Anomalous quantum glass of bosons in a random potential in two dimensions

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References:
arXiv:1110.3213, revised 2014(v3)

outlines

Introduction

QMC simulations of site-disordered BHM in 2D

Percolation scenario and avoided degeneracies

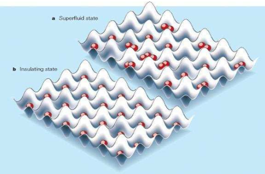
MG-BG crossover and Critical behavior at SF-QG transition

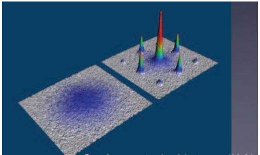
Summary

Interacting lattice bosons: clean system

$$H = -t \sum_{\langle ij \rangle} (b_i^+ b_j + b_j^+ b_i) + \frac{U}{2} \sum_{i=1}^{N} n_i(n_i - 1) - \mu \sum_{i=1}^{N} n_i,$$

two ground states SF and MI

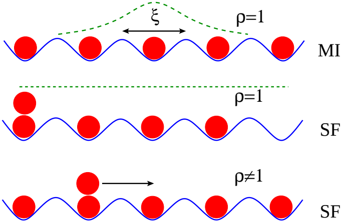


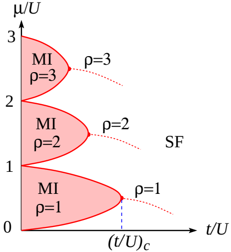


Greiner et al., Nature ('02)

- MI: integer filling, insulating, gaped
- SF: any filling fraction, gapless

two ways from MI to SF





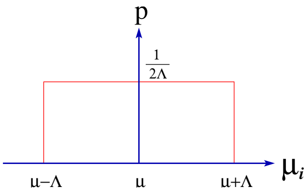
Interacting lattice bosons: disorder present

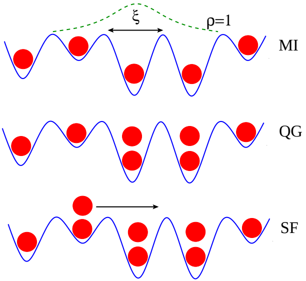
Fascinating interplay between disorder, interactions and SF

- two kinds of disorder: hopping disorder and site disorder
- we focus on site disorder: no explicit particle-hole symmetry

$$H = -t\sum_{\langle ij\rangle}(b_i^+ b_j + b_j^+ b_i) + \frac{U}{2}\sum_{i=1}^{N} n_i(n_i - 1) - \sum_{i=1}^{N}\mu_i n_i,$$

Bounded disorder

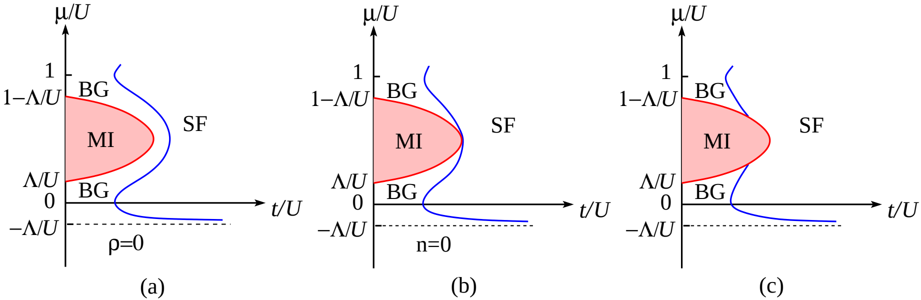




- **Quantum Glass**: insulating but Gapless;
  Current prevailing notion: always compressible in 2D with random potentials $\rightarrow$ BG

Previous studies of the site-disordered BH model

$T = 0$ phase diagram: in the presence of disorder, QG always intervenes SF and MI ?



Fisher *et al*, PRB, 1989

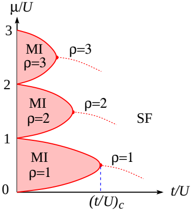
(a) Söyler '11; Prokof'ev, '04; Pollet, '09; Herbut, '97; '98; Weichman, '96, '08; Svistunov, '96;

(b) Singh; '92; Pazmandi; '98; Pai, '96;

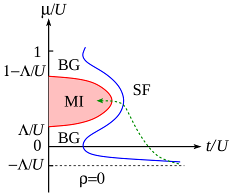
(c) Scalettar, '91; Krauth, '91; Kisker; '97; Sen, '01; Lee, '01; Wu, '08; Bissbort, '09

Recent progresses: theorem of inclusion

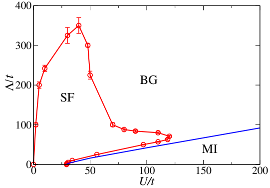
Without disorder



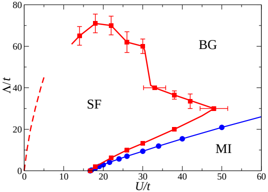
With disorder



$\rho = 1$, in the $U - \Lambda$ plane



QG always between SF and MI, 3D

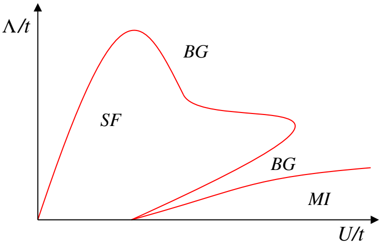


QG always between SF and MI, 2D

Pollet *et al*, PRL, 2009; Söyler *et al*, 2011; Rieger, meanfield phase diagram, NJP, 2013

$\rho = 1$ 2D BHM with site disorder

The phase diagram is well established, the properties of QG state for 2D BHM with site disorder are not well understood



two types of glass states are known;

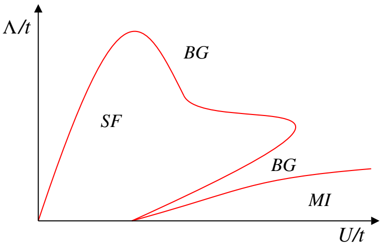
- the compressible Bose glass (BG)
- the incompressible Mott glass (MG)

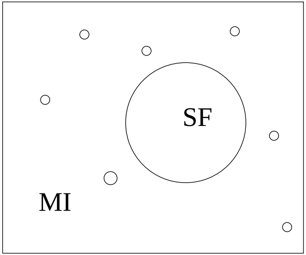
Commonly believed:

- MG only at commensurate filling with particle-hole symmetry
- BG in the 2D site-disordered BHM, always compressible

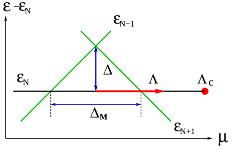
pollet et al PRL 2009,Gurarie et al PRB 2009,Söyler, PRL 2011.

Standard Scenario





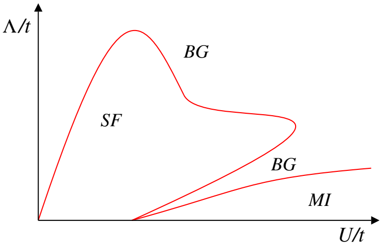
The "finger region" is a Griffiths phase: **rare large regions** of phase A (SF) inside phase B (MI) lead to singularities.

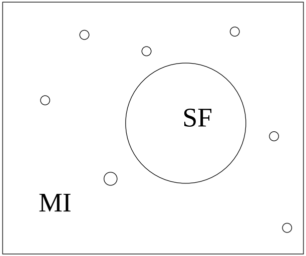


Griffiths arguments: Fisher 89,Freericks 96

- MI-QG boundary based on $\Lambda = \Delta_\mathrm{M}/2$
- $\Lambda > \Delta_\mathrm{M}/2$, arbitrarily large SF puddles can appear
- $\Lambda < \Lambda_c$, SF puddles NOT percolating, insulating
- Fundamentally different from the MI:
  gapless due to arbitrarily large SF region

Standard scenario





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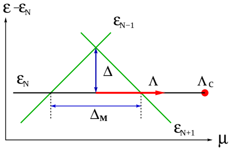
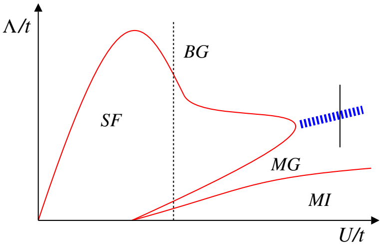


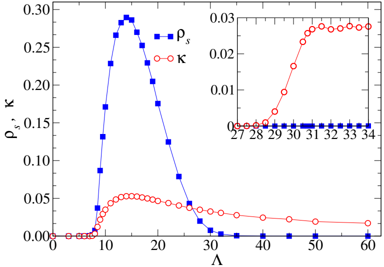
- fluctuations of the overall chemical potential within the **SF domains** lead to near degeneracies of different particle-number sectors $\rightarrow$ nonzero compressibility BG

Pollet *et al*, PRL, 2009; Gurare *et al* PRB, 2009

$\rho = 1$ 2D BHM with site disorder, QMC study



$L = 16, \beta = 8$



QG for $4.2 < \Lambda < 7.8$

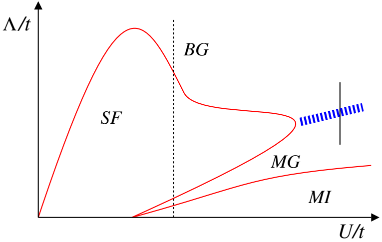
Using SSE QMC, we study

- compressibility κ: particle-number fluctuations
- superfluid stiffness ρ_s: winding number fluctuations

Parameters:

- Adjust μ to ensure $\rho = \langle n \rangle / N = 1$
- Fix (a) $U/t = 22$ (b) $U/t = 60$
- Average over up to hundreds of realizations

Our MC results



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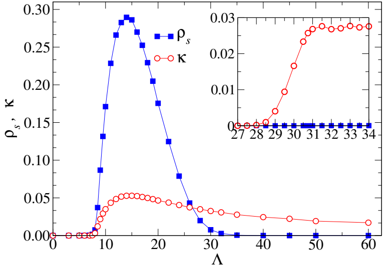


QG for $4.2 < \Lambda < 7.8$

- ρ_s:
  - sharp increase at $\Lambda \approx 8$, enter SF
  - decreases to zero at $\Lambda \approx 30$, enter QG.
- κ
  - substantial in SF and QG re-entered at large Λ.
  - However, it is very small before entering the SF phase, not only in the Mott phase but also in QG

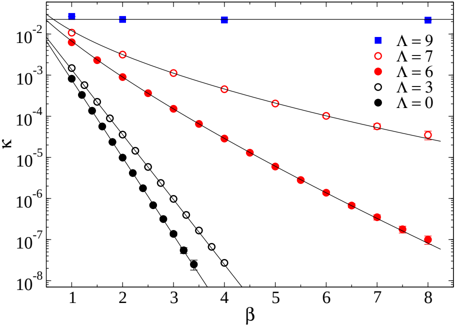
QMC simulations : $\kappa - T$ behavior

- Quantum Glass region

$$\kappa \sim \exp(-b/T^{\alpha}) + c, \qquad \alpha < 1, c = 0$$

- comparing to MI: the above form with $\alpha = 1$ and $b = \Delta$

$L = 32, U/t = 22$



- ▸ κ in **QG (MG)** follows the exponential form

$$\kappa \sim \exp(-b/T^{\alpha})$$

$\alpha \approx 0.77$ for $\Lambda = 6$
$\alpha \approx 0.53$ for $\Lambda = 7$

- ▸ MI points ($\Lambda = 0$ and 3): $\kappa \propto \mathrm{e}^{-\Delta/T}$
- ▸ SF point ($\Lambda = 9$): $\kappa(T)$ converges rapidly to a non-zero value

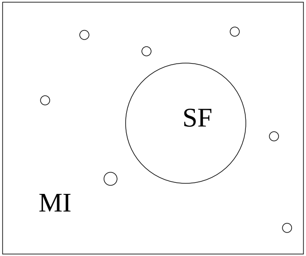
Temperature behavior of κ

- χ in random quantum spin systems, corresponding to κ, vanishes as $T \to 0$ due to spin-inversion symmetry, corresponding to particle-hole symmetry for bosons

  Roscilde, PRL 2007, Ma 2014
- Such an incompressible and insulating QG is termed an MG and has also been shown to exist in variants of the 2D random BHM where particle-hole symmetry is explicitly built in Altman PRB, 2010, Iyer PRB 2012
- In the presence of random potentials there is no explicit particle-hole symmetry. But, in principle there could be emergent particle-hole symmetry, as in the clean BHM at the tips of the Mott lobes
- $\kappa = 0$ may not hold strictly, the physics behind is similar to a true Mott Glass.

Percolation scenario

Consider an ensemble of SF domains below the percolation threshold

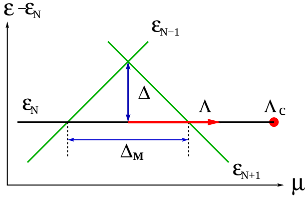


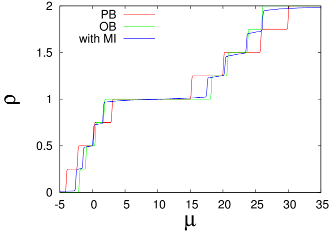
- Finite-size (s) gap $\propto \frac{1}{s^a}$, a unknown exponent.
- Given $T = \frac{1}{m^a}$, all domains of sizes $s < m$ gaped, NO contribution to κ
- Only domains of sizes $s > m$ contribute to κ
  Prob. of a site belong to an SF domain with $s > m$ is $\propto \exp(-bm^c)$, c unknown exponent.

thus

$$\kappa \propto \exp(-bT^{-c/a}) = \exp(-bT^{-\alpha})$$

How about the degeneracy of different particle-number sectors?

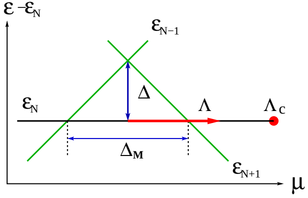


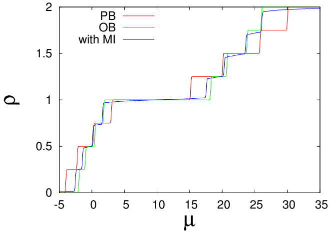


- The percolation scenario neglects the arbitrarily close degeneracy of different particle-number sectors due to fluctuations of the average chemical potential of the domains, which lead to $\kappa(T=0) > 0$ in the standard BG scenario

- How can these degeneracies be avoided?

Avoided degeneracy

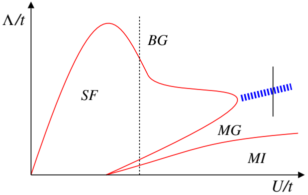




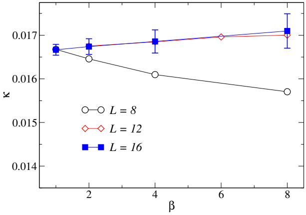
study isolated domains embedded in Mott background

- finite-size effects: particle-number degeneracies only occur when the domains are very large with the critical size diverging at the Mott phase boundary
- All domains below the critical size have vanishing compressibility and should not be regarded as superfluid
- rare large domains should also have an altered spectral structure due to quantum-criticality when the SF boundary is approached.
- both small and large typical domains (the latter of which are fractals) may not contribute to the $T = 0$ compressibility.

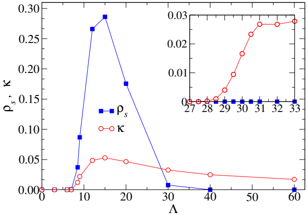
BG and Crossover between MG and BG



$\Lambda = 60$



$U = 60, \beta = 8$

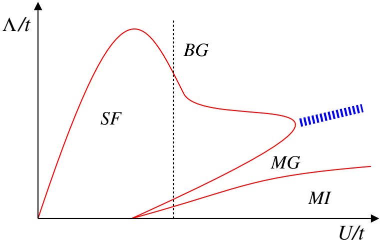


- compressible BG in the re-entrance region
- dramatic variation in the compressibility along a vertical line at $U = 60$. κ increases rapidly with Λ between 28 to 31, Mott boundary at $\Lambda \approx 24$ enhancement is more than four orders of magnitude

a sharp cross over, not a phase transition

a change from a MG: typical non-Mott domains are not superfluid to a BG: the domains are superfluid but do not form a coherent global state.

Critical behavior at SF-MG transition ($T = 0$)



- Scaling at the quantum transition point

$$\kappa(\Lambda) \propto (\Lambda - \Lambda_c)^{\nu(2-z)}$$

if $z = 2$ as often argued (Söyler, PRL, 2013), then $\kappa \neq 0$ at Λ_c and inside glass closing to Λ_c.
if $z < 2$ then $\kappa = 0$ at Λ_c.

- A key question then is whether $z = 2$ or $z < 2$ In the former case divergent SF clusters in the MI background close to the percolation point would be compressible, while in the latter case they should be incompressible.

- There are arguments for $z = 2$ but no rigorous proofs.
- Some works on models related to BHM have $z < 2$

  Meier, PRL 2012; Priyadarshee, PRL 2006;

  some suggest $z = 2$, but with large error, also consistent with $z < 2$ Krauth, PRL 1991, Alet, PRE, 2003; Prokof'ev, PRL 2004

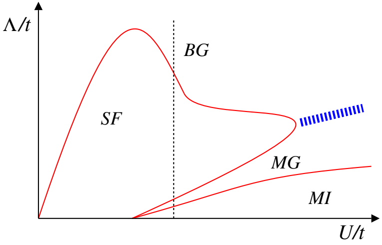
Critical behavior at SF-MG transition ($T = 0$)

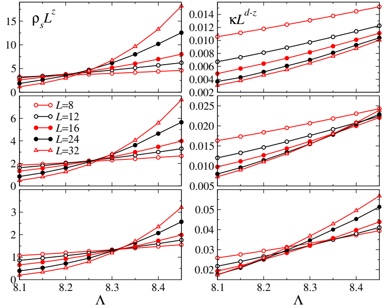
- Determine the dynamic exponent z according to FSS

$$\kappa_u(\Lambda, L) \propto L^{z-d}(\Lambda - \Lambda_c), \qquad \rho_s(\Lambda, L) \propto L^{-z}(\Lambda - \Lambda_c),$$

$\kappa_u(\Lambda, L)$ and $\rho_s(\Lambda, L)$ are calculated at $\beta \approx L^z$.

- Comparing three conjectured $z = 2, 1.75$ and 1.5





- At $T = 0$, $\kappa \sim (\Lambda - \Lambda_c)^{\nu(2-z)}$, $z \approx 1.75$, continuous

Critical behavior at SF-MG transition ($T = 0$)

- it has been implicitly assumed that any non-singular contributions to κ can be neglected.
- If regular contributions arise from SF domains larger than a critical size, then we would expect these contributions to increase with L, leads to an apparent enhancement of z.
  We find a reduction from $z = 2$, non-singular background contributions are not responsible for this effect and $z < 2$ should be a robust result.
- consistent with the drop of κ at SF-MG from right, while at SF-BG there are no strong variations, suggesting $z = 2$

Summary

- Based on plausible arguments and unbiased QMC results: there is an novel QG state with extremely small κ decaying exponentially with temperature for commensurate filling and moderate disorder strength in 2D.
- Percolation scenario: finite-size gap of SF domains explains the exponentially decay of κ
- Finite-size effects also make particle-number degeneracies avoided
- A dynamic exponent $z < 2$ provides an explanation for an anomalously small, or possibly vanishing, $T = 0$ compressibility in the finger region
- The sharp cross-over from anomalously small to normal compressibility away from the SF phase at larger U also shows that there are two distinct types of glass phases
- The scenario in this work applies only to integer filling fractions

Thank You