Entanglement Effect in a Two-channel Kondo System under Bias

Jongbae Hong (POSTECH & APCTP)

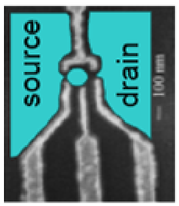
Sep. 1, 2014 Nordita

Two-reservoir Mesoscopic Kondo Systems

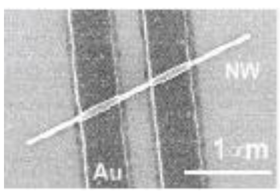
& extensions (From Quantum Dot to Cuprate)

Extension: correlated samples

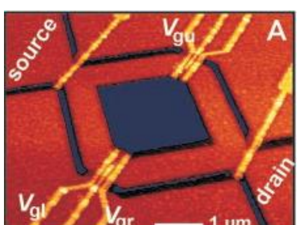
QD Single-electron Transistor



Semicond.

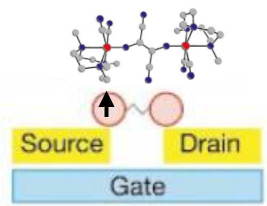


Nano-rod

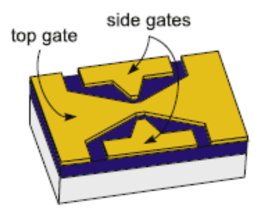


A-B ring

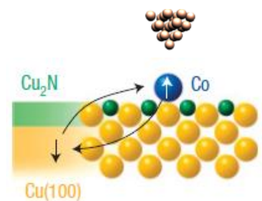
Single-molecule Transistor



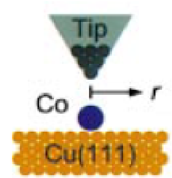
Quantum Point Contact



Adsorbed magnetized atom on metallic substrate



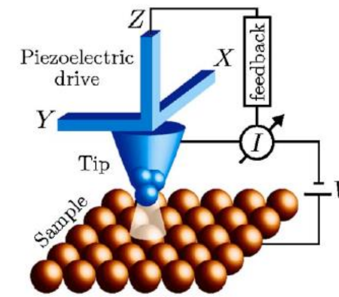
with insulating layer



without insulating layer

Correlated Graphenes

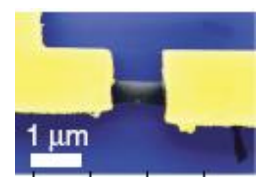
High Tc Supercond.



Doped Mono-layer

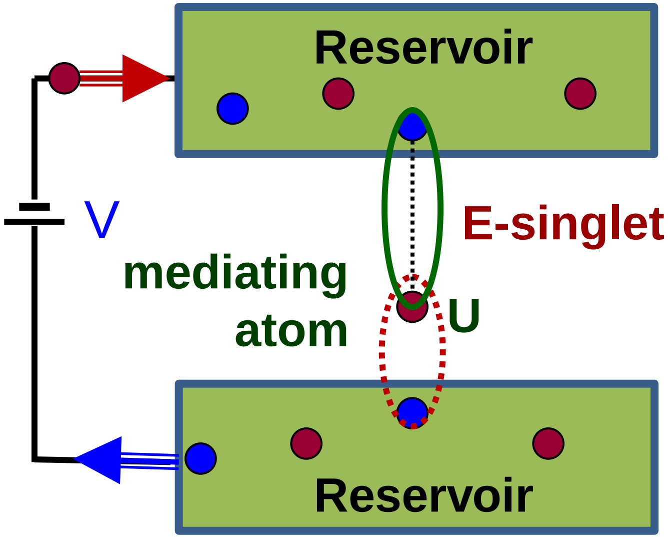
Bilayer

Trilayer

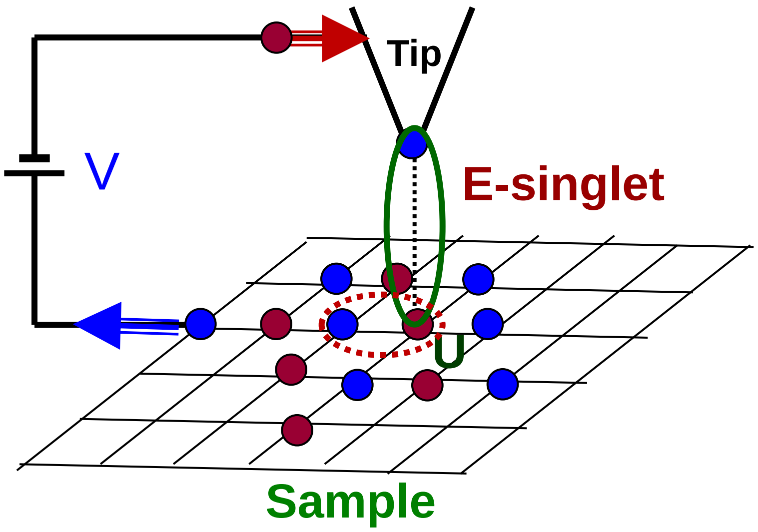


BSCCO et al.

Tunneling Current near V≈0



Mediating atom
is isolated



Mediating atom
is in the sample

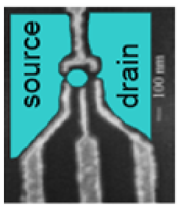
Both entangled singlets perform coherent co-tunneling

Two-reservoir Mesoscopic Kondo Systems

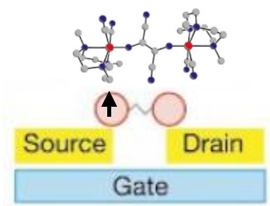
& extensions (From Quantum Dot to Cuprate)

Correlated samples

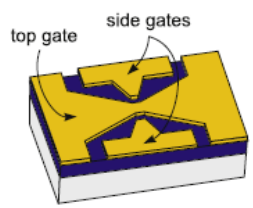
QD Single-electron Transistor



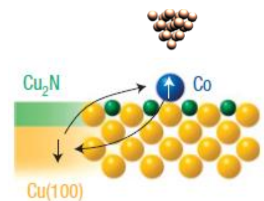
Single-molecule Transistor



Quantum Point Contact

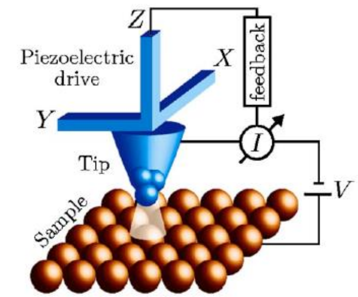


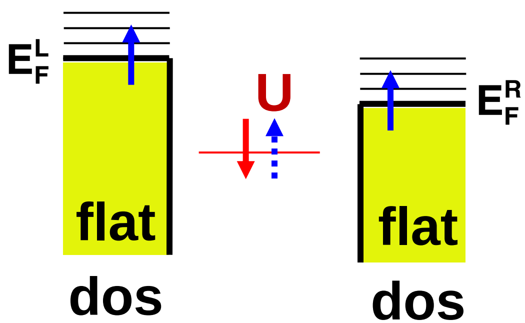
Adsorbed magnetized atom on metallic substrate

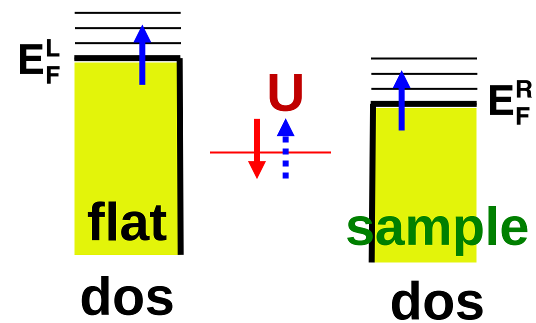


Correlated Graphenes

High Tc Supercond.





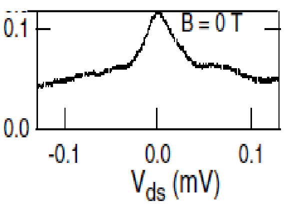


dI/dV formula for a meso. Kondo system ➔ Replace one flat dos by sample dos

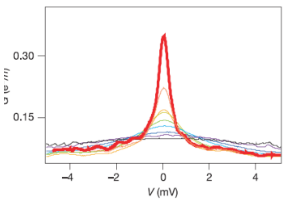
Task: Understanding Tunneling Conductances

Correlated samples

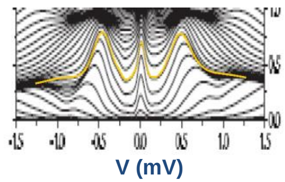
QD Single-electron Transistor	Single-molecule Transistor	Quantum Point Contact	Adsorbed magnetized atom	Correlated Graphenes	High Tc Supercond. BSCCO



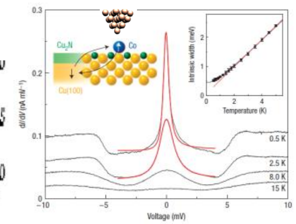
Kogan, PRL (2004)



Liang, Nature (2002)

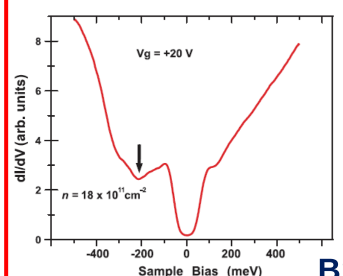


Sarkozy, PRB (2009)

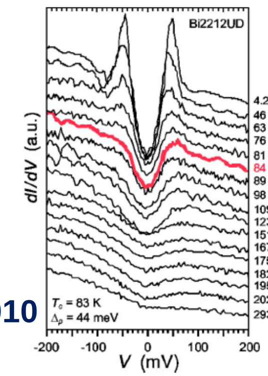


Otte, N. Phys. 2008

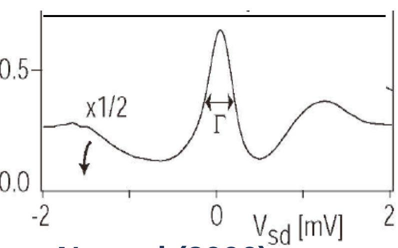
Doped Mono-layer



Brar, PRL. 2010

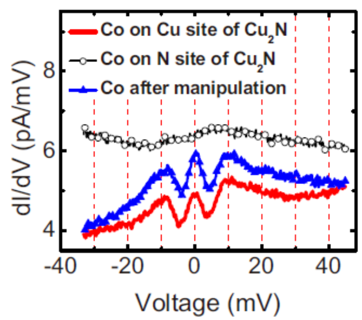


Renner, PRL 1998

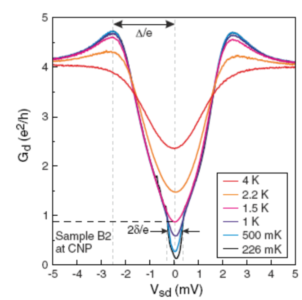


Nygard (2006)

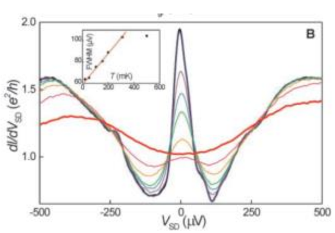
Choi, J. Vac. Sci. Tech. (2009)



Bilayer

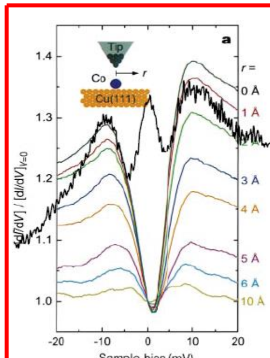


Freitag, PRL 2012

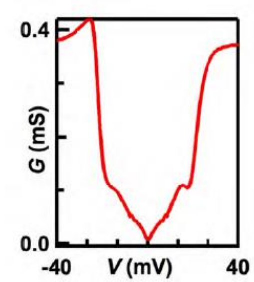


van der Wiel, Science (2000)

Manoharan, Nature (2000)



Trilayer



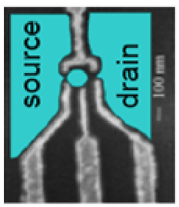
Bao, N. Phys. 2011

Two-reservoir Mesoscopic Kondo Systems

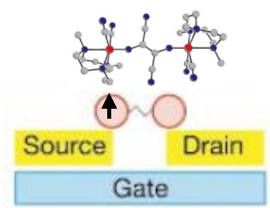
& extensions (From Quantum Dot to Cuprate)

Correlated samples

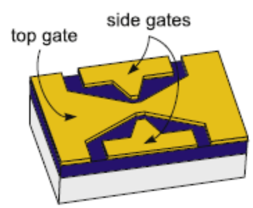
QD Single-electron Transistor



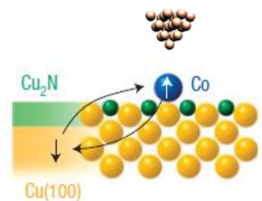
Single-molecule Transistor



Quantum Point Contact

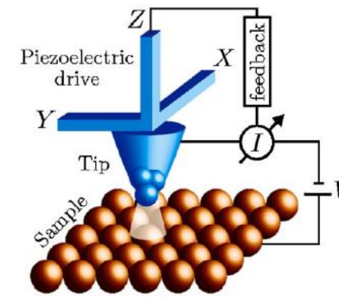


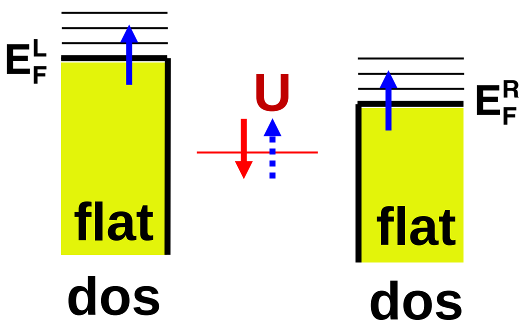
Adsorbed magnetized atom on metallic substrate

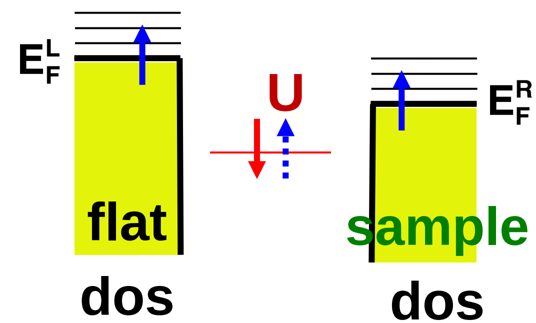


Correlated Graphenes

High Tc Supercond.



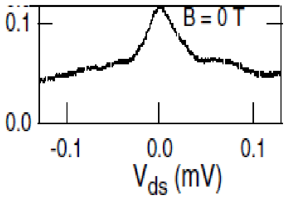




dI/dV vs. V for a correlated system has in principle three coherent peaks

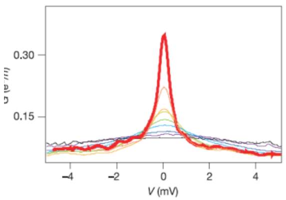
Correlated samples

QD Single-electron Transistor



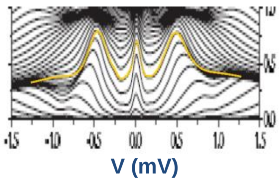
Kogan, PRL (2004)

Single-molecule Transistor



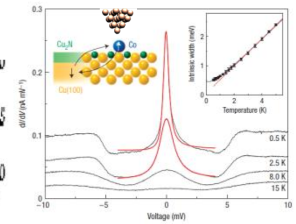
Liang, Nature (2002)

Quantum Point Contact



Sarkozy, PRB (2009)

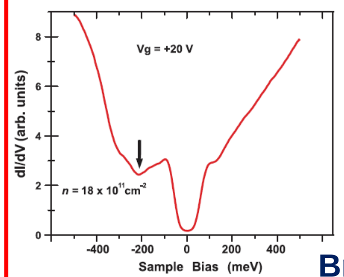
Adsorbed magnetized atom



Otte, N. Phys. 2008

Correlated Graphenes

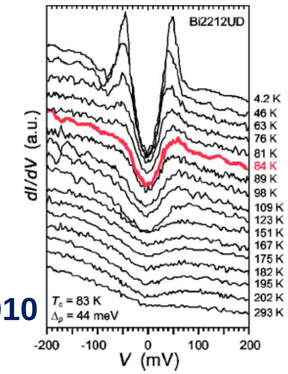
Doped Mono-layer



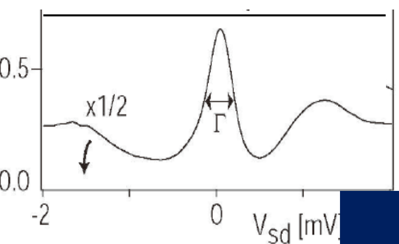
Brar, PRL. 2010

High Tc Supercond.

BSCCO

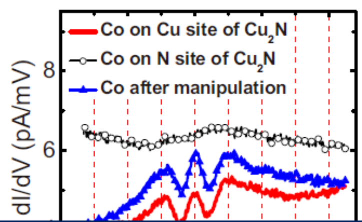


Renner, PRL 1998

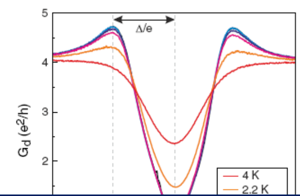


Nygard (2006)

Choi, J. Vac. Sci. Tech.

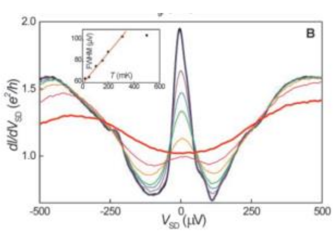


Bilayer

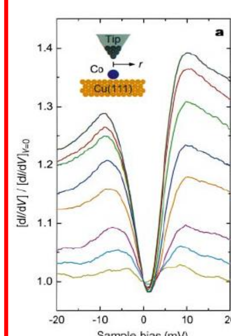


Freitag, PRL 2012

Q1: Why no ZBP on the right side?

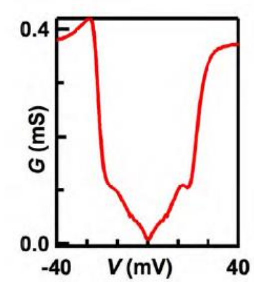


van der Wiel, Science (2000)



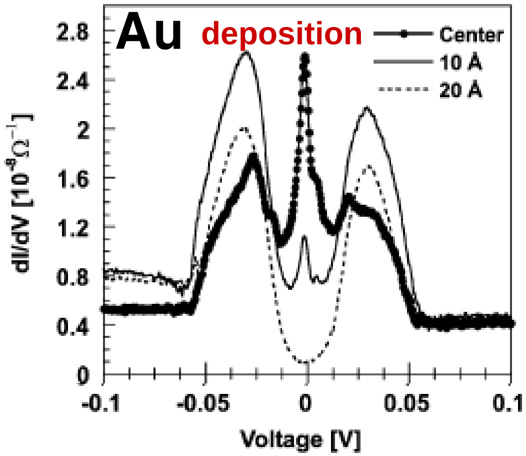
Manoharan, Nature (2000)

Trilayer

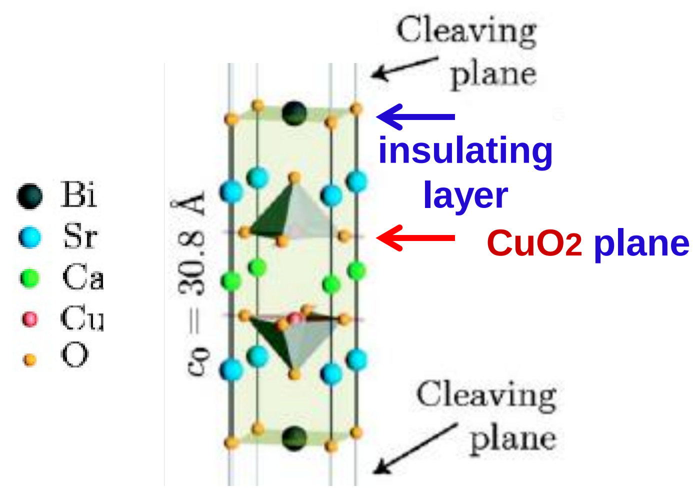


Bao, N. Phys. 2011

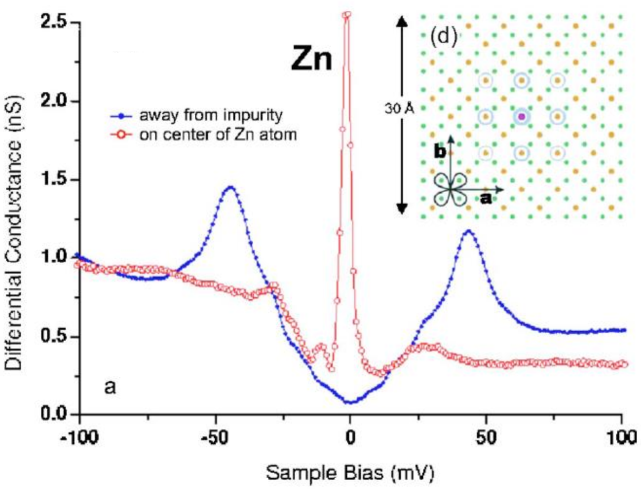
Revived zero-bias peak in BSCCO

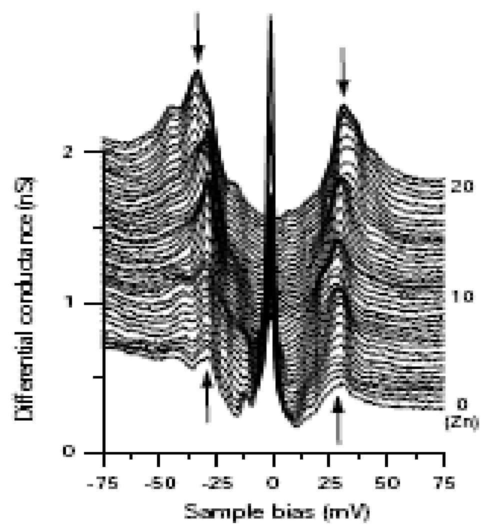


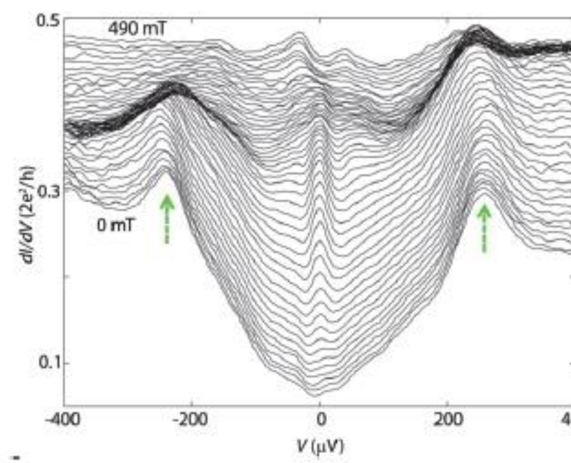
Yazdani,
PRL (1999)



Zn replaces Cu

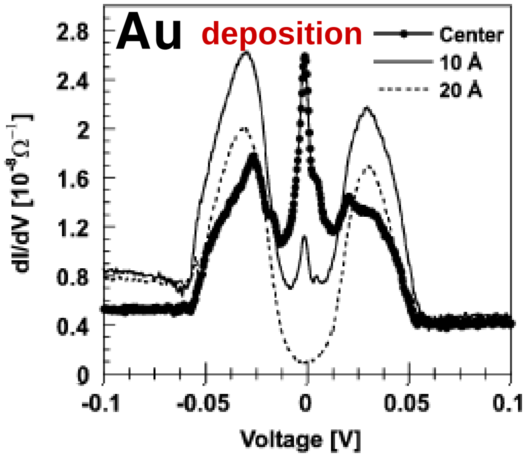




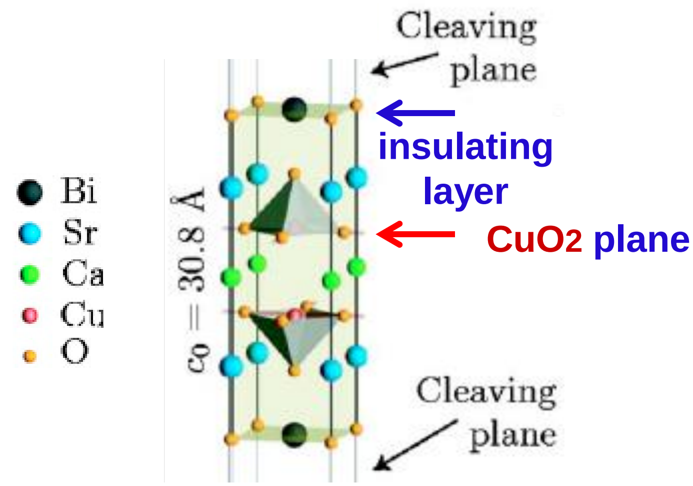


Pan, Nature (2000)

Revived zero-bias peak in BSCCO



Yazdani, PRL (1999)



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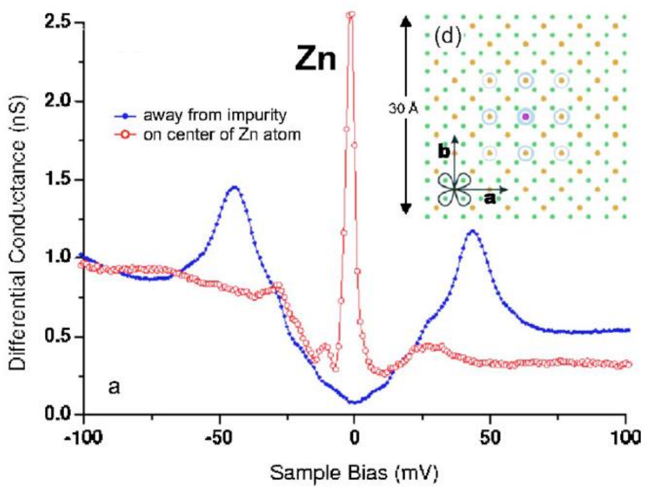
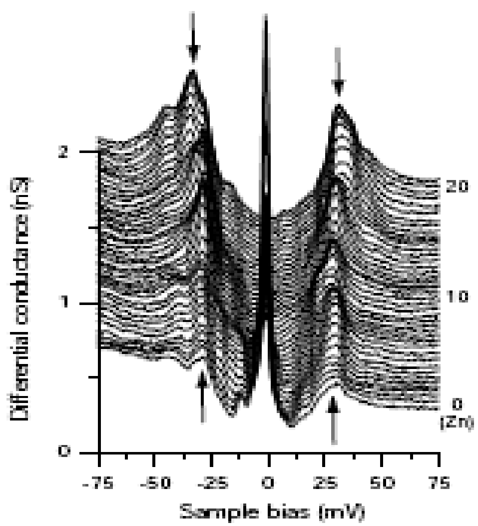
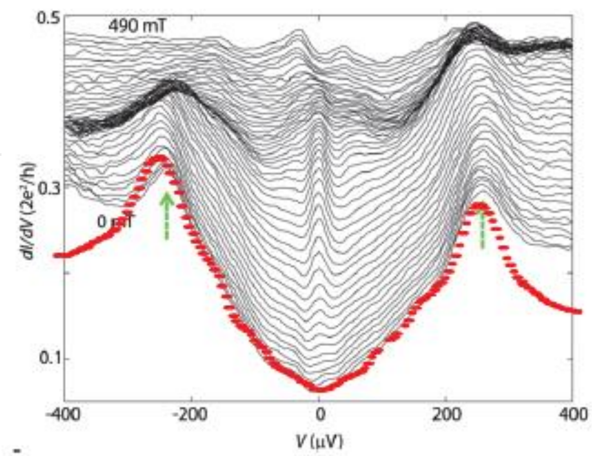


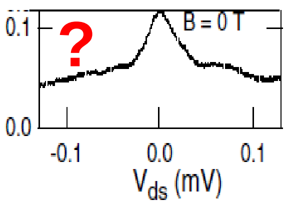




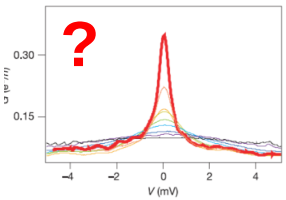
Pan, Nature (2000)

Correlated samples

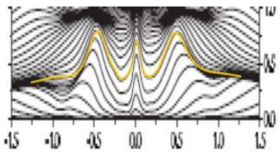
QD Single-electron Transistor	Single-molecule Transistor	Quantum Point Contact	Adsorbed magnetized atom	Correlated Graphenes	High Tc Supercond.



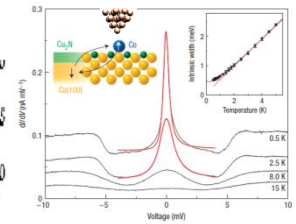
Kogan, PRL (2004)



Liang, Nature (2002)

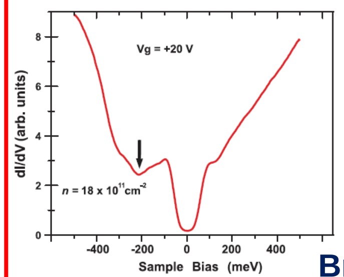


Sarkozy, PRB (2009)



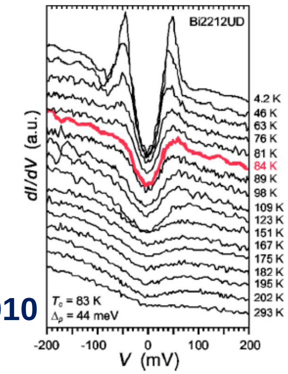
Otte, N. Phys. 2008

Doped Mono-layer

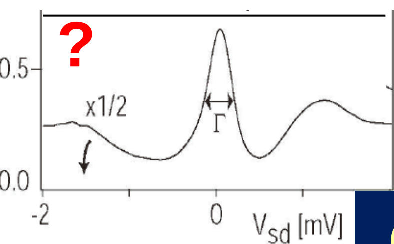


Brar, PRL. 2010

BSCCO

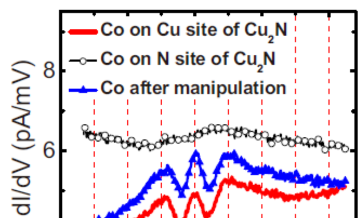


Renner, PRL 1998

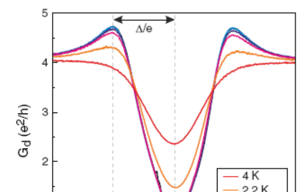


Nygard (2006)

Choi, J. Vac. Sci. Tech.

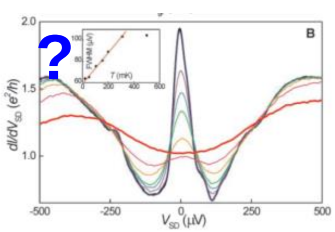


Bilayer



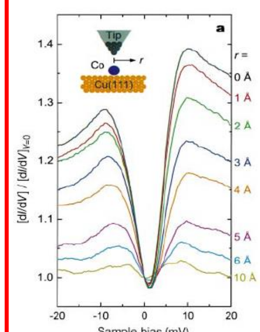
Q2: Where are side peaks in those ?s?

ag, 2012

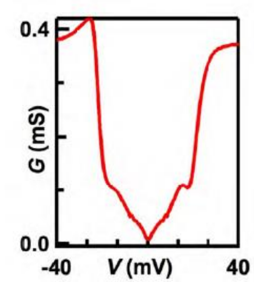


van der Wiel, Science (2000)

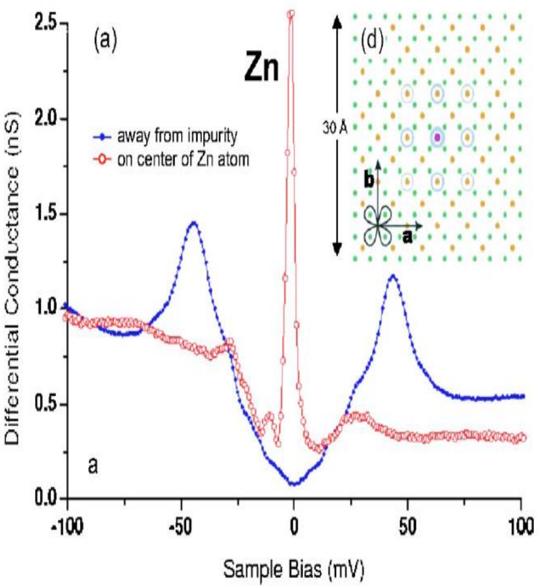
Manoharan, Nature (2000)

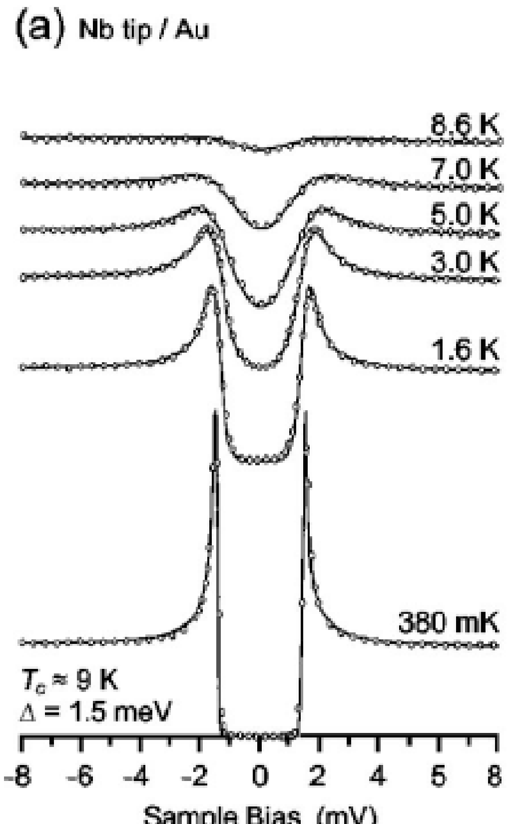


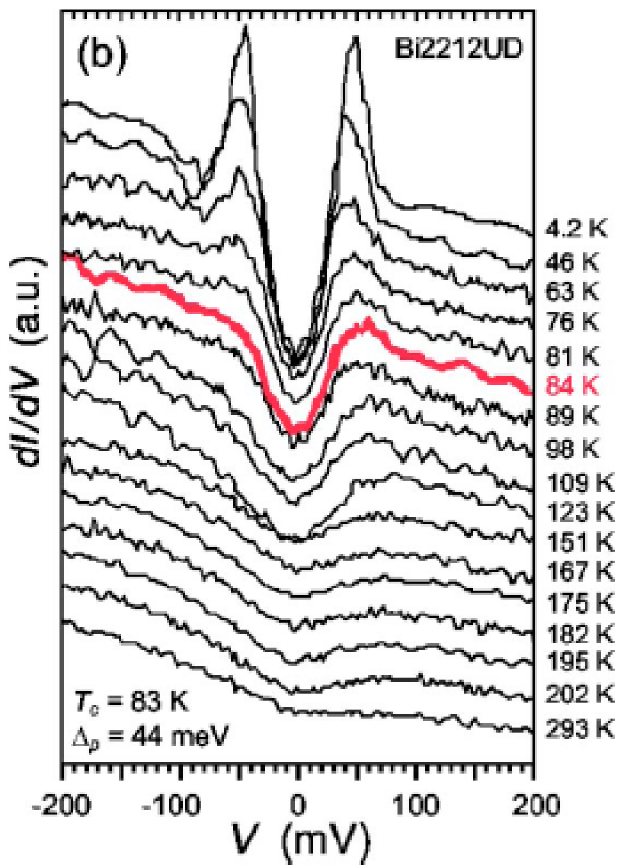
Trilayer



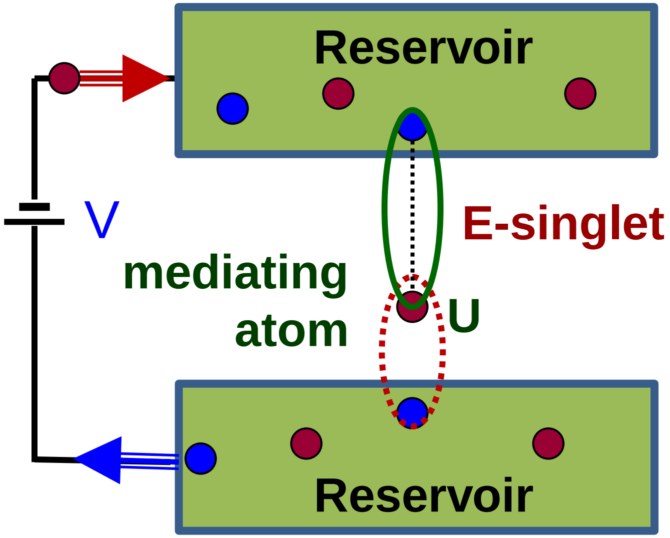
Bao, N. Phys. 2011


(a)
Zn
(d)
30 Å
b
a
away from impurity
on center of Zn atom
Differential Conductance (nS)
2.5
2.0
1.5
1.0
0.5
0.0
a
-100
-50
0
50
100
Sample Bias (mV)

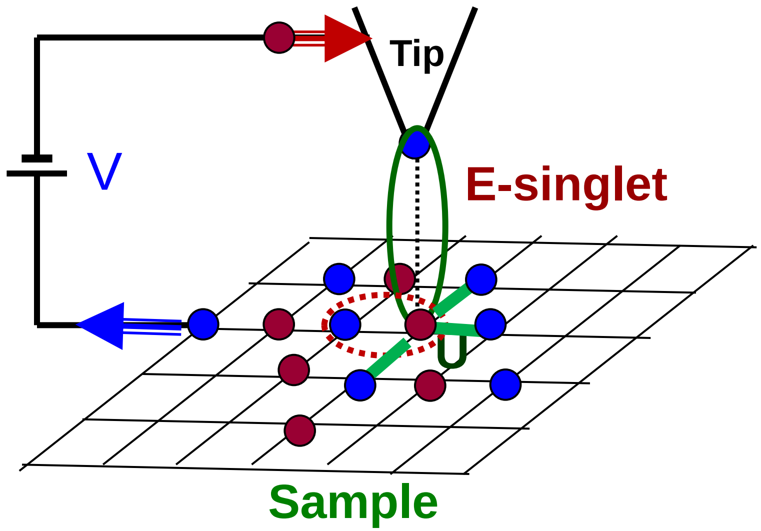

(a) Nb tip / Au
8.6 K
7.0 K
5.0 K
3.0 K
1.6 K
380 mK
$T_c \approx 9$ K
$\Delta = 1.5$ meV
-8
-6
-4
-2
0
2
4
6
8
Sample Bias (mV)


(b)
Bi2212UD
dI/dV (a.u.)
4.2 K
46 K
63 K
76 K
81 K
84 K
89 K
98 K
109 K
123 K
151 K
167 K
175 K
182 K
195 K
202 K
293 K
$T_c = 83$ K
$\Delta_p = 44$ meV
-200
-100
0
100
200
V (mV)

Tunneling Current near V≈0



Mediating atom
is isolated

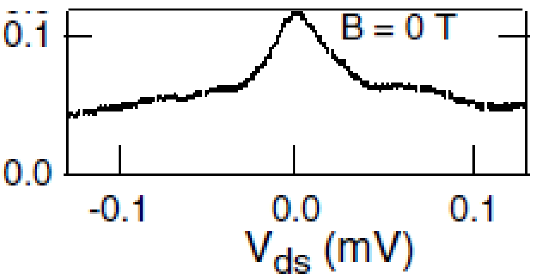


Mediating atom
is in the sample

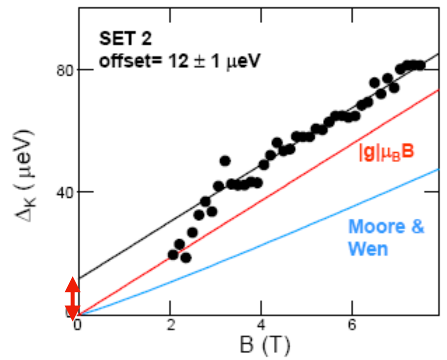
Ans. 1: Those green channels are free and their fluctuations induce double occupancy at the mediating atom

Q2 is related to the experiments applying B field

MIT Group



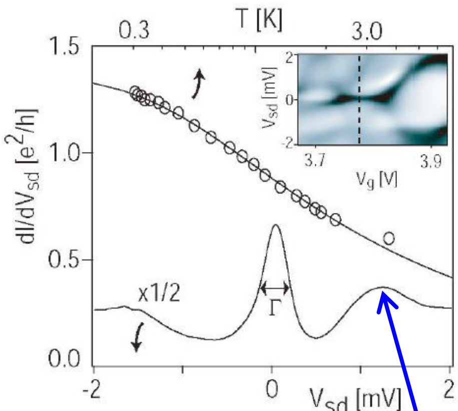
Off-set in Field-induced Splitting



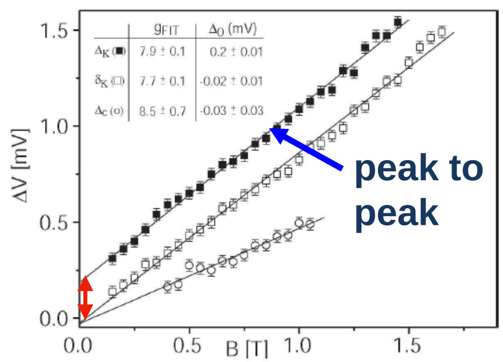
Ans. 2:

?-1 are ?-2 are weakly coupled Kondo system

Neals Bohr Group

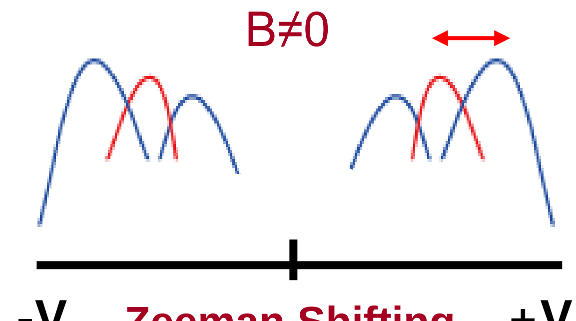


Coulomb Blockade Peak



peak to peak

Three coherent peaks!



Zeeman Shifting

Weakly & strongly coupled Kondo systems

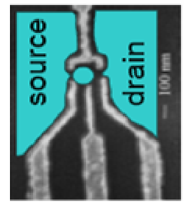
$$T_K = \frac{\sqrt{\Gamma U}}{2} e^{\pi\varepsilon_0(\varepsilon_0 + U)/\Gamma U}$$

Dimensionless form:

$$2T_K/\Gamma = (U/\Gamma)^{1/2}\, e^{\pi\varepsilon_0(\varepsilon_0 + U)/\Gamma U}$$

Parameter of coupling strength

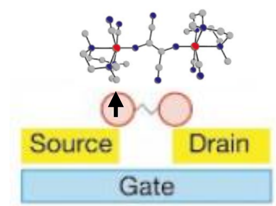
QD Single-electron Transistor



$\Gamma/U \sim 0.15$

Kogan, PRL (2004)

Single-molecule Transistor



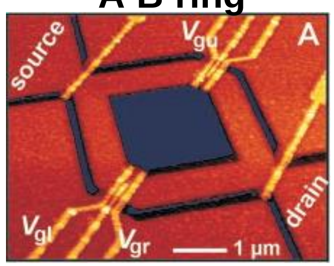
$\Gamma/U \sim 0.03$

Liang, Nature (2002)

Weakly coupled QD Kondo systems

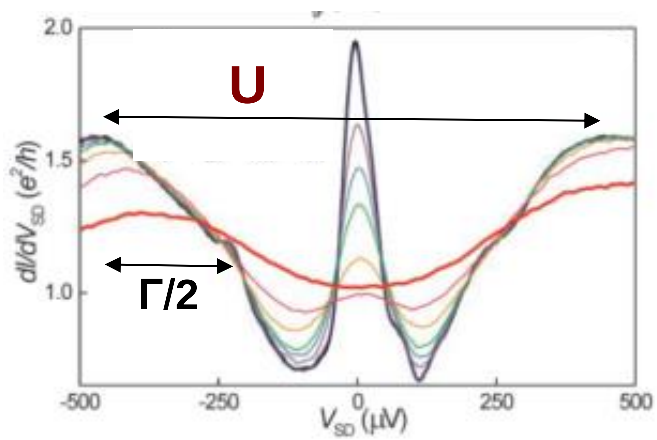
Strongly coupled QD Kondo system

A-B ring

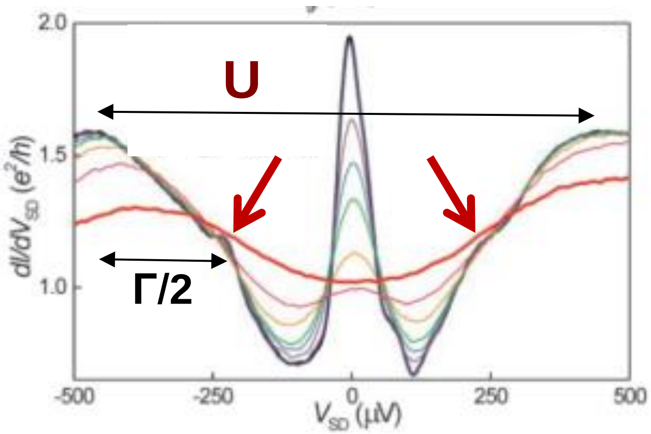


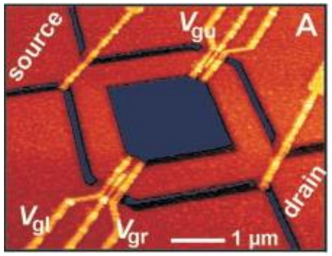
$\Gamma/U \sim 0.5$

van der Wiel, Science (2000)



Where are two side peaks?

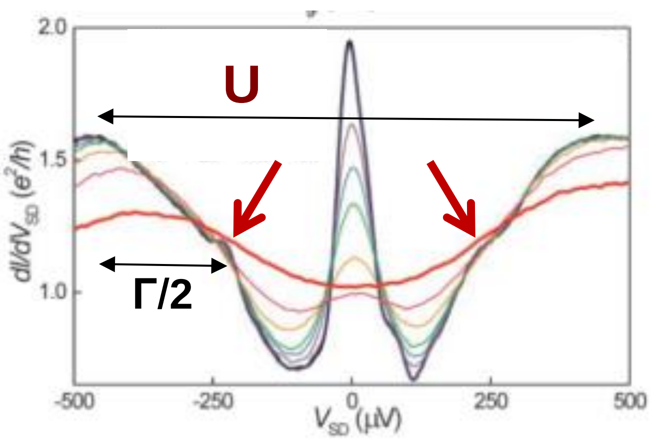


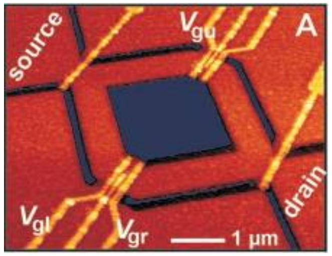


Γ/U ~ 0.5

van der Wiel, Science (2000)

Identifying two side peaks (↙ ↘) is another big issue.



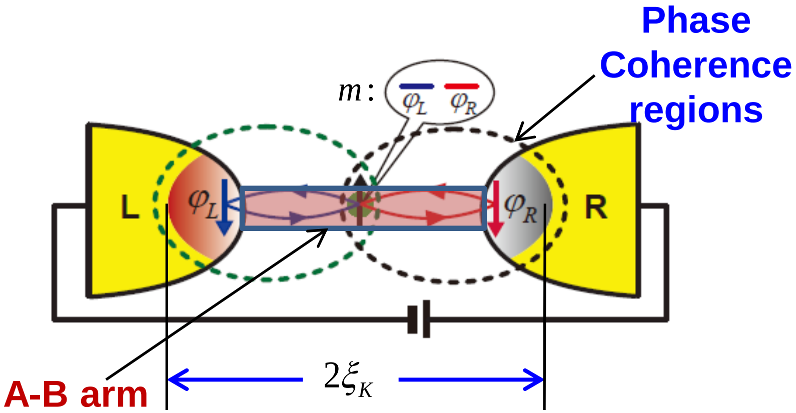


Γ/U ~ 0.5

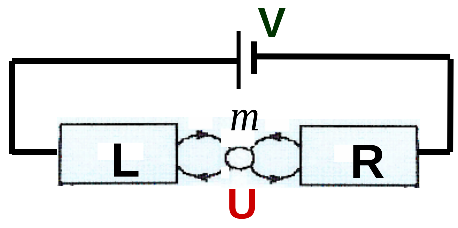
van der Wiel, Science (2000)

Identifying two side humps (↙ ↘) is another big issue.

New Type: **Different-phase Kondo Clouds**



Two-channel Kondo system under bias



Previous theoretical studies using

Keldysh + NCA

Keldysh Perturbation

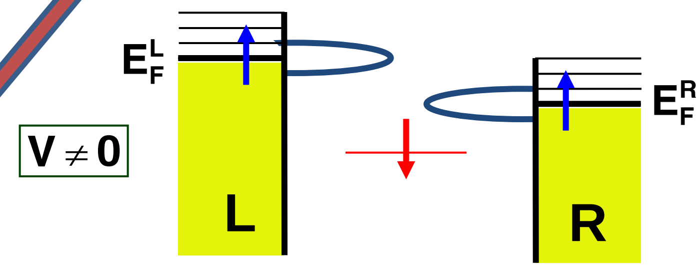
Real Time RG

Q. Monte Carlo

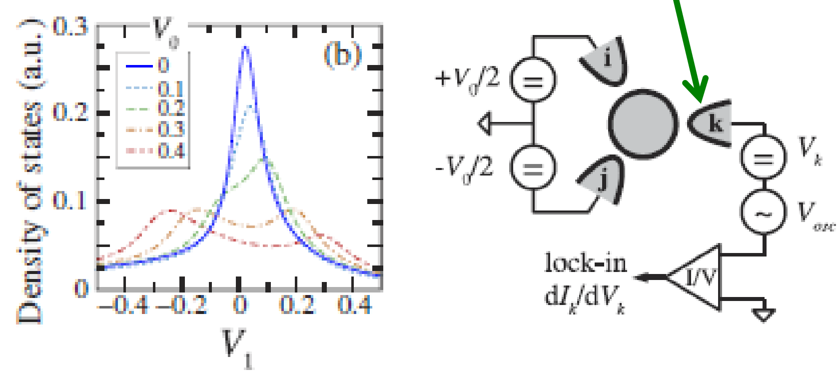
Scattering-state NRG

They studied independent Kondo clouds sharing an impurity!!

Bias-dependent Kondo peak splitting



phase-breaking scatterer



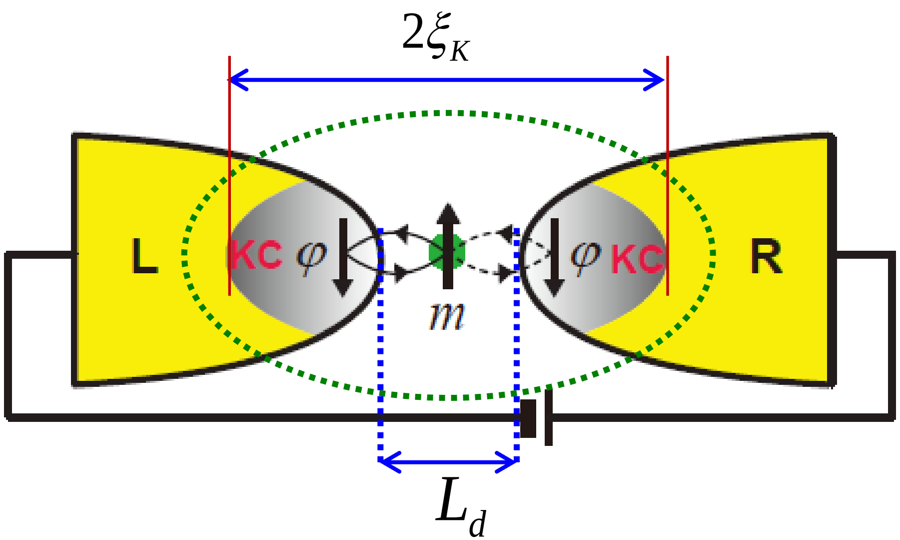
R. Leturcq et al., Phys. Rev. Lett. **95**, 126603 (2005)

Realistic mesoscopic Kondo systems:

Phase Coherence Region, Device Dimension, & Kondo Screening Length

L_d ξ_K

Same-phase Kondo Clouds



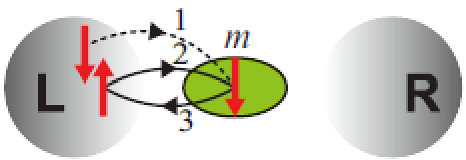
In this circumstance, two Kondo clouds are no longer independent.

They are coherently supposed: Entangled singlet

Dynamical

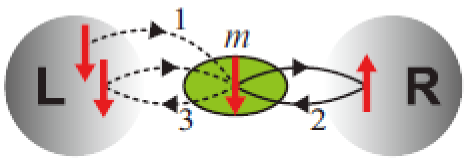
Descriptions of L-R coherent superpositions

Kondo coupling strength of L-side:



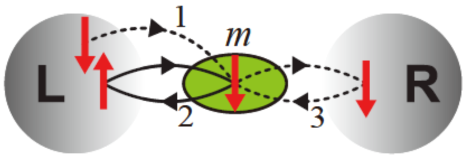
2 and 3 deote

exchange



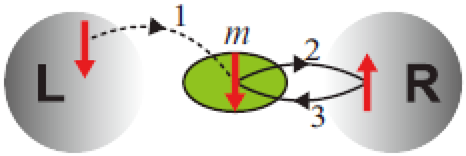
L-R coherent superposition

Transport:



Singlet hopping

L-R coherent superposition



exchange

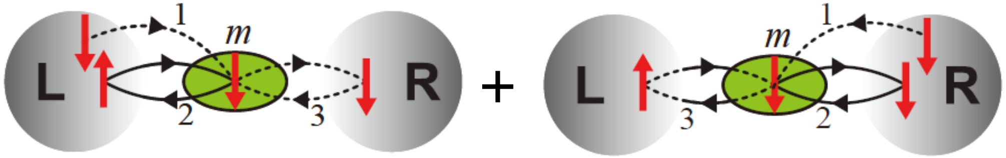
L-R coherent superposition

Dynamical

Descriptions of L-R coherent superpositions

* L-R symmetric superposition

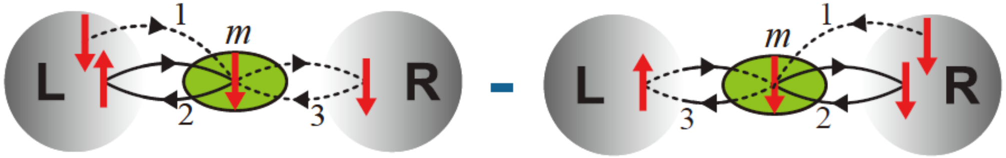
ex:



➔ Kondo peak at equilibrium

* L-R antisymmetric superposition

ex:



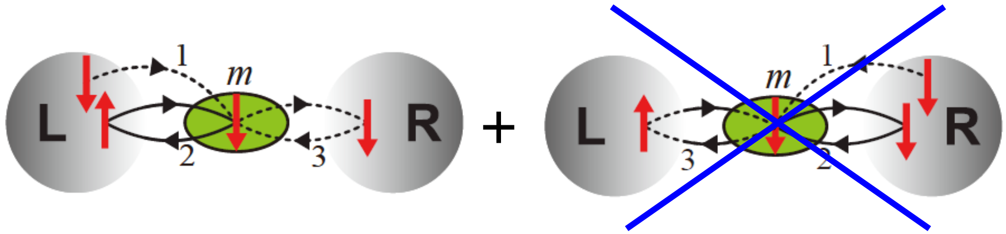
At equilibrium, it vanishes. But under bias?

Dynamical

Descriptions of L-R coherent superpositions

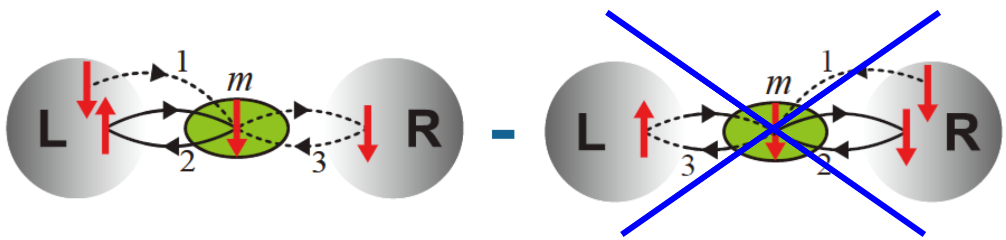
* L-R symmetric superposition

ex:



* L-R antisymmetric superposition

ex:



Under bias, the second parts vanish due to unidirectional motion of the entangled singlet

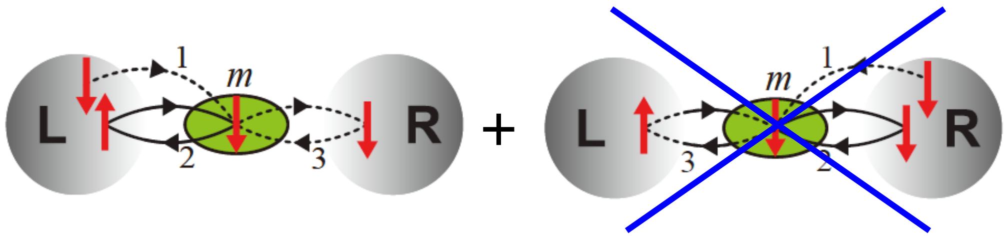
➔ Sym. and antisym. parts become same!!

Dynamical

Descriptions of L-R coherent superpositions

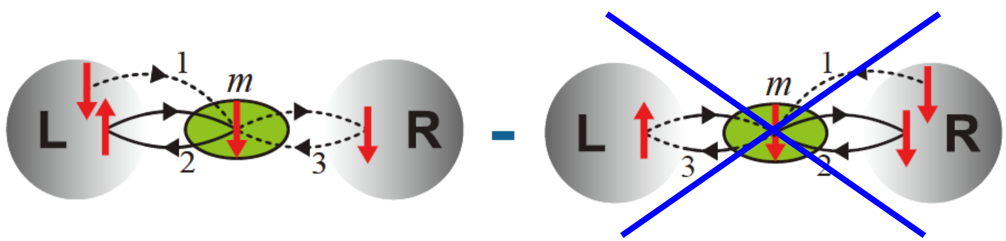
* L-R symmetric superposition

ex:



* L-R antisymmetric superposition

ex:



This is the effect of entanglement: The KEY for understanding non-equilibrium Kondo transport

Under bias, the second parts vanish due to unidirectional motion of the entangled singlet

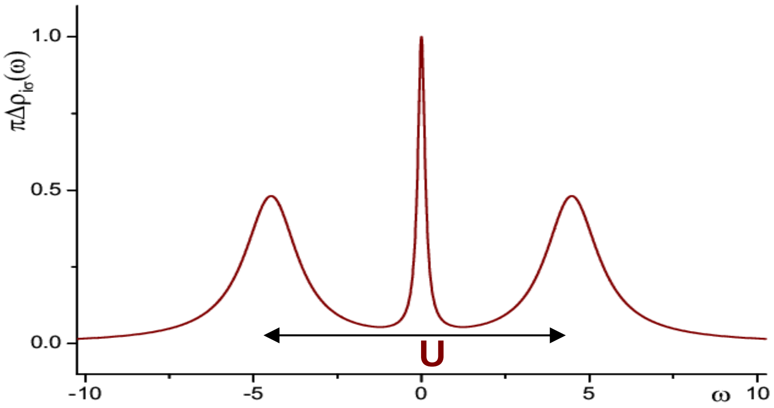
Strong E field

➔ Sym. and antisym. parts become same!!

In summary,

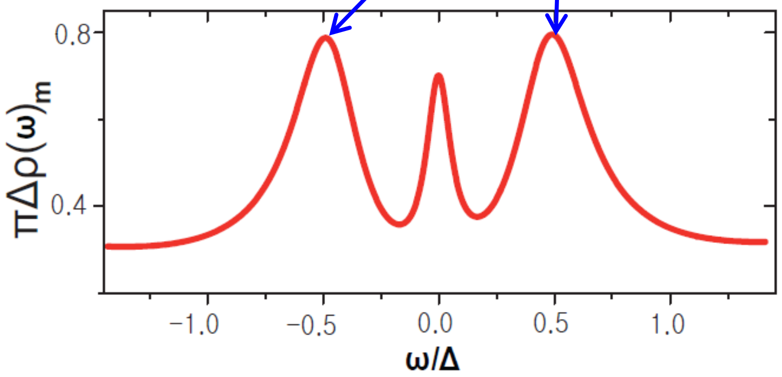
Under bias, nonvanishing antisymm. superposition produces two coherent side peaks

Spectral function at equilibrium



(1 coherent peak + 2 Coul.-peaks)

Spectral function under bias



(3 coherent peaks + 2 Coul.-peaks)

Tunneling Conductance

Tunneling Current at T=0K:

steady-state

$$I(V)=\frac{e}{\hbar}\sum_{\sigma}\int_0^{eV}d\omega\frac{\Gamma^{\mathrm{L}}(\omega)\Gamma^{\mathrm{R}}(\omega)}{\Gamma^{\mathrm{L}}(\omega)+\Gamma^{\mathrm{R}}(\omega)}\rho_{m\sigma}^{ss}(\omega)\,|_{\omega=eV}$$

Meir & Wingreen;
Hershfield et al. (1992)

This formula is valid for $\Gamma^{L}(\omega)\propto\Gamma^{R}(\omega)$

Tunneling Conductance at T=0K:

$$\frac{dI}{dV}=\frac{e^2}{\hbar}\left[\frac{\Gamma^{\mathrm{L}}(\omega)\Gamma^{\mathrm{R}}(\omega)}{\Gamma^{\mathrm{L}}(\omega)+\Gamma^{\mathrm{R}}(\omega)}\rho_{m}^{ss}(\omega)\right]_{\omega=eV}$$

$$\rho_m^{ss}(\omega)\neq\rho_m^{eq}(\omega)$$

$$\rho_m^{ss}(\omega):bias\text{-}independent$$

$$\rho_{m\sigma}^{ss}(\omega)=(\text{-}1/\pi)\,\mathrm{Im}\,\underline{G_{mm\sigma}^{+ss}(\omega)}$$

The quantity we obtain

Theoretical Procedure for

getting $G^{+}_{mm\sigma}(\omega)$

Resolvent Green's function

(1) $G^{\pm}_{mm\sigma}(\omega) = \langle u_{m\sigma} | (\omega \pm i\eta - \hat{H})^{-1} | u_{m\sigma} \rangle$ **Schrödinger**

(2) $G^{\pm}_{mm\sigma}(\omega) = \langle c_{m\sigma} | (\omega \pm i\eta - \mathbf{L})^{-1} | c_{m\sigma} \rangle$ **Heisenberg** $\mathbf{L} c_{m\sigma} = [H, c_{m\sigma}]$

(1) ➔ Construct **Hamiltonian** matrix in terms of a complete set of basis **state** vectors

(2) ➔ Construct **Liouvillian** matrix in terms of a complete set of basis **operators**

Hamiltonian approach vs. Liouvillian approach

Nontrivial Toy Model

Two-site Hubbard Model:

$$H = -t\,\Sigma_{i\neq j,\sigma} c^{+}_{i\sigma} c_{j\sigma} + I\,\Sigma_i\, n_{i\uparrow} n_{i\downarrow}$$

Hamiltonian Formulation:

Harris & Lange, PR 157, 295 (1967)

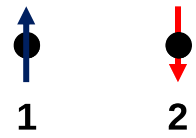
$$\rho_{1\sigma}(\omega) = \sum_n \delta(\omega - E_n^{N+1} + E_0^N)\left|\left\langle \Psi_0^N \,|\, c_{1\sigma} \,|\, \Psi_n^{N+1} \right\rangle\right|^2 - \sum_n \delta(\omega + E_n^{N-1} - E_0^N)\left|\left\langle \Psi_0^N \,|\, c^{+}_{1\sigma} \,|\, \Psi_n^{N-1} \right\rangle\right|^2$$

$$\rho^{\dagger}{}_{(\uparrow\downarrow)}(E) = \frac{(1-\lambda_-)^2}{4(1+\lambda_-^2)}\,\delta(E-I+t+E_-) + \frac{(1+\lambda_-)^2}{4(1+\lambda_-^2)}\,\delta(E-I-t+E_-)$$

$$+\frac{(1+\lambda_-)^2}{4(1+\lambda_-^2)}\,\delta(E-E_-+t) + \frac{(1-\lambda_-)^2}{4(1+\lambda_-^2)}\,\delta(E-E_--t)$$

$$\lambda_- = \frac{1}{4t}\left(I - \sqrt{16t^2 + I^2}\right)$$

Liouvillian Formulation:



$$j_{1\downarrow}^{-} = c_{2\downarrow}^{+} c_{1\downarrow} - c_{1\downarrow}^{+} c_{2\downarrow}$$

(1) Complete set of basis vectors:

$$c_{1\uparrow} \qquad c_{2\uparrow} \qquad 2(\delta \mathrm{n}_{1\downarrow} c_{1\uparrow} + j_{1\downarrow}^{-} c_{2\uparrow}) \qquad 2(\delta \mathrm{n}_{1\downarrow} c_{2\uparrow} - j_{1\downarrow}^{-} c_{1\uparrow})$$

(2) Construct matrix $z\mathbf{I} + i\mathbf{L}$

$$z\mathbf{I} + i\mathbf{L} = \begin{pmatrix} z & -it & 0 & iI/2 \\ -it & z & iI/2 & 0 \\ 0 & iI/2 & z & 3it \\ iI/2 & 0 & 3it & z \end{pmatrix}$$

(3) Take matrix inverse

$$\rho_{\{\uparrow\downarrow\}}^{\uparrow}(\omega) = -\frac{1}{\pi} \mathrm{Re}[(z\mathbf{I} + i\mathbf{L})^{-1}]_{11}$$

$$\text{Weight}: \frac{1}{4} \pm \frac{t}{\sqrt{16t^2 + I^2}}$$

$$\rho^{\dagger}{}_{\{\uparrow\downarrow\}}(E) = \frac{(1-\lambda_-)^2}{4(1+\lambda_-^2)} \delta(E-I+t+E_-) + \frac{(1+\lambda_-)^2}{4(1+\lambda_-^2)} \delta(E-I-t+E_-)$$

$$+ \frac{(1+\lambda_-)^2}{4(1+\lambda_-^2)} \delta(E-E_-+t) + \frac{(1-\lambda_-)^2}{4(1+\lambda_-^2)} \delta(E-E_--t)$$

$$\lambda_- = \frac{1}{4t}\left(I - \sqrt{16\mathrm{t}^2 + I^2}\right)$$

Harris & Lange, PR 157, 295 (1967)

Two-reservoir Anderson impurity model

Hamiltonian formulation vs. Liouvillian formulation

Obtaining **basis state vectors** is hopeless

Obtaining **basis operators** is possible

(1) JH, J. Phys.: Condens, Matter vol. 23, 225601 (2011)

(2) JH, J. Phys.: Condens, Matter vol. 23, 275602 (2011)

Determining basis vectors:

$$\mathrm{iG}^{+}_{\mathrm{mm}\uparrow} = \langle c_{m\uparrow} | (\mathrm{z}\mathbf{I} + \mathrm{i}\mathbf{L}_I + \mathrm{i}\mathbf{L}_C)^{-1} | c_{m\uparrow} \rangle = \langle c_{m\uparrow} | (\hat{A} + \hat{B})^{-1} | c_{m\uparrow} \rangle$$

$$\hat{A} = (z\mathbf{I} + i\mathbf{L}_{\mathrm{I}}), \;\; \hat{B} = i\mathbf{L}_{\mathrm{C}},$$

I : isolated part $\qquad$ c : connecting part

Using operator identity $\dfrac{1}{\hat{A}+\hat{B}} = \dfrac{1}{\hat{A}} - \dfrac{1}{\hat{A}}\hat{B}\dfrac{1}{\hat{A}} + \dfrac{1}{\hat{A}}\hat{B}\dfrac{1}{\hat{A}}\hat{B}\dfrac{1}{\hat{A}} - \cdots,$

$$iG^+_{mm\uparrow}(\omega) = \left\langle c_{m\uparrow} \left| \frac{1}{(z\mathbf{I} + i\mathbf{L}_\mathrm{I}) + i\mathbf{L}_\mathrm{C}} \right| c_{m\uparrow} \right\rangle$$

$$= \left\langle c_{m\uparrow} \left| \frac{1}{(z\mathbf{I} + i\mathbf{L}_\mathrm{I})} \right| c_{m\uparrow} \right\rangle \quad \Leftarrow \frac{1}{\hat{A}}$$

$$- \left\langle c_{m\uparrow} \left| \frac{1}{(z\mathbf{I} + i\mathbf{L}_\mathrm{I})} i\mathbf{L}_\mathrm{C} \frac{1}{(z\mathbf{I} + i\mathbf{L}_\mathrm{I})} \right| c_{m\uparrow} \right\rangle \quad \Leftarrow -\frac{1}{\hat{A}} \hat{B} \frac{1}{\hat{A}}$$

$$+ \left\langle c_{m\uparrow} \left| \frac{1}{(z\mathbf{I} + i\mathbf{L}_\mathrm{I})} i\mathbf{L}_\mathrm{C} \frac{1}{(z\mathbf{I} + i\mathbf{L}_\mathrm{I})} i\mathbf{L}_\mathrm{C} \frac{1}{(z\mathbf{I} + i\mathbf{L}_\mathrm{I})} \right| c_{m\uparrow} \right\rangle + \cdots$$

$$iG^+_{mm\uparrow}(\omega) = \left\langle c_{m\uparrow} \left| \hat{G}_\mathrm{I} \right| c_{m\uparrow} \right\rangle - \left\langle c_{m\uparrow} \left| \hat{G}_\mathrm{I} \right| \Phi_m \right\rangle + \left\langle \Phi_m \left| \hat{G} \right| \Phi_m \right\rangle$$

where

$$\hat{G}_\mathrm{I} = (z\mathbf{I} + i\mathbf{L}_\mathrm{I})^{-1}$$

$$\Phi_m = i\mathbf{L}_C (z\mathbf{I} + i\mathbf{L}_\mathrm{I})^{-1} c_{m\uparrow}$$

Expressing this in a matrix form yields

$$iG^+_{mm\uparrow}(\omega) = \left(\langle c_{m\uparrow} | \quad \langle \Phi_m | \right) \begin{pmatrix} \hat{G}_\mathrm{I} & -\hat{G}_\mathrm{I} \\ 0 & \hat{G} \end{pmatrix} \begin{pmatrix} | c_{m\uparrow} \rangle \\ | \Phi_m \rangle \end{pmatrix}$$

Similarity transformation using $\mathrm{U} = \begin{pmatrix} 1 & -\frac{\hat{G}_I}{\hat{G}-\hat{G}_I} \\ 0 & 1 \end{pmatrix}$

diagonalizes $\hat{G}$: $\hat{G}_d = \mathrm{U}\hat{G}\mathrm{U}^{-1} = \begin{pmatrix} \hat{G}_I & 0 \\ 0 & \hat{G} \end{pmatrix}$

Then, we obtain

$$iG^{\pm}_{mm\uparrow}(\omega) = (\langle \tilde{c}_{m\uparrow} | \ \ \langle \Phi_m |) \begin{pmatrix} \hat{G}_I & 0 \\ 0 & \hat{G} \end{pmatrix} \begin{pmatrix} | \tilde{c}_{m\uparrow} \rangle \\ | \Phi_m \rangle \end{pmatrix}$$

where

$$\tilde{c}_{m\uparrow} = c_{m\uparrow} + \frac{z\mathbf{I} + i\mathbf{L}}{i\mathbf{L}_C} \Phi_m, \qquad \Phi_m = i\mathbf{L}_C (z\mathbf{I} + i\mathbf{L}_\mathrm{I})^{-1} c_{m\uparrow}$$

A complete set of linearly independent basis vectors is given by collecting the linearly independent components of the vector $| \tilde{c}_{m\uparrow} \rangle$.

Complete Set of basis vectors for $c_{m\uparrow}(\mathrm{t})$:

	poles	self-energy
	$c_{m\uparrow},\quad n_{m\downarrow}c_{m\uparrow}$	$c^{L,R}_{k\uparrow},\ n_{m\downarrow}c^{L,R}_{k\uparrow},\quad k=1,2,\cdots,\infty$
1	$\dot{j}^{-L,R}_{m\downarrow}c_{m\uparrow},\ \dot{j}^{+L,R}_{m\downarrow}c_{m\uparrow},$	$\dot{j}^{-L,R}_{m\downarrow}c^{L,R}_{k\uparrow},\quad \dot{j}^{+L,R}_{m\downarrow}c^{L,R}_{k\uparrow}$
2	$(\mathbf{L}_\mathrm{C}\,\dot{j}^{-L,R}_{m\downarrow})c_{m\uparrow},\ (\mathbf{L}_\mathrm{C}\,\dot{j}^{+L,R}_{m\downarrow})c_{m\uparrow},$	$(\mathbf{L}_\mathrm{C}\,\dot{j}^{-L,R}_{m\downarrow})c^{L,R}_{k\uparrow},\ (\mathbf{L}_\mathrm{C}\,\dot{j}^{+L,R}_{m\downarrow})c^{L,R}_{k\uparrow}$
3	$(\mathbf{L}^2_\mathrm{C}\,\dot{j}^{-L,R}_{m\downarrow})c_{m\uparrow},\ (\mathbf{L}^2_\mathrm{C}\,\dot{j}^{+L,R}_{m\downarrow})c_{m\uparrow},$	$(\mathbf{L}^2_\mathrm{C}\,\dot{j}^{-L,R}_{m\downarrow})c^{L,R}_{k\uparrow},\ (\mathbf{L}^2_\mathrm{C}\,\dot{j}^{+L,R}_{m\downarrow})c^{L,R}_{k\uparrow}$
$\vdots$	$\vdots$	$\vdots$
∞	$(\mathbf{L}^\infty_\mathrm{C}\,\dot{j}^{-L,R}_{m\downarrow})c_{m\uparrow},\ (\mathbf{L}^\infty_\mathrm{C}\,\dot{j}^{+L,R}_{m\downarrow})c_{m\uparrow},$	$(\mathbf{L}^\infty_\mathrm{C}\,\dot{j}^{-L,R}_{m\downarrow})c^{L,R}_{k\uparrow},\ (\mathbf{L}^\infty_\mathrm{C}\,\dot{j}^{+L,R}_{m\downarrow})c^{L,R}_{k\uparrow}$

number of hopping of $\downarrow$-spin electron

Neglect multiple hoppings: Effect of entanglement under bias

	poles	self-energy
	$c_{m\uparrow}, \quad n_{m\downarrow} c_{m\uparrow}$	$c_{k\uparrow}^{L,R}, \ n_{m\downarrow} c_{k\uparrow}^{L,R}, \quad k = 1, 2, \cdots, \infty$
1	$j_{m\downarrow}^{-L,R} c_{m\uparrow}, \ j_{m\downarrow}^{+L,R} c_{m\uparrow},$	$j_{m\downarrow}^{-L,R} c_{k\uparrow}^{L,R}, \quad j_{m\downarrow}^{+L,R} c_{k\uparrow}^{L,R}$
2	$(\mathbf{L}_{\mathrm{C}} j_{m\downarrow}^{-L,R}) c_{m\uparrow}, \ (\mathbf{L}_{\mathrm{C}} j_{m\downarrow}^{+L,R}) c_{m\uparrow},$	$(\mathbf{L}_{\mathrm{C}} j_{m\downarrow}^{-L,R}) c_{k\uparrow}^{L,R}, \ (\mathbf{L}_{\mathrm{C}} j_{m\downarrow}^{+L,R}) c_{k\uparrow}^{L,R}$
3	$(\mathbf{L}_{\mathrm{C}}^2 j_{m\downarrow}^{-L,R}) c_{m\uparrow}, \ (\mathbf{L}_{\mathrm{C}}^2 j_{m\downarrow}^{+L,R}) c_{m\uparrow},$	$(\mathbf{L}_{\mathrm{C}}^2 j_{m\downarrow}^{-L,R}) c_{k\uparrow}^{L,R}, \ (\mathbf{L}_{\mathrm{C}}^2 j_{m\downarrow}^{+L,R}) c_{k\uparrow}^{L,R}$
$\vdots$	$\vdots$	$\vdots$
∞	$(\mathbf{L}_{\mathrm{C}}^\infty j_{m\downarrow}^{-L,R}) c_{m\uparrow}, \ (\mathbf{L}_{\mathrm{C}}^\infty j_{m\downarrow}^{+L,R}) c_{m\uparrow},$	$(\mathbf{L}_{\mathrm{C}}^\infty j_{m\downarrow}^{-L,R}) c_{k\uparrow}^{L,R}, \ (\mathbf{L}_{\mathrm{C}}^\infty j_{m\downarrow}^{+L,R}) c_{k\uparrow}^{L,R}$

number of hopping
of ↓-spin electron

APPROXIMATION: Neglect in **Kondo regime** & self-energy

poles	self-energy
$c_{m\uparrow}$, $n_{m\downarrow}c_{m\uparrow}$	$c_{k\uparrow}^{L,R}$, $n_{m\downarrow}c_{k\uparrow}^{L,R}$, $k=1,2,\cdots,\infty$
1 $\dot{j}_{m\downarrow}^{-L,R}c_{m\uparrow}$, $\dot{j}_{m\downarrow}^{+L,R}c_{m\uparrow}$,	$\dot{j}_{m\downarrow}^{-L,R}c_{k\uparrow}^{L,R}$, $\dot{j}_{m\downarrow}^{+L,R}c_{k\uparrow}^{L,R}$

1: number of hopping of ↓-spin electron

Orthonormalized basis vectors:

$$c_{m\uparrow},\ \frac{(\delta\,\dot{j}_{m\downarrow}^{-L,R})c_{m\uparrow}}{\sqrt{\langle(\delta\,\dot{j}_{m\downarrow}^{-L,R})^2\rangle}},\ \frac{(\delta\,\dot{j}_{m\downarrow}^{+L,R})c_{m\uparrow}}{\sqrt{\langle(\delta\,\dot{j}_{m\downarrow}^{+L,R})^2\rangle}}$$

➡ **5 peaks**

$$c_{k\uparrow}^{L,R},\ \frac{(\delta n_{m\downarrow})c_{k\uparrow}^{L,R}}{\sqrt{\langle(\delta n_{m\downarrow})^2\rangle}},\ k=1,2,\cdots,\infty$$

➡ **peak widening**

Constructing Liouvillian matrix: $\mathbf{M} = z\mathbf{I} + i\mathbf{L}$

Basis Vectors: $c_{k\uparrow}^{L}$, $\delta n_{m\downarrow} c_{k\uparrow}^{L}$, $\delta j_{m\downarrow}^{-L} c_{m\uparrow}$, $\delta j_{m\downarrow}^{+L} c_{m\uparrow}$, $c_{m\uparrow}$, $\delta j_{m\downarrow}^{+R} c_{m\uparrow}$, $\delta j_{m\downarrow}^{-R} c_{m\uparrow}$, $\delta n_{m\downarrow} c_{k\uparrow}^{R}$, $c_{k\uparrow}^{R}$

	$c_{k\uparrow}^{L}$	$\delta n_{m\downarrow} c_{k\uparrow}^{L}$	$\delta j_{m\downarrow}^{-L} c_{m\uparrow}$	$\delta j_{m\downarrow}^{+L} c_{m\uparrow}$	$c_{m\uparrow}$	$\delta j_{m\downarrow}^{+R} c_{m\uparrow}$	$\delta j_{m\downarrow}^{-R} c_{m\uparrow}$	$\delta n_{m\downarrow} c_{k\uparrow}^{R}$	$c_{k\uparrow}^{R}$
$c_{k\uparrow}^{L}$	$\begin{pmatrix} z+i\varepsilon_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & z+i\varepsilon_\infty \end{pmatrix}$	0	0	0	$\begin{pmatrix} iV_{km}^{*} \\ \vdots \\ iV_{km}^{*} \end{pmatrix}$	0	0	0	
$\delta n_{m\downarrow} c_{k\uparrow}^{L}$	0	$\begin{pmatrix} z+i\varepsilon_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & z+i\varepsilon_\infty \end{pmatrix}$	$\begin{pmatrix} -\xi_m^{-}V_{km} \\ \vdots \\ -\xi_m^{-}V_{km} \end{pmatrix}$	$\begin{pmatrix} -\xi_m^{+}V_{km} \\ \vdots \\ -\xi_m^{+}V_{km} \end{pmatrix}$	0	$\begin{pmatrix} -\xi_m^{+}V_{km} \\ \vdots \\ -\xi_m^{+}V_{km} \end{pmatrix}$	$\begin{pmatrix} -\xi_m^{-}V_{km} \\ \vdots \\ -\xi_m^{-}V_{km} \end{pmatrix}$		
$\delta j_{m\downarrow}^{-L} c_{m\uparrow}$	0	$(\xi_m^{+*}V_{km}^{*} \cdots \xi_m^{+*}V_{km}^{*})$	$-i\omega$	$-\gamma_{LL}$	$U_{j^-}^{L}$	γ_{LR}	γ_j	(∞X5)	
$\delta j_{m\downarrow}^{+L} c_{m\uparrow}$	0	$(\xi_m^{-*}V_{km}^{*} \cdots \xi_m^{-*}V_{km}^{*})$	γ_{LL}	$-i\omega$	$U_{j^+}^{L}$	γ_j	γ_{LR}		
$c_{m\uparrow}$	$(iV_{km} \cdots iV_{km})$	0	$-U_{j^-}^{L*}$	$-U_{j^+}^{L*}$	$-i\omega$	$-U_{j^+}^{R*}$	$-U_{j^-}^{R*}$		
$\delta j_{m\downarrow}^{+R} c_{m\uparrow}$	0	$(\xi_m^{+*}V_{km}^{*} \cdots \xi_m^{+*}V_{km}^{*})$	$-\gamma_{LR}$	$-\gamma_j$	$U_{j^+}^{R}$	$-i\omega$	γ_{RR}		
$\delta j_{m\downarrow}^{-R} c_{m\uparrow}$	0	$(\xi_m^{-*}V_{km}^{*} \cdots \xi_m^{-*}V_{km}^{*})$	$-\gamma_j$	$-\gamma_{LR}$	$U_{j^-}^{R}$	$-\gamma_{RR}$	$-i\omega$		
$\delta n_{m\downarrow} c_{k\uparrow}^{R}$	0		(5X∞)					$\begin{pmatrix} z+i\varepsilon_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & z+i\varepsilon_\infty \end{pmatrix}$	0
$c_{k\uparrow}^{R}$								0	$\begin{pmatrix} z+i\varepsilon_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & z+i\varepsilon_\infty \end{pmatrix}$

Constructing Liouvillian matrix: $\mathbf{M} = z\mathbf{I} + i\mathbf{L}$

Basis Vectors: $c^L_{k\uparrow}$, $\delta n_{m\downarrow} c^L_{k\uparrow}$, $\delta j^{-L}_{m\downarrow} c_{m\uparrow}$, $\delta j^{+L}_{m\downarrow} c_{m\uparrow}$, $c_{m\uparrow}$, $\delta j^{+R}_{m\downarrow} c_{m\uparrow}$, $\delta j^{-R}_{m\downarrow} c_{m\uparrow}$, $\delta n_{m\downarrow} c^R_{k\uparrow}$, $c^R_{k\uparrow}$

L-Reservoir

$$
\left(\begin{array}{c|c|ccc|cc|c|c}
\begin{pmatrix} z+i\varepsilon_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & z+i\varepsilon_\infty \end{pmatrix} & 0 & 0 & 0 & \begin{pmatrix} iV^*_{km} \\ \vdots \\ iV^*_{km} \end{pmatrix} & 0 & 0 & 0 & 0 \\
0 & \begin{pmatrix} z+i\varepsilon_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & z+i\varepsilon_\infty \end{pmatrix} & \begin{pmatrix} -\xi^-_m V_{km} \\ \vdots \\ -\xi^-_m V_{km} \end{pmatrix} & \begin{pmatrix} -\xi^+_m V_{km} \\ \vdots \\ -\xi^+_m V_{km} \end{pmatrix} & 0 & \begin{pmatrix} -\xi^+_m V_{km} \\ \vdots \\ -\xi^+_m V_{km} \end{pmatrix} & \begin{pmatrix} -\xi^-_m V_{km} \\ \vdots \\ -\xi^-_m V_{km} \end{pmatrix} & 0 & 0 \\
\hline
0 & (\xi^{+*}_m V^*_{km} \cdots \xi^{+*}_m V^*_{km}) & -i\omega & -\gamma_{LL} & U^L_{j^-} & \gamma_{LR} & \gamma_j & 0 & 0 \\
0 & (\xi^{-*}_m V^*_{km} \cdots \xi^{-*}_m V^*_{km}) & \gamma_{LL} & -i\omega & U^L_{j^+} & \gamma_j & \gamma_{LR} & 0 & 0 \\
(iV_{km} \cdots iV_{km}) & 0 & -U^{L*}_{j^-} & -U^{L*}_{j^+} & -i\omega & -U^{R*}_{j^+} & -U^{R*}_{j^-} & 0 & 0 \\
0 & (\xi^{+*}_m V^*_{km} \cdots \xi^{+*}_m V^*_{km}) & -\gamma_{LR} & -\gamma_j & U^R_{j^+} & -i\omega & \gamma_{RR} & 0 & 0 \\
0 & (\xi^{-*}_m V^*_{km} \cdots \xi^{-*}_m V^*_{km}) & -\gamma_j & -\gamma_{LR} & U^R_{j^-} & -\gamma_{RR} & -i\omega & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & \begin{pmatrix} z+i\varepsilon_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & z+i\varepsilon_\infty \end{pmatrix} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \begin{pmatrix} z+i\varepsilon_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & z+i\varepsilon_\infty \end{pmatrix}
\end{array}\right)
$$

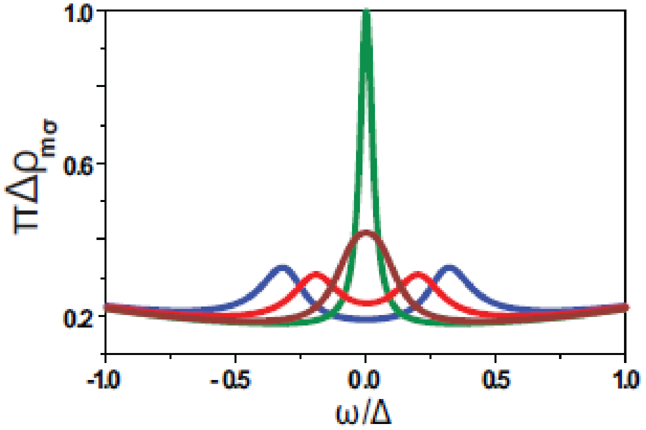
(∞X5)

(5X∞)

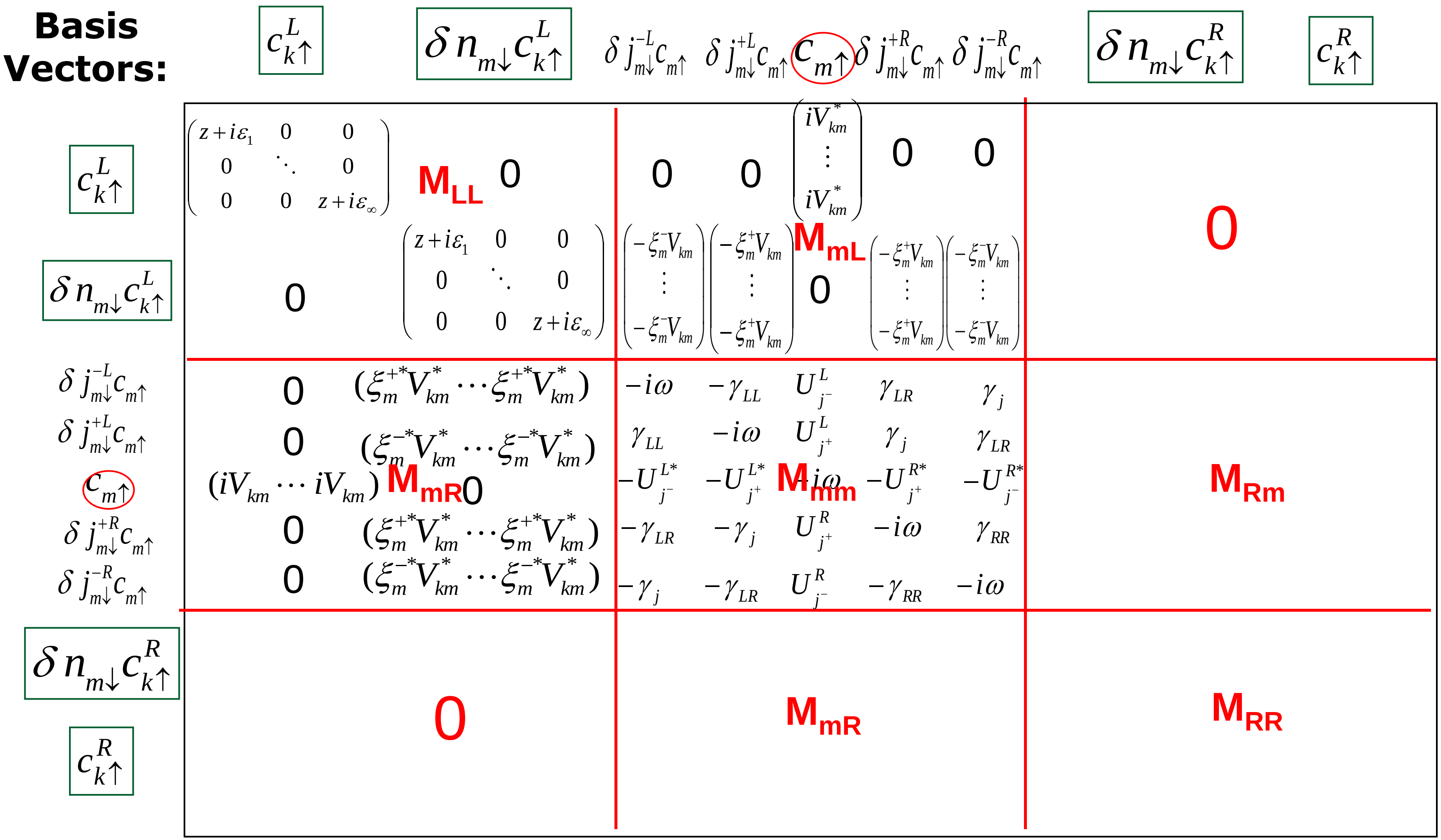
R-Reservoir

Neglecting

parts



Constructing Liouvillian matrix: $\mathbf{M} = z\mathbf{I} + i\mathbf{L}$



Matrix Reduction: Löwdin's Partitioning technique

P.O. Löwdin, J. Math. Phys. Vol. 3, 969 (1962)

Matrix equation:

$$\begin{pmatrix} M_{LL} & M_{mL} & 0 \\ M_{Lm} & M_{mm} & M_{Rm} \\ 0 & M_{mR} & M_{RR} \end{pmatrix} \begin{pmatrix} C_k^L \\ C_m \\ C_k^R \end{pmatrix} = 0$$

$$k = 0, 1, 2, \cdots, \infty$$

$$C_m = (C_1, C_2, C_3, C_4, C_5)^T$$

$$(M_{mm} - M_{Lm} M_{LL}^{-1} M_{mL} - M_{Rm} M_{RR}^{-1} M_{mR}) C_m \equiv M_r C_m$$

$$M_r = M_{mm} - \underline{\underline{M_{Lm} M_{LL}^{-1} M_{mL} - M_{Rm} M_{RR}^{-1} M_{mR}}}$$: 5X5 matrix

self-energy

$\Downarrow$

On-site Green's function: $iG_{mm}^{+}(\omega) = (\mathbf{M}_{\mathbf{r}}^{-1})_{33}$

$$\rho_m^{ss}(\omega) = (1/\pi)\,\mathrm{Re}\,(\mathbf{M}_{\mathbf{r}}^{-1})_{33}$$

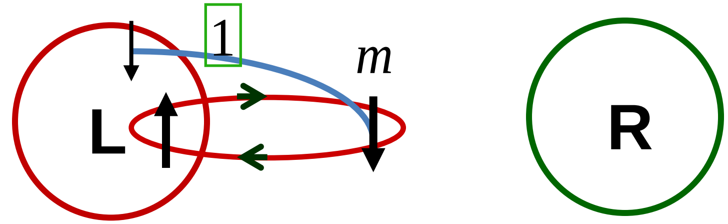
Reservoir degrees of freedom are transformed into **self-energy** by matrix reduction process

$$\rho_m^{ss}(\omega) = (1/\pi)\,\mathrm{Re}\left(\mathbf{M}_r^{-1}\right)_{33}$$

$$\mathbf{M}_r = \begin{bmatrix} -i\omega & -\gamma_{LL} & U^{L}_{j^-} & \gamma_{LR} & \gamma_j \\ \gamma_{LL} & -i\omega & U^{L}_{j^+} & \gamma_j & \gamma_{LR} \\ -U^{L*}_{j^-} & -U^{L*}_{j^+} & -i\omega & -U^{R*}_{j^+} & -U^{R*}_{j^-} \\ -\gamma_{LR} & -\gamma_j & U^{R}_{j^+} & -i\omega & \gamma_{RR} \\ -\gamma_j & -\gamma_{LR} & U^{R}_{j^-} & -\gamma_{RR} & -i\omega \end{bmatrix}$$

with additional $i\beta_{ij}[\Sigma^L_0(\omega)+\Sigma^R_0(\omega)]$ **except U-terms**

$$\gamma_{LL} = \langle i\Sigma_k V_{km}(c^L_{k\uparrow} + c^R_{k\uparrow})c^+_{m\uparrow}[j^{-L}_{m\downarrow}, j^{+L}_{m\downarrow}]\rangle$$

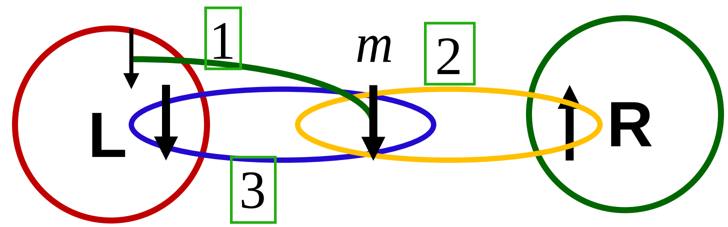


$$\langle [c^+_{m\uparrow} c^{+L}_{k'\downarrow} c^L_{k\uparrow} c_{m\downarrow}, c^+_{m\downarrow} c^L_{k'\downarrow}]\rangle$$

exchange

$$\langle [c^+_{m\uparrow} c^{+L}_{k'\downarrow} c^R_{k\uparrow} c_{m\downarrow}, c^+_{m\downarrow} c^L_{k'\downarrow}]\rangle$$

singlet hopping



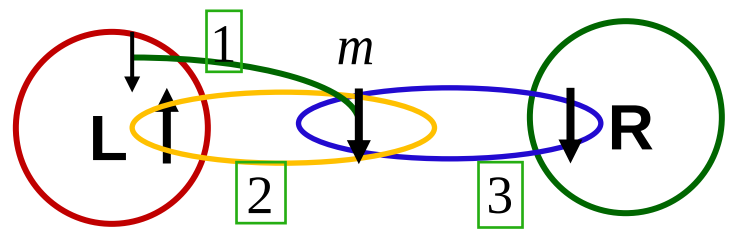
$$\gamma_{LR} = \langle i\Sigma_k V_{km}(c^L_{k\uparrow} + c^R_{k\uparrow})c^+_{m\uparrow}[j^{-L}_{m\downarrow}, j^{+R}_{m\downarrow}]\rangle$$

$+\, sign$

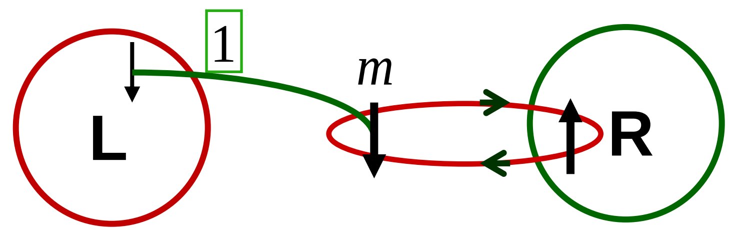
$$\gamma_j = \langle i\Sigma_k V_{km}(c^L_{k\uparrow} + c^R_{k\uparrow})c^+_{m\uparrow}[j^{\mp L}_{m\downarrow}, j^{\mp R}_{m\downarrow}]\rangle$$

$-\, sign$

$$\langle [c^+_{m\uparrow} c^{+R}_{k''\downarrow} c^L_{k\uparrow} c_{m\downarrow}, c^+_{m\downarrow} c^L_{k'\downarrow}]\rangle \pm \langle [c^+_{m\uparrow} c^{+L}_{k''\downarrow} c^R_{k\uparrow} c_{m\downarrow}, c^+_{m\downarrow} c^R_{k'\downarrow}]\rangle$$



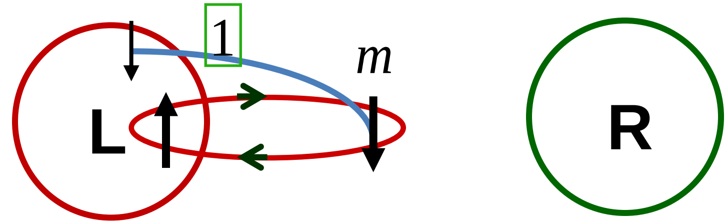
$$\langle [c^+_{m\uparrow} c^{+R}_{k''\downarrow} c^R_{k\uparrow} c_{m\downarrow}, c^+_{m\downarrow} c^L_{k'\downarrow}]\rangle \pm \langle [c^+_{m\uparrow} c^{+L}_{k''\downarrow} c^L_{k\uparrow} c_{m\downarrow}, c^+_{m\downarrow} c^R_{k'\downarrow}]\rangle$$



$$\gamma_{LL} = \langle i\Sigma_k V_{km}(c^L_{k\uparrow} + c^R_{k\uparrow})c^+_{m\uparrow}[j^{-L}_{m\downarrow}, j^{+L}_{m\downarrow}]\rangle$$

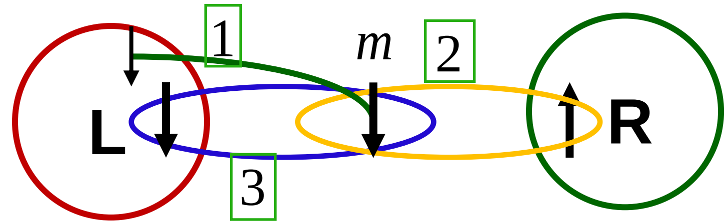
$$\langle [c^+_{m\uparrow} c^{+L}_{k'\downarrow} c^L_{k\uparrow} c_{m\downarrow}, c^+_{m\downarrow} c^L_{k'\downarrow}]\rangle$$

exchange



$$\langle [c^+_{m\uparrow} c^{+L}_{k'\downarrow} c^R_{k\uparrow} c_{m\downarrow}, c^+_{m\downarrow} c^L_{k'\downarrow}]\rangle$$

singlet hopping

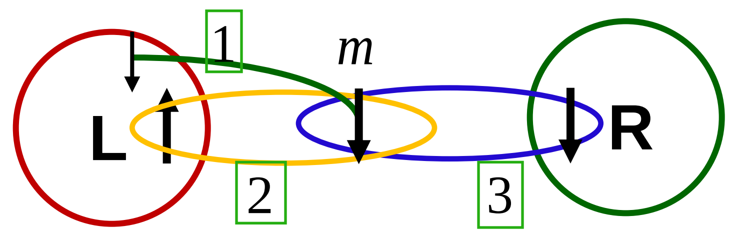


$$\gamma_{LR} = \langle i\Sigma_k V_{km}(c^L_{k\uparrow} + c^R_{k\uparrow})c^+_{m\uparrow}[j^{-L}_{m\downarrow}, j^{+R}_{m\downarrow}]\rangle \quad + sign$$

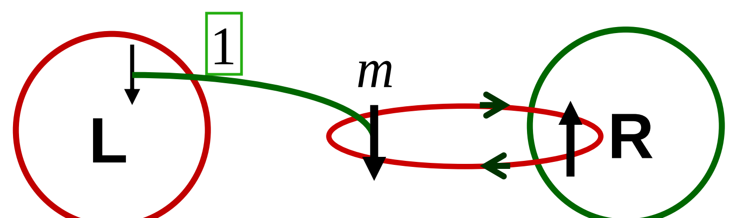
$$\gamma_j = \langle i\Sigma_k V_{km}(c^L_{k\uparrow} + c^R_{k\uparrow})c^+_{m\uparrow}[j^{\mp L}_{m\downarrow}, j^{\mp R}_{m\downarrow}]\rangle \quad - sign$$

$\gamma_j = 0$ *at equil.*

$$\langle [c^+_{m\uparrow} c^{+R}_{k''\downarrow} c^L_{k\uparrow} c_{m\downarrow}, c^+_{m\downarrow} c^L_{k'\downarrow}]\rangle \pm \langle [c^+_{m\uparrow} c^{+L}_{k''\downarrow} c^R_{k\uparrow} c_{m\downarrow}, c^+_{m\downarrow} c^R_{k'\downarrow}]\rangle$$



$$\langle [c^+_{m\uparrow} c^{+R}_{k''\downarrow} c^R_{k\uparrow} c_{m\downarrow}, c^+_{m\downarrow} c^L_{k'\downarrow}]\rangle \pm \langle [c^+_{m\uparrow} c^{+L}_{k''\downarrow} c^L_{k\uparrow} c_{m\downarrow}, c^+_{m\downarrow} c^R_{k'\downarrow}]\rangle$$



Under bias, $\gamma_{LR} = \gamma_j$ **: Condition for steady-state non-equil.**

$$\rho_m^{ss}(\omega) = (1/\pi)\,\mathrm{Re}\left(\mathbf{M}_r^{-1}\right)_{33}$$

3X3

$$\mathbf{M}_r = \begin{bmatrix} -i\omega & -\gamma_{LL} & U^{L}_{j^-} & \gamma_{LR} & \gamma_j \\ \gamma_{LL} & -i\omega & U^{L}_{j^+} & \gamma_j & \gamma_{LR} \\ -U^{L*}_{j^-} & -U^{L*}_{j^+} & -i\omega & -U^{R*}_{j^+} & -U^{R*}_{j^-} \\ -\gamma_{LR} & -\gamma_j & U^{R}_{j^+} & -i\omega & \gamma_{RR} \\ -\gamma_j & -\gamma_{LR} & U^{R}_{j^-} & -\gamma_{RR} & -i\omega \end{bmatrix}$$

with additional $i\beta_{ij}[\Sigma^L_0(\omega)+\Sigma^R_0(\omega)]$ except U-terms

*** Conventional Kondo System:**
(Single Reservoir)

3X3

Single Reservoir:

$$\mathbf{M}_{\mathbf{r}} = \begin{bmatrix} -\mathrm{i}\omega & -\gamma_{LL} & U^{L}_{j^-} \\ \gamma_{LL} & -\mathrm{i}\omega & U^{L}_{j^+} \\ -U^{L*}_{j^-} & -U^{L*}_{j^+} & -\mathrm{i}\omega \end{bmatrix}$$

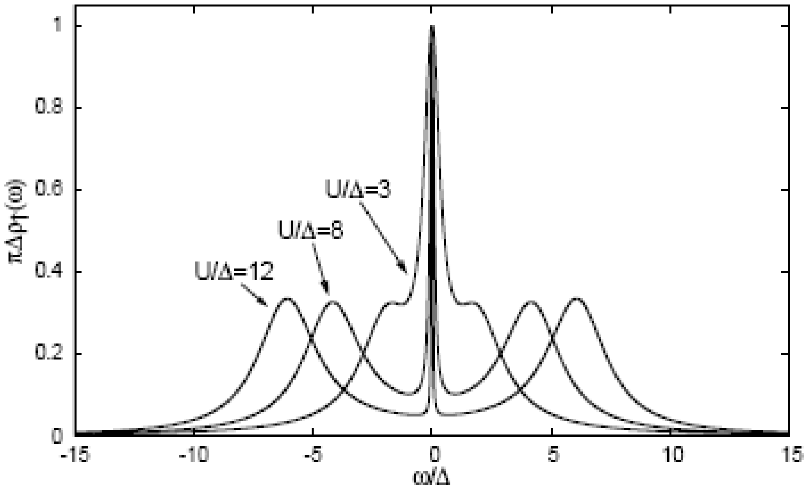
$$U^{L}_{j^{\pm}} = U/\,2\sqrt{2}$$

with additional self-energy terms $i\beta_{ij}\Sigma^{L}{}_{0}(\omega)$ except U-elements

$$\gamma_{LL} = \langle \sum_{k} V^{*}_{km}\, c_{k\uparrow} c^{+}_{m\uparrow}\, \dot{j}^{-L}_{m\downarrow} \dot{j}^{+L}_{m\downarrow} \rangle$$

$$\rho_{\sigma}(\omega) = (1/\pi)\,\mathrm{Re}\,(\mathbf{M}_{\mathbf{r}}^{-1})_{33}$$

$$\gamma_{LL} = (U/2)\sqrt{\frac{Z_S}{1-Z_S}}$$



$$Z_S = Z_S^{BA} = (4/\pi)\sqrt{U/\Gamma}\; e^{-\frac{\pi U}{4\Gamma}}$$

Our APPROXIMATION is reasonable!!

Information from Liouvillian Matrix: $i\mathbf{L}$

Two-reservoir under bias

$$
i\mathbf{L} = \begin{bmatrix} 0 & -\gamma & \tilde{U} & \gamma' & \gamma' \\ \gamma & 0 & \tilde{U} & \gamma' & \gamma' \\ -\tilde{U} & -\tilde{U} & 0 & -\tilde{U} & -\tilde{U} \\ -\gamma' & -\gamma' & \tilde{U} & 0 & \gamma \\ -\gamma' & -\gamma' & \tilde{U} & -\gamma & 0 \end{bmatrix}
$$

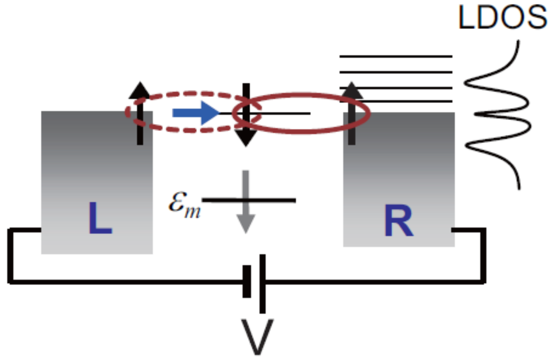
Eigenvalues
(tunneling levels):

Fermi level	Side Peaks	Coulomb Peaks
$0,$	$\mp i\gamma,$	$\mp i\sqrt{4\tilde{U}^2+\gamma^2}$

Eigenvectors
(tunneling modes):

$$
\begin{matrix} \dot{j}^{-L}_{m\downarrow} c_{m\uparrow} \\ \dot{j}^{+L}_{m\downarrow} c_{m\uparrow} \\ c_{m\uparrow} \\ \dot{j}^{+R}_{m\downarrow} c_{m\uparrow} \\ \dot{j}^{-R}_{m\downarrow} c_{m\uparrow} \end{matrix} \qquad \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}
$$

L-R symmetric



singlet co-tunneling
with exchange

Information from Liouvillian Matrix: $i\mathbf{L}$

Two-reservoir under bias

$$
i\mathbf{L} = \begin{bmatrix} 0 & -\gamma & \tilde{U} & \gamma' & \gamma' \\ \gamma & 0 & \tilde{U} & \gamma' & \gamma' \\ -\tilde{U} & -\tilde{U} & 0 & -\tilde{U} & -\tilde{U} \\ -\gamma' & -\gamma' & \tilde{U} & 0 & \gamma \\ -\gamma' & -\gamma' & \tilde{U} & -\gamma & 0 \end{bmatrix}
$$

Eigenvalues

(tunneling levels):

Side Peaks

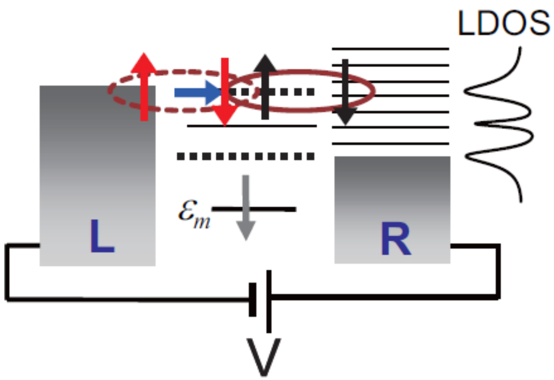
$$
0,\quad \mp i\gamma,\quad \mp i\sqrt{4\tilde{U}^2+\gamma^2}
$$

Eigenvectors

(tunneling modes):

$$
\begin{matrix} j^{-L}_{m\downarrow} c_{m\uparrow} \\ j^{+L}_{m\downarrow} c_{m\uparrow} \\ c_{m\uparrow} \\ j^{+R}_{m\downarrow} c_{m\uparrow} \\ j^{-R}_{m\downarrow} c_{m\uparrow} \end{matrix}
\quad
\begin{bmatrix} -1 \\ i \\ 0 \\ -i \\ 1 \end{bmatrix}
\quad \& \quad
\begin{bmatrix} -1 \\ -i \\ 0 \\ i \\ 1 \end{bmatrix}
$$

L-R antisymmetric

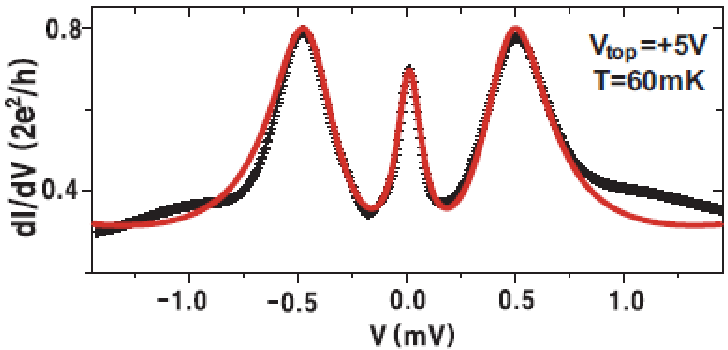


singlet co-tunneling

Reproducing dI/dV line shapes (some of my results)

Quantum Point Contact

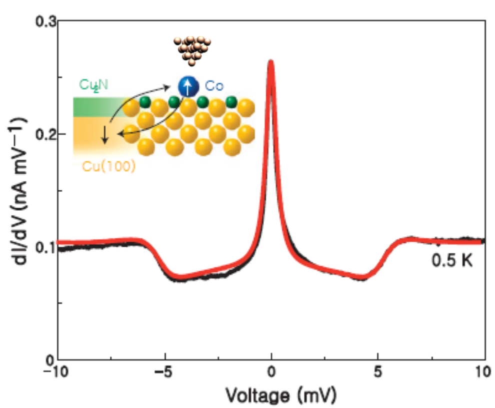
Sarkozy, PRB (2009)



γ_{LL}	γ_{RR}	$\gamma_{j,LR}$	$\mathrm{Re}U_{j-}^{L,R}$	$\mathrm{Re}U_{j+}^{L}$	$\mathrm{Re}U_{j+}^{R}$	$\mathrm{Im}U_{j\pm}^{L,R}$
0.5	0.5	0.65	1.05	1.26	1.26	0

Adsorbed magnetized atom on an insulating layer covering metallic substrate

Otte, N. Phys. 2008

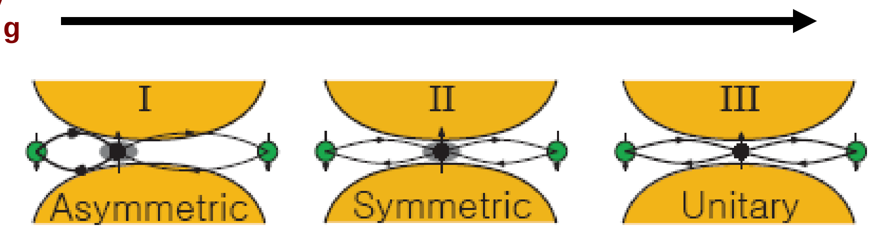


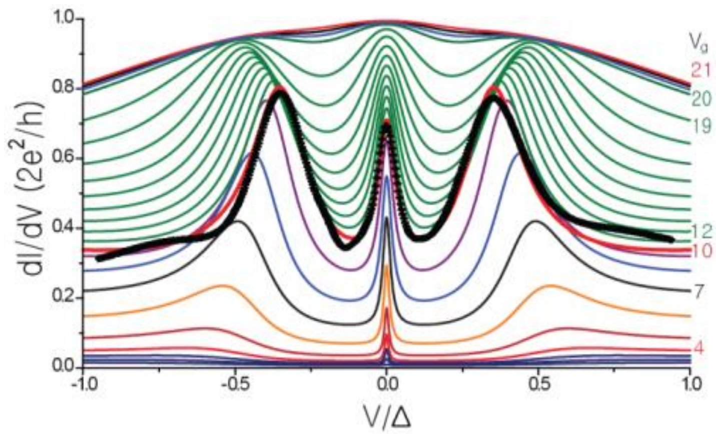
γ_{LL}	γ_{RR}	$\gamma_{j,LR}$	$\mathrm{Re}U_{j-}^{L,R}$	$\mathrm{Re}U_{j+}^{L}$	$\mathrm{Re}U_{j+}^{R}$	$\mathrm{Im}U_{j\pm}^{L,R}$
0.8	0.7	0.43	2.8	7.0	1.62	0

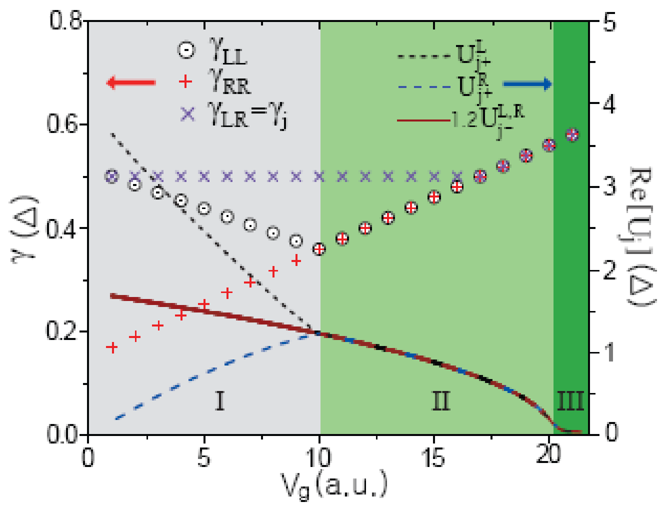
Gate voltage-dependent dI/dV

Scenario by spin-DFT calculation

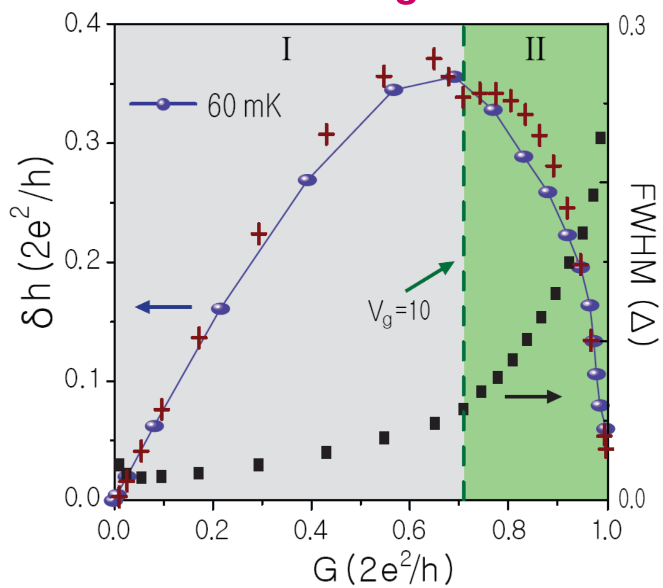
V_g





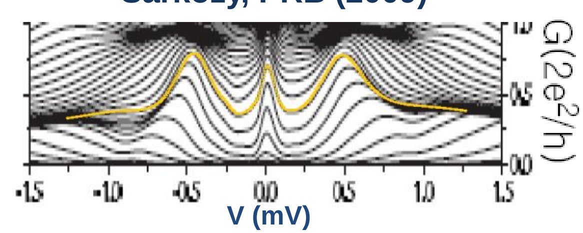


ZBP-height



Quantum Point Contact

Sarkozy, PRB (2009)

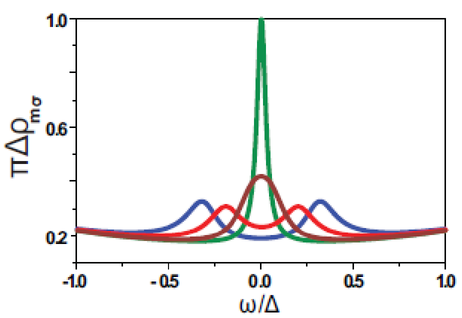


Conclusions

(1) We study mesoscopic Kondo systems and find that the entanglement effect (coherent superposition between two Kondo clouds) is critical to explain the experimental line shapes of tunneling conductance.

(2) We show that antisymmetric superposition does not vanish under bias and gives two coherent side peaks.

(3) We recover the bias-dependent Kondo peak splitting by treating a system having independent Kondo clouds.



(4) We reproduce the experimental dI/dV line shapes obtained for various mesoscopic Kondo systems.

(5) More things are left to fully appreciate the entangled singlet dynamics imbedded in mesoscopic Kondo systems and strongly correlated samples.