Uncovering the density of Dark Matter at the centre of dwarf spheroidal galaxies

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MITP Workshop
Cosmic-Rays and Photons from Dark Matter Annihalation: Theoretical Issues
June 29, 2013

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Motivation: Why Study dSph Kinematics?

- Clean target for indirect detection
- Large mass-luminosity ratio
- Proximity and Sphericity
- Earth-incident flux strongly dependent on DM density

$$\frac{d\phi}{dE} = \frac{1}{4\pi}\frac{<\sigma v>}{2m_{DM}^2}\frac{dN}{dE} \times \int_{\Delta\Omega}\int \rho_{DM}^2(l,\Omega)\,dl\,d\Omega$$

Right hand term is the astrophysical J-Factor which depends on the square of the halo density profile.

Dwarf Spheroidal Observables

- Photometry. The projected radii R and 2D surface brightness $\Sigma(R)$ of the stars.
- Stellar Kinematics: The projected velocities of order $10^2 - 10^3$ stars along the line of sight.
- Metallicity: Can now identify distinct metal rich and poor stellar subcomponents.

How can we use this data to constrain the density?

- Assuming spherical symmetry and no collisions a condition for dynamic equilibrium is the time-independent collisionless Boltzmann equation (CBE)

$$
\begin{aligned}
\frac{\partial f}{\partial t} &= v_r \frac{\partial f}{\partial r} + \left(\frac{v_\theta^2 + v_\phi^2}{r} - \frac{d\Phi}{dr} \right) \frac{\partial f}{\partial v_r} \\
&+ \frac{1}{r} (v_\phi^2 \cot\theta - v_r v_\theta) \frac{\partial f}{\partial v_\theta} \\
&- \frac{1}{r} (v_\phi v_r + v_\phi v_\theta \cot\theta) \frac{\partial f}{\partial v_\phi} \\
&= 0
\end{aligned}
$$

- This places conditions on the potential for compatibility with the stellar phase space $f(r, \vec{v})$ and thus to the observables upon projection along the line of sight to f_{los}.

The CBE is difficult to solve even with current numeric capabilities. Several key techniques:

- Change coordinates to integrals of motion e.g. $f(E)$ for isotropic and $f(E, L)$ for anisotropic spherical models to automatically satisfy CBE.
- Integrate over velocity space for Jeans equations that place local constraints on velocity moments.
- Further integrating over position space yields global virial constraints.

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Multiplying the CBE by v_r and integrating over velocity space gives the second order Jeans equation

$$\frac{d(\nu\sigma_r^2)}{dr} + \frac{2\beta}{r}\nu\sigma_r^2 + \nu\frac{d\Phi}{dr} = 0.$$

Can calculate the local variance from given gravitational potential $\Phi(r)$, stellar density distribution $\nu(r)$ and anisotropy parameter $\beta(r)$

$$\phi(r) = \frac{4\pi G}{r}\int_0^r r^2\left[\nu(r) + \rho_{DM}(r)\right]dr\ , \quad \beta(r) = 1 - \frac{\sigma_t^2(r)}{2\sigma_r^2(r)},$$

which measures the deviation from the isotropic system.

Projection

The projected surface density profile is readily obtained from $\nu(r)$,

$$\Sigma(R) = 2\int_R^\infty \frac{\nu(r)r}{\sqrt{r^2 - R^2}}dr.$$

Projecting the dispersion however entangles it in an integral equation with the unknown anisotropy parameter,

$$\sigma_{los}^2 = \frac{2}{\Sigma(R)}\int_R^\infty (1 - \beta\frac{R^2}{r^2})\sigma_r^2 \frac{\nu(r)r}{\sqrt{r^2 - R^2}}dr$$

mass-anisotropy degeneracy $\rightarrow$ large uncertainty in J-factor.

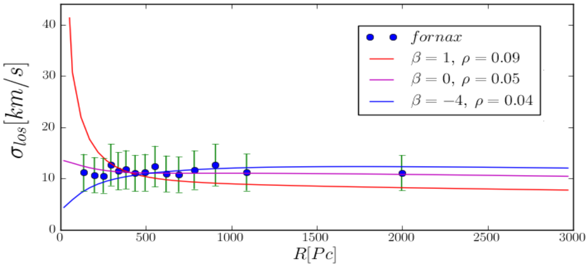

Figure: Mass-Anisotropy Degeneracy

Recipe

- Pick a density model e.g NFW profile,
- Model the anisotropy, e.g

$$\beta(r) = (\beta_\infty - \beta_0)\frac{r^2}{r_\beta^2 + r^2} + \beta_0$$

- Generate a theoretical dispersion via the Jeans equation with a set of parameters e.g $p = \{\beta_0, \beta_\infty, r_\beta, \rho_0, r_{\rm nfw}\}$
- Fit to the binned variances of velocity data d with likelihood function e.g

$$-\ln \mathcal{L}(d|p) = \frac{1}{2}\sum_{i=1}^{N}\frac{(\sigma^2 - \widehat{\sigma}^2)^2}{\alpha_{\sigma^2}^2}$$

- Run MCMC for posteriors of density parameters

Breaking the degeneracy

New methods from larger data sets and resolution of stellar subcomponents

- Mass slope method for multiple subcomponents. Estimates for Fornax and Sculptor are inconsistent with cuspy profiles.
- In Sculptor multiple populations embedded in a shared NFW cannot jointly satisfy virial constraints (Evans 2012) or the Jeans equations (Battaglia 08).
- With large data sets one could also look to the higher moments of the velocity distribution.

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There are three unique moments at fourth order $\overline{v_r^4}$, $\overline{v_t^4}$ and $\overline{v_r^2 v_t^2}$ related by only two fourth order Jeans equations.

$$\frac{d(\nu\overline{v_r^4})}{dr} - \frac{6}{r}\nu\overline{v_r^2 v_t^2} + \frac{2}{r}\nu\overline{v_r^4} + 3\nu\sigma_r^2\frac{d\Phi}{dr} = 0.$$

$$\frac{d(\nu\overline{v_r^2 v_t^2})}{dr} - \frac{4}{3r}\nu\overline{v_t^4} + \frac{4}{r}\nu\overline{v_r^2 v_t^2} + \nu\sigma_t^2\frac{d\Phi}{dr} = 0.$$

We therefore introduce an extra degree of freedom,

$$\beta'(r) = 1 - \frac{3}{2}\frac{\overline{v_r^2 v_t^2}}{\overline{v_r^4}}$$

to close the equations. The projected fourth moment is then,

$$\overline{v_{los}^4}(R) = \frac{2}{\Sigma}\int_R^\infty \left(g\overline{v_r^4} + \frac{3R^4}{4r^3}(\beta' - \beta)\sigma_r^2\frac{d\phi}{dr}\right)\frac{\nu(r)r}{\sqrt{r^2 - R^2}}dr$$

$$g(\beta', r, R) = 1 - 2\beta'\frac{R^2}{r^2} + \frac{\beta'(1-\beta')}{2}\frac{R^4}{r^4} - \frac{R^4}{4r^3}\frac{d\beta'}{dr}.$$

Kurtosis

- Need to bin observed LOS data to sample the nth moment, statistical errors increase rapidly with n.
- For ease of interpretation the fourth moment is standardised for the kurtosis

$$\kappa_{los} = \frac{\overline{v_{los}^4}}{(\sigma_{los}^2)^2}$$

- Extending the likelihood function with β' parameters to jointly fit the line of sight dispersions and kurtosis,

$$-\ln \mathcal{L}(d|p) = \frac{1}{2}\left\{\sum_{i=1}^{N}\frac{(\sigma_{\text{los}}^2 - \widehat{\sigma}_{\text{los}}^2)^2}{\alpha_{\sigma^2}^2} + \sum_{i=1}^{N}\frac{(\kappa_{\text{los}} - \widehat{\kappa}_{\text{los}})^2}{\alpha_{\kappa}^2}\right\}$$

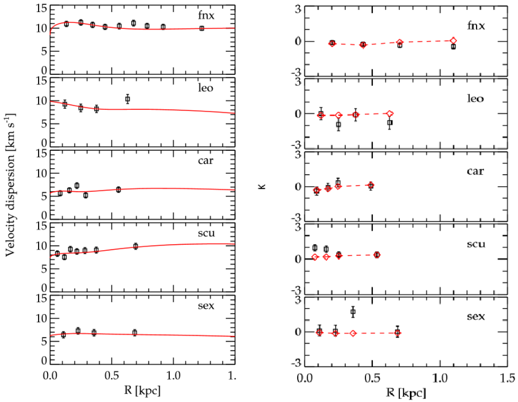

Figure: DSph velocity moments (Strigari et al 2010)

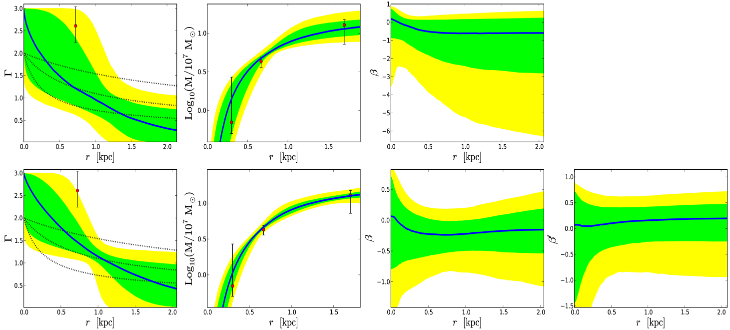

Figure: Fornax MCMC Analysis

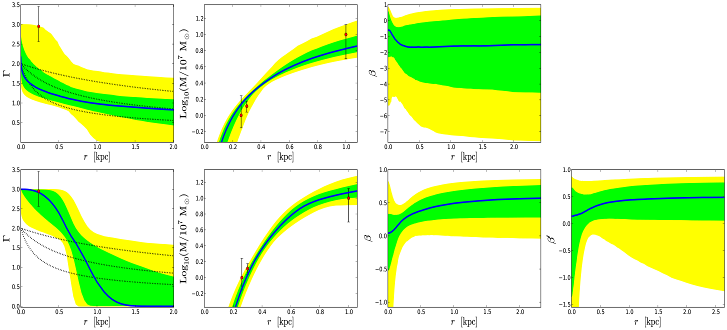

Figure: Sculptor MCMC Analysis

- The kurtosis data does tighten the constraints! The fourth order degeneracy is less affecting than the traditional one
- In Fornax the improvement is small and is not able to distinguish between cusped and cored solutions. Don't predict large core suggested by multiple populations mass slope method.
- In Sculptor a cored solution is clearly favoured. This is line with evidence from the mass slope method, virial theorem and Jeans analysis of stellar subcomponents.

So why does the inclusion of the kurtosis make such a dramatic difference in Sculptor but not Fornax?.......

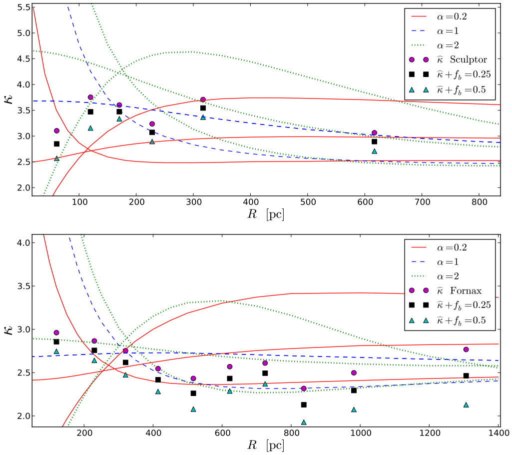

Figure: Fornax and Sculptor kurtosis data and Jeans solutions

Key issues

- Foreground contaminants
- Binary stars
- Potential rotation in Sculptor.
- Uncertainty in stellar density profile $\nu(r)$

Whilst each of these could reduce the tension of the Sculptor measurement with the cusped ΛCDM halo none of them is likely to increase the core size in Fornax in line with the literature.

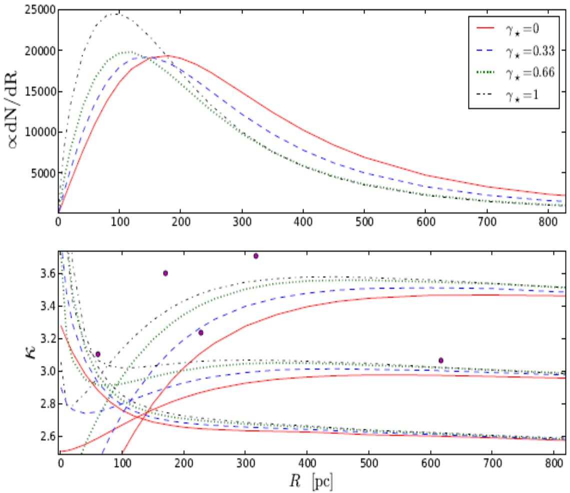

Figure: NFW Jeans solutions with different stellar densities

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Conclusions

- DSphs are a clean target for indirect detection and as DM dominated objects are good tests for N-body simulation predictions for halo density profiles
- The mass-anisotropy degeneracy in the traditional Jeans analysis generates large uncertainties in the density and J-factor.
- Higher order moments can partially relieve the degeneracy and predict a cored profile in Sculptor in line with other methods in the literature. For Fornax however there is tension with the large cores predicted by studies of stellar subcomponents.
- Already with two samples there is variety. To explore whether astrophysical effects (e.g supernova feedback/star formation), unexplored systematics or alternative cosmologies (e.g WDM, SIDM) are the cause of these discrepancies needs more time and data.