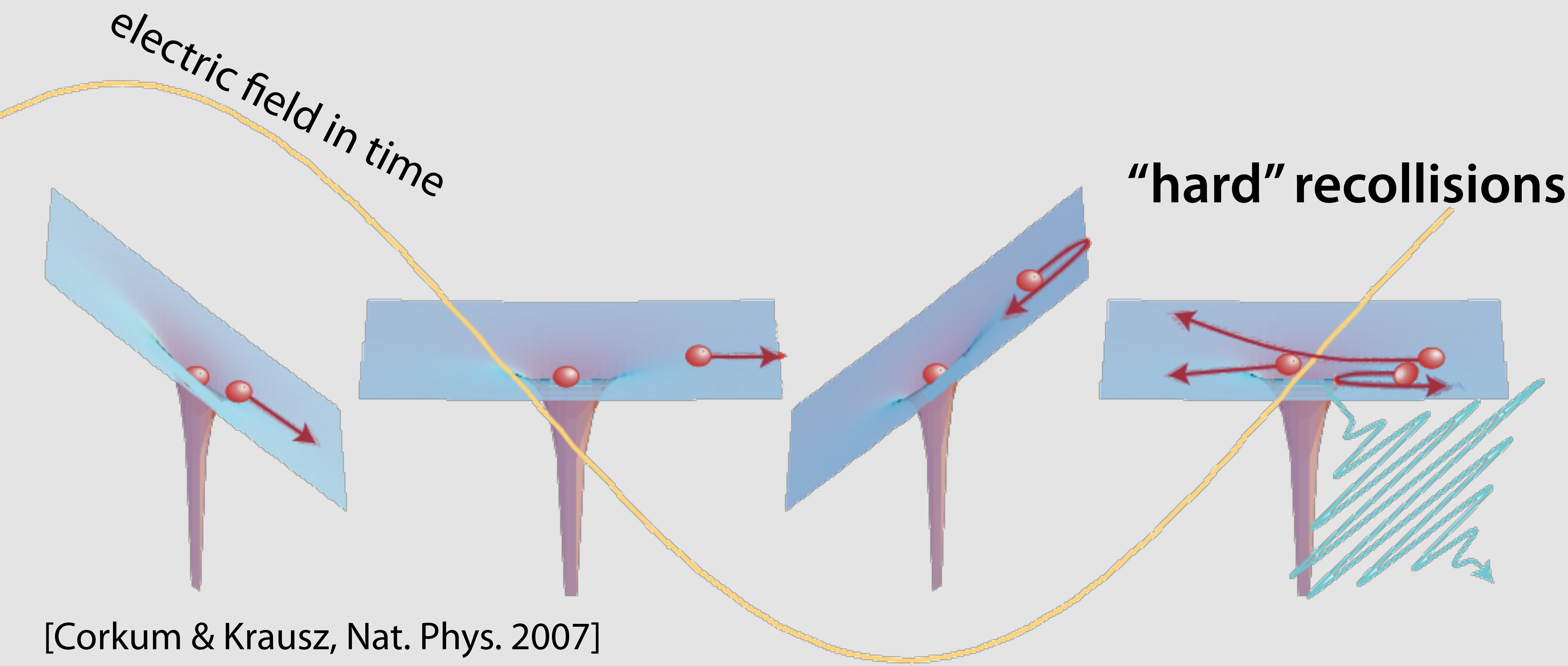
Slow, slower, and even slower electrons from strong-field ionization

Ulf Saalmann
Max-Planck-Institute for the Physics of Complex Systems • Dresden

Alexander Kästner, Elias Diesen & Jan-Michael Rost

- **low-energy structures (LESs)**
- **very-low-energy structure (VLES)**
- **“zero”-energy structure (ZES)**

recollisions in strong-field ionisation

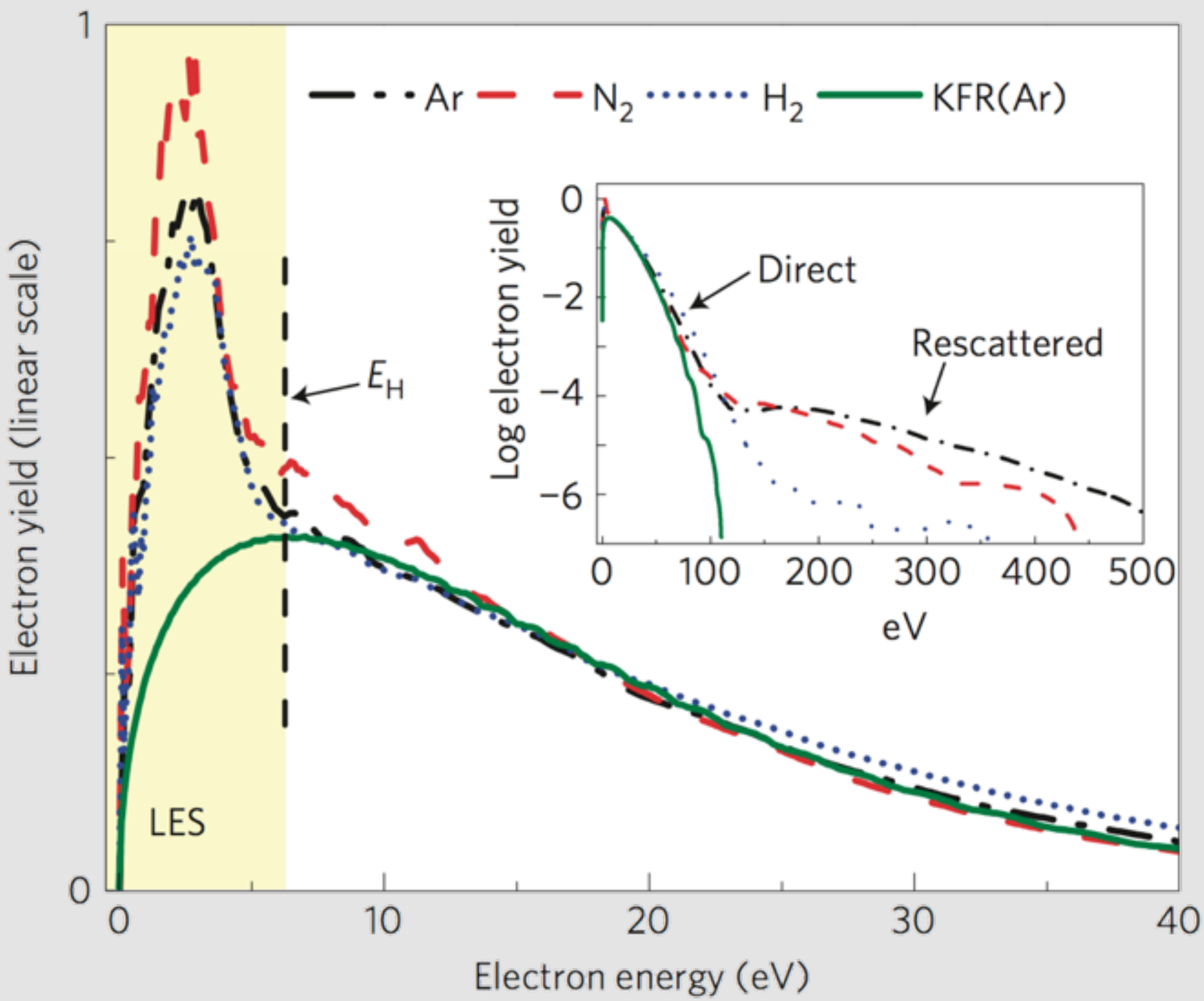


[Corkum & Krausz, Nat. Phys. 2007]

observation of the LES

data from Blaga et al. [Nat. Phys. 5 (2009) 335]

photo-electrons for λ=2µm and I=1.5×10^{14}W/cm^2



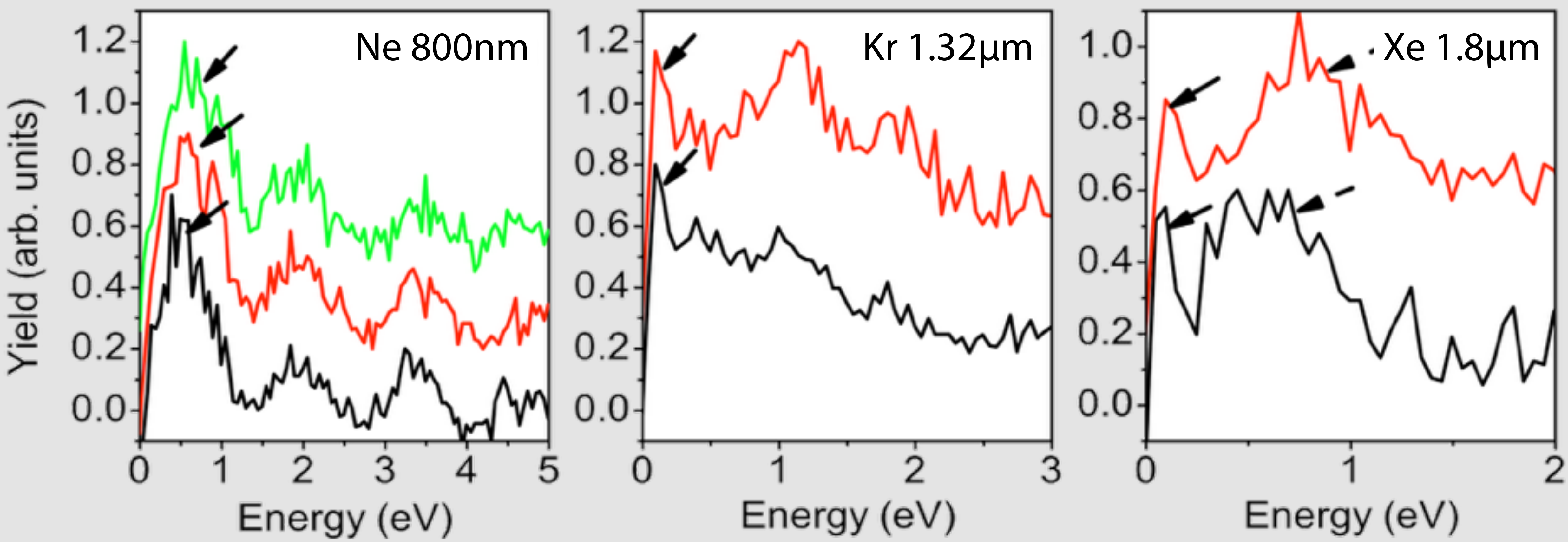
why E_H not E_{peak} ?

behavior at E=0 ?

observation of a VLES

data from Wu et al. [PRL 109 (2012) 043001]

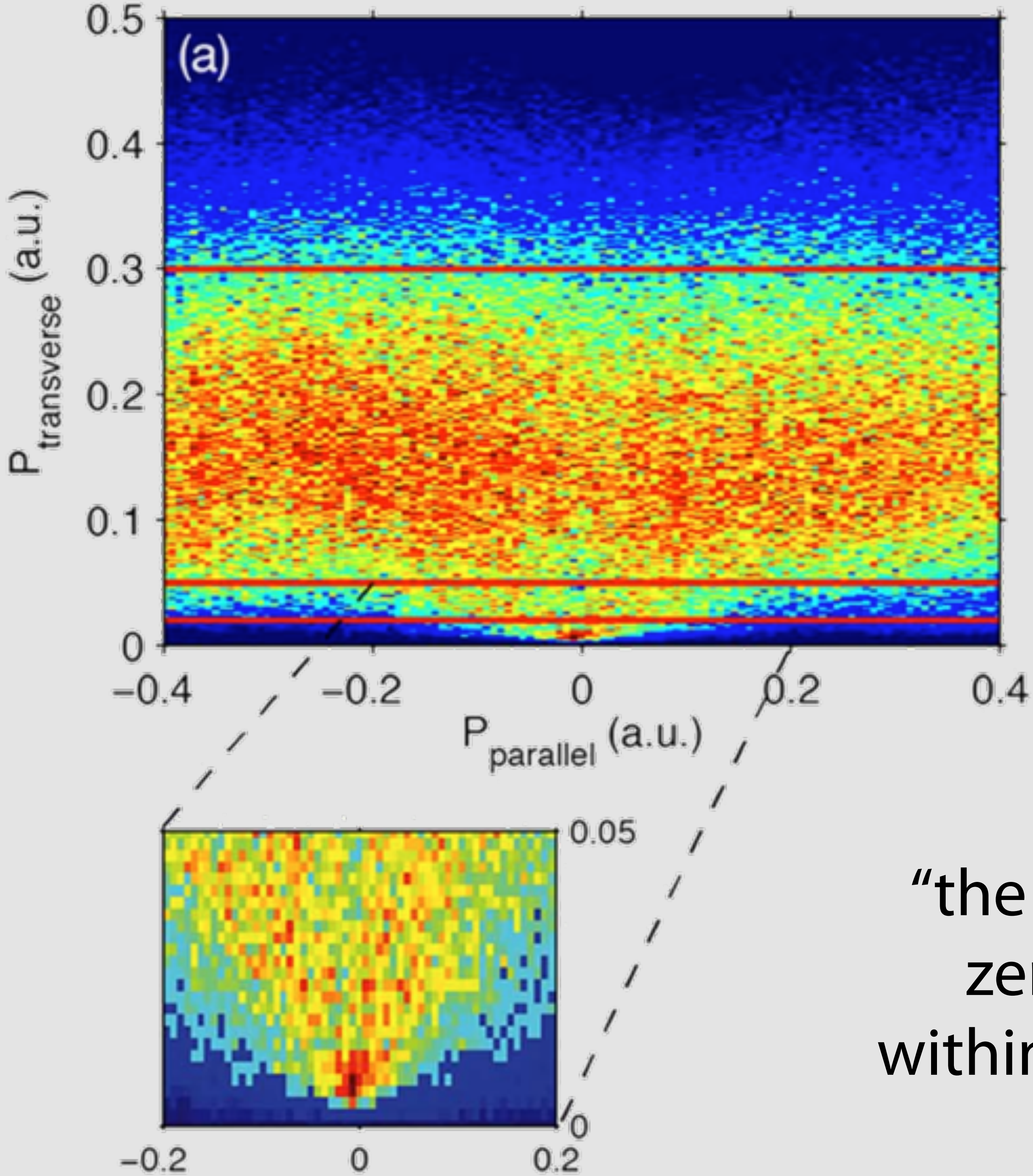
“denote this as the high-energy low-energy structure (HLES)”
“for visual convenience, the HLES and the VLES are marked”



behavior at *E*=0 ?

observation of a “zero-peak”

data from Dura et al. [Sci. Rep. 3 (2013) 2675]



“the offset of the ‘zero-peak’ from zero transverse momentum is within our measurement resolution” slow

- reproduction of the original LES
  by various quantum and classical calculations

  Blaga et al., Nat. Phys. 2009,
  Quan et al., Phys. Rev. Lett. 2009.
  Catoire et al. , Las. Phys. 2009.
  Liu and Hatsagortsyan, Phys. Rev. Lett. 2010.
  Lemell et al., Phys. Rev. A 2012.
  ...

agreement on forward scattering

classical trajectories

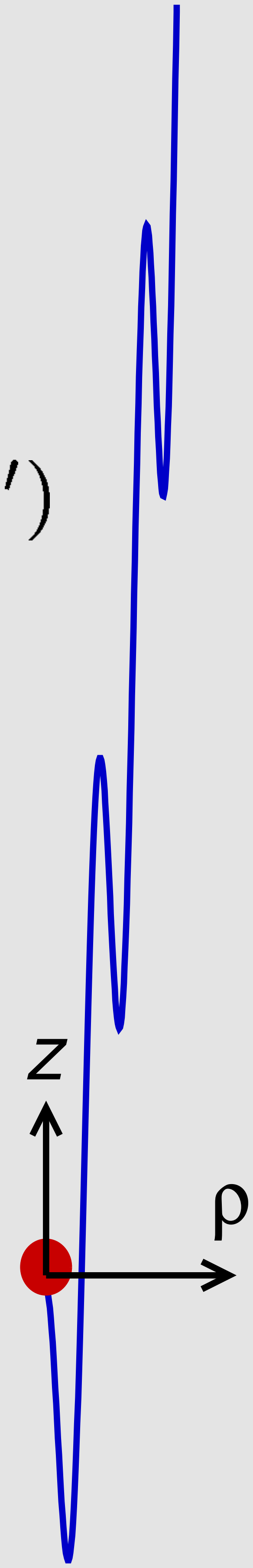
$$H = \frac{{p_\rho}^2}{2} + \frac{{p_z}^2}{2} - \frac{1}{\sqrt{z^2+\rho^2}} + z\,F_0 \cos(\omega t)$$

initial values (time t'): field $F_0 \cos(\omega t') \to A_0 \sin(\omega t')$

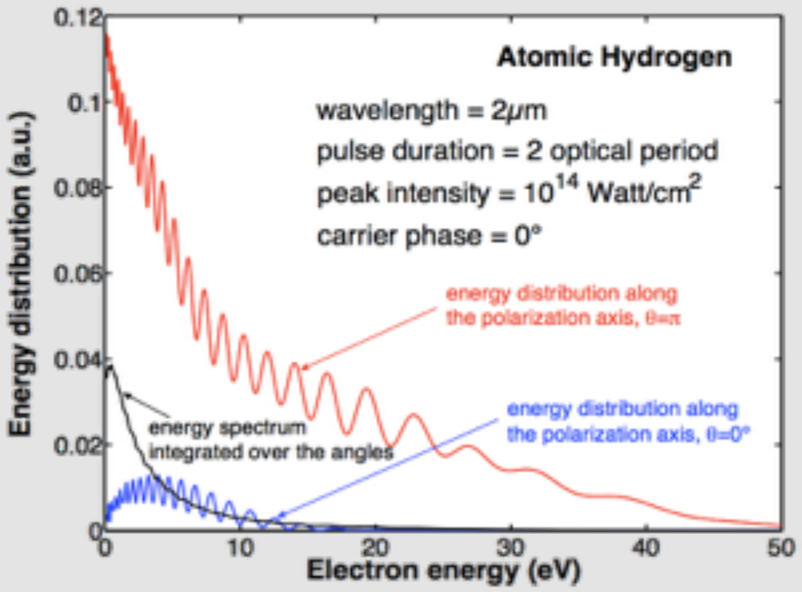
$$z(t') = z' \quad p_z(t') = 0$$
$$\rho(t') = 0 \quad p_\rho(t') = p'_\rho$$

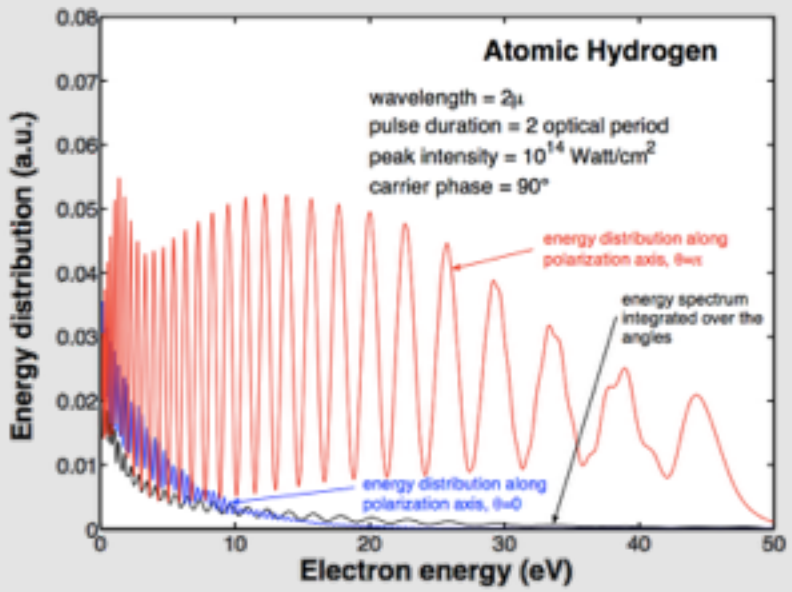
probability for $F = F(t')$

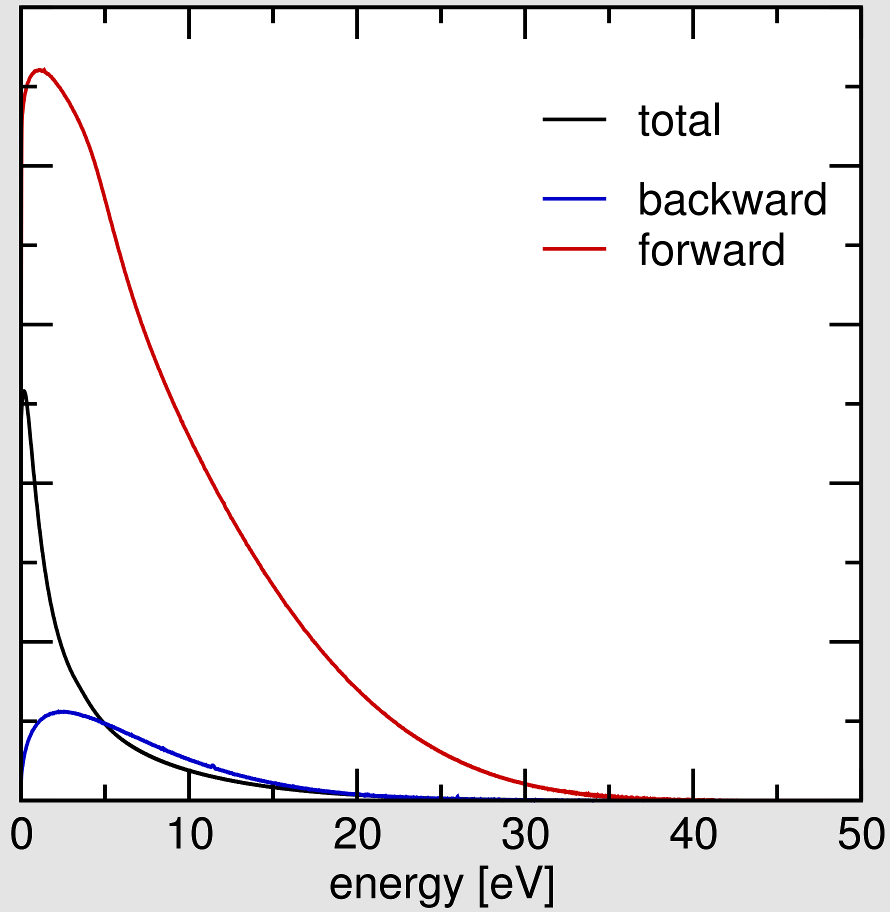
$$W(F, p'_\rho) \propto \frac{p'_\rho}{\sqrt{1+p'^2_\rho/2E_{\mathrm{ip}}}} \frac{\mathrm{e}^{-2(2E_{\mathrm{ip}}+p'^2_\rho)^{3/2}/3F}}{F^2}$$

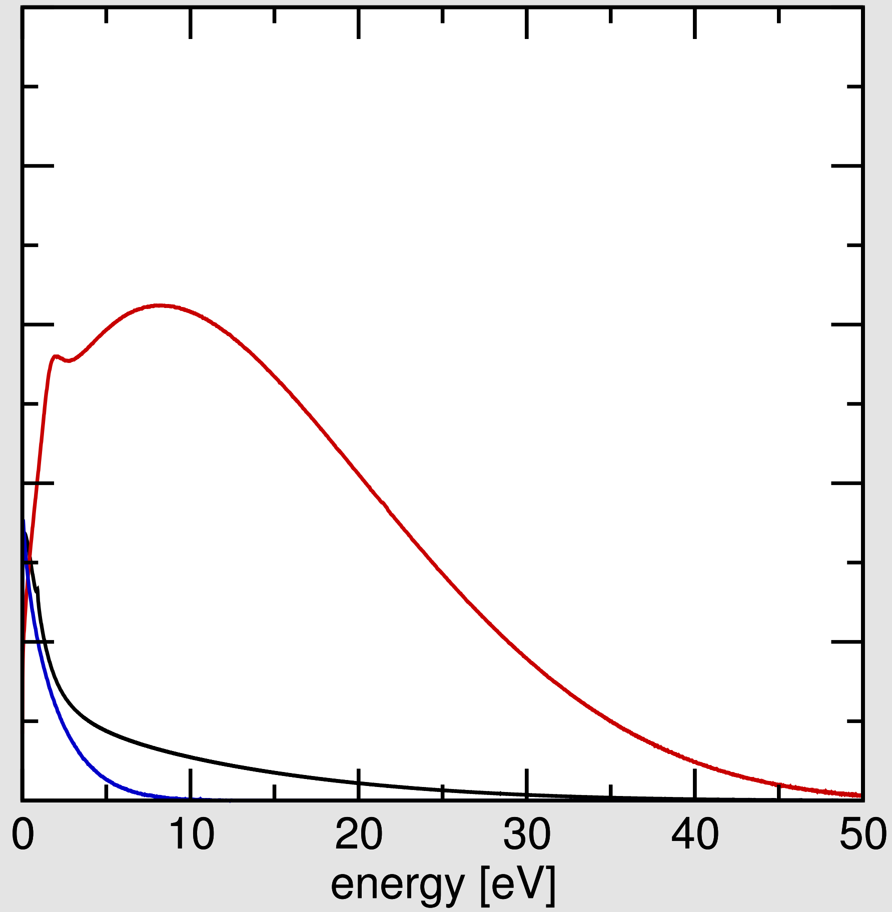


classical vs. Bernard Piraux @ NORDITA May 21st

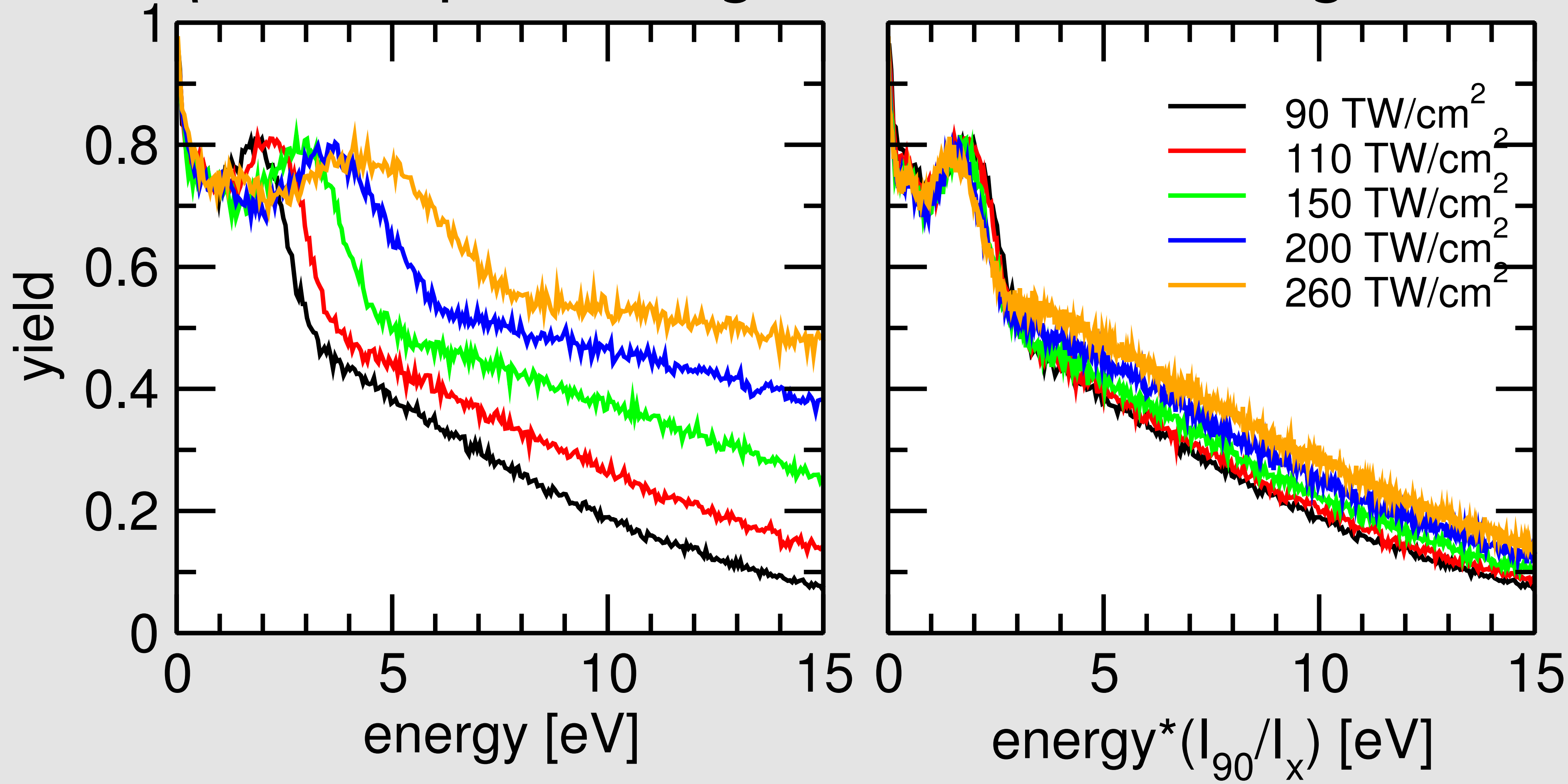








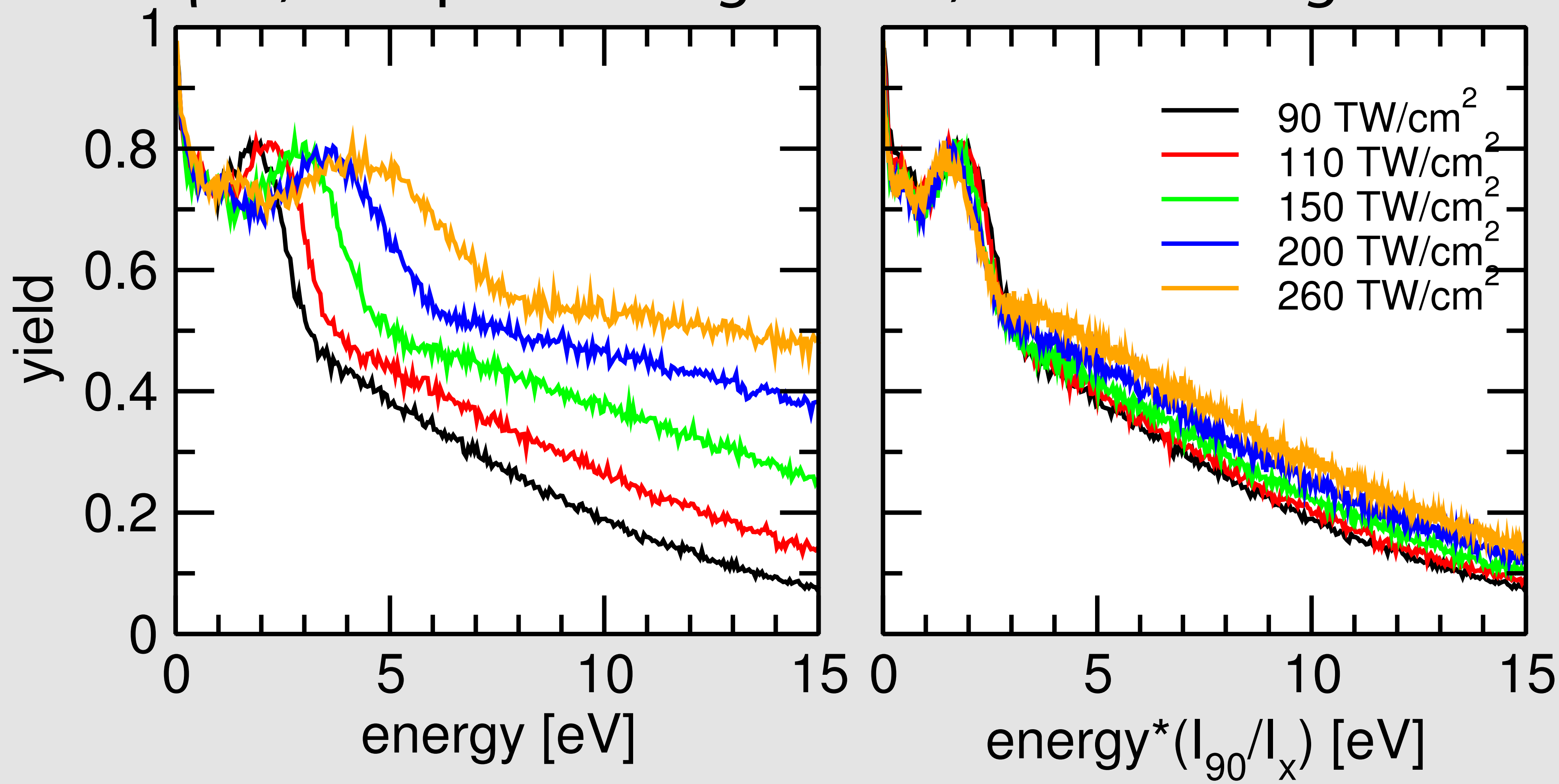
argon, ADK tunneling rates + classical propagation
wavelength λ=2μm, acceptance angle θ<5°, focus-averaged



What forms the peak structure?

numerical calculations

argon, ADK tunneling rates + classical propagation
wavelength λ=2μm, acceptance angle θ<5°, focus-averaged

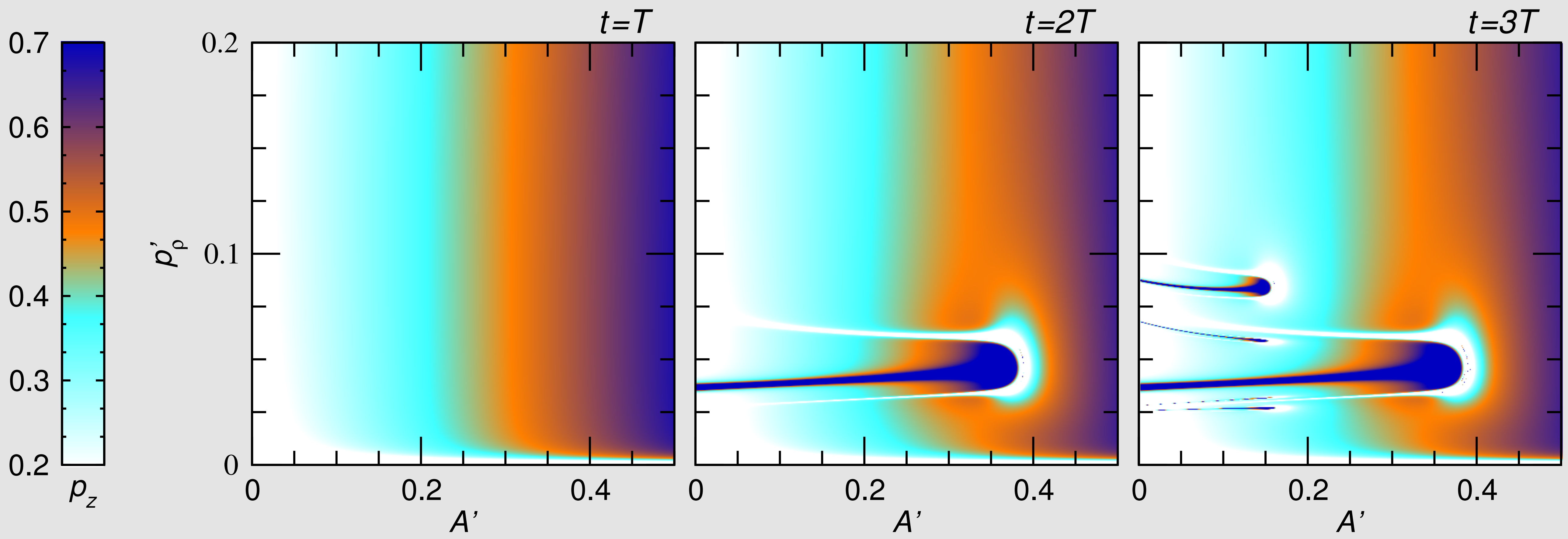


What forms the peak structure?

trajectories: initial conditions → final observables
or in between (but time-dependent fields)

deflection function: dependence on initial variables

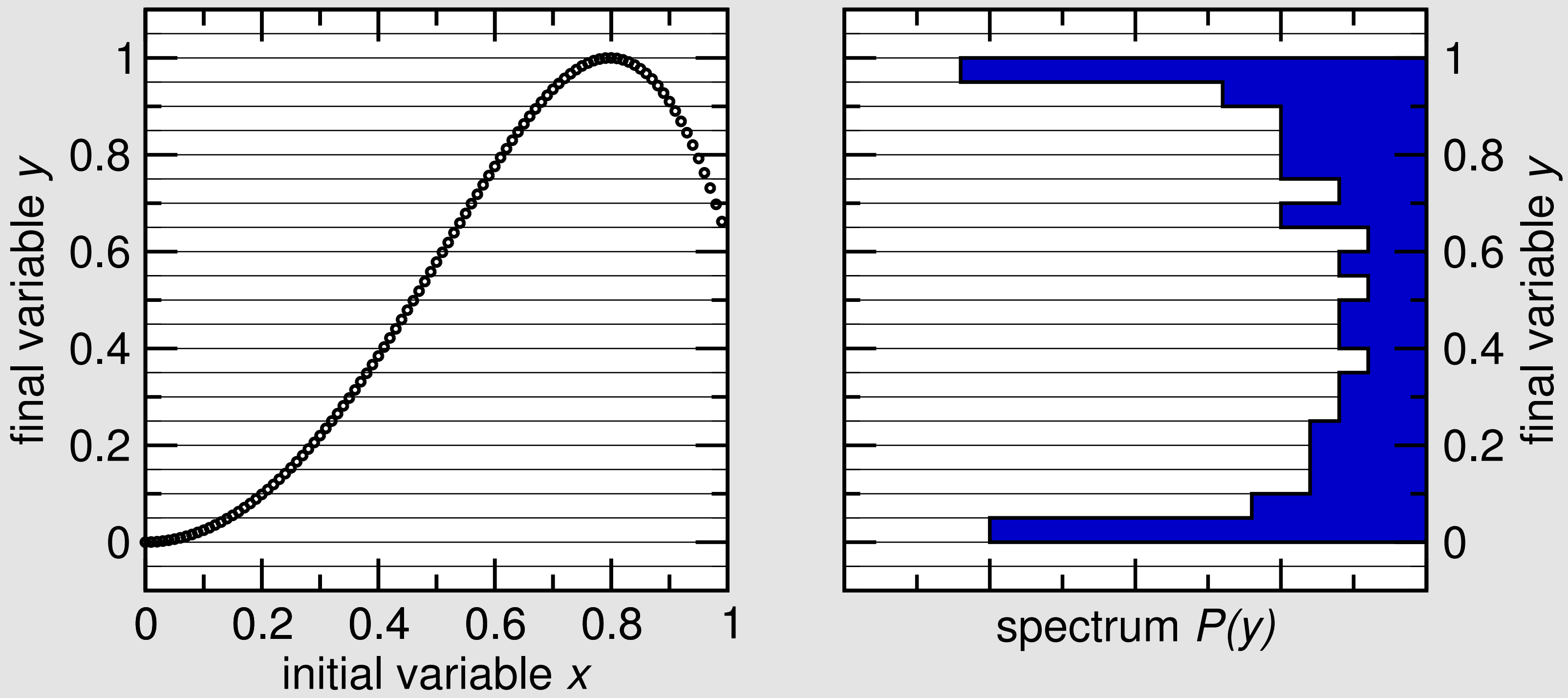
longitudinal momentum $p_z(t{=}nT;\,A',\,p'_\rho)$



(multiple) recollisions

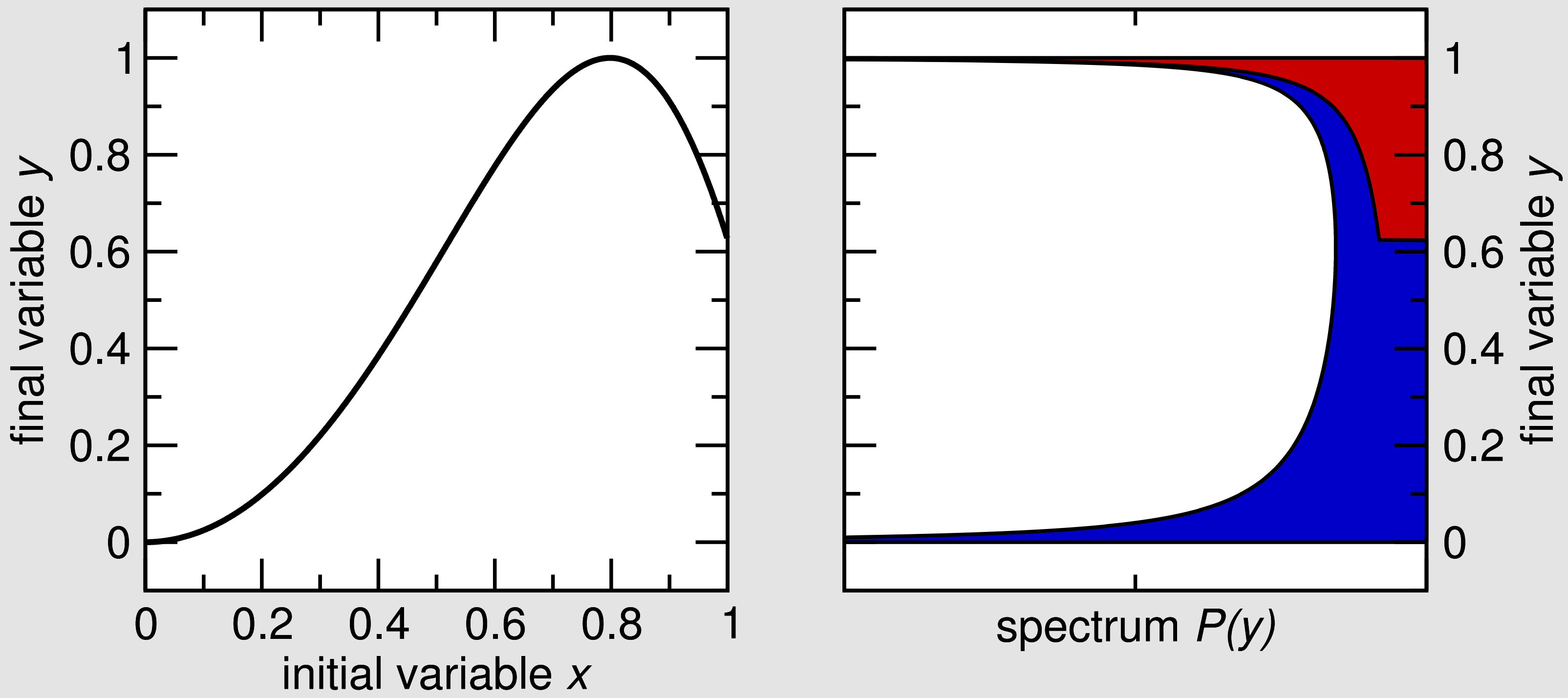
formation of peaks (LES)?

1-dim deflection function



spectrum from “binning” of final variables

1-dim deflection function



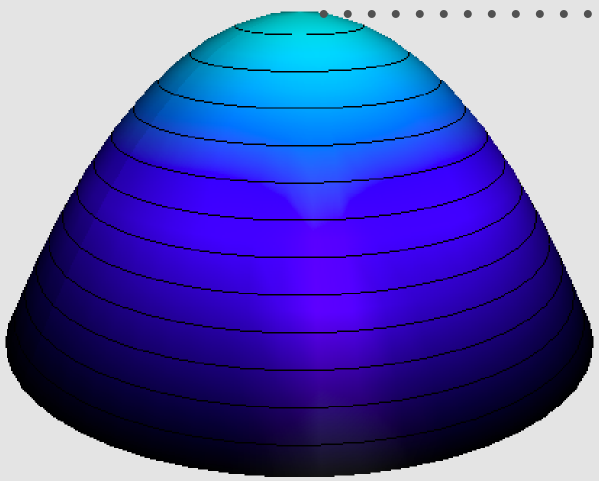
deflection function Y: relation between initial x and final variable y

→ defines the spectrum $$P(y) = \int \mathrm{d}x\, \delta(y - Y(x))$$
$$= \sum_j \left| \frac{\partial Y(x)}{\partial x} \right|^{-1}_{x = X_j(y)}$$

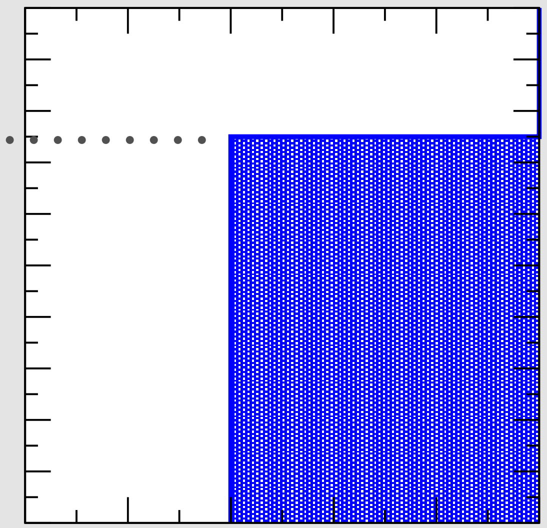
extrema of the deflection function → peaks in the spectrum

2-dim deflection function

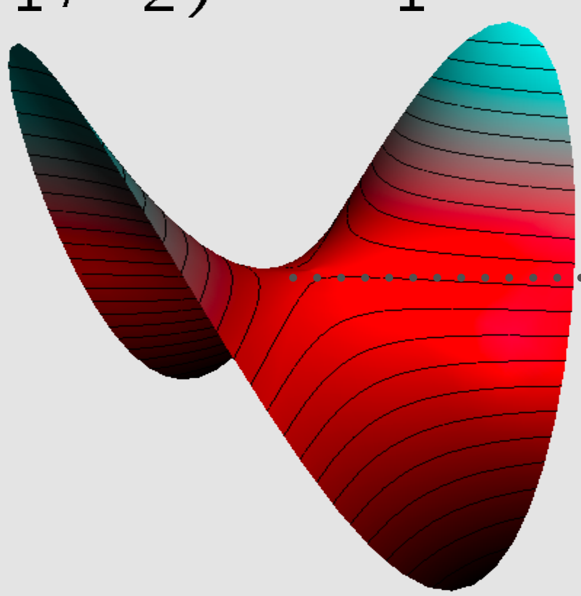
$$Y(x_1, x_2) = -{x_1}^2 - {x_2}^2$$



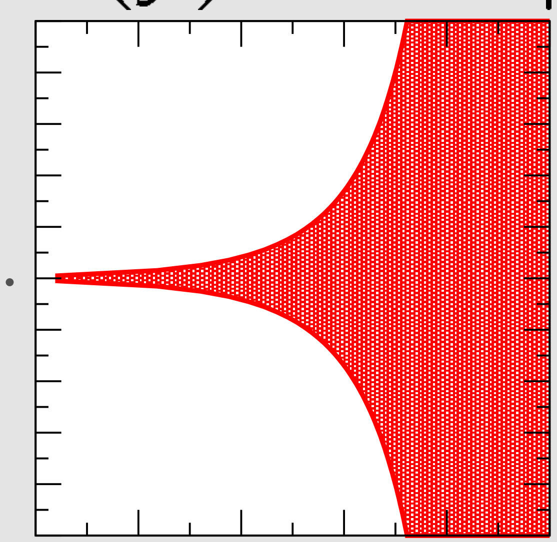
$$P(y) = \Theta(-y)$$



$$Y(x_1, x_2) = {x_1}^2 - {x_2}^2$$



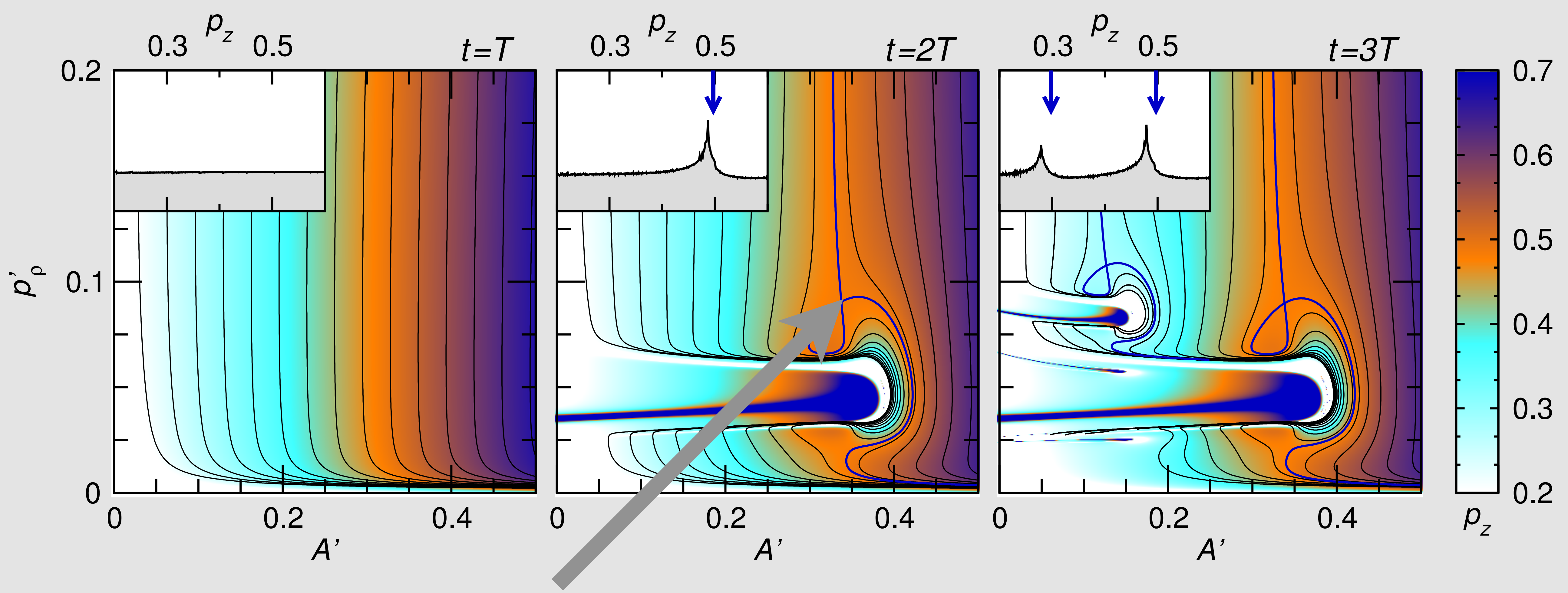
$$P(y) \sim -\ln|y|$$



→ peak structure for **saddle points** (cf. van-Hove singularities)

deflection function for λ=2μm and I=10^{14}W/cm^2

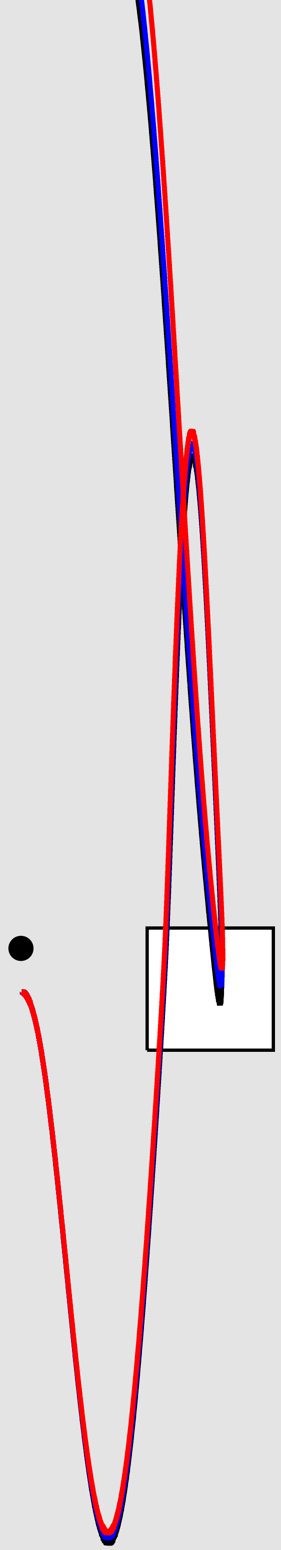
longitudinal momentum $p_z(t = nT; A', p'_\rho)$

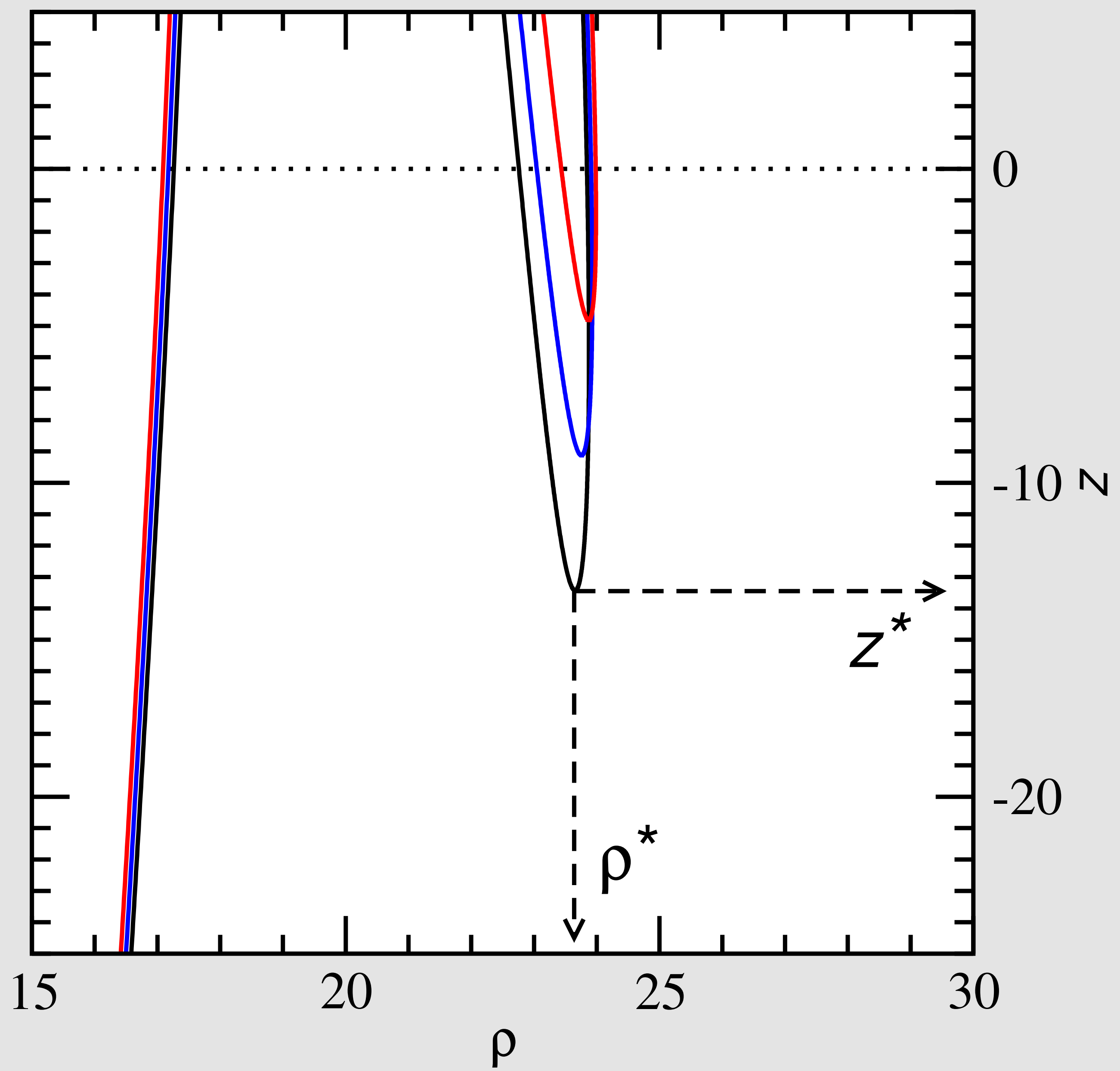


mechanism of saddle-point formation?

→ trajectories at saddle point

saddle-point trajectories

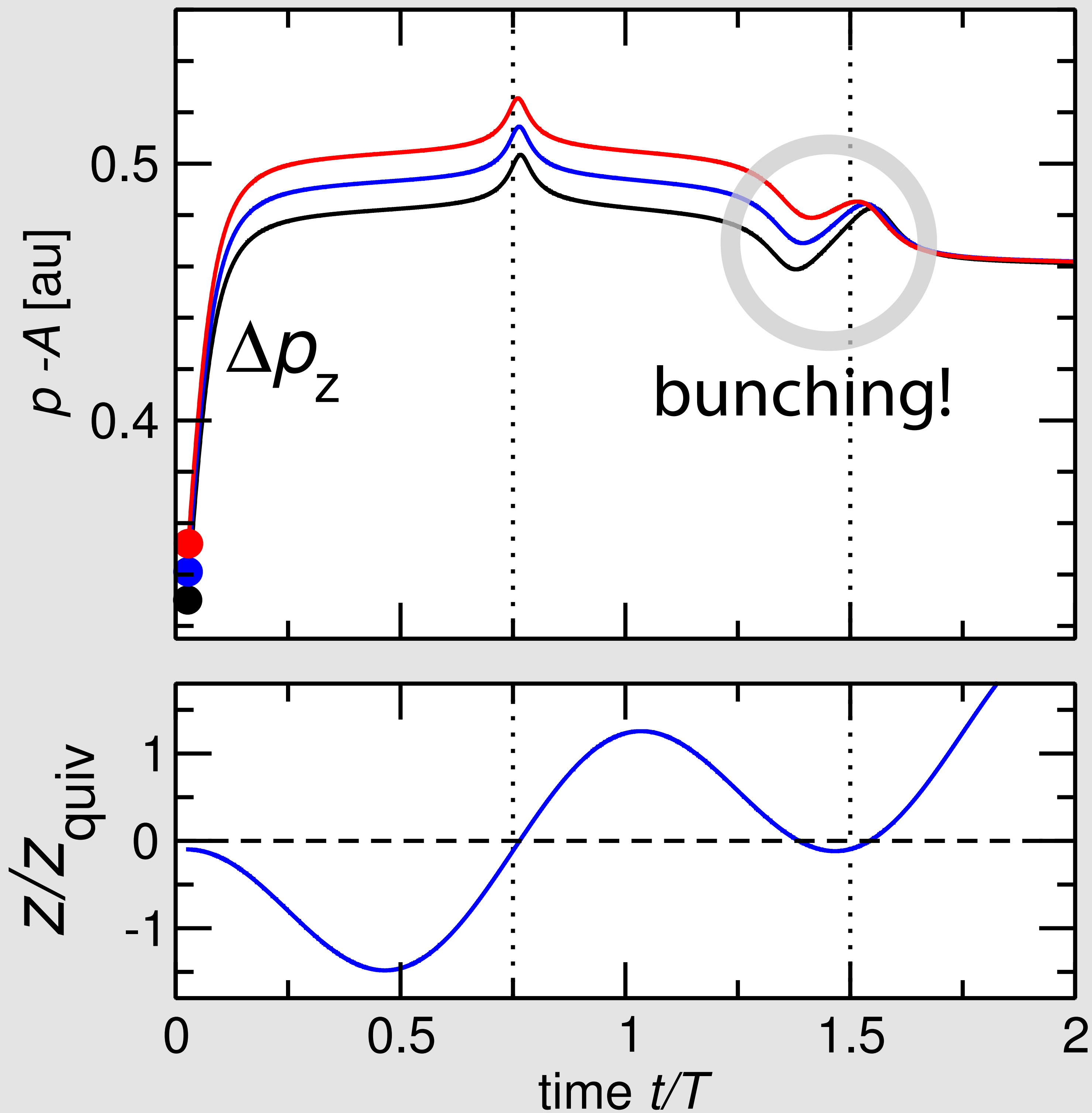




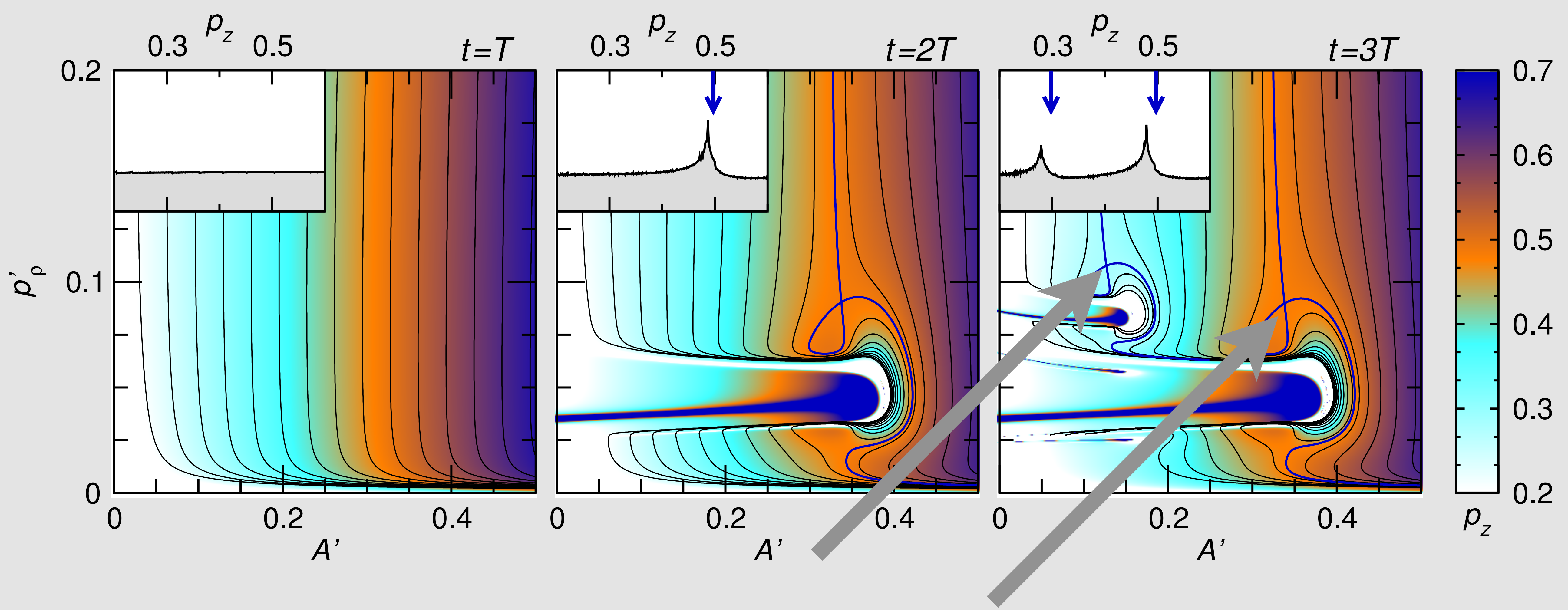
recollisions aside the ion, i.e. weak perturbation

→ soft recollisions

bunching in soft recollisions

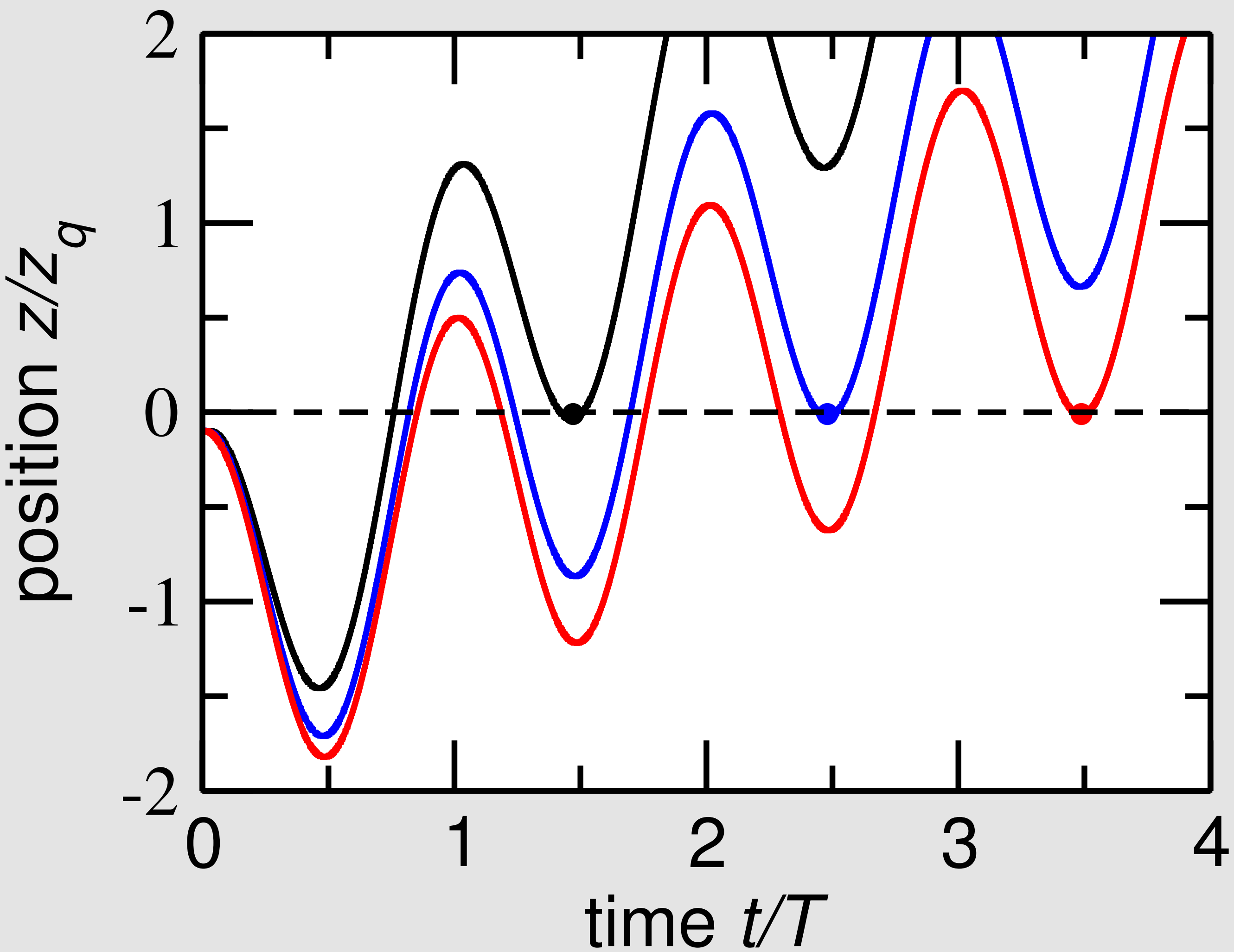


trajectories with different initial drift momenta end up with the same final momentum longitudinal momentum $p_z(t=nT; A', p'_\rho)$



saddle point(s) from bunching of trajectories

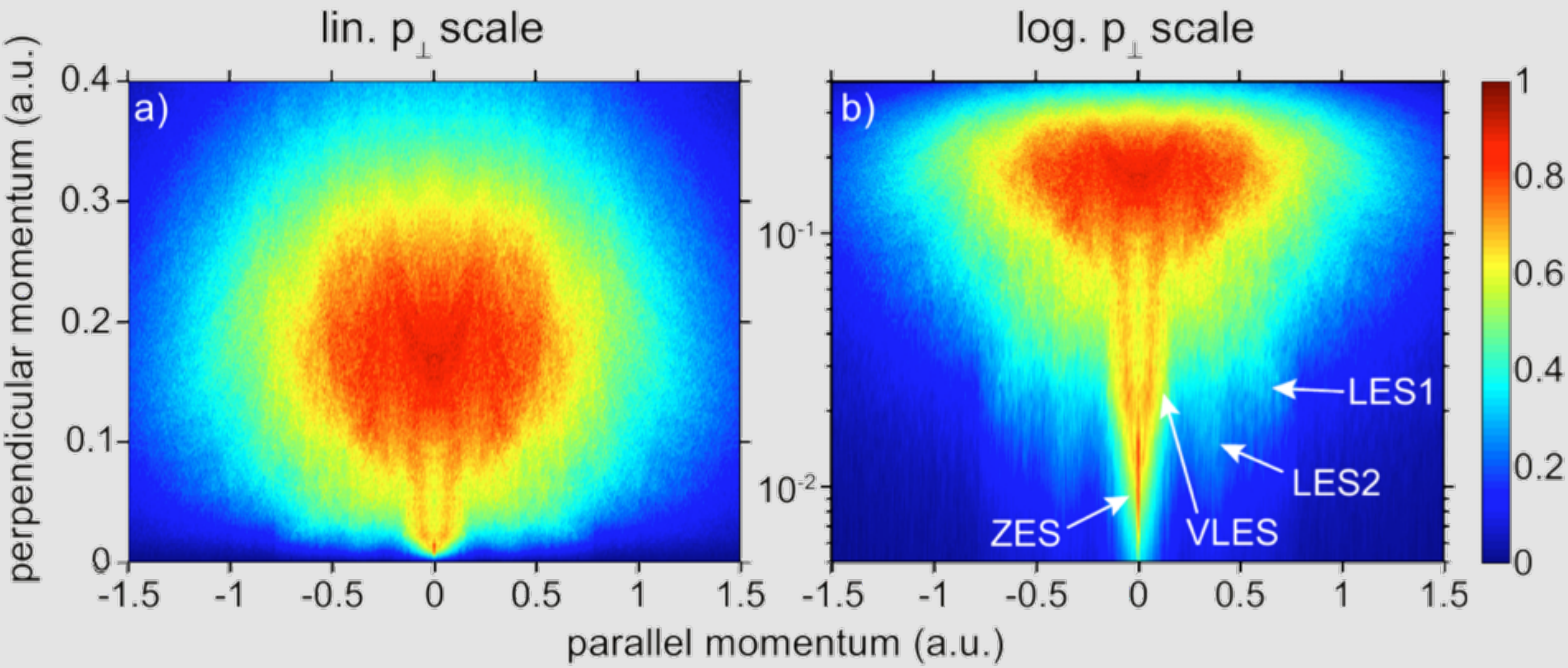
series of LES peaks



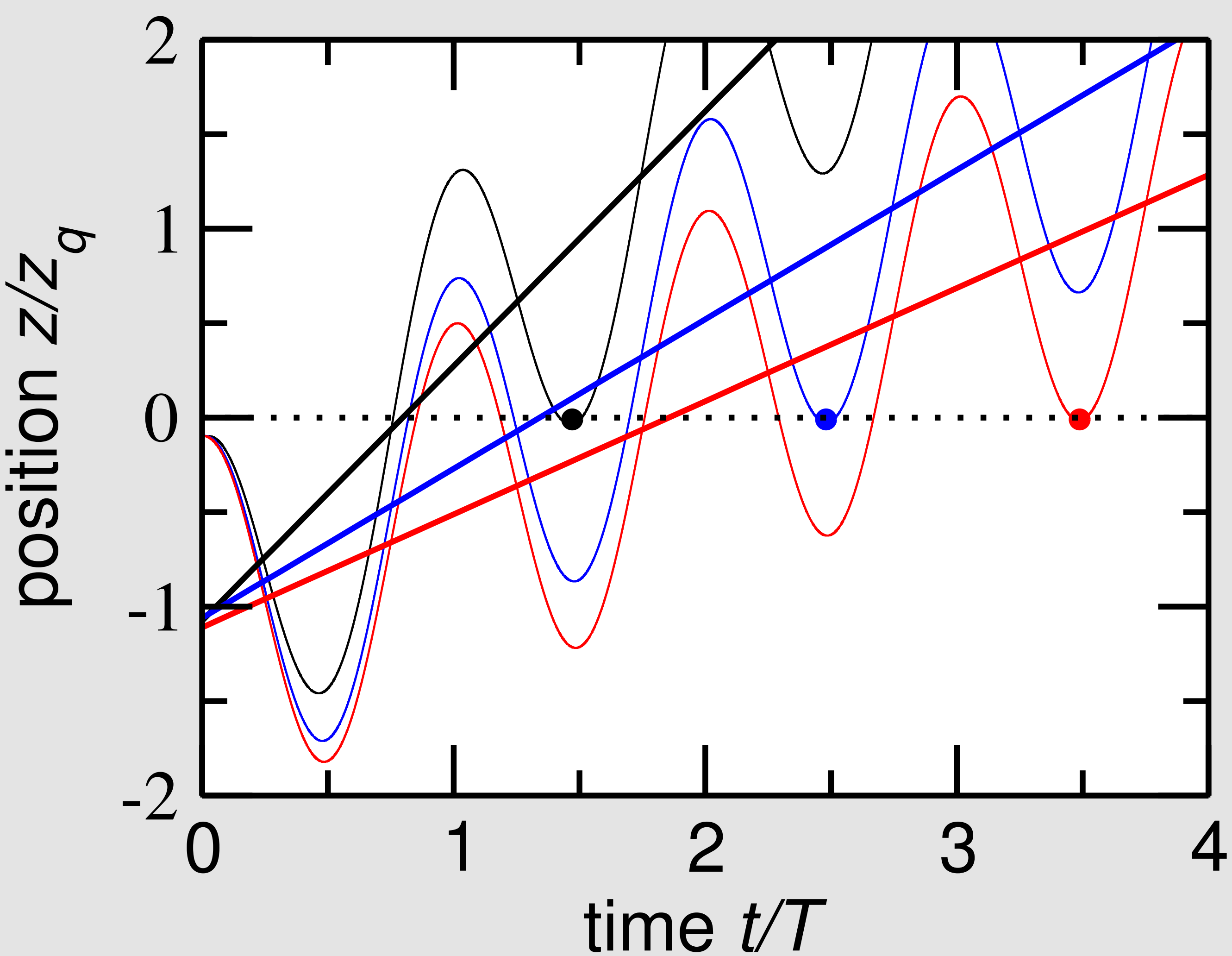
soft recollisions at later times $\quad t_k^* = \frac{2k+1}{2} T$

higher-order LES in experiment

recent data from Wolter et al. [Phys. Rev. A 2014]



series of LES peaks



explicit analytical expression for LES energy

$$E_k = \frac{8}{(2k+1)^2\pi^2} E_{\mathrm{pond}}$$

$$E_1 \approx \frac{E_{\mathrm{pond}}}{10}$$

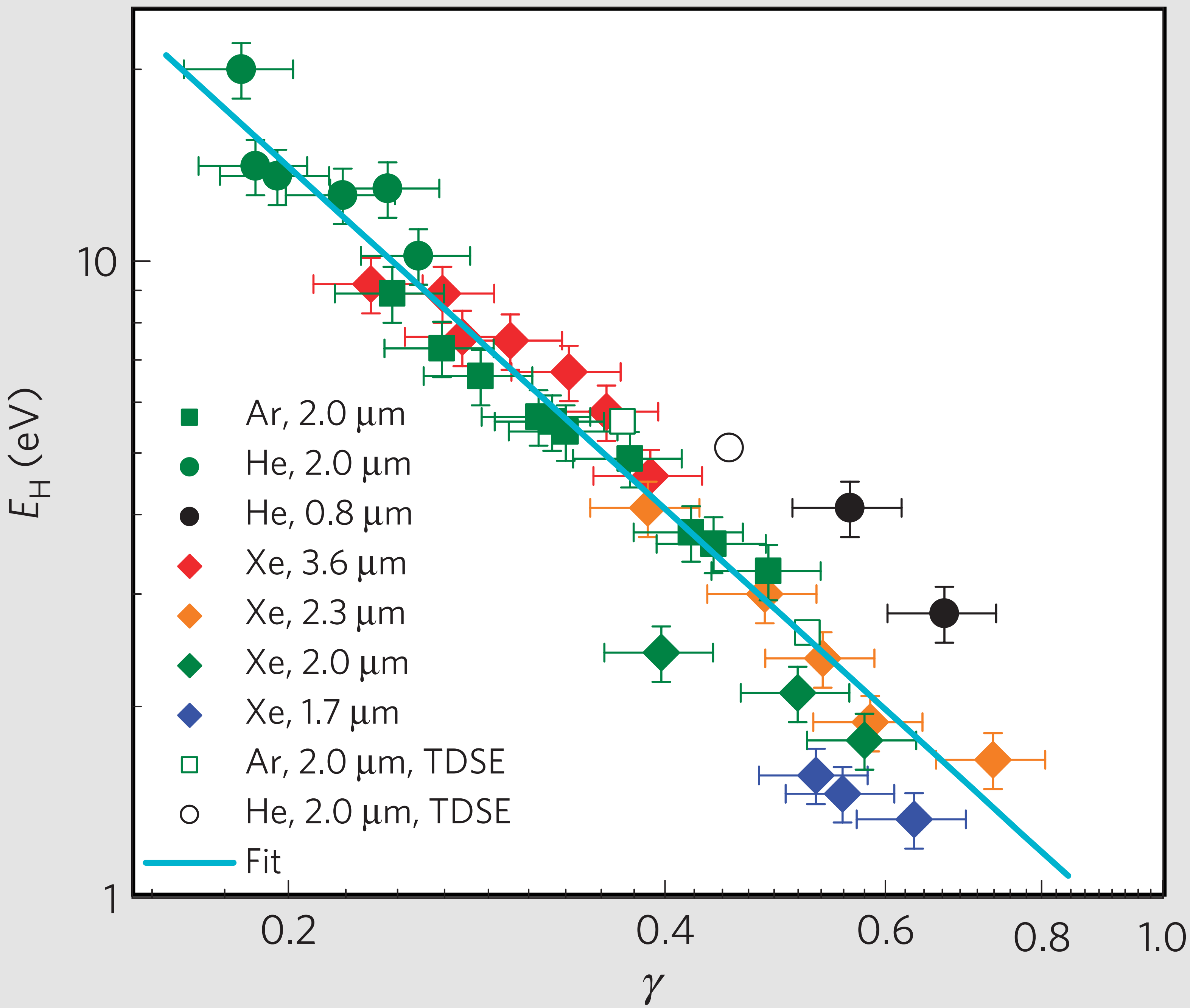
soft recollisions at later times $\quad t_k^* = \frac{2k+1}{2} T$

$$z(t) = p\, t + \frac{F_0}{\omega^2}\left[\cos(\omega t) - 1\right]$$

$$z(t_k^*) = 0 \quad \rightarrow \quad p_k = -\frac{1}{t_k^*}\frac{F_0}{\omega^2}[-2] \qquad = \frac{F_0}{\omega}\frac{2}{(2k+1)\pi}$$

ponderomotive energy or Keldysh parameter?

Blaga et al. Nat. Phys. 5 (2009) 335



ponderomotive energy

$$E_{\mathrm{pond}} \equiv \frac{F_0}{4\omega^4}$$

Keldysh parameter

$$\gamma \equiv \sqrt{\frac{E_{\mathrm{ip}}}{2E_{\mathrm{pond}}}}$$

peak position for few-cycle pulses

$$z(t) = p\,t + \int_0^t \mathrm{d}t'\,\mathcal{A}(t')$$

soft recollision

$$z(t^* = 3\pi/\omega) = 0 \qquad p = -\frac{1}{t^*}\int_0^{t^*} \mathrm{d}t\,\mathcal{A}(t_0 + t)$$

$$\mathcal{A}(t) = -\frac{\mathcal{A}_0}{\omega}\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{e}^{-2\ln 2\,[t/T]^2}\cos(\omega t - \phi) \qquad T = n\frac{2\pi}{\omega}$$

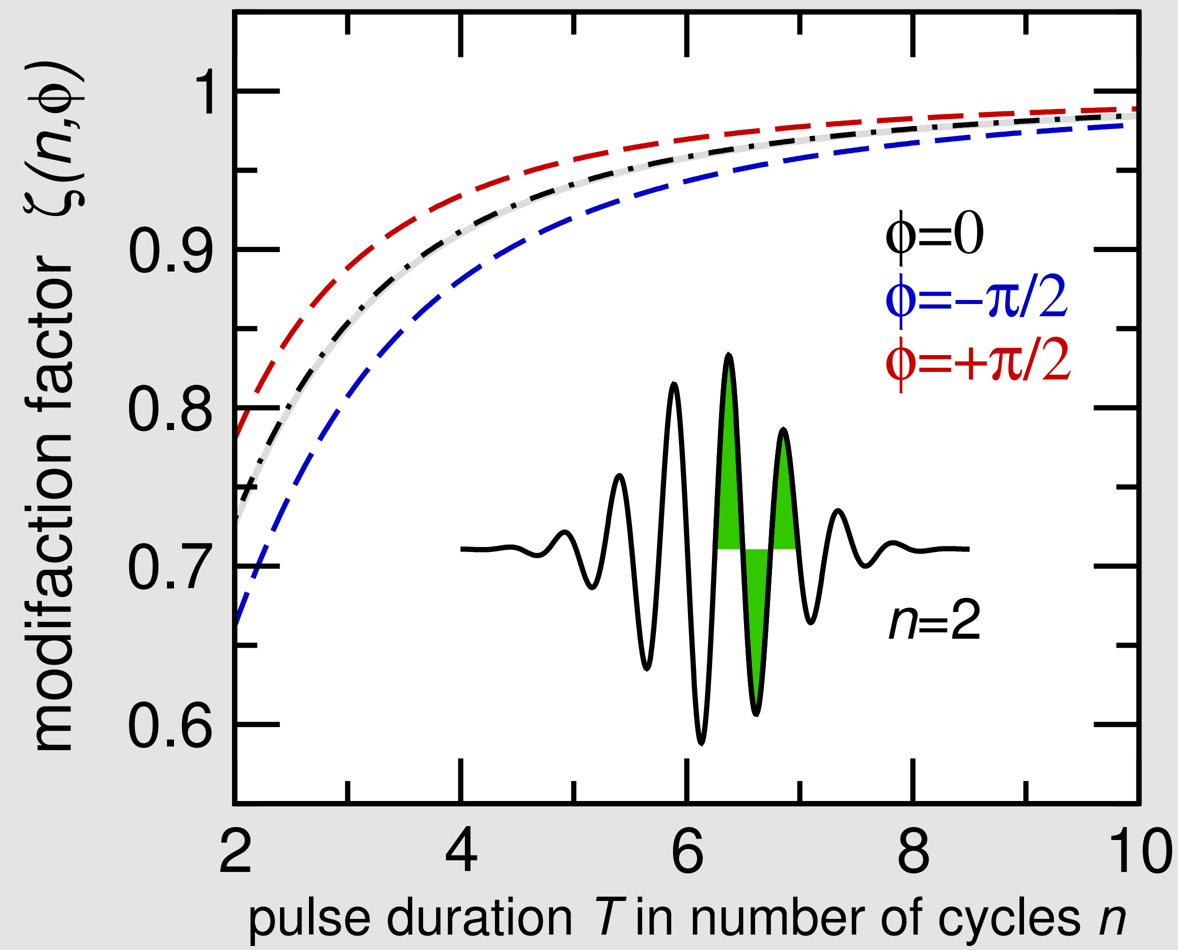
$$p(n,\phi) = \zeta(n,\phi)p_\infty \qquad p_\infty \equiv \frac{2\mathcal{A}_0}{3\pi}$$

$$\zeta(n,\phi) = \frac{1}{2}\left[\mathrm{e}^{-[\ln 2/2][\phi/\pi]^2/n^2} + \mathrm{e}^{-[\ln 2/2][3+\phi/\pi]^2/n^2}\right]$$

CEP average $\bar{\zeta}(n) \approx \frac{1}{2}\left[1 + \mathrm{e}^{-[\ln 2/2]\,9/n^2}\right] < 1$

peak position for few-cycle pulses

lower low-energy peaks are expected !



$$p(n,\phi) = \zeta(n,\phi)p_\infty \qquad p_\infty \equiv \frac{2\mathcal{A}_0}{3\pi}$$

$$\zeta(n,\phi) = \frac{1}{2}\left[e^{-[\ln 2/2][\phi/\pi]^2/n^2} + e^{-[\ln 2/2][3+\phi/\pi]^2/n^2}\right]$$

CEP average $\bar{\zeta}(n) \approx \frac{1}{2}\left[1 + e^{-[\ln 2/2]\,9/n^2}\right] < 1$

focus average

$$P_{\tilde{F}}(I) = \mathrm{e}^{-1/\tilde{F}\sqrt{I/I_{\mathrm{max}}}} \frac{2I/I_{\mathrm{max}}+1}{[I/I_{\mathrm{max}}]^3} \sqrt{1 - I/I_{\mathrm{max}}}$$

$$\tilde{F} = F/[2E_{\mathrm{ip}}]^{3/2}$$

$$p \propto \sqrt{I} \qquad \langle p \rangle_{\mathrm{focus}} = \frac{p(I_{\mathrm{max}})}{1 + [3/2]^2 \tilde{F}}$$

$$\frac{\bar{p}^2(n)}{2} = \frac{{p_\infty}^2}{2} \frac{\bar{\zeta}^2(n)}{\left[1 + [3/2]^2 \tilde{F}\right]^2}$$

$$\frac{\bar{E}_{\mathrm{LES}}(n)}{E_{\mathrm{pond}}} = \frac{2}{9\pi^2} \frac{[1 + \mathrm{e}^{-[\ln 2/2]\, 9/n^2}]^2}{1 + [9/2]\, F/[2E_{\mathrm{ip}}]^{3/2}}$$

universal dependence on

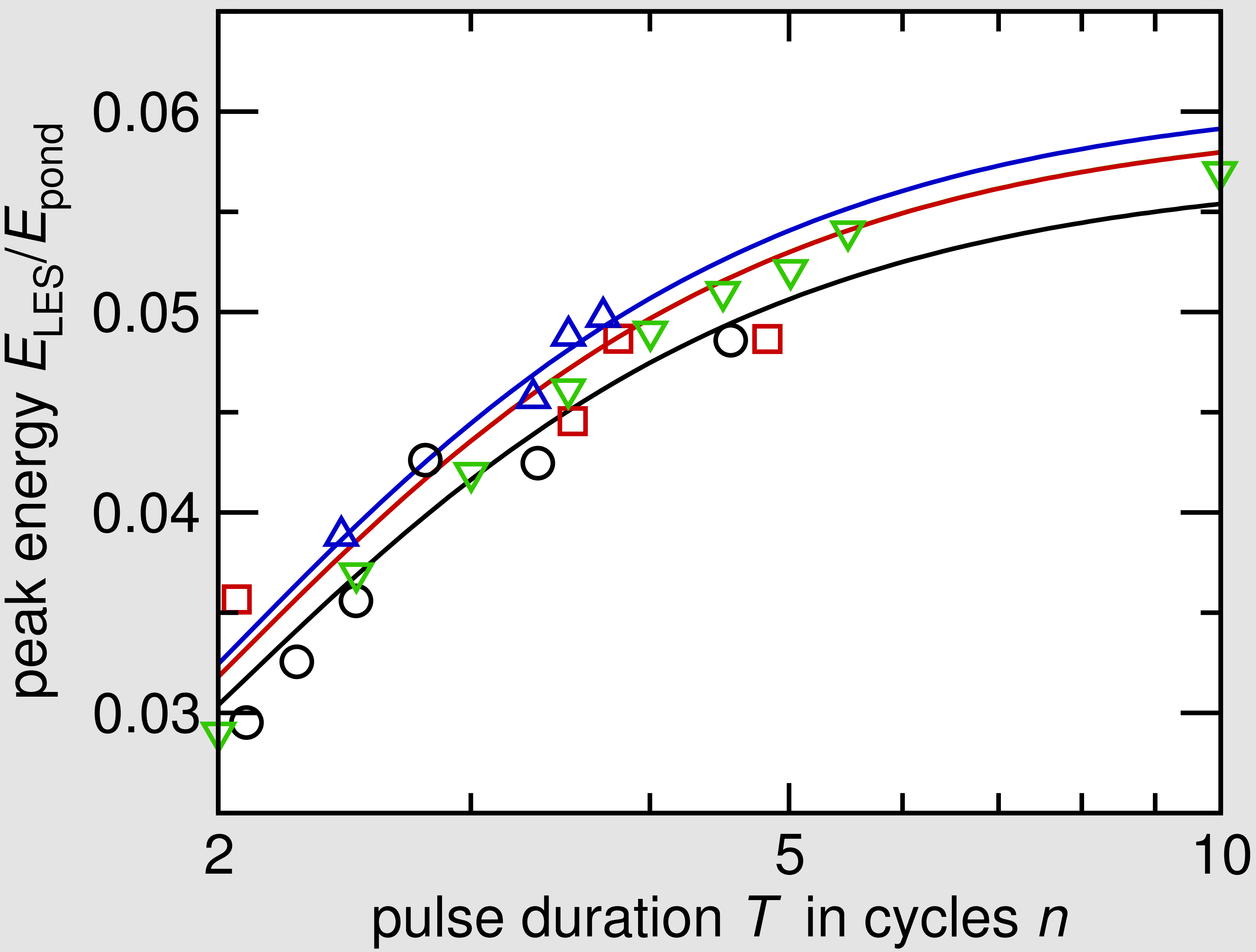
pulse duration (cycles)

and

effective field strength

experiments in the DiMauro group

rare-gas atoms in
few-cycle pulses
1.8μm 10^{14} W/cm^2



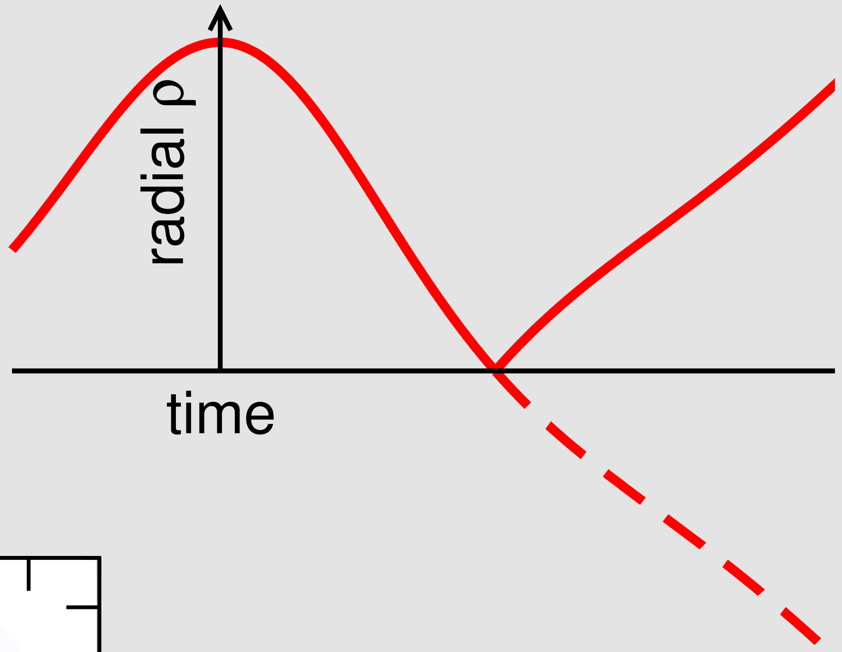
Ar 31eV
Ar 55eV
Kr 20eV
Kr 25eV

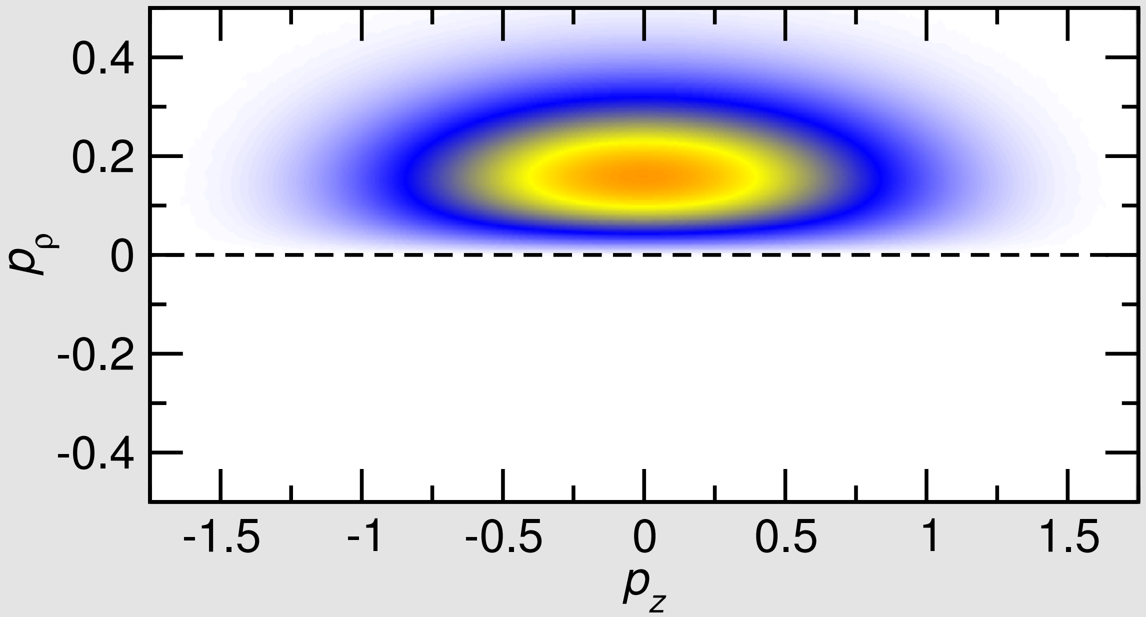
slower

momentum distribution

$p_\rho \hat{=} (-1)^c p_\rho$

$c =$ crossings of laser axis



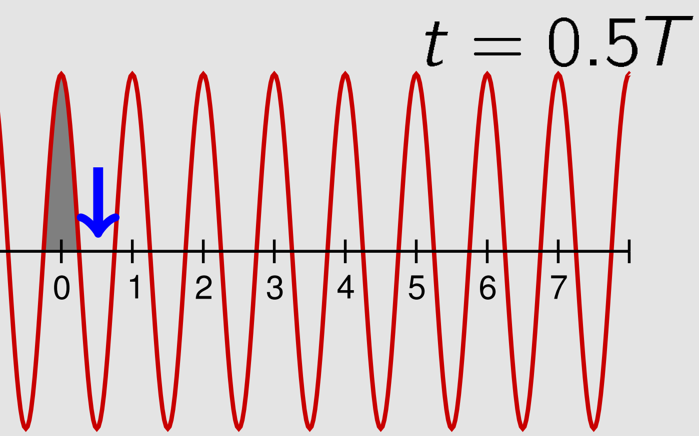


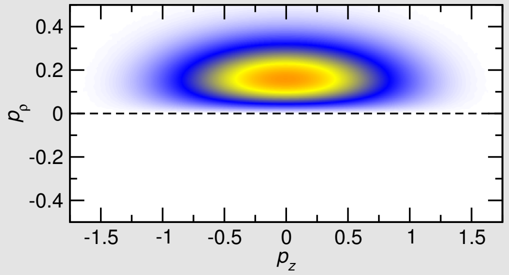
E_{ip}=15eV
λ=2000nm
I=10^{14}W/cm^2

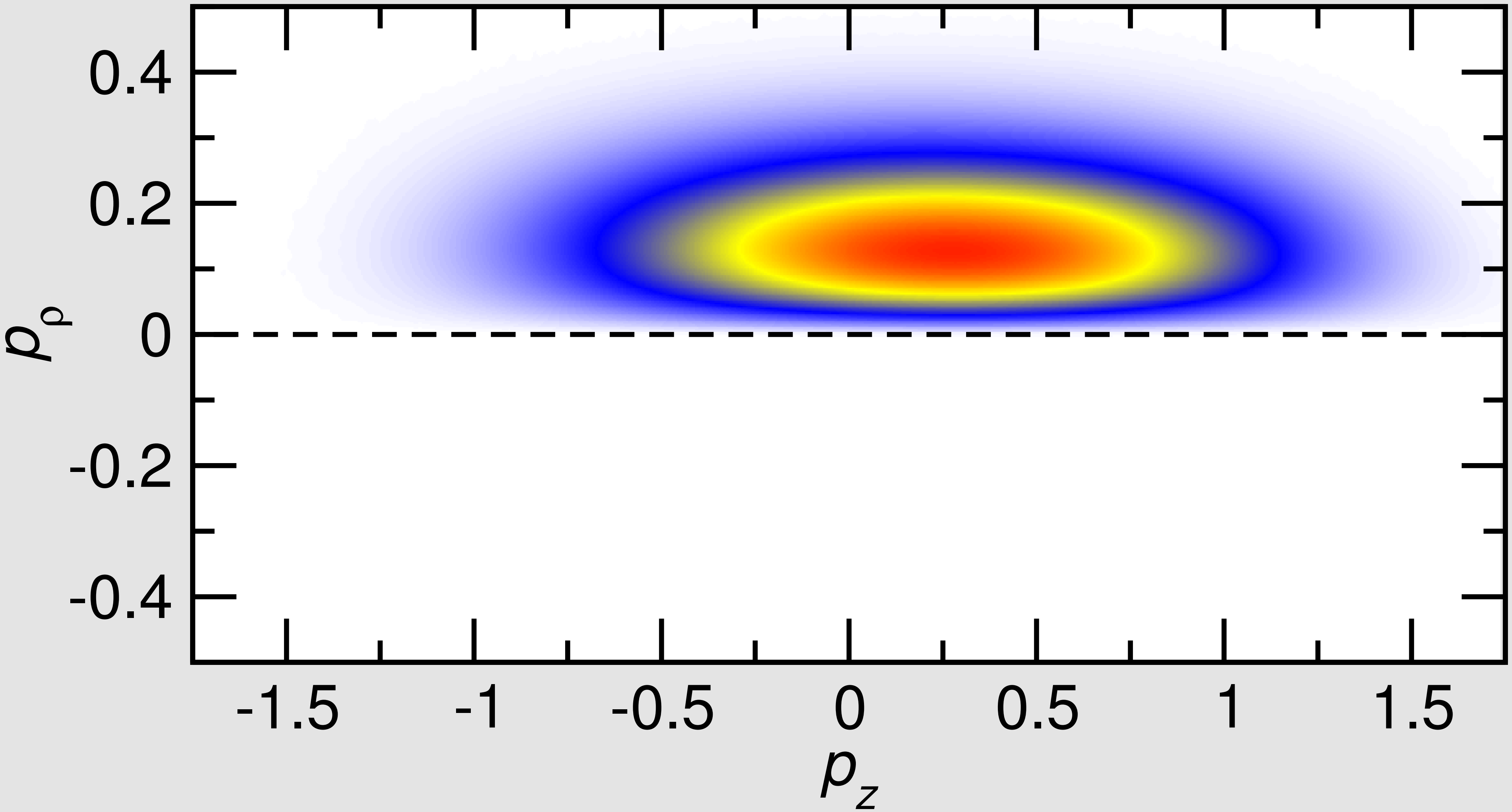
probability for $F = F(t')$

$$W(F, p'_\rho) \propto \frac{p'_\rho}{\sqrt{1+p'^2_\rho/2E_{\mathrm{ip}}}} \frac{e^{-2(2E_{\mathrm{ip}}+p'^2_\rho)^{3/2}/3F}}{F^2}$$

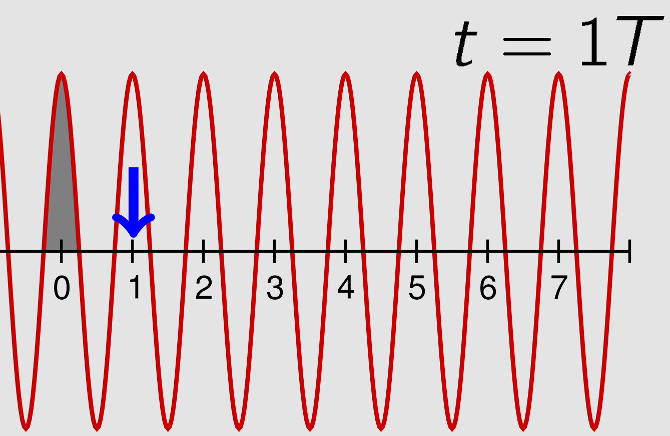
momentum distribution

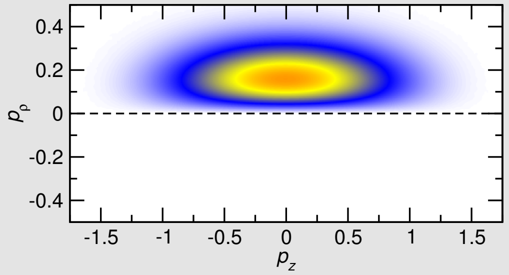


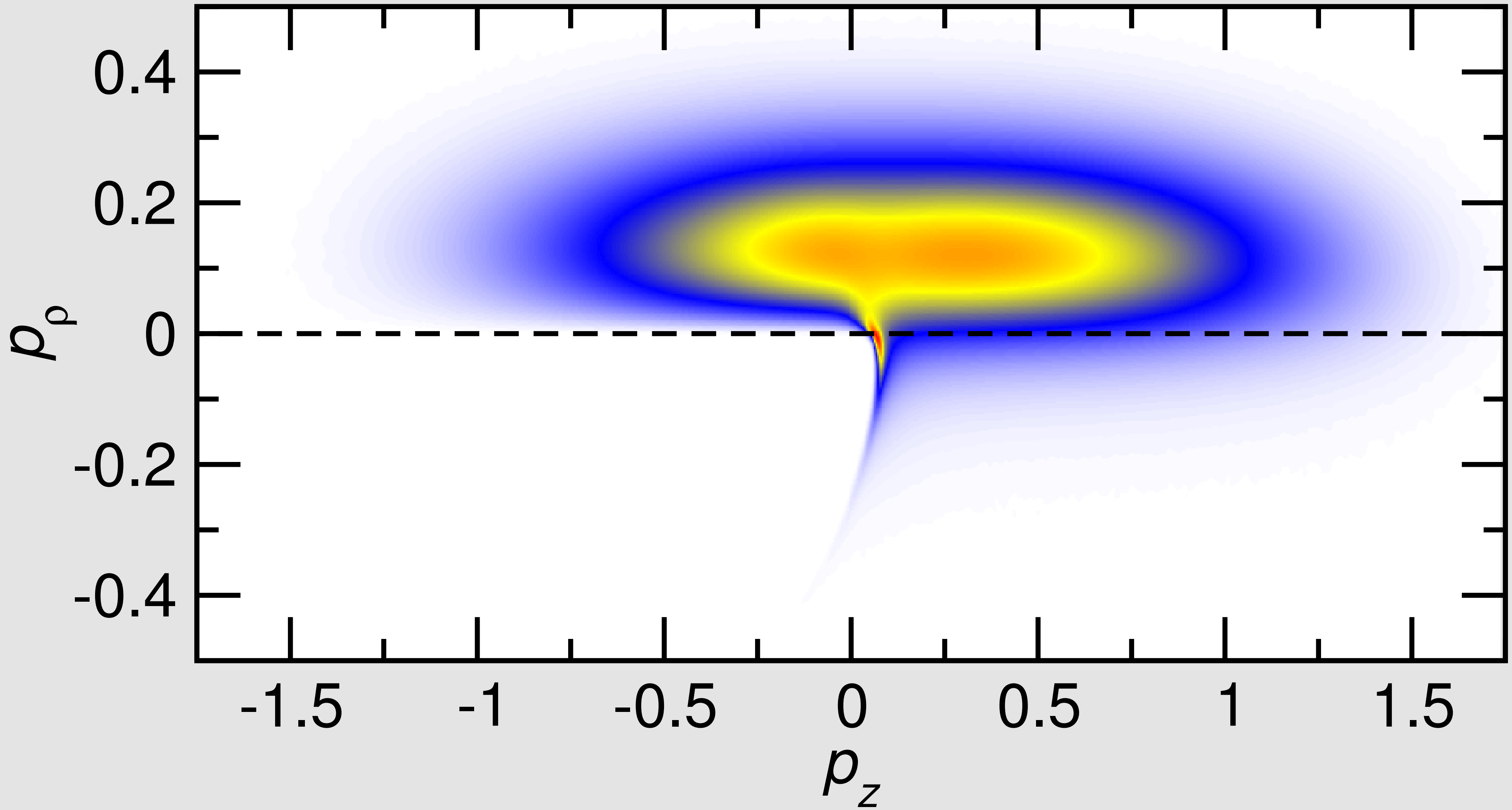




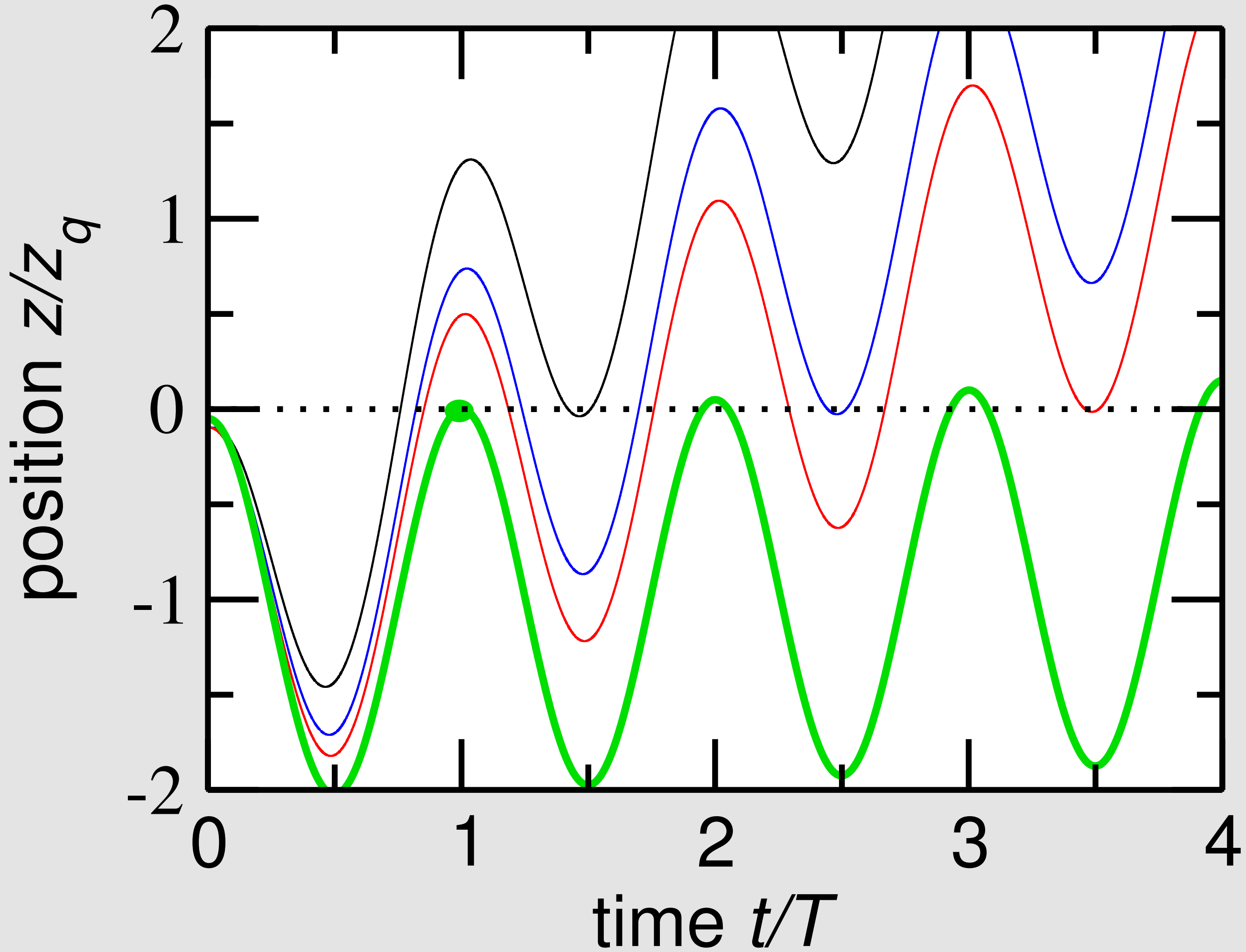
momentum distribution





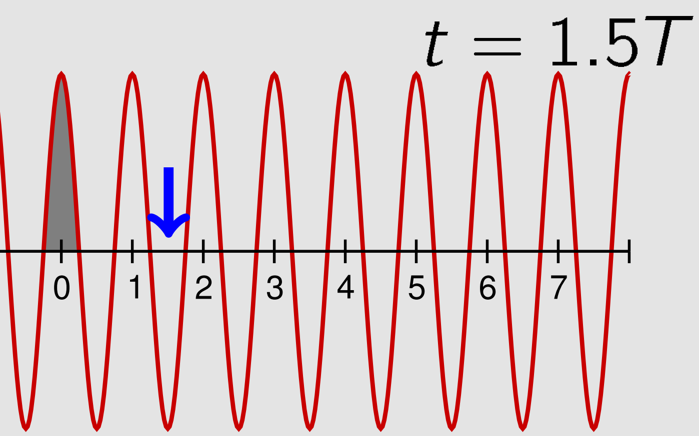


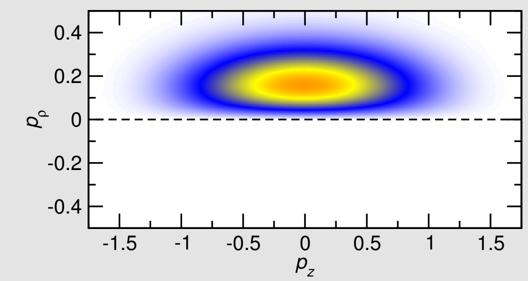
series of LES peaks



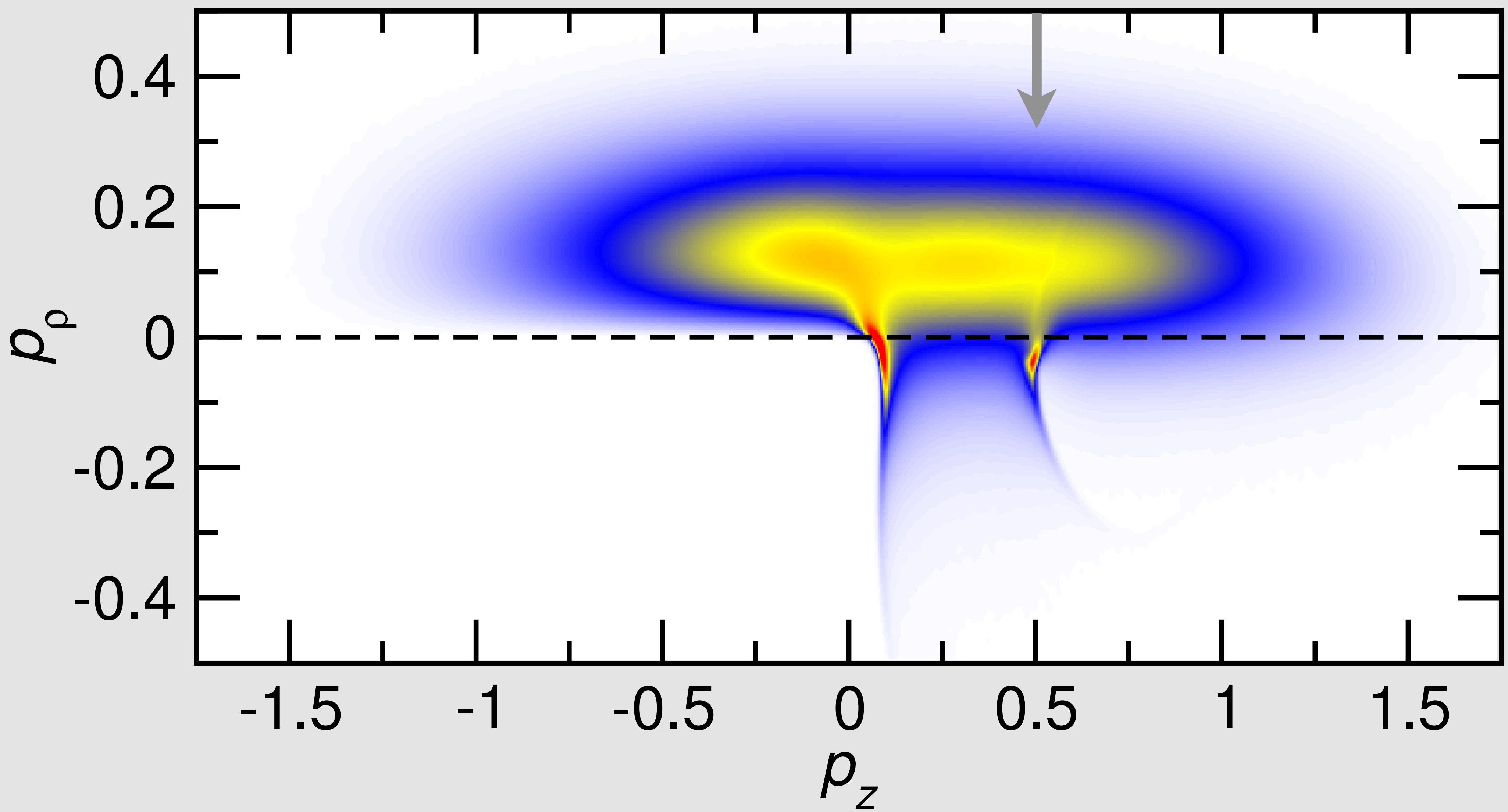
- **analogous bunching**
- **no field strength dependence**
- **at very low energies → VLES**

momentum distribution

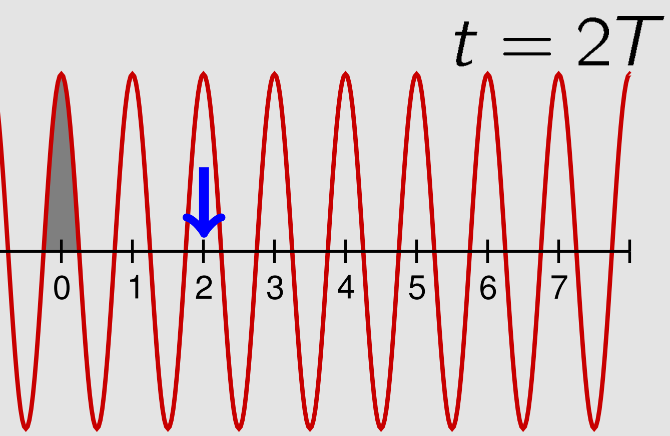


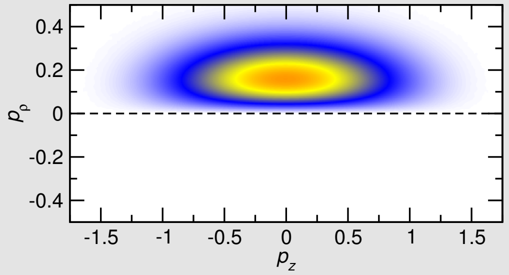


$$p_k = \frac{F}{\omega} \frac{2}{(2k+1)\pi}$$

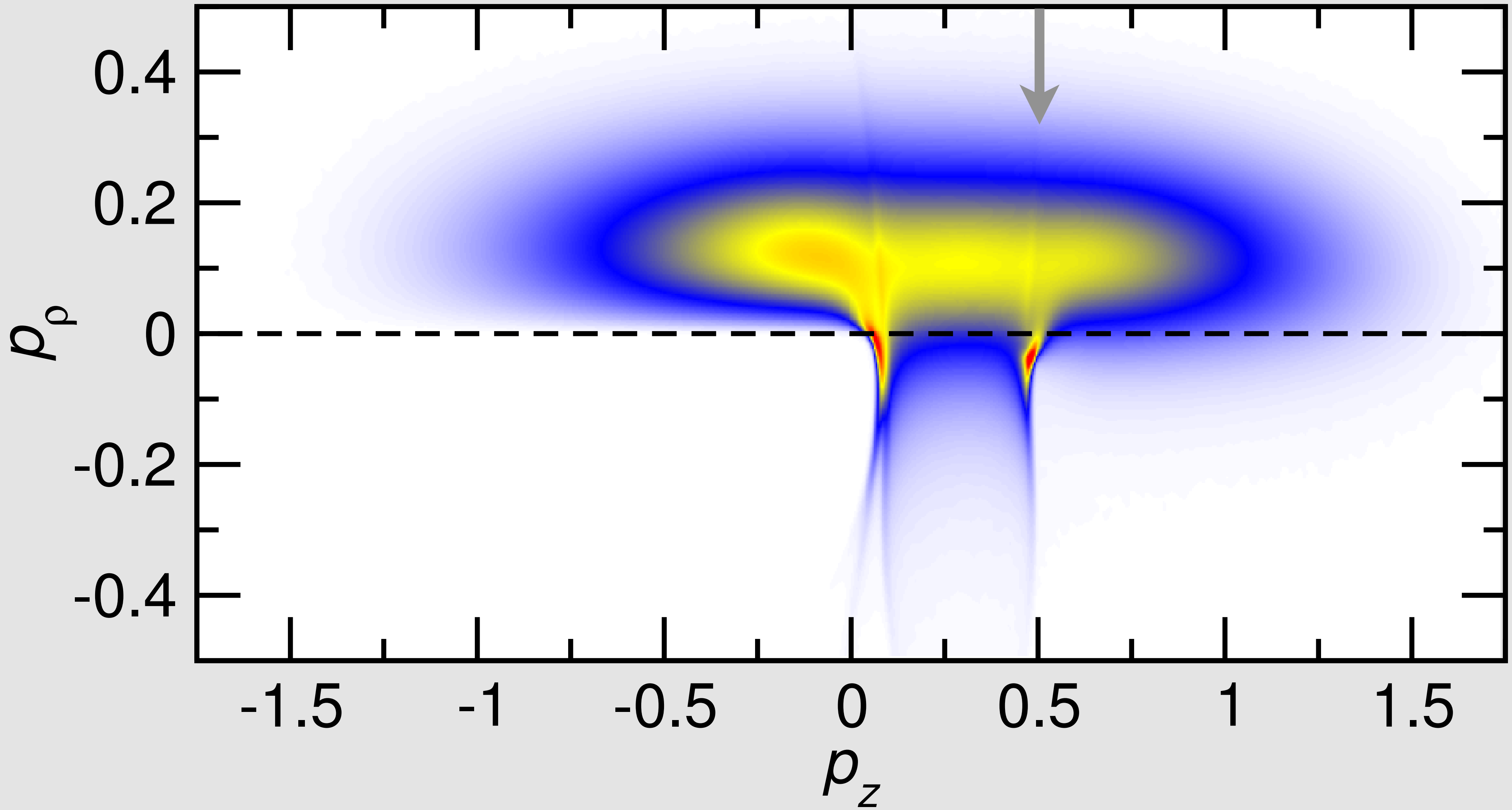


momentum distribution

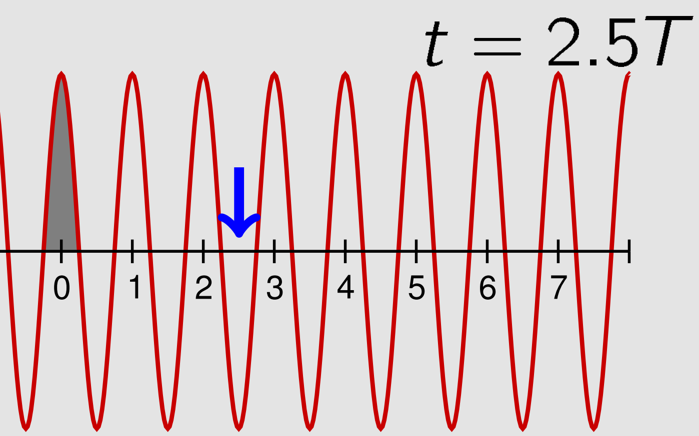


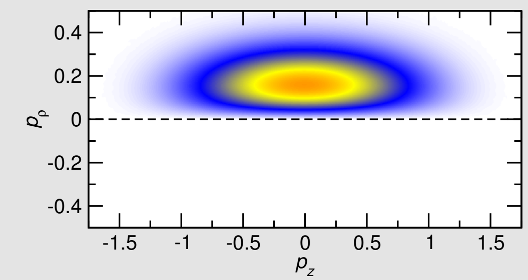


$$p_k = \frac{F}{\omega}\frac{2}{(2k+1)\pi}$$

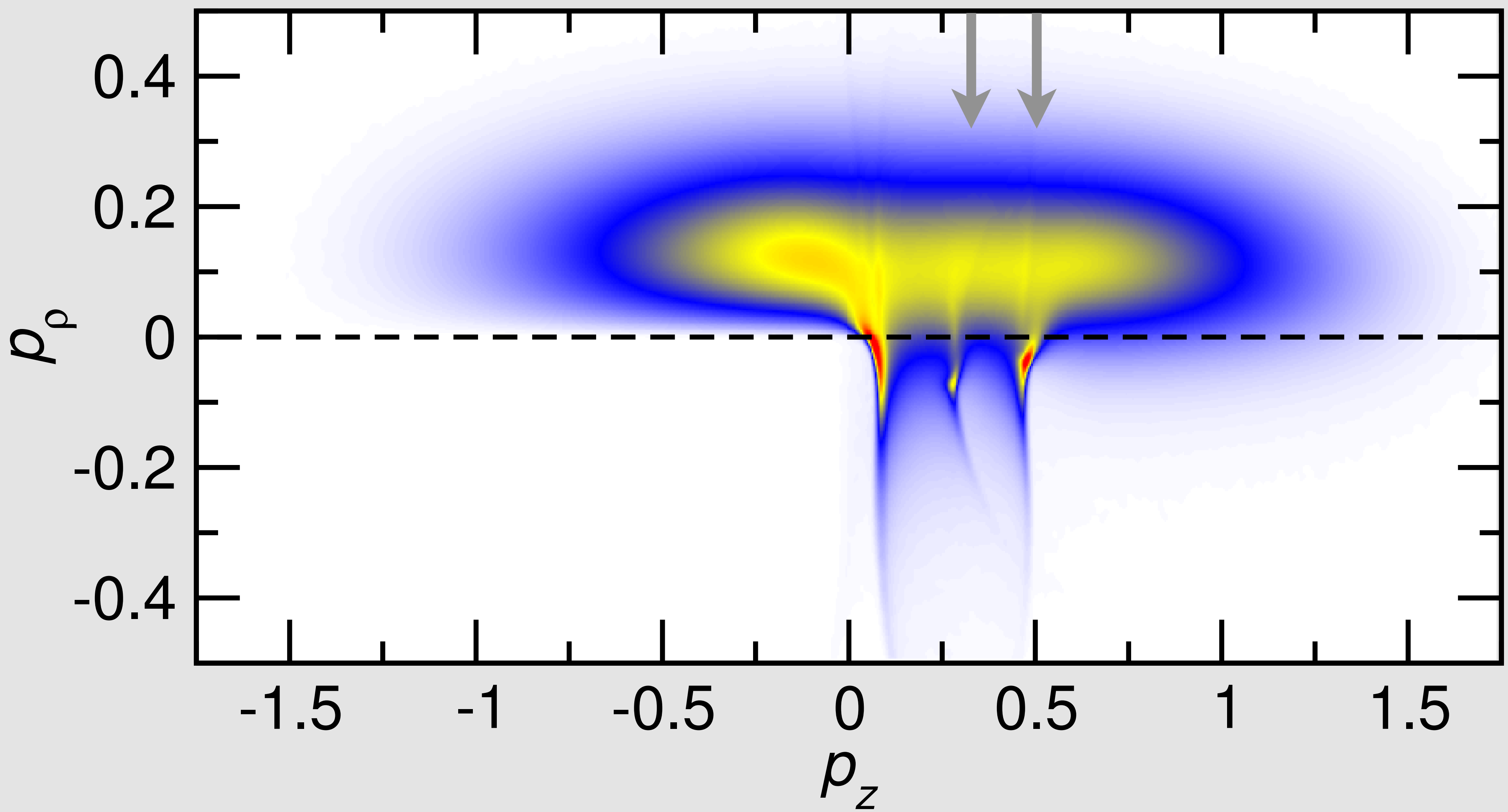


momentum distribution

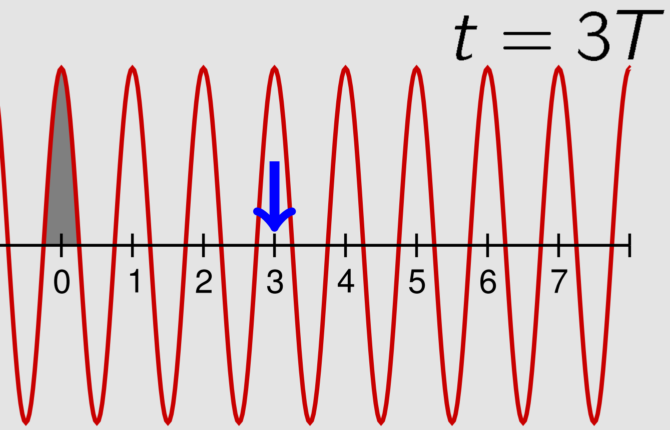


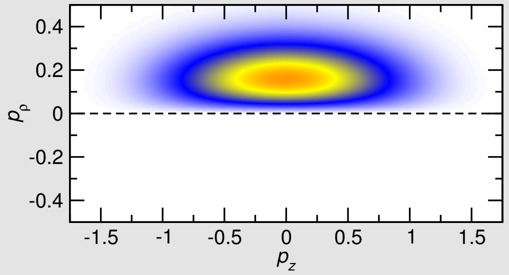


$$p_k = \frac{F}{\omega}\frac{2}{(2k+1)\pi}$$

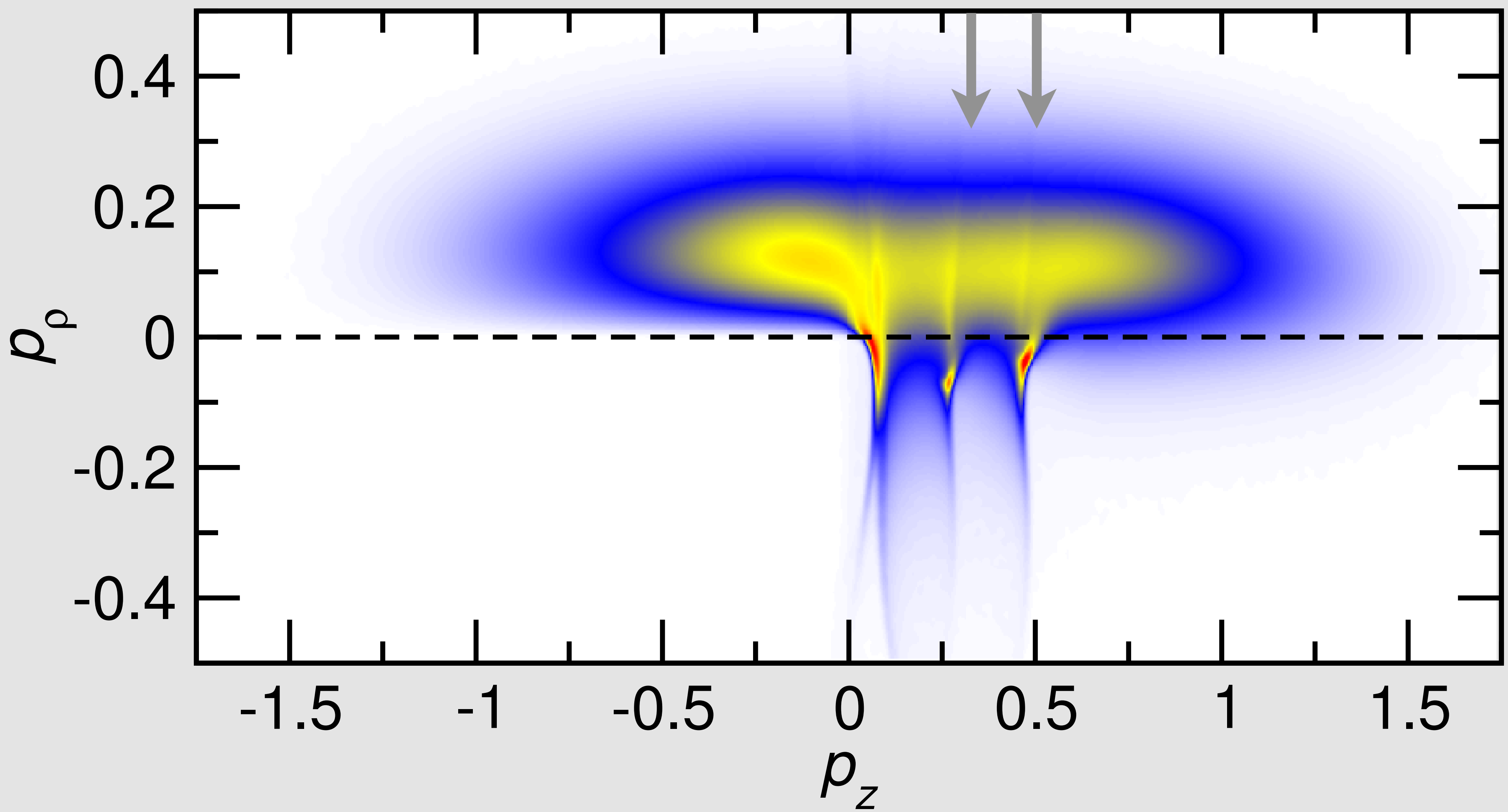


momentum distribution

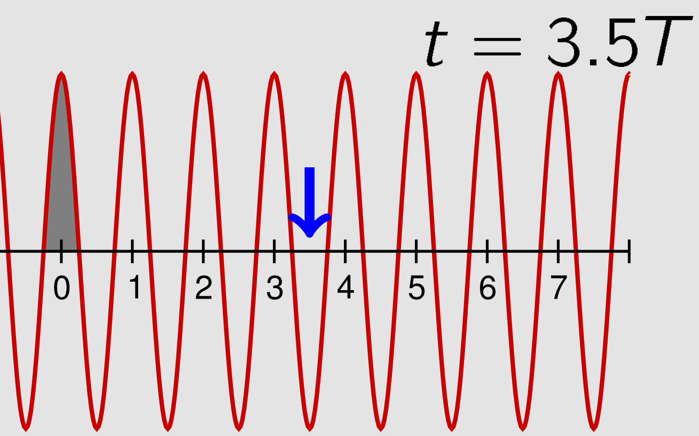


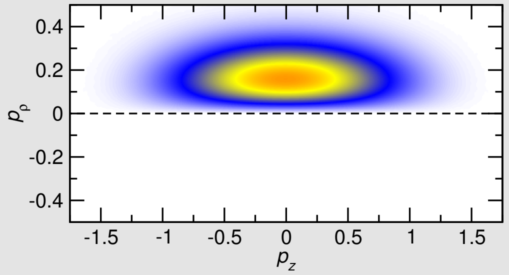


$$p_k = \frac{F}{\omega} \frac{2}{(2k+1)\pi}$$

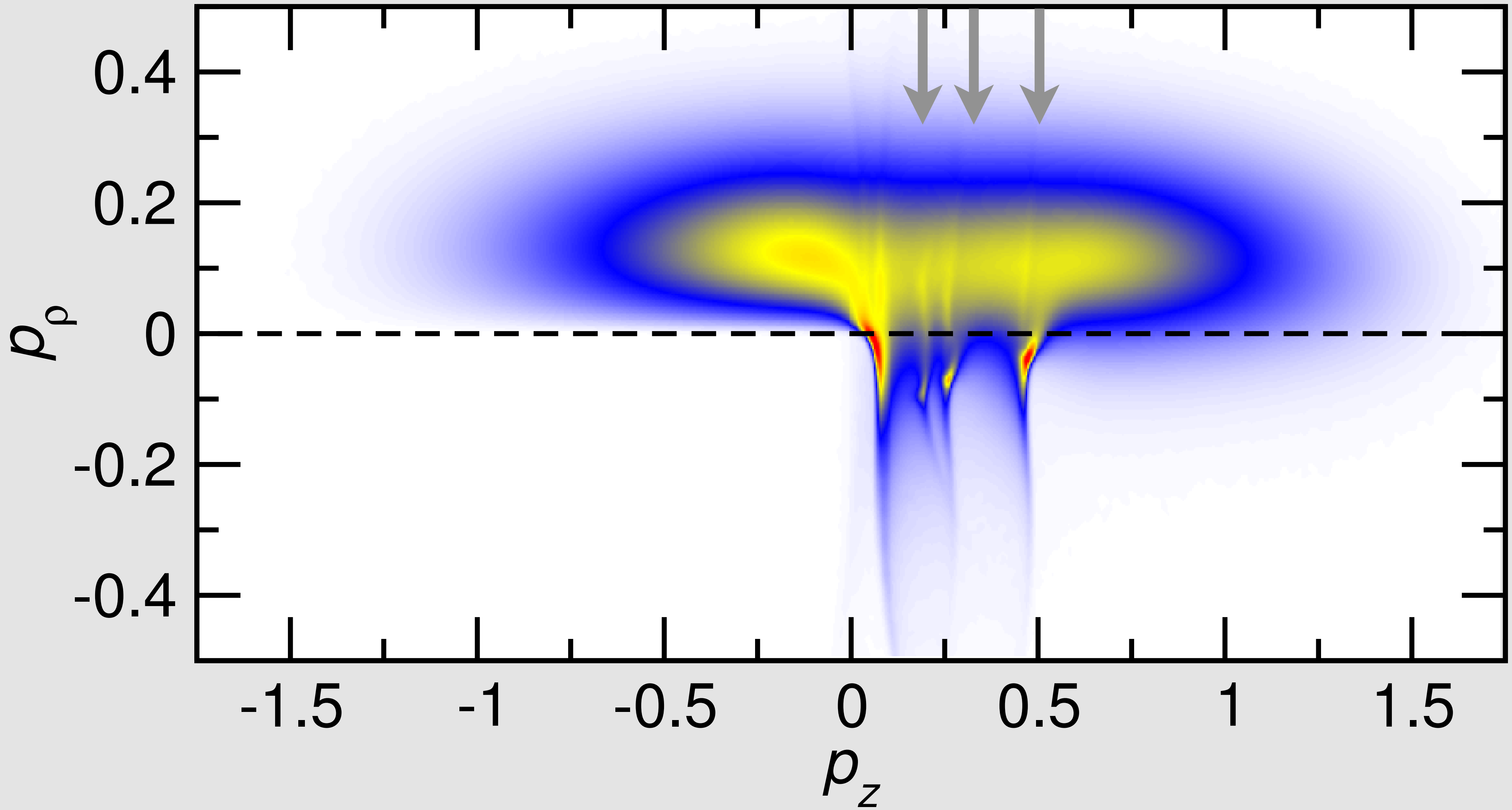


momentum distribution

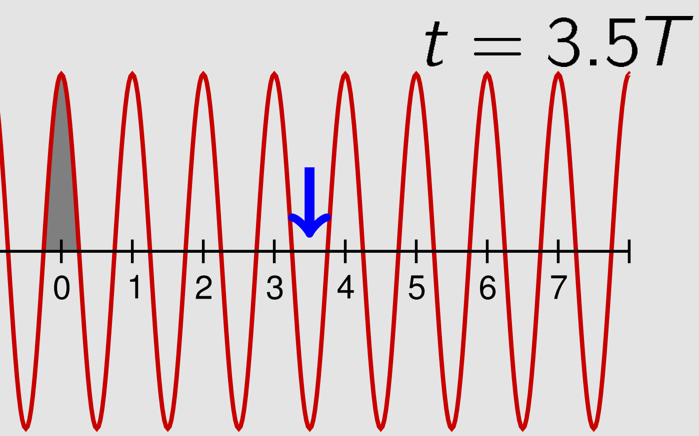




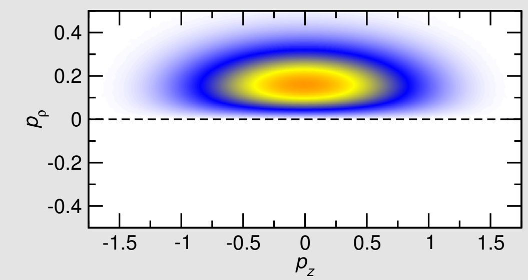
$$p_k = \frac{F}{\omega}\frac{2}{(2k+1)\pi}$$



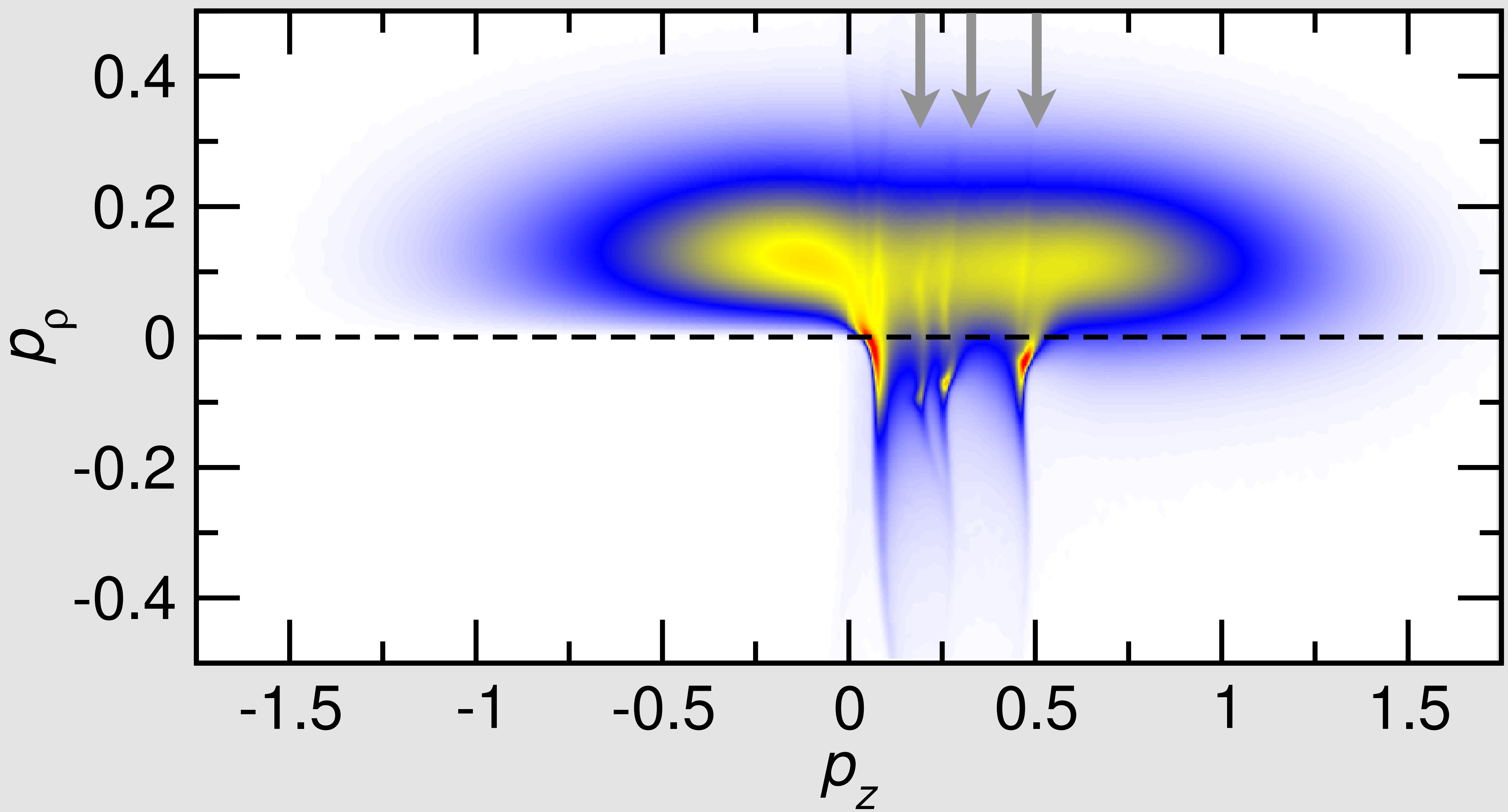
momentum distribution



no ZES here!

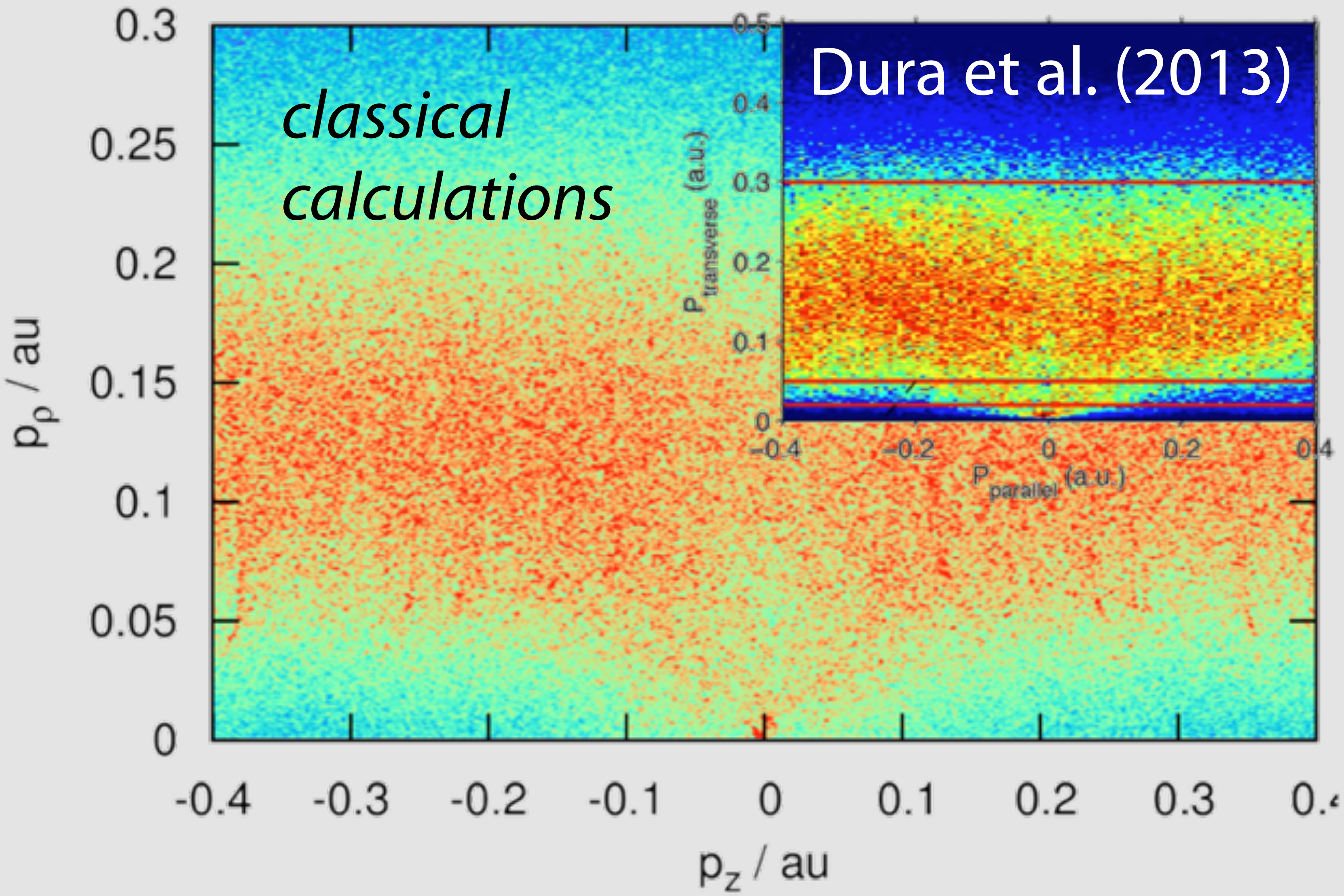


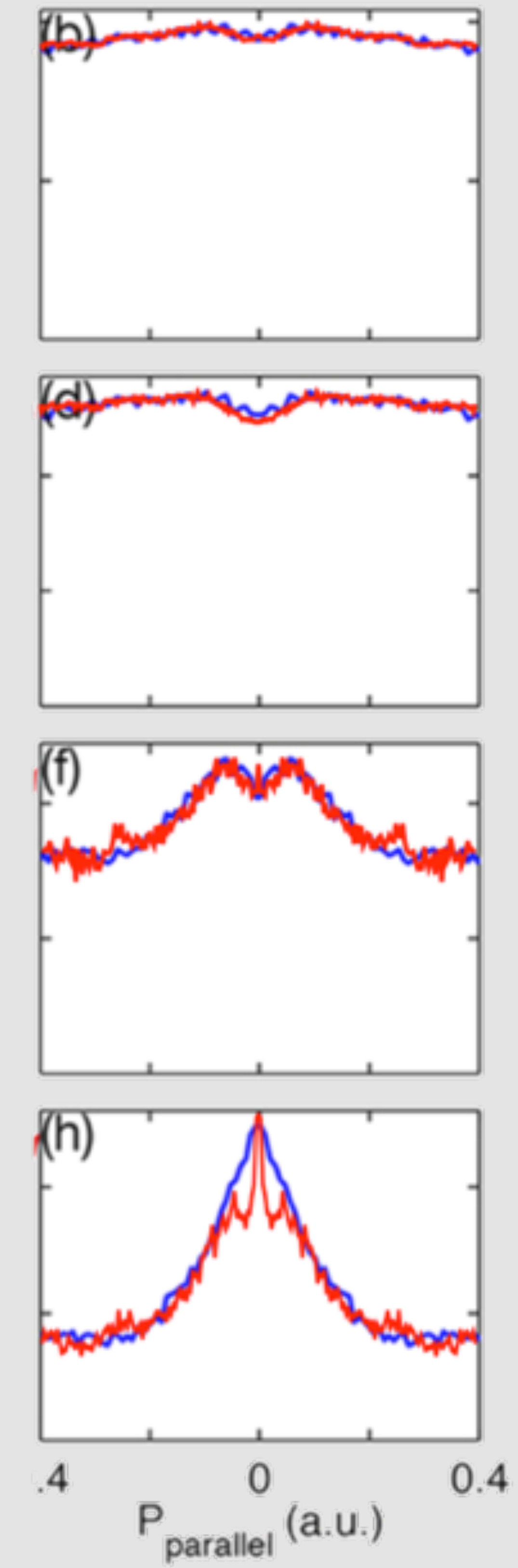
$$p_k = \frac{F}{\omega} \frac{2}{(2k+1)\pi}$$



even slower

calculations vs. measurement

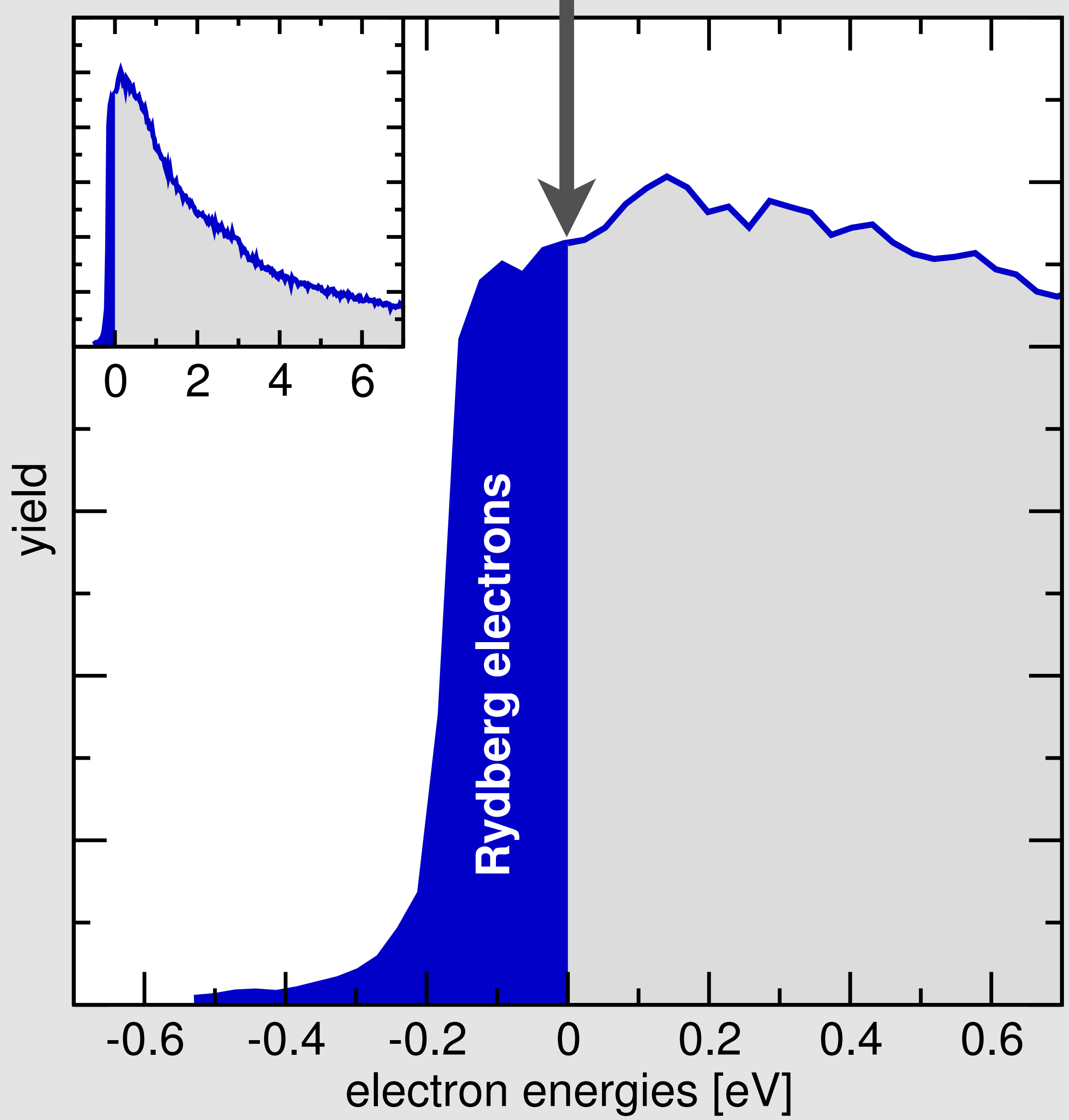




agreement if extraction field included!

“zero”-energy electrons

only electrons with *E*>-1meV
are additionally extracted

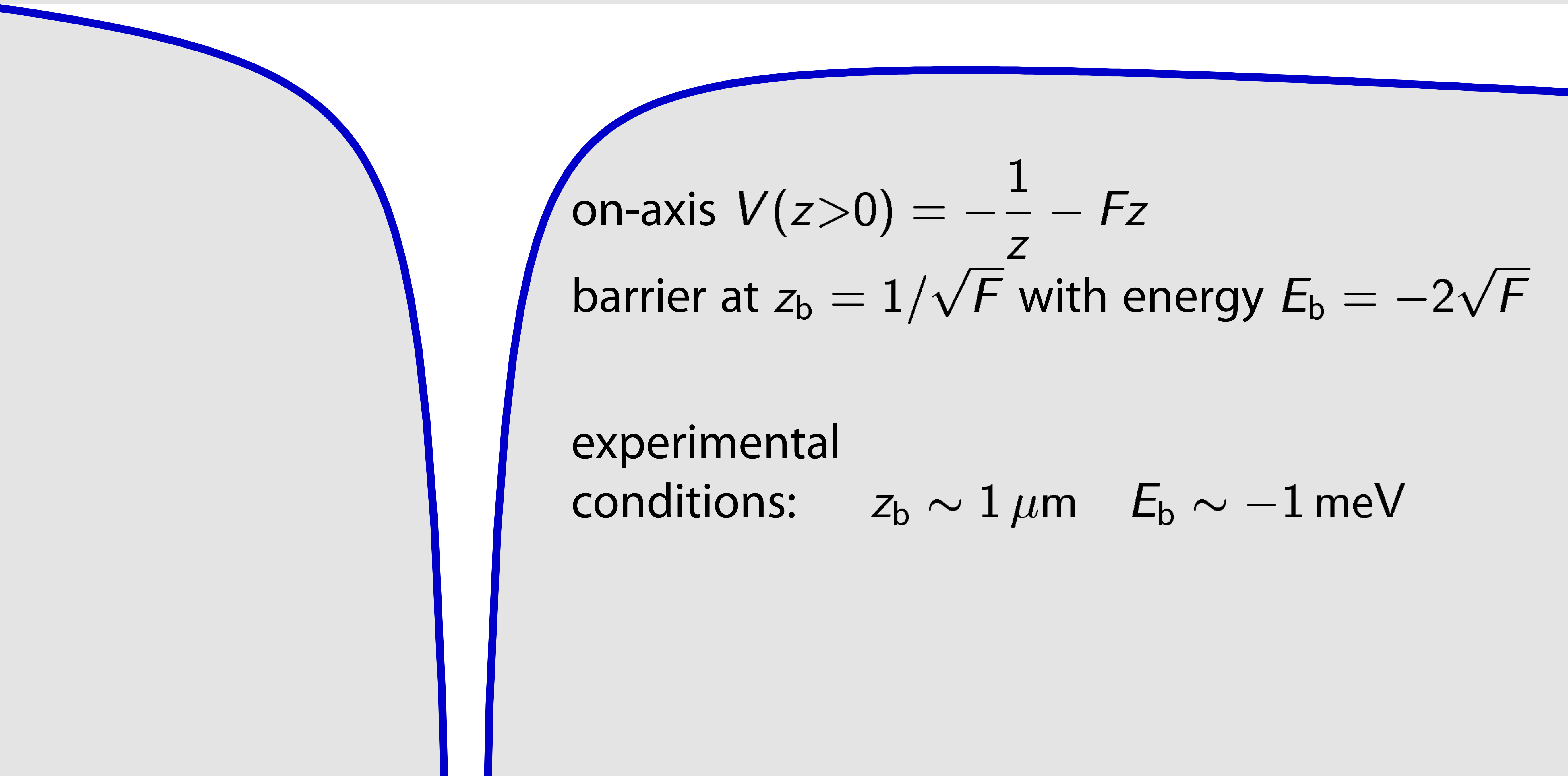


“frustrated tunneling”
Nubbemeyer et al.
PRL 101, 233001 (2008)

Why do they form a peak?

"zero"-energy electrons

extraction field matters,
despite being weak F~1...10V/cm~10^{-9}au

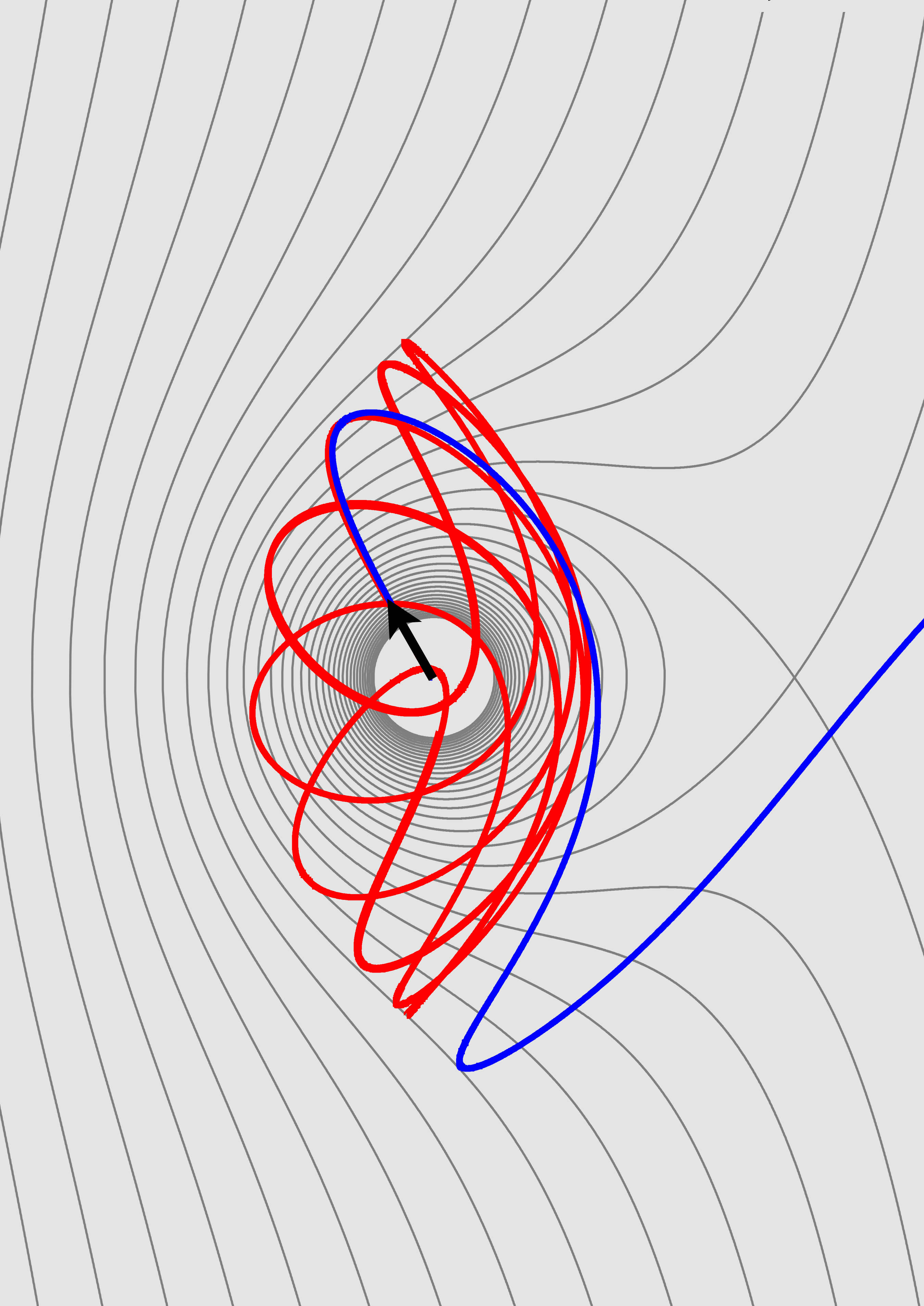


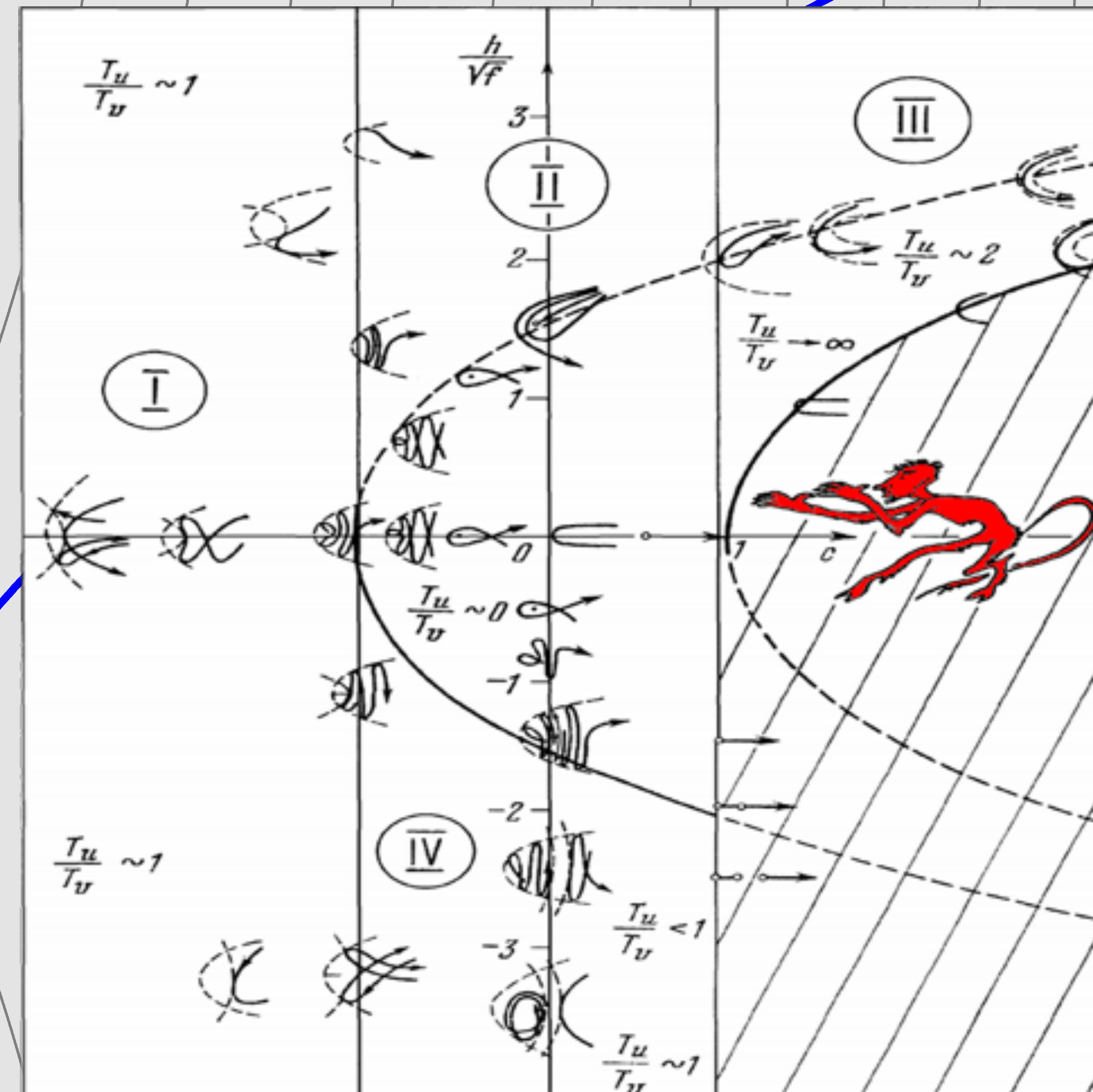
on-axis $V(z\!>\!0) = -\dfrac{1}{z} - Fz$

barrier at $z_\mathrm{b} = 1/\sqrt{F}$ with energy $E_\mathrm{b} = -2\sqrt{F}$

experimental
conditions: $z_\mathrm{b} \sim 1\,\mu\mathrm{m} \quad E_\mathrm{b} \sim -1\,\mathrm{meV}$

typical above-the-barrier trajectories





“Essays on the Motion of Celestial Bodies” V. V. Beletsky

standard Stark problem

$$\tilde{H} = \frac{\tilde{p}_\rho{}^2}{2} + \frac{\tilde{p}_z{}^2}{2} - \frac{1}{\sqrt{\tilde{\rho}^2+\tilde{z}^2}} - F\tilde{z}$$

original Hamiltonian

proper scaling $\frac{1}{F^{1/2}}\begin{pmatrix}\rho \\ z\end{pmatrix}, \boxed{F^{1/4}\begin{pmatrix}p_\rho \\ p_z\end{pmatrix}}, \; F^{1/2}E, \; \frac{1}{F^{3/4}}t$

$$H = \frac{p_\rho{}^2}{2} + \frac{p_z{}^2}{2} - \frac{1}{\sqrt{\rho^2+z^2}} - z$$

generic Hamiltonian

standard Stark problem

$$H = \frac{p_\rho{}^2}{2} + \frac{p_z{}^2}{2} - \frac{1}{\sqrt{\rho^2+z^2}} - z$$

generic Hamiltonian

$$u \equiv \sqrt{r+z},\ v \equiv \sqrt{r-z} \qquad \left(r = \sqrt{\rho^2+z^2}\right)$$

separated Hamiltonian

$$H_u = \frac{p_u{}^2}{2} - E\,u^2 - \frac{u^4}{2} - 1$$

$$H_v = \frac{p_v{}^2}{2} - E\,v^2 + \frac{v^4}{2} - 1$$

$$E_u + E_v = 0$$

$$t = \int_0^\tau \mathrm{d}\tau' \left[u^2(\tau') + v^2(\tau')\right]$$

standard Stark problem

separated Hamiltonian

$$H_u = \frac{{p_u}^2}{2} - E\,u^2 - \frac{u^4}{2} - 1$$

$$H_v = \frac{{p_v}^2}{2} - E\,v^2 + \frac{v^4}{2} - 1$$

$$E_u + E_v = 0$$

$$t = \int_0^\tau \mathrm{d}\tau' \left[u^2(\tau') + v^2(\tau')\right]$$

standard Stark problem

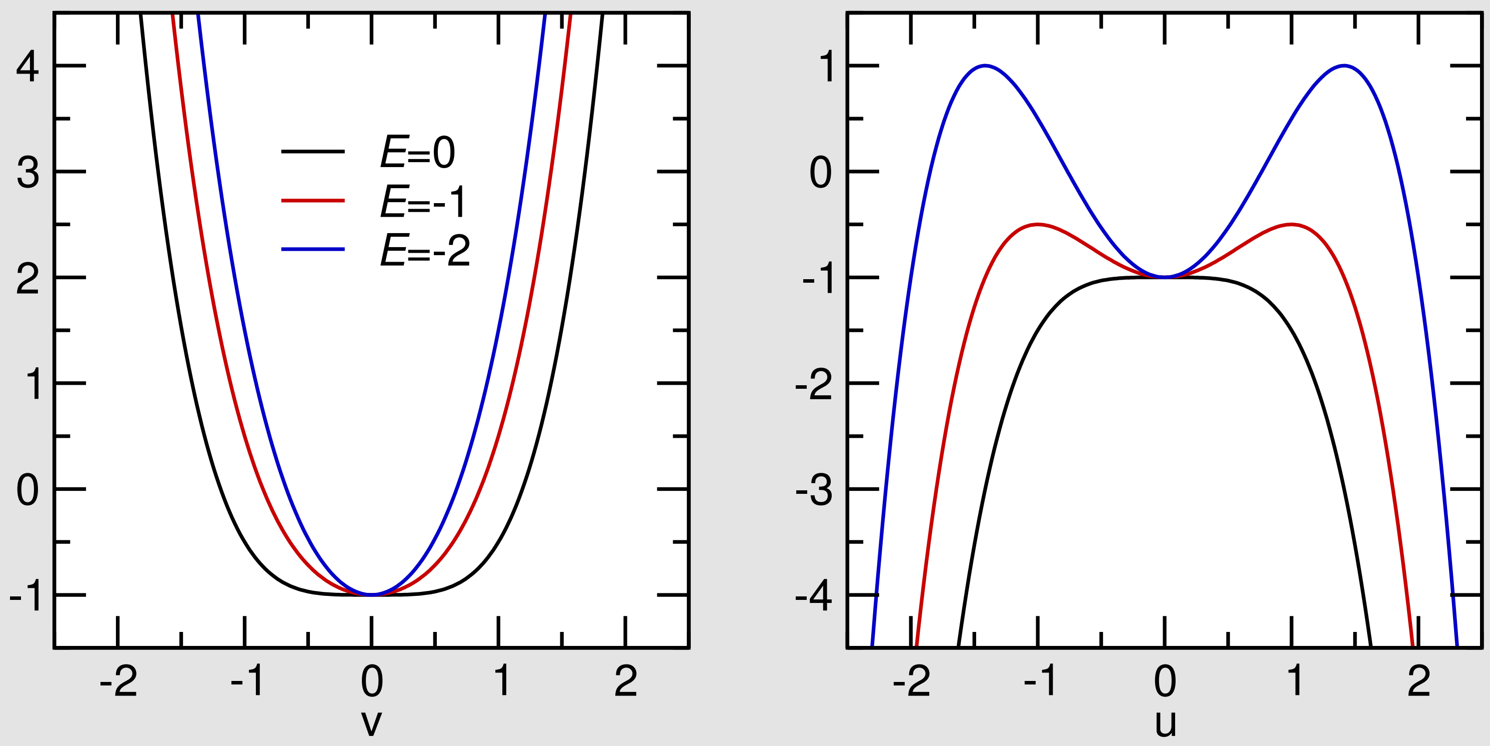
separated
Hamiltonian

$$H_u = \frac{{p_u}^2}{2} - E\,u^2 - \frac{u^4}{2} - 1$$

$$H_v = \frac{{p_v}^2}{2} - E\,v^2 + \frac{v^4}{2} - 1$$

$$E_u + E_v = 0$$

$$t = \int_0^\tau \mathrm{d}\tau' \left[u^2(\tau') + v^2(\tau')\right]$$



for $E < 0$

barrier at $u_\mathrm{b} = \sqrt{-E}$

with energy $E_\mathrm{b} = \frac{E^2}{2} - 1$

standard Stark problem

separated Hamiltonian

$$H_u = \frac{{p_u}^2}{2} - E\,u^2 - \frac{u^4}{2} - 1$$

$$H_v = \frac{{p_v}^2}{2} - E\,v^2 + \frac{v^4}{2} - 1$$

$$E_u + E_v = 0$$

$$t = \int_0^{\tau} \mathrm{d}\tau' \left[u^2(\tau') + v^2(\tau')\right]$$

quiver amplitude (~nm)

barrier distance (~ µm)

only trajectories starting at the Coulomb center characterized by energy *E* and angle θ

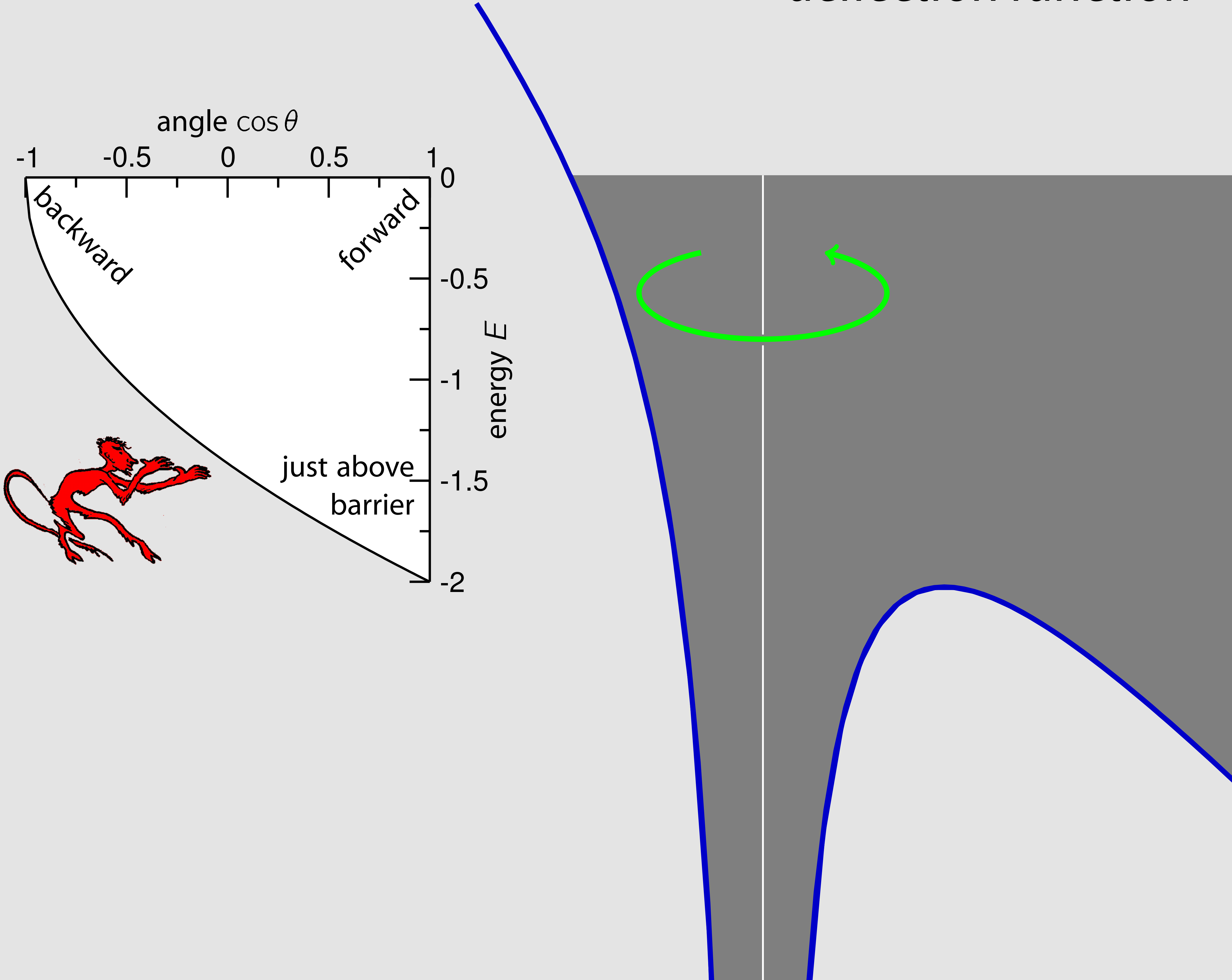
$$u(0) = 0 \qquad v(0) = 0$$

$$p_u(0) = \cos\left(\tfrac{\theta}{2}\right) \qquad p_v(0) = \sin\left(\tfrac{\theta}{2}\right)$$

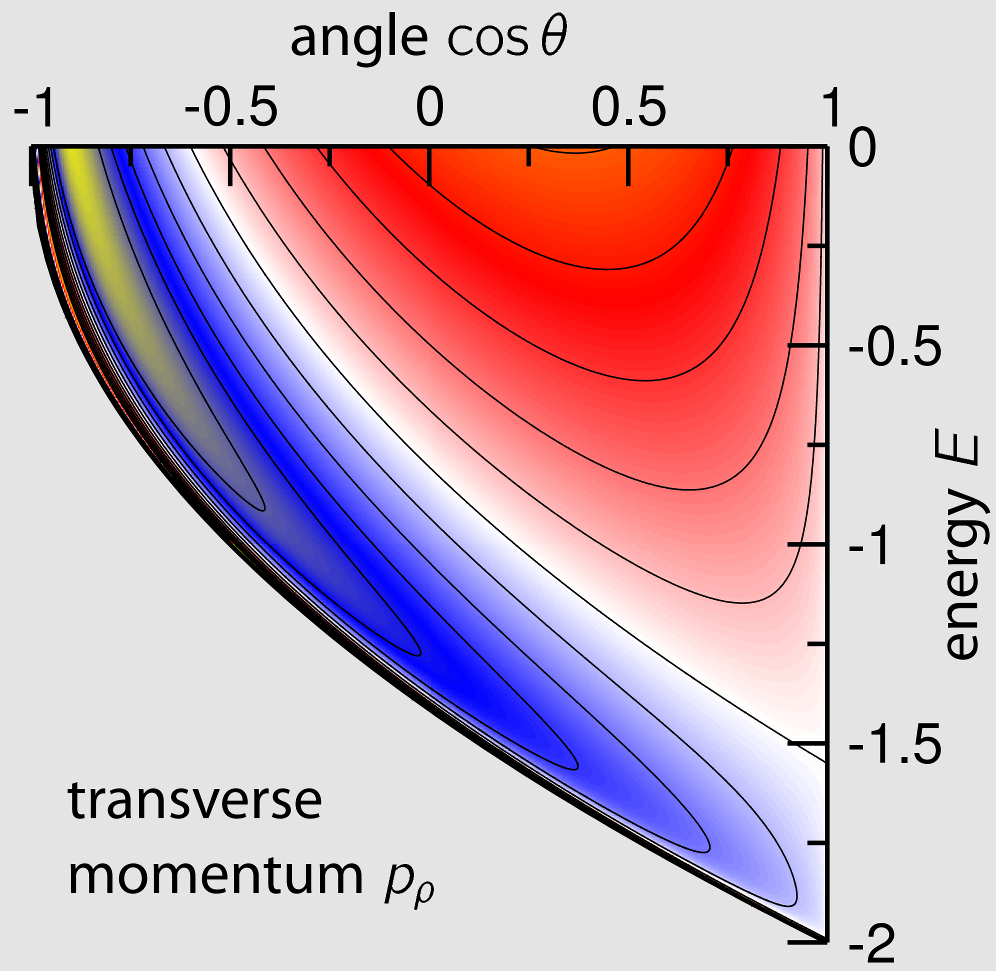
$$E_u = +\cos\theta \qquad E_v = -\cos\theta$$

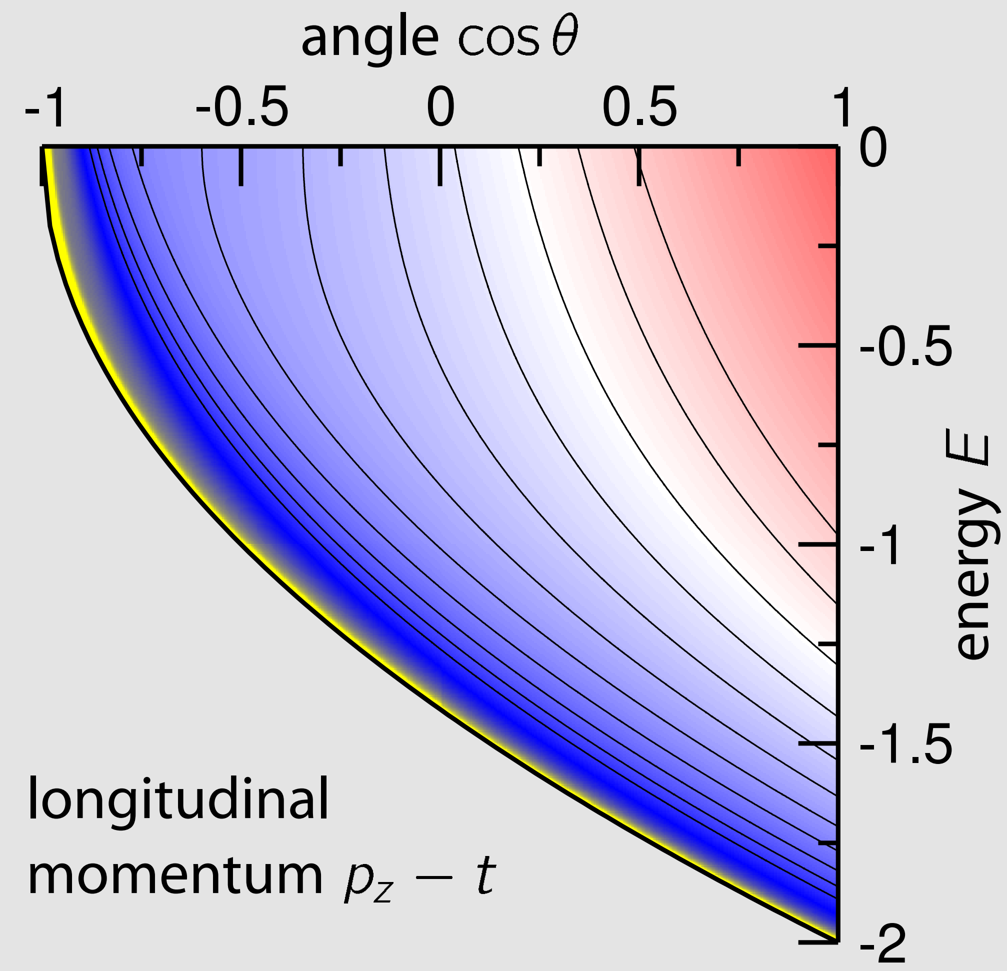
trajectories are being trapped if $E_u = \cos\theta \le E^2/2 - 1$

deflection function



2D deflection function





$$p_z(t) = p_z^{\text{init}} + Ft \quad \text{or} \quad p_z^{\text{init}} = p_z(t) - Ft \to \text{const}$$

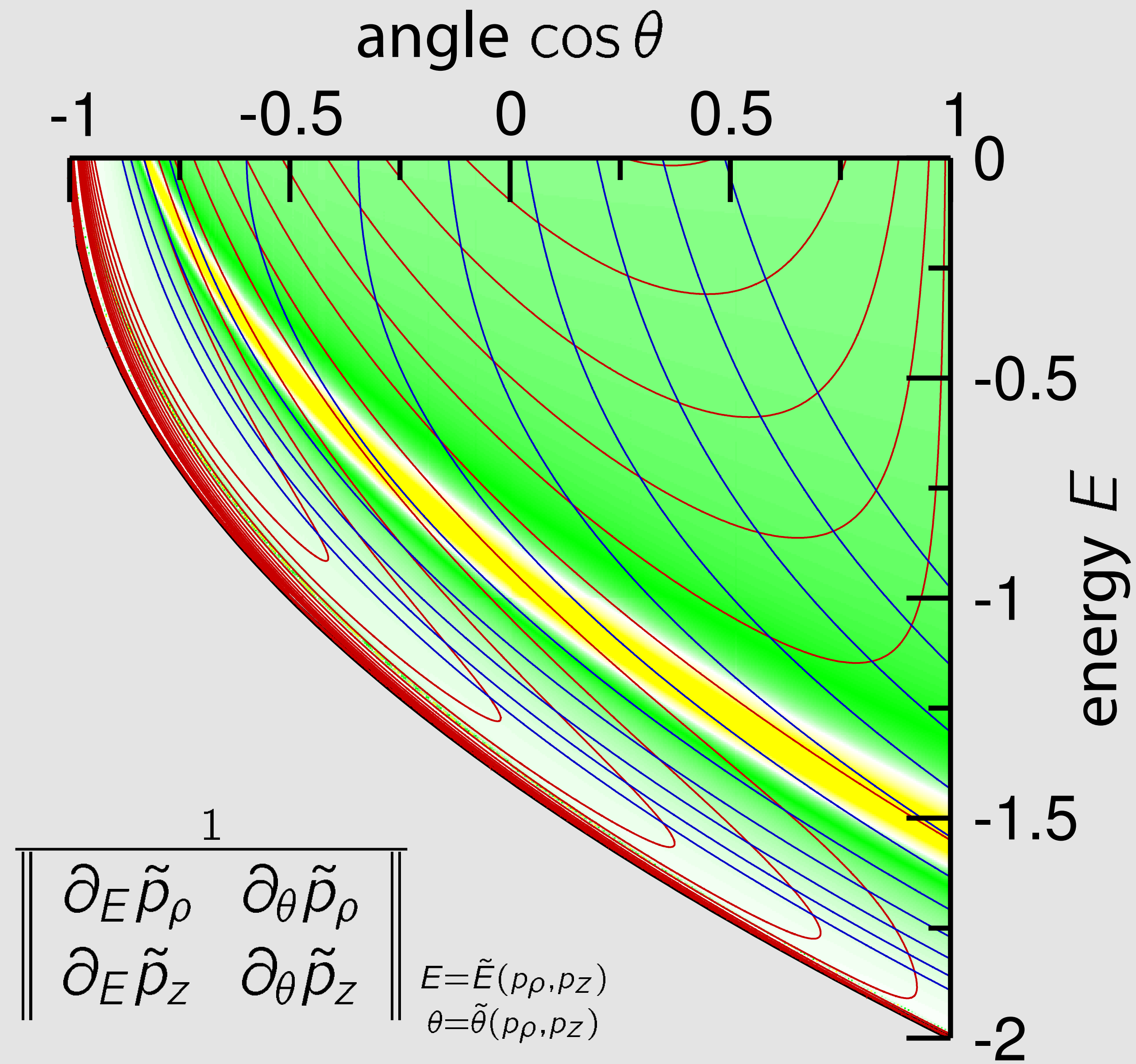
2D deflection function

$$P(p_\rho, p_z) = \int \mathrm{d}E \int \mathrm{d}\theta \; \delta\big(p_\rho - \tilde{\mathsf{p}}_\rho(E,\theta)\big)\, \delta\big(p_z - \tilde{\mathsf{p}}_z(E,\theta)\big)$$

$$= \frac{1}{\left\| \begin{array}{cc} \partial_E \tilde{\mathsf{p}}_\rho & \partial_\theta \tilde{\mathsf{p}}_\rho \\ \partial_E \tilde{\mathsf{p}}_z & \partial_\theta \tilde{\mathsf{p}}_z \end{array} \right\|_{\substack{E=\tilde{E}(p_\rho, p_z) \\ \theta=\tilde{\theta}(p_\rho, p_z)}}}$$

peak for $\dfrac{\partial_E \tilde{\mathsf{p}}_\rho}{\partial_\theta \tilde{\mathsf{p}}_\rho} = \dfrac{\partial_E \tilde{\mathsf{p}}_z}{\partial_\theta \tilde{\mathsf{p}}_z}$, i. e. parallel contour lines

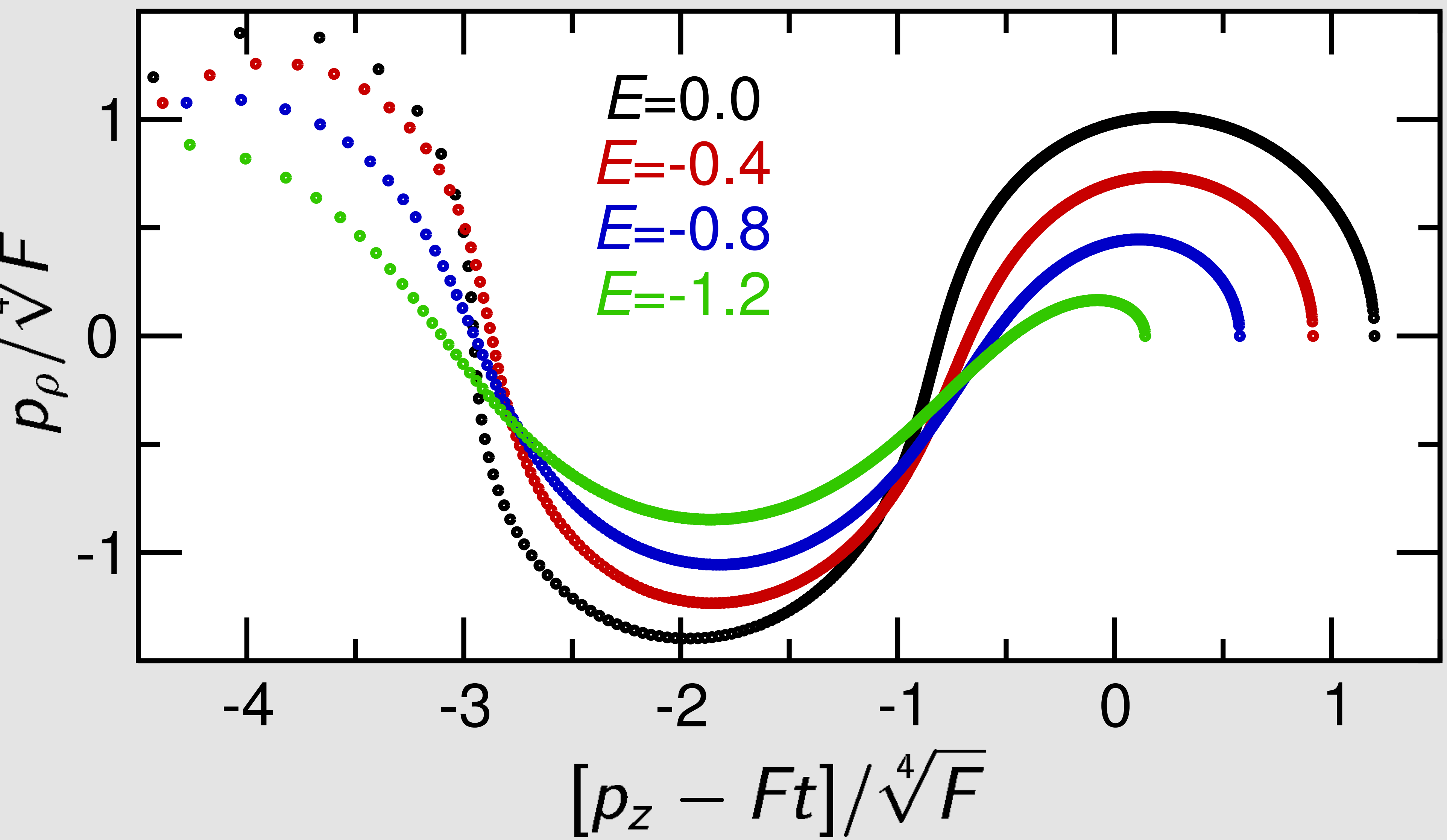
2D deflection function



→ ridge in the (p_ρ, p_z) spectrum

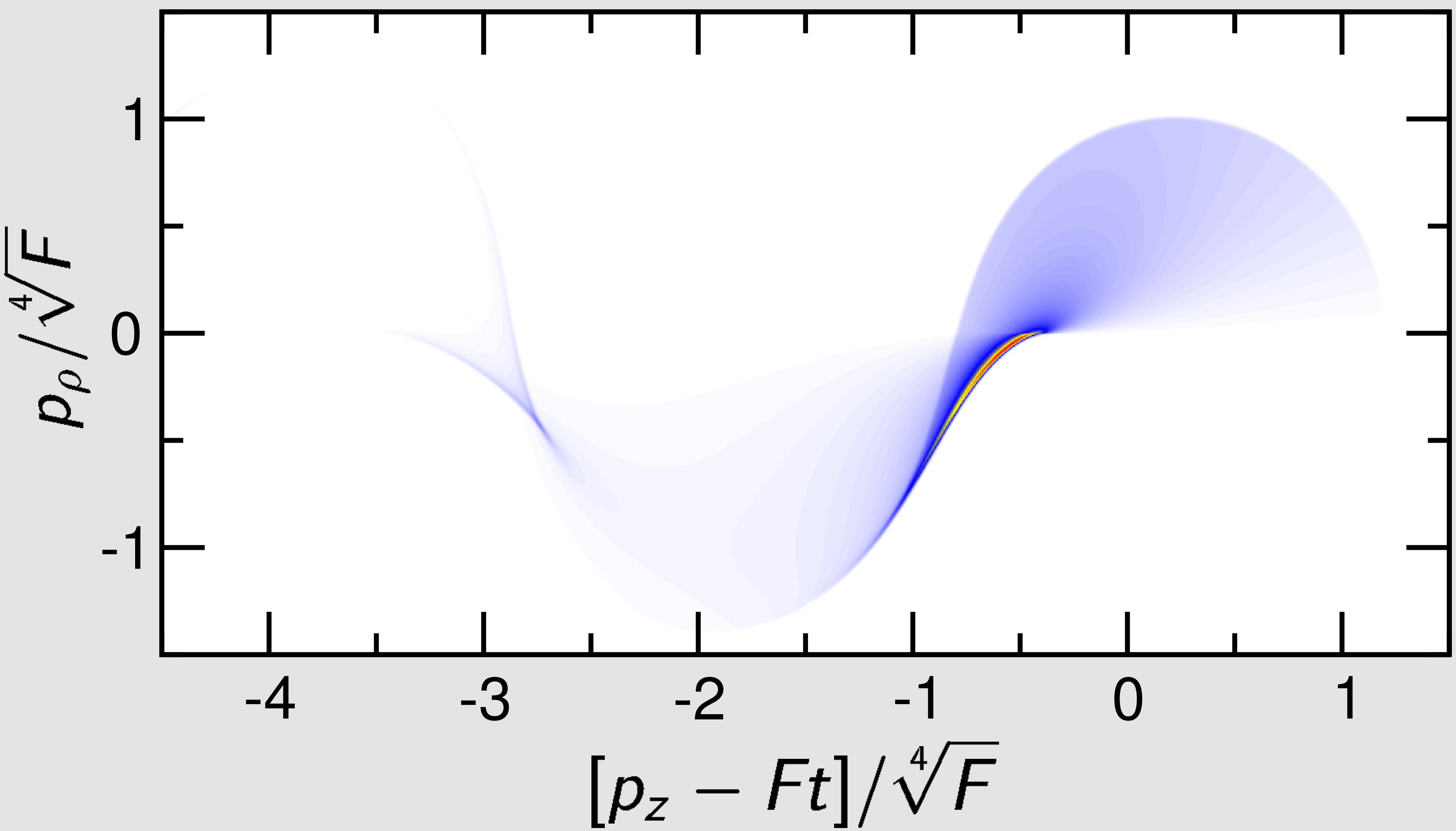
momentum distribution from Stark geometry

transverse and longitudinal momenta
for specific energies E and various angles θ



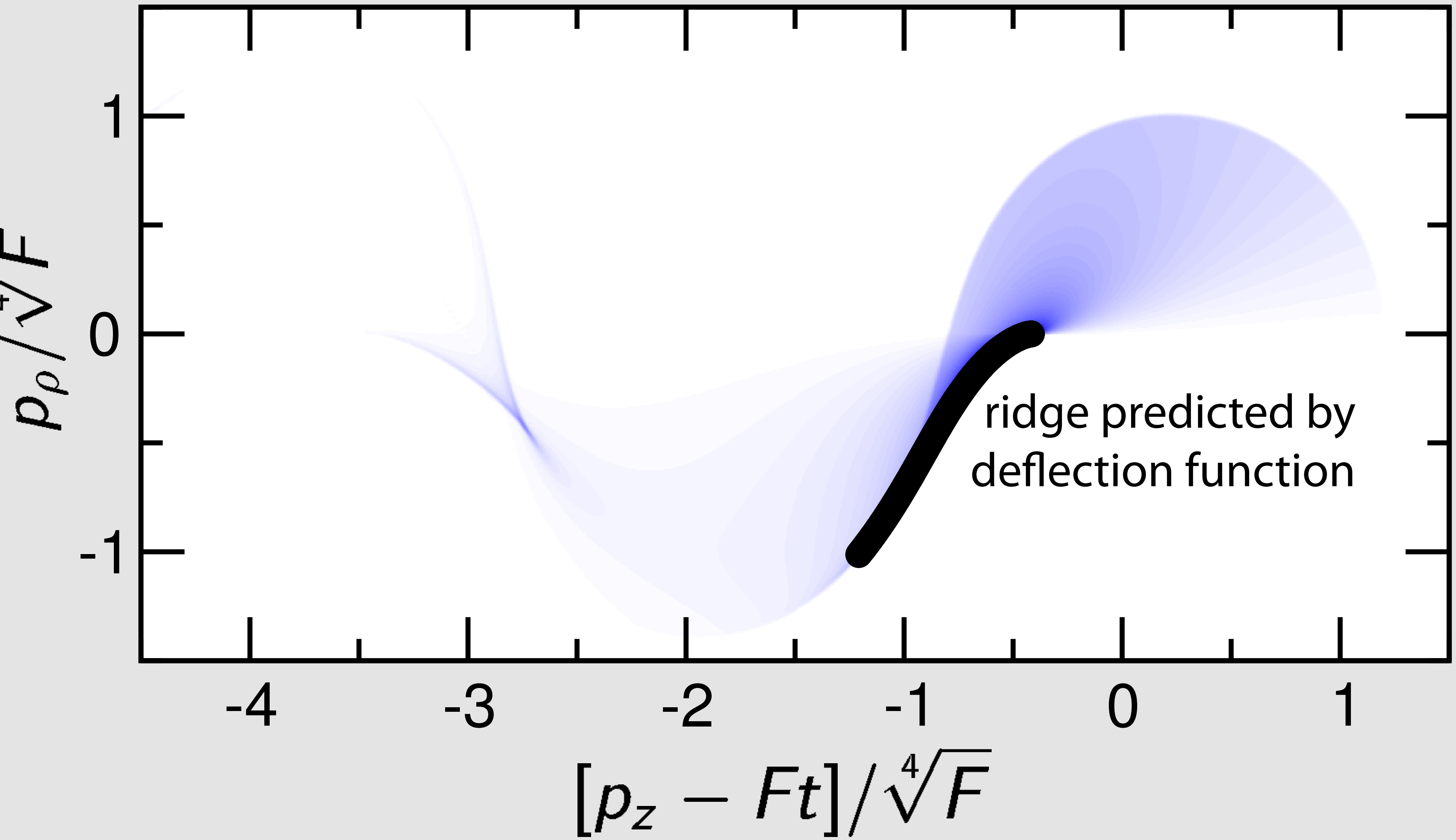
momentum distribution from Stark geometry

spectrum for uniform distribution
of energies *E* and angles θ

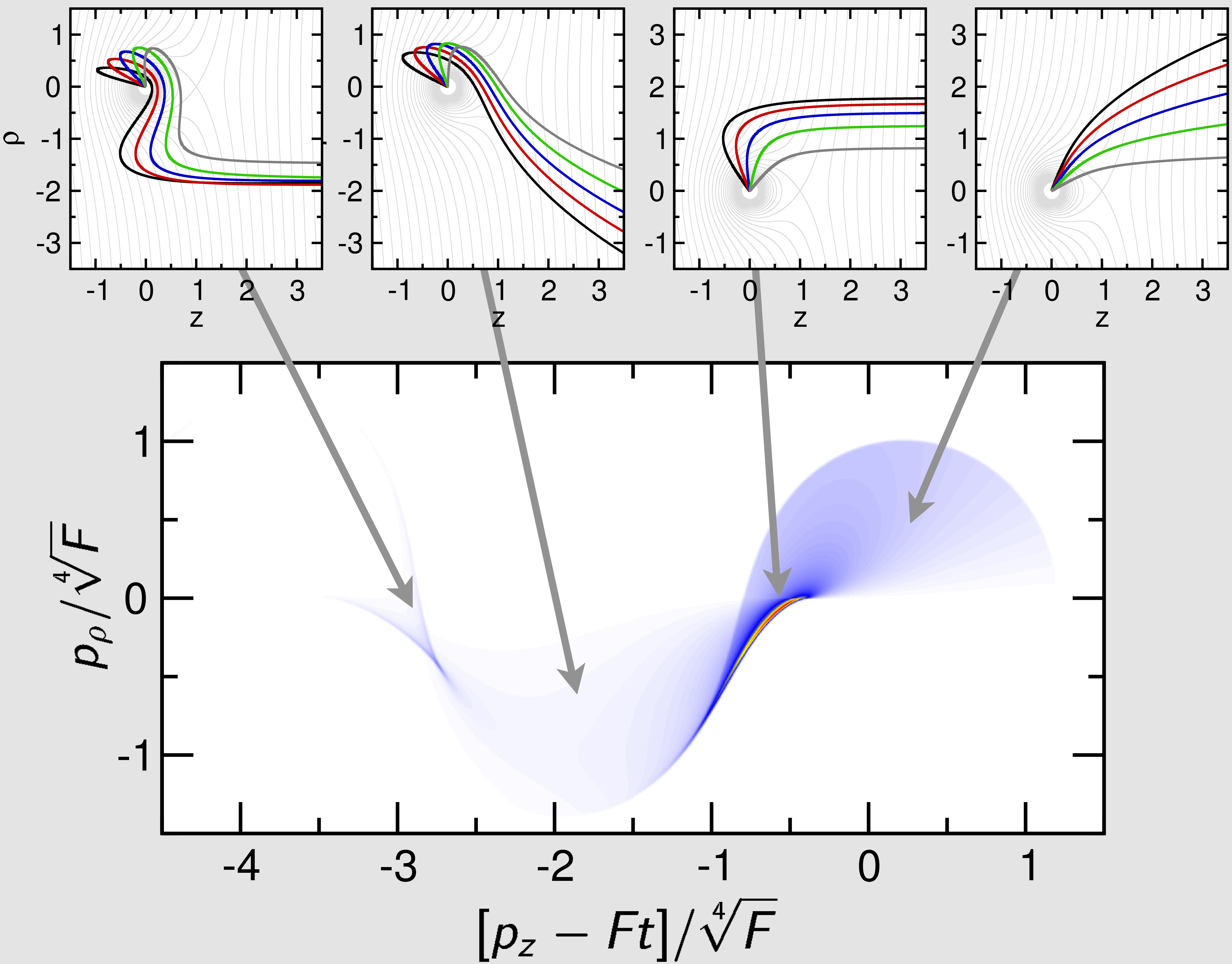


momentum distribution from Stark geometry

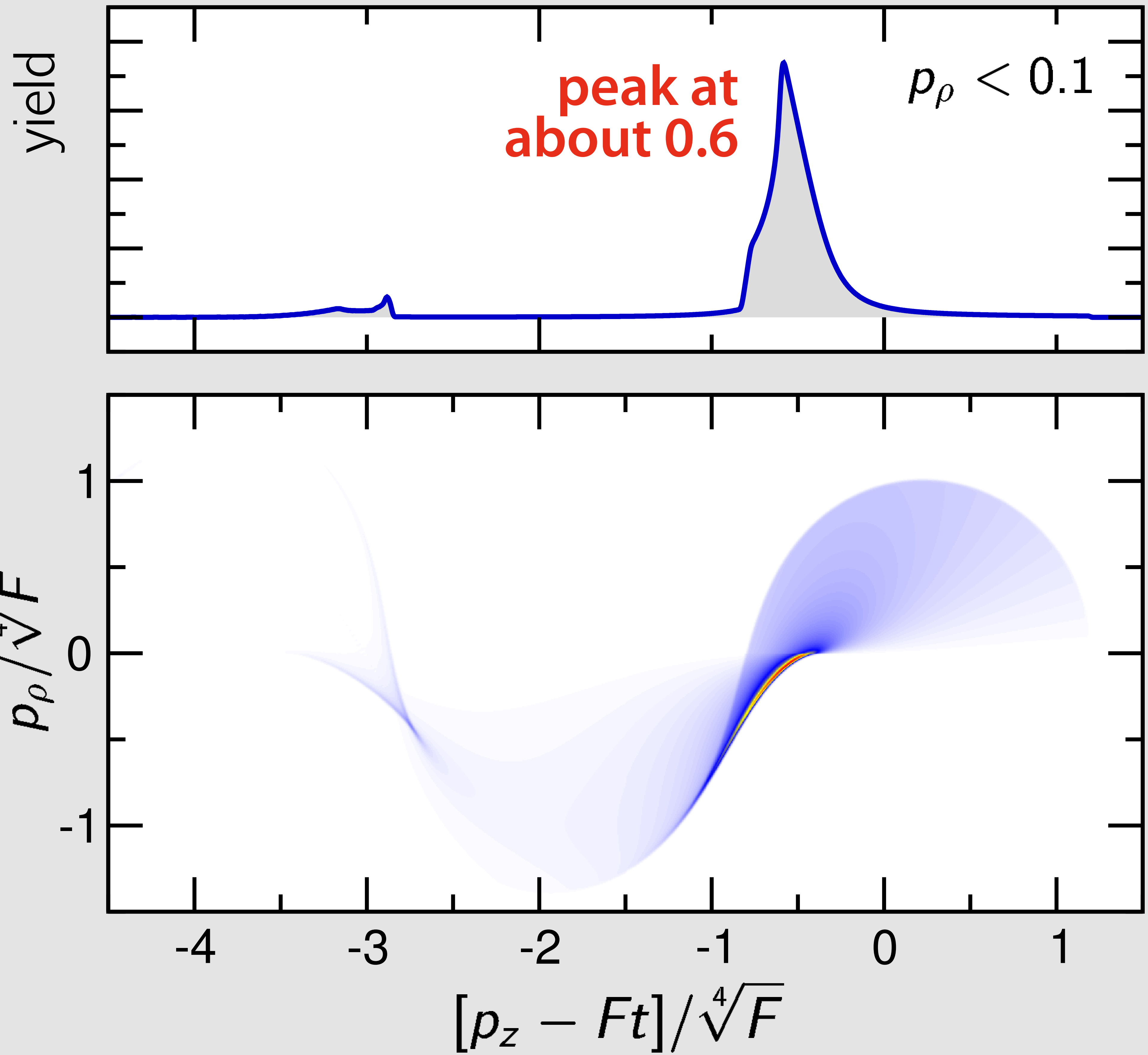
spectrum for uniform distribution
of energies E and angles θ



momentum distribution from Stark geometry



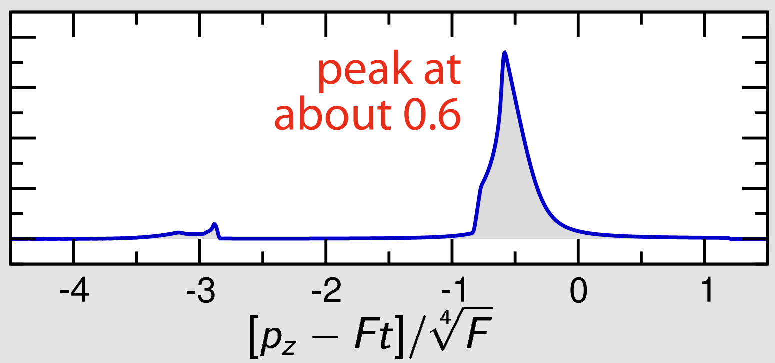
momentum distribution from Stark geometry



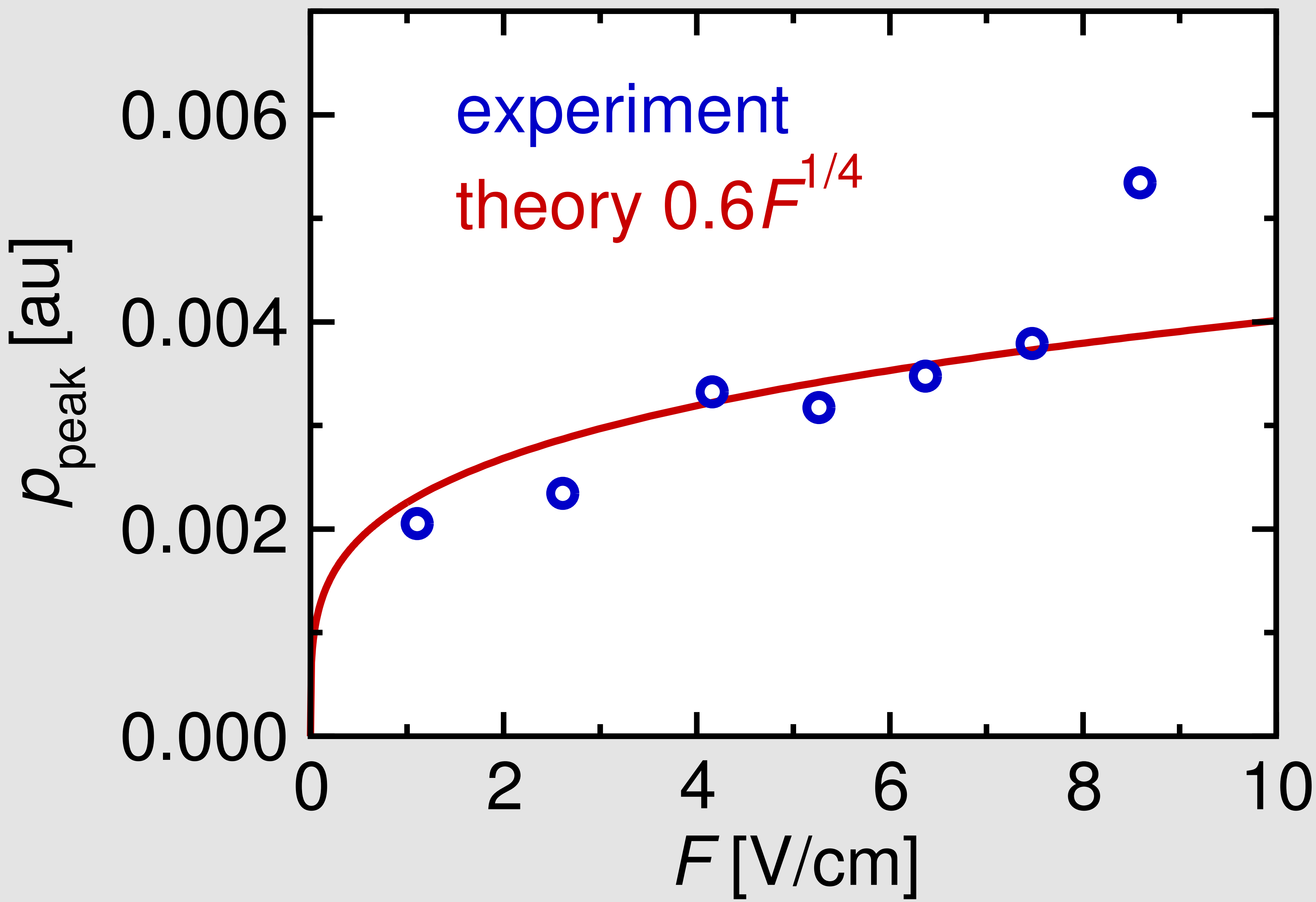
recent ZES measurements

Richter, Kunitski, Dörner (Frankfurt)

N_2, 800nm

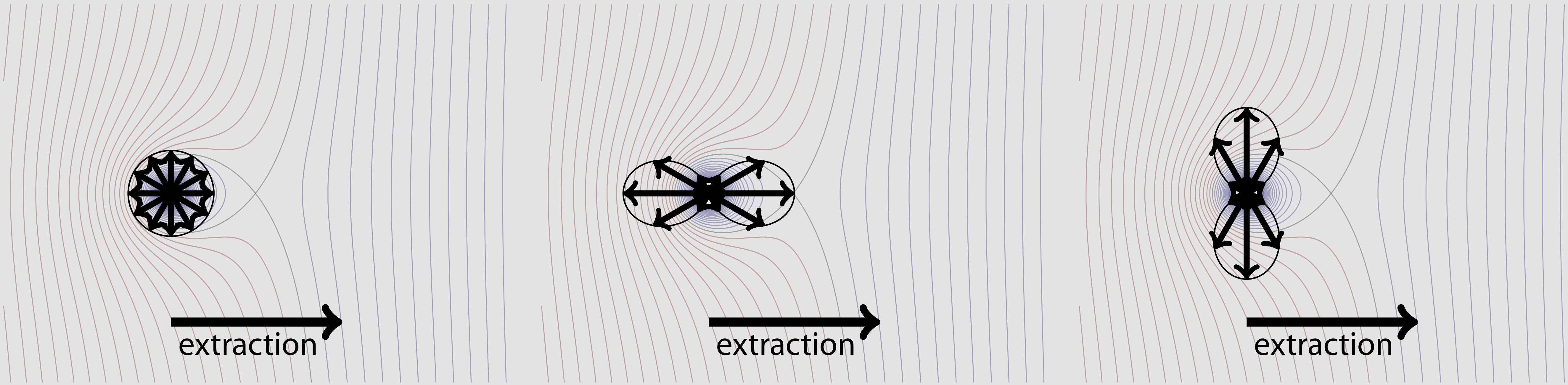


distance of the "zero-peak" from zero



influence of angular distribution

$$R = \frac{1}{2}\int_{-2}^{0} \mathrm{d}E \, \frac{\int_{\xi_{\min}}^{+1} \mathrm{d}\xi \, P(\xi)}{\int_{-1}^{+1} \mathrm{d}\xi \, P(\xi)} \qquad \xi \equiv \cos\theta \qquad \xi_{\min} \equiv E^2/2 - 1$$



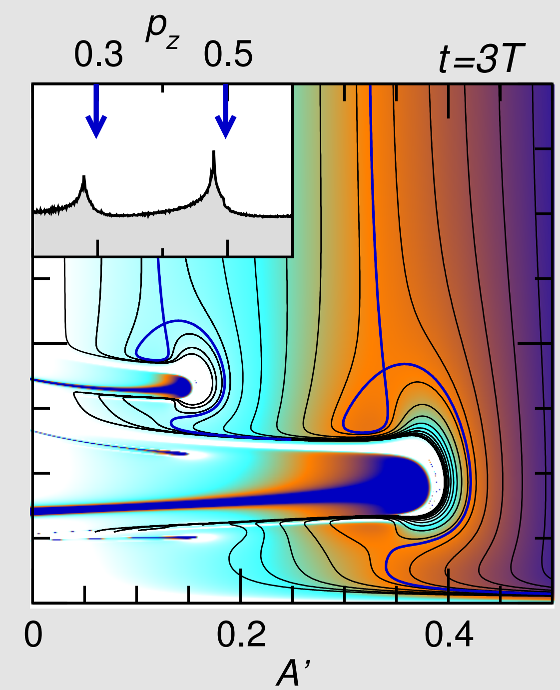
$P(\xi) = 1$ $\qquad$ $P(\xi) = \xi^2 \sim \cos^2\theta$ $\qquad$ $P(\xi) = 1 - \xi^2 \sim \sin^2\theta$

$R = \frac{2}{3}$ $\qquad$ $R = \frac{22}{35}$ $\qquad$ $R = \frac{24}{35}$

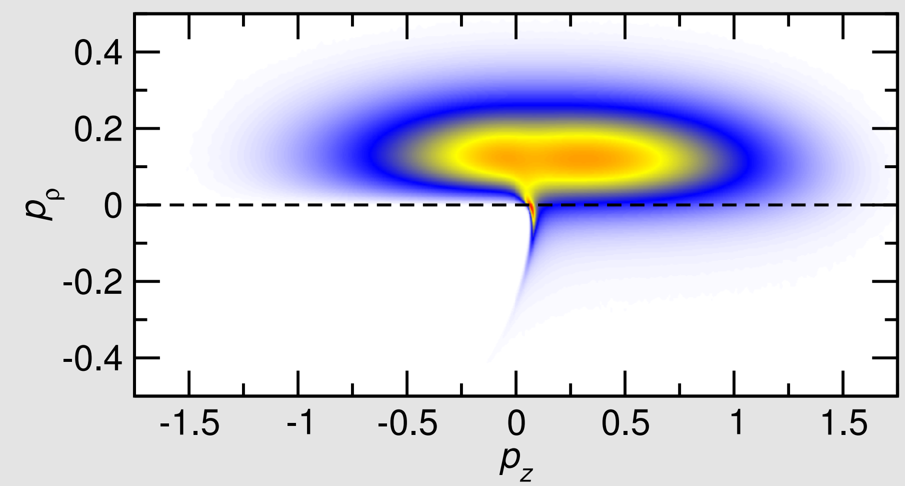
almost independent of angular distribution !

summary

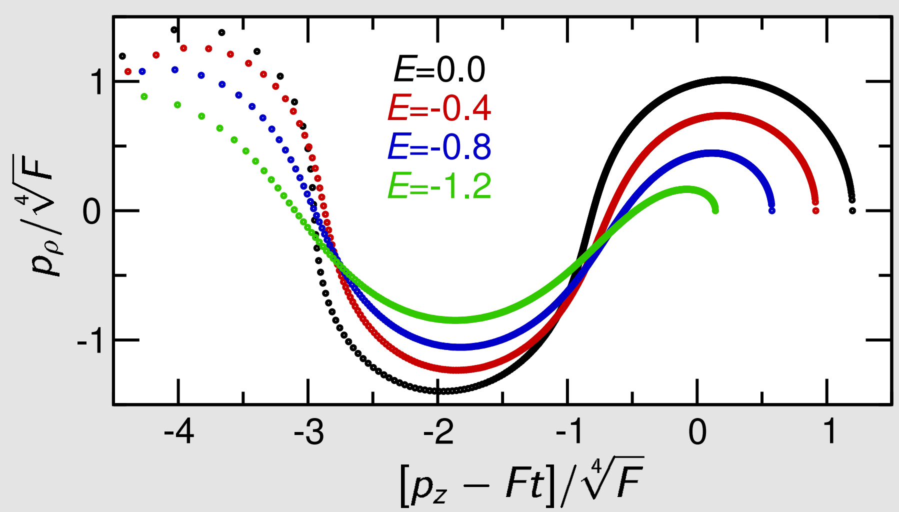
- **LES**
  mechanism from deflection function
  → longitudinal bunching in soft recollisions
  → higher-order (lower-energy) peaks



- **VLES**
  alternative soft recollisions



- **"Z"ES**
  analysis of Stark trajectories
  due to extraction field
  → meachnism for peak formation
  → scaling with field strength,
  confirmed experimentally




Thank you!