UHE neutrino and cosmic ray emission from GRBs: revising the models and clarifying the cosmic ray-neutrino connection

Mauricio Bustamante

In collaboration with Philipp Baerwald and Walter Winter

Institut für Theoretische Physik und Astrophysik
Universität Würzburg

Very Large Volume Neutrino Telescope Workshop
Stockholm, August 05, 2013
The origin of UHE CRs ($\gtrsim 10^9$ GeV) and $\nu$'s is still unknown:

- *how* are they produced?
- *where* are they produced?

GRBs are among the best candidate sources:

- radiated energy of $\sim 10^{52} - 10^{53}$ erg
- intense magnetic fields of $\sim 10^5$ G
- magnetically-confined $p$'s shock-accelerated to $\sim 10^{12}$ GeV

**Problem:** experiments (IceCube, ANTARES) are starting to strongly constrain the simplest emission models

**Solution:** we need to build more realistic models!
Long-duration GRB ($\geq 2$ s): a compact object ($\sim 10^3$ km) emits relativistically expanding baryonic-loaded matter ejecta.
The standard “neutron model” of emission

Joint production of UHECRs, $\nu$’s, and $\gamma$’s:

$$ p\gamma \rightarrow \Delta^+ (1232) \rightarrow \left\{ \begin{array}{l} n\pi^+ , \text{ BR } = \frac{1}{3} \\ p\pi^0 , \text{ BR } = \frac{2}{3} \end{array} \right. $$

$$ \pi^+ \rightarrow \mu^+ \nu_\mu \rightarrow \bar{\nu}_\mu e^+ \nu_e \nu_\mu $$

$$ \pi^0 \rightarrow \gamma\gamma $$

$$ n \text{ (escapes) } \rightarrow pe^- \bar{\nu}_e $$

($$\Delta^+ : \sim 50\% \text{ of all } p\gamma \text{ interactions}$$)

After propagation, with flavour mixing:

$$ \nu_e : \nu_\mu : \nu_\tau : p = 1 : 1 : 1 : 1 $$

(“one $\nu_\mu$ per cosmic ray”)

The simplest neutron model is now strongly disfavoured ➤
IceCube Collaboration:

- $\nu$ flux normalised to GRB $\gamma$ fluence:
  \[
  \int_0^\infty dE_\nu E_\nu F_\nu (E_\nu) \propto \int_{1 \text{ keV}}^{10 \text{ MeV}} d\varepsilon\gamma \varepsilon\gamma F_\gamma (\varepsilon\gamma)
  \]

- quasi-diffuse $\nu$ flux from 117 GRBs
- analytical calculation – in tension with upper bounds

IceCube Coll., Nature 484, 351 (2012)
The neutron model under tension?

More detailed particle physics (NeuCosmA):
- extra multi-\(\pi\), \(K\), \(n\) production modes
- synchrotron losses of secondaries
- adiabatic cooling
- full photon spectrum

\(\nu\) flux \(\sim\) one order of magnitude lower


IceCube Collaboration:
- \(\nu\) flux normalised to GRB \(\gamma\) fluence:
  \[
  \int_0^\infty dE_\nu E_\nu F_\nu (E_\nu) \propto \int_{1 \text{ keV}}^{10 \text{ MeV}} d\varepsilon \gamma \varepsilon \gamma F_\gamma (\varepsilon \gamma)
  \]
- quasi-diffuse \(\nu\) flux from 117 GRBs
- analytical calculation – in tension with upper bounds

IceCube Coll., Nature 484, 351 (2012)
The neutron model under tension?

IceCube Collaboration:

- $\nu$ flux normalised to GRB $\gamma$ fluence:
  \[
  \int_{0}^{\infty} dE \nu E_{\nu} F_{\nu}(E_{\nu}) \propto \int_{10 \text{MeV}}^{1 \text{keV}} d\epsilon \gamma \epsilon_{\gamma} F_{\gamma}(\epsilon_{\gamma})
  \]

- quasi-diffuse $\nu$ flux from 117 GRBs
- analytical calculation – in tension with upper bounds

IceCube Collaboration,

- More detailed particle physics (NeuCosmA):
  - extra multi-$\pi$, $K$, $n$ production modes
  - synchrotron losses of secondaries
  - adiabatic cooling
  - full photon spectrum
  - $\nu$ flux $\sim$ one order of magnitude lower

ANTARES Collab., 1307.0304 – see talk by J. Schmid

Recent search by ANTARES optimised for NeuCosmA:

- ANTARES NeuCosMA
- ANTARES Guetta
- IceCube IC40+59
- ANTARES 2007

ANTARES Collab., 1307.0304 – see talk by J. Schmid

Recent search by ANTARES optimised for NeuCosmA:

- extra multi-$\pi$, $K$, $n$ production modes
- synchrotron losses of secondaries

More detailed particle physics (NeuCosmA):

- extra multi-$\pi$, $K$, $n$ production modes
- synchrotron losses of secondaries
- adiabatic cooling
- full photon spectrum
- $\nu$ flux $\sim$ one order of magnitude lower

More detailed particle physics (NeuCosmA):

- extra multi-$\pi$, $K$, $n$ production modes
- synchrotron losses of secondaries
- adiabatic cooling
- full photon spectrum
- $\nu$ flux $\sim$ one order of magnitude lower

ibliography:

- IceCube Coll., Nature 484, 351 (2012)

Mauricio Bustamante
Universität Würzburg
Revising the neutron model: NeuCosmA

- Detailed $p\gamma$ cross section

**Graphical Content:**

- DESY, SLAC, Cornell late 60's, early 70's
- Fermilab, 1978
- DESY, 199X Baksan, 2003

**Equation:**

$$\sigma(\epsilon) \text{[\mu barn]}$$

**Axis:**

- $\epsilon_r$ [GeV]
- $1000 \rightarrow 500 \rightarrow 50 \rightarrow 10 \rightarrow 1 \rightarrow 0.1$ on a logarithmic scale.

**Data Points:**

- Data points for different energy ranges.

**Notes:**

- Implemented as fast SOPHIA-based parametrisation

**References:**

Revising the neutron model: NeuCosmA

• Contributions to the full photohadronic cross section

“WB flux”:
traditional, analytical
Waxman-Bahcall prediction

\[ E^2 \phi_\nu = 0.45 \times 10^{-8} \frac{f_\pi}{0.2} \]

Use this to normalise the proton and photon spectra – and to study spectral changes

“WB \Delta^+–approx.": explicit synchrotron cooling of pions
Revising the neutron model: NeuCosmA

- Contributions to the full photohadronic cross section

![Graph showing the comparison between WB and NeuCosmA fluxes](image)

Especially "Multi π" contribution leads to a change of flux shape; neutrino flux higher by up to a factor of 3 compared to WB treatment.
Contributions to \((\nu_\mu + \bar{\nu}_\mu)\) flux from \(\pi^\pm\) decay divided in:

- \(\Delta(1232)\)-resonance

\[ \Delta(1232) \] especially "Multi \(\pi\)" contribution leads to change of flux shape; neutrino flux higher by up to a factor of 3 compared to WB treatment

Contributions to \((\nu_\mu + \bar{\nu}_\mu)\) flux from \(\pi^\pm\) decay divided in:

- \(\Delta(1232)\)-resonance
- Higher resonances

\[ E^2 \phi_{\nu_\mu + \bar{\nu}_\mu} \text{ (GeV}^2 \text{ sr}^{-1} \text{ s}^{-1} \text{ cm}^{-2}) \]

\[ 10^{-8} \quad 10^{-7} \]

\[ 10^{-10} \quad 10^{-9} \quad 10^{-8} \quad 10^{-7} \quad 10^{-6} \quad 10^{-5} \quad 10^{-4} \quad 10^{-3} \quad 10^0 \quad 10^3 \quad 10^4 \quad 10^5 \quad 10^6 \quad 10^7 \quad 10^8 \]

E/GeV

Contributions to \( (\nu_{\mu} + \bar{\nu}_{\mu}) \) flux from \( \pi^{\pm} \) decay divided in:

- \( \Delta(1232) \)-resonance
- Higher resonances
- \( t \)-channel (direct production)

Contributions to $(\nu_{\mu} + \bar{\nu}_{\mu})$ flux from $\pi^{\pm}$ decay divided in:

- $\Delta(1232)$-resonance
- Higher resonances
- $t$-channel (direct production)
- High energy processes (multiple $\pi$)

Especially "Multi $\pi$" contribution leads to change of flux shape; neutrino flux higher by up to a factor of 3 compared to WB treatment
Revising the neutron model: NeuCosmA

- Further particle decays

\[ \pi^+ \rightarrow \mu^+ + \nu_\mu \]
\[ \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \]

\[ \pi^- \rightarrow \mu^- + \bar{\nu}_\mu \]
\[ \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \]

\[ K^+ \rightarrow \mu^+ + \nu_\mu \]

\[ n \rightarrow p + e^- + \bar{\nu}_e \]
Revising the neutron model: NeuCosmA

- Further particle decays

\[
\begin{align*}
\pi^+ &\rightarrow \mu^+ + \nu_\mu \\
\mu^+ &\rightarrow e^+ + \nu_e + \bar{\nu}_\mu \\
\pi^- &\rightarrow \mu^- + \bar{\nu}_\mu \\
\mu^- &\rightarrow e^- + \bar{\nu}_e + \nu_\mu \\
K^+ &\rightarrow \mu^+ + \nu_\mu \\
n &\rightarrow p + e^- + \bar{\nu}_e
\end{align*}
\]

Resulting \( \nu_e \) flux (at the observer)

\[E^2 \phi_{\nu_e} \text{(GeV sr}^{-1} \text{s}^{-1} \text{cm}^{-2})]\]

---

\[ \pi^+ \rightarrow \mu^+ + \nu_\mu \]
\[ \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \]
\[ \pi^- \rightarrow \mu^- + \bar{\nu}_\mu , \]
\[ \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \]
\[ K^+ \rightarrow \mu^+ + \nu_\mu \]
\[ n \rightarrow p + e^- + \bar{\nu}_e \]

**Resulting \( \nu_\mu \) flux (at the observer)**

\[
E^2 \phi_{\nu_\mu}(E) \text{ (GeV sr}^{-1} \text{ s}^{-1} \text{ cm}^{-2})
\]

\[ 10^{-10} \quad 10^{-9} \quad 10^{-8} \quad 10^{-7} \]

\[ 10^2 \quad 10^3 \quad 10^4 \quad 10^5 \quad 10^6 \quad 10^7 \quad 10^8 \quad 10^9 \quad 10^{10} \]

Revising the neutron model: NeuCosmA

• Neutrino spectra including flavour mixing

Electron neutrino spectrum

Muon neutrino spectrum

Characteristic double peak structure from $\mu$ and $\pi$ decay in both flavours, additional peak from $K^+$ decay at $10^8$ to $10^9$ GeV

Revising the neutron model: NeuCosmA

• How the spectrum changes...

Corrections to the analytical model:

► shape revised:

▶ shift of first break (correction of photohadronic threshold)

▶ different cooling breaks for μ’s and π’s

▶ \((1 + z)\) correction on the variability scale of the GRB

► Correction \(c f_\pi\) to π prod. efficiency:

▶ \(f_{C,\gamma}\): full spectral shape of photons

▶ \(f_\approx = 0.69\): rounding error in analytical calculation

▶ \(f_{\sigma} \approx 2/3\): from neglecting the width of the \(\Delta\)-resonance

► Correction \(c_{S}\):

▶ energy losses of secondaries

▶ energy dependence of the mean free path of protons

Revising the neutron model: NeuCosmA

- How the spectrum changes ... (cont.)

At example of GRB080603A:

1. Correction to analytical model (IC-FC → RFC)

2. Change due to full numerical calculation

\[ E^2 \langle \dot{E} \rangle \left[ \text{GeV cm}^{-2} \right] \]

\[ E_\gamma \ \text{[GeV]} \]

IC-FC: IceCube-Fireball Calculation
RFC: Revised Fireball Calculation
NFC: Numerical Fireball Calculation

Revising the neutron model: NeuCosmA

- The new prediction of the quasi-diffuse GRB $\nu$ flux

- Same $n = 117$ GRBs, effective area, and parameters as used by the IC-40 analysis

- Calculate the associated neutrino flux for each burst and the stacked flux $F_\nu (E_\nu)$

- Quasidiffuse flux:

  $$\phi_\nu (E_\nu) = F_\nu (E_\nu) \frac{1}{4\pi} \frac{1}{n} \frac{1}{\text{667 bursts}} \frac{1}{\text{yr}}$$

- Statistical uncertainty: extrapolation of a few bursts to a quasidiffuse flux

- Astrophysical uncertainty:
  - $0.001 \leq t_\nu [s] \leq 0.1$
  - $200 \leq \Gamma \leq 500$
  - $1.8 \leq \alpha_p \leq 2.2$
  - $0.1 \leq \epsilon_e / \epsilon_B \leq 10$

Further revisions: direct proton escape

The neutron model hinges on:

1. $p$’s magnetically confined, only $n$’s escape
2. $p$’s interact at most once, $n$’s do not (*optically thin source*)

However, under the “one $\nu_\mu$ per CR” hypothesis, GRBs are disfavoured to be the sole source of UHECRs (*Ahlers et al.*).

Further revisions: direct proton escape

The neutron model hinges on:

1. \( p \)'s magnetically confined, only \( n \)'s escape
2. \( p \)'s interact at most once, \( n \)'s do not (optically thin source)

However, under the “one \( \nu_\mu \) per CR” hypothesis, GRBs are disfavoured to be the sole source of UHECRs (AHLERS et al.).

What if 1 and 2 are violated?

- \( p \)'s “leak out”, not accompanied by (direct) \( \nu \) production
- multiple \( p \) interactions enhance the \( \nu \) flux
- in optically thick sources, only \( n \)'s at the borders escape

A two-component model of CR emission

Optical depth:

\[ \tau_n = \left| \frac{t_p^{-1} \gamma}{t_{\text{dyn}}^{-1}} \right|_{E_{p,\text{max}}} = \begin{cases} \lesssim 1, & \text{optically thin source} \\ > 1, & \text{optically thick source} \end{cases} \]

\( E_{p,\text{max}} \) determined from a competition of processes:

\[ t'_{\text{acc}} (E'_{p,\text{max}}) = \min \left[ t'_{\text{dyn}}, t'_{\text{syn}}, t'_{p\gamma} (E'_{p,\text{max}}) \right] \]

Acceleration efficiency, \( \eta \):

\[ t'_{\text{acc}} (E'_{p}) = \frac{E'_{p}}{\eta ce B'} \]

Particles can escape from within a shell of thickness \( \lambda'_{\text{mfp}} \):

\[ \lambda'_{p,\text{mfp}} (E') = \min \left[ \Delta r', R'_L (E'), ct'_{p\gamma} (E') \right] \]
\[ \lambda'_{n,\text{mfp}} (E') = \min \left[ \Delta r', ct'_{p\gamma} (E') \right] \]

\[ f_{\text{esc}} = \frac{\lambda'_{\text{mfp}}}{\Delta r'} \]

fraction of escaping particles
A two-component model of CR emission

Optically **thin** source:

- $L_{\gamma,\text{iso}} = 10^{50}$ erg s$^{-1}$
- $\tau_n = 3.04 \times 10^{-2}$

Optically **thick** source:

- $L_{\gamma,\text{iso}} = 10^{52}$ erg s$^{-1}$
- $\tau_n = 3.37$

$E^2 f_{\gamma} / \text{GeV cm}^{-2}$

---

A two-component model of CR emission

Scan of the GRB emission parameter space:

acceleration efficiency $\eta = 0.1$

$\eta = 1.0$

We use a **Boltzmann equation** to transport protons to Earth:

- **Comoving number density of protons (GeV$^{-1}$ cm$^{-3}$):**

  \[
  Y_p(E, z) = \frac{n_p(E, z)}{(1 + z)^3},
  \]

  with $n_p$ the real number density

- **Transport equation (comoving source frame):**

  \[
  \dot{Y}_p = \partial_E (H E Y_p) + \partial_E (b_{e^+ e^-} Y_p) + \partial_E (b_{p\gamma} Y_p) + L_{\text{CR}}
  \]

  - adiabatic losses
  - photohadronic losses
  - pair production losses
  - CR injection from sources

  \[
  Q_{\text{CR}}(E) \propto E^{-\alpha_p} e^{-E/E_p,\text{max}}
  \]
UHECR flux at Earth from $n$ and direct $p$ escape:

- HiRes–I
- HiRes–II

\[ \alpha_p = 2.5, \text{two comp. model} \]
\[ \alpha_p = 2.5, \text{neutron escape only} \]
\[ \alpha_p = 2.3, \text{neutron escape only} \]
\[ \alpha_p = 2.0, \text{neutron escape only} \]

\[ \log_{10} \left( \frac{E}{\text{GeV}} \right) \]

\[ \mathrm{E}^3 J(E) \left[ \frac{\text{GeV}^2 \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1}}{\text{ } } \right] \]
The UHECR and UHE $\nu$ fluxes at Earth

UHECR flux at Earth from $n$ and direct $p$ escape:

- $\alpha_p = 2.5$, two comp. model
- $\alpha_p = 2.5$, neutron escape only — dip model
- $\alpha_p = 2.3$, neutron escape only — dip model
- $\alpha_p = 2.0$, neutron escape only — transition model

$E^3 J(E)$ [GeV$^2$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$]

$\log_{10} \left( \frac{E}{\text{GeV}} \right)$

$\uparrow$ the direct $p$ escape enhances the high-energy peak
The UHECR and UHE $\nu$ fluxes at Earth

UHECR flux at Earth from $n$ and direct $p$ escape:

Our two-component model is able to fit the UHECR data
The UHECR and UHE $\nu$ fluxes at Earth

neutron model vs. two-component model: prompt and cosmogenic $\nu$'s

\[ f_e^{-1} \approx 107 \]

\[ f_e^{-1} \approx 144 \]

\[ \alpha_p = 2 \]

\[ \chi^2/\text{d.o.f.} = 2.59 \]

\[ \chi^2/\text{d.o.f.} = 11.94 \]
The big (multi-messenger) picture

\[ \gamma \dot{N}, E_{\gamma,\text{iso}} \]

\[ \frac{1}{f_e} \times \frac{1}{f_{\text{thresh}}} \times \frac{1}{f_{\text{dark}}} \times f_{\pi} \]

\[ \sim \frac{f_{\pi}}{f_{\text{CR}} \times f_{\text{bol}}} \]
We have revised the GRB $n$ model of $\nu$ emission:
- corrected, full numerical calculation with detailed particle physics
- yields a quasi-diffuse flux $\sim 1$ order magnitude below the analytical one by IceCube

We have explored a GRB emission model with:
- the standard $n$ escape component, plus
- an explicit direct $p$ escape component
  - improves the fit to the UHECR observations

The directly-escaping protons . . .
- affect the prompt $\nu$ flux,
- but not (much) the cosmogenic flux

By clarifying the UHE $\gamma$–CR–$\nu$ connection, we might rule out large regions of emission + propagation parameter space
Backup slides
UHE $\nu$'s in the GRB internal shock model

Secondary injection of neutrons, neutrinos ($\text{GeV}^{-1} \text{ cm}^{-3} \text{ s}^{-1}$)

$$Q' (E') = \int_{E'}^{\infty} \frac{dE'_p}{E'_p} N'_p (E'_p) \int_{0}^{\infty} c d\varepsilon' N'_\gamma (\varepsilon') R (E', E'_p, \varepsilon')$$

Normalisation to the observed GRB photon flux $F_\gamma$

$$\int \varepsilon' N'_\gamma (\varepsilon') d\varepsilon' = \frac{E'_{\text{sh}}}{V'_{\text{iso}}} \propto F_\gamma,$$
$$\frac{1}{f_e} \frac{E'_{\text{sh}}}{V'_{\text{iso}}} \propto \frac{F_\gamma}{f_e}$$

Fluence per shell, at Earth ($\text{GeV}^{-1} \text{ cm}^{-2}$)

$$\mathcal{F}^{\text{sh}} = t_v V'_{\text{iso}} \frac{(1 + z)^2}{4\pi d_L^2} Q'$$
Secondary injection of neutrons, neutrinos (GeV$^{-1}$ cm$^{-3}$ s$^{-1}$)

\[
Q'(E') = \int_{E'}^{\infty} \frac{dE'_p}{E'_p} N'_p (E'_p) \int_0^\infty c d\varepsilon' N'_\gamma (\varepsilon') R (E', E'_p, \varepsilon')
\]

- Photon density, shock rest frame (GeV$^{-1}$ cm$^{-3}$):

\[
N'_\gamma (\varepsilon') \propto \begin{cases} 
(\varepsilon')^{-\alpha_\gamma}, & \varepsilon'_{\gamma,\text{min}} = 0.2 \text{ eV} \leq \varepsilon' \leq \varepsilon'_{\gamma,\text{break}} \\
(\varepsilon')^{-\beta_\gamma}, & \varepsilon'_{\gamma,\text{break}} \leq \varepsilon' \leq \varepsilon'_{\gamma,\text{max}} = 300 \times \varepsilon'_{\gamma,\text{min}}
\end{cases}
\]

\[
\varepsilon'_{\gamma,\text{break}} = O (\text{keV}), \alpha_\gamma \approx 1, \beta_\gamma \approx 2
\]

- Proton density:

\[
N'_p (E'_p) \propto (E'_p)^{-\alpha_p} \times \exp \left[ - \left( \frac{E'_p}{E'_{p,\text{max}}} \right)^2 \right] \quad (\alpha_p \approx 2)
\]

Maximum proton energy limited by energy losses:

\[
t'_{\text{acc}} (E'_{p,\text{max}}) = \min \left[ t'_{\text{dyn}}, t'_{\text{syn}} (E'_{p,\text{max}}), t'_{p\gamma} (E'_{p,\text{max}}) \right]
\]
Secondary injection of neutrons, neutrinos (GeV\(^{-1}\) cm\(^{-3}\) s\(^{-1}\))

\[ Q'(E') = \int_{E'}^\infty \frac{dE'_p}{E'_p} \frac{N'_p(E'_p)}{E'_p} \int_0^\infty c d\epsilon' N'_\gamma(\epsilon') R(E', E'_p, \epsilon') \]

Normalisation to the observed GRB photon flux \( F_\gamma \)

\[ \int \epsilon' N'_\gamma(\epsilon') d\epsilon' = \frac{E'_{\text{sh}}} {V'_\text{iso}} \propto F_\gamma, \quad \int E'_p N'_p(E'_p) dE'_p = \frac{1}{f_e} \frac{E'_{\text{sh}}}{V'_\text{iso}} \propto \frac{F_\gamma}{f_e} \]
Secondary injection of neutrons, neutrinos (GeV$^{-1}$ cm$^{-3}$ s$^{-1}$)

$$Q'(E') = \int_{E'}^{\infty} \frac{dE'}{E'_p} N'_p(E'_p) \int_{E'_p}^{\infty} c d\epsilon' N'_\gamma(\epsilon') R(E', E'_p, \epsilon')$$

Normalisation to the observed GRB photon flux $F_\gamma$

$$\int \epsilon' N'_\gamma(\epsilon') d\epsilon' = \frac{E'_{sh}}{V'_iso} \propto F_\gamma, \quad \int E'_p N'_p(E'_p) dE'_p = \frac{1}{f_e} \frac{E'_{sh}}{V'_iso} \propto \frac{F_\gamma}{f_e}$$

Fluence per shell, at Earth (GeV$^{-1}$ cm$^{-2}$)

$$\mathcal{F}^{sh} = t_v V'_iso \frac{(1 + z)^2}{4\pi d^2_L} Q'$$
Optically thin sources ($\tau_n < 1$):

Optically thick sources ($\tau_n > 1$):

P. BAERWALD, MB, W. WINTER,
Three emission regimes

**Optically thin to neutron escape regime**
- the standard emission scenario
- $p$'s magnetically confined: $n$'s and $\nu$'s from $p\gamma$ interactions
- $n$'s escape and decay to produce UHECRs

**Direct escape regime**
- directly-escaping $p$'s from the borders dominate
- subdominant $n$ production
- more CRs emitted, so “one $\nu_\mu$ per CR” no longer valid

**Optically thick to neutron escape regime**
- $n$'s and $p$'s in the bulk trapped by multiple $p\gamma$ interactions
- they only escape from the borders
- $\nu$ production enhanced
We use a sophisticated prediction of the GRB neutrino flux (Hümmers et al.):

- full background photon spectrum (not only peak energy)
- energy dependence of the mean free path of protons
- cooling of secondaries
- high-energy photopion processes
- neutrinos from decay of $\mu^\pm$, $\pi^\pm$, $K$, $n$
- helicity dependence of $\mu$ decays
- flavour mixing

One order of magnitude below prediction of benchmark models used by IceCube


Interaction with the photon backgrounds

- **Energy loss rate** (GeV s$^{-1}$):

  \[ b(E) \equiv \frac{dE}{dt} \]

- **For pair production** $p\gamma \rightarrow pe^+ e^-$:

  \[
  b_{e^+e^-}(E, z) = -\alpha r_0^2 (m_e c^2)^2 c \int_2^\infty d\xi n_\gamma \left( \frac{\xi m_e c^2}{2\gamma}, z \right) \frac{\phi(\xi)}{\xi^2}
  \]

- $n_\gamma$: isotropic photon background (GeV$^{-1}$ cm$^{-3}$)
- $\xi$: photon energy in units of $m_e c^2$
- proton energy: $E = \gamma m_p c^2$ ($\gamma \gg 1$)
- $\phi(\xi)$: (tabulated) integral in energy of outgoing $e^-$


Interaction with the photon backgrounds

Photohadronic interactions – $p\gamma$ interaction rate ($s^{-1}$ per particle):

$$\Gamma_{p\gamma\rightarrow p'b}(E, z) = \frac{1}{2} \frac{m_p^2}{E^2} \int_{\epsilon_{\text{th}} m_p / 2E}^{\infty} d\epsilon \frac{n_{\gamma}(\epsilon, z)}{\epsilon^2} \int_{\epsilon_{\text{th}}}^{2E\epsilon/m_p} d\epsilon_r \sigma_{p\gamma\rightarrow p'b}^{\text{tot}}(\epsilon_r)$$

For given values of $E$ and $z$, NeuCosmA calculates the cooling rate $t_{p\gamma}^{-1} \equiv - (1/E) b_{p\gamma} (s^{-1})$ as

$$t_{p\gamma}^{-1}(E, z) = \sum_i \Gamma_{p\rightarrow p}^i(E, z) K^i,$$

with $K^i E$ the loss of energy per interaction

From this, we calculate back $b_{p\gamma}$ (GeV s$^{-1}$) . . .

. . . and the corresponding energy-loss term in the transport equation, $\partial_E (b_{p\gamma} Y_p)$. 

Interaction lengths

Note that $L_{\text{CIB}} \gg L_{\text{CMB}}$: