Pairing and quarteting in proton-neutron systems

Nicolae Sandulescu

National Institute of Physics and Nuclear Engineering, Bucharest

Main message: proton-neutron pairing is properly described by alpha-like quartets, not by Cooper pairs

Outline

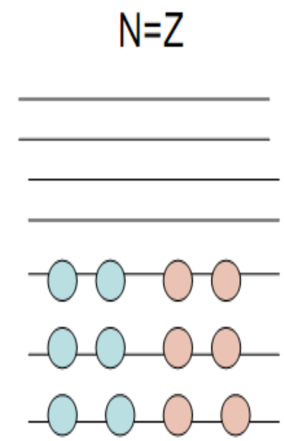
A) Isovector pairing with alpha-like quartets

- *quartet model for N=Z and N>Z systems*
- *isovector pairing and Wigner energy*

B) Isoscalar- isovector pairing with alpha-like quartets

- *competition between J=1 and J=0 pairing*

Isovector pairing of protons and neutrons



$$H = \sum_i \varepsilon_i (N_i^{(\nu)} + N_i^{(\pi)}) + \sum_{ij,\tau} V(i,j) P_{i,\tau}^+ P_{j,\tau}$$

$$P_{i0}^+ \propto \nu_i^+ \pi_{\bar{i}}^+ + \pi_i^+ \nu_{\bar{i}}^+ \qquad P_{i1}^+ \propto \nu_i^+ \nu_{\bar{i}}^+ \qquad P_{i-1}^+ \propto \pi_i^+ \pi_{\bar{i}}^+$$

- **Exact solution:** $g = V(i,j)$ Richardson, 1966 J. Links et al, J.P(2002) J. Dukelsky et al, PRL(2006)

  SO(5)

$$|\Psi> = \left[\prod_{\alpha=1}^{M} B_{-1}^+(e_\alpha)\right] \prod_\beta \left[T_+(\omega_\beta) - \sum_\alpha \frac{I_{+,\alpha}}{\omega_\beta - e_\alpha}\right] |\Lambda> \qquad B_\tau^+(e_\alpha) = \sum_i \frac{P_{i,\tau}^+}{2\varepsilon_i - e_\alpha}$$

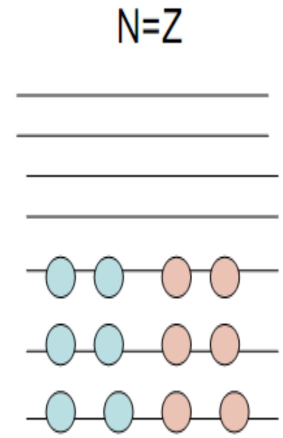
- BCS approximation: two degenerate solutions (which do not coexist)

$$\Delta_{\nu\pi} \neq 0,\ \Delta_\nu = \Delta_\pi = 0 \quad \text{or} \quad \Delta_{\nu\pi} = 0,\ \Delta_\nu = \Delta_\pi \neq 0$$

typical errors in correlation energies 30-40%

isovector pairing in the PBCS approximation

$$H = \sum_i \varepsilon_i (N_i^{(\nu)} + N_i^{(\pi)}) + \sum_{ij,\tau} V(i,j) P^+_{i,\tau} P_{j,\tau}$$



$$P^+_{i0} \propto \nu_i^+ \pi_{\bar{i}}^+ + \pi_i^+ \nu_{\bar{i}}^+ \qquad P^+_{i1} \propto \nu_i^+ \nu_{\bar{i}}^+ \qquad P^+_{i-1} \propto \pi_i^+ \pi_{\bar{i}}^+$$

collective pairs

$$\Gamma^+_{\nu\pi} = \sum_i x_i P^+_{i0} \qquad \Gamma^+_{\pi\pi} = \sum_i x_i P^+_{i-1} \qquad \Gamma^+_{\nu\nu} = \sum_i x_i P^+_{i1}$$

PBCS

$$| PBCS0 > \propto (\Gamma^+_{\nu\pi})^{\frac{N+Z}{2}} | - > \qquad | PBCS1 > \propto (\Gamma^+_{\nu\upsilon})^{N/2} \Gamma^+_{\pi\pi})^{Z/2} | - >$$

	SM	PBCS1	PBCS0
^{44}Ti	5.973	5.487 (8.134%)	4.912 (17.763%)
^{48}Cr	9.593	8.799 (8.277%)	7.885 (17.805%)
^{52}Fe	10.768	9.815 (8.850%)	8.585 (20.273%)

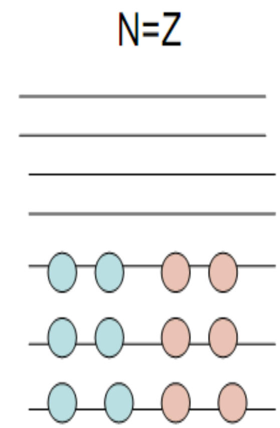
- no mixing of pn with nn and pp pairing !

restoration of the isospin symmetry ?

Isospin conservation and quarteting

$$H = \sum_i \varepsilon_i (N_i^{(\nu)} + N_i^{(\pi)}) + \sum_{ij,\tau} V(i,j) P_{i,\tau}^{+} P_{j,\tau}$$

$$P_{i1}^{+} \propto \nu_i^{+} \nu_{\bar{i}}^{+} \qquad P_{i-1}^{+} \propto \pi_i^{+} \pi_{\bar{i}}^{+} \qquad P_{i0}^{+} \propto \nu_i^{+} \pi_{\bar{i}}^{+} + \pi_i^{+} \nu_{\bar{i}}^{+}$$



non-collective quartets

$$Q_{ij}^{+} = [P_{i\tau}^{+} P_{j\tau'}^{+}]^{T=0} \propto P_{\nu\nu,i}^{+} P_{\pi\pi,j}^{+} + P_{\pi\pi,i}^{+} P_{\nu\nu,j}^{+} - P_{\nu\pi,i}^{+} P_{\nu\pi,j}^{+}$$

collective quartet

$$Q^{+} = \sum_{ij} x_{ij} (P_{\nu\nu,i}^{+} P_{\pi\pi,j}^{+} + P_{\pi\pi,i}^{+} P_{\nu\nu,j}^{+} - P_{\nu\pi,i}^{+} P_{\nu\pi,j}^{+})$$

quartet condensate

$$|QCM\rangle = Q^{+n_q} |-\rangle$$ (has T=0, J=0)

Quartet condensation and Cooper pairs

$$|QCM> = Q^{+n_q}|->$$

ansatz:

$$Q^+ \approx \sum_{ij} x_i x_j (P^+_{\nu\nu,i} P^+_{\pi\pi,j} + P^+_{\pi\pi,i} P^+_{\nu\nu,j} - P^+_{\nu\pi,i} P^+_{\nu\pi,j})$$

$$Q^+ \equiv (2\Gamma^+_{\nu\nu}\Gamma^+_{\pi\pi} - \Gamma^+_{\nu\pi}\Gamma^+_{\nu\pi}) \qquad \Gamma^+_\tau = \sum_i x_i P^+_{i,\tau}$$

collective Cooper pairs

$$|QCM> = (2\Gamma^+_{\nu\nu}\Gamma^+_{\pi\pi} - \Gamma^+_{\nu\pi}\Gamma^+_{\nu\pi})^{n_q}|->$$

'coherent' mixing of condensates formed by nn, pp and pn pairs

$$|QCM> = \sum_{k=0}^{n_q} \ldots (\Gamma^+_{\nu\nu}\Gamma^+_{\pi\pi})^{n_q-k}(\Gamma^+_{\nu\pi}\Gamma^+_{\nu\pi})^k \; |->$$

PBCS condensates are only two terms in QCM

$$|PBCS0> \propto (\Gamma^{+2}_{\nu\pi})^{n_q}|-> \qquad |PBCS1> \propto (\Gamma^+_{\nu\upsilon}\Gamma^+_{\pi\pi})^{n_q}|->$$

Quartet condensation: calculation scheme

$$H = \sum_i \varepsilon_i (N_i^{(\nu)} + N_i^{(\pi)}) + \sum_{ij,\tau} V(i,j) P_{i,\tau}^+ P_{j,\tau}$$

$$|\Psi(x)\rangle = (2\Gamma_{\nu\nu}^+ \Gamma_{\pi\pi}^+ - \Gamma_{\nu\pi}^+ \Gamma_{\nu\pi}^+)^{n_q} |-\rangle \qquad \Gamma_\tau^+ = \sum_i x_i P_{i,\tau}^+$$

$$\delta_x \langle \Psi | H | \Psi \rangle = 0 \quad \text{with} \quad \langle \Psi | \Psi \rangle = 1$$

Method of recurrence relations

auxiliary states $\quad |n_1, n_2, n_3\rangle \equiv \Gamma_{\nu\nu}^{+n_1} \Gamma_{\pi\pi}^{+n_2} \Gamma_{\nu\pi}^{+n_3} |0\rangle$

$$|\Psi\rangle = \sum_k \binom{n_q}{k} (-1)^{n_q - k} 2^k |k, k, 2(n_q - k)\rangle.$$

$$\langle n_1' n_2' n_3' | \hat{H} | n_1, n_2, n_3 \rangle$$

coupled, recursive relations for

SO(5) $\quad \langle | P_{i\tau}^+ | \rangle \quad \langle | N_{i\tau} | \rangle \quad \langle | T_{i\tau} | \rangle$

$$\langle | P_{i\tau}^+ P_{j\tau'} | \rangle$$

Recursive relations: example

$$
\begin{aligned}
&\langle n_1'n_2'n_3'|P_{i,1}^{+}P_{j,1}|n_1n_2n_3\rangle = n_1x_j\langle n_1-1n_2n_3|P_{i,1}|n_1'n_2'n_3'\rangle\\
&-x_ix_j^2(n_1'n_1n_3\langle n_1-1n_2n_3-1|P_{j,0}|n_1'-1n_2'n_3'\rangle\\
&+n_1'n_1(n_1-1)\langle n_1-2n_2n_3|P_{j,1}|n_1'-1n_2'n_3'\rangle\\
&+\tfrac{1}{2}n_1'n_3(n_3-1)\langle n_1n_2n_3-2|P_{j,-1}|n_1'-1n_2'n_3'\rangle)\\
&+x_i^2x_j^2[n_1'n_3'n_1n_3[\langle n_1'-1n_2'n_3'-1|P_{j,0}^{+}P_{i,0}|n_1-1n_2n_3-1\rangle\\
&+\delta_{ij}(\langle n_1'-1n_2'n_3'-1|n_1-1n_2n_3-1\rangle\\
&-\tfrac{1}{2}\langle n_1'-1n_2'n_3'-1|N_{i,0}|n_1-1n_2n_3-1\rangle)]\\
&+n_1'(n_1'-1)n_1n_3[\langle n_1'-2n_2'n_3'|P_{j,0}^{+}P_{i,1}|n_1-1n_2n_3-1\rangle\\
&-\delta_{ij}\langle n_1'-2n_2'n_3'|T_{i,-1}|n_1-1n_2n_3-1\rangle]\\
&+\tfrac{1}{2}n_3'(n_3'-1)n_1n_3[\langle n_1'n_2'n_3'-2|P_{j,0}^{+}P_{i,-1}|n_1-1n_2n_3-1\rangle\\
&+\delta_{ij}\langle n_1'n_2'n_3'-2|T_{i,1}|n_1-1n_2n_3-1\rangle]\\
&+n_1'n_3'n_1(n_1-1)[\langle n_1-2n_2n_3|P_{i,0}^{+}P_{j,1}|n_1'-1n_2'n_3'-1\rangle\\
&-\delta_{ij}\langle n_1-2n_2n_3|T_{i,-1}|n_1'-1n_2'n_3'-1\rangle]\\
&+n_1'(n_1'-1)n_1(n_1-1)[\langle n_1'-2n_2'n_3'|P_{j,1}^{+}P_{i,1}|n_1-2n_2n_3\rangle\\
&+\delta_{ij}(\langle n_1'-2n_2'n_3'|n_1-2n_2n_3\rangle-\langle n_1'-2n_2'n_3'|N_{i,1}|n_1-2n_2n_3\rangle)]\\
&+\tfrac{1}{2}n_3'(n_3'-1)n_1(n_1-1)\langle n_1'n_2'n_3'-2|P_{j,1}^{+}P_{i,-1}|n_1-2n_2n_3\rangle\\
&+\tfrac{1}{2}n_1'n_3'n_3(n_3-1)[\langle n_1n_2n_3-2|P_{i,0}^{+}P_{j,-1}|n_1'-1n_2'n_3'-1\rangle\\
&+\delta_{ij}\langle n_1n_2n_3-2|T_{i,1}|n_1'-1n_2'n_3'-1\rangle]\\
&+\tfrac{1}{2}n_1'(n_1'-1)n_3(n_3-1)\langle n_1n_2n_3-2|P_{i,1}^{+}P_{j,-1}|n_1'-2n_2'n_3'\rangle\\
&+\tfrac{1}{4}n_3'(n_3'-1)n_3(n_3-1)[\langle n_1'n_2'n_3'-2|P_{j,-1}^{+}P_{i,-1}|n_1n_2n_3-2\rangle\\
&+\delta_{ij}(\langle n_1'n_2'n_3'-2|n_1n_2n_3-2\rangle-\langle n_1'n_2'n_3'-2|N_{i,-1}|n_1n_2n_3-2\rangle)]].
\end{aligned}
$$

Accuracy of quartet condensation model

$$H = \sum_i \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i,j) \sum_t \; P_{it}^+ P_{jt}$$

pairing forces extracted from SM interactions

$$|QCM> \equiv (Q^+)^{n_q} |->$$ $$|PBCS1> \propto (\Gamma_{\nu\upsilon}^+ \Gamma_{\pi\pi}^+)^{n_q} |->$$ $$|PBCS0> \propto (\Gamma_{\nu\pi}^{+2})^{n_q} |->$$

	SM	QCM	PBCS1	PBCS0
^{20}Ne	9.173	9.170 (0.033%)	8.385 (8.590%)	7.413 (19.187%)
^{24}Mg	14.460	14.436 (0.166%)	13.250 (8.368%)	11.801 (18.389%)
^{28}Si	15.787	15.728 (0.374%)	14.531 (7.956%)	13.102 (17.008%)
^{32}S	15.844	15.795 (0.309%)	14.908 (5.908%)	13.881 (12.389%)
^{44}Ti	5.973	5.964 (0.151%)	5.487 (8.134%)	4.912 (17.763%)
^{48}Cr	9.593	9.569 (0.250%)	8.799 (8.277%)	7.885 (17.805%)
^{52}Fe	10.768	10.710 (0.539%)	9.815 (8.850%)	8.585 (20.273%)
^{104}Te	3.831	3.829 (0.052%)	3.607 (5.847%)	3.356 (12.399%)
^{108}Xe	6.752	6.696 (0.829%)	6.311 (6.531%)	5.877 (12.959%)
^{112}Ba	8.680	8.593 (1.002%)	8.101 (6.670%)	13.064 (13.064%)

N. S, Daniel Negrea, J. Dukelsky, C.W. Johnson, PRC85, 061303(R) (2012)

how important is the isospin restoration ?

Isospin restoration versus quartet correlations

$$H = \sum_i \varepsilon_i \left(N_i^{(\nu)} + N_i^{\pi}\right) - g \sum_{ij,\tau} P_{i,\tau}^{+} P_{j,\tau}$$

$$|QCM> \equiv (Q^{+})^{n_q} |->$$

$$|PBCS(N,T)\rangle = \hat{P}_T \hat{P}_N |BCS\rangle$$

$$E_{corr} = E_0 - E$$

^{52}Fe

Exact value: 8.29 MeV

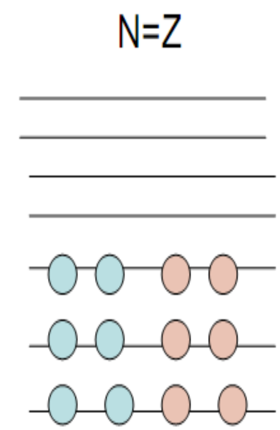
PBCS(N,T): 7.63 MeV (8%) (Chen et al , Nucl. Phys.A 1978)

QCM: 8.25 MeV (0.5%)

QCM state describes additional quartet-type correlations !

Isospin pairing with distinct quartets

$$H = \sum_i \varepsilon_i (N_i^{(\nu)} + N_i^{(\pi)}) + \sum_{ij,\tau} V(i,j) P_{i,\tau}^+ P_{j,\tau}$$



non-collective quartets

$$A_{ij}^+ = [P_{i\tau}^+ P_{j\tau'}^+]^{T=0} \propto P_{\nu\nu,i}^+ P_{\pi\pi,j}^+ + P_{\pi\pi,i}^+ P_{\nu\nu,j}^+ - P_{\nu\pi,i}^+ P_{\nu\pi,j}^+$$

collective quartet

$$Q^+ = \sum_{ij} x_{ij} Q_{ij}^+$$

quartet condensate

$$|QCM> = Q^{+n_q} |->$$

collective quartets

$$Q_\nu^+ = \sum_{ij} q_{ij}^{(\nu)} Q_{ij}^+$$

product of distinct quartets

$$|QM> = Q_1^+ Q_2^+ \ldots Q_{n_q}^+ |->$$

M. Sambataro and N.S, PRC88 (2013)

analogy with Richardson solution for like-particle pairing !

Isovector pairing with distinct quartets: accuracy

$$H = \sum_i \varepsilon_i (N_i^{(\nu)} + N_i^{(\pi)}) + \sum_{ij,\tau} V(i,j) P_{i,\tau}^{+} P_{j,\tau}$$

$$|QM> = Q_1^{+} Q_2^{+} \ldots Q_{n_q}^{+} |->$$

$$|QCM> = Q^{+n_q} |->$$

Spherical single-particle basis

V_{ij} extracted from standard shell model interactions

	exact	QM	QCM
^{20}Ne	-9.174	-9.174 (-)	-9.170 (0.04%)
^{24}Mg	-14.461	-14.458 (0.02%)	-14.436 (0.17%)
^{28}Si	-15.787	-15.780 (0.04%)	-15.728 (0.37%)
^{32}S	-15.844	-15.844 (-)	-15.795 (0.31%)
^{44}Ti	-5.965	-5.965 (-)	-5.964 (0.02%)
^{48}Cr	-9.579	-9.573 (0.06%)	-9.569 (0.10%)
^{52}Fe	-10.750	-10.725 (0.23%)	-10.710 (0.37%)
^{104}Te	-3.832	-3.832 (-)	-3.829 (0.08%)
^{108}Xe	-6.752	-6.752 (-)	-6.696 (0.83%)
^{112}Ba	-8.680	-8.678 (0.02%)	-8.593 (1.00%)

M. Sambataro and N.S., PRC88 (2013) 061303(R)

Isovector pairing with distinct quartets: accuracy

$$H = \sum_i \varepsilon_i \left(N_i^{(\nu)} + N_i^{\pi} \right) - g \sum_{ij,\tau} P_{i,\tau}^{+} P_{j,\tau}$$

$$|QM> = Q_1^+ Q_2^+ \ldots Q_{n_q}^+ |->$$

$$|QCM> = Q^{+n_q} |->$$

Deformed single-particle basis (HF + Skyrme force SLy4)

V_{ij}=-24/A

	exact	QM	QCM
^{20}Ne	-6.5505	-6.5505	-6.539 (0.18%)
^{24}Mg	-8.4227	-8.4227	-8.388 (0.41%)
^{28}Si	-9.6610	-9.6610	-9.634 (0.28%)
^{32}S	-10.2629	-10.2629	-10.251 (0.12%)
^{44}Ti	-3.1466	-3.1466	-3.142 (0.15%)
^{48}Cr	-4.2484	-4.2484	-4.227 (0.50%)
^{52}Fe	-5.4532	-5.4531	-5.426 (0.50%)
^{104}Te	-1.0837	-1.0837	-1.082 (0.16%)
^{108}Xe	-1.8696	-1.8696	-1.863 (0.35%)
^{112}Ba	-2.7035	-2.7034	-2.688 (0.57%)

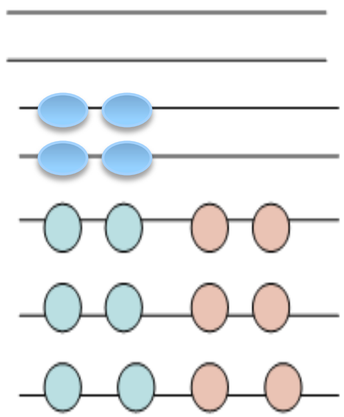
QM gives the exact solution for isovector pairing !?

Conclusions for isovector pairing in N=Z nuclei

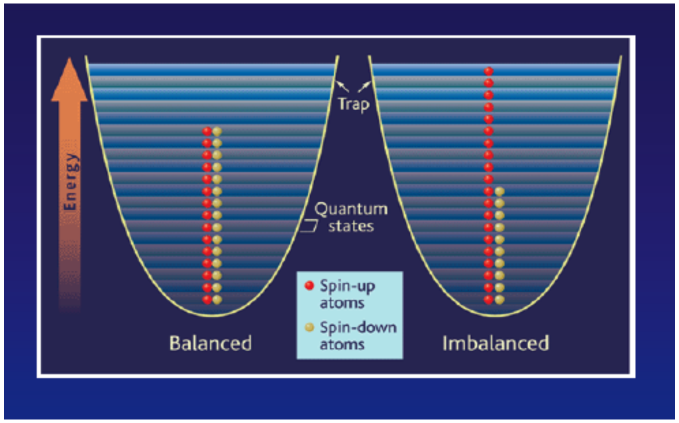
- *T=1 pairing is accurately described by quartets, not by pairs*
- there is not a pure condensate of isovector pn pairs in N=Z nuclei

Pairing and quarteting in N>Z systems

N>Z



Imbalanced fermionic systems



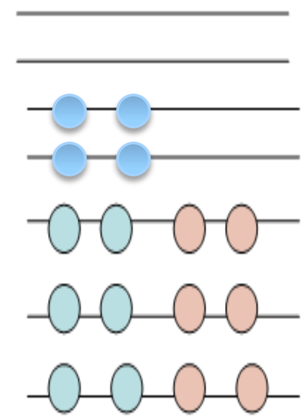
gapless/breached pair superfludity

LOFF phase (non-zero CM momentum)

Proton-neutron pairing and quartets for N>Z nuclei

nuclei with N-Z=2n_N

N>Z



ansatz

- all protons are correlated in alpha-like quartets
- neutrons in excess form a pair condensate

$$|\Psi\rangle = (\tilde{\Gamma}^{+}_{\nu\nu})^{n_N} (Q^{+})^{n_q} \, |-\rangle$$

$$|\Psi\rangle = (\tilde{\Gamma}^{+}_{\nu\nu})^{n_N} (2\Gamma^{+}_{\nu\nu}\Gamma^{+}_{\pi\pi} - \Gamma^{+}_{\nu\pi}\Gamma^{+}_{\nu\pi})^{n_q} \, |-\rangle$$

$$\tilde{\Gamma}^{+}_{\nu\nu} = \sum_i \tilde{x}_i \nu_i^{+} \nu_{\bar{i}}^{+} \qquad \Gamma^{+}_{\nu\nu} = \sum_i x_i \nu_i^{+} \nu_{\bar{i}}^{+}$$

no quarteting: $|PBCS1\rangle = (\tilde{\Gamma}^{+}_{\nu\nu})^{N/2} (\Gamma^{+}_{\pi\pi})^{Z/2} \, |-\rangle$

Alpha-like condensation for N>Z nuclei: results

$$H = \sum_i \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i,j) \sum_t P_{it}^{+} P_{jt}$$

pairing forces extracted from SM interactions

$$|QCM>=(\tilde{\Gamma}_{\nu\nu}^{+})^{n_N}(Q^{+})^{n_q}|0>$$

$$|PBCS1>=(\tilde{\Gamma}_{\nu\nu}^{+})^{N/2}(\Gamma_{\pi\pi}^{+})^{Z/2}$$

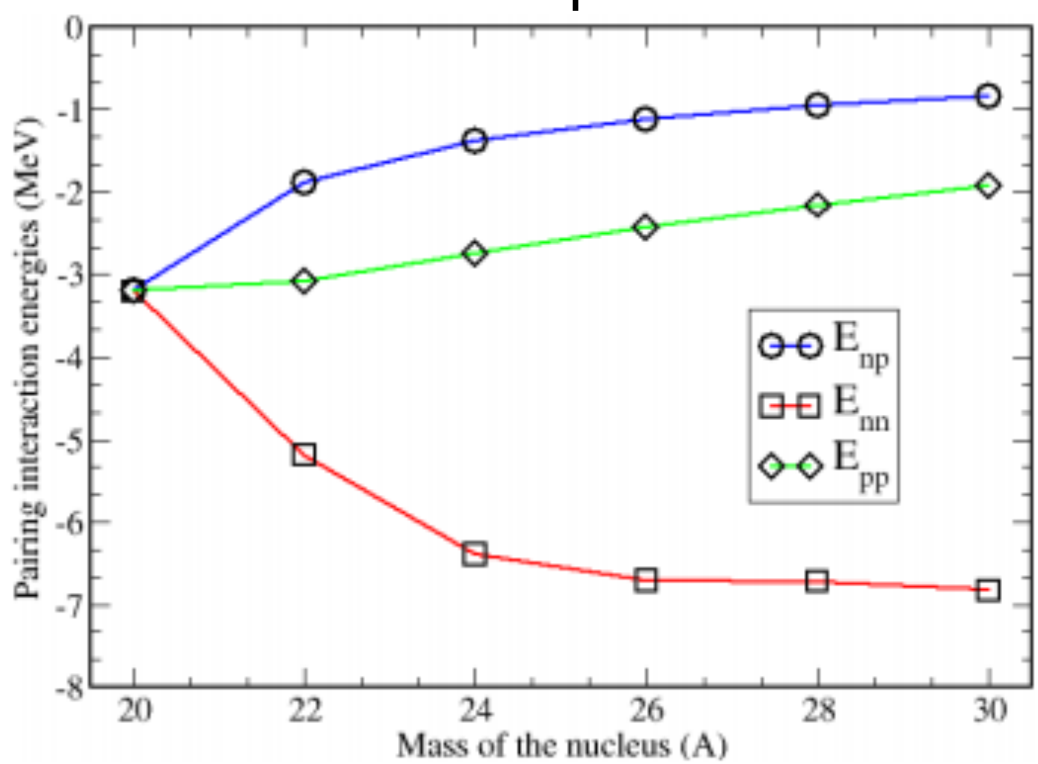
	Exact	QCM	PBCS1		Exact	QCM	PBCS1
^{20}Ne	6.550	6.539 (0.17%)	5.752 (12.18%)	^{24}Mg	8.423	8.388 (0.41%)	7.668 (8.96%)
^{22}Ne	6.997	6.969 (0.40%)	6.600 (5.67%)	^{26}Mg	8.680	8.654 (0.30%)	8.258 (4.86%)
^{24}Ne	7.467	7.426 (0.55%)	7.226 (3.23%)	^{28}Mg	8.772	8.746 (0.30%)	8.531 (2.75%)
^{26}Ne	7.626	7.592 (0.45%)	7.486 (1.84%)	^{30}Mg	8.672	8.656 (0.18%)	8.551 (1.39%)
^{28}Ne	7.692	7.675 (0.22%)	7.622 (0.91%)	^{32}Mg	8.614	8.609 (0.06%)	8.567 (0.55%)
^{30}Ne	7.997	7.994 (0.04%)	7.973 (0.30%)	^{28}Si	9.661	9.634 (0.28%)	9.051 (6.31%)
^{30}Si	9.310	9.296 (0.15%)	9.064 (2.64%)	^{32}Si	9.292	9.283 (0.10%)	9.196 (1.03%)

N. S, D. Negrea, C. W. Johnson, PRC86 (2012) 041302(R)

Proton-neutron pairing far from N=Z line

$$|\Psi\rangle = (\tilde{\Gamma}^{+}_{\nu\nu})^{n_N}(2\Gamma^{+}_{\nu\nu}\Gamma^{+}_{\pi\pi} - \Gamma^{+}_{\nu\pi}\Gamma^{+}_{\nu\pi})^{n_q}|-\rangle$$

Ne isotopes



pn pairing and alpha-like quartets persist far from N=Z line !

Isovector pairing in Skyrme-HF+QCM

$$H = \sum_i (\varepsilon_i^{(\nu)} N_i^{(\nu)} + \varepsilon_i^{(\pi)} N_i^{(\pi)}) + g \sum_{i,j,t} P_{it}^{+} P_{jt}$$

pairing is treated as a residual interaction relative to a HF mean field

single-particle energies: from Skyrme-HF

$$|QCM\rangle = (\tilde{\Gamma}_{\nu\nu}^{+})^{n_N} (Q^{+})^{n_q} |core\rangle$$

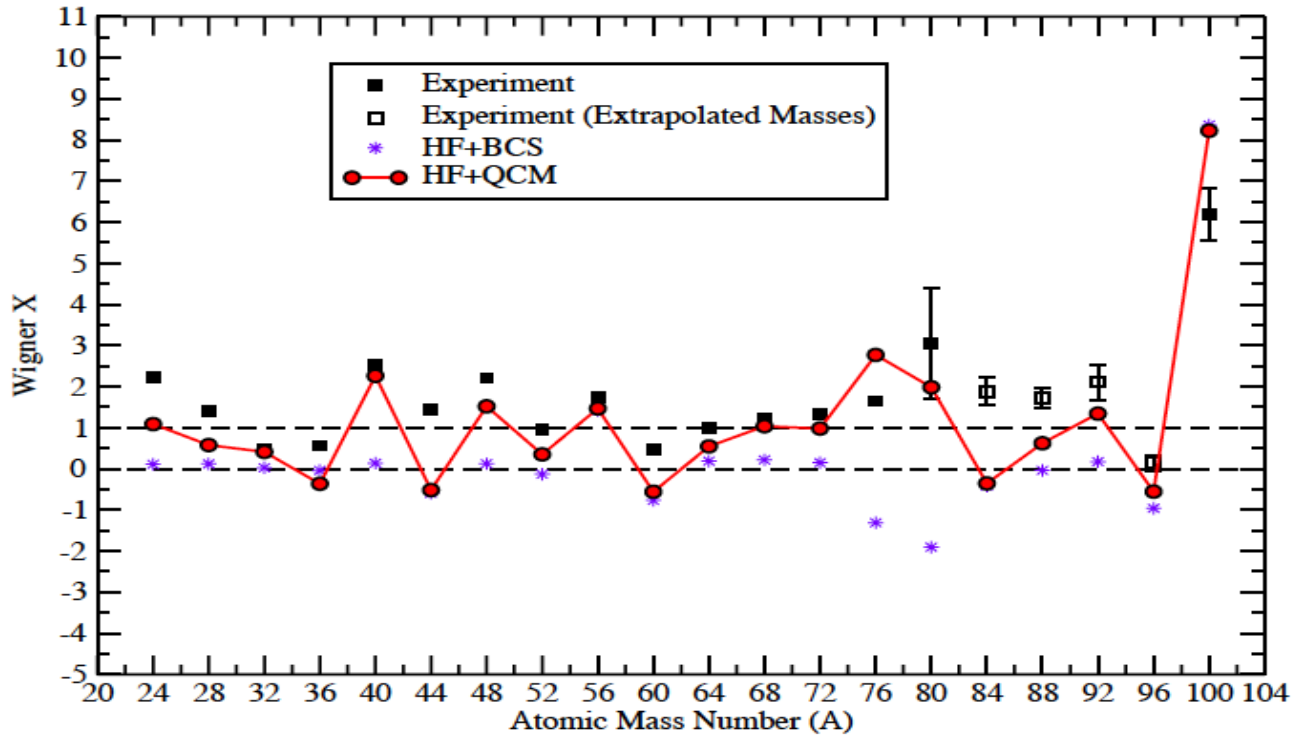
$\mathrm{v}_i^{(\nu)2}$ $\mathrm{v}_i^{(\pi)2}$ $\Rightarrow$ densities $\Rightarrow$ HF

D. Negrea and N. S, Phys Rev C90,024322 (2014)

Wigner energy from HF+QCM

$$E(N,Z) = E(N=Z) + a_s \frac{(N-Z)^2}{A} + a_W \frac{|N-Z|}{A} + \delta E_{shell} + \delta E_P$$ (no Coulomb)

$$E(N,Z) = E(N=Z) + \frac{T_z(T_z+X)}{2\Theta} \qquad T_z = 0,2,4$$



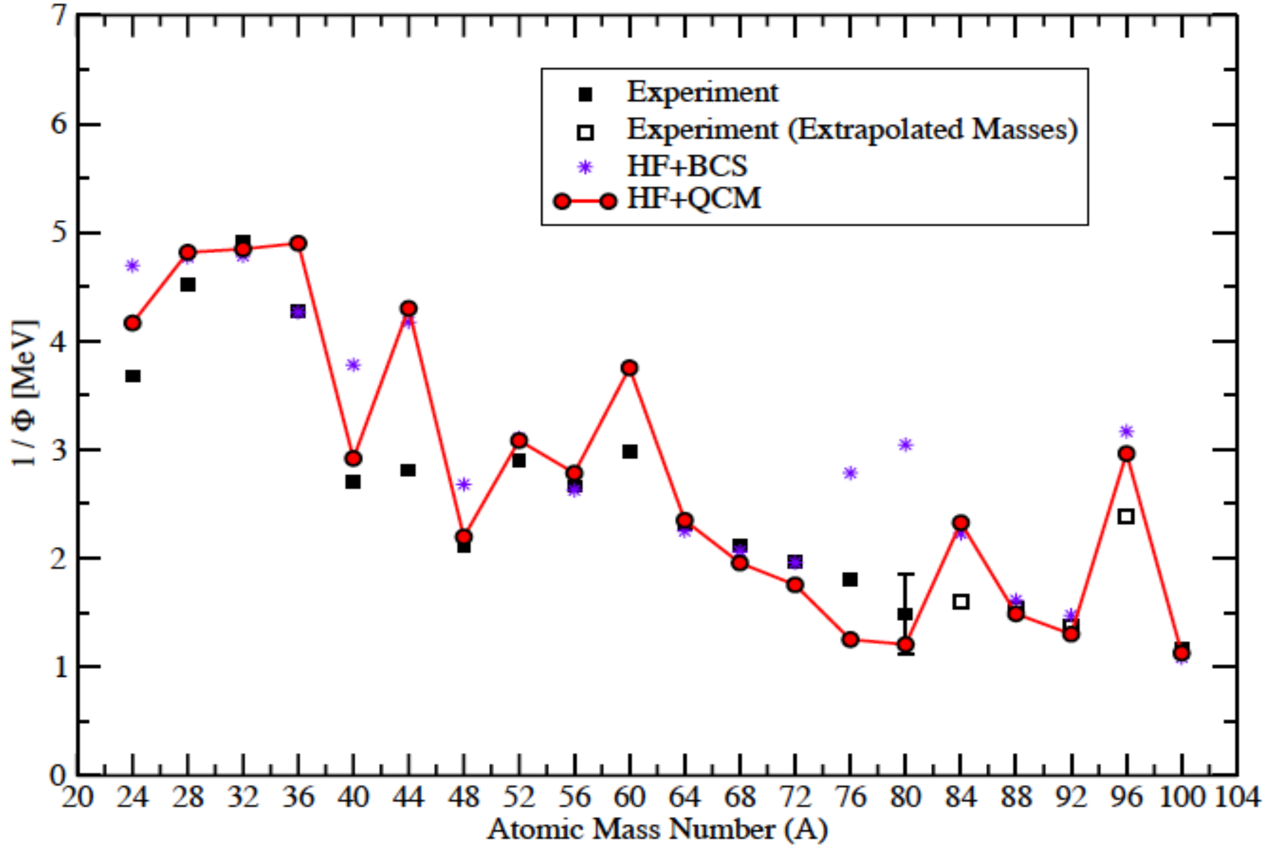
BCS fails to describe the Wigner energy

T=1 pairing has a significant contribution to Wigner energy !

D. Negrea and N. S, PRC90 (2014)

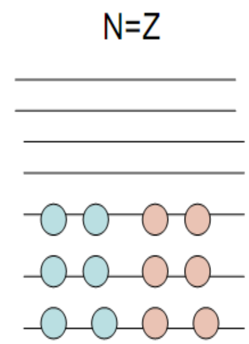
Symmetry energy from HF+QCM

$$E(N,Z) = E(N=Z) + \frac{T_z(T_z + X)}{2\Theta}$$



Isovector and isoscalar (J=1) pairing: quartet model

$$\hat{H} = \sum_i (2j_i+1)\epsilon_i N_i + \sum_{i,j} V^{T=1}_{J=0}(i,j) \sum_{T_z} P^+_{i,T_z} P_{j,T_z} + \sum_{ij,kl} V^{T=0}_{J=1}(ij,kl) \sum_{J_z} D^+_{ij,J_z} D_{kl,J_z}.$$



$$P^+_{i,T_z} = \sqrt{2j_i+1}[a^+_i a^+_i]^{T=1,J=0}_{T_z} \qquad D^+_{ij,J_z} = [a^+_i a^+_j]^{J=1,T=0}_{J_z}$$

collective quartets

$$Q^{+(iv)}_\nu = \sum_{i,j} x^{(\nu)}_{ij} [P^+_i P^+_j]^{T=0} \qquad Q^{+(is)}_\nu = \sum_{ij,kl} y^{(\nu)}_{ij,kl} [D^+_{ij} D^+_{kl}]^{J=0}$$

generalised quartet

$$Q^+_\nu = Q^{+(iv)}_\nu + Q^{+(is)}_\nu$$

ground state

$$|QM\rangle = \prod_{\nu=1}^{N_Q} Q^\dagger_\nu |0\rangle.$$

superposition of T=0 and T=1 quartets

$$|is\rangle = \prod_{\nu=1}^{N_Q} Q^{\dagger(is)}_\nu |0\rangle \quad \cdots\cdots\cdots \quad |iv\rangle = \prod_{\nu=1}^{N_Q} Q^{\dagger(iv)}_\nu |0\rangle$$

Isovector and isoscalar (J=1) pairing: accuracy of QM

$$H = \sum \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i,j) \sum_{\tau} P_{i\tau}^{+} P_{j\tau} + \sum_{ij} V_{J=1}^{T=0}(i,j) \sum_{\sigma} D_{i\sigma}^{+} D_{j\sigma}$$

$$|QM>== Q_1^{+} Q_2^{+} \ldots Q_{n_q}^{+} \,|->$$

$$|QM(l.o)>== Q^{+} \,|QCM(n_q - 1)>$$

	N_Q	exact	QM	QM(l.o.)	|<QM|QM(l.o.)>|
^{24}Mg	2	-28.694	-28.626 (0.24%)	-28.592 (0.35%)	0.9993
^{28}Si	3	-35.600	-35.396 (0.57%)	-35.307 (0.82%)	0.9980
^{32}S	4	-38.965	-38.865 (0.25%)	-38.668 (0.76%)	0.9942
^{48}Cr	2	-11.649	-11.624 (0.21%)	-11.614 (0.30%)	0.9996
^{52}Fe	3	-13.887	-13.823 (0.46%)	-13.804 (0.60%)	0.9994
^{108}Xe	2	-5.505	-5.495 (0.18%)	-5.490 (0.27%)	0.9995
^{112}Ba	3	-7.059	-7.035 (0.34%)	-7.025 (0.48%)	0.9987

M. Sambataro, N.S. and C.W.Johnson, in preparation

Isovector versus isoscalar pairing correlations

$$H = \sum \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i,j) \sum_{\tau} P_{i\tau}^{+} P_{j\tau} + \sum_{ij} V_{J=1}^{T=0}(i,j) \sum_{\sigma} D_{i\sigma}^{+} D_{j\sigma}$$

$$|QM> = = Q_1^+ Q_2^+ ... Q_{N_q}^+ |->$$

$$Q_\nu^+ = Q_\nu^{+(iv)} + Q_\nu^{+(is)}$$

$$|is\rangle = \prod_{\nu=1}^{N_Q} Q_\nu^{\dagger(is)} |0\rangle$$

$$|iv\rangle = \prod_{\nu=1}^{N_Q} Q_\nu^{\dagger(iv)} |0\rangle$$

	QM	iv	is	$< QM\|iv >$	$< QM\|is >$	$< iv\|is >$
^{20}Ne	15.985	14.402 (9.9%)	15.130 (5.35%)	0.884	0.953	0.843
^{24}Mg	28.625	23.269 (18.71%)	26.925 (5.94%)	0.650	0.910	0.336
^{28}Si	35.386	28.896 (18.34%)	33.377 (5.68%)	0.590	0.910	0.341
^{32}S	38.844	33.958 (12.58%)	37.881 (2.48%)	0.640	0.974	0.587
^{44}Ti	7.02	6.27 (10.6%)	4.92 (30%)	0.90	0.68	0.3
^{48}Cr	11.624	10.59 (8.9%)	7.38 (36.5%)	0.906	0.497	0.22
^{52}Fe	13.823	12.814 (7.3%)	9.98 (27.83%)	0.927	0.753	0.74
^{104}Te	3.147	3.041 (3.37%)	1.549 (50.78%)	0.978	0.489	0.314
^{108}Xe	5.495	5.240 (4.64%)	2.627 (52.19%)	0.958	0.354	0.234
^{112}Ba	7.035	6.614 (5.98%)	4.466 (36.52%)	0.939	0.375	0.376

isoscalar and isovector pairing *always* coexist together

isoscalar pairing is suppresed by spin-orbit

Summary and Conclusions

<u>Main message</u>: *isovector and isoscalar pairing are accurately described by alpha-like quartets, not by Cooper pairs*

Isovector pairing in the quartet formalism

- *isovector pairing gives a significant contribution to Wigner energy*
- *isovector proton-neutron pairing and the quartet structure persist far from N=Z line*

Isoscalar-isovector pairing in the quartet formalism

- *isoscalar and isovector pairing always coexist in the ground state of N=Z nuclei*
- *J=1 pairing has an important contribution in sd-shell nuclei and is strongly supressed in pf-shell nuclei*