odd interactions in quantum magnets and liquids

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plan

I. interactions of magnons

II. triangular AF, $S(\boldsymbol{q},\omega)$

III. ^{4}He, roton

IV. XY magnets, lifetime

Mike Zhitomirsky

Sasha Chernyshev

MZ and **SC**, PRL **82**, 4536 (1999);
SC and **MZ**, PRL **97**, 207202 (2006);
SC and **MZ**, PRB **79**, 144416 (2009);
M. Mourigal, **MZ**, and **SC**, PRB **82** 144402 (2010);
W. Fuhrman, M. Mourigal, **MZ**, and **SC**, PRB **88**, 184405 (2012);

I. and II.

MZ and **SC**, RMP **85**, 219 (2013);
M. Mourigal, W. Fuhrman, **SC**, and **MZ**, PRB **88**, 094407 (2013)

REVIEWS OF MODERN PHYSICS, VOLUME 85, JANUARY–MARCH 2013

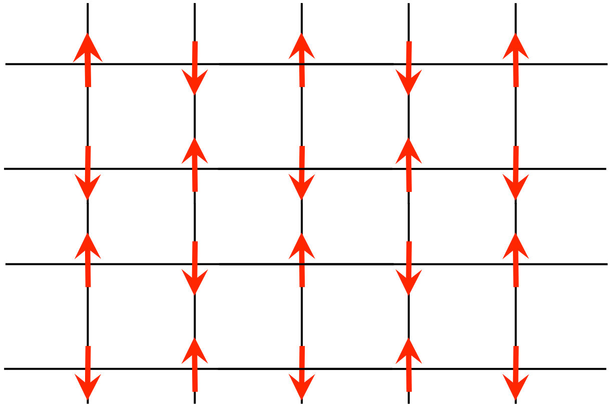
***Colloquium*: Spontaneous magnon decays**

M. E. Zhitomirsky

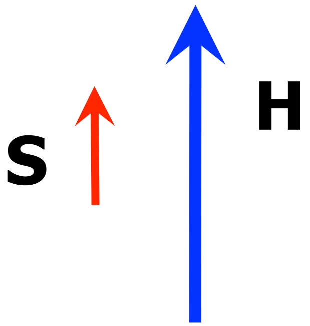
Service de Physique Statistique, Magnétisme et Supraconductivité, UMR-E9001,

According to conventional wisdom a quasiparticle is presumed well defined until proven not to be. The textbook

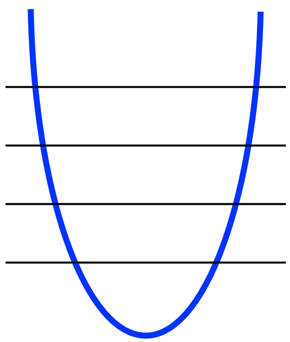
magnons ≈ bosons, why?



- spins commute on different sites
- effective Weiss field for each spin
- local raising (lowering) operators

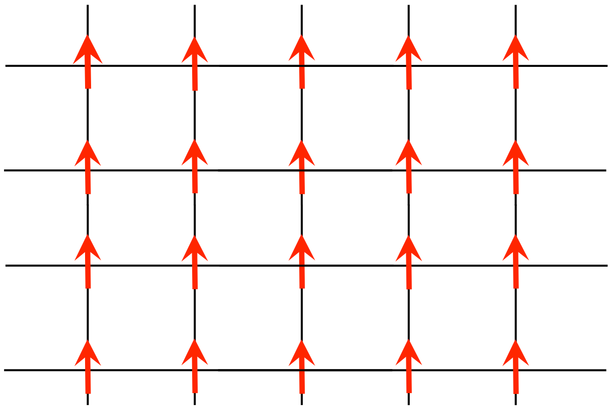


$$\mathcal{H} = \mathbf{H} \cdot \mathbf{S}$$



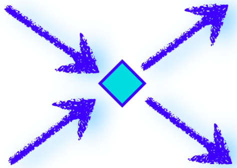
$$S^z = S - b^\dagger b,$$
$$S^+ \approx b, \quad S^- \approx b^\dagger$$

what would bosons do?



- Hamiltonian of the Bose gas
  - # of bosons preserved

$$\hat{\mathcal{H}}_0 = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} b^\dagger_{\mathbf{k}} b_{\mathbf{k}} + \frac{1}{4} \sum_{\mathbf{k}_i} V^{(1)}_{\mathbf{k}_1,\mathbf{k}_2;\mathbf{k}_3,\mathbf{k}_4} b^\dagger_{\mathbf{k}_1} b^\dagger_{\mathbf{k}_2} b_{\mathbf{k}_3} b_{\mathbf{k}_4}$$



- exactly the same for a ferromagnet
  - GS and excitations are e-states of $S^z_{\rm tot}$
  - GS $S^z_{\rm tot}\,|GS\rangle = SN\,|GS\rangle$
  - magnon $\Rightarrow \Delta S^z = -1$
  - magnon # preserved

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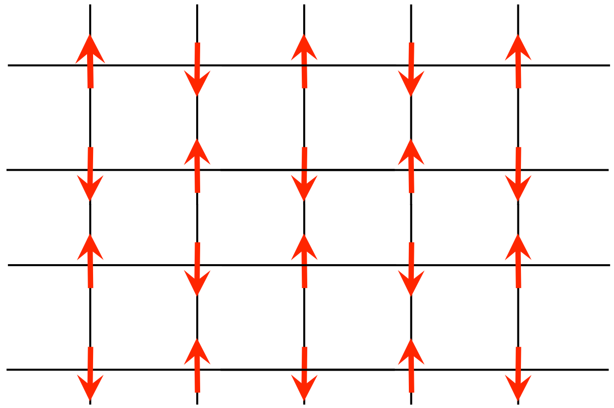
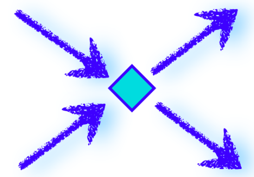
what would bosons do in an AF?

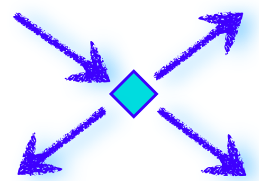
- in an antiferromagnet
  - GS is the superposition of states with different $S_{\rm tot}$ [PWA'82]
  - no definite spin for a magnon!
  - magnon # **not** preserved: $b \Rightarrow ub + vb^\dagger$



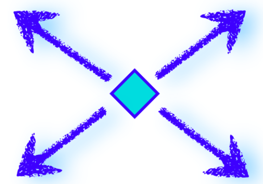
$$\hat{\mathcal{H}}_0 = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} b^\dagger_{\mathbf{k}} b_{\mathbf{k}} + \frac{1}{4} \sum_{\mathbf{k}_i} V^{(1)}_{\mathbf{k}_1,\mathbf{k}_2;\mathbf{k}_3,\mathbf{k}_4} b^\dagger_{\mathbf{k}_1} b^\dagger_{\mathbf{k}_2} b_{\mathbf{k}_3} b_{\mathbf{k}_4}$$



$$+ \frac{1}{3!} \sum_{\mathbf{k}_i} V^{(2)}_{\mathbf{k}_1,\mathbf{k}_2;\mathbf{k}_3,\mathbf{k}_4} b^\dagger_{\mathbf{k}_1} b^\dagger_{\mathbf{k}_2} b^\dagger_{\mathbf{k}_3} b_{\mathbf{k}_4} + \mathrm{H.c.}$$



$$+ \frac{1}{4!} \sum_{\mathbf{k}_i} V^{(3)}_{\mathbf{k}_1,\mathbf{k}_2;\mathbf{k}_3,\mathbf{k}_4} b^\dagger_{\mathbf{k}_1} b^\dagger_{\mathbf{k}_2} b^\dagger_{\mathbf{k}_3} b^\dagger_{\mathbf{k}_4} + \mathrm{H.c.}$$



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formally …

PHYSICAL REVIEW B VOLUME 3, NUMBER 3 1 FEBRUARY 1971

Dynamics of an Antiferromagnet at Low Temperatures: Spin-Wave Damping and Hydrodynamics*

A. B. Harris and D. Kumar
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104

and

B. I. Halperin and P. C. Hohenberg

HKHH, 71

V. G. Bar'yakhtar, V. L. Sobolev, and A. G. Kvirikadze, ZhETP **65**, 790 (1973);

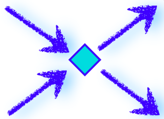
S. M. Rezende and R. M. White, PRB **14**, 2939 (1976); **18**, 2346 (1978);

Yu. A. Kosevich and A. V. Chubukov, Sov. Phys. JETP **64**, 654 (1986)];

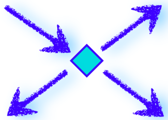
S. Tyc and B. I. Halperin, PRB **42**, 2096 (1990);

P. Kopietz, PRB **41**, 9228 (1990).

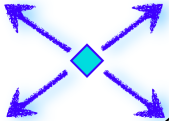
$$\hat{\mathcal{H}}_0 = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} b^\dagger_{\mathbf{k}} b_{\mathbf{k}} + \frac{1}{4}\sum_{\mathbf{k}_i} V^{(1)}_{\mathbf{k}_1,\mathbf{k}_2;\mathbf{k}_3,\mathbf{k}_4}\, b^\dagger_{\mathbf{k}_1} b^\dagger_{\mathbf{k}_2} b_{\mathbf{k}_3} b_{\mathbf{k}_4}$$



$$+ \frac{1}{3!}\sum_{\mathbf{k}_i} V^{(2)}_{\mathbf{k}_1,\mathbf{k}_2;\mathbf{k}_3,\mathbf{k}_4}\, b^\dagger_{\mathbf{k}_1} b^\dagger_{\mathbf{k}_2} b^\dagger_{\mathbf{k}_3} b_{\mathbf{k}_4}$$



$$+ \frac{1}{4!}\sum_{\mathbf{k}_i} V^{(3)}_{\mathbf{k}_1,\mathbf{k}_2;\mathbf{k}_3,\mathbf{k}_4}\, b^\dagger_{\mathbf{k}_1} b^\dagger_{\mathbf{k}_2} b^\dagger_{\mathbf{k}_3} b^\dagger_{\mathbf{k}_4}$$

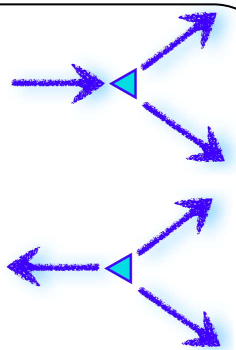


- are they valid?
- is this all?

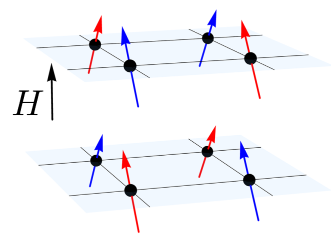
what else would bosons do?

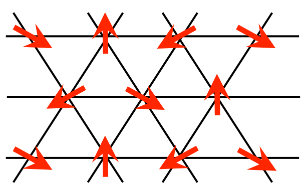
- condense! $\langle b_{\mathbf{k}=0}\rangle \neq 0$
  - # of bosons **not** preserved
  - Bogolyubov substitution $\quad b^\dagger_{\mathbf{k}_1} b^\dagger_{\mathbf{k}_2} b_{\mathbf{k}_3} b_{\mathbf{k}_4} \Rightarrow b^\dagger_{\mathbf{k}_1} b^\dagger_{\mathbf{k}_2} b_{\mathbf{k}_3} \langle b_0\rangle$
  - cubic anharmonicities

$$\mathcal{H} = \ldots \; + \frac{1}{2!}\sum_{\mathbf{k}_i} V^{(3,1)}_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3}\, b^\dagger_{\mathbf{k}_1} b^\dagger_{\mathbf{k}_2} b_{\mathbf{k}_3} + \text{H.c.}$$
$$+ \frac{1}{3!}\sum_{\mathbf{k}_i} V^{(3,2)}_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3}\, b^\dagger_{\mathbf{k}_1} b^\dagger_{\mathbf{k}_2} b^\dagger_{\mathbf{k}_3} + \text{H.c.}$$



- corresponds to a **non-collinear** antiferromagnet
  - GS: spin rotational symmetry broken completely
    - AFs in a field
    - frustrated AFs





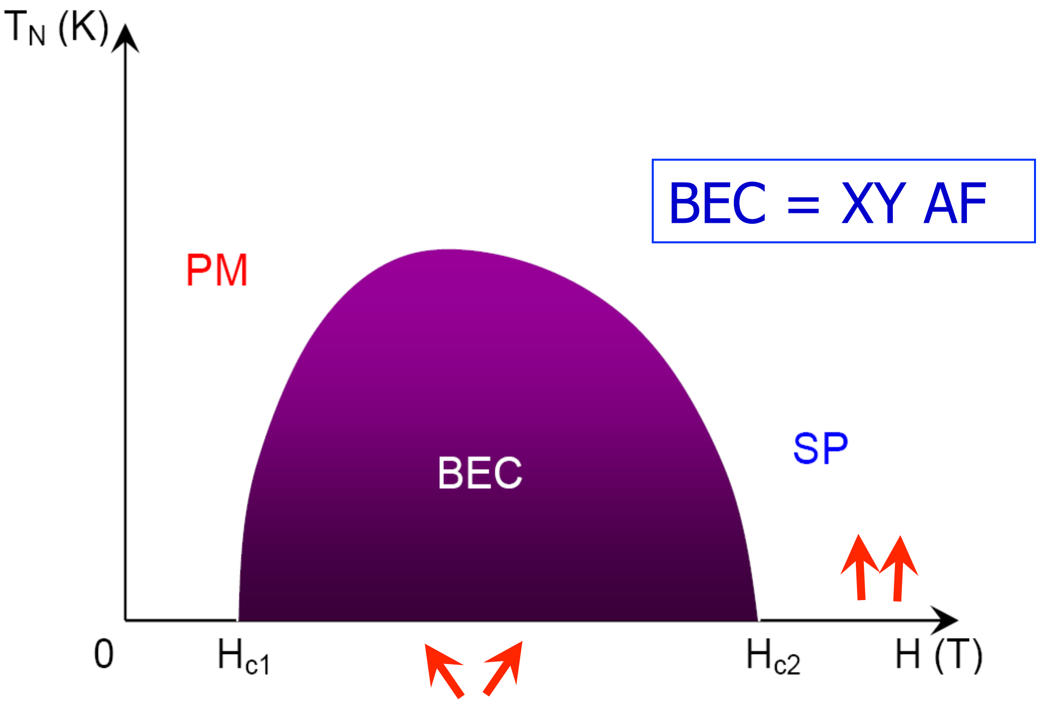
MZ and **SC**, RMP **85**, 219 (2013)

BEC antiferromagnets

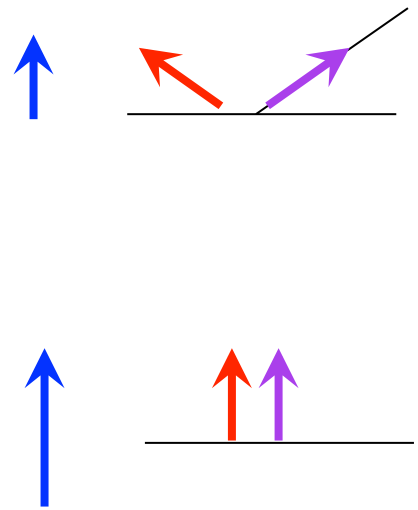
T. Matsubara and H. Matsuda, Prog. Theor. Phys. **16**, 569 (1956);
E. G. Batyev and L. S. Braginskii, Sov. Phys. JETP **60**, 781 (1984).

BEC: U(1) symmetry breaking, order parameter

$$\mathbf{\Delta} = |\Delta| e^{i\varphi}$$

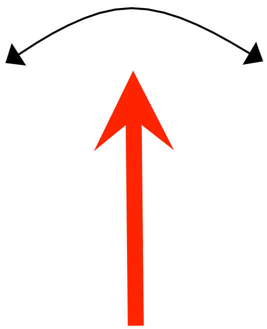


BEC AF: "transverse" magnetization



collinear vs non-collinear

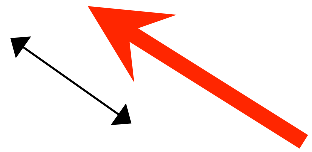
frustration (competing interactions) → **non-collinearity**
→ $\boxed{\text{transverse-longitudinal coupling}}$ → 3-boson terms



transverse (T)

"one-magnon"

$$S^- \approx \sqrt{2S}\, a^\dagger$$

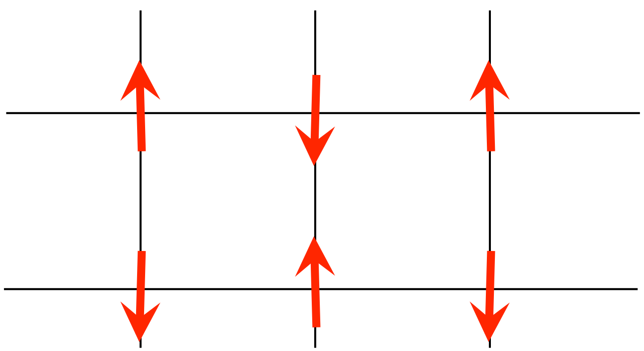


longitudinal (L)

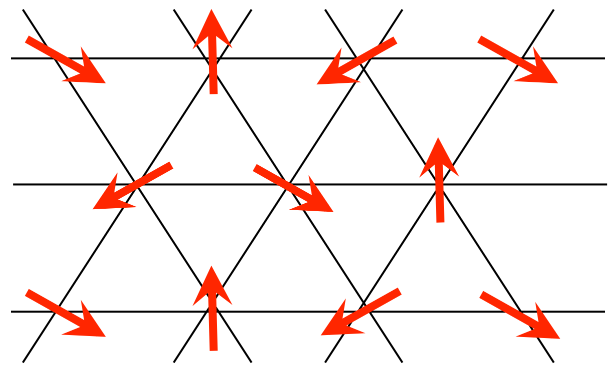
"two-magnon"

$$S^z = S - a^\dagger a$$

only T-T or L-L



strong T-L



remaining $U(1)$ symmetry forbids $S^z S^\pm$ terms

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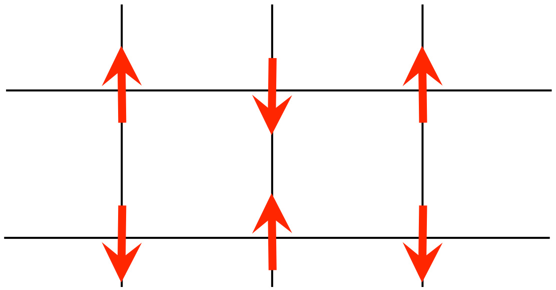
two classes of (quantum) [AF]magnets

collinear (quartic anharmonicity)



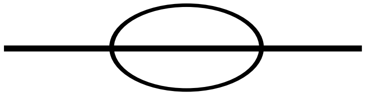
no coupling with 2-magnon continuum

$\Downarrow$

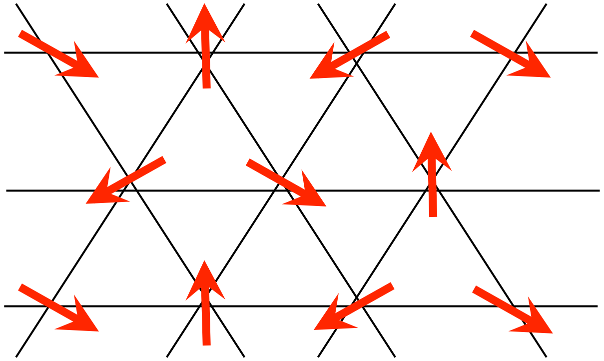
weakly-interacting excitations

$\Downarrow$

small renormalizations even
in low D and *S=1/2*



non-collinear (qubic anharmonicity)



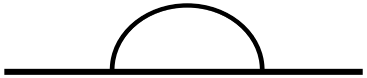
direct coupling 1-to-2-magnon states

$\Downarrow$

strong(er)-interacting excitations

$\Downarrow$

large spectrum renormalizations
in low D and *S=1/2*



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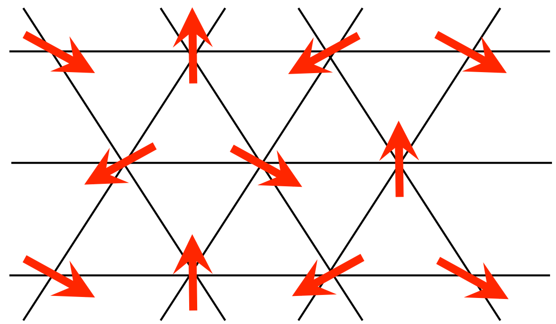
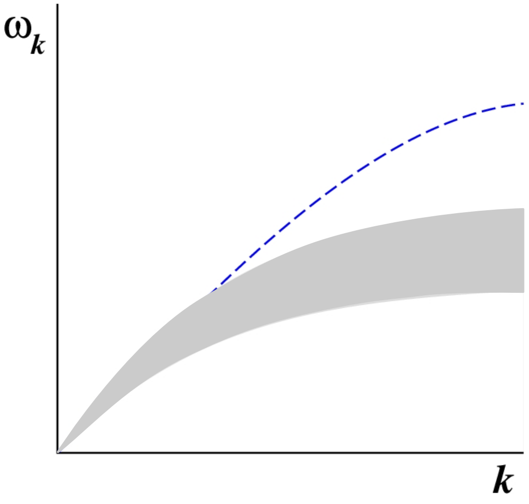
most important qualitative difference

magnons in **non-collinear*** AFs:

- often decay even at *T=0* (yield broad peaks)



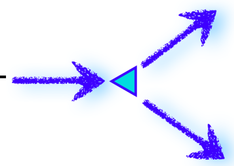


* and others with cubic anharmonicities

decays

frustration (competing interactions) → **non-collinearity**
→ transverse-longitudinal coupling → 3-boson terms → $\boxed{\text{decays}}$

$$\mathcal{H} = \ldots + \frac{1}{2!}\sum_{\mathbf{k}_i} V^{(3,1)}_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3}\, b^\dagger_{\mathbf{k}_1} b^\dagger_{\mathbf{k}_2} b_{\mathbf{k}_3} +$$



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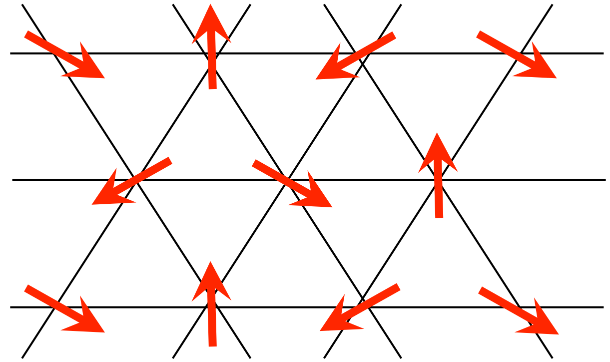

- three-boson terms are necessary for (1-in-2) decays
- "kinematic" conditions (*E*- and *k*- conservations) make it sufficient
- same conditions that favor cubic terms favor decay kinematics

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in contrast to …

PHYSICAL REVIEW B VOLUME 3, NUMBER 3 1 FEBRUARY 1971

Dynamics of an Antiferromagnet at Low Temperatures: Spin-Wave Damping and Hydrodynamics*

A. B. Harris and D. Kumar
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104

and

B. I. Halperin and P. C. Hohenberg
Bell Telephone Laboratories, Murray Hill, New Jersey 07974

(Received 14 July 1970)

Regime A: $\epsilon_{\vec{k}} \ll \tau^3 \ll 1$,

$$\Gamma_{\vec{k}} = \hbar^{-1}\Sigma''(\vec{k}, \omega_E \epsilon_{\vec{k}})$$

$$= (2\omega_E/S^2)\epsilon_{\vec{k}}^2\, \tau^3 (2\pi)^{-3} (a\,|\ln\tau| + a') ; \qquad (9.1a)$$

Regime B: $\tau^3 \ll \epsilon_{\vec{k}} \ll \tau \ll 1$,

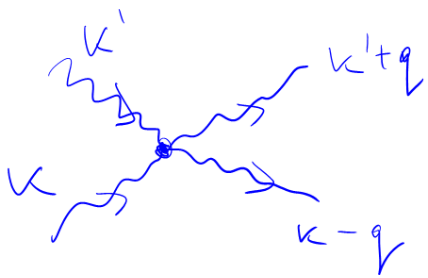
$$\Gamma_{\vec{k}} = (8\omega_E/3S^2)\, \epsilon_{\vec{k}}^2 \tau^3\, (2\pi)^{-3}\,[b\ln(\tau/k) + b'] ; \qquad (9.1b)$$

Regime C: $\tau \ll \epsilon_{\vec{k}} \ll \tau^{1/3} \ll 1$,

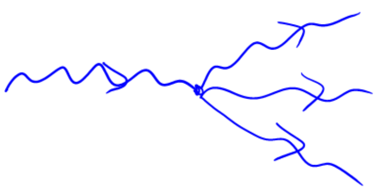
$$\Gamma_{\vec{k}} = (\pi\omega_E/108S^2)\epsilon_{\vec{k}}\tau^4 ; \qquad (9.1c)$$

Regime D: $\tau^{1/3} \ll \epsilon_{\vec{k}} \ll 1$,

$$\Gamma_{\vec{k}} = (\omega_E/2S^2\pi^3)\, \tau^5 \zeta(5)\, [\, g(\hat{k})\, \epsilon_{\vec{k}}^2]^{-1} . \qquad (9.1d)$$

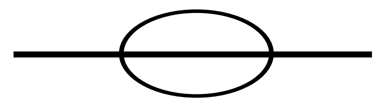


T > 0



T ≥ 0, ?

T = 0 forbidden



- in **collinear** AFs 1-to-3 decay conditions are uncommon
- even if --- their effect is weak

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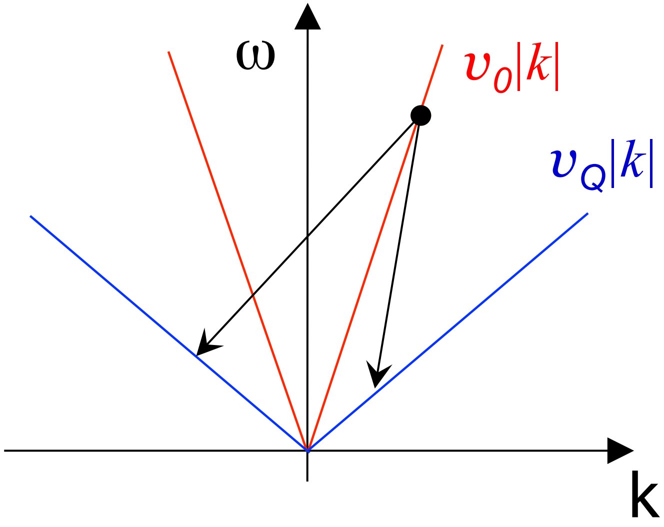
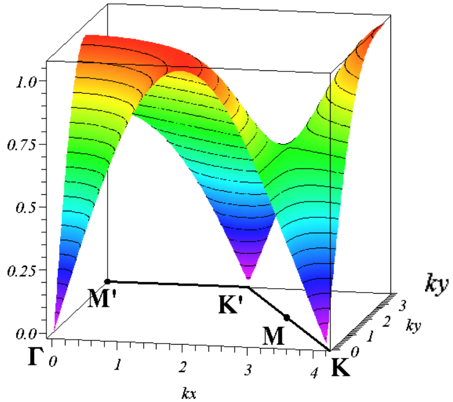
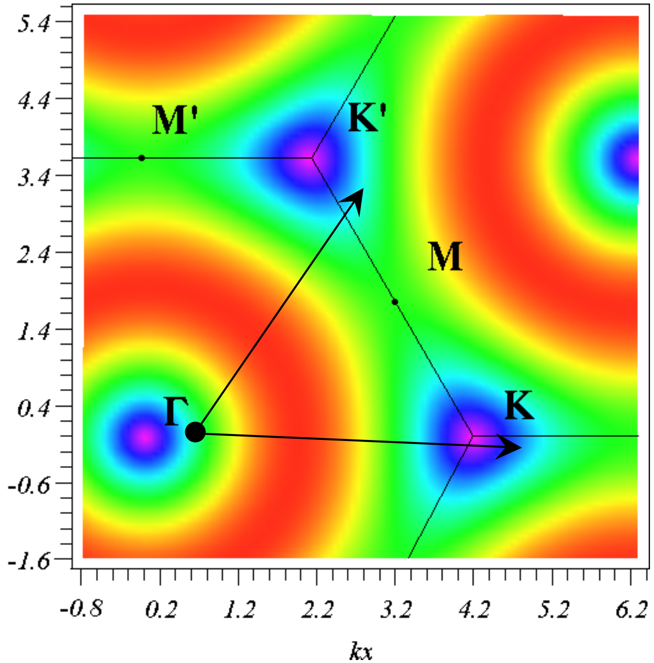
common?

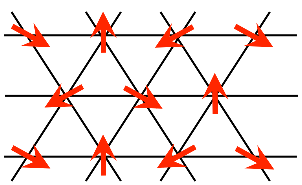


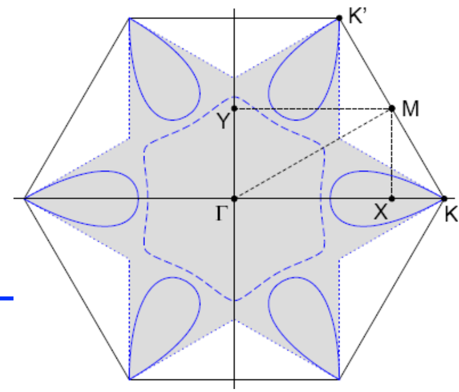




$$\varepsilon_{\mathbf{k}} = \varepsilon_{\mathbf{q}} + \varepsilon_{\mathbf{k}-\mathbf{q}}$$

- for a **spiral-like state** (more than one type of Goldstone mode) decay conditions are **always** fulfilled for a (large) range of k

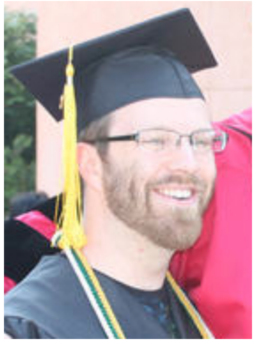




MZ and **SC**, RMP **85**, 219 (2013)

part II.

Martin Mourigal



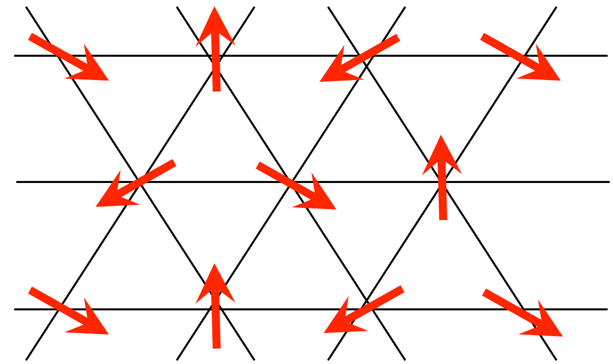
Wesley Fuhrman

Mike Zhitomirsky

Sasha Chernyshev

 M. Mourigal, W. Fuhrman, **SC** and **MZ**, PRB **88**, 094407 (2013)

$S(\mathbf{q},\omega)$, triangular lattice AF



- why? previous attempts* not intuitive …
  … and contain too many terms …
- show effects of decays in $S(\mathbf{q},\omega)$

$$\boxed{\mathcal{S}^{\alpha_0\beta_0}(\mathbf{q},\omega)=\int_{-\infty}^{\infty}\frac{dt}{2\pi}\,e^{i\omega t}\left\langle S^{\alpha_0}_{\mathbf{q}}(t)S^{\beta_0}_{-\mathbf{q}}(0)\right\rangle}$$

$$\boxed{\mathcal{S}^{\text{tot}}(\mathbf{q},\omega)=\alpha\,\mathcal{S}^{x_0x_0}(\mathbf{q},\omega)+\beta\,\mathcal{S}^{y_0y_0}(\mathbf{q},\omega)+\gamma\,\mathcal{S}^{z_0z_0}(\mathbf{q},\omega)}$$

$$\begin{aligned}
\mathcal{S}^{\text{tot}}(\mathbf{q},\omega) &= \mathcal{S}^{\text{diag}}(\mathbf{q},\omega)+\cancel{\mathcal{S}^{\text{mix}}(\mathbf{q},\omega)},\\
\mathcal{S}^{\text{diag}}(\mathbf{q},\omega) &= \mathcal{S}^{\perp}(\mathbf{q},\omega)+\mathcal{S}^{L}(\mathbf{q},\omega)\ ,\\
\mathcal{S}^{\perp}(\mathbf{q},\omega) &= \mathcal{S}^{yy}_{\mathbf{q},\omega}+\frac{1}{2}\left(\mathcal{S}^{xx}_{\mathbf{q}_+,\omega}+\mathcal{S}^{xx}_{\mathbf{q}_-,\omega}\right),\\
\mathcal{S}^{L}(\mathbf{q},\omega) &= \frac{1}{2}\left(\mathcal{S}^{zz}_{\mathbf{q}_+,\omega}+\mathcal{S}^{zz}_{\mathbf{q}_-,\omega}\right),\\
\cancel{\mathcal{S}^{\text{mix}}(\mathbf{q},\omega)} &= \cancel{\frac{i}{2}\left(\mathcal{S}^{xz}_{\mathbf{q}_-,\omega}-\mathcal{S}^{zx}_{\mathbf{q}_-,\omega}-\mathcal{S}^{xz}_{\mathbf{q}_+,\omega}+\mathcal{S}^{zx}_{\mathbf{q}_+,\omega}\right)}.
\end{aligned}$$

1-magnon part

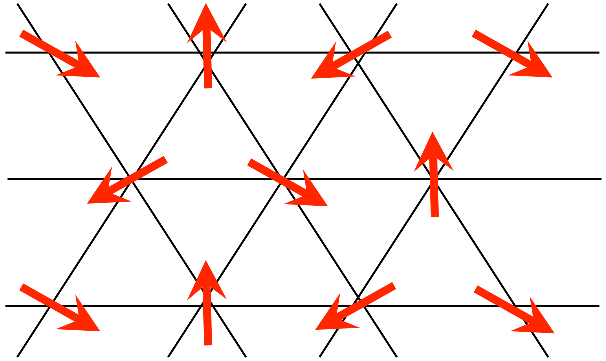
2-magnon part

- measure in laboratory
- think in local ref. frame

$$\boxed{\begin{aligned}\mathcal{S}^{\text{tot}}(\mathbf{q},\omega)&\approx\mathcal{S}^{\text{diag}}(\mathbf{q},\omega)\\ \mathbf{q}_\pm&=\mathbf{q}\pm\mathbf{Q}\end{aligned}}$$

* T. Ohyama and H. Shiba, JPSJ **62**, 3277 (1993).

$S(\mathbf{q},\omega)$, II



- transverse (1-magnon) components

$$\begin{aligned}\mathcal{S}^{xx}(\mathbf{q},\omega) &= \frac{S}{2}\Lambda_+^2(u_\mathbf{q}+v_\mathbf{q})^2 A_{11}(\mathbf{q},\omega)\\ \mathcal{S}^{yy}(\mathbf{q},\omega) &= \frac{S}{2}\Lambda_-^2(u_\mathbf{q}-v_\mathbf{q})^2 A_{11}(\mathbf{q},\omega)\end{aligned}$$

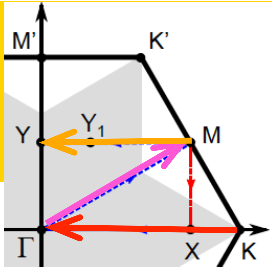
decays are here

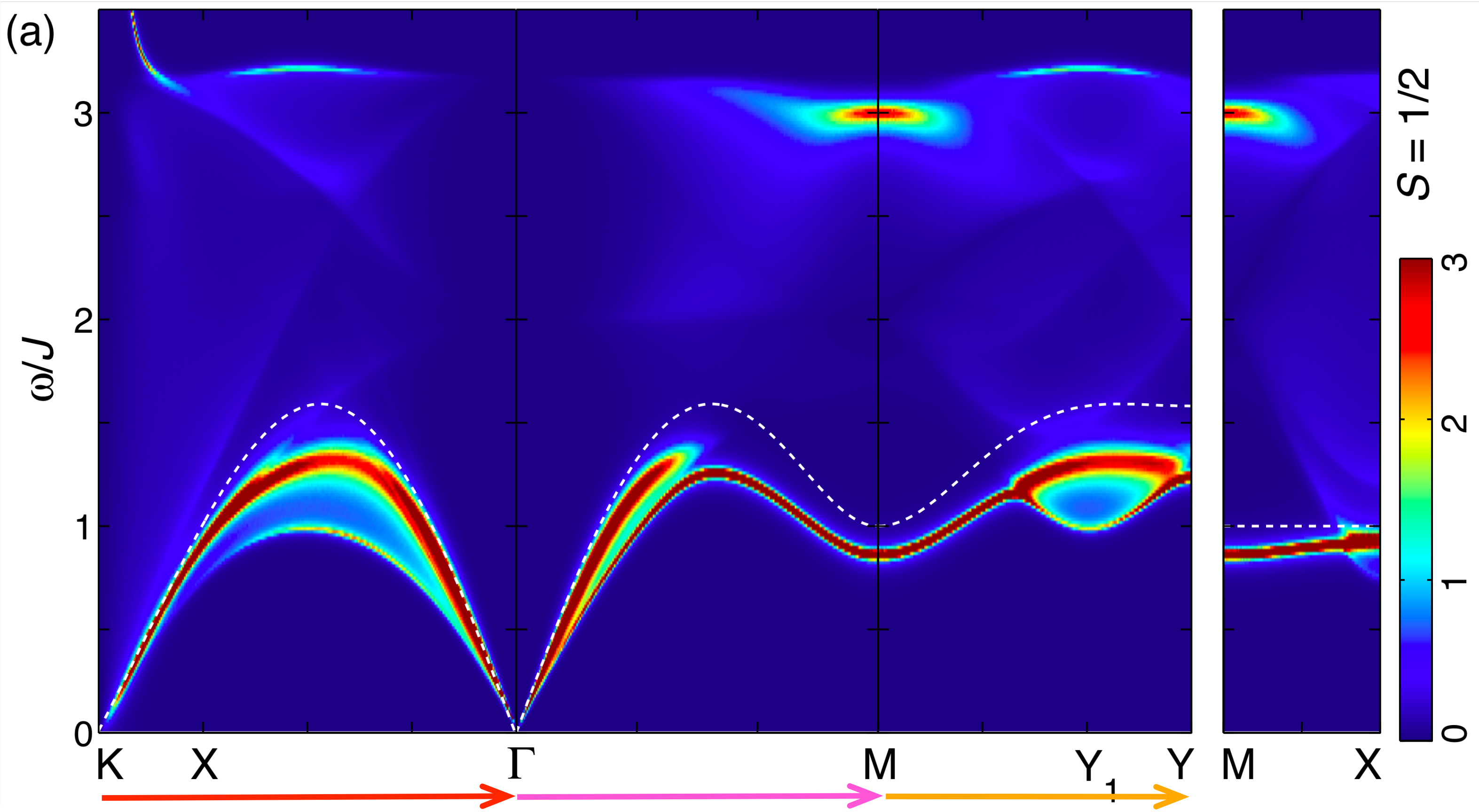
- longitudinal (2-magnon) part (broad in ω)

$$\mathcal{S}^{zz}(\mathbf{q},\omega) = \frac{1}{2}\sum_{\mathbf{k}}(u_\mathbf{k}v_{\mathbf{k}-\mathbf{q}}+v_\mathbf{k}u_{\mathbf{k}-\mathbf{q}})^2\delta(\omega-\varepsilon_\mathbf{k}-\varepsilon_{\mathbf{k}-\mathbf{q}})$$

M. Mourigal, W. Fuhrman, **SC** and **MZ**, PRB **88**, 094407 (2013)

$A(\mathbf{q},\omega)$, $S=1/2$

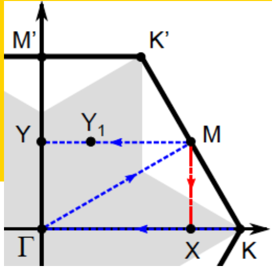


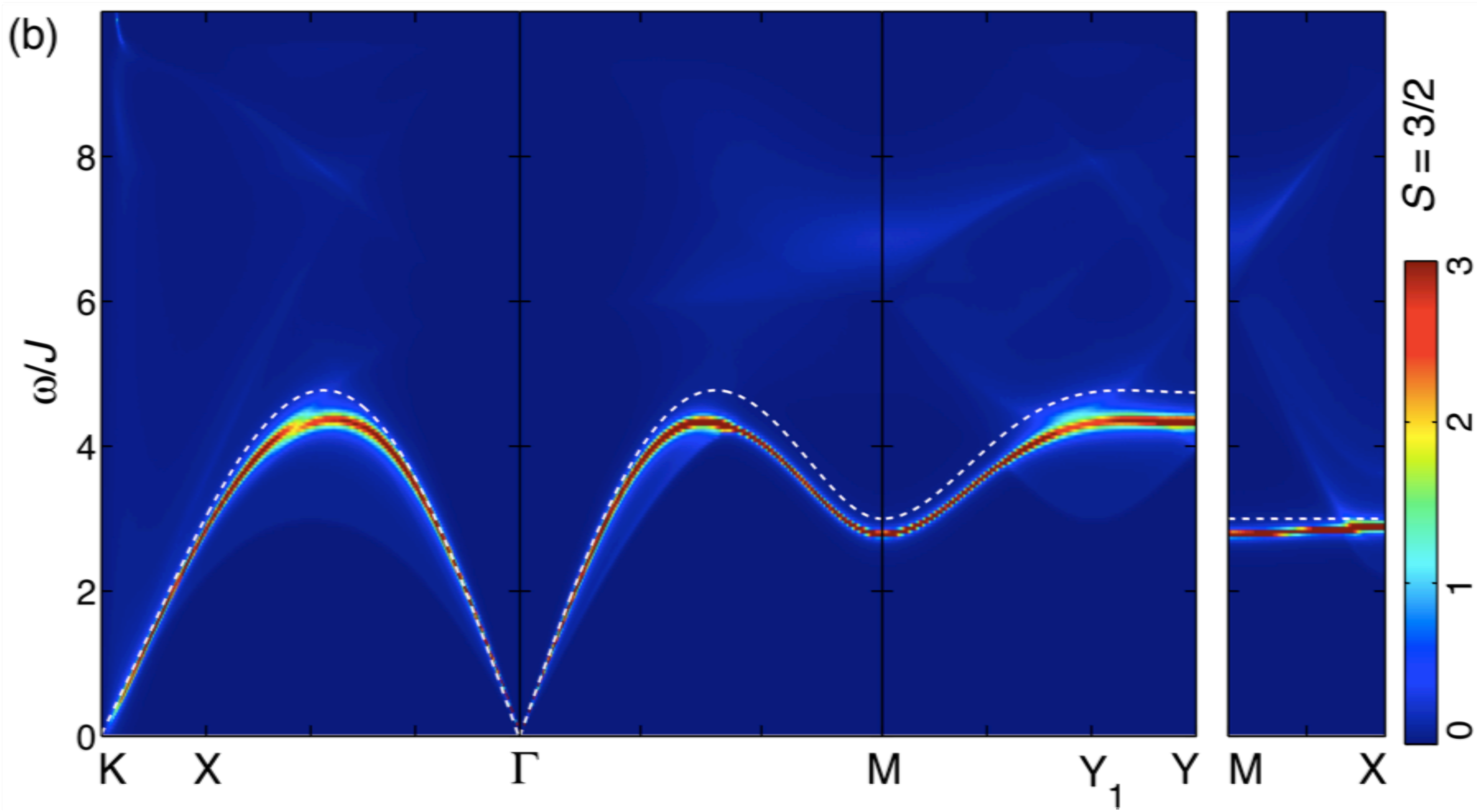


- broad peaks + double-peak features + non-lorentzian
- "stable" regions, "unstable" regions

 M. Mourigal, W. Fuhrman, **SC** and **MZ**, PRB **88**, 094407 (2013)

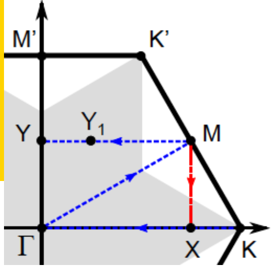
$A(\mathbf{q},\omega)$, $S=3/2$

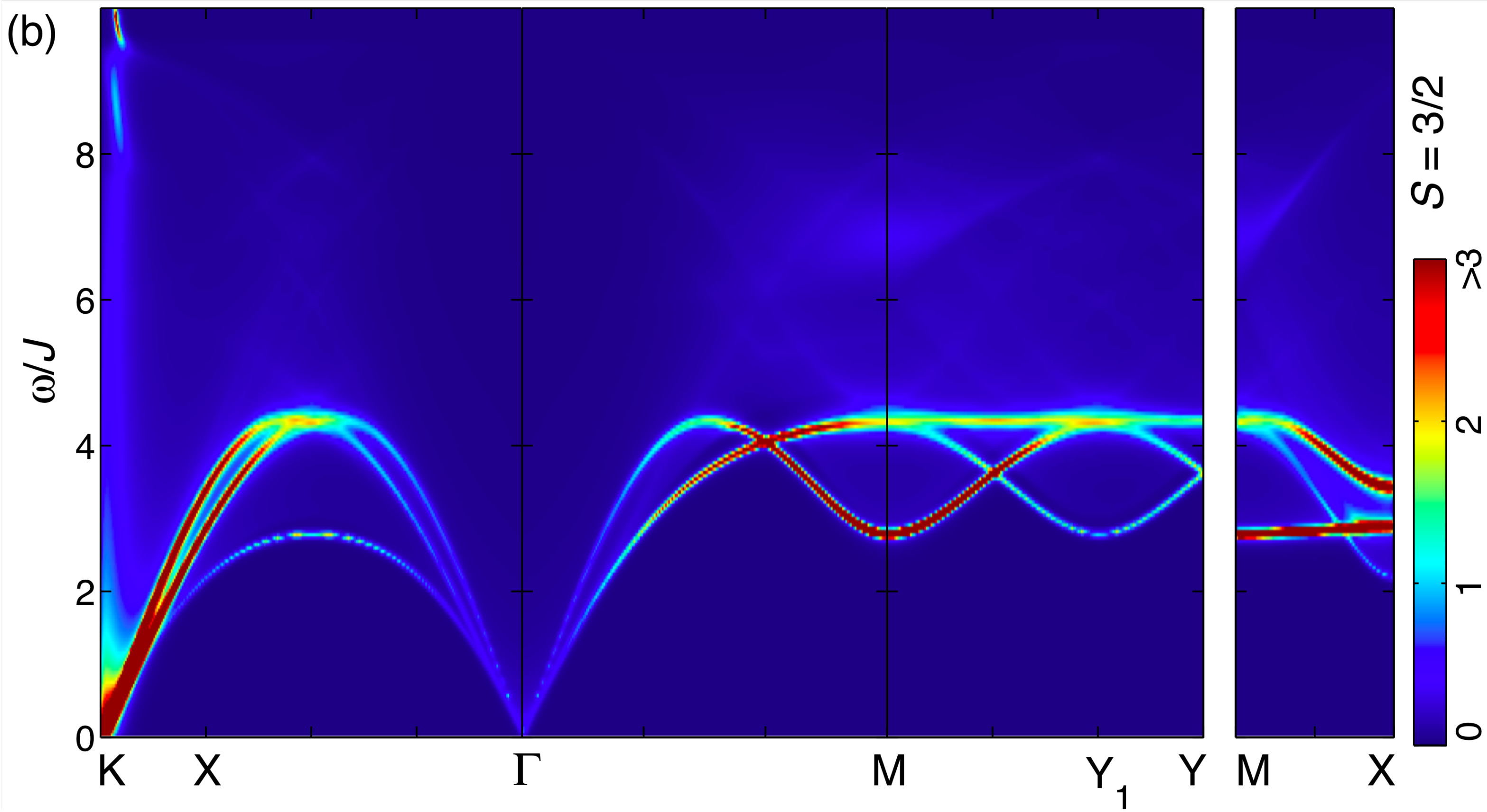




- broadening, two-magnon continuum
- to obtain $S(\mathbf{q},\omega)$ need "folding" of $A(\mathbf{q},\omega)$

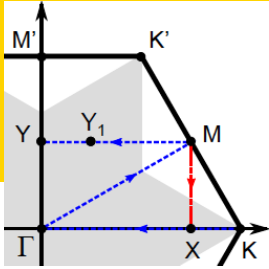
$S(\mathbf{q},\omega)$, S=3/2

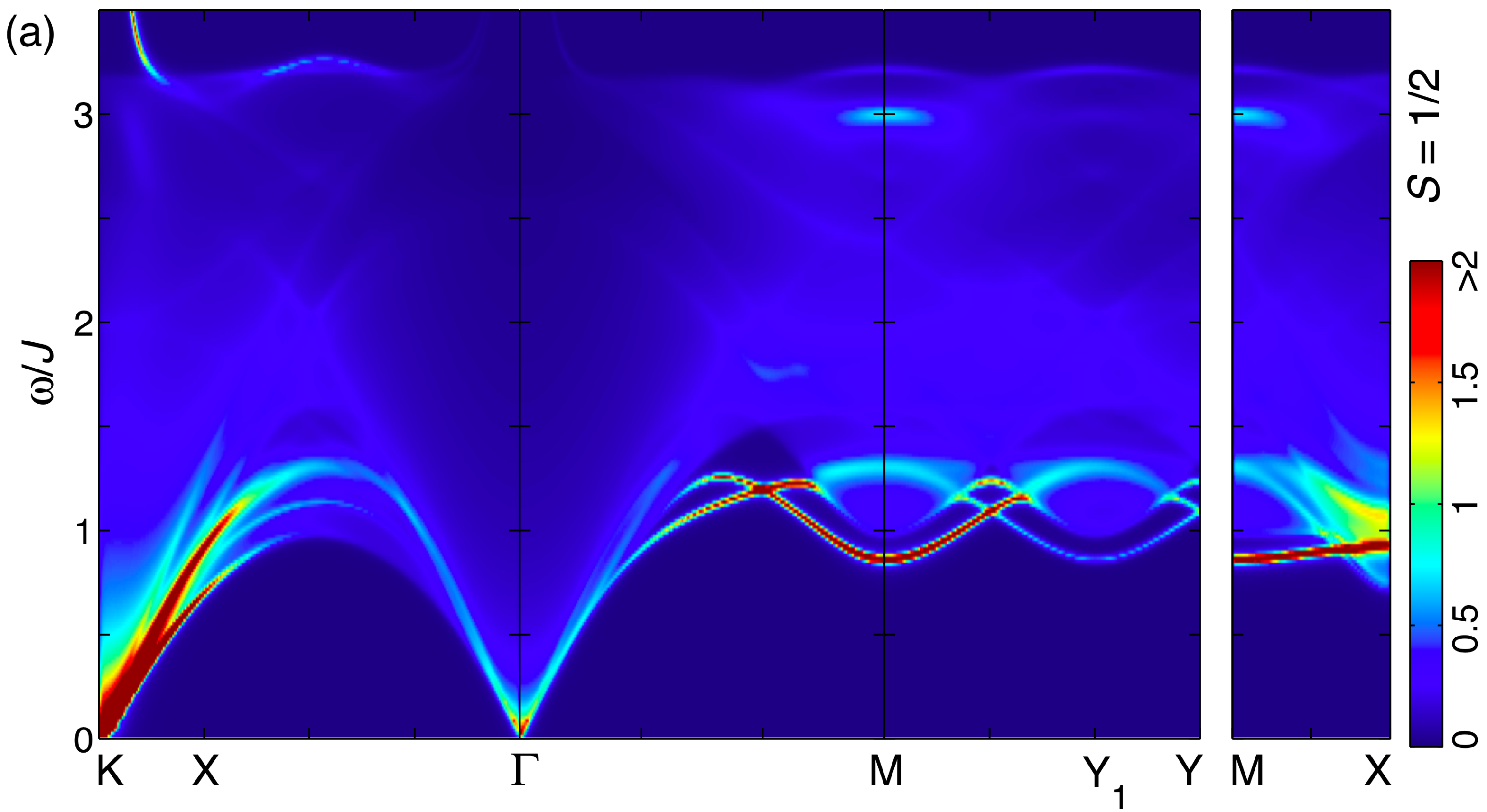




 M. Mourigal, W. Fuhrman, **SC** and **MZ**, PRB **88**, 094407 (2013)

$S(\mathbf{q},\omega)$, $S=1/2$

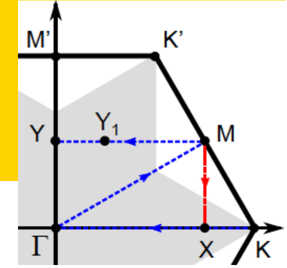


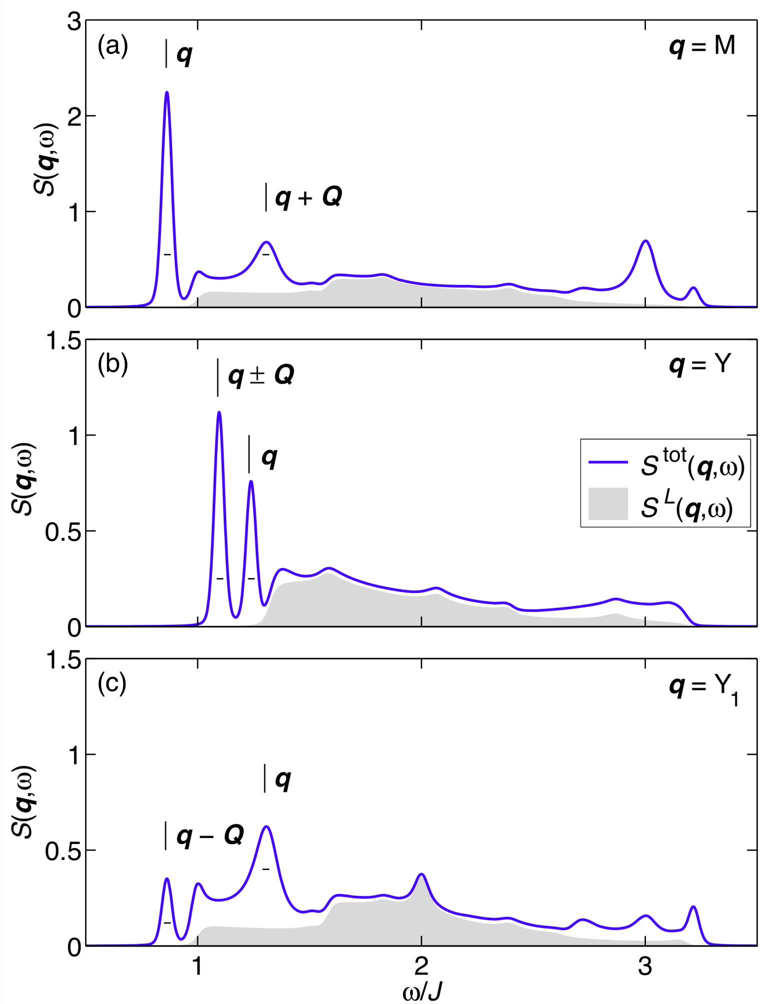


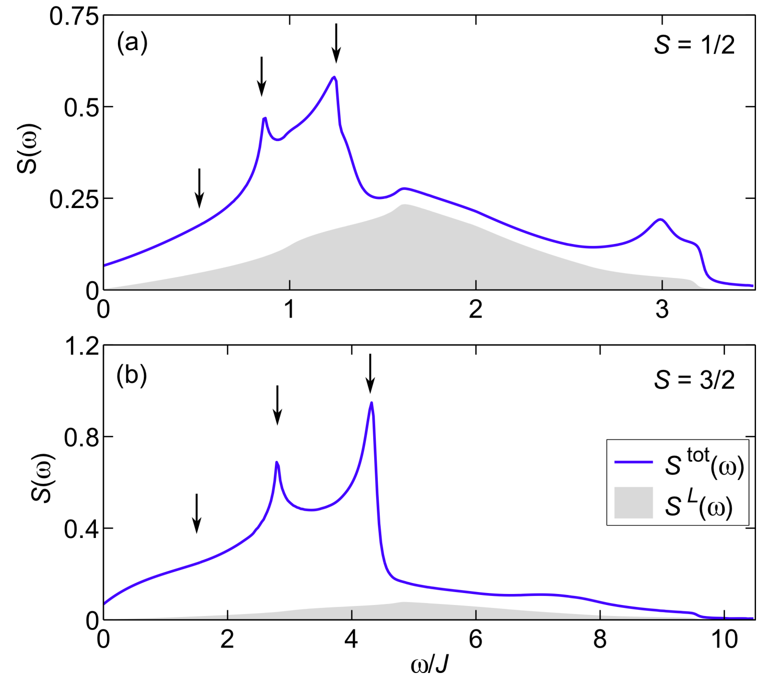
- complicated features persist

 M. Mourigal, W. Fuhrman, **SC** and **MZ**, PRB **88**, 094407 (2013)

$S(\boldsymbol{q},\omega)$, S=1/2, 3/2



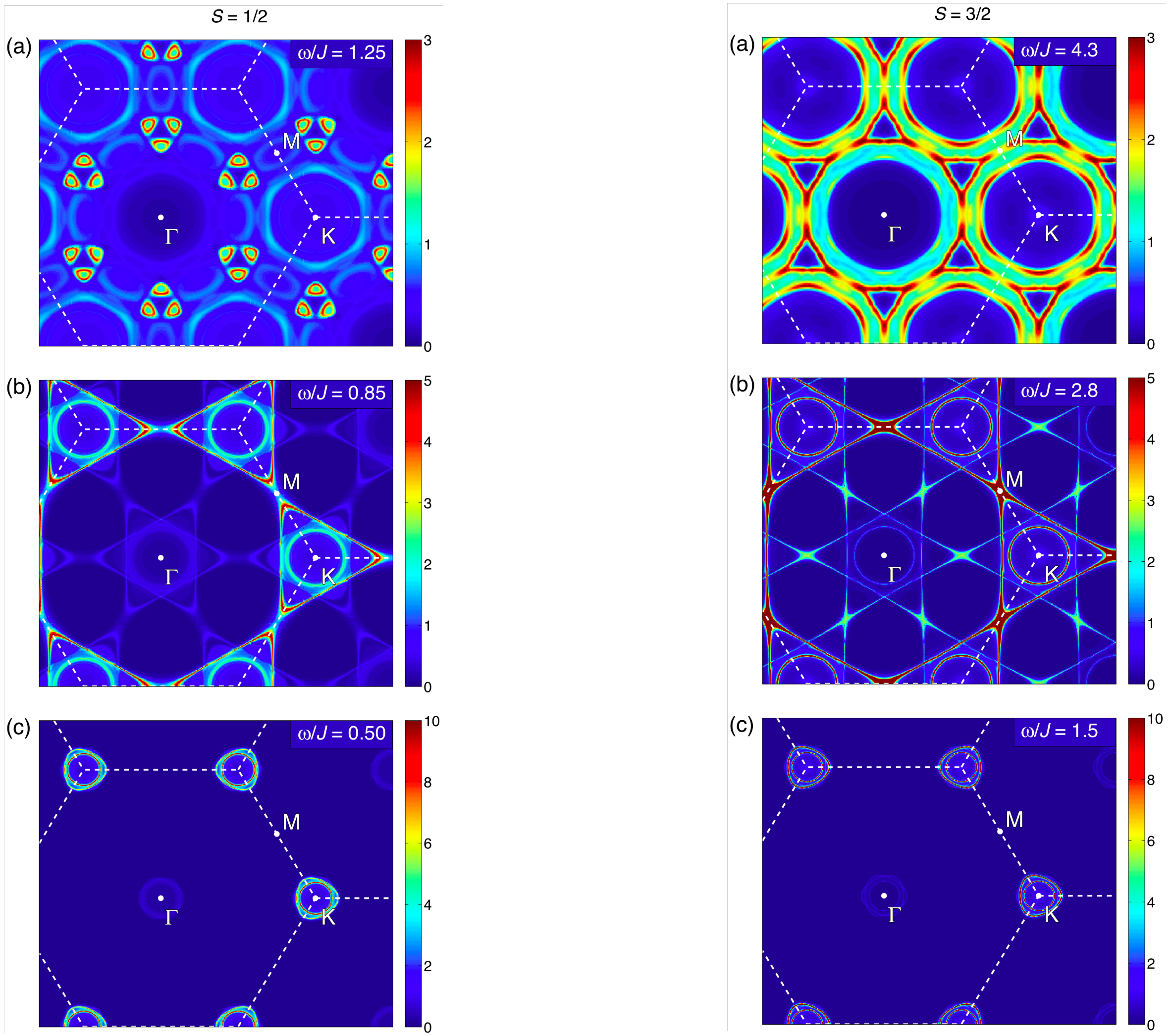




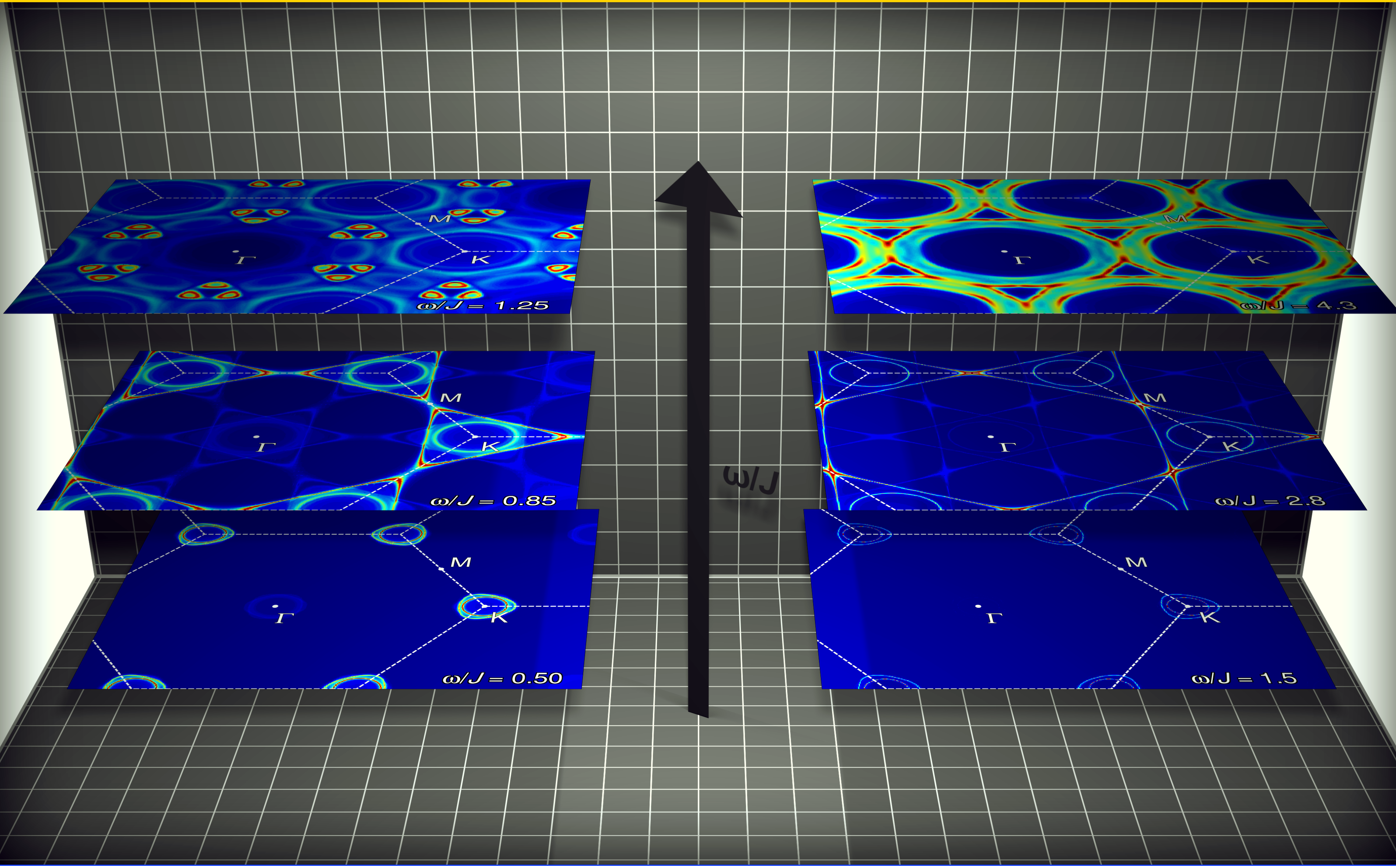
- integrated structure factor *S(ω)*

- roles of transverse and longitudinal components

$S(\mathbf{q},\omega)$, constant energy scans



$S(\boldsymbol{q},\omega)$, constant energy scans

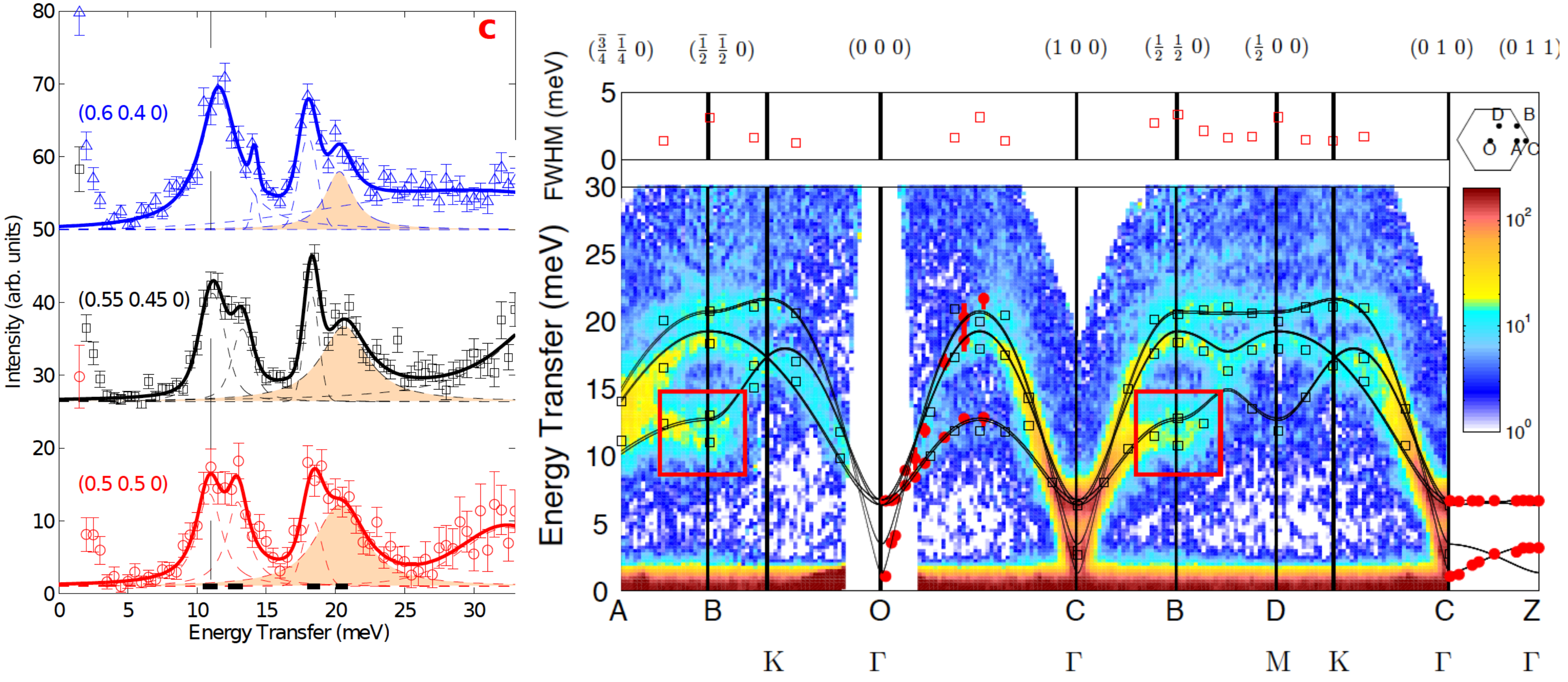


Nordita, NDFCM, M. Mourigal, W. Fuhrman, **SC** and **MZ**, PRB **88**, 094407 (2013)

experiments

Magnon breakdown in a two dimensional triangular lattice Heisenberg antiferromagnet of multiferroic $LuMnO_3$

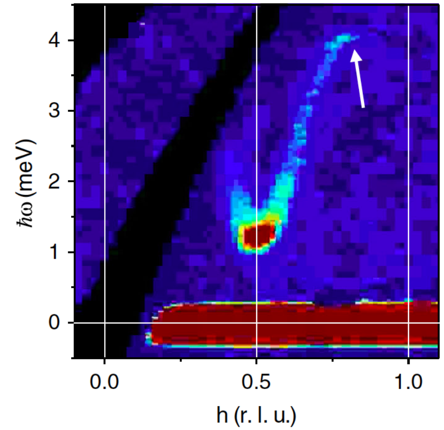
Joosung Oh,[1,2] Manh Duc Le,[1,2] Jaehong Jeong,[1,2] Jung-hyun Lee,[3,4] Hyungje Woo,[5,6] Wan-Young Song,[3,4] T. G. Perring,[5] W. J. L. Buyers,[7] S-W. Cheong,[8] and Je-Geun Park[1,2,3,*]



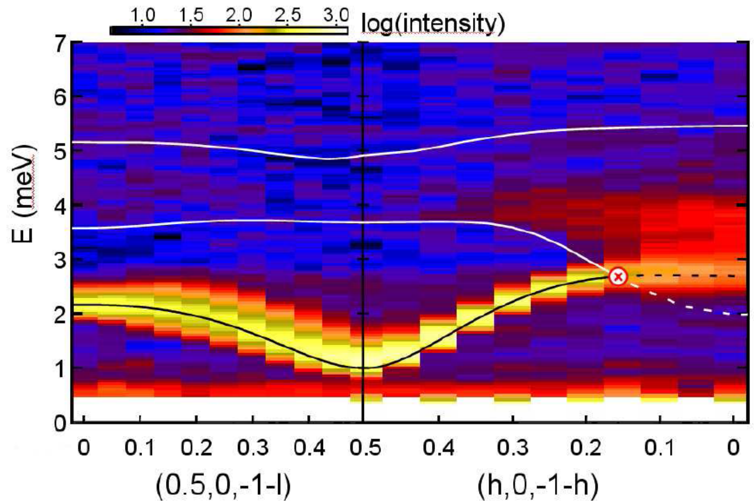
PRL **111**, 257202 (2013)

spin-gap systems, triplet decays

☑ order not necessary to have decays



M. Stone, *etal.,* Nature **440,** 187-190 (2006).



T. A. Masuda, *etal.*, PRL **96**, 047210 (2006)

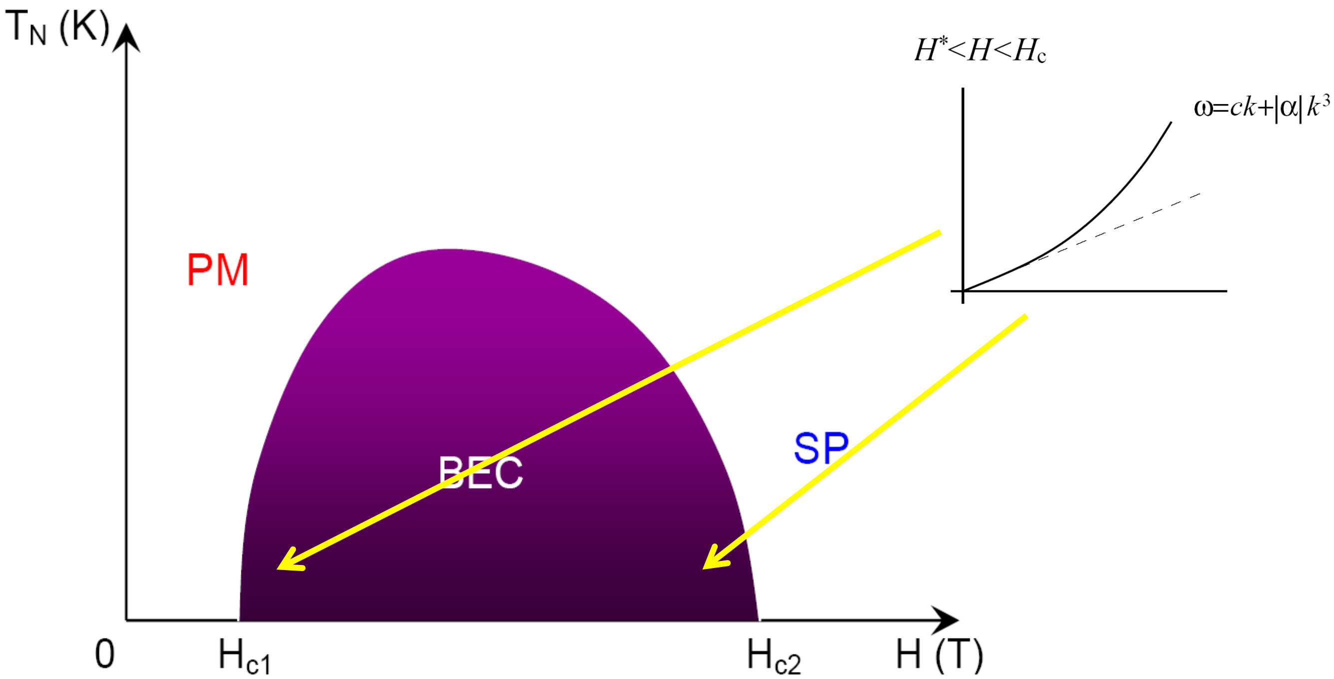
IPA-CuCl$_3$

piperazinium hexachlorodicuprate (PHCC)

A. Kolezhuk and S. Sachdev, Phys. Rev. Lett. **96**, 087203 (2006),
M. E. Zhitomirsky, Phys. Rev. B **73**, 100404 (2006)

MZ and **SC**, RMP **85**, 219 (2013)

other systems/experiments: BEC



- ☑ BEC in a field = XY AF
- ☑ thermal conductivity

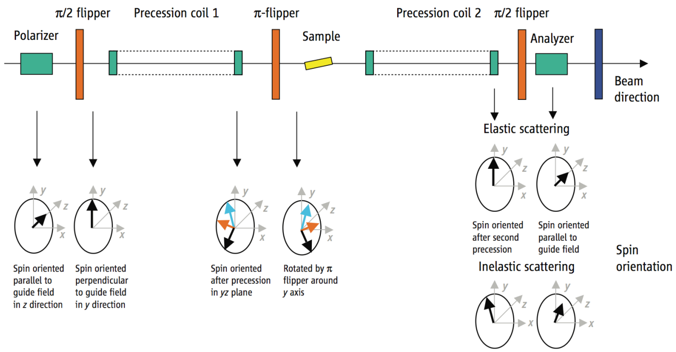
conclusions

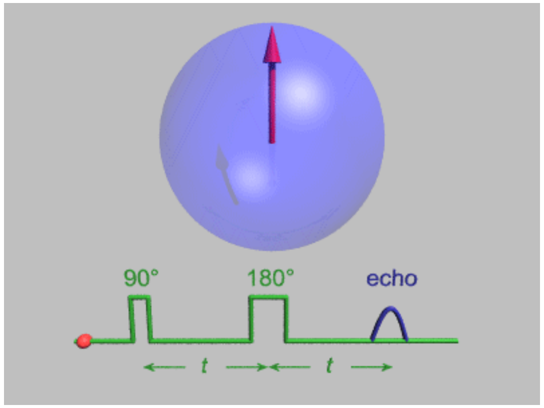
- ☑ **decays** in magnon spectra of non-collinear magnets are generic
- ☑ enhanced by lower D and small spin
- ☑ non-Lorenzian features in $S(\mathbf{k},\omega)$ and decay regions can be used for fingerprinting

- ☑ finite-T ?
- ☑ more experiments ?

parts III. and IV.

neutron-scattering spin-echo





APPLIED PHYSICS 1888 VOL 312 SCIENCE 2006

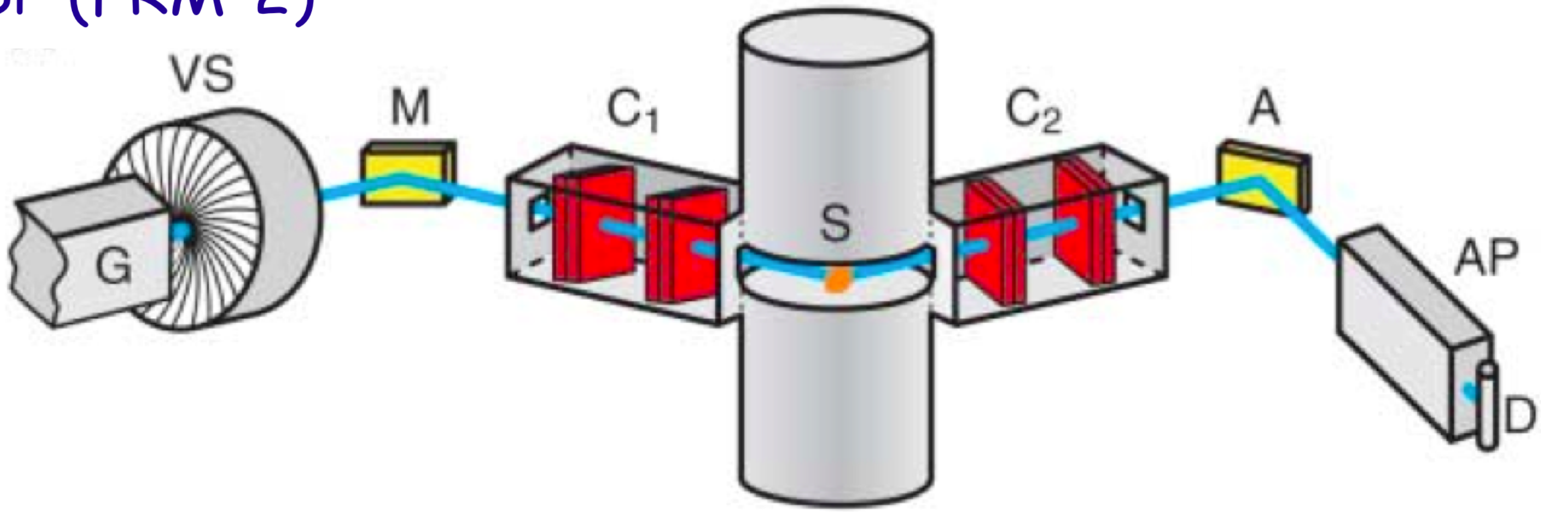
The Neutron Spin-Echo Technique at Full Strength

Joël Mesot

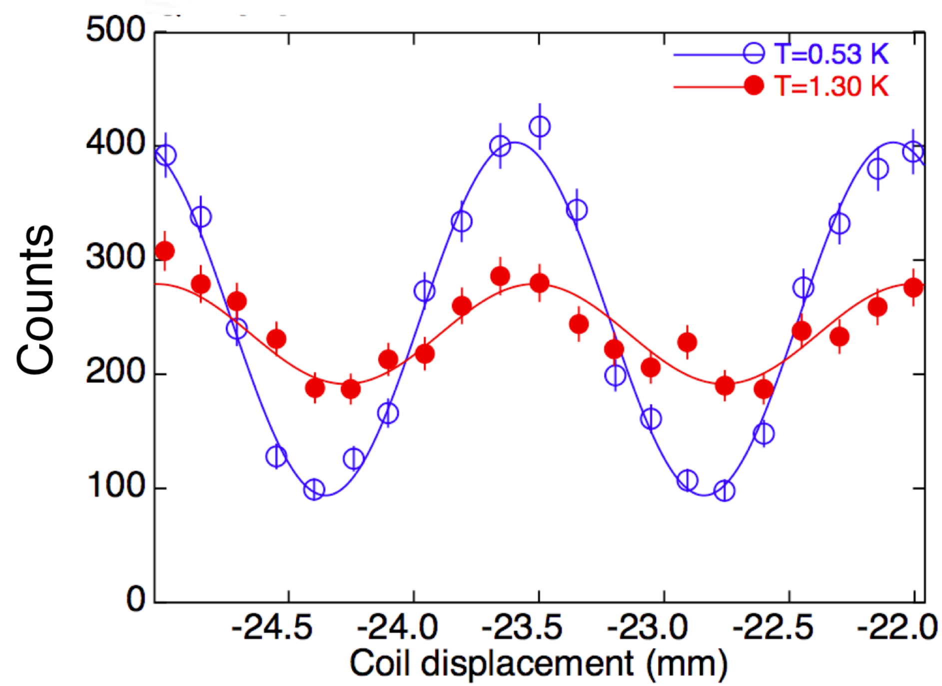
- resolution ~ 1μeV (≈0.01K) !

Neutron Resonant Spin Echo

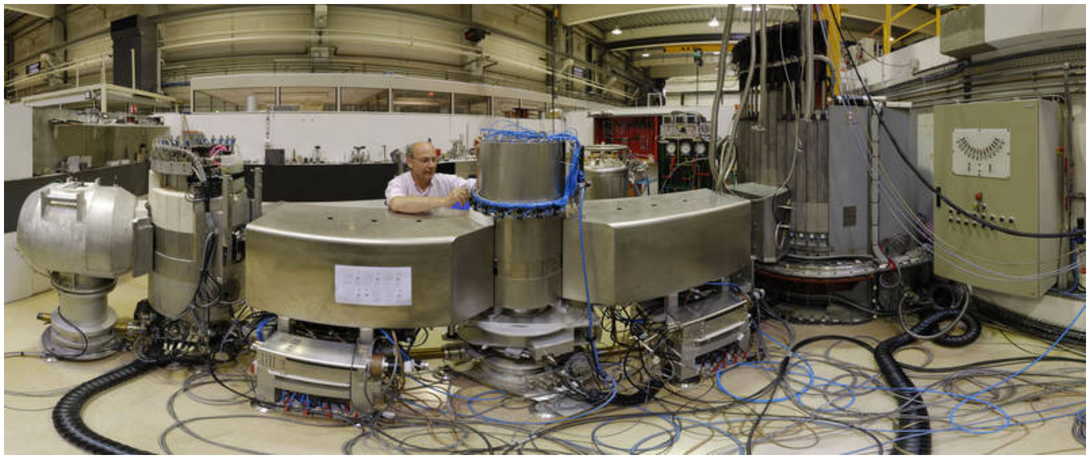
TRISP (FRM-2)

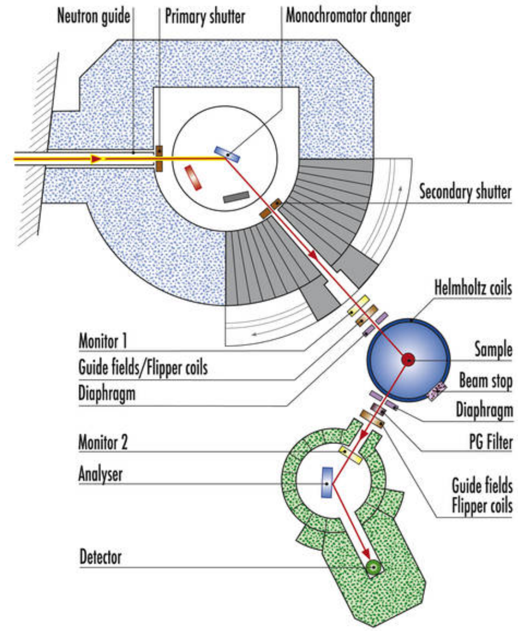


NRSE:
Line widths of dispersive modes
Energy shifts (phase stability)



ILL, IN22, and ZETA





Thermal neutron three-axis spectrometer with polarization analysis IN22

IN22 is a three-axis spectrometer (CRG) equipped for full polarization analysis. The option CRYOPAD and a 15 Tesla cryomagnet are optimised for inelastic scattering. The option ZETA provides neutron resonance spin echo (NRSE).

NEUTRONS FOR SCIENCE®

Institut Laue-Langevin

CRG three-axis spectrometer IN22 with the ZETA resonant neutron spin-echo setup

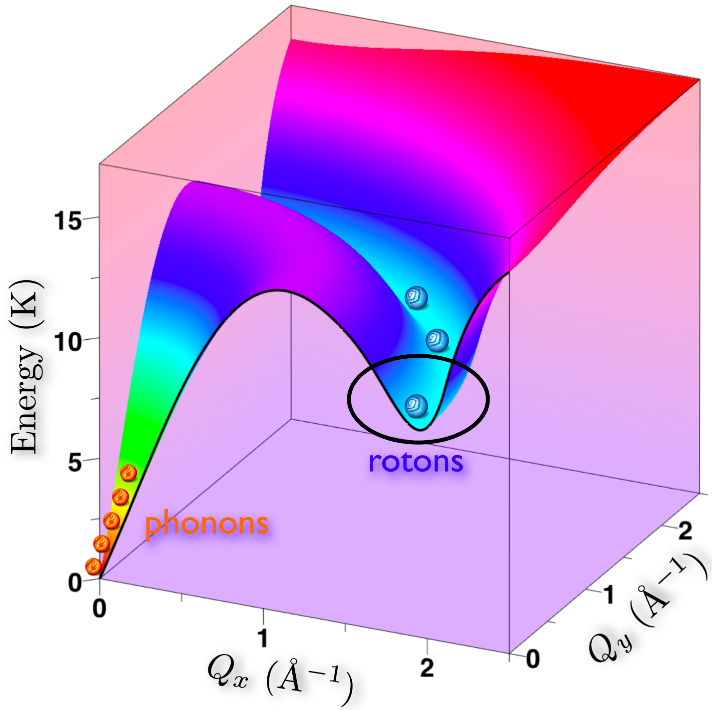
III. odd interactions in superfluid ^{4}He

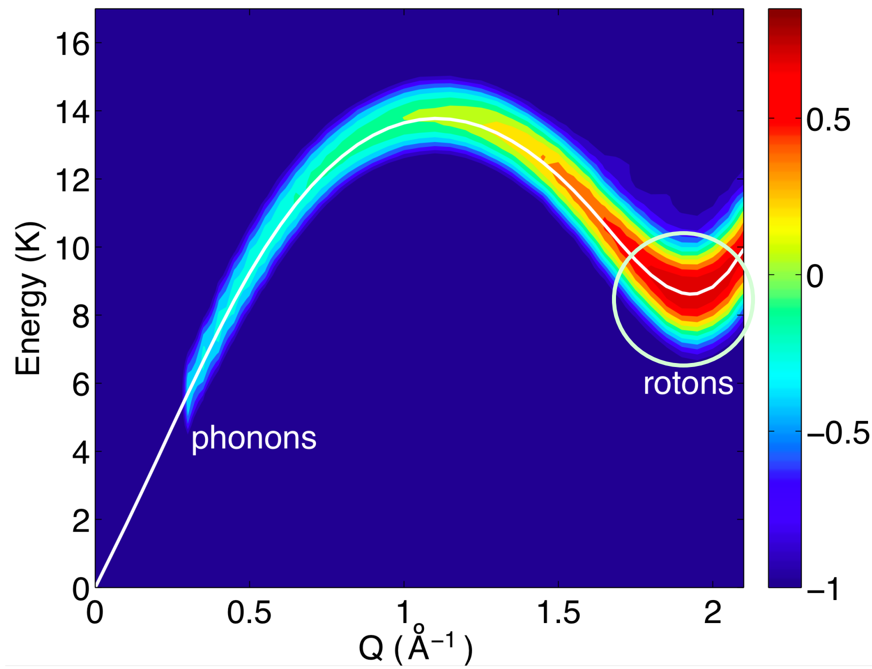
experiments: Björn Fåk (CEA, ILL)
Thomas Keller (Munich, Stuttgart)

theory: Mike Zhitomirsky (CEA)
Sasha Chernyshev (UC Irvine)

arXiv:1206.1498; PRL **109**, 155305 (2012)

spectrum of ^{4}He



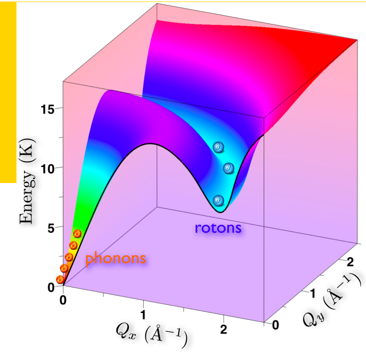


K. H. Andersen *et al.*, J. Phys. Cond. Mat. **6**, 821 (1994).

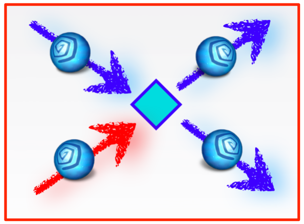
- *T*-dependence of roton's:
  - lifetime
  - energy

BF, TK, MZ, **SC**, PRL **109**, 155305 (2012).

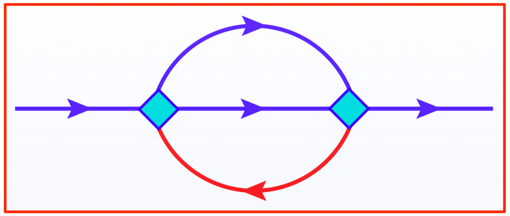
(long) history …



- L. D. Landau and I. M. Khalatnikov, *The theory of the viscosity of helium II: I. Collisions of elementary excitations in helium II*, Zh. Eksp. Teor. Fiz. **19**, 637- 650 (1949).
- roton lifetime (linewidth)



$$\Gamma(T) \propto N_r(T) \propto \sqrt{T}\, e^{-\frac{\Delta(T)}{T}}$$



- K. Bedell, D. Pines, and A. Zawadowskii, PRB **29**, 102 (1984).
- prefactors + energy shift

$$\delta(T) = \Delta(T) - \Delta_0 \propto N_r(T) \propto \sqrt{T}\, e^{-\frac{\Delta(T)}{T}}$$

* **no** three-boson interaction needed/directly involved

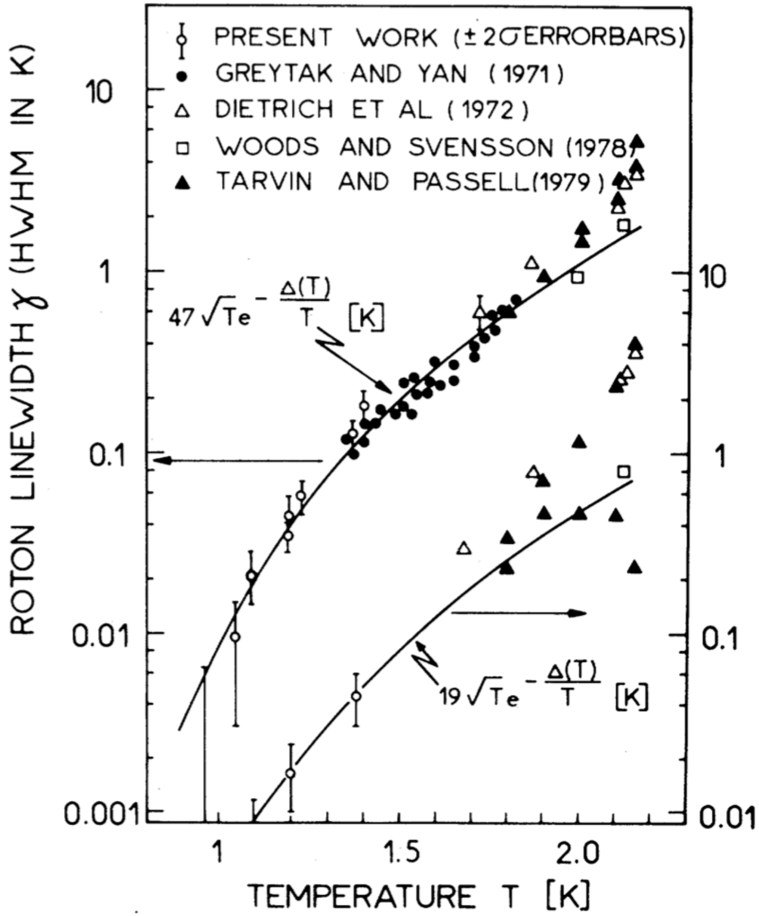
** Hartree-term gives the same for $\delta(T)$

neutron-scattering spin-echo, (>1.2K)

VOLUME 44, NUMBER 24 PHYSICAL REVIEW LETTERS 16 JUNE 1980

High-Resolution Study of Excitations in Superfluid ^{4}He by the Neutron Spin-Echo Technique

F. Mezei



- LK-theory fits:
  - quantitative

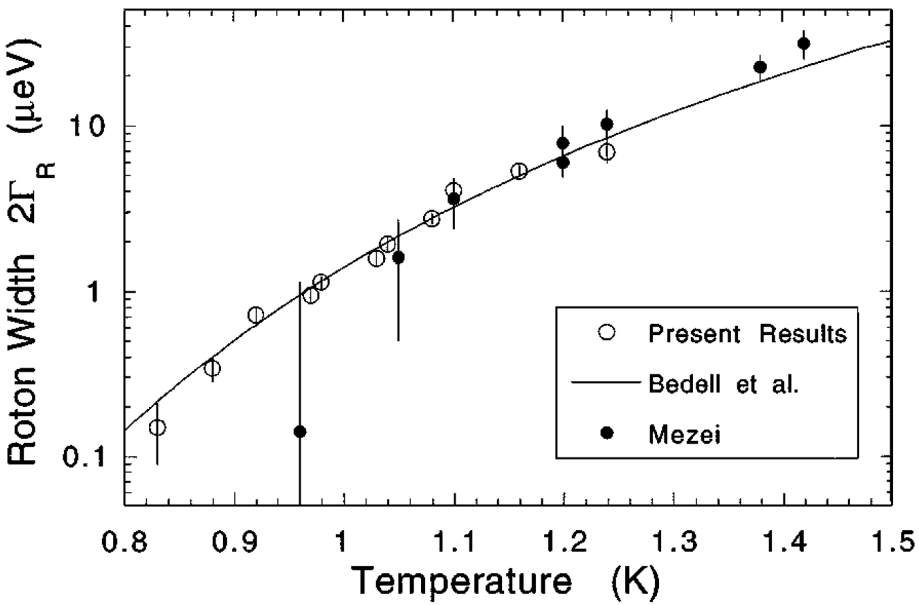
is this the end of the story?

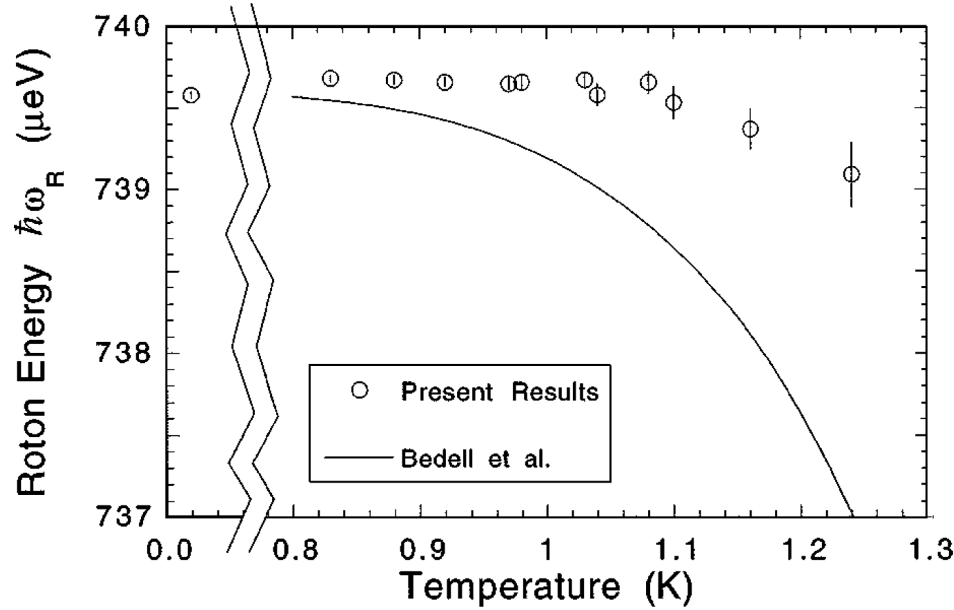
neutron-scattering spin-echo, (>0.88K)

VOLUME 77, NUMBER 19 PHYSICAL REVIEW LETTERS 4 NOVEMBER 1996

High-Resolution Measurements of Rotons in ^{4}He

K. H. Andersen, J. Bossy, J. C. Cook, O. G. Randl, and J.-L. Ragazzoni
Institut Laue-Langevin, B.P. 156, 38042 Grenoble Cedex 9, France
(Received 22 March 1996)



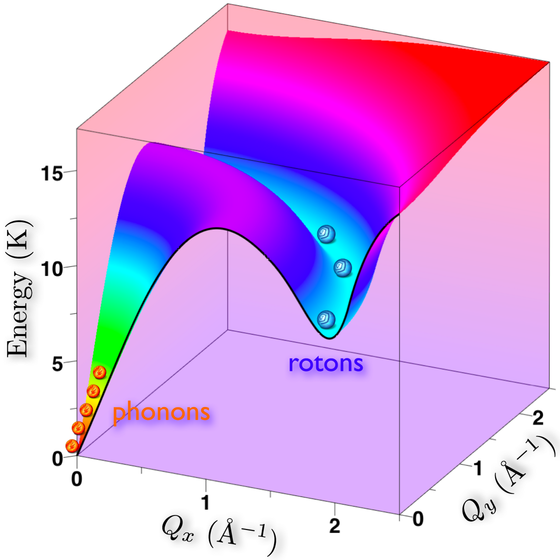


- linewidth:
  LK-theory continues to work

$$\Gamma(T) \propto N_r(T) \propto \sqrt{T}\, e^{-\frac{\Delta(T)}{T}}$$

et al. and (2) that the temperature dependence of the roton energy cannot be described by the same theory in this temperature range. In fact, the rather good agreement between theory and experiment previously shown by the NSE technique [3], is now seen to break down below $T = 1.2$ K, while the theory is expected to work best in the $T \rightarrow 0$ limit.

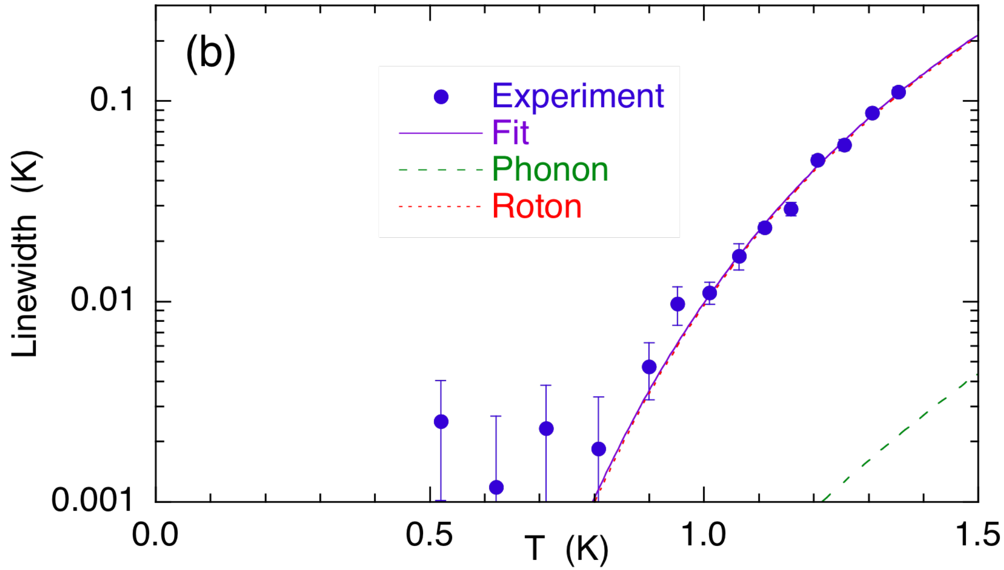
phonons ?



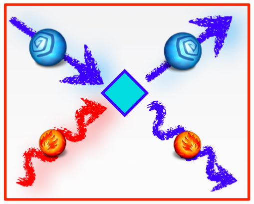
$$\left.\begin{array}{l}\Gamma(T)\\ \\ \delta(T)\end{array}\right\} \propto\ T^3 \times T^4 = T^7 \left(= \widetilde{A} \cdot \frac{T^7}{c^7}\right)$$

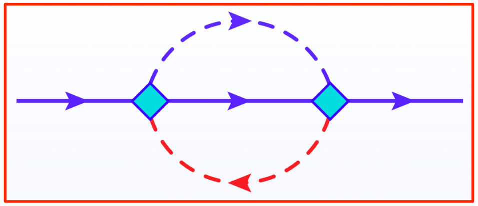
population × Rayleigh scattering

- using numbers: very small effect



- where are the phonons ?
- roton-*phonon* scattering:





- linewidth: LK-theory is fine !

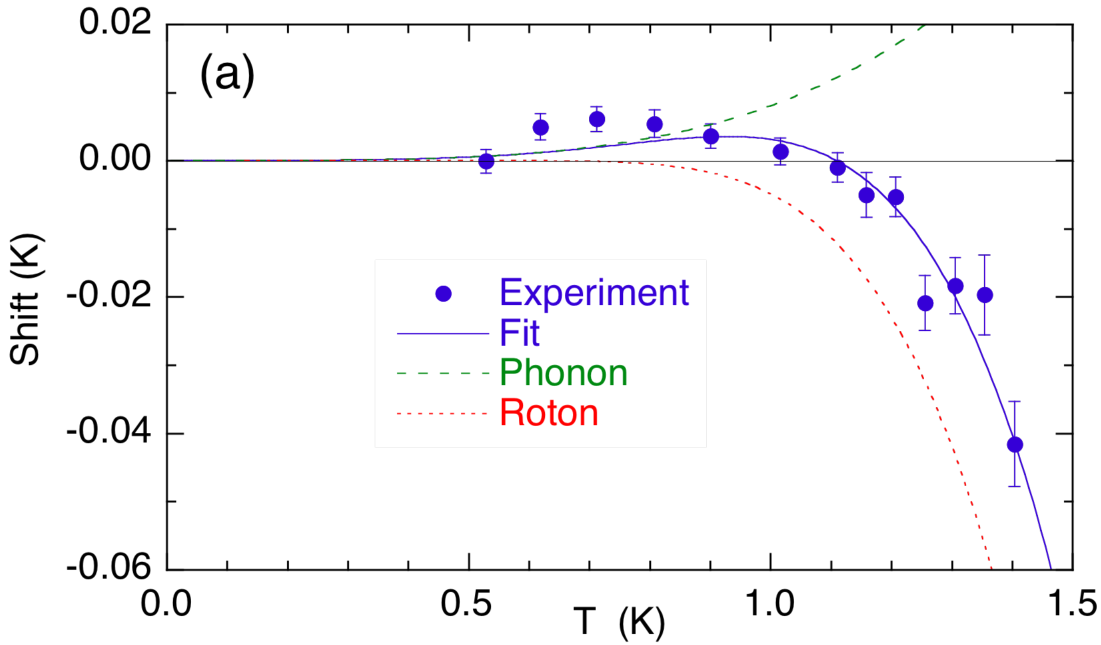
$$\Gamma(T) \propto N_r(T) \propto \sqrt{T}\, e^{-\frac{\Delta(T)}{T}}$$

BF, TK, MZ, **SC**, PRL **109**, 155305 (2012).

* (= r-r at T=0.5K, where both ~10^{-6} K)

neutron-scattering spin-echo, (>0.5K)

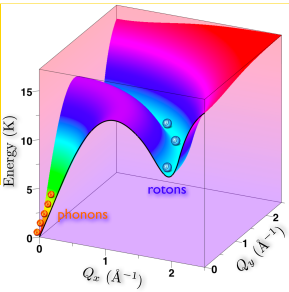
resolution ~ 0.2 μeV (≈0.002K) !



- energy shift:
  - LK-theory fails
  - positive shift at low *T*

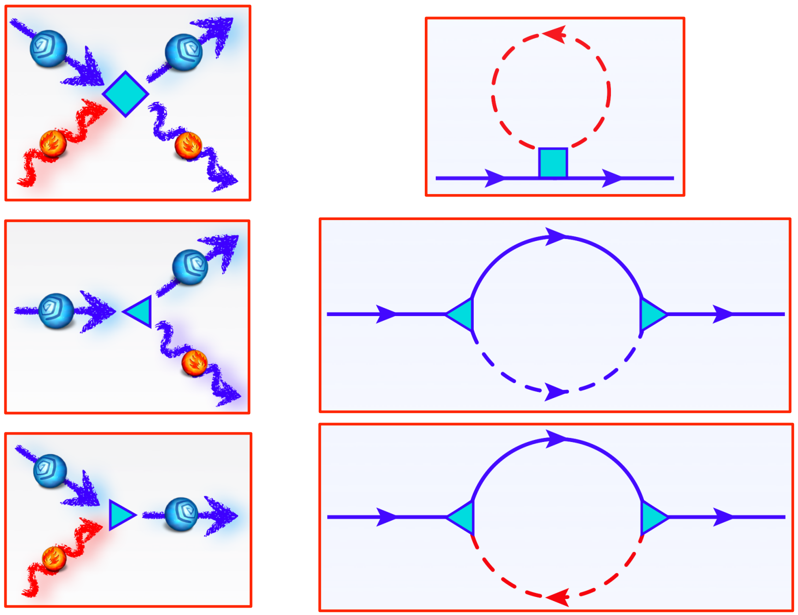
$$\delta(T) \propto \sqrt{T}\, e^{-\Delta(T)/T} + ???$$

BF, TK, MZ, **SC**, PRL **109**, 155305 (2012).



phonons to the rescue

- **more** roton-*phonon* scatterings:



Hartree

decay

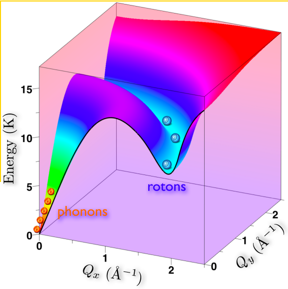
coalescence

$$\Gamma \equiv 0$$

$$\Rightarrow \delta(T) \propto T^4$$

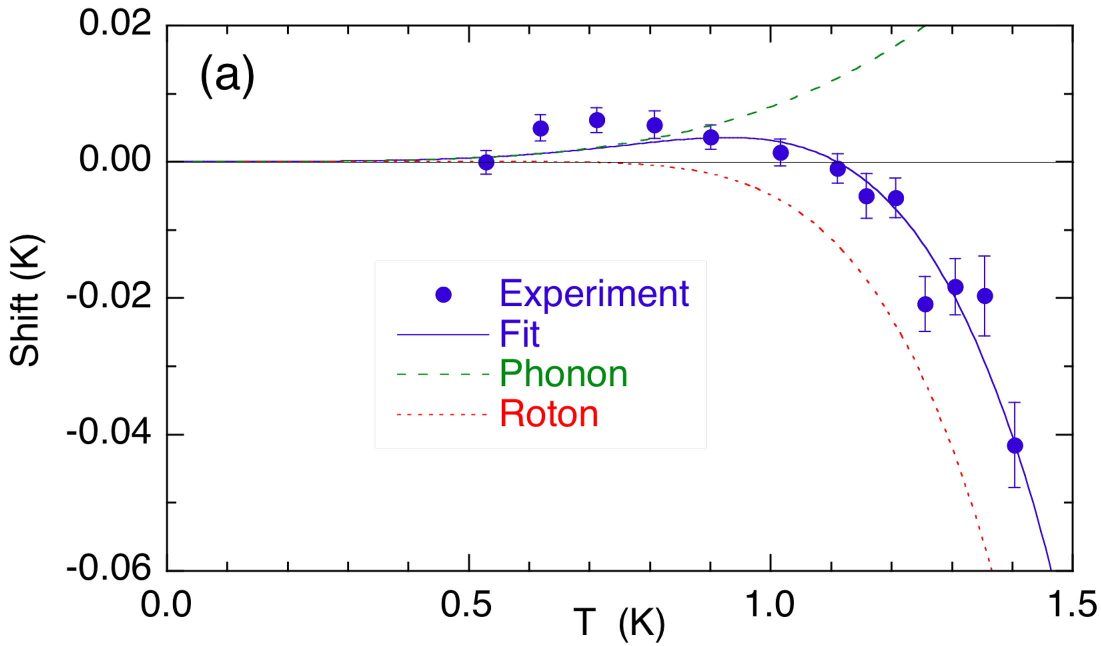
- three-particle (decay, coalescence) are necessarily positive

BF, TK, MZ, **SC**, PRL **109**, 155305 (2012).



theory vs experiment

$$\delta(T) = -\delta_r \sqrt{T}\left(1 + \alpha\sqrt{T}\right) e^{-\Delta(T)/T} + \delta_{ph} T^4$$



- theory:
  - quantitatively (!) explains positive shift at low *T*
  - provides reasonable fit
  - three-particle interactions dominate the new effect

BF, TK, MZ, **SC**, PRL **109**, 155305 (2012).

conclusions/outlook

- ☑ clear sign of the odd (3-particle) interactions in the roton energy shift
- ☑ LK ++ [LK-ZC (?)] theory is formulated
- ☑ neutron spin-echo allows to reach new regimes

- ☯ even lower T?
- ☯ phonon-phonon scattering, pressure dependence, etc.

IV. lifetime of gapped excitations in (collinear) antiferromagnets

theory: Sasha Chernyshev (UC Irvine)
Mike Zhitomirsky (CEA)

experiments: Louis-Pierre Regnault,
Nicolas Martin (CEA, ILL)

arXiv: 1206.4690; PRL **109**, 097201 (2012)

material

L. P. REGNAULT AND J. ROSSAT-MIGNOD

PHASE TRANSITIONS IN QUASI TWO-DIMENSIONAL PLANAR MAGNETS

L. J. De Jongh (Ed.), Magnetic Properties of Layered Transition Metal Compounds 271–321.

$BaNi_2(PO_4)_2$

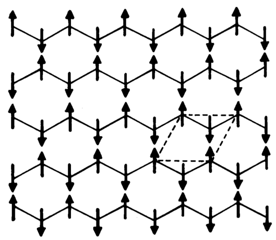
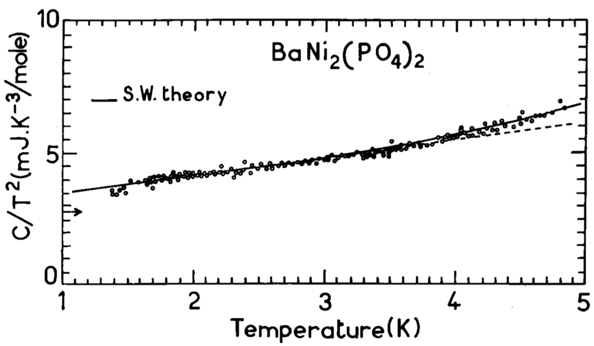
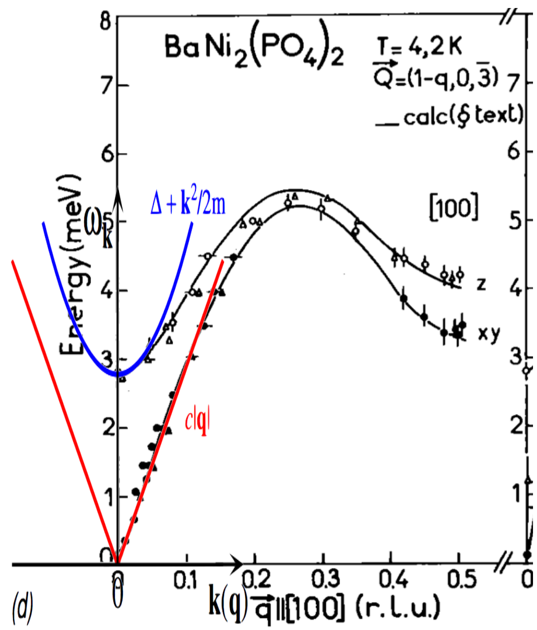
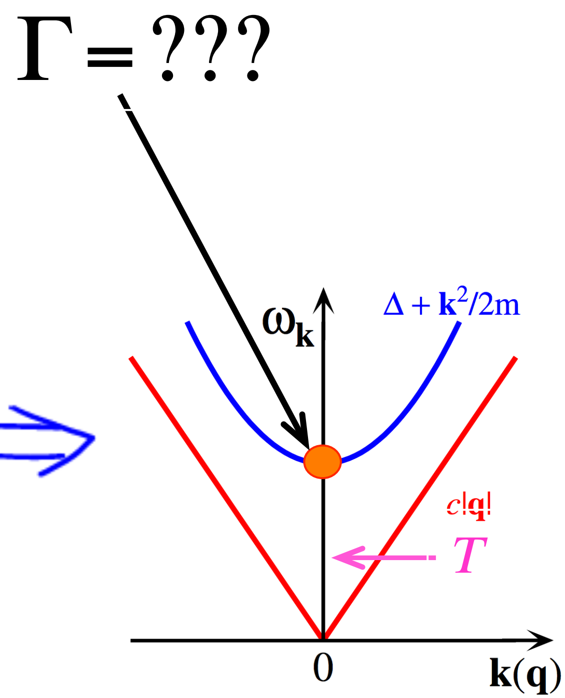


Fig. 3. Antiferromagnetic structure of $BaNi_2(PO_4)_2$.



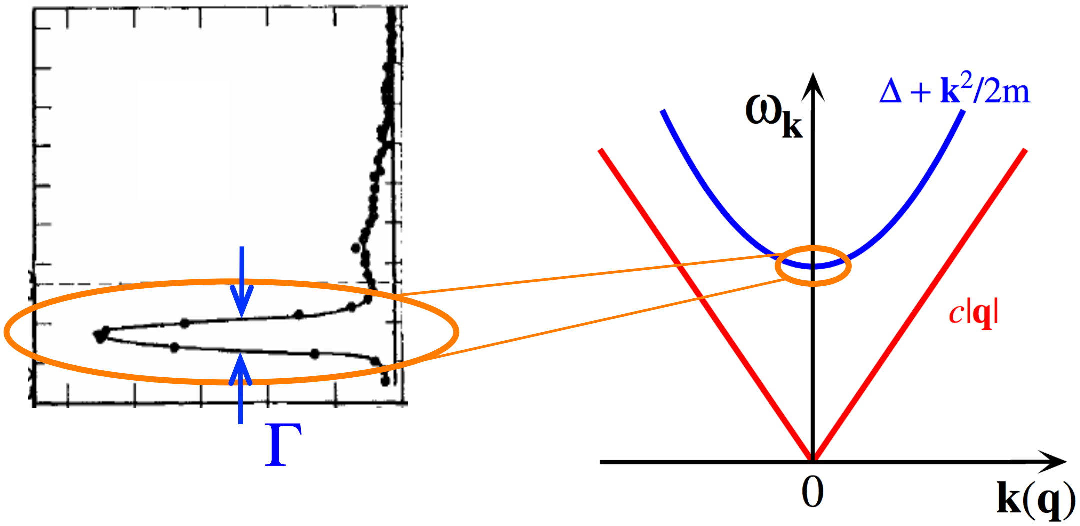




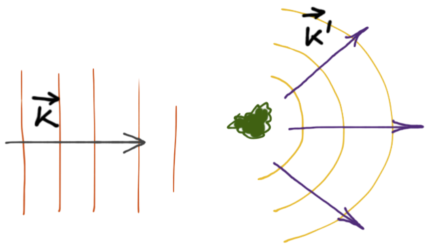
- $S = 1$, 2D, planar (XY)
- has gapped
- and acoustic modes

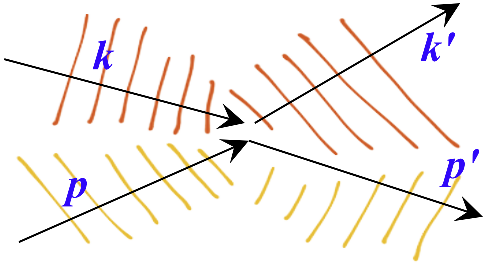
$$\boxed{\mathcal{H} = \sum_{\langle i,j \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + D \sum_i (S^z)^2}$$

lifetime/linewidth/damping/decay rate



- lifetime^{-1} = linewidth in "simple" AFs
- spin waves: scattering on?
  - impurities
  - themselves
  - combination of the two (impurity-assisted)



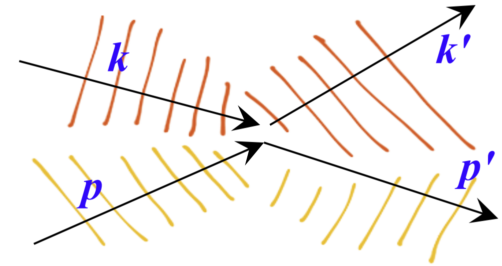


where from?

- by the nature of spin-to-bosons mapping:

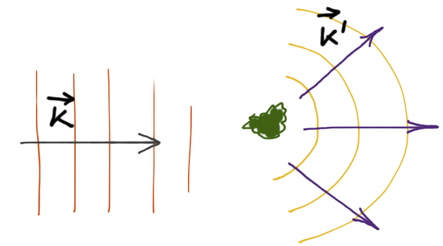
- $\Rrightarrow$ magnon-magnon scattering

$$J\mathbf{S}\cdot\mathbf{S},\, D\left(S^z\right)^2 \Rightarrow \varepsilon_{\mathbf{k}} b^\dagger_{\mathbf{k}} b_{\mathbf{k}} + V^{\rm m-m}_{\mathbf{k},\mathbf{p},\mathbf{k}',\mathbf{p}'} b^\dagger_{\mathbf{k}'} b^\dagger_{\mathbf{p}'} b_{\mathbf{p}} b_{\mathbf{k}}$$



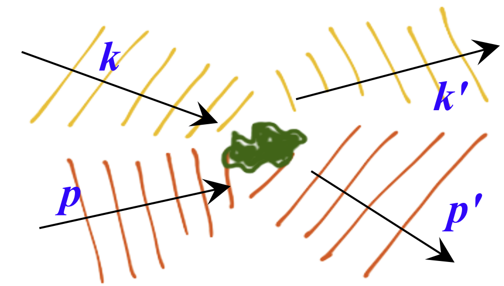
- $\Rrightarrow$ impurity [site/bond defect] scattering

$$\delta J\left[\delta D\right] \Rightarrow \delta J\left[\delta D\right] b^\dagger_{\mathbf{k}'} b_{\mathbf{k}}$$

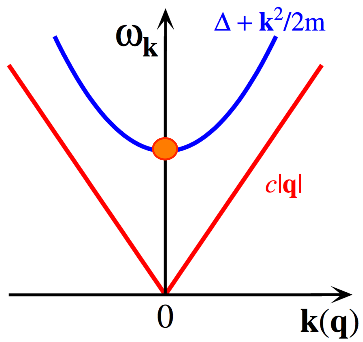


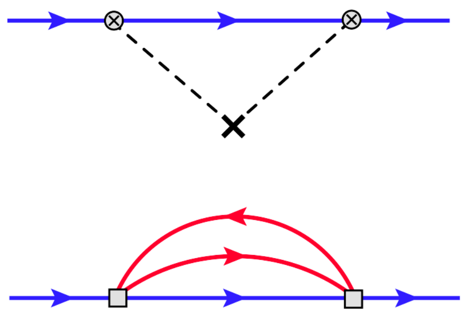
- $\Rrightarrow$ ***and*** impurity-assisted magnon-magnon scattering

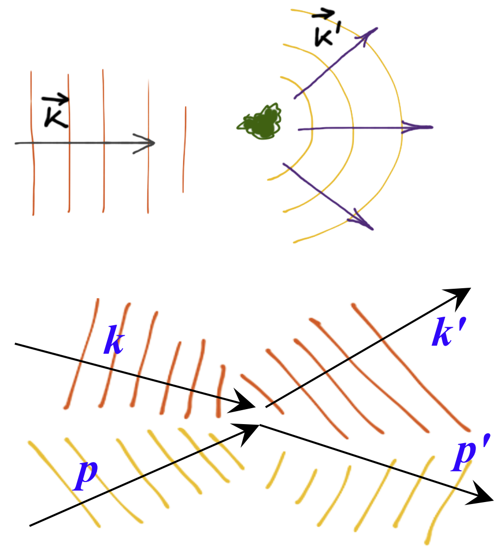
$$\delta J\left[\delta D\right] \Rightarrow V^{\rm imp}_{\mathbf{k},\mathbf{p},\mathbf{k}',\mathbf{p}'} b^\dagger_{\mathbf{k}'} b^\dagger_{\mathbf{p}'} b_{\mathbf{p}} b_{\mathbf{k}}$$



damping, theory expectations, I







- local distortions → δD, δJ → conventional impurity scattering (2D):

$$\Gamma_{\mathbf{k}}^{\mathrm{imp}} \approx \Gamma_0 \propto n_{\mathrm{imp}} \overline{\delta D}^2 \, \frac{m\,\omega_{\mathrm{max}}^2}{\Delta^2}$$

- gapped on thermally excited gapless: $\Gamma_{\mathbf{k}\to 0}^{\mathrm{m-m}} \approx \frac{\pi^3}{15} \frac{\tilde{g}^2}{c} \left(\frac{T}{c}\right)^5$

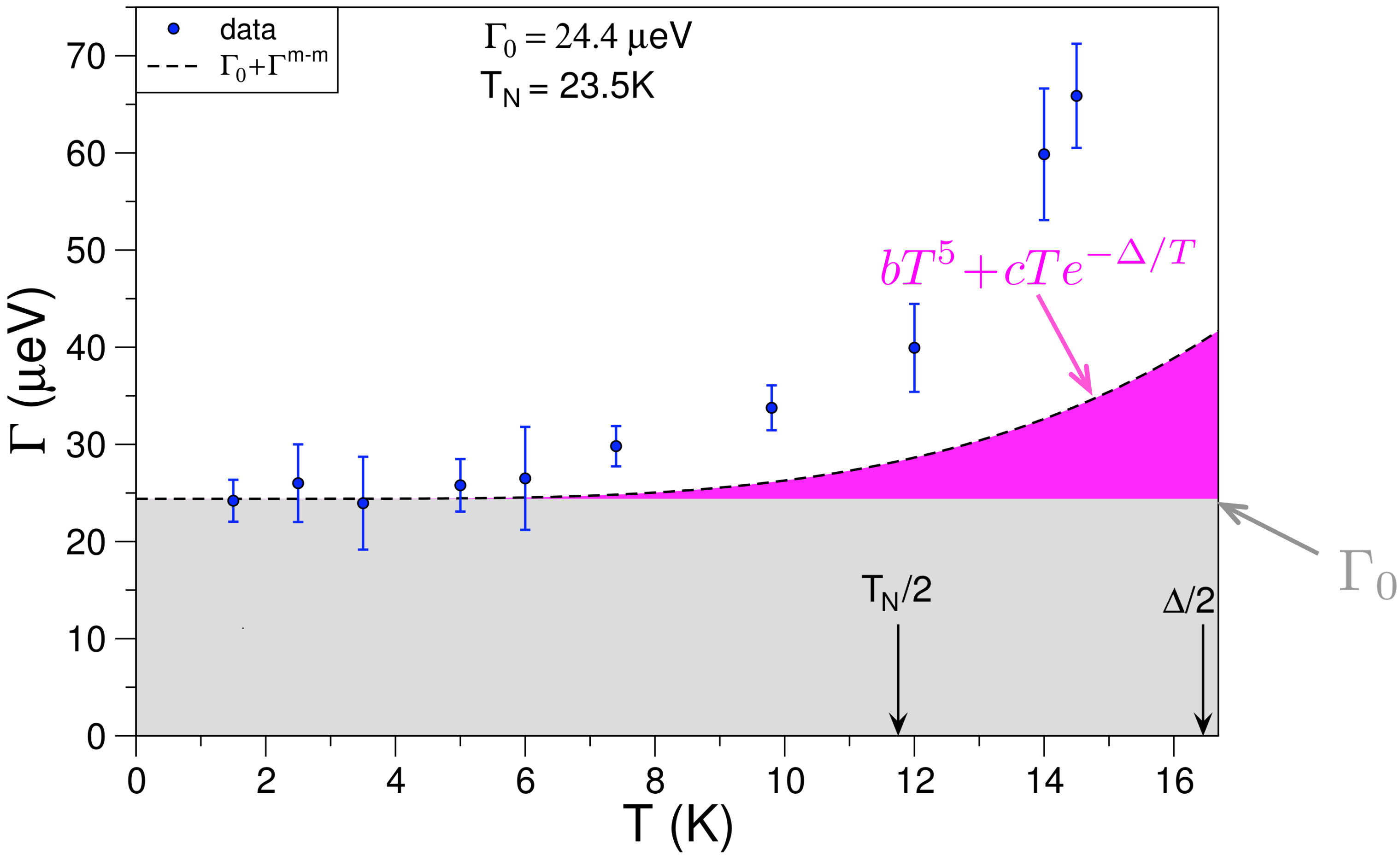
- (and on gapped): $\Gamma_{\mathbf{k}\to 0}^{\beta\beta\to\beta\beta} \approx \frac{g_\beta^2 m^2 T}{4\pi} \, e^{-\Delta/T}$

- numbers for m-m scattering are known/derivable!

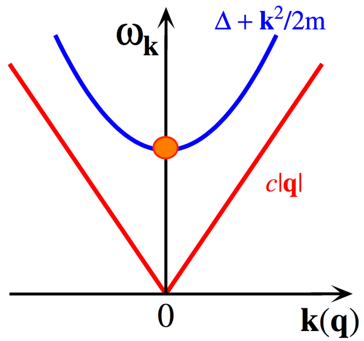
* [3D → T^7]

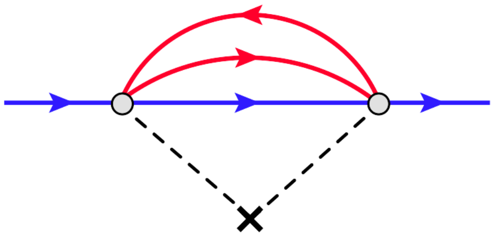
standard lore: $\varrho = \varrho_0^{\mathrm{imp}} + \varrho^{ee}(T)$

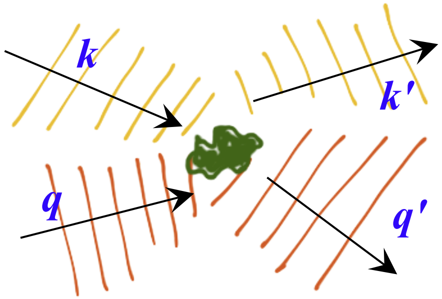
$$\Gamma = \underbrace{\Gamma_0}_{\mathrm{imp}} + \underbrace{bT^5 + cTe^{-\Delta/T}}_{\mathrm{m-m}}$$



damping, theory expectations, II







- impurity facilitates stronger m-m scattering

$$V^{\mathrm{imp}}_{\mathbf{k},\mathbf{q};\mathbf{k}',\mathbf{q}'} \approx \tilde{g}_{\mathrm{imp}}/\sqrt{\tilde{q}\tilde{q}'} \qquad \text{vs} \qquad V^{\mathrm{m-m}}_{\mathbf{kq};\mathbf{k}'\mathbf{q}'} \propto \sqrt{qq'}$$

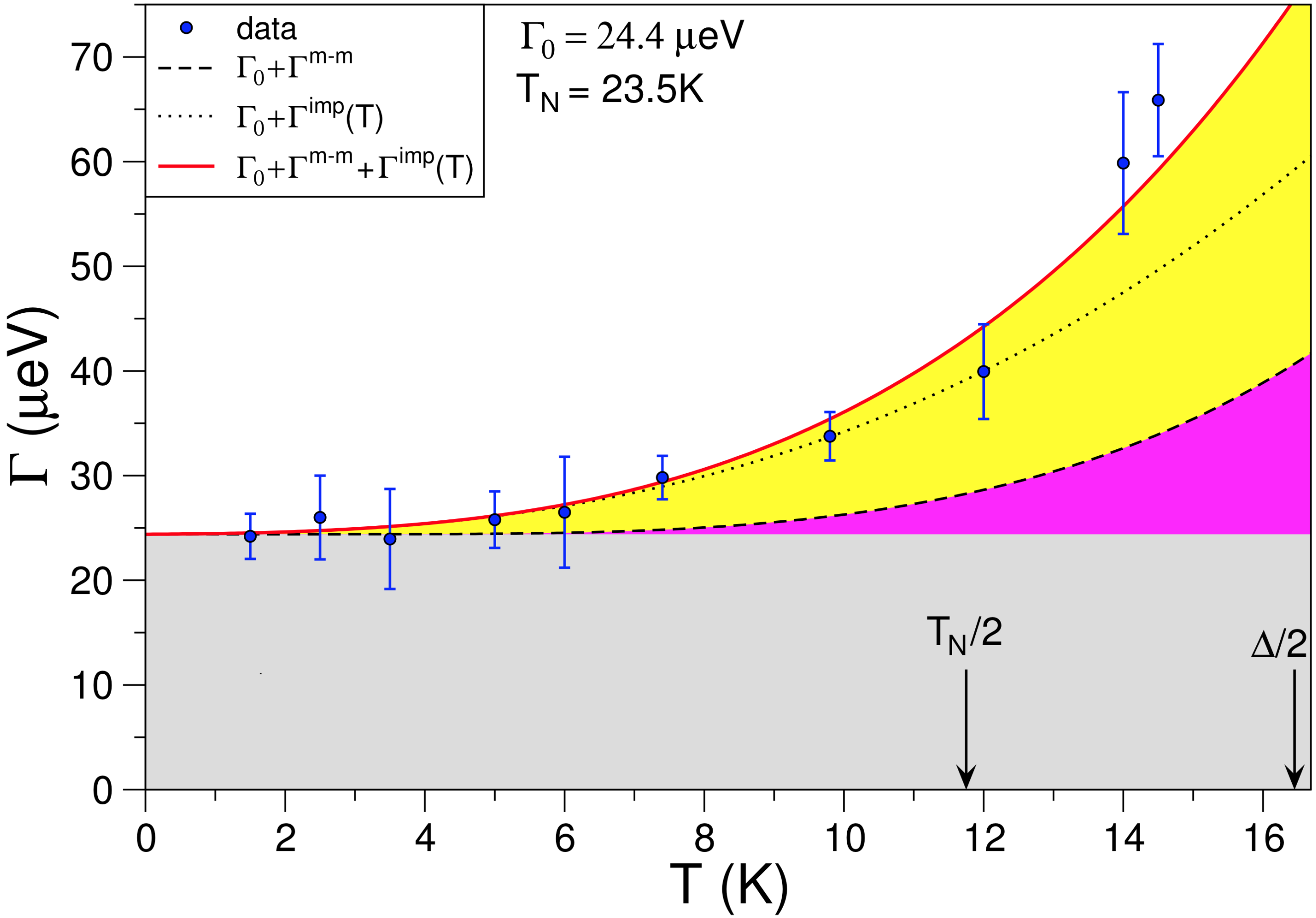
- → lower power of T in Γ

$$\boxed{\Gamma^{\mathrm{imp},T}_{\mathbf{k}\to 0} \approx \widetilde{A}\left(\frac{T}{c}\right)^2\left[\left(\ln\frac{T}{\omega_0}\right)^2 + \frac{\pi^2}{3}\right]} \quad \boxed{\widetilde{A} \sim n_{\mathrm{imp}}\overline{\delta D}^2 m}$$

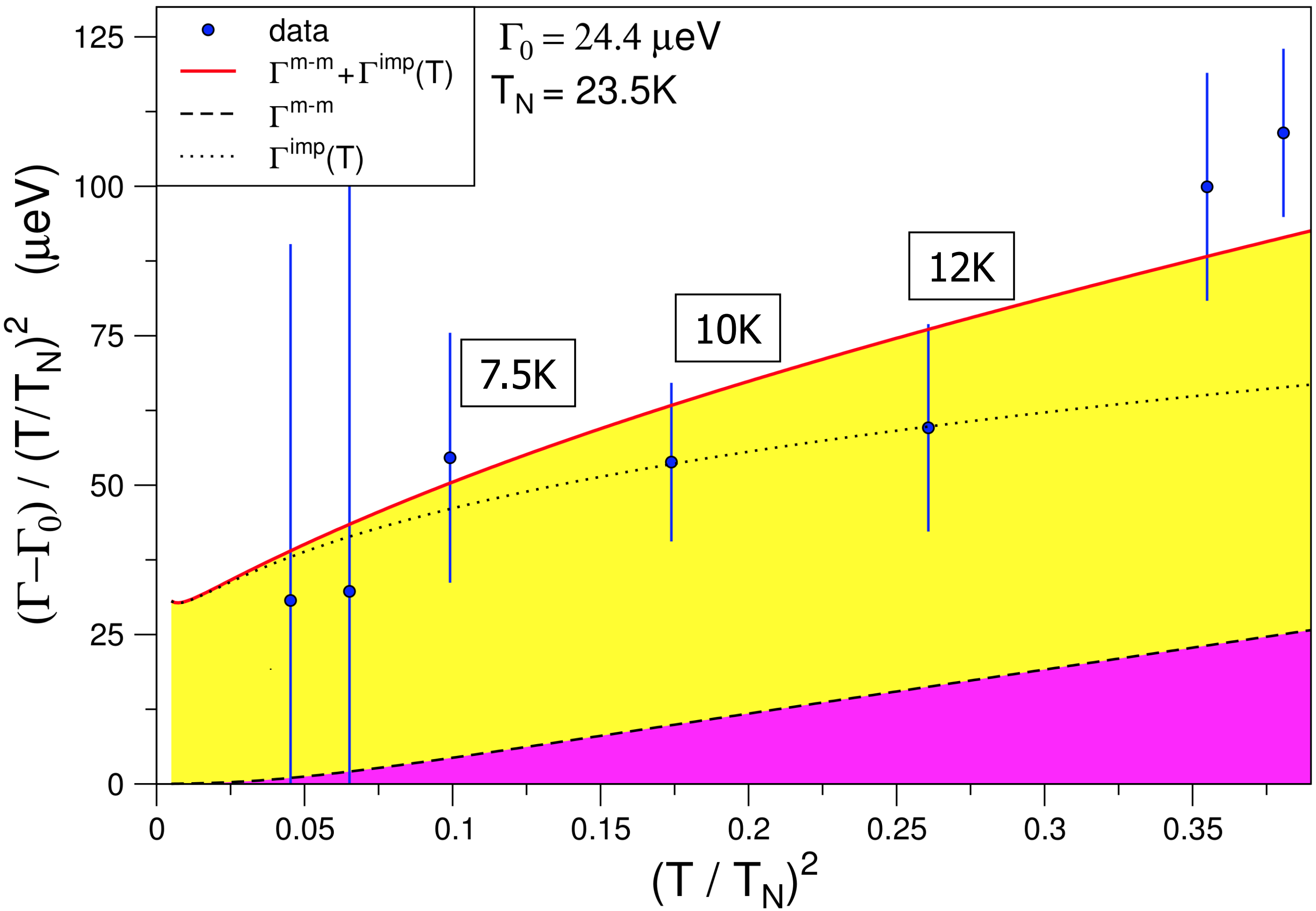
- reasons for "stronger" potential:
  - m-m interactions are singular but cancel out
  - impurities violate that cancellation
  - dynamically-induced "strong" disorder: optical spin-flip "sits" at impurity, scatters acoustic mode stronger …

beyond the standard model …

$$\Gamma = \underbrace{\Gamma_0}_{\text{imp}} + \underbrace{aT^2\left[\left(\ln\frac{T}{\omega_0}\right)^2 + \frac{\pi^2}{3}\right]}_{\text{imp. finite}-T} + \underbrace{bT^5 + cTe^{-\Delta/T}}_{\text{m}-\text{m}}$$



better picture …



cross-checks, predictions

T=0 and *T>0* impurity terms must be related ($\Gamma_0 \sim \widetilde{A} \sim n_{\rm imp}\overline{\delta D}^2 m$)

☑ true, in our fit: $\Gamma_0 \approx \widetilde{A} \approx 25\ \mu{\rm eV}$

does disorder strength make sense?

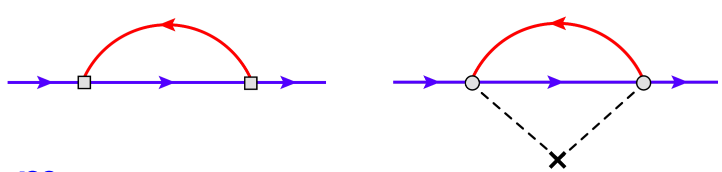
☑ estimate: $n_{\rm imp}(\overline{\delta D}/D)^2 \approx \Gamma_0/\omega_{max} \approx 10^{-2}$

translates into a (very reasonable) statement that in $BaNi_2(PO4)_2$, strong modulation of magnetic couplings of order 1 is spread over 1 in 100 unit cells

predictions:

- 3D: $\Gamma_{3D}^{\rm imp} \propto n_{\rm imp} T^{9/2}$
- AFs with non-collinear order → 3-magnon coupling $\quad {\rm BaCo_2(AsO_4)_2}$

new diagrams:



no change in m-m

lower power of T in impurity-induced: $\boxed{\Gamma_{{\bf k}\to 0}^{{\rm imp},T} \approx \widetilde{A}_3 \left(\frac{T}{c}\right) \ln\frac{T}{\omega_0}}$ $\quad \widetilde{A}_3 \propto n_{\rm imp}|\delta g_3|^2$

conclusions

- ☑ general case: low-T lifetime of a magnetic excitation is completely dominated by the effects induced by a simple structural disorder
- ☑ support from experiments
- ☑ further predictions are made
- ☑ should be relevant to other systems