Delta^2}} \Rightarrow \Psi_{fin}(Q) \sim e^{-\frac{(Q-kC_w)^2}{2\Delta^2}}$$

$$\langle\phi|\psi\rangle$$

$$\Rightarrow |\Psi_{fin}(Q)|^2 \sim e^{-\frac{(Q-k\mathrm{Re}C_w)^2}{\Delta^2}}$$

$$\Updownarrow$$

$$\Psi_{in}(P) \sim e^{-\frac{P^2\Delta^2}{2}} \Rightarrow \Psi_{fin}(P) \sim e^{-\frac{P^2\Delta^2}{2}-ikC_wP}$$

$$\boxed{\delta P = 2g\mathrm{Var}(P)\mathrm{Im}C_w}$$

$$\Rightarrow |\Psi_{fin}(P)|^2 \sim e^{-(P-k\Delta^{-2}\mathrm{Im}C_w)^2\Delta^2}$$

Weak Measurements of Light Chirality with a Plasmonic Slit

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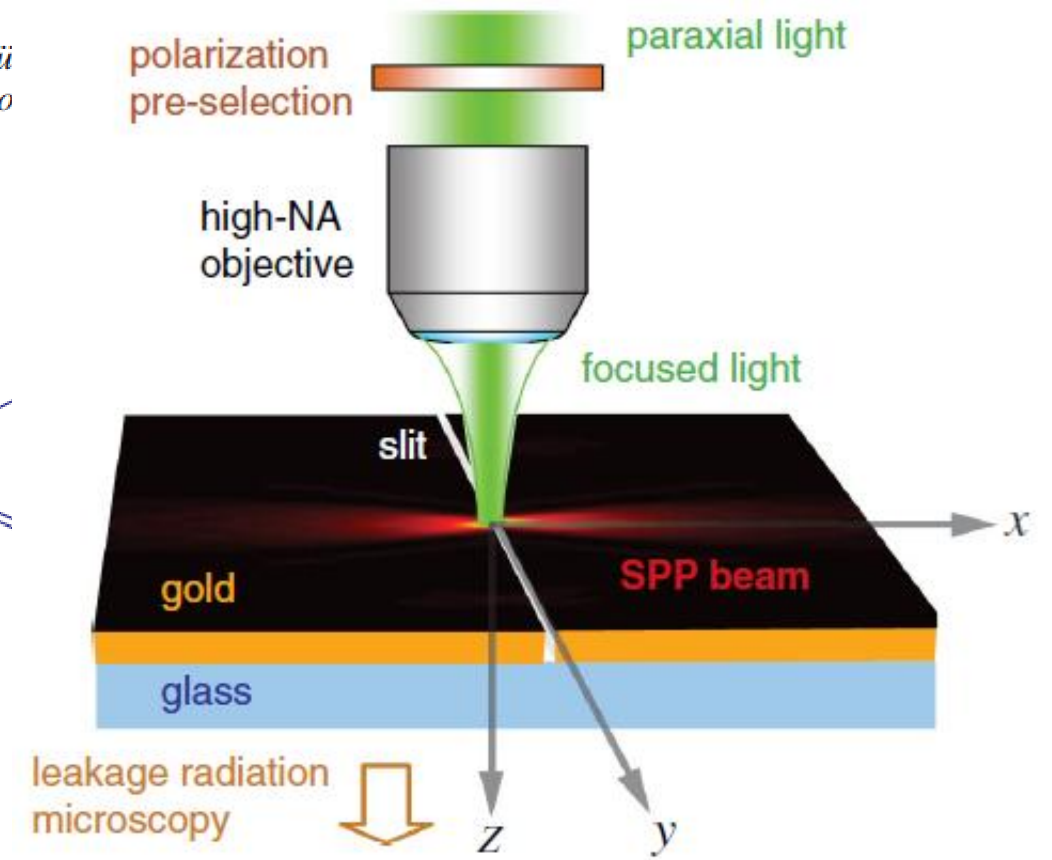
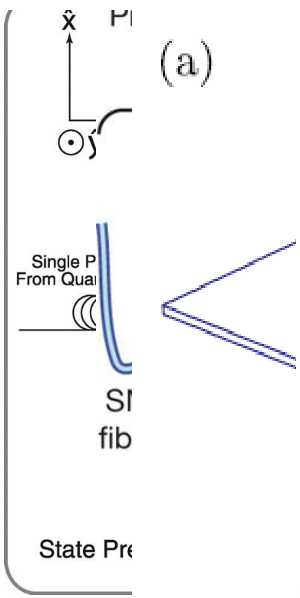
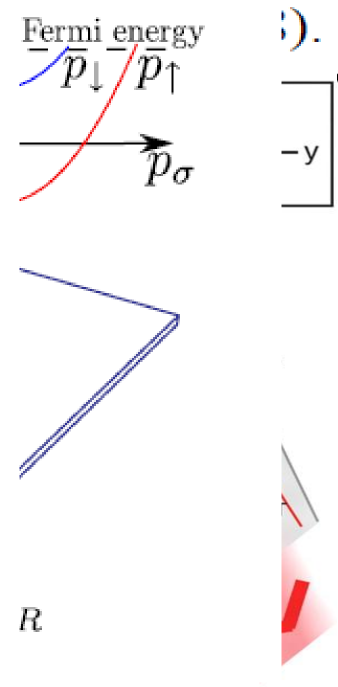
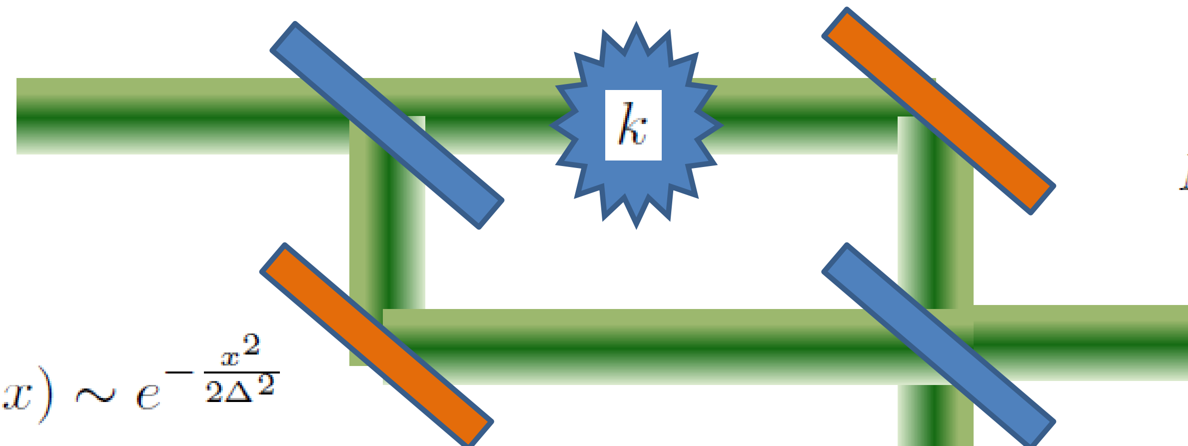






SNR in weak measurements

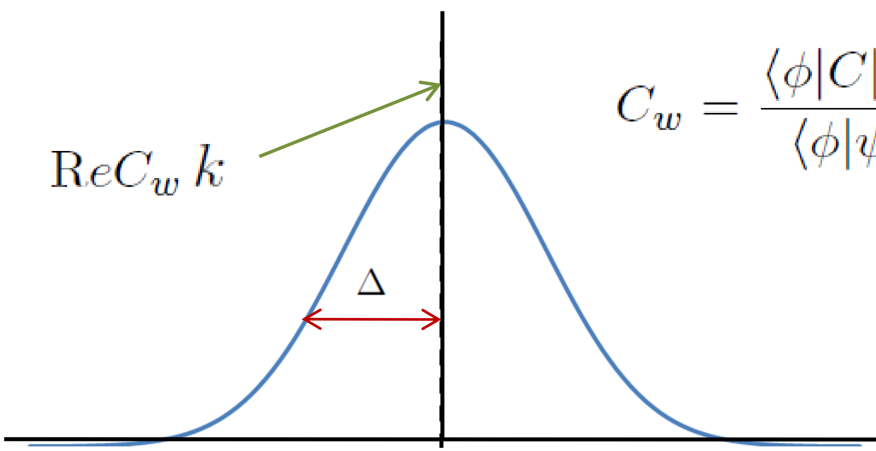


$$N_\phi = N\left|\langle\phi|\psi\rangle\right|^2 \sim 1/\left|C_w\right|^2$$

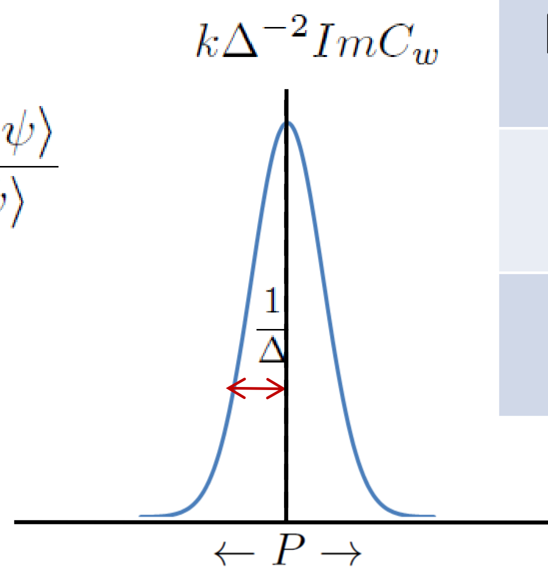
$$\Psi(x) \sim e^{-\frac{x^2}{2\Delta^2}}$$

$$U = e^{-ik\hat{P}\hat{C}}$$

$$U = e^{-ik\hat{P}C_w}$$

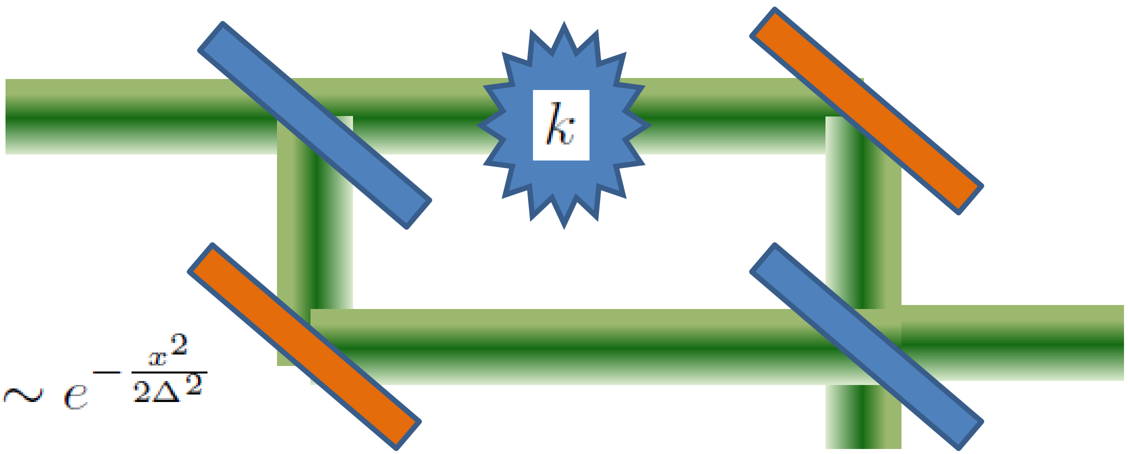


$$C_w = \frac{\langle\phi|C|\psi\rangle}{\langle\phi|\psi\rangle}$$



Method	SNR
Normal	$\sqrt{N}\,\frac{k}{\Delta}$
Re WV	$\sqrt{N_\phi}\,\frac{k\mathrm{Re}C_w}{\Delta}$
Im WV	$\sqrt{N_\phi}\,\frac{k\mathrm{Im}C_w}{\Delta}$

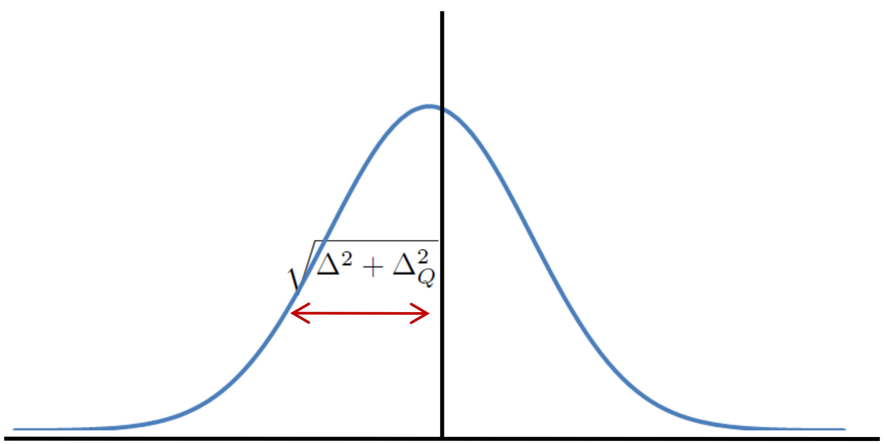
Technical ”x” noise

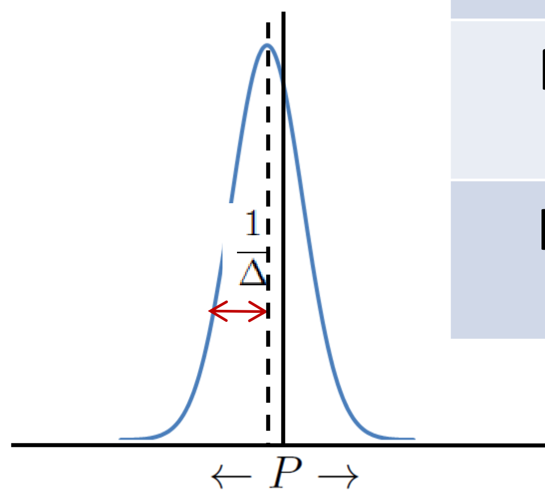


$$\Psi(x) \sim e^{-\frac{x^2}{2\Delta^2}}$$

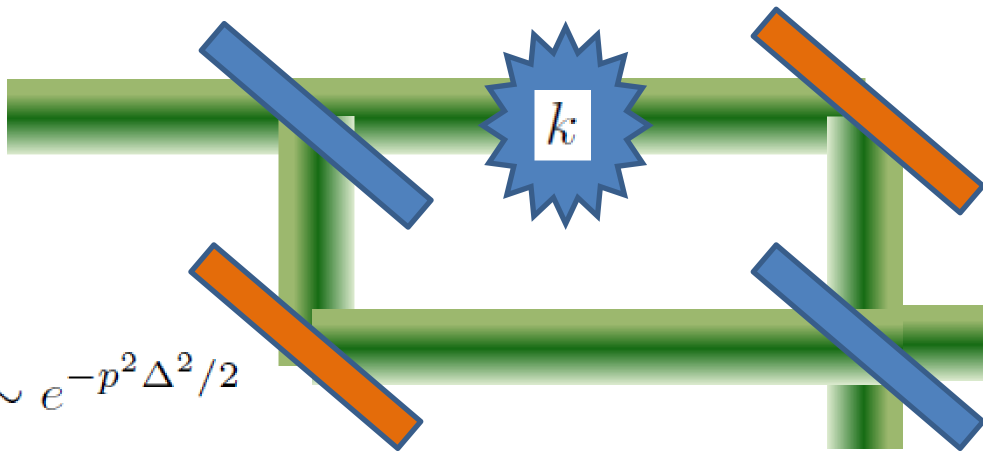
$$\Psi(x) \sim e^{-\frac{(x+Q(t))^2}{2\Delta^2}}$$

Method	SNR
Normal	$\frac{\sqrt{N}k}{\sqrt{\Delta^2+\Delta_Q^2}}$
Re WV	$\frac{\sqrt{N_\phi}k\mathrm{Re}C_w}{\sqrt{\Delta^2+\Delta_Q^2}}$
Im WV	$\sqrt{N_\phi}\frac{k\mathrm{Im}C_w}{\Delta}$





Technical "P" noise

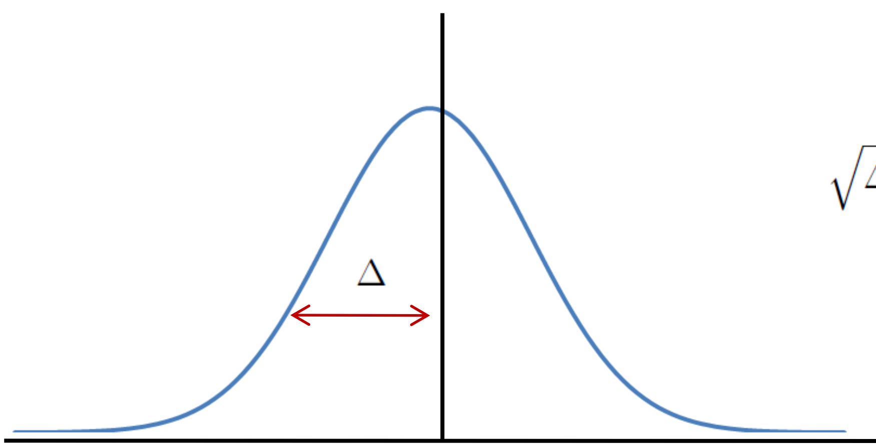


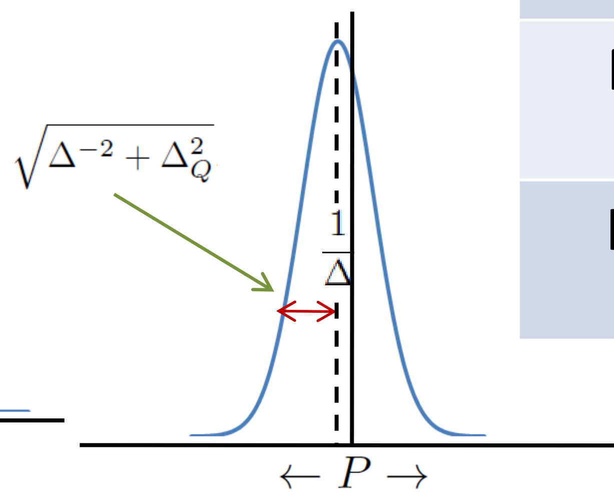
$$\boxed{\sqrt{N_\phi} k \mathrm{Im} C_w \sqrt{\Delta^{-2} + \Delta_Q^2}}$$

$$\Psi(p) \sim e^{-p^2 \Delta^2 / 2}$$

$$\Psi(p) \sim e^{-(p+Q(t))^2 \Delta^2 / 2}$$

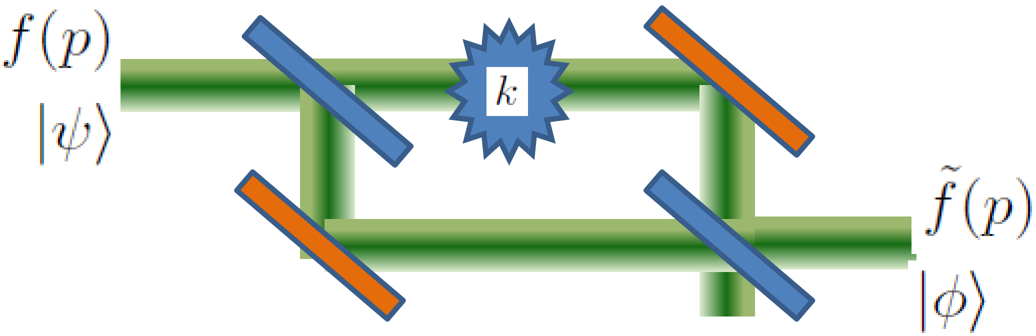
$$k \left(\Delta^{-2} + \Delta_Q^2 \right) \mathrm{Im} C_w$$





Method	SNR
Normal	$\sqrt{N} \frac{k}{\Delta}$
Re WV	$\sqrt{N_\phi} \frac{k \mathrm{Re} C_w}{\Delta}$
Im WV	

"Mathematical" derivation



$$\equiv e^{-ik\hat{P}\hat{C}}$$

$$Prob\left(|\phi\rangle|p\right) = \left|\langle\phi|e^{-ikp\hat{C}}|\psi\rangle\right|^2 = |\langle\phi|\psi\rangle|^2\left(1 + 2k\mathrm{Im}C_w p\right) + O(kp)^2$$

$$f(p) \to \tilde{f}(p) \simeq f(p)\left(1 + 2k\mathrm{Im}C_w p\right)$$

$$\langle p\rangle_F = \int dp\tilde{f}(p)p \simeq \frac{\int dp f(p)(1 + 2k\mathrm{Im}C_w p)p}{\int dp f(p)(1 + 2k\mathrm{Im}C_w p)} \simeq \langle p\rangle_I + 2k\mathrm{Im}C_w\left(\langle p^2\rangle_I - \langle p\rangle_I^2\right)$$

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$$\boxed{\delta p = 2k\mathrm{Im}C_w(\Delta p)^2}$$

Conclusions

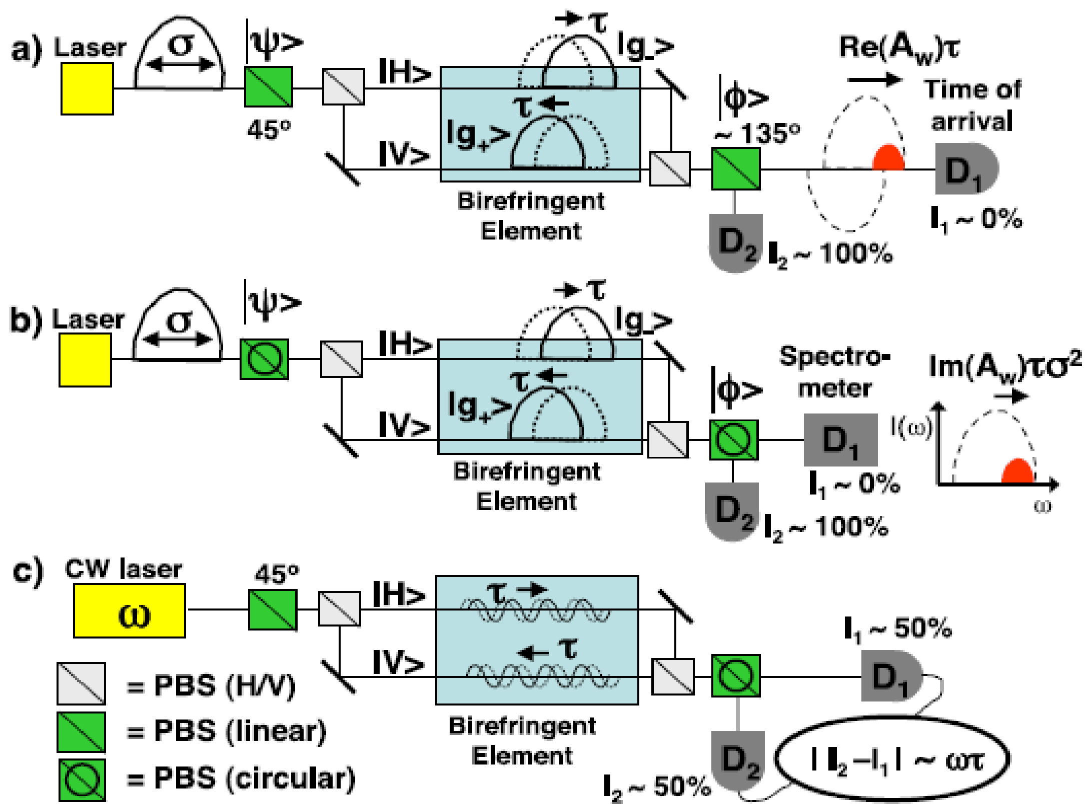
- The concept of Weak Value offers intuition regarding a measurement process
- A clear view of an experimental setup can help overcoming technical difficulties
- Uncertainty can be seen as a resource in some scenarios
- Results derived from the basic structure of QM are applicable in many contexts

Can time be a meter?

PRL **105,** 010405 (2010) PHYSICAL REVIEW LETTERS week ending 2 JULY 2010

Measuring Small Longitudinal Phase Shifts: Weak Measurements or Standard Interferometry?

Nicolas Brunner[1] and Christoph Simon[2]



Can time be a meter?

PRL **111,** 023604 (2013) PHYSICAL REVIEW LETTERS week ending 12 JULY 2013

Demonstration of Weak Measurement Based on Atomic Spontaneous Emission

Itay Shomroni, Orel Bechler, Serge Rosenblum, and Barak Dayan*

PRL **111,** 033604 (2013) PHYSICAL REVIEW LETTERS week ending 19 JULY 2013

Phase Estimation with Weak Measurement Using a White Light Source

Xiao-Ye Xu,[1] Yaron Kedem,[2] Kai Sun,[1] Lev Vaidman,[2] Chuan-Feng Li,[1,*] and Guang-Can Guo[1]

August 15, 2013 / Vol. 38, No. 16 / OPTICS LETTERS 2949

Weak-values technique for velocity measurements

Gerardo I. Viza,[1,*] Julián Martínez-Rincón,[1] Gregory A. Howland,[1] Hadas Frostig,[2]
Itay Shomroni,[3] Barak Dayan,[3] and John C. Howell[1,2]

Alternative derivation

Consider the simple(st?) Hamiltonian:

$$H = g(t)C, \quad \int g(t)dt = k \quad \Rightarrow \quad U = e^{-ikC}$$

Starting with $|\Psi\rangle$, the probability to find $|\Phi\rangle$, in the end is

$$P\left(|\Phi\rangle\middle|k\right) = \left|\langle\Phi|e^{-ikC}|\Psi\rangle\right|^2 = \left|\langle\Phi|\Psi\rangle\right|^2\left(1 + 2k\mathrm{Im}C_w\right) + O(k^2)$$

If k had an initial distribution $f(k)$, after a postselection to $|\Phi\rangle$, it is distributed as

$$f_\Phi(k) = \frac{f(k)\,P\left(|\Phi\rangle\middle|k\right)}{P\left(|\Phi\rangle\right)} \simeq \frac{f(k)\left(1 + 2k\mathrm{Im}C_w\right)}{1 + 2\langle k\rangle\mathrm{Im}C_w}$$

The new average of k is given by

$$\boxed{\delta k \simeq 2\mathrm{Im}C_w\mathrm{Var}(k)}$$

$$\langle k\rangle_\Phi \simeq \frac{\langle k\rangle + 2\langle k^2\rangle\mathrm{Im}C_w}{1 + 2\langle k\rangle\mathrm{Im}C_w} \simeq \langle k\rangle + 2\mathrm{Im}C_w\left(\langle k^2\rangle - \langle k\rangle^2\right)$$

Applying to experiments

PRL 111, 023604 (2013) PHYSICAL REVIEW LETTERS week ending 12 JULY 2013

Demonstration of Weak Measurement Based on Atomic Spontaneous Emission

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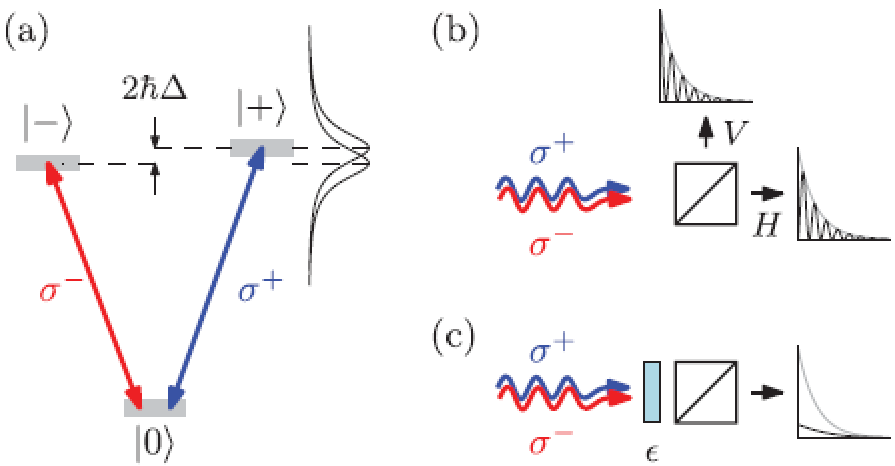
(Received 22 October 2012; published 12 July 2013)



$$H = \Delta C \qquad k = \Delta t$$

t is the time the atom was in an excited state

$$\boxed{\delta k \simeq 2\mathrm{Im}C_w \mathrm{Var}\left(k\right)}$$

$$\Rightarrow \quad \delta t \simeq 2\mathrm{Im}C_w \Delta \mathrm{Var}\left(t\right)$$

$$\int tP(t)dt - \int tP_0(t)dt \simeq \frac{2\Delta \cot\epsilon}{\Gamma^2} = 2\Delta\, \mathrm{Var}P_0\, \mathrm{Im}\langle\hat{\sigma}_z\rangle_W \qquad (4)$$

Applying to experiments

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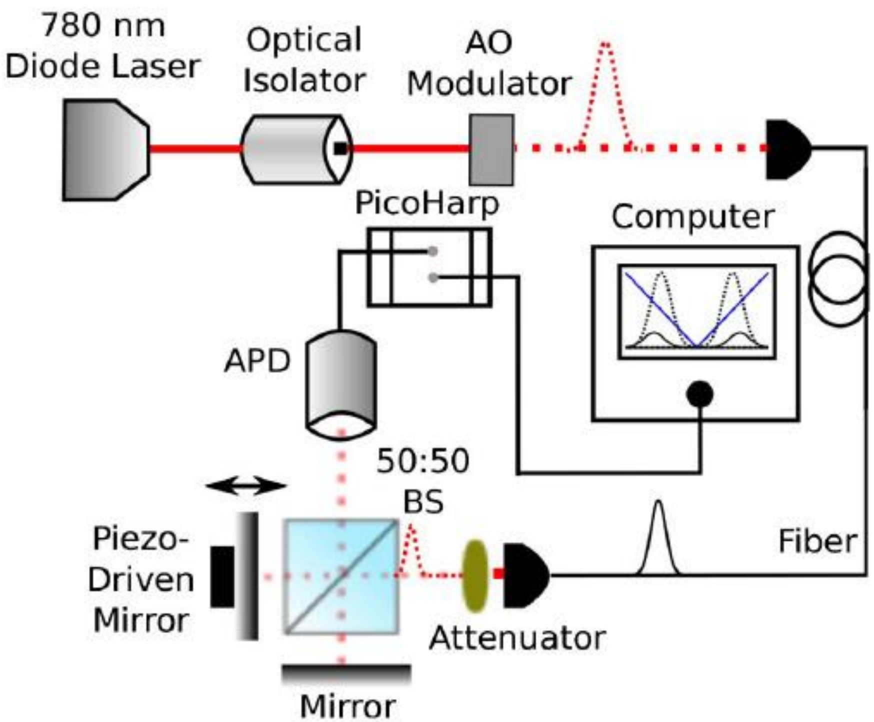

$$k = vt\frac{2\pi}{\lambda}$$

t is when the light hits the mirror

$$\boxed{\delta k \simeq 2\mathrm{Im}C_w \mathrm{Var}\left(k\right)}$$

$$\Rightarrow \delta t \simeq 2\mathrm{Im}C_w v\frac{2\pi}{\lambda}\mathrm{Var}\left(t\right)$$

Applying to experiments

PRL **111,** 033604 (2013) PHYSICAL REVIEW LETTERS week ending 19 JULY 2013

Phase Estimation with Weak Measurement Using a White Light Source

Xiao-Ye Xu,[1] Yaron Kedem,[2] Kai Sun,[1] Lev Vaidman,[2] Chuan-Feng Li,[1,*] and Guang-Can Guo[1]
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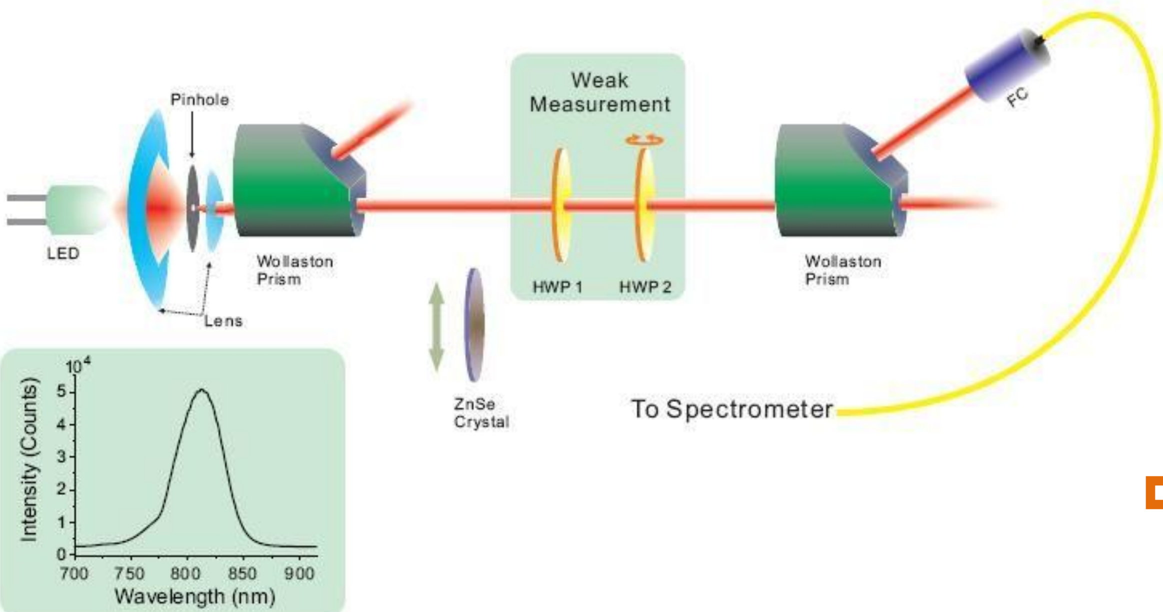


$$H = \omega C \qquad k = \omega \tau$$

$$\boxed{\delta k \simeq 2\mathrm{Im} C_w \mathrm{Var}\left(k\right)}$$

$$\Rightarrow \quad \delta \omega \simeq 2\mathrm{Im} C_w \tau \mathrm{Var}\left(\omega\right)$$

Phase Estimation with Weak Measurement Using a White Light Source (LED)

Two problems: 1. Linear polarizers → real weak value

2. The average frequency is large

$$\frac{\omega_0}{2\pi} \sim 400\ \mathrm{T}Hz, \qquad \tau \sim 1\ \mathrm{a}s\ (10^{-18}\mathrm{s}) \qquad \Rightarrow\ \omega_0\tau \sim \frac{1}{500}$$

$$|C_w| \sim 1000 \quad \Rightarrow \omega_0\tau|C_w| > 1$$

One solution: we separate the known average and the uncertainty. The average produce an imaginary part and the uncertainty is smaller than the average

$$U = e^{-ig\hat{C}\hat{P}} = e^{-ig\hat{C}(\hat{P}-P_0)}e^{-ig\hat{C}P_0} \qquad |\psi\rangle \to |\psi'\rangle = e^{-ig\hat{C}P_0}|\psi\rangle$$

$$C_w = \frac{\langle\phi|C|\psi'\rangle}{\langle\phi|\psi'\rangle} = \frac{\langle\phi|Ce^{-ig\hat{C}P_0}|\psi\rangle}{\langle\phi|e^{-ig\hat{C}P_0}|\psi\rangle} \simeq \frac{1}{2\langle\phi|\psi\rangle - igP_0}$$

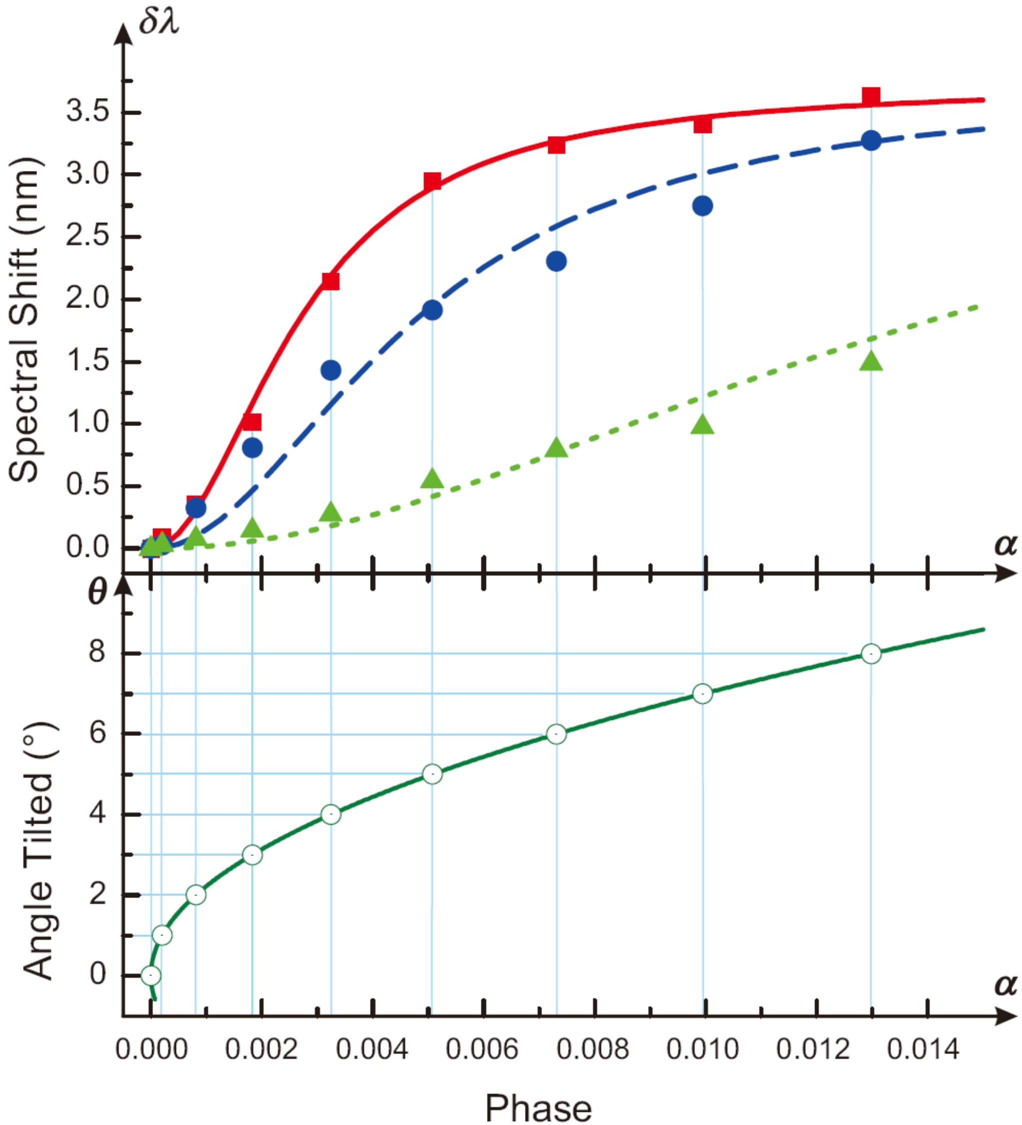
Phase Estimation with Weak Measurement Using a White Light Source (LED)

$$C_w \simeq \frac{1}{2\langle\phi|\psi\rangle - igP_0} = \frac{1}{\beta - i\alpha}$$

⇨ $$\mathrm{Im}C_w \simeq \frac{\alpha}{\beta^2+\alpha^2}$$

$$\delta\omega \simeq 2\mathrm{Im}C_w\tau\mathrm{Var}\left(\omega\right)$$

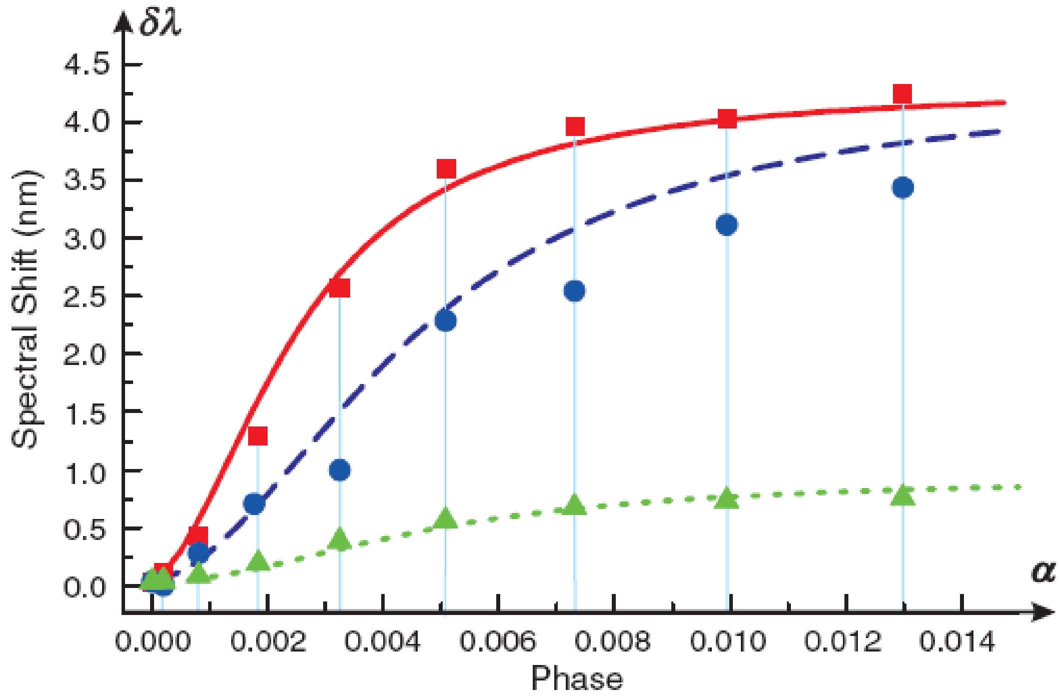
⇨ $$\delta\lambda \simeq 2\frac{\mathrm{Var}\left(\lambda\right)}{\lambda_0}\frac{\alpha^2}{\beta^2+\alpha^2}$$

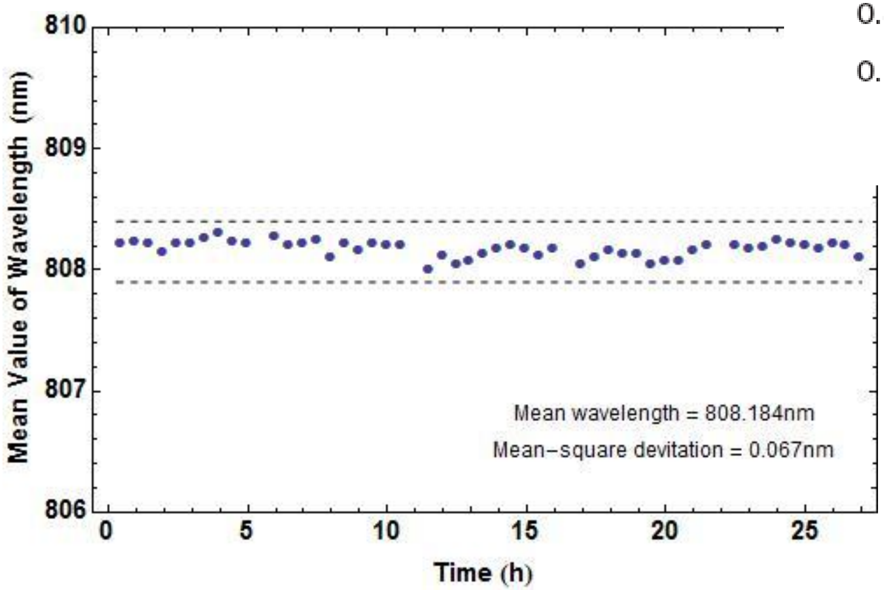


Phase Estimation with Weak Measurement Using a White Light Source (LED)

$$\delta\lambda \simeq 2\frac{\mathrm{Var}\,(\lambda)}{\lambda_0}\frac{\alpha^2}{\beta^2+\alpha^2}$$

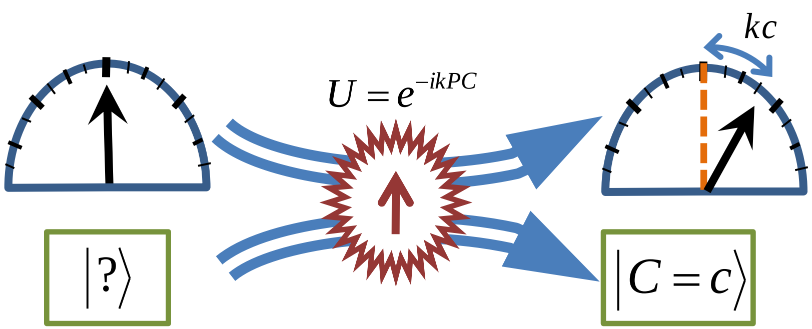
- ➢Dispersion have (almost) no effect
- ➢Spectral filter decrease the shift drastically



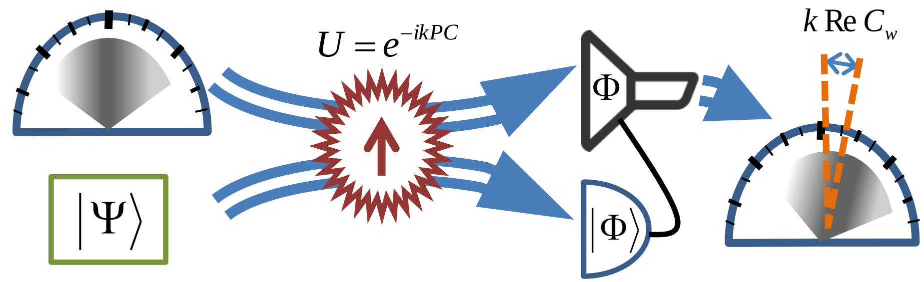


- ➢Long stability time

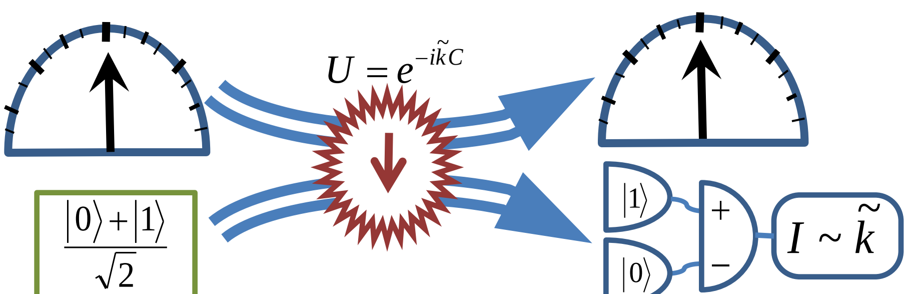
New concept, New technique



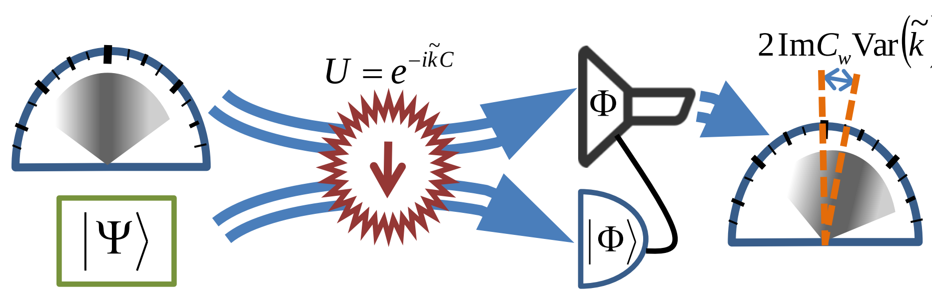
Von Neumann
Scheme



Weak measurements
with real weak value



Interferometry



Weak measurements with
imaginary weak value

Conclusions

- An alternative derivation for the effect of an imaginary weak value
- The new formalism does not require the meter to be a quantum variable
- A method based on this effect is described by a new measurement model
- Possible applications for a wide range of tasks

Any Questions?