Junction of two topological insulators and a p-wave superconductor

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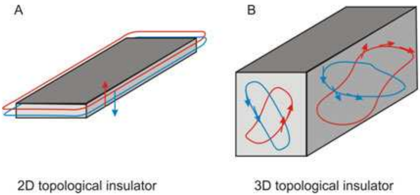
Soori, Deb, Sengupta and Sen, Phys. Rev. B 87, 245435 (2013)

Organization of the talk

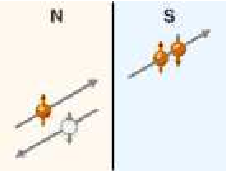
- Introduction
- Scattering at the junction
- Hamiltonian and energy dispersion
- Probability and charge current
- A general boundary condition
- Charge conductance
- Spin conductance
- Summary

- Topological insulators are gapped in the bulk but have gapless states on the surface. So the surface states are metallic.



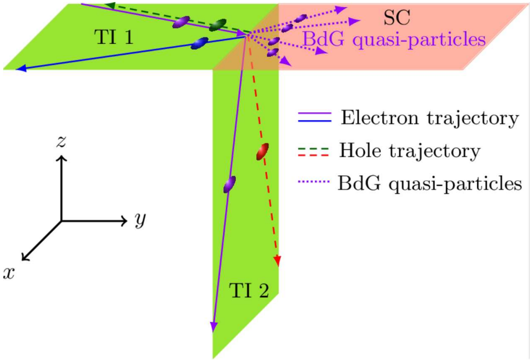
- Described by massless Dirac equation, edge states are robust against static (Time-reversal-invariant) disorder.

- Transmission across a junction of a normal metal (NM) and a superconductor (SC) has been extensively studied (Blonder et al, Phys. Rev. B 25, 4515 (1982)1982, K. Sengupta et al,Phys. Rev. B 63, 144531 (2001))
- The sub-gap transport in such junctions is governed by Andreev reflection



- A junction between a topological insulator (TI) and superconductor (SC) is different and more complex because :
  1. Surface states of a TI display spin-momentum locking
  2. Particle and hole are coupled in a superconductor
- Four component spinor formalism to describe both spin and particle-hole degrees of freedom

Scattering at the junction



- Schematic diagram (3D-pic) of the set-up. Normal or Andreev scatterings of the incident electron on TI-1 and TI-2 and BdG quasiparticle (evanescent) transmissions on the SC side

Hamiltonian and energy dispersion

- Hamiltonian on a TI-surface:

$$H_1 = \int\int dk_i\, dk_j\, \psi^\dagger_{\vec{k}}\, [\hbar v_F \hat{n} \cdot (\sigma \times \overrightarrow{k}) - \mu_{TI} I]\, \psi_{\vec{k}}$$

  where ψ is a two component spinor, v_F is the Fermi velocity , $\hat{n}$ points normal to the 2-D surface , k_i, k_j are momenta in the 2-D plane

- $E = -\mu_{TI} \pm \hbar v_F \sqrt{k_i^2 + k_j^2}$
- Wave fns on the SC side are described by a four component

  BdG spinor: $\Psi_{SC} = \begin{pmatrix} \Psi_\uparrow \\ \Psi_\downarrow \\ \Psi^\star_\uparrow \\ \Psi^\star_\downarrow \end{pmatrix}$

- SC Hamiltonian :

$$H_{SC} = \int_{-\infty}^{\infty} dx \int_0^{\infty} dy\, \Psi^\dagger(x,y) \Big[\Big(-\frac{\hbar^2 \vec{\nabla}^2}{2m} - \mu_{SC} \Big) \tau^z + \Delta(x,y) \Big] \Psi(x,y),$$

- SC pair potential $\Delta(x,y)$ can be s-wave (singlet) or p-wave (triplet)
- SC pair potential $\Delta(x,y)$ for p-wave (triplet) SC:

$$\Delta(x,y) = \begin{pmatrix} 0 & \Delta_0 f(\vec{k})(\vec{d}\cdot\vec{\sigma})i\sigma^y \\ -\Delta_0 f^*(\vec{k})(\vec{d}\cdot\vec{\sigma}^*)i\sigma^y & 0 \end{pmatrix}$$

where $f(\vec{k}) = -i(a_x\partial_x + a_y\partial_y)$

We choose $a_x = 0$ and $a_y = 1$ making the pair potential anisotropic

- The choice of $\vec{d}$ $(=\hat{x},\hat{y},\hat{z})$ determines the spin-state of the Cooper pair

- $E = \pm\sqrt{(\frac{\hbar^2}{2m}(k_x^2+k_y^2)-\mu_{SC})^2+\Delta_0^2(\overrightarrow{a}\cdot\overrightarrow{k})(\overrightarrow{a^*}\cdot\overrightarrow{k})}$

- Let's assume : $\Psi = \begin{pmatrix} \psi \\ \phi \end{pmatrix}$ where ψ and ϕ are the upper two and lower two components of the BdG spinor Ψ
- Probability density : $\rho_p = \psi^\dagger\psi + \phi^\dagger\phi = \Psi^\dagger\Psi$
  Charge density : $\rho_c = e(\psi^\dagger\psi - \phi^\dagger\phi) = e\Psi^\dagger\tau^z\Psi$
- Corresponding currents $\vec{J_p}$ and $\vec{J_c}$ satisfy the equation of continuity : $\partial_t\rho + \vec{\nabla}\cdot\vec{J} = 0$
- Using the equations of motion and continuity we can find the currents :

$$\vec{J_p}^{TI} = v_F[\psi^\dagger\hat{n}\times\vec{\sigma}\psi + \phi^\dagger\hat{n}\times\vec{\sigma}^*\phi]$$
$$\vec{J_p}^{SC} = \frac{\hbar}{m}Im(\Psi^\dagger\tau^z\vec{\nabla}\Psi) + \vec{J}_{pair}$$

- $\vec{J}_{pair} = \hat{x}\frac{2\Delta_0}{\hbar}Re[a_x\psi^\dagger(\vec{d}\cdot\vec{\sigma})i\sigma^y\phi] + \hat{y}\frac{2\Delta_0}{\hbar}Re[a_y\psi^\dagger(\vec{d}\cdot\vec{\sigma})i\sigma^y\phi]$

A general boundary condition

- To get the boundary condition we use conservation of probability current normal to the junction
- $(\hat{y}\cdot\overrightarrow{J}_{p1})_{y\to 0_-} - (\hat{z}\cdot\overrightarrow{J}_{p2})_{z\to 0_-} = (\hat{y}\cdot\overrightarrow{J}_{p3})_{y\to 0_+}$
- We find that a general boundary condition involves three time-reversal invariant barriers near the junction
- Physically these barriers can arise due to gate voltages and also a lattice mismatch between TIs and SC
- General boundary condition :
  1. $\Psi_3 = M(\chi_1)\Psi_1 + \beta M^\dagger(\chi_2)\Psi_2$
  2. $\frac{\hbar}{mv_1}\partial_y\Psi_3 - 2\chi_3\Psi_3 + \frac{\Delta_0}{\hbar v_1}\begin{pmatrix} 0 & -a_y(\overrightarrow{d}\cdot\overrightarrow{\sigma})\sigma^y \\ a_y^*(\overrightarrow{d}\cdot\overrightarrow{\sigma}^*)\sigma^y & 0 \end{pmatrix}$
     $= i\sigma^x\otimes\tau^z[M(\chi_1)\Psi_1 - \beta M^\dagger(\chi_2)\Psi_2]$

$\beta = \sqrt{\frac{v_2}{v_1}}$ and $M(\chi) = \cos(\chi) - i\sin(\chi)\sigma^x\otimes\tau^z$

- We want to solve the scattering problem and find the conductance
- Consider an electron incident on the junction from TI-1 with energy E which lies in the SC gap
- The wavefunctions on TI and SC sides will look like :

$$\Psi_1 = \psi_{1p} + r_N \psi_{1p} + r_A \psi_{1h}, \quad \Psi_2 = t_N \psi_{2p} + t_A \psi_{2h}.$$

$$\Psi_3 = t_1 \psi_{sc}^{\uparrow +} + t_3 \psi_{sc}^{\uparrow -} + t_2 \psi_{sc}^{\downarrow +} + t_4 \psi_{sc}^{\downarrow -},$$

- The scattering amplitudes $r_{N(A)},\ t_{N(A)},\ t_{1,2,3,4}$ can be determined using the boundary conditions

- Conservation of the probability current at the junction implies :
  $v_1 \sin\theta_1 = v_1(\sin\theta_1 R_N + \sin\theta_{1h} R_A) + v_2(\sin\theta_2 T_N + \sin\theta_{2h} T_A)$

- Charge current conservation perpendicular to the junction :
  $J_3 = J_{1,in} - J_{2,out}$

- Incoming charge current along $\hat{y}$ on TI-1 :
  $J_{1,in} = ev_1[\sin\theta_1(1 - R_N) + \sin\theta_{1h} R_A]$

- Outgoing charge current along $-\hat{z}$ on TI-2 :
  $J_{2,out} = ev_2[\sin\theta_2 T_N - \sin\theta_{2h} T_A]$

- Hence using conservation of probability current we get :
  $J_3 = 2e(v_1 \sin\theta_{1h} R_A + v_2 \sin\theta_{2h} T_A)$

- A voltage bias V is applied on the TI-1 side maintaining the TI-2 and SC at the same Fermi energy.
- The differential conductance $G_3 = dI_3/dV$ on SC-side at an applied bias voltage V.
- Integrating over all angles of incidence θ_1, the differential conductance is:

$$G_3(V) = G_0(V) \int_0^{\pi} d\theta_1 \, [\sin\theta_{1h} R_A + \sin\theta_{2h} T_A],$$

where $R_{N(A)} = |r_{N(A)}|^2$, $T_{N(A)} = |t_{N(A)}|^2$ and

$G_0(V) = e^2 W(\mu_{TI} + eV)/(hv_1\mu_{TI})$
and W is the width of the sample

Zero bias peak

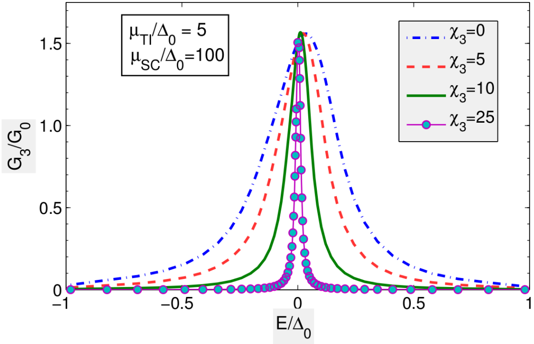


Figure: The sub-gap conductance of a p_y-wave SC in units of G_0, for $\vec{d} = \hat{z}$ and $\chi_1 = \chi_2 = 0$

- The zero bias peak is due to the mid-gap bound state i.e E$=$ 0 state on SC side
- At E$=$ 0 transmission is independent of χ_3
- Peak is sharper at large χ_3

- Conductance is different for different spin triplet states i.e for different $\overrightarrow{d}$
- For $\overrightarrow{d} = \hat{z}$ at E$=$ 0: $r_A = t_N = 0$ and t_A and hence the conductance depends only on $(\chi_1 - \chi_2)$
- For $\overrightarrow{d} = \hat{y}$ we found the similar result as $\overrightarrow{d} = \hat{z}$
- For $\overrightarrow{d} = \hat{x}$ we found $t_A = t_N = 0$, while $|r_A|^2 = (\sin\theta_1)^2$ and $|r_N|^2 = (\cos\theta_1)^2$ are independent of χ_1 , χ_2
- Electrons and holes see opposite barrier strengths χ_i and $-\chi_i$ respectively

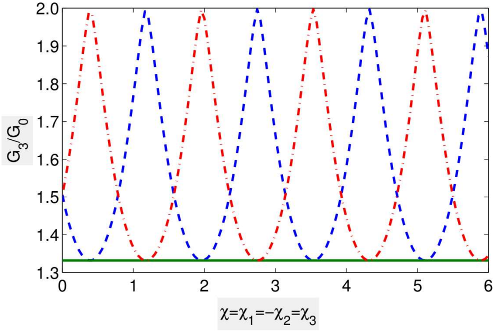


Figure: Conductance of a p_y-wave SC at $E = 0$ for different spin pairings: $\vec{d} = \hat{z}$ (blue dashed line), $\vec{d} = \hat{x}$ (green solid line) and $\vec{d} = \hat{y}$ (red dot-dashed line)

- This can be used to distinguish between different spin-triplet states of the p-wave SC.

- Typically, in a Schrödinger system, the conductance decays with the barrier strength, while in a Dirac system, conductance is a periodic function of the barrier strength.
- In our setup, the TI-sides are Dirac-like while the SC is Schrödinger-like.
- So, the conductance depends on barriers $\chi_{1,2}$ periodically and decays with barrier χ_3 on SC side

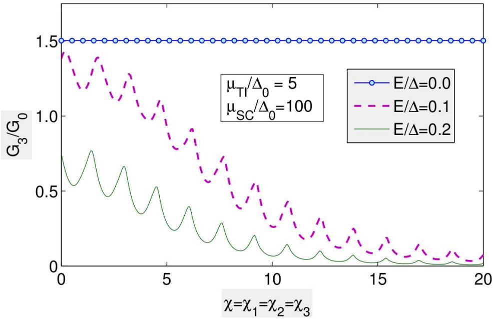


Figure: Conductance of a p_y-wave SC with $\vec{d} = \hat{z}$ at different energies in the SC gap

Spin conductance

- For $a = x, y, z$, the a component of the spin density is given by
$$\rho^a = \frac{\hbar}{2} \Psi_3^\dagger \tau^z \otimes \sigma^a \Psi_3$$

- The spin current $\vec{J}^a$ corresponding to the spin density ρ^a can be calculated on the SC side using the equations of motion.
- The corresponding spin conductance is given by $G_s^a = G_{s0} \int_0^\pi d\theta_1 \, J_y^a$, where $G_{s0} = eW(\mu_{TI} + eV)/(hv_1)^2$.
- Now, there are nine currents corresponding to three spin components and three possible choices of the $\vec{d}$ vector.

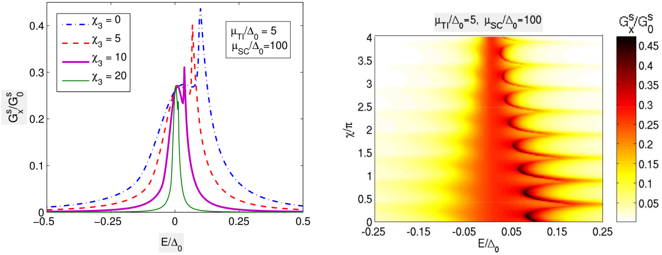
	$\vec{d} = \hat{x}$	$\vec{d} = \hat{y}$	$\vec{d} = \hat{z}$
J_s^x	0	non-zero	non-zero
J_s^y	0	non-zero	0
J_s^z	non-zero	non-zero	0

Table: Expressions for y component of spin currents for different spin pairings of the p-wave SC.

	$\vec{d} = \hat{x}$	$\vec{d} = \hat{y}$	$\vec{d} = \hat{z}$
G_s^x	0	non-zero	non-zero
G_s^y	0	0	0
G_s^z	0	0	0

Table: Spin conductances for different spin pairings.

- The x-spin conductance is non-zero for the cases $\vec{d} = \hat{y}$ and $\vec{d} = \hat{z}$.
- Let us look at the features of the x-spin conductance for the case $\vec{d} = \hat{z}$.



Left panel: G_S^x as a function of $E = eV$ for $\chi_1 = \chi_2 = 0$, $\mu_{TI}/\Delta_0 = 5$ and $\mu_{sc}/\Delta_0 = 100$.

Right panel: G_S^x as a function of $\chi = \chi_1 = \chi_2 = \chi_3$ and E.

- The unusual satellite peak (SP) in addition to the ZBP is a novel feature.

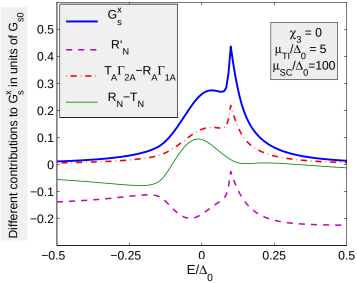
- To understand the SP, we relate the spin-currents on SC side to the physical quantities on TI-1 and TI-2 using the boundary condition.
- The x-spin current on the SC side of the junction is linearly related to the steady state charge densities on the TI-1 and TI-2 sides, all evaluated at the junction:

$$\hat{y}\cdot\vec{J}_s^x = \frac{\hbar v_F}{2}[\Psi_1^\dagger \tau^z \Psi_1 - \Psi_2^\dagger \tau^z \Psi_2] = 1 + R_N + R_N' - R_A\Gamma_{1A} - T_N + T_A\Gamma_{2A}$$

where $R_N' = Re(r_N + r_N e^{-i2\theta_1})$,
$\Gamma_{1A} = \nu_E^2 \cos^2\theta_1 - \nu_E \cos\theta_1 \sqrt{\nu_E^2\cos^2\theta_1 - 1}$, $\Gamma_{2A} = |\nu_E \cos\theta_1|$ when $\nu_E \equiv (\mu_{TI} + E)/(\mu_{TI} - E) > 1$.

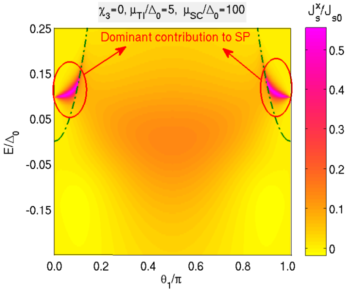
- It is interesting to note that the spin conductance on the SC side depends on both the phase and magnitude of the reflection amplitude r_N.

- The following figure shows the contributions to the spin conductance from the different terms in the previous equation integrated over all incident angles θ_1.



- From the figure, it is evident that the Andreev scattering term, $T_A\Gamma_{2A} - R_A\Gamma_{1A}$, and the phase term R'_N contribute the most to the SP.

- From the contour plot of spin-current $\hat{y} \cdot J_s^x$ as a function of E and angle of incidence θ_1, the contribution to the SP from a range of θ_1 is clearly visible.



- If $\nu_E \cos\theta_1 > 1$, the Andreev modes on TI-1 and TI-2 become evanescent and do not carry any current.

- At a given (E, θ_1), momentum of the Andreev reflected hole is: $k_{yh} = \sqrt{(\mu_{TI} - E)^2 - (\mu_{TI} + E)^2 \cos^2\theta_1}/(\hbar v_1) = \frac{\mu_{TI} - E}{\hbar v_1}\sqrt{1 - \nu_E^2 \cos^2\theta_1}$.
- The contribution to the SP comes from region where the Andreev modes exist.
- And therefore, the SP appears at a positive bias.
- Also, the phase of the reflection amplitude r_N changes from $-\pi$ to π through 0 when E is varied at a fixed θ_1.

Summary

- We described a formalism to study the transport across junctions of topological insulators and superconductors and derived the appropriate boundary conditions.
- A general boundary condition involves three time-reversal invariant barriers
- Charge conductance shows a zero bias peak
- Tunneling conductance of TI-SC junction provides a novel method for detection of different directions of $\vec{d}$ of SC (spin-state of Cooper pairs in SC)
- Such junctions can inject spin current into SC for certain triplet-pairings of the Cooper pairs
- The spin conductance shows a satellite peak at finite bias voltages in addition to the zero bias peak

References

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