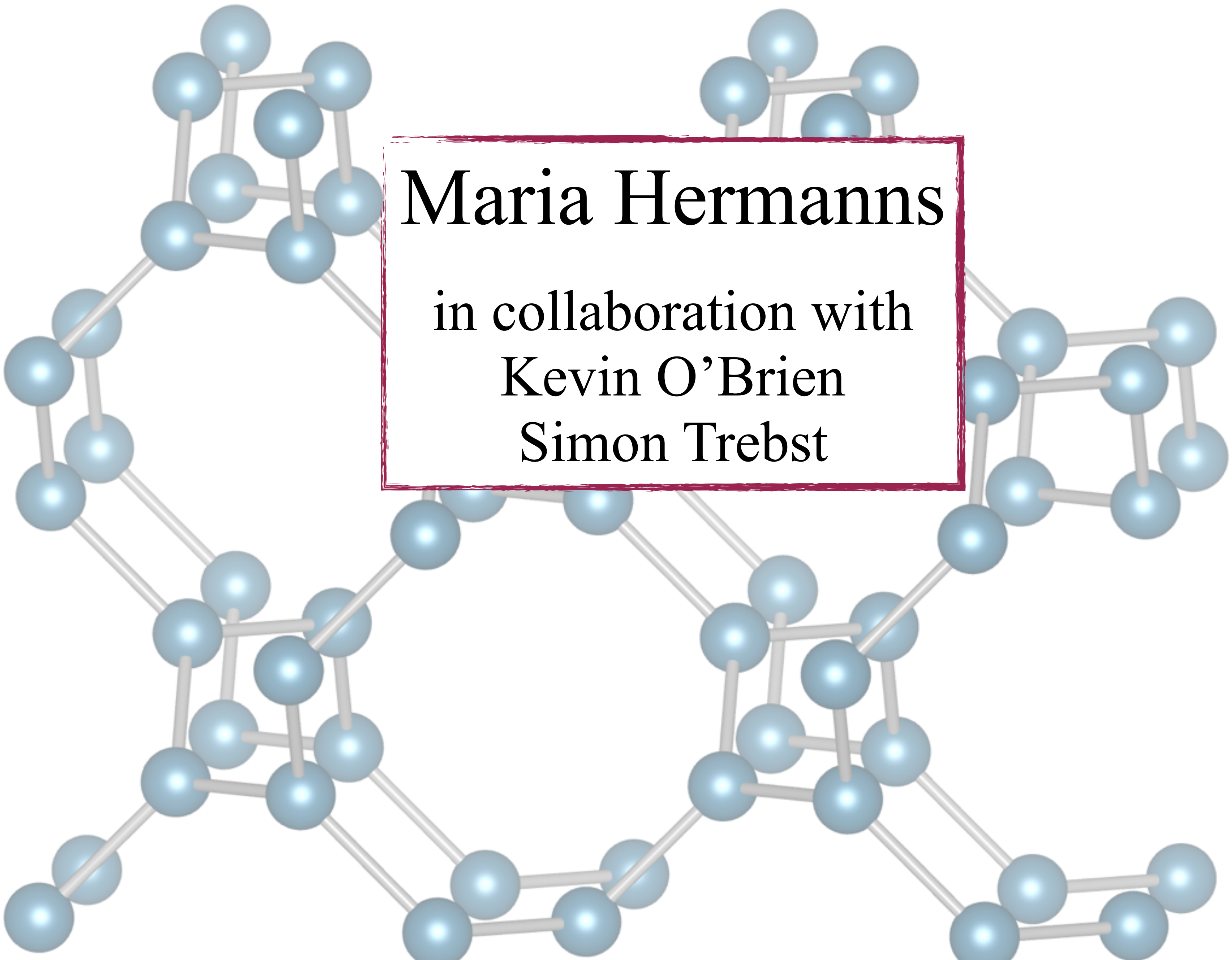
Quantum spin liquid with a Majorana Fermi surface



Maria Hermanns

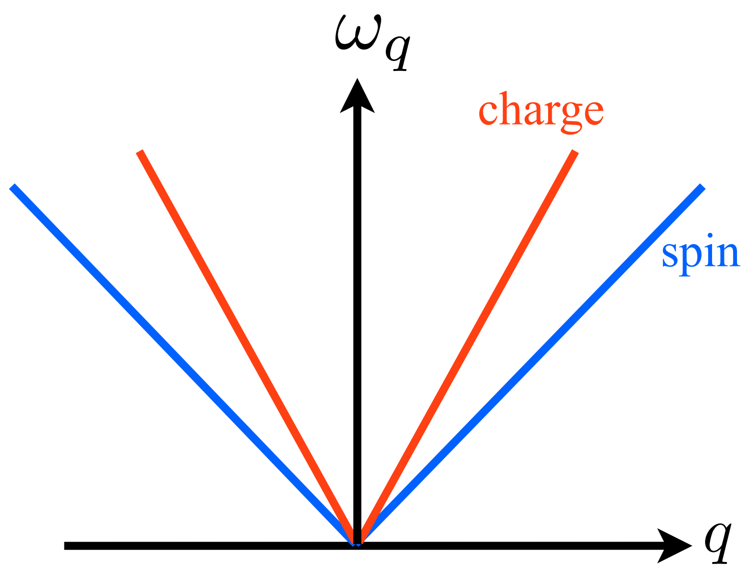
in collaboration with
Kevin O'Brien
Simon Trebst

Fractionalization in strongly correlated systems

(Strong) interactions can lead to emergent quasiparticles
with 'fractional' quantum numbers

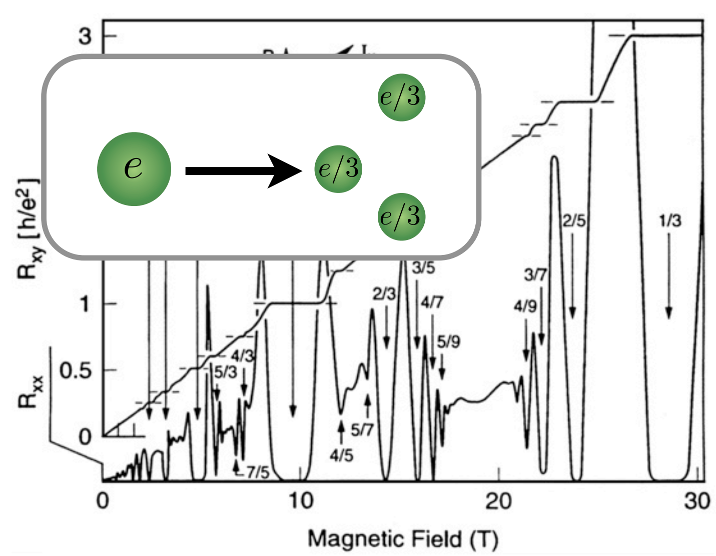
Spin-charge separation

Fermi gas in 1D



Electron fractionalization

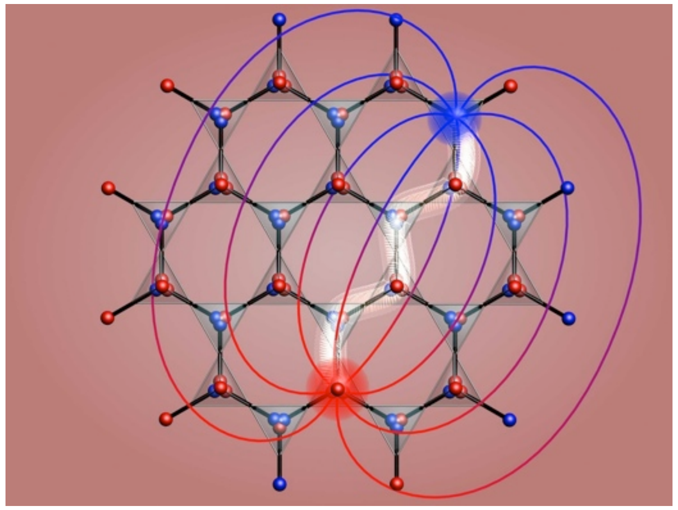
Quantum Hall liquids



Tsui, Störmer, Gossard, PRL **48**, 1559 (1982)

Magnetic monopoles

Spin ice



Castelnovo etal, Nature **451**, 22-23 (2007)

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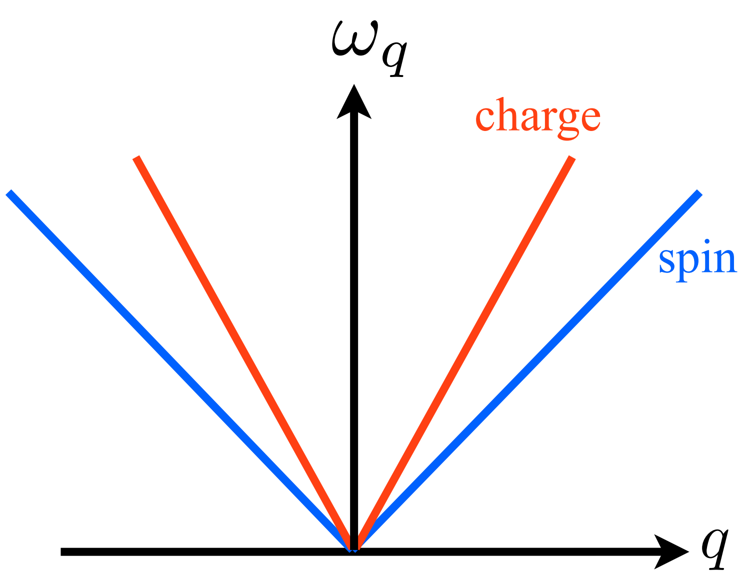

Electron fractionalization

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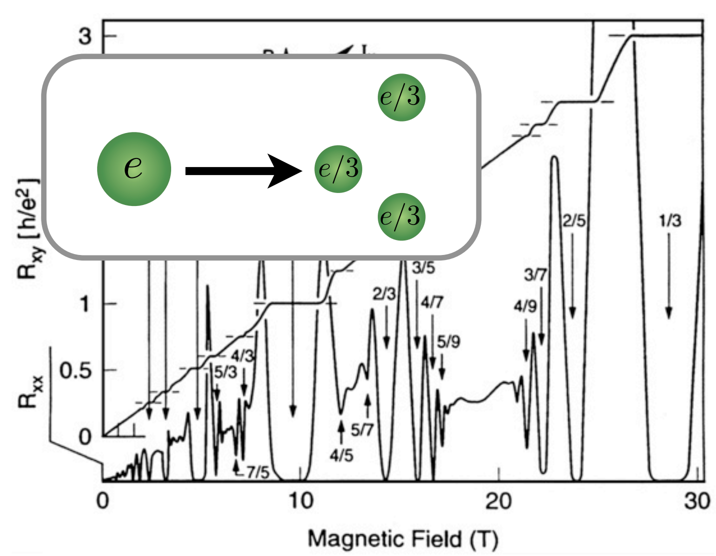
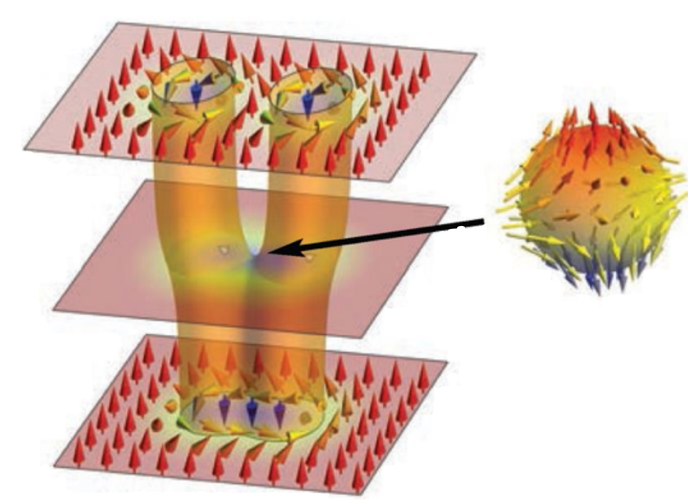


Tsui, Störmer, Gossard, PRL **48**, 1559 (1982)

Magnetic monopoles

Skyrmions in chiral metals

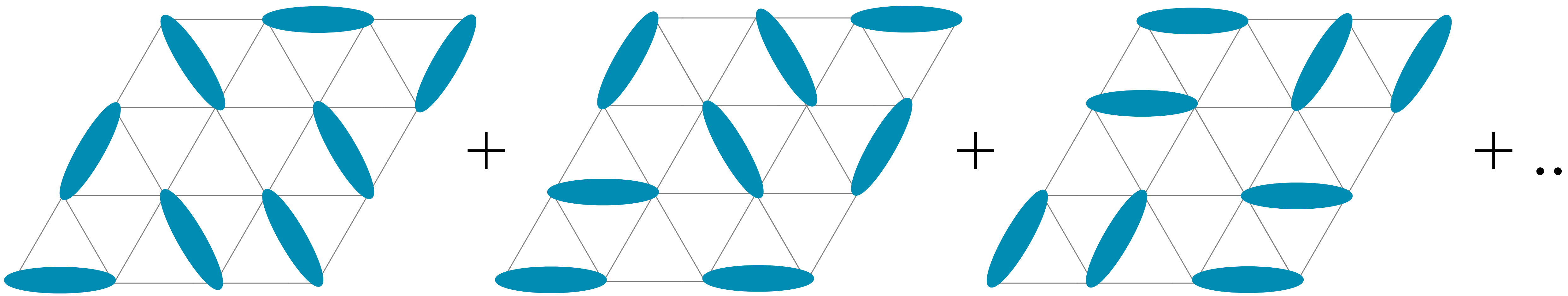


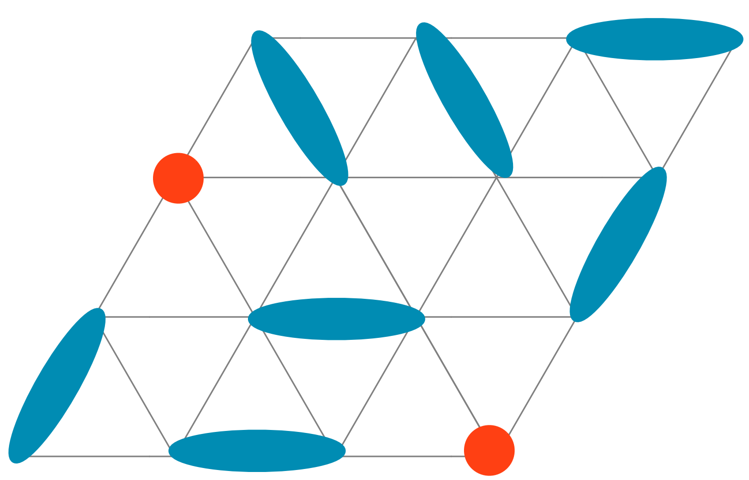
Milde et al., Science **340,** 1076 (2013)

Fractionalization in magnetic systems

Moessner, Sondhi PRL 86, 1881 (2000)

Resonating **V**alence **B**ond liquid:

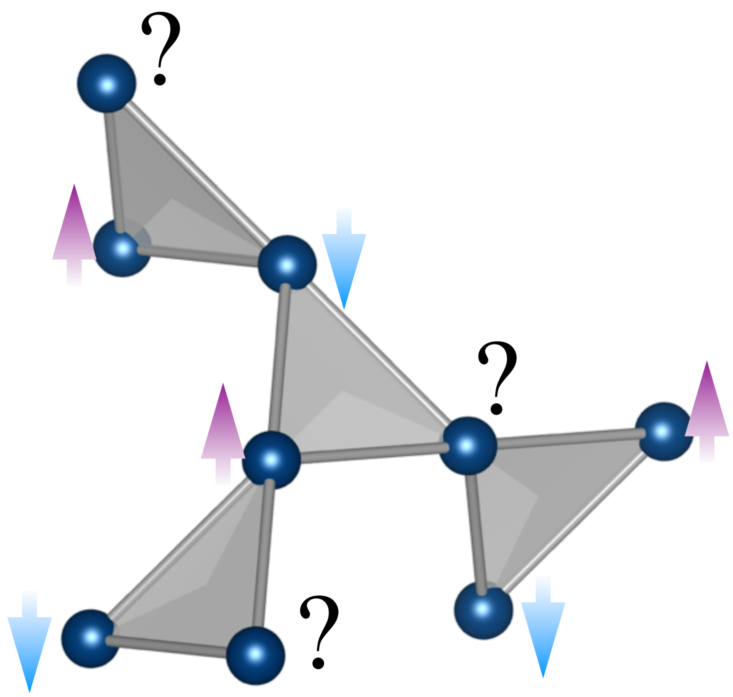




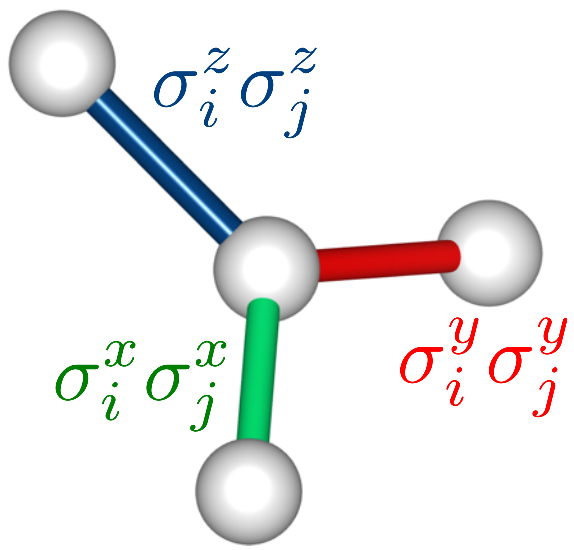
- short-range dimer correlations
- no magnetic order
- gapped, deconfined spinon excitations – spin 1/2

Towards spin liquids – frustration

geometric frustration



exchange frustration



Towards spin liquids – frustration

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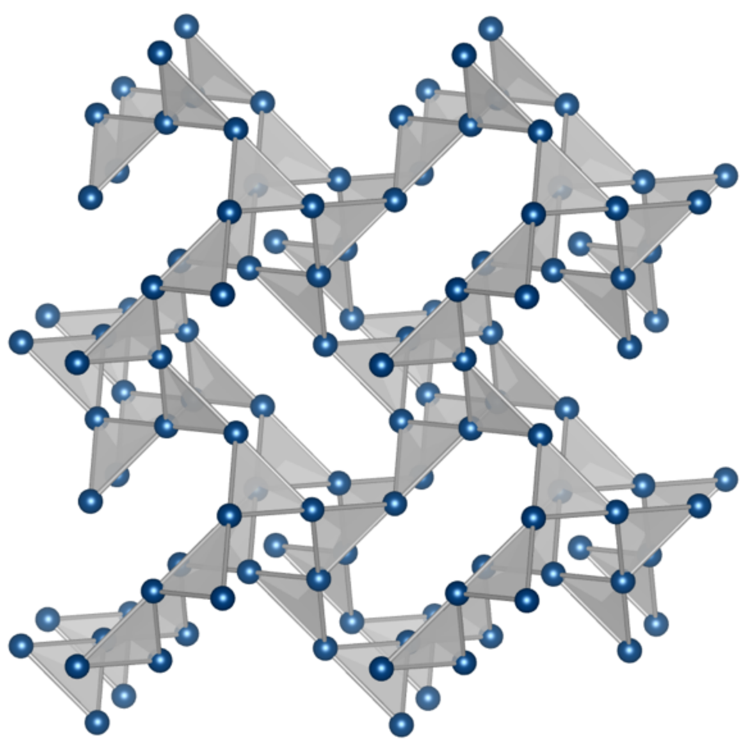
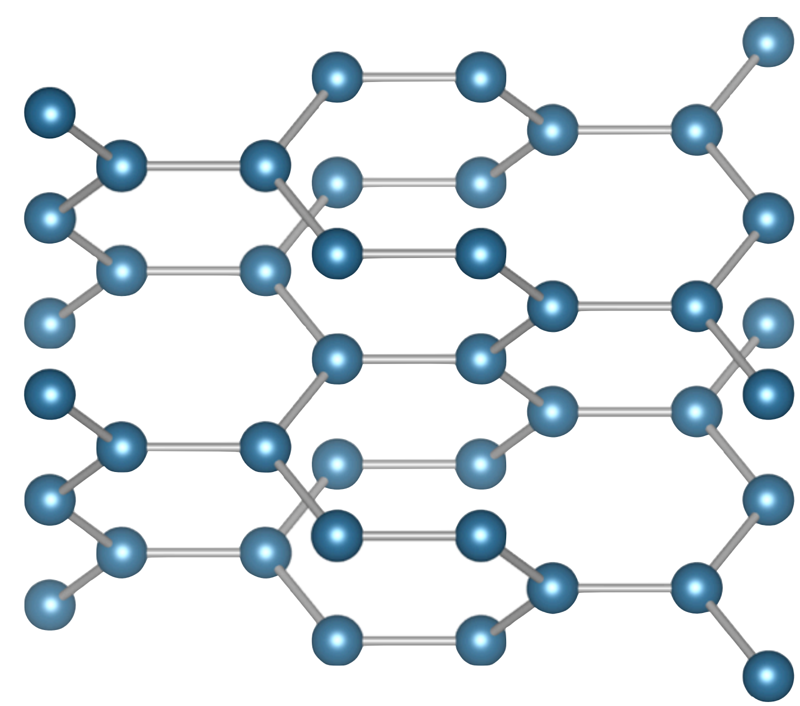


hyperkagome lattice
$Na_4Ir_3O_8$

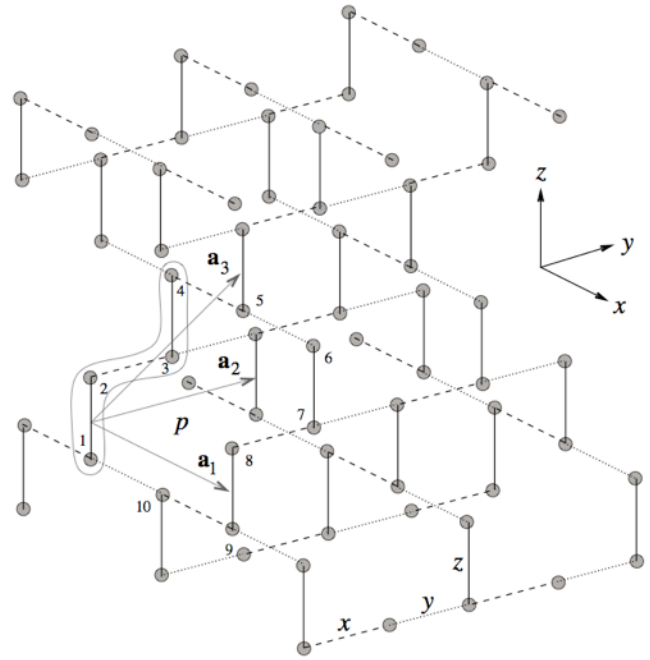


hyperhoneycomb lattice
Li_2IrO_3

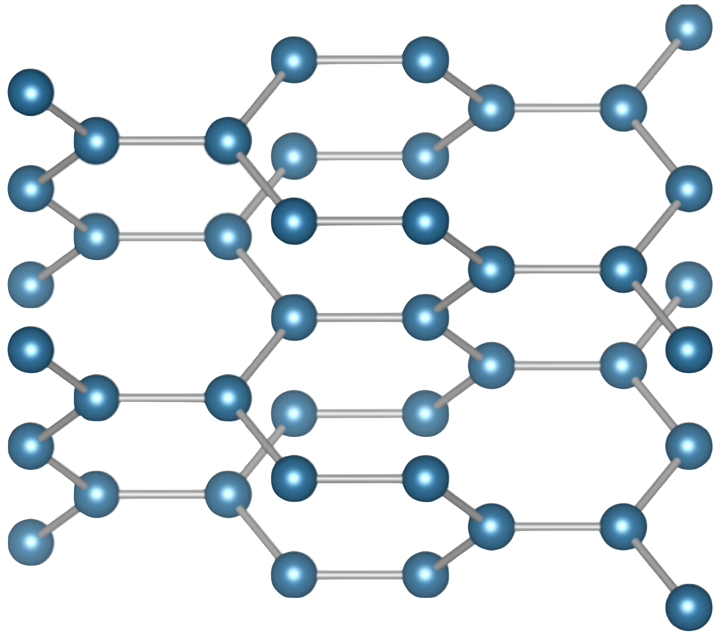
Y. Okamoto, M. Nohara, H. Aruga-Katori, and H. Takagi,
PRL **99**, 137207 (2007)

T. Takayama etal., arXiv:1403.3296 (2014)
K.A. Modic etal. arXiv:1402.3254 (2014)

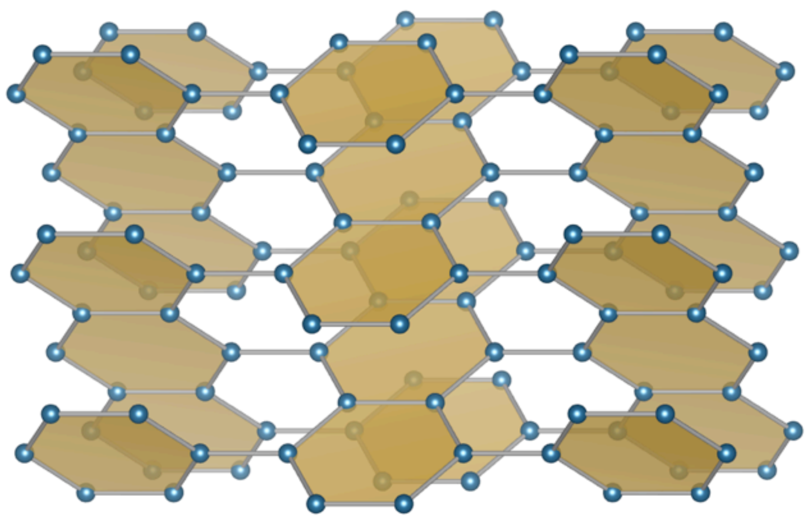
Zoo of 3D tri-coordinated lattices



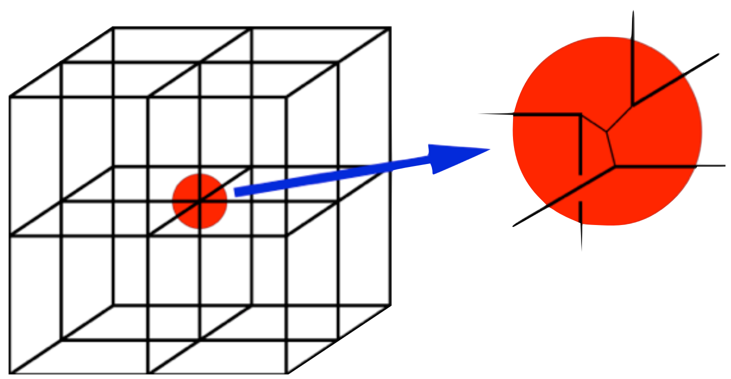
Mandal, Surendran, PRB 79, 024426 (2009)



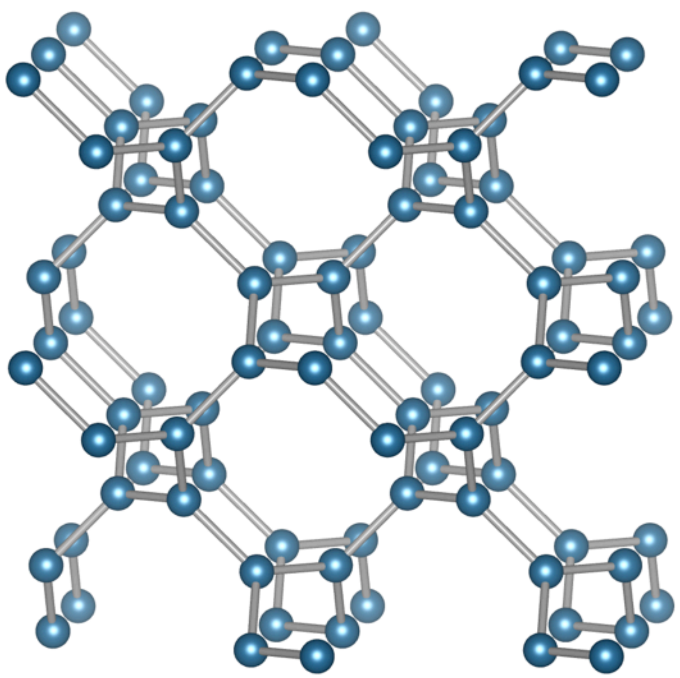
Takayama etal., arXiv:1403.3296 (2014)



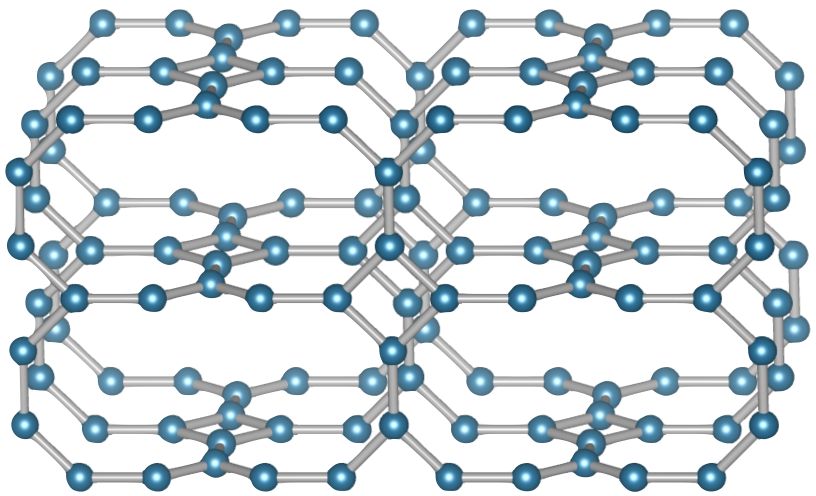
Modic etal. arXiv:1402.3254 (2014)



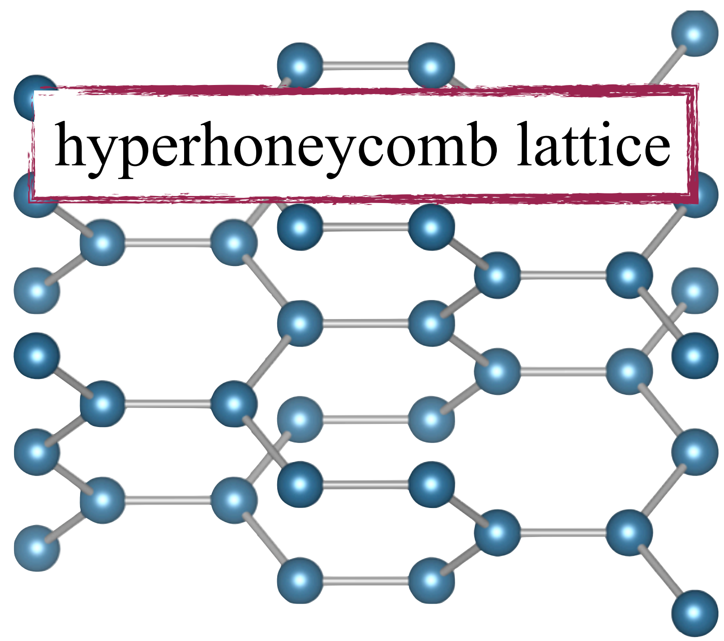
Levin, Wen, PRB 71, 045110 (2005)



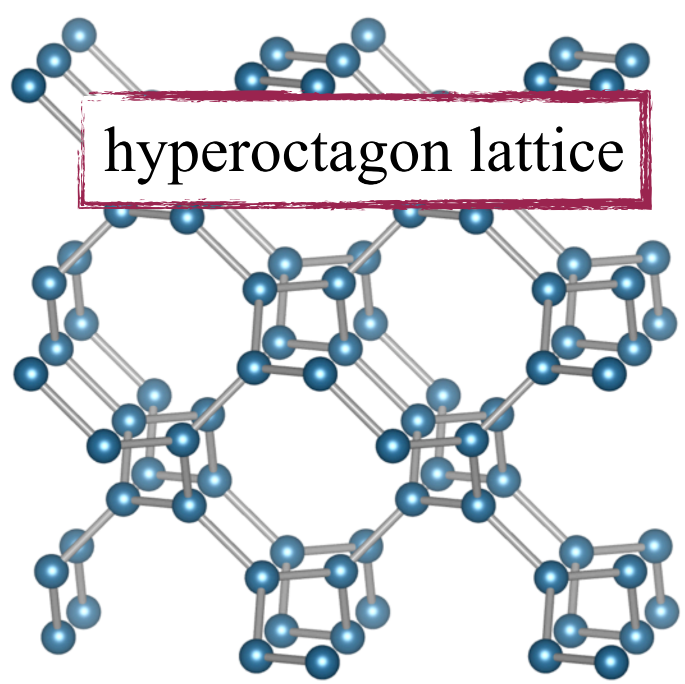
Hermanns, Trebst, PRB 89, 235102 (2014)



Zoo of 3D tri-coordinated lattices

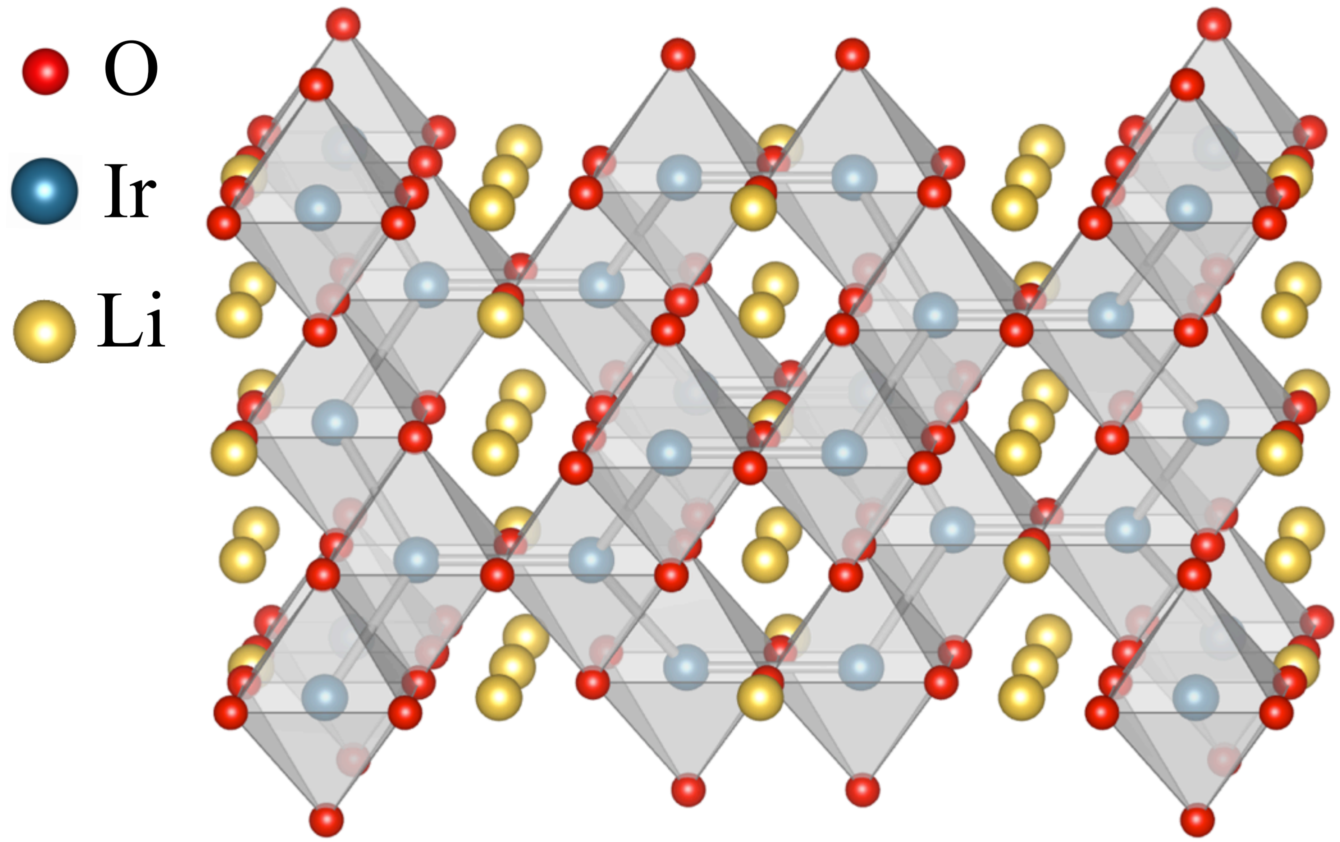


Takayama etal., arXiv:1403.3296 (2014)



Hermanns, Trebst, PRB 89, 235102 (2014)

Hyperhoneycomb material β-Li_2IrO_3



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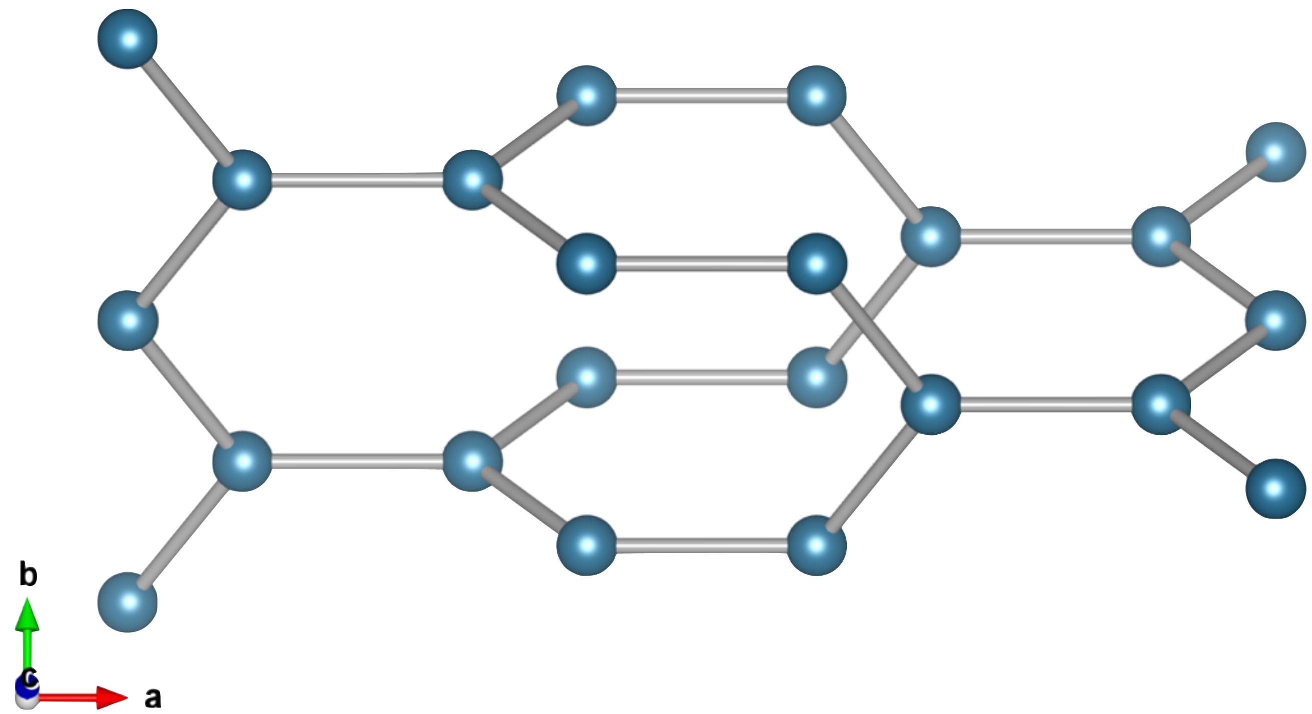
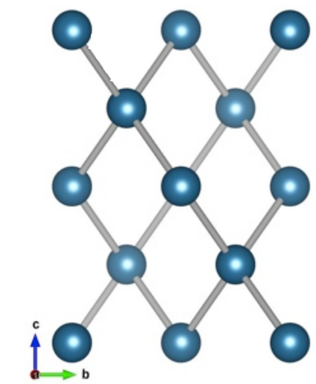
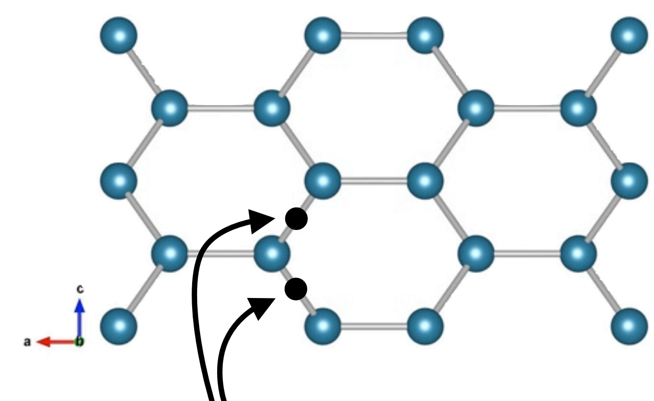




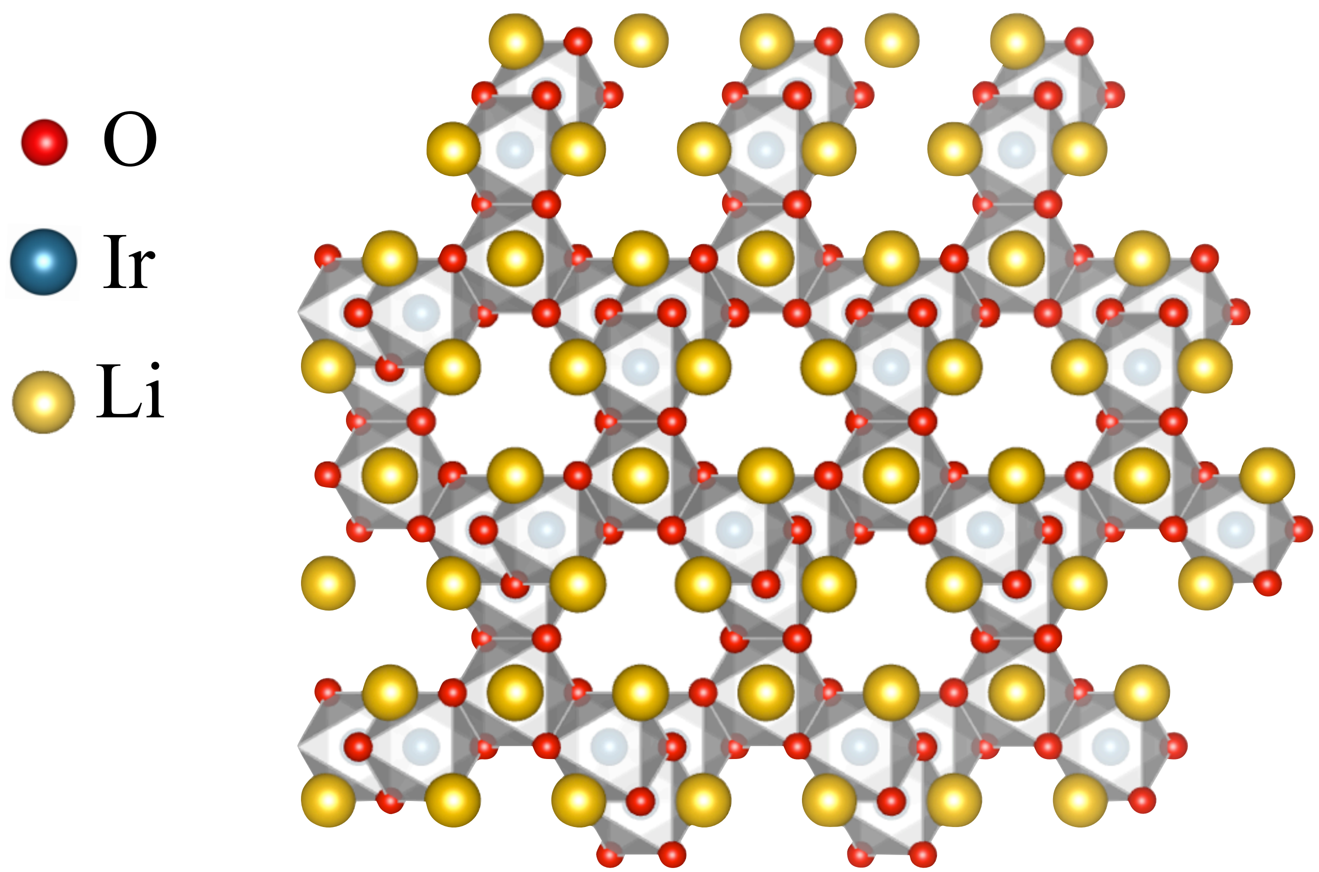
180° rotation



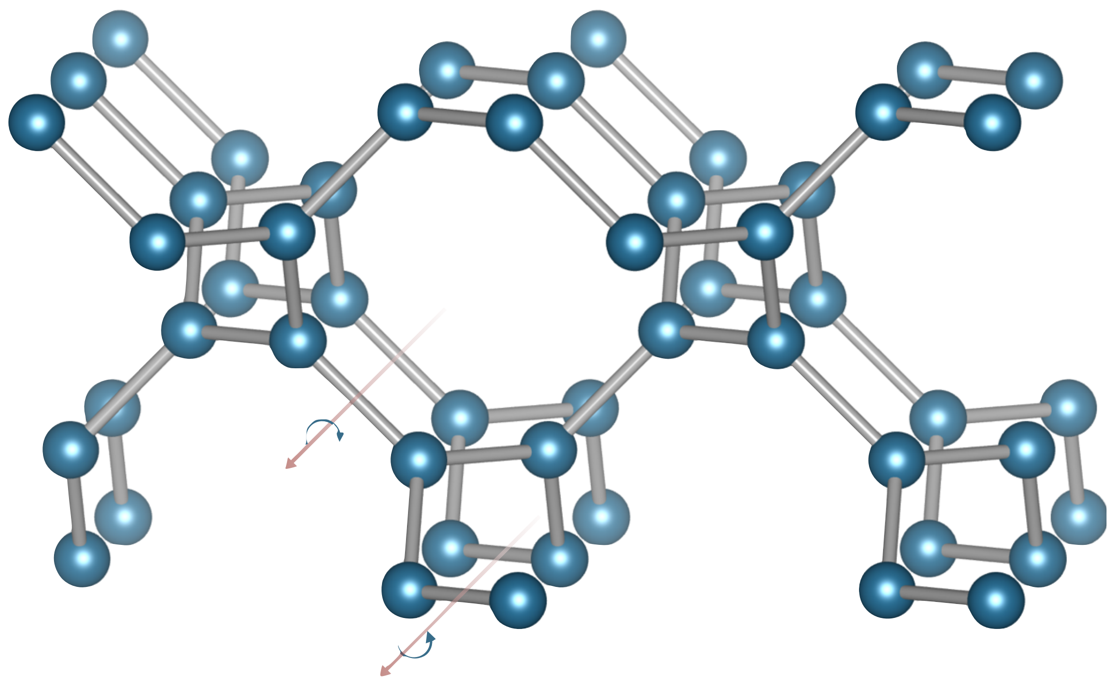
Inversion centers

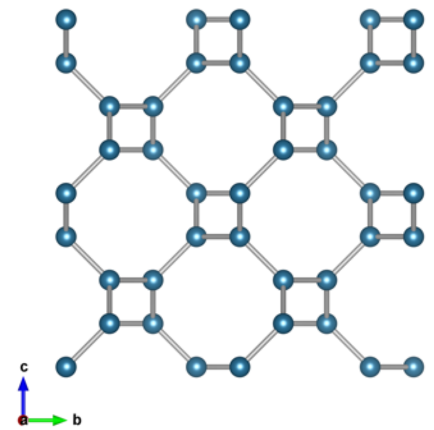
- tri-coordinated
- inversion symmetric
- preferred direction
- synthesized by - H. Takagi arxiv:1403.3296
  - J. Analytis arxiv:1402.3254

Hyperoctagon material δ-Li_2IrO_3 (?)

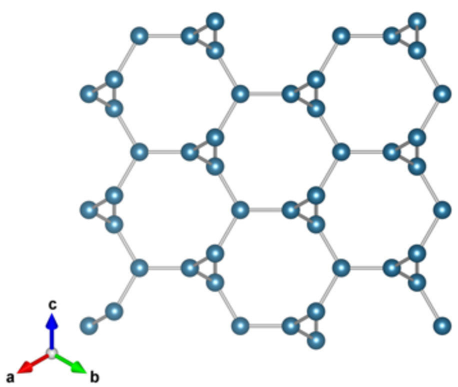


Hyperoctagon material δ-Li_2IrO_3 (?)

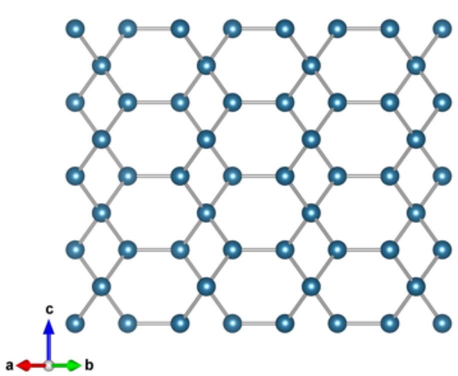




90° screw-rotation



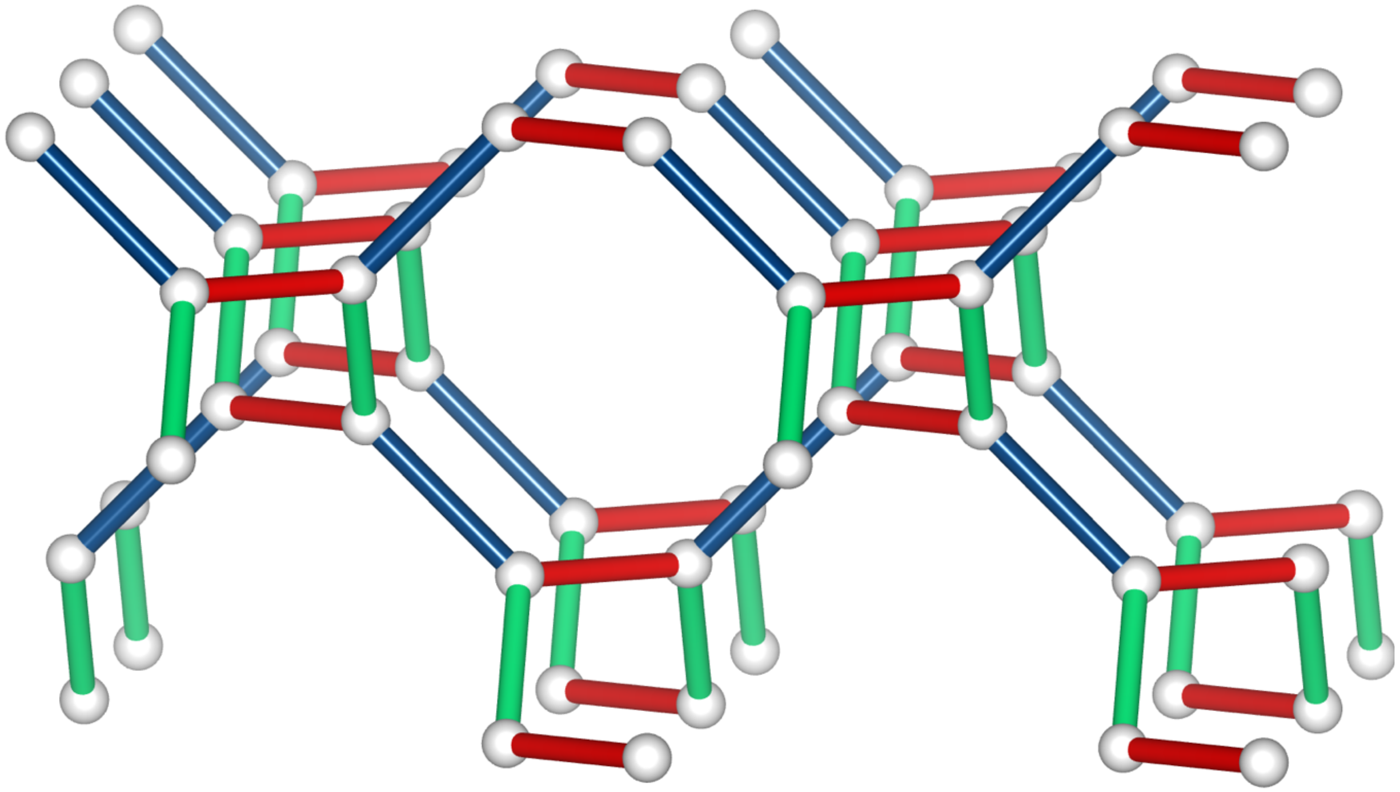
120° rotation



180° rotation

- tri-coordinated
- chiral
- cubic symmetry
- possible fourth crystalline form of Li_2IrO_3

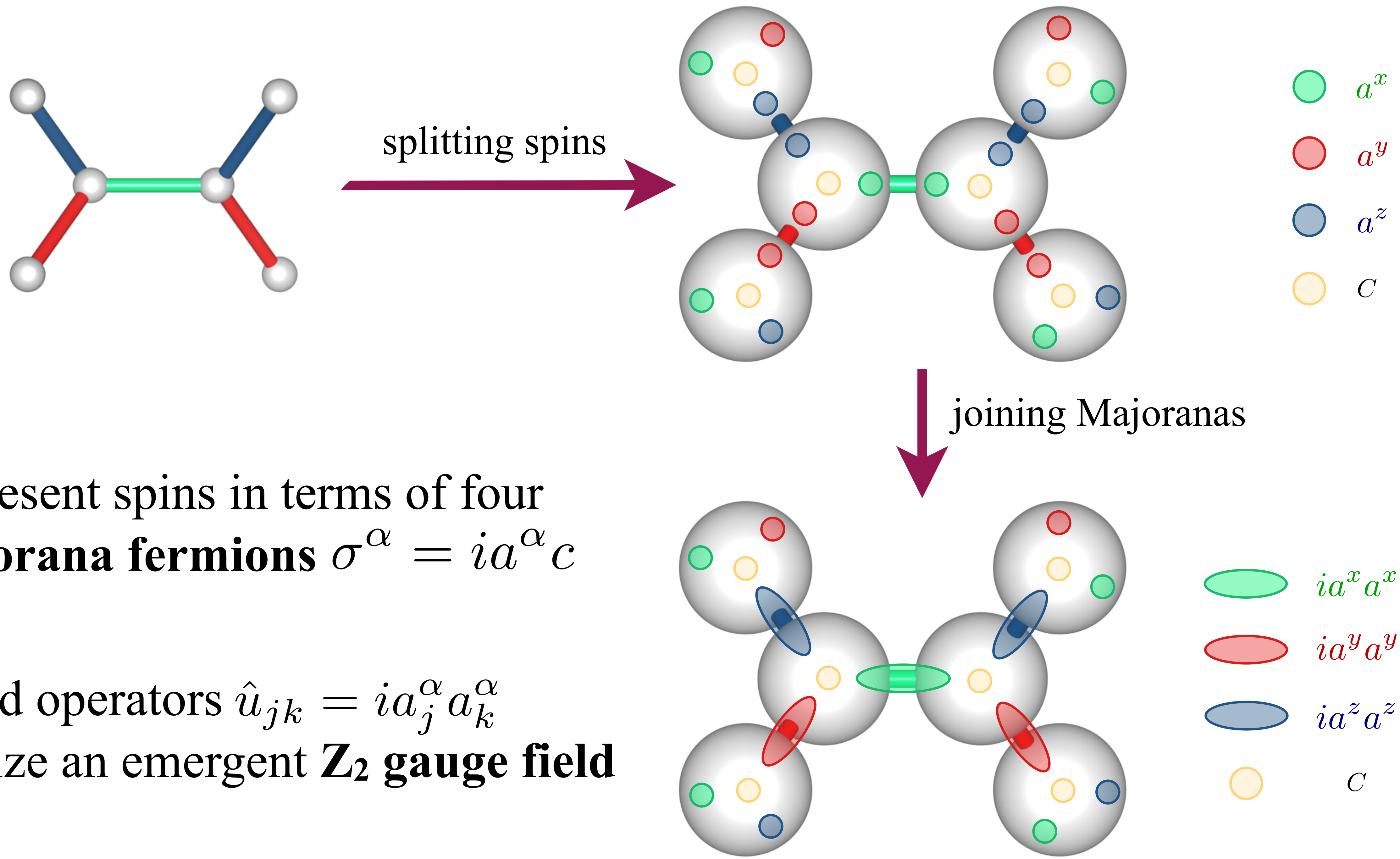
Exchange frustration on the hyperoctagon lattice



$$H = -\sum_{\text{x-bonds}} J_x \sigma_j^x \sigma_k^x - \sum_{\text{y-bonds}} J_y \sigma_j^y \sigma_k^y - \sum_{\text{z-bonds}} J_z \sigma_j^z \sigma_k^z$$

3D generalization of Kitaev's honeycomb model

Spin fractionalization and Majorana fermions



- Represent spins in terms of four **Majorana fermions** $\sigma^\alpha = ia^\alpha c$

- Bond operators $\hat{u}_{jk} = ia_j^\alpha a_k^\alpha$ realize an emergent $\mathbf{Z_2}$ **gauge field**

A. Kitaev, Annals of Physics **321**, 2 (2006)

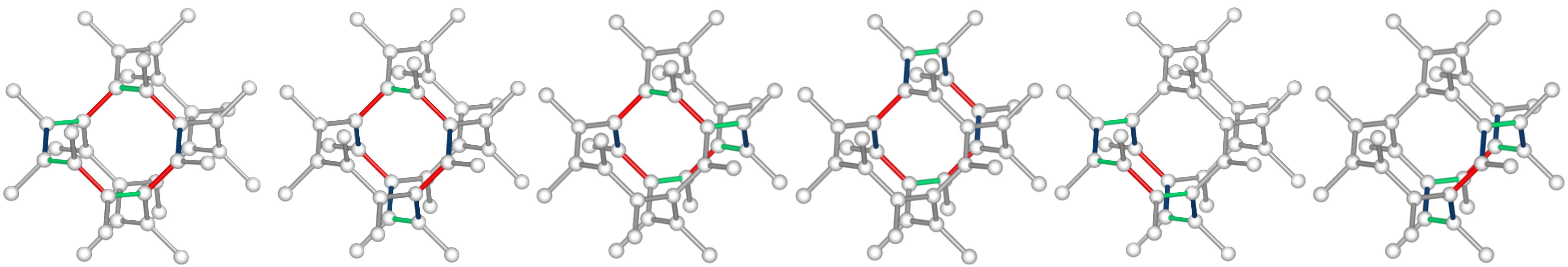
Physics of the Z_2 gauge field

Z_2 gauge field is **static** due to presence of additional conserved quantities

Six fundamental **loop operators** (per unit cell) $W_l = \prod_{\langle \alpha,\beta \rangle \in l} \sigma_\alpha^{\gamma_{\alpha\beta}} \sigma_\beta^{\gamma_{\alpha\beta}}$

↓

conserved quantities



loop operators define closed Z_2 flux loops – **no monopoles**

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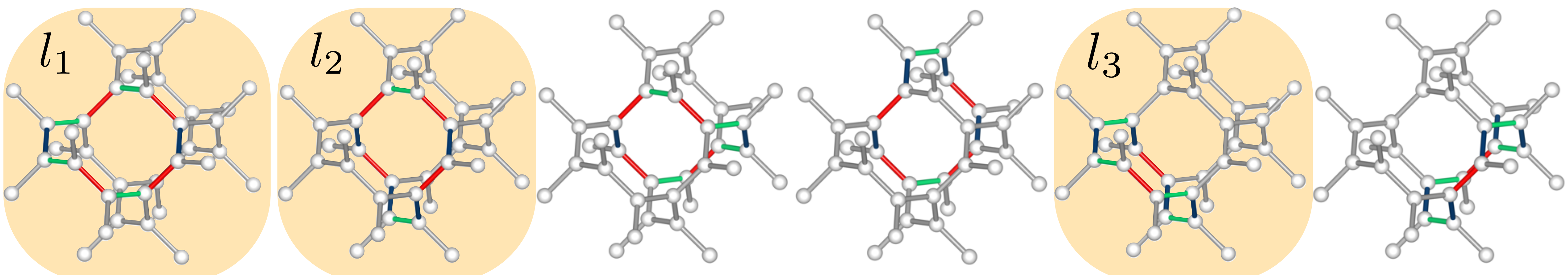
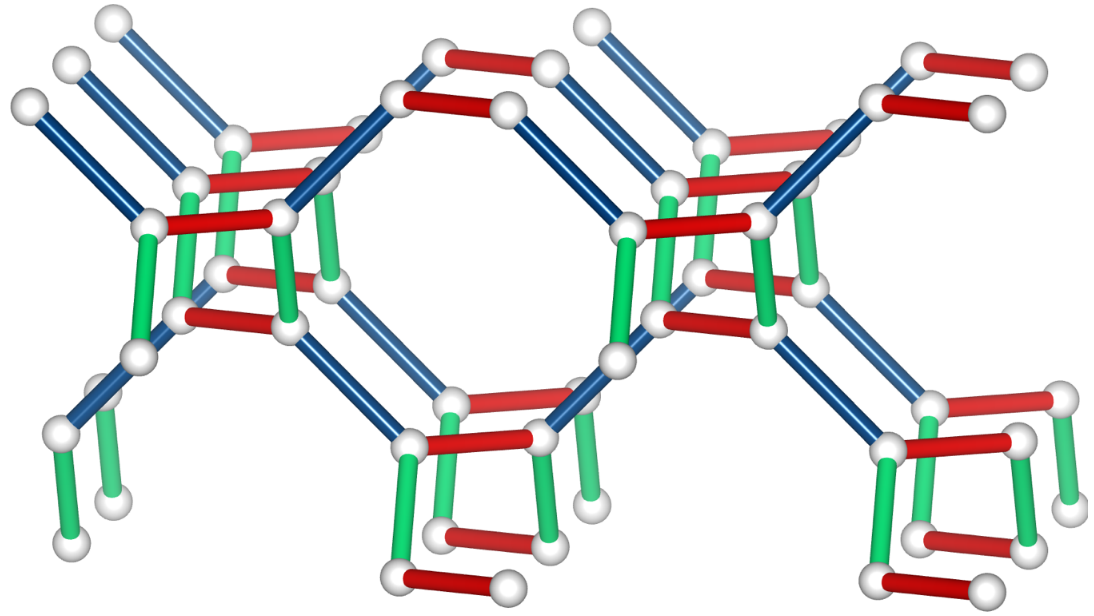


loop operators define closed Z_2 flux loops – **no monopoles**

only two loop operators per unit cell are linearly independent due to constraints

e.g. $W_{l_1} W_{l_2} W_{l_3} = 1$

Exchange frustration on the hyperoctagon model



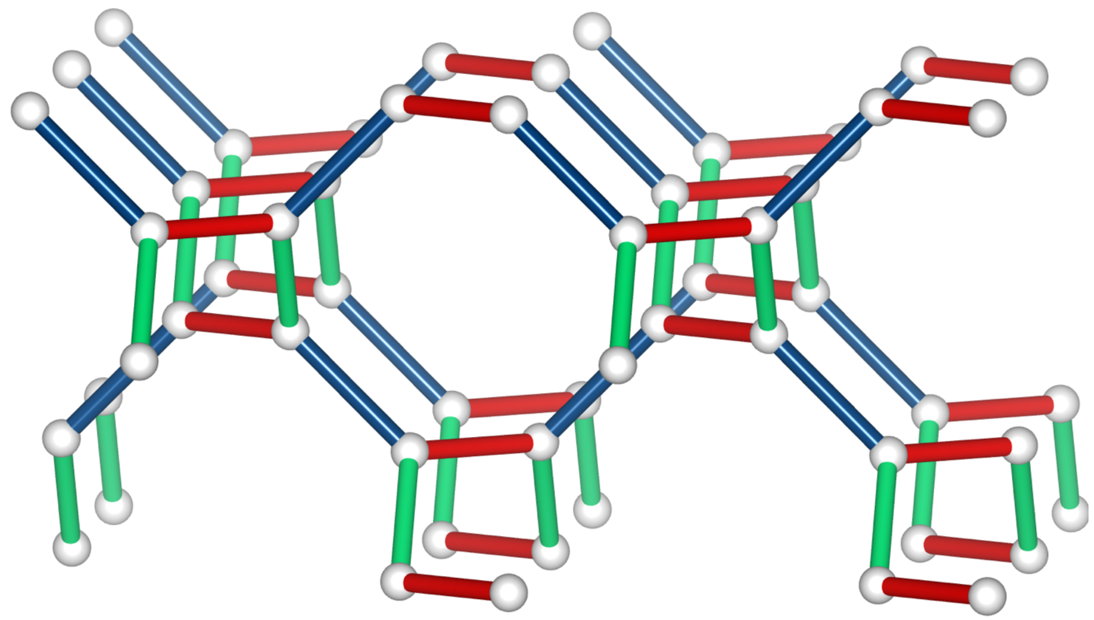
$$H = -\sum_{\text{x-bonds}} J_x \sigma_j^x \sigma_k^x - \sum_{\text{y-bonds}} J_y \sigma_j^y \sigma_k^y - \sum_{\text{z-bonds}} J_z \sigma_j^z \sigma_k^z$$

Hilbert space split into two separate sectors: $2^N = 2^{N/2} \times 2^{N/2}$

Majorana fermions c_j
"**spinons**"

flux loops "**visons**"
(static and gapped)

Exchange frustration on the hyperoctagon model



$$H = i \sum_{\gamma-\mathrm{bond}} J_\gamma c_j c_k$$

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Majorana fermions c_j
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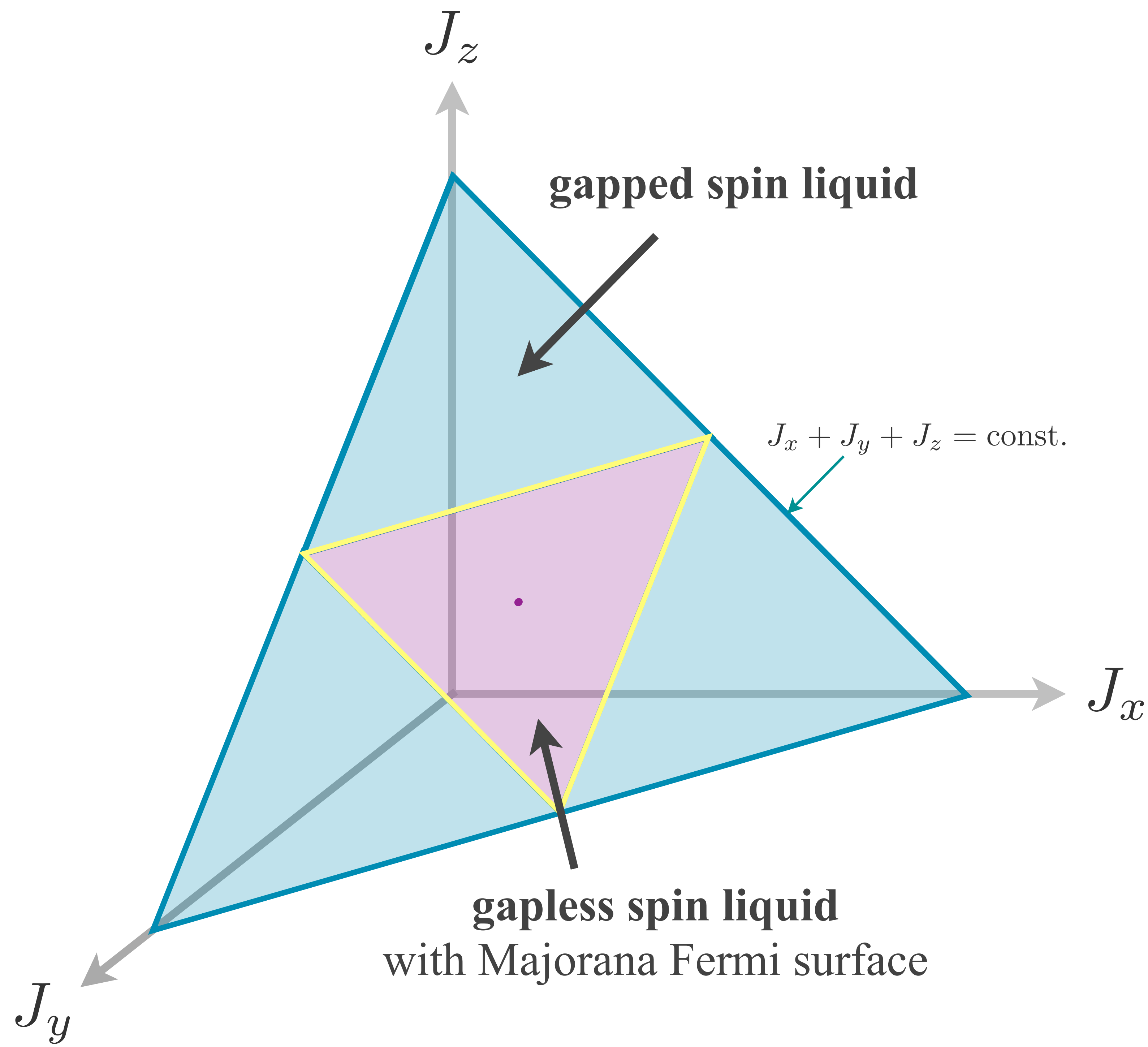
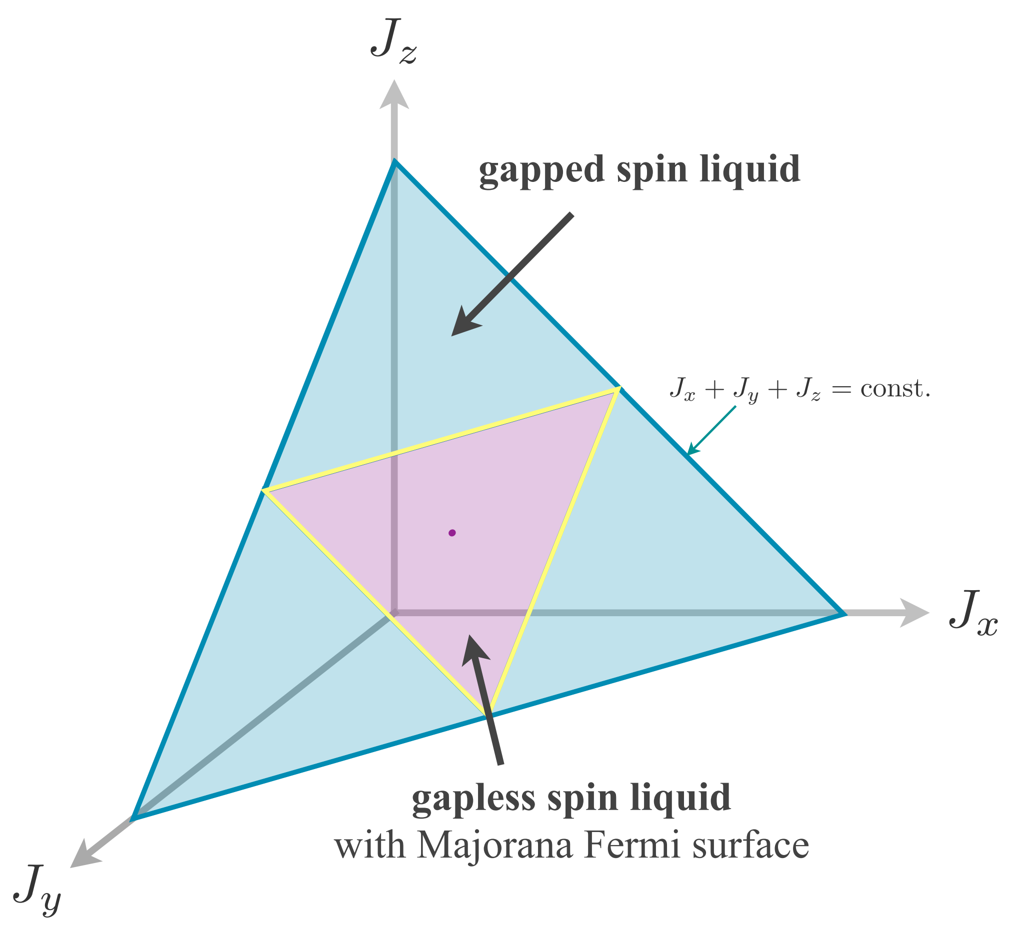
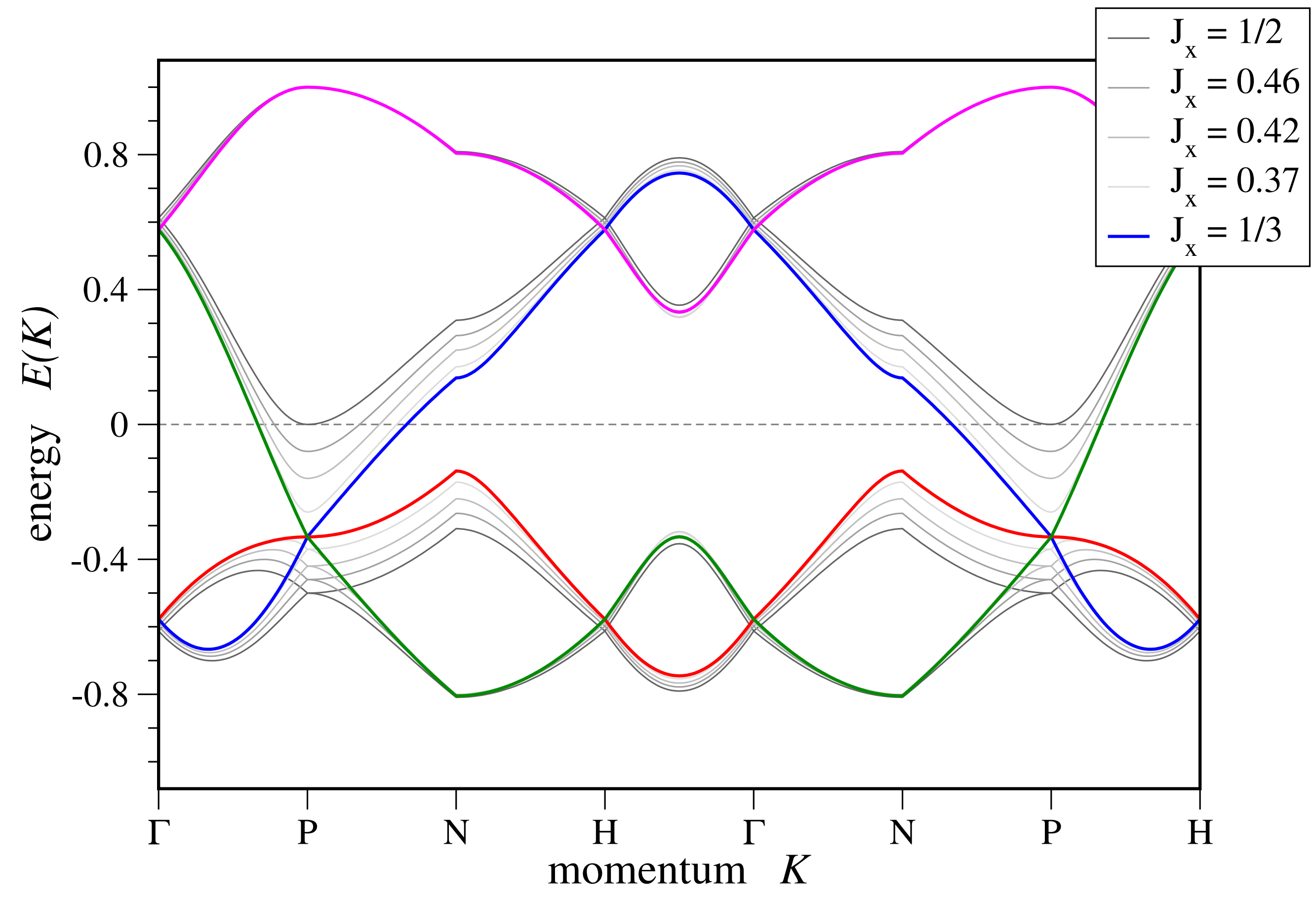
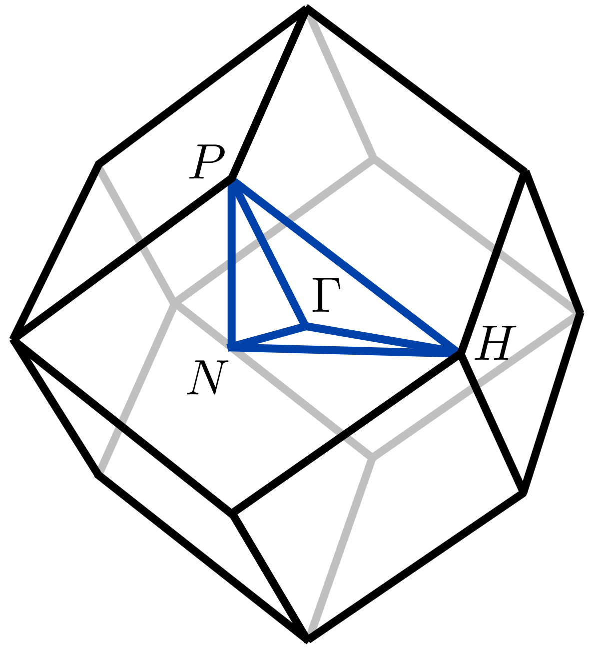
The phase diagram



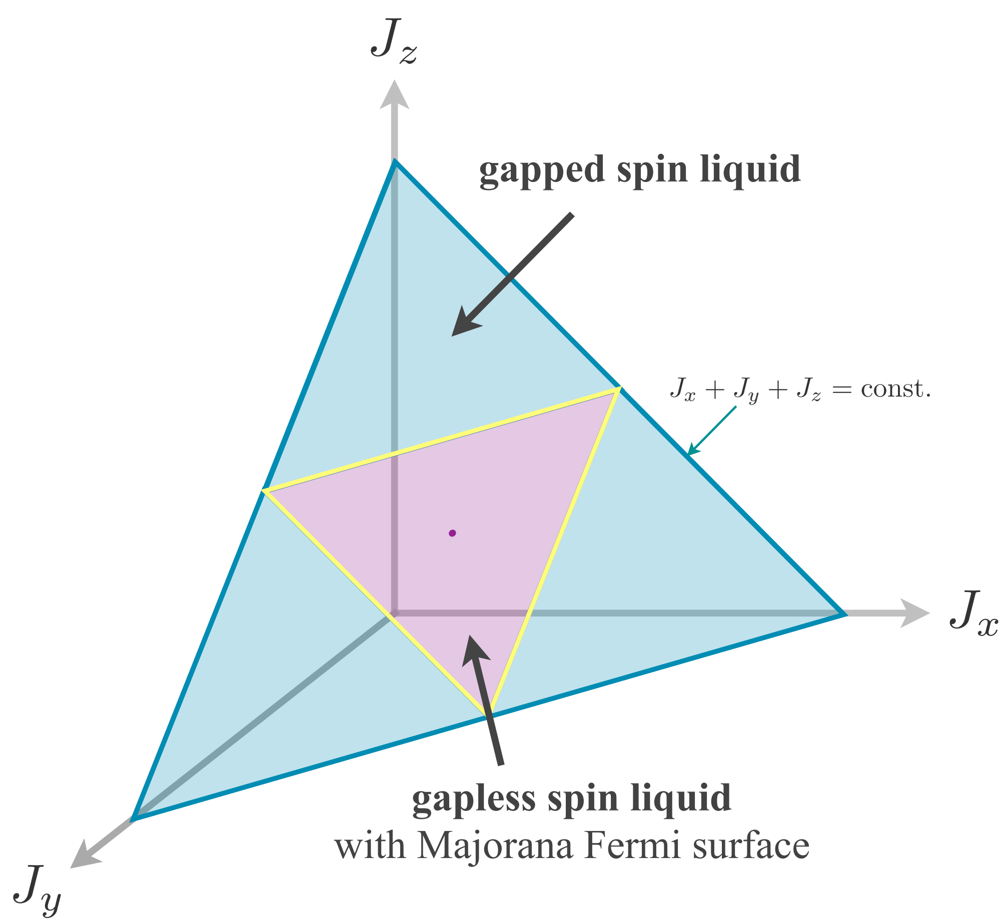
Majorana Fermi surface

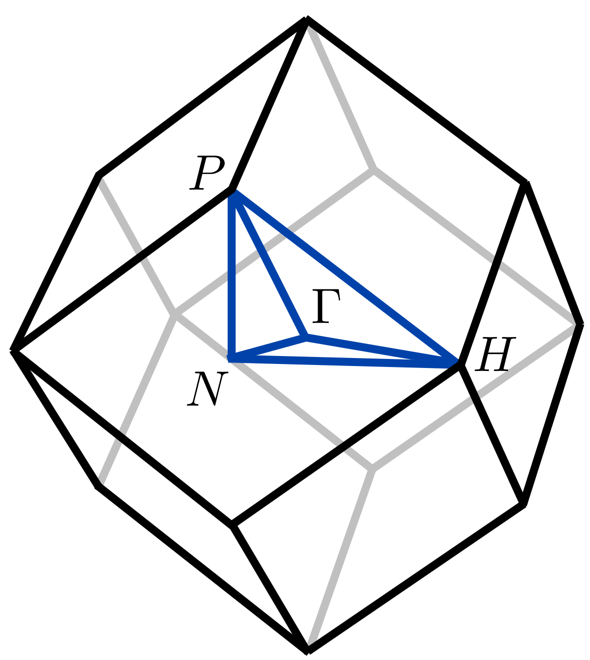


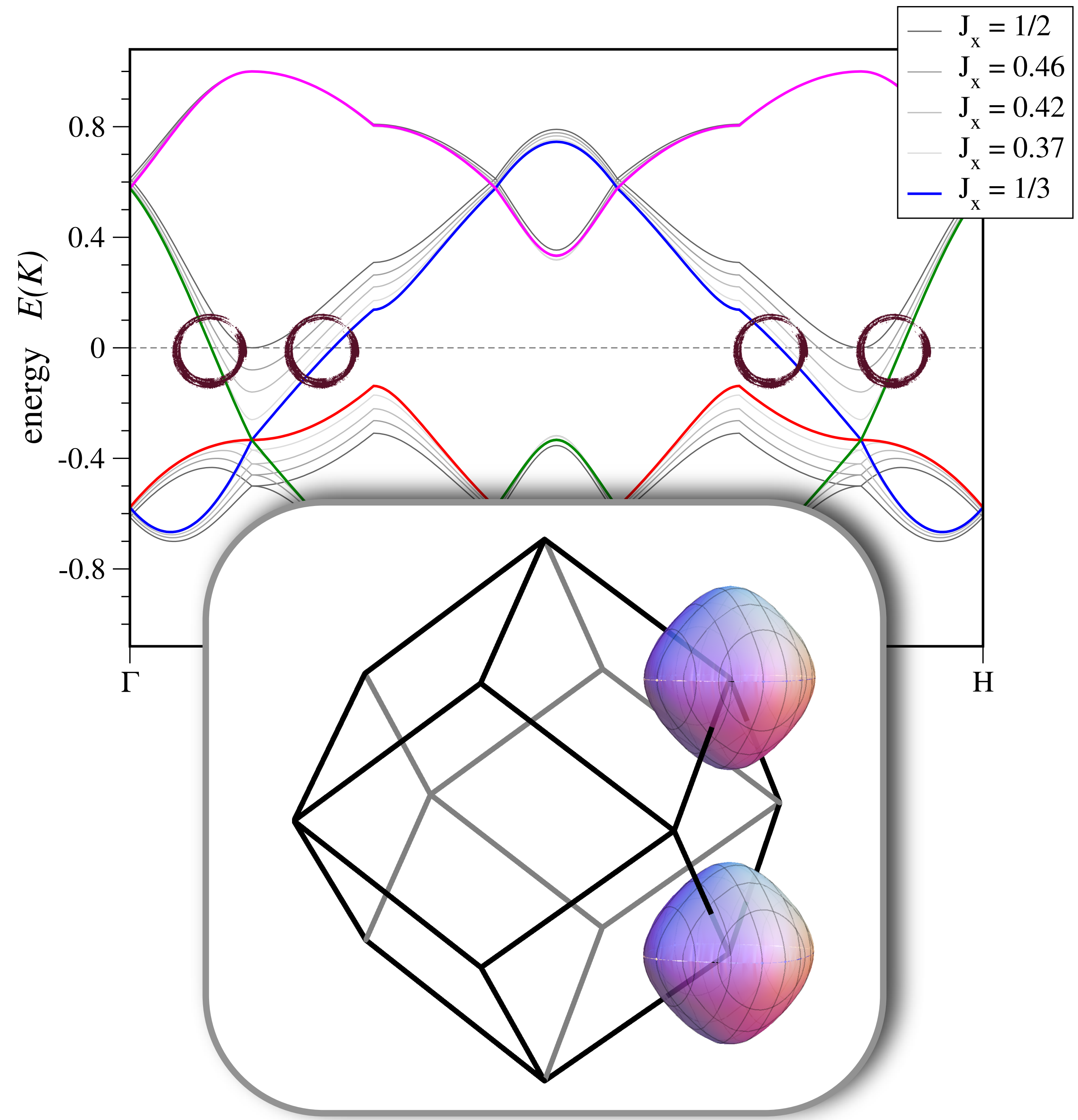




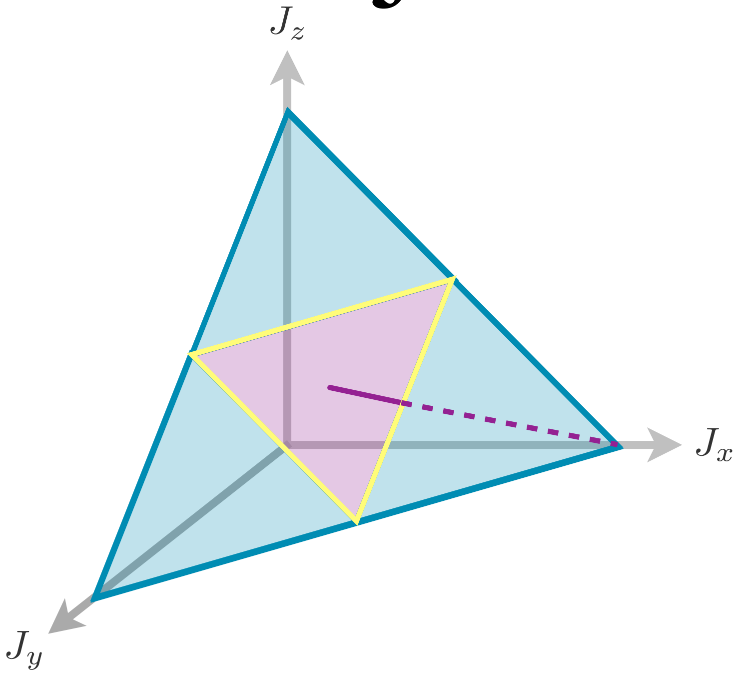
Majorana Fermi surface

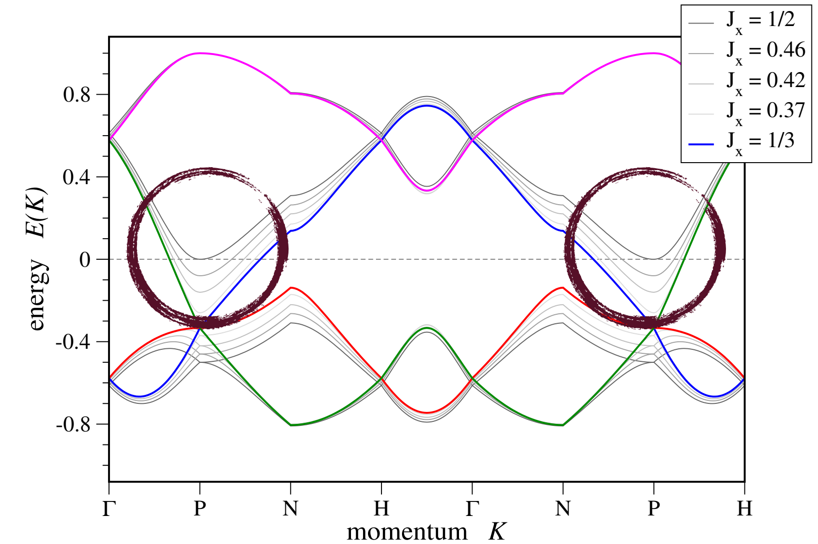


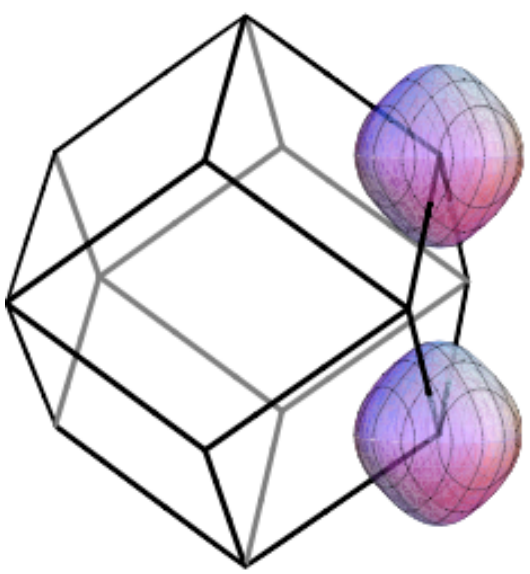




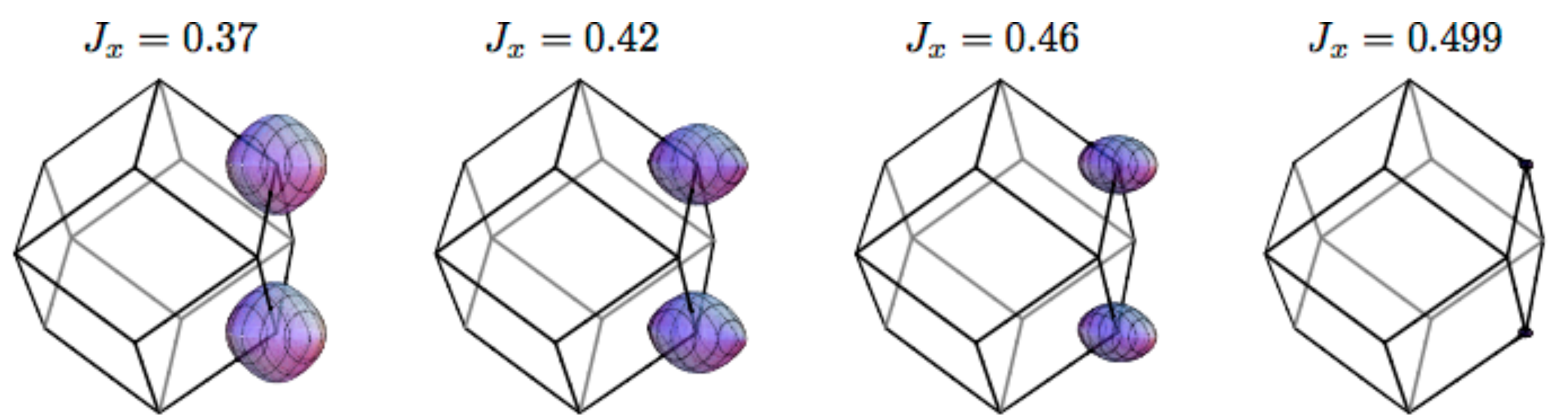
Majorana Fermi surface in gapless region







$J_x = 1/3$



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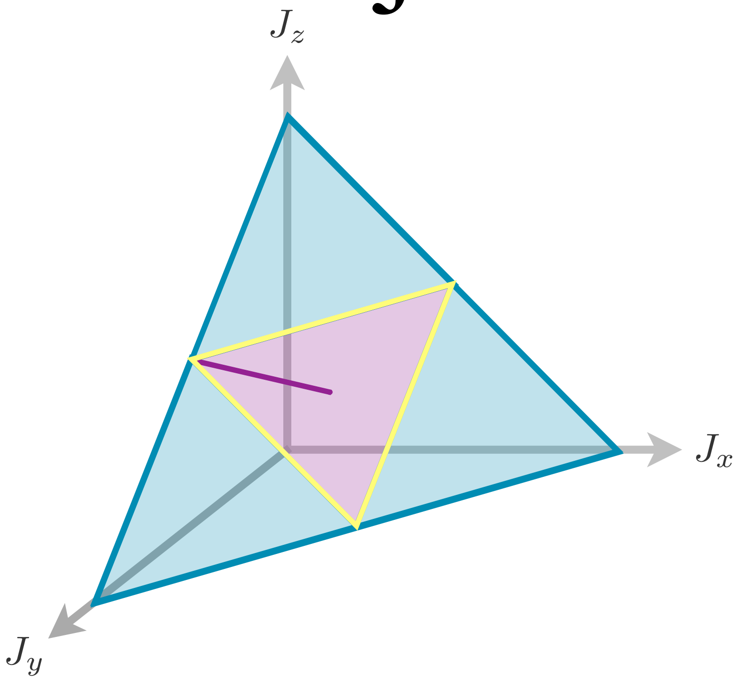
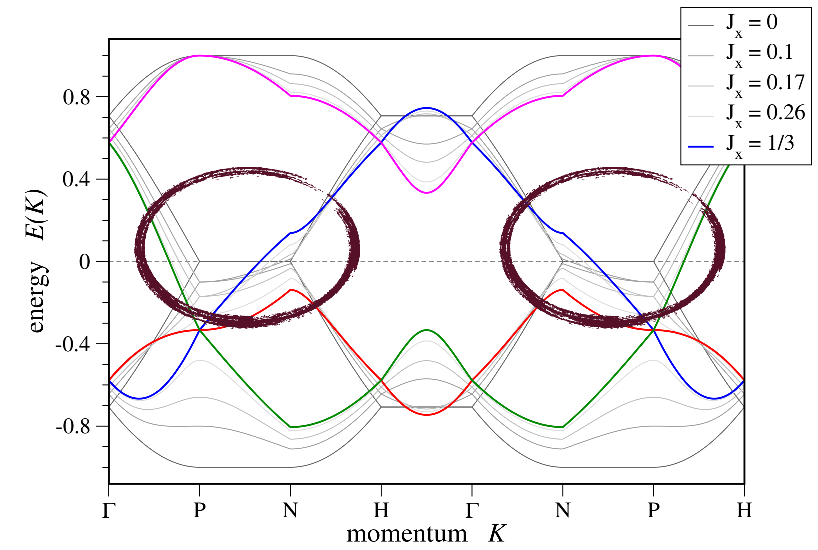
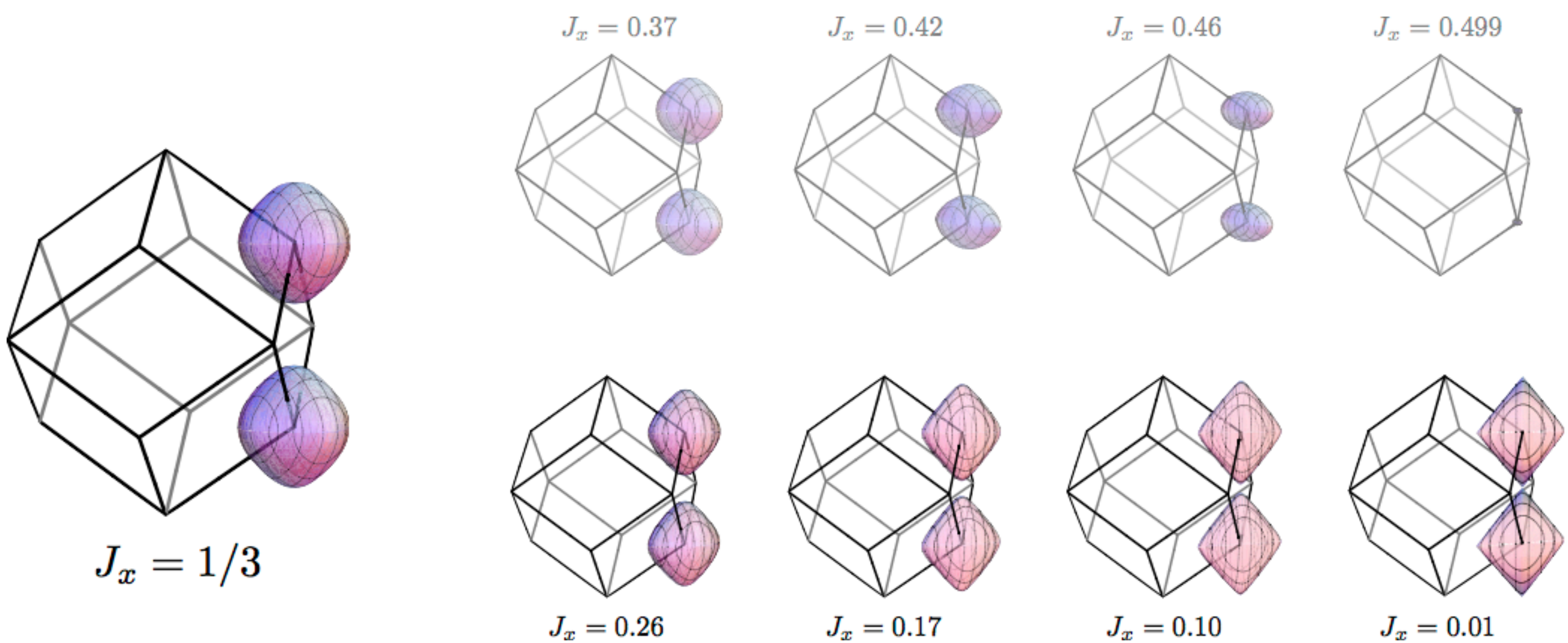
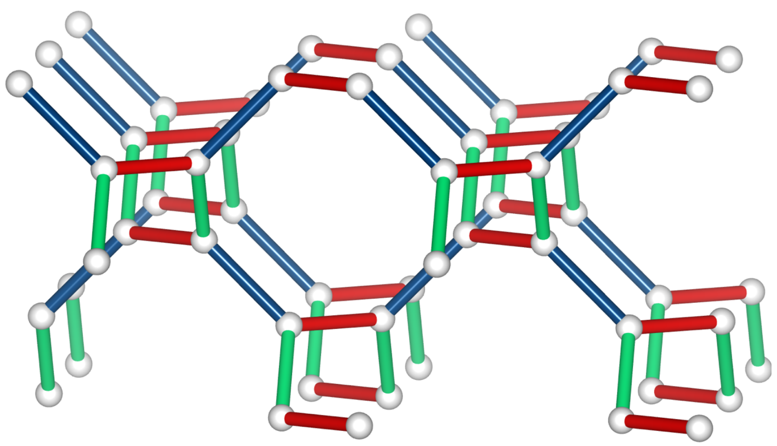
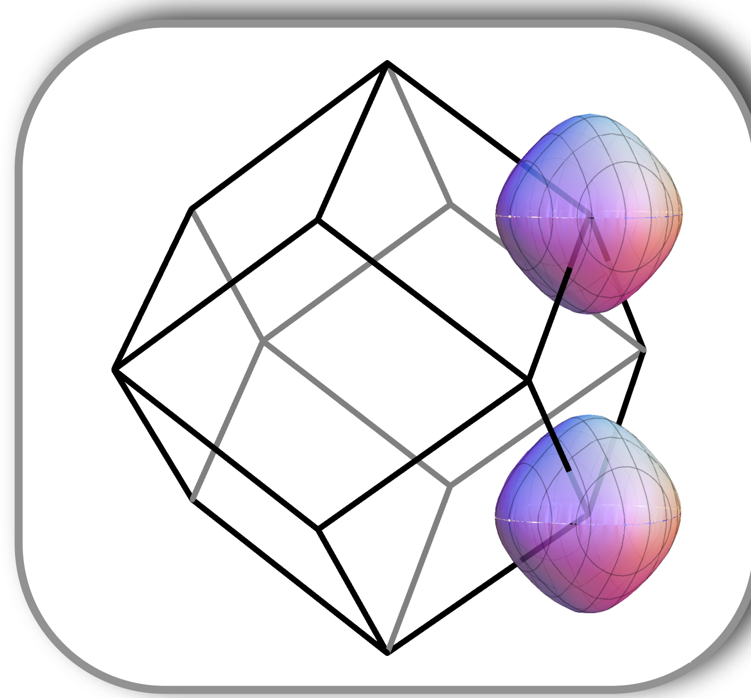


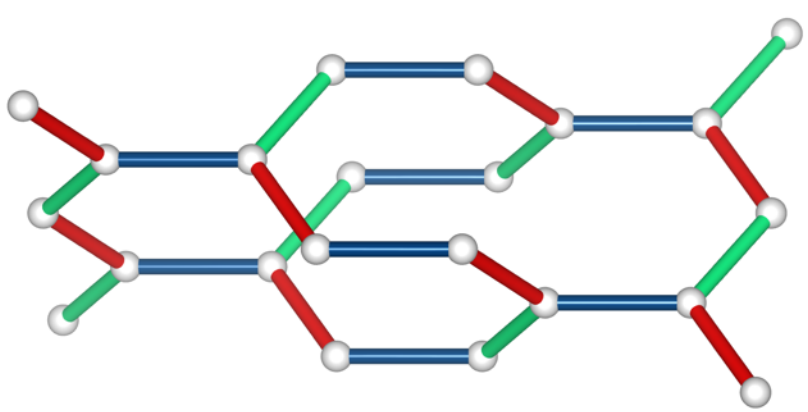


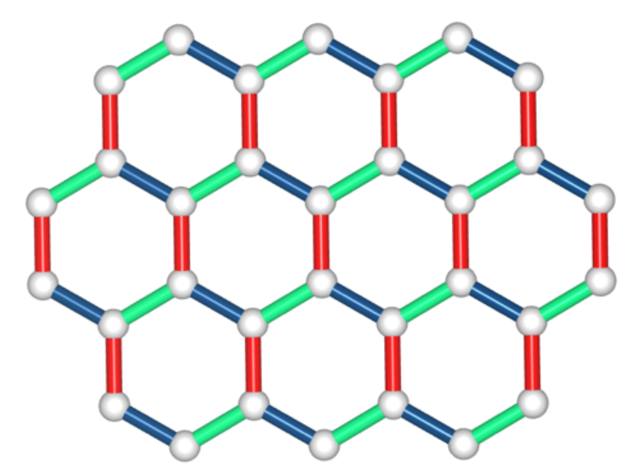


Zoo of gapless spin liquids





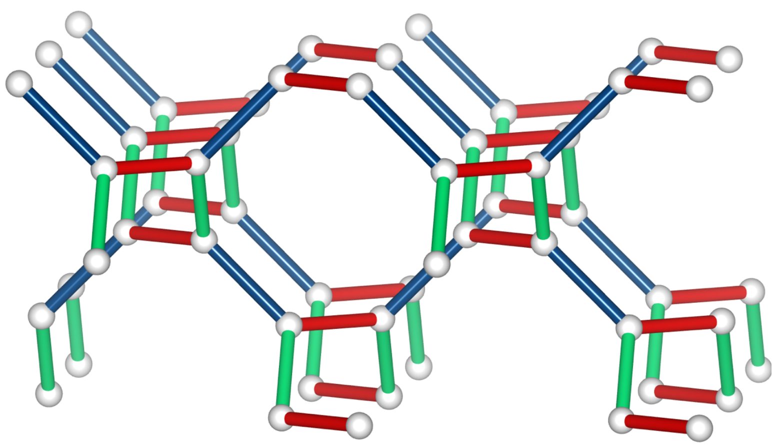


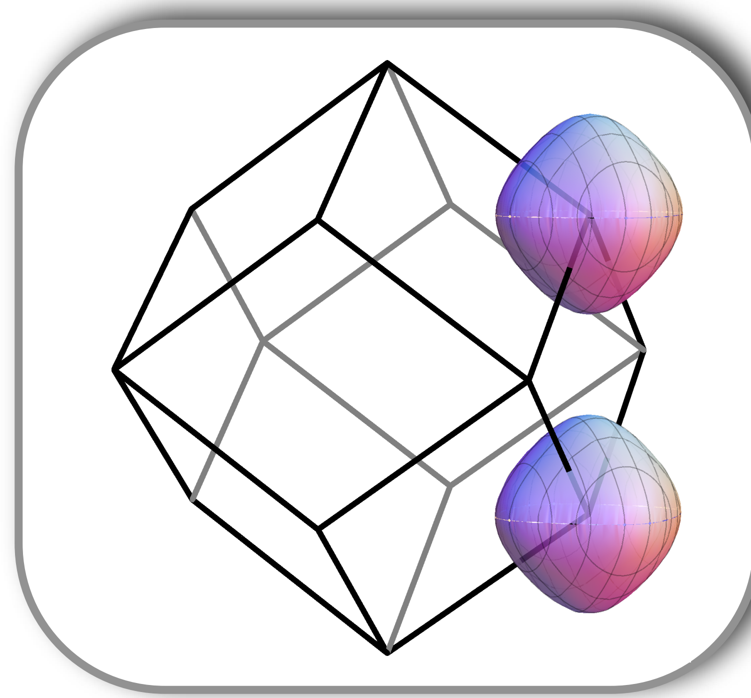


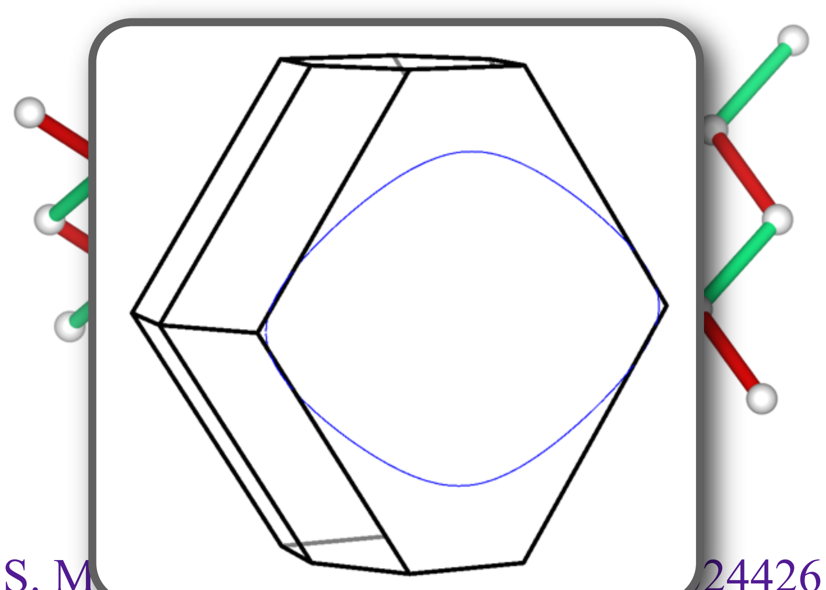
S. Mandal, N. Surendran, PRB 79, 024426 (2009)
E. K.-H. Lee etal., PRB 89, 045117 (2014)
I. Kimchi, J.G. Analytis, A. Vishwanath, arXiv:1309.1171

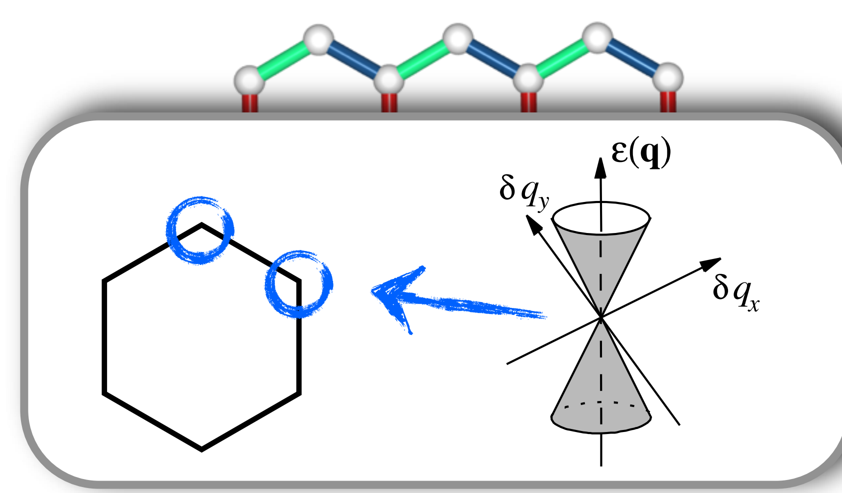
A. Kitaev, Annals of Physics **321**, 2 (2006)

Zoo of gapless spin liquids









S. M 24426 (2009)
E. K.-H. Lee etal., PRB 89, 045117 (2014)
I. Kimchi, J.G. Analytis, A. Vishwanath, arXiv:1309.1171

A. Kitaev, Annals of Physics **321**, 2 (2006)

Thermodynamics

static spin correlation functions $\langle \sigma_i^\alpha(\vec{r})\sigma_j^\beta(0)\rangle$ decay exponentially

Bond-energy correlation functions $\langle \mathcal{B}_\gamma(0)\mathcal{B}_\gamma(\vec{r})\rangle - \langle \mathcal{B}_\gamma(0)\rangle\langle \mathcal{B}_\gamma(\vec{r})\rangle$

exhibit algebraic divergence at spinon Fermi surface ($\mathcal{B}_\gamma = \sigma_i^\gamma \sigma_j^\gamma$)

Different 3D gapless spin liquids can be **experimentally distinguished** by measuring the **specific heat coefficient** C(T)/T

Z_2 spin liquid with spinon Fermi line (hyperhoneycomb)	$C(T) \sim T^2$	$C(T)/T \to 0$
Z_2 spin liquid with spinon Fermi surface (hyperoctagon)	$C(T) \sim T$	$C(T)/T \to const.$
U(1) spin liquid with spinon Fermi surface (slave-fermion + MF)	$C(T) \sim T \ln T$	$C(T)/T \to \infty$

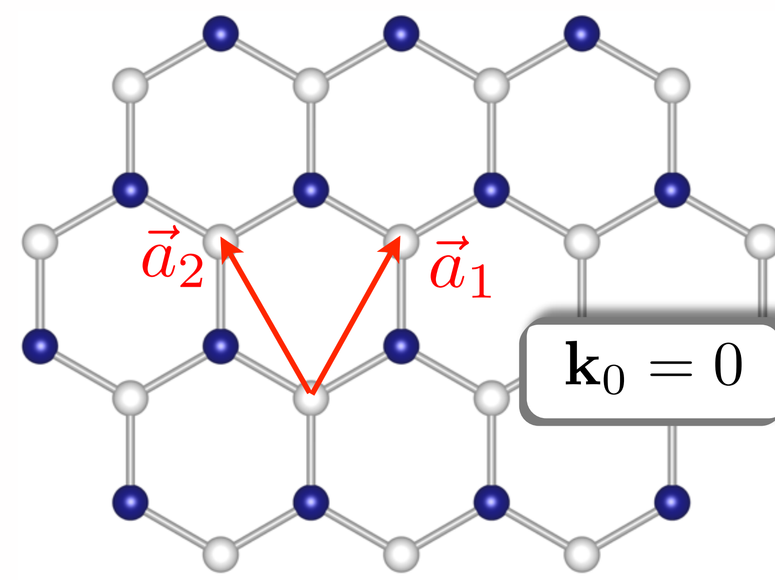
Majorana metal!

Symmetries in Majorana Systems

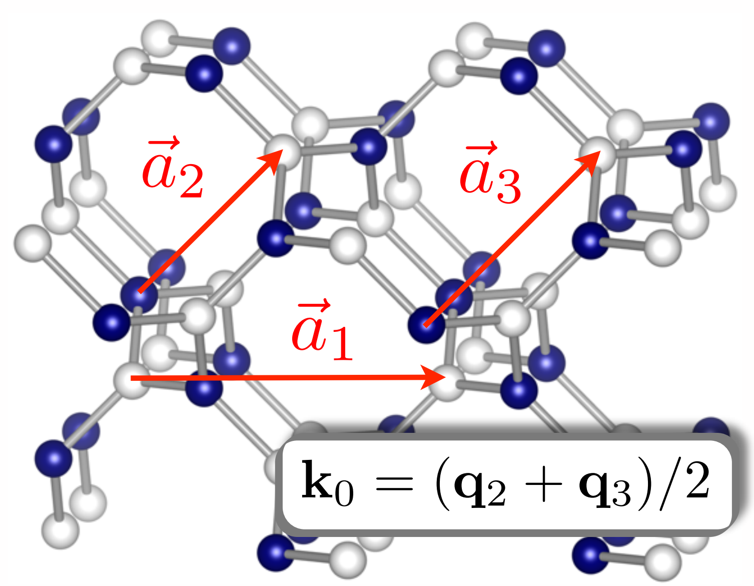
Particle-hole symmetry	$\epsilon(\mathbf{k}) = -\epsilon(-\mathbf{k})$
Sublattice symmetry	$\epsilon(\mathbf{k}) = -\epsilon(\mathbf{k} - \mathbf{k}_0)$
Time-reversal symmetry	$\epsilon(\mathbf{k}) = \epsilon(-\mathbf{k} + \mathbf{k}_0)$
Inversion symmetry	$\epsilon(\mathbf{k}) = \epsilon(-\mathbf{k} + \mathbf{k}_0)$

$\mathbf{k}_0$ is the reciprocal lattice vector of the translation vector of the sublattice

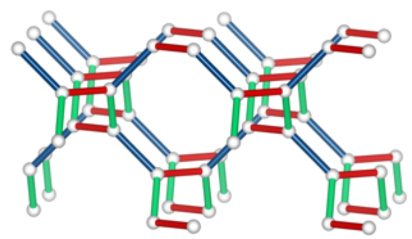
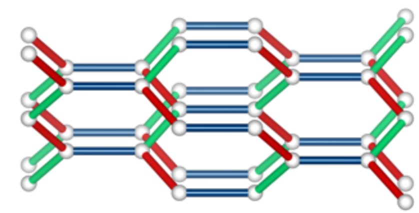
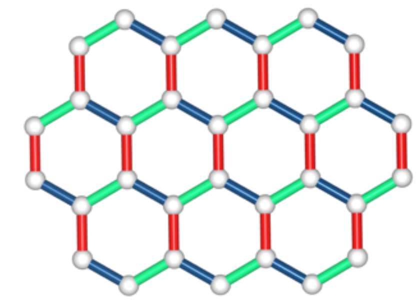
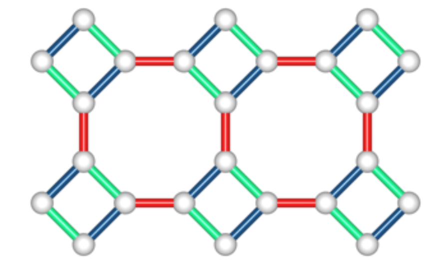
honeycomb lattice



hyperoctagon lattice



Symmetry classification

				
Particle-hole	$\epsilon(\mathbf{k}) = -\epsilon(-\mathbf{k})$	$\epsilon(\mathbf{k}) = -\epsilon(-\mathbf{k})$	$\epsilon(\mathbf{k}) = -\epsilon(-\mathbf{k})$	$\epsilon(\mathbf{k}) = -\epsilon(-\mathbf{k})$
Sublattice (=PH+TR)	$\epsilon(\mathbf{k}) = -\epsilon(\mathbf{k} - \mathbf{k}_0)$	$\epsilon(\mathbf{k}) = -\epsilon(\mathbf{k})$	$\epsilon(\mathbf{k}) = -\epsilon(\mathbf{k})$	$\epsilon(\mathbf{k}) = -\epsilon(\mathbf{k} - \mathbf{k}_0)$
Time-reversal	$\epsilon(\mathbf{k}) = \epsilon(-\mathbf{k} - \mathbf{k}_0)$	$\epsilon(\mathbf{k}) = \epsilon(-\mathbf{k})$	$\epsilon(\mathbf{k}) = \epsilon(-\mathbf{k})$	$\epsilon(\mathbf{k}) = \epsilon(-\mathbf{k} - \mathbf{k}_0)$
Inversion		$\epsilon(\mathbf{k}) = -\epsilon(-\mathbf{k})$	$\epsilon(\mathbf{k}) = -\epsilon(-\mathbf{k})$	$\epsilon(\mathbf{k}) = \epsilon(-\mathbf{k} - \mathbf{k}_0)$

$\mathbf{k}_0$ is the reciprocal lattice vector of the translation vector of the sublattice

Symmetries $\leftrightarrow$ Fermi surface

Particle-hole symmetry	$\epsilon(\mathbf{k}) = -\epsilon(-\mathbf{k})$
Time-reversal symmetry	$\epsilon(\mathbf{k}) = \epsilon(-\mathbf{k} - \mathbf{k}_0)$
Inversion symmetry	$\epsilon(\mathbf{k}) = \epsilon(-\mathbf{k} - \mathbf{k}_0)$

$\mathbf{k}_0 = 0$: honeycomb / hyperhoneycomb

particle-hole symmetry at every **k** point
doubly degenerate zero-modes at same **k**

$$h(\mathbf{k}) = \begin{pmatrix} \mathbf{0} & \mathbf{A} \\ \mathbf{A}^\dagger & \mathbf{0} \end{pmatrix}$$

protected by time-reversal

stable zero-mode manifolds are **separated points** (2D) and **lines** (3D)

$\mathbf{k}_0 \neq 0$: hyperoctagon

generic band hamiltonian at given **k**
zero-modes are at different **k**

$$h(\mathbf{k}) = \begin{pmatrix} 0 & & \mathbf{A} \\ & \ddots & \\ \mathbf{A}^\dagger & & 0 \end{pmatrix}$$

stable zero-mode manifolds are **lines** (2D) and **surfaces** (3D)

P-wave pairing instability on the hyperoctagon lattice

Center of Majorana Fermi surfaces are **high-symmetry** points defined by $\mathbf{k}_0/2$

$$\mathbf{K}_{1/2} = (\pi, \pi, \pm\pi)$$

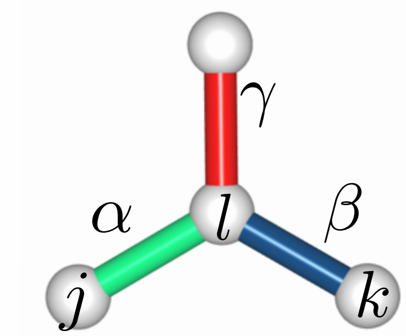
Time-reversal symmetry: $\epsilon(\mathbf{q} + \mathbf{K}_j) = \epsilon(-\mathbf{q} + \mathbf{K}_j)$

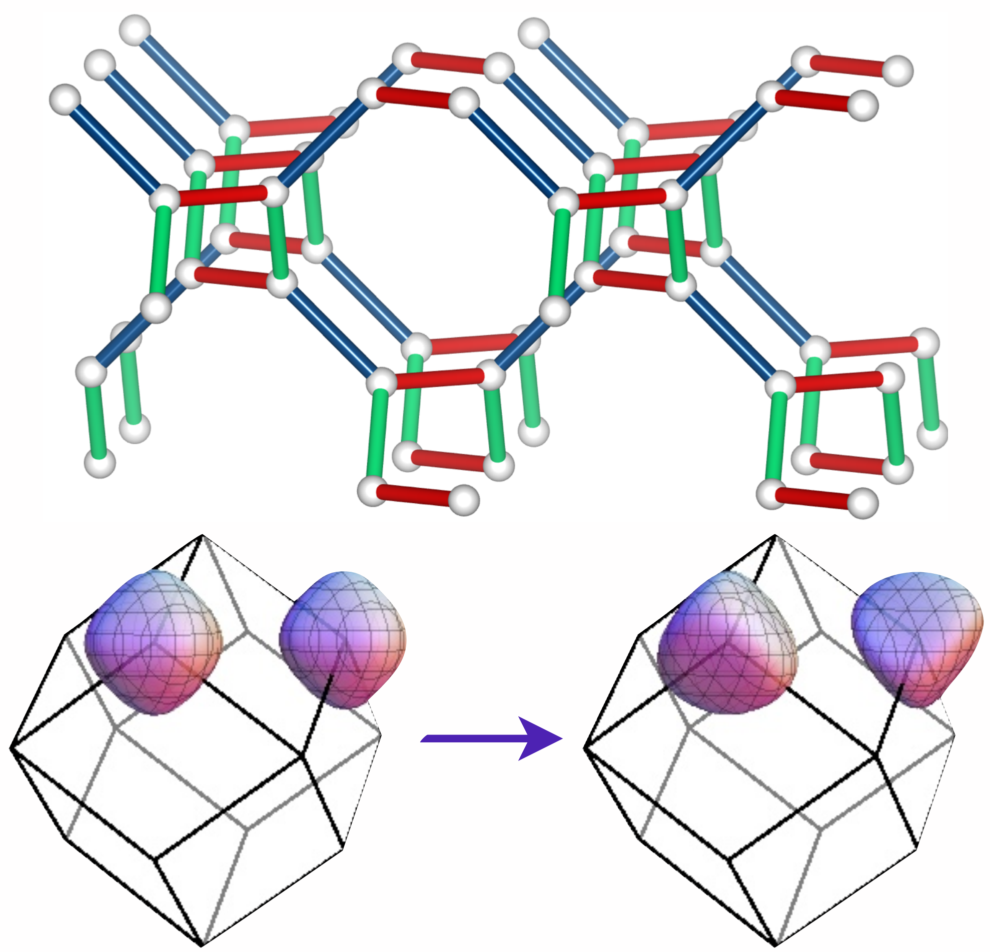
p-wave pairing instability in effective spinless Fermion model
(broken translation symmetry)
relevant for additional nearest-neighbor Heisenberg coupling

Breaking time-reversal symmetry

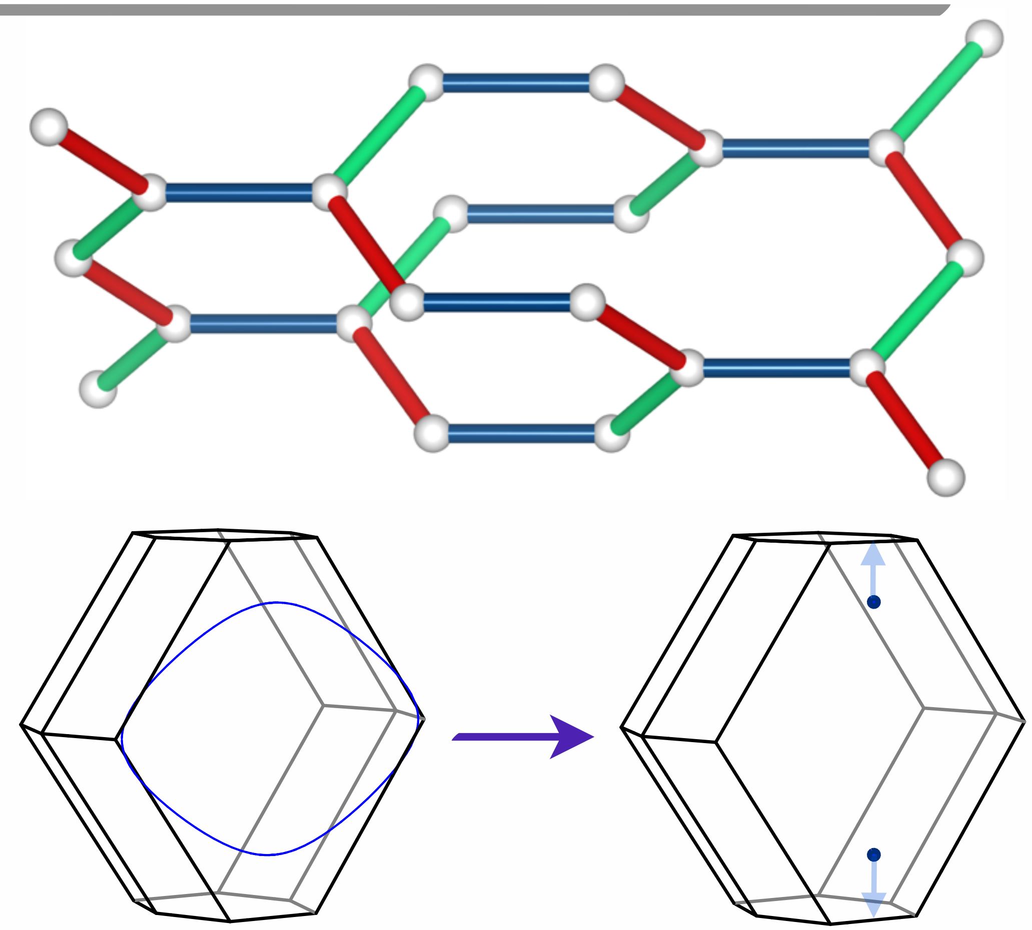
external **magnetic field** in (1,1,1)-direction

$$H_{eff} = -J \sum_{\gamma-bond} \sigma_j^\gamma \sigma_k^\gamma - \kappa \sum_{\langle j,l,k \rangle} \sigma_j^\alpha \sigma_k^\beta \sigma_l^\gamma$$





Breaking TR **stabilizes Fermi surface**



Breaking TR reduces line to **pair of Weyl nodes**

Weyl nodes and Weyl semi-metals

Touching of two bands in 3D is generically **linear**

$$\hat{H} = \vec{v}_0 \cdot \vec{q}\,\mathbb{1} + \sum_{i=1}^{3} \vec{v}_j \cdot \vec{q}\,\sigma_j$$ **Weyl nodes**

Weyl nodes are **sources/sinks of Berry flux**

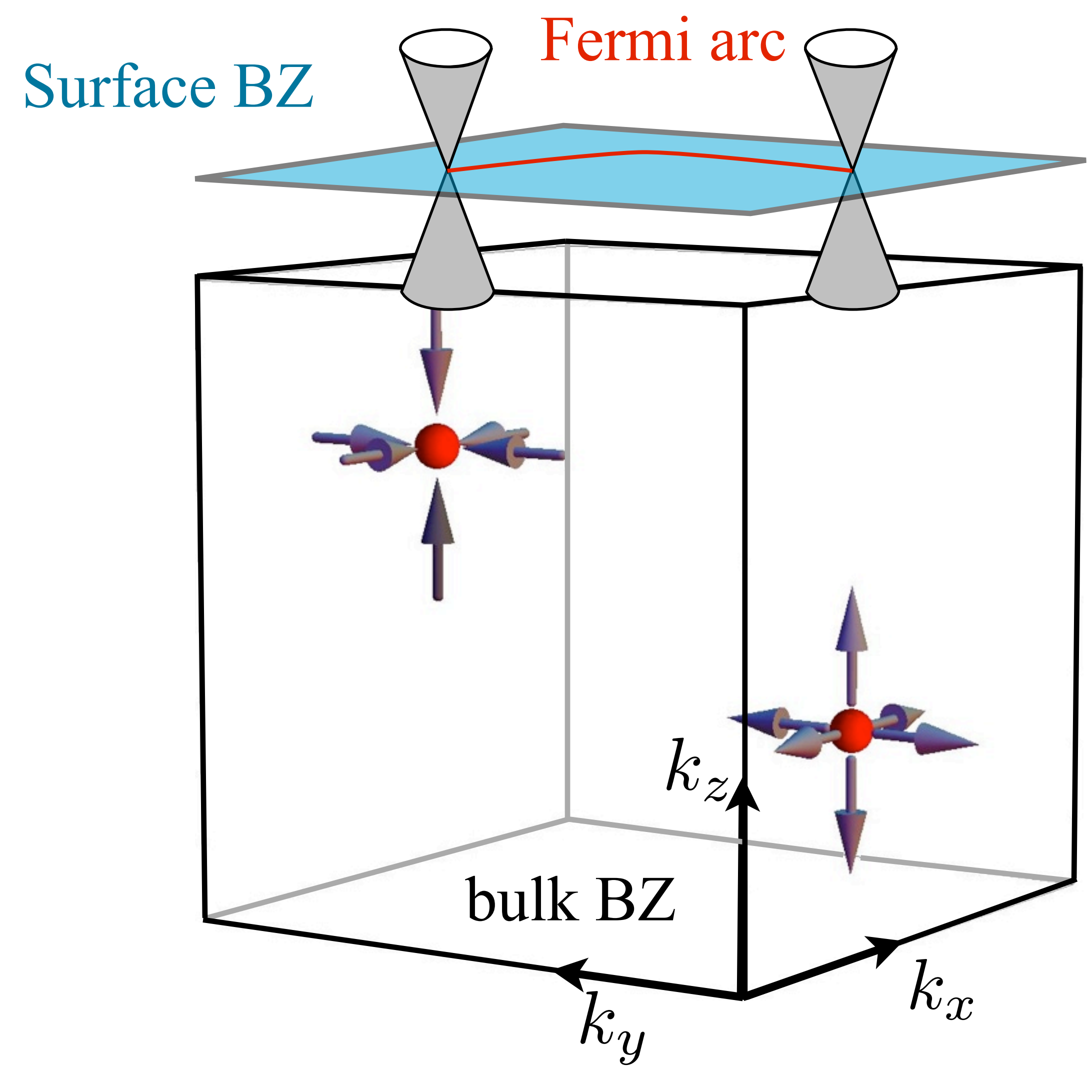
$$\vec{B}_n(\vec{k}) = \nabla_{\vec{k}} \times \left(i \langle n(\vec{k}) | \nabla_{\vec{k}} | n(\vec{k}) \rangle \right)$$

with chirality $\text{sign}[\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3)]$

here: Weyl nodes pinned at zero energy!

unusal surface states: **Fermi arcs**

topological semi-metal with **protected** surface states
(metallic cousin of the topological insulator)

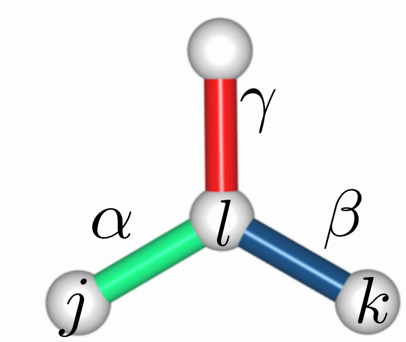


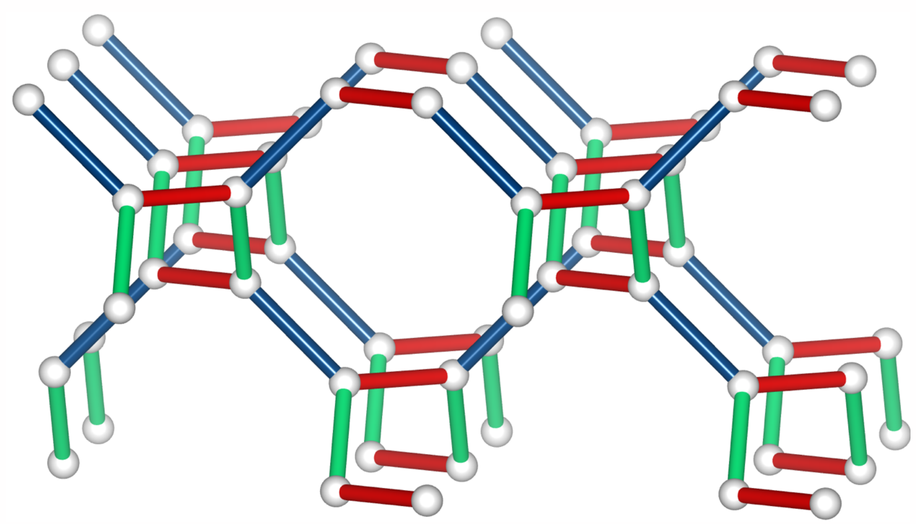
Wan etal., PRB 83, 205101 (2011)

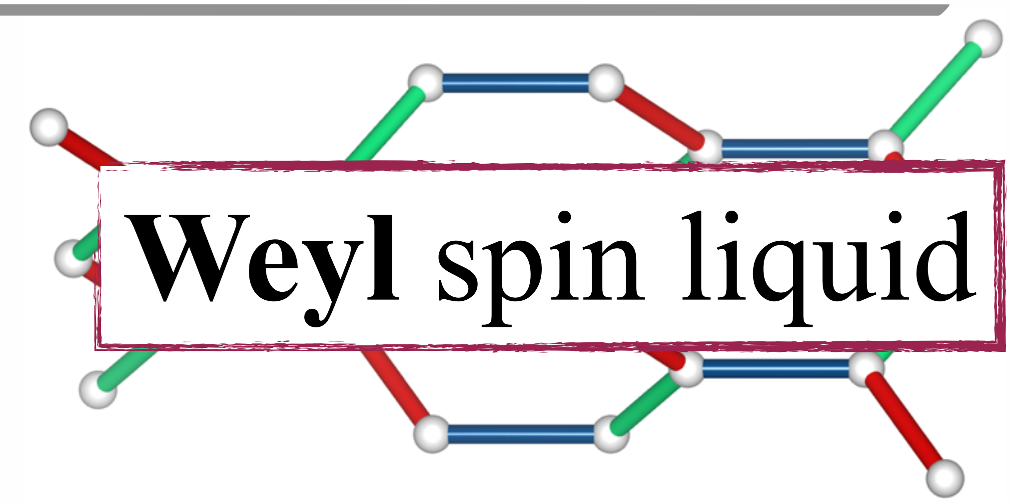
Breaking time-reversal symmetry

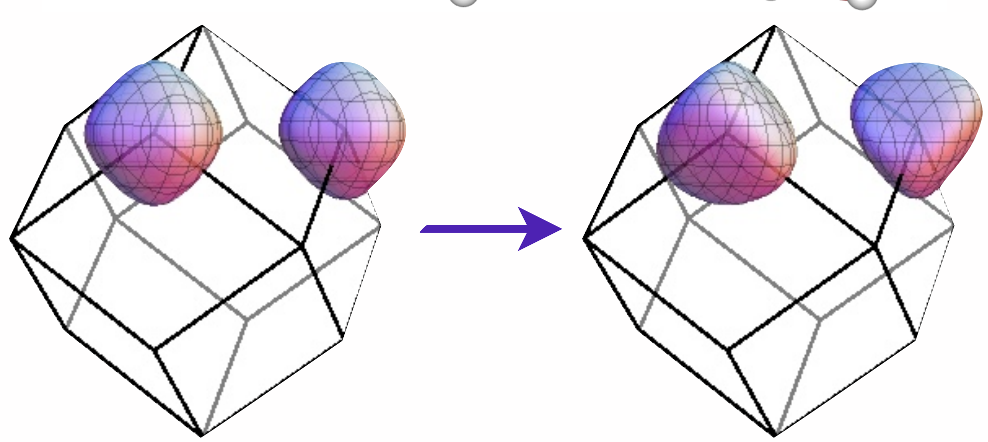
external **magnetic field** in (1,1,1)-direction

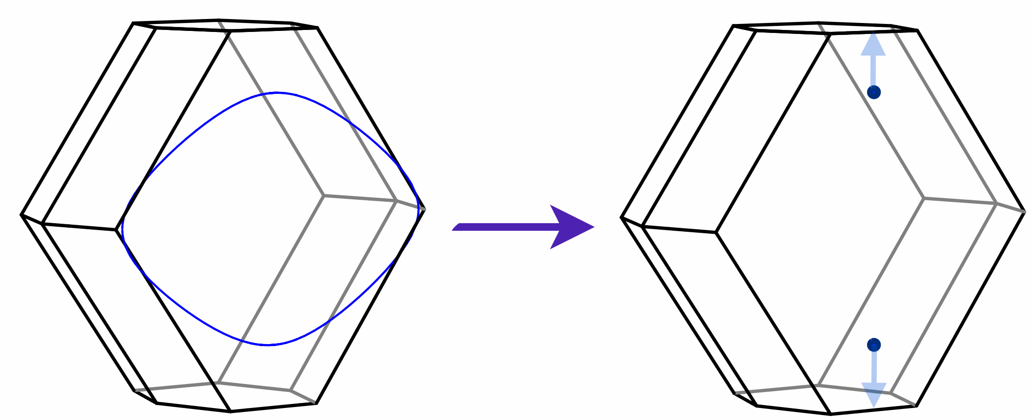
$$H_{eff} = -J \sum_{\gamma-bond} \sigma_j^\gamma \sigma_k^\gamma - \kappa \sum_{\langle j,l,k \rangle} \sigma_j^\alpha \sigma_k^\beta \sigma_l^\gamma$$











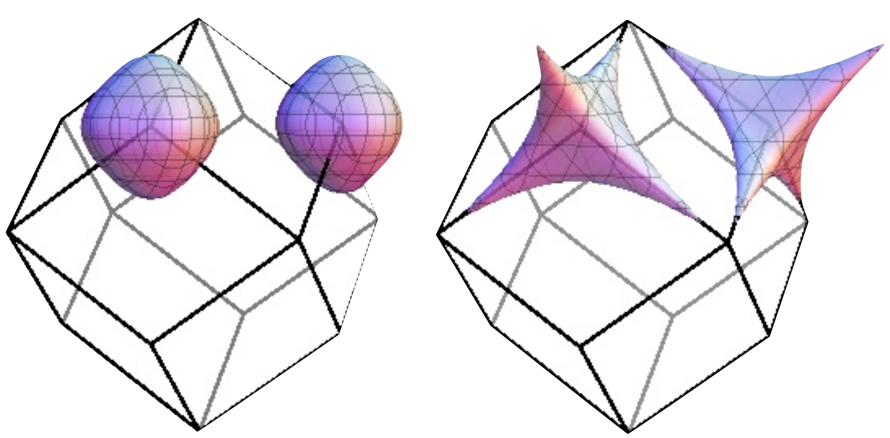
Breaking TR **stabilizes Fermi surface**

Breaking TR reduces line to **pair of Weyl nodes**

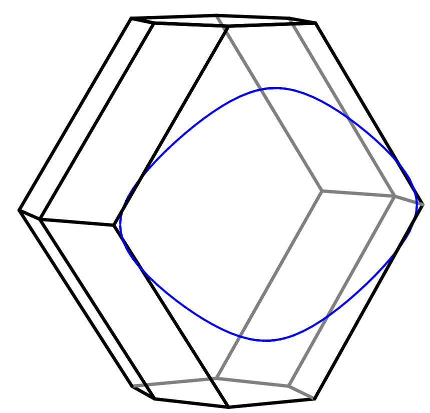
Summary

(Strong) interactions can lead to emergent quasiparticles
with 'fractional' quantum numbers

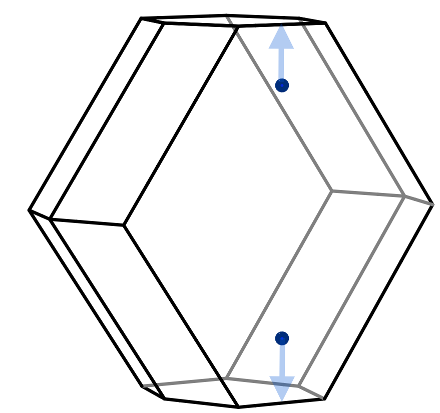
Spin liquid with emergent spinon Fermi surface



Spin liquid with emergent spinon Fermi line



Weyl spin liquid



Majorana Metal ⟷ **Majorana semi-metal**

How big is the zoo of Majorana metals?

M. Hermanns and S. Trebst, PRB 89, 235102 (2014)
M. Hermanns, K. O'Brien, and S. Trebst, in preparation