Alpha-decay: a computational challenge

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Extreme Light Infrastructure (ELI):
the future laser facility @ Bucharest-Măgurele

Outline

I. Basic laws in α-decay

II. Microscopic approach to describe α-decay width

III. α-decay spectroscopy

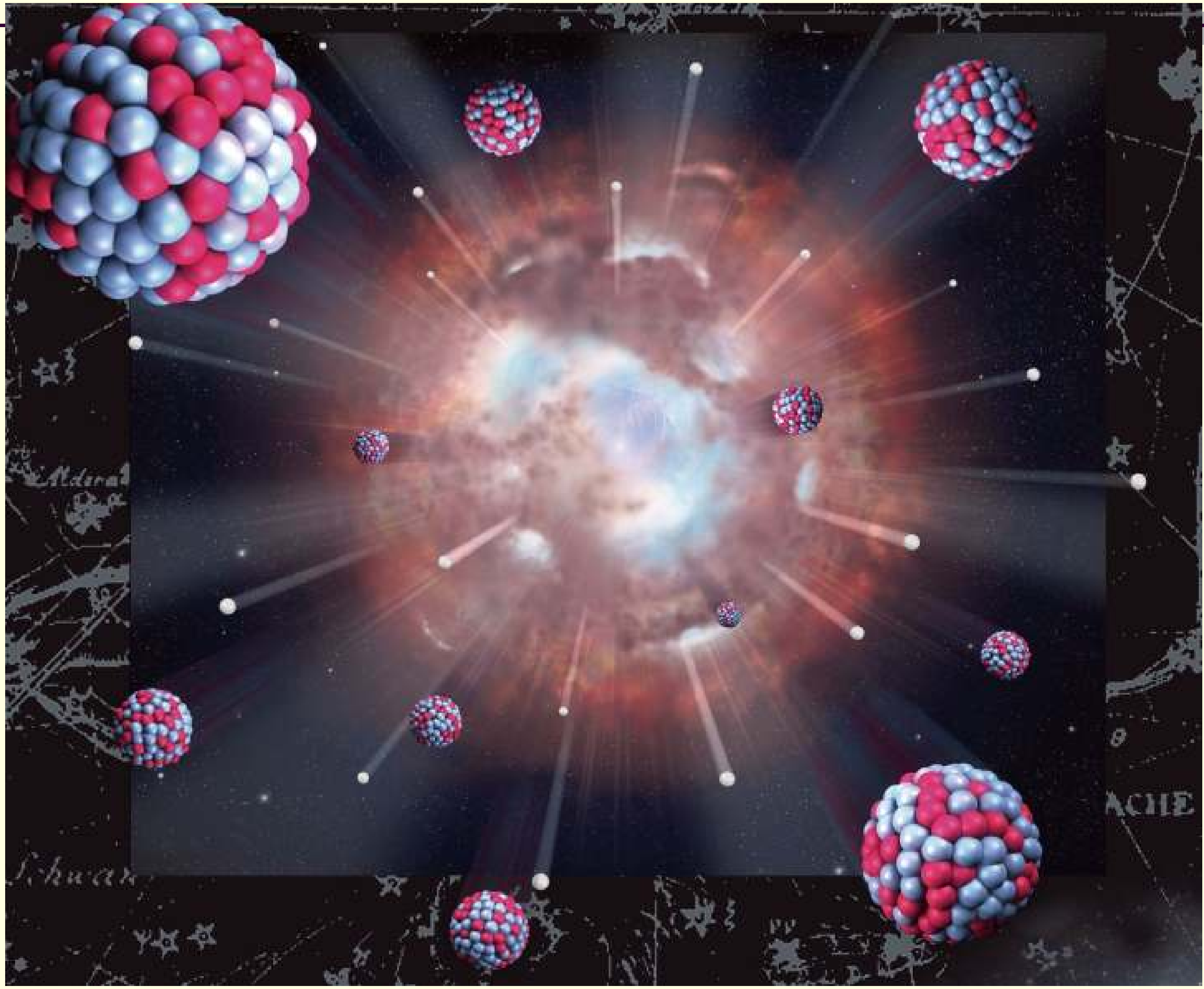
IV. Coupled channels approach of α-transitions

V. Surface α-clustering in ^{212}Po

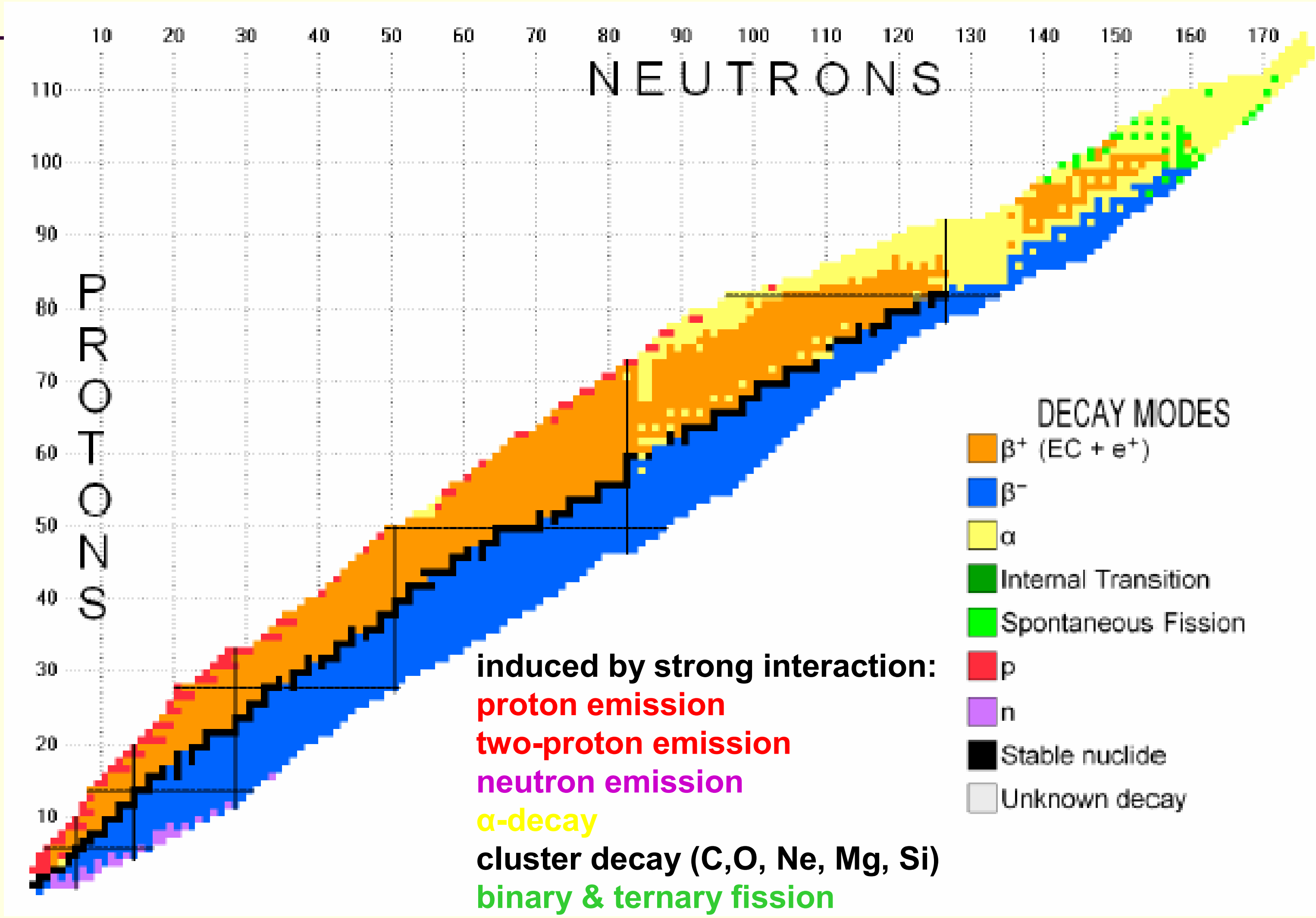
VI. Probing shape coexistence by α-decay to excited 0^+ states

VII. Conclusions

Heavy nuclei were created in supernovae explosions

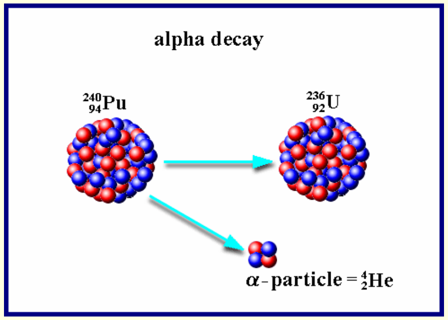


Most of nuclei are unstable
by decaying through various nodes



I. Basic laws in alpha-decay

A. Geiger-Nuttall law for half lives



$$\log_{10} T = a\frac{Z_D}{\sqrt{E}} + b$$

- **H. Geiger and J.M. Nuttall** "The ranges of the α particles from various radioactive substances and a relation between range and period of transformation," *Philosophical Magazine*, Series 6, vol. 22, no. 130, 613-621 (1911).

- **H. Geiger and J.M. Nuttall** "The ranges of α particles from uranium," *Philosophical Magazine*, Series 6, vol. 23, no. 135, 439-445 (1912).

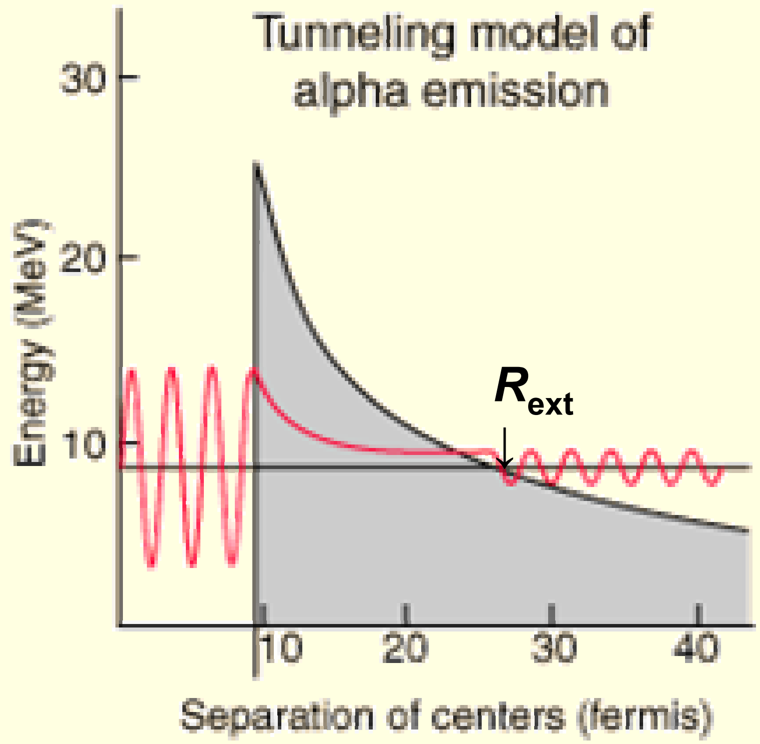
George Gamow in 1909,
two years before
the discovery of the G-N law

… and in 1930,
two years after
his explanation

G. Gamow "Zur Quantentheorie des Atomkernes" (On the quantum theory of the atomic nucleus), *Zeitschrift für Physik*, vol. 51, 204-212 (1928).

The first probabilistic interpretation of the wave function

$\Psi_{\text{int}} \rightarrow$



$\leftarrow A\Psi_{ext}$

A **is called** **scattering amplitude**

Internal region **External region**

External wave function describes a decaying state

Decay law

$$\left|\Psi_{ext}(R,t)\right|^2 \rightarrow N(t) = N_0 e^{-\lambda t}$$

implies a wave function

$$\Psi_{ext}(R,t) = \Psi_{ext}(R) e^{-i(E/\hbar)t}$$

with complex energy

$$E = E_0 - i\frac{\Gamma}{2}$$

Decay constant is proportional to decay width:

$$\lambda = \frac{\Gamma}{\hbar}$$

External radial wave function is called Gamow state

Its radial part is an outgoing
spherical Coulomb wave

$$\Psi_{ext}(R) = \frac{H_l^{(+)}(kR)}{R} \xrightarrow[R\to\infty]{} \frac{e^{i(kR+\sigma_l)}}{R}$$

Internal radial wave function is a narrow resonace

It is "almost bound" by the external Coulomb potential and it can be normalized to unity in the internal region

$$\int_0^{R_{ext}} \left| R\Psi_{\text{int}}(R) \right|^2 dR = 1$$

It has very small values outside the barrier

Scattering amplitude is given by the matching condition:

$$|A| = \left| \frac{\Psi_{\text{int}}(R_{ext})}{\Psi_{ext}(R_{ext})} \right| < 10^{-10}$$

Decay width is the flux of outgoing particles

$$\Gamma = \hbar \mathrm{v} \left| A \right|^2 = \hbar \mathrm{v} \left| \frac{\Psi_{\mathrm{int}}(R)}{\Psi_{ext}(R)} \right|^2$$

The width does not depend on the matching radius *R*
because both functions satisfy
the same Schrödinger equation

Half life is given by: $$T = \frac{\hbar \ln 2}{\Gamma}$$

Decay width can be rewritten

as a product between

$$\Gamma = 2\gamma^2 P$$

reduced width squared

$$\gamma^2 = \frac{\hbar^2}{2mR}\left|\Psi_{\text{int}}(R)\right|^2$$

and penetrability on the matching radius *R*

$$P = \frac{\kappa R}{\left|H_0^{(+)}(\chi, kR)\right|^2} = ce^{-d\chi}$$

depending exponentially opon the Coulomb parameter

$$\chi = \frac{2Z_D Z_C}{\hbar \mathrm{v}} = \frac{2Z_D Z_C}{\hbar\sqrt{2E/m}}$$

Geiger-Nuttall law is given by the penetrability

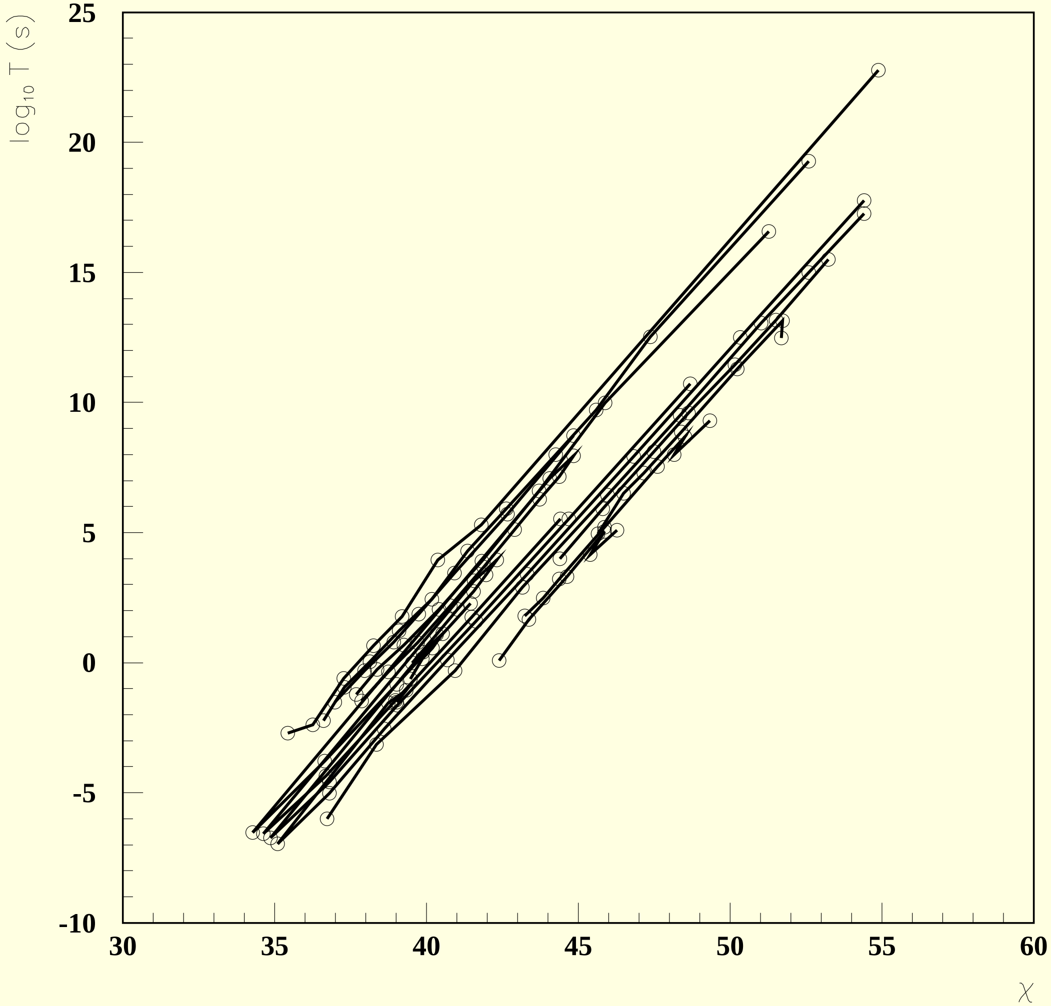
$$\log_{10} \Gamma = \log_{10} P + \log_{10} 2\gamma^2$$

Geiger-Nuttal law supposes a constant reduced width

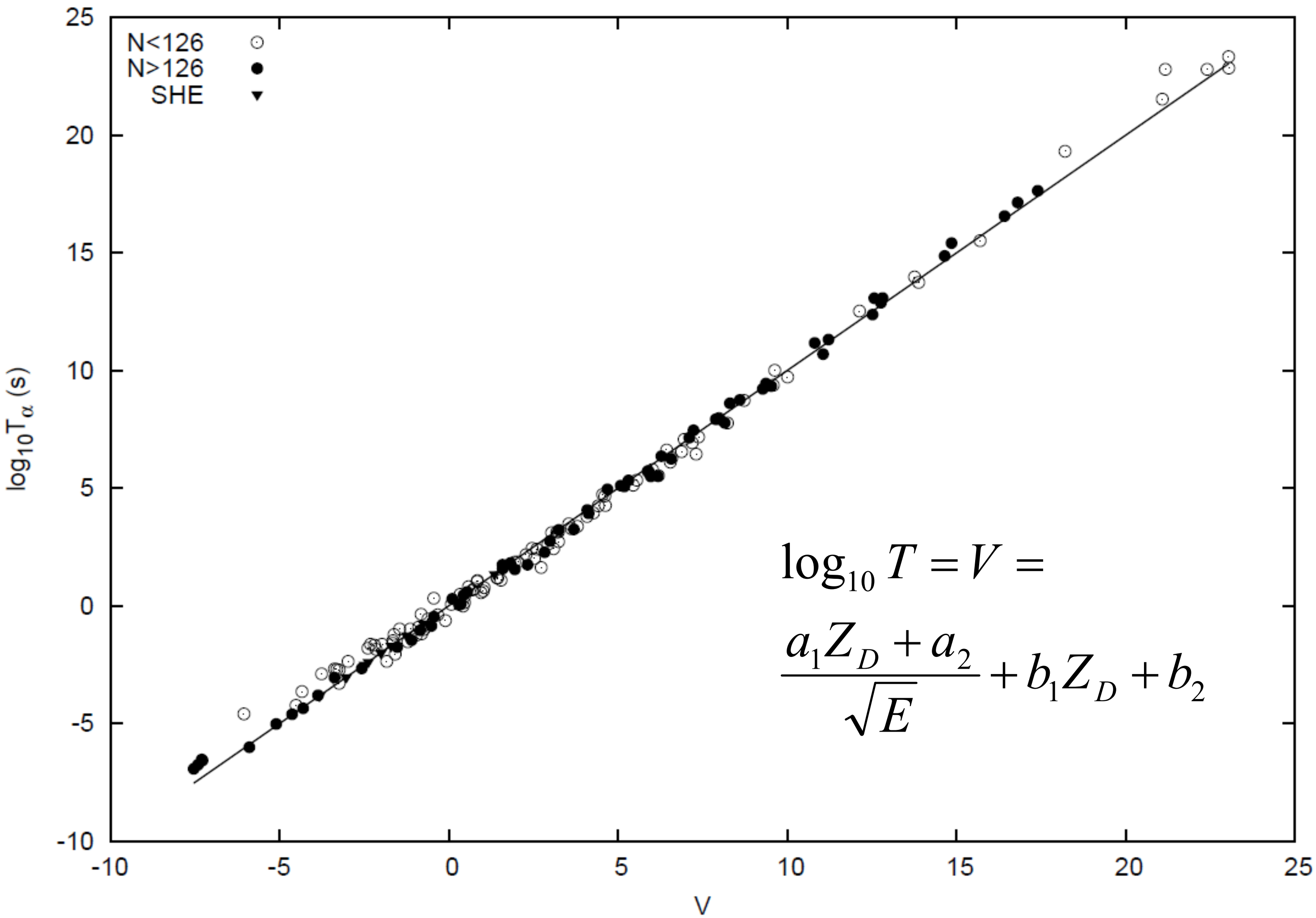
$$\log_{10} P = a\chi + b$$

$$\chi = c\frac{Z_D}{\sqrt{E}}$$

Geiger-Nuttall law for α-decay gives several parallel lines corresponding to various isotope chains

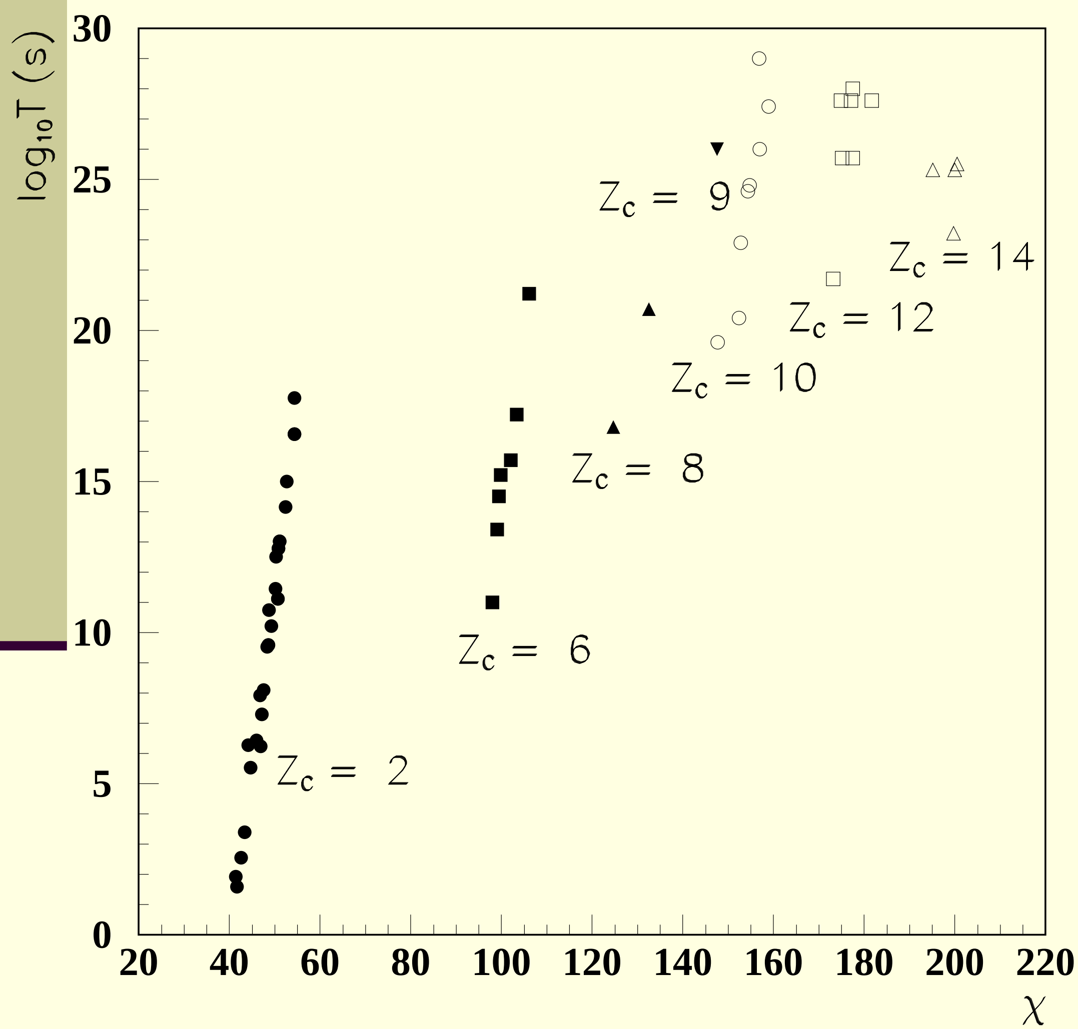


Viola-Seaborg graph
reduces parallel lines to a single linear dependence



$$\log_{10} T = V = \frac{a_1 Z_D + a_2}{\sqrt{E}} + b_1 Z_D + b_2$$

Geiger-Nuttall law can be generalized for cluster-decays



Magic radioactivity
"Pb decay"
Z_D~82

Viola-Seaborg rule generalized to cluster decays

$$\log_{10} T = \frac{a_1 Z_D Z_C + a_2}{\sqrt{E}} + b_1 Z_D Z_C + b_2$$

B. The law for reduced widths

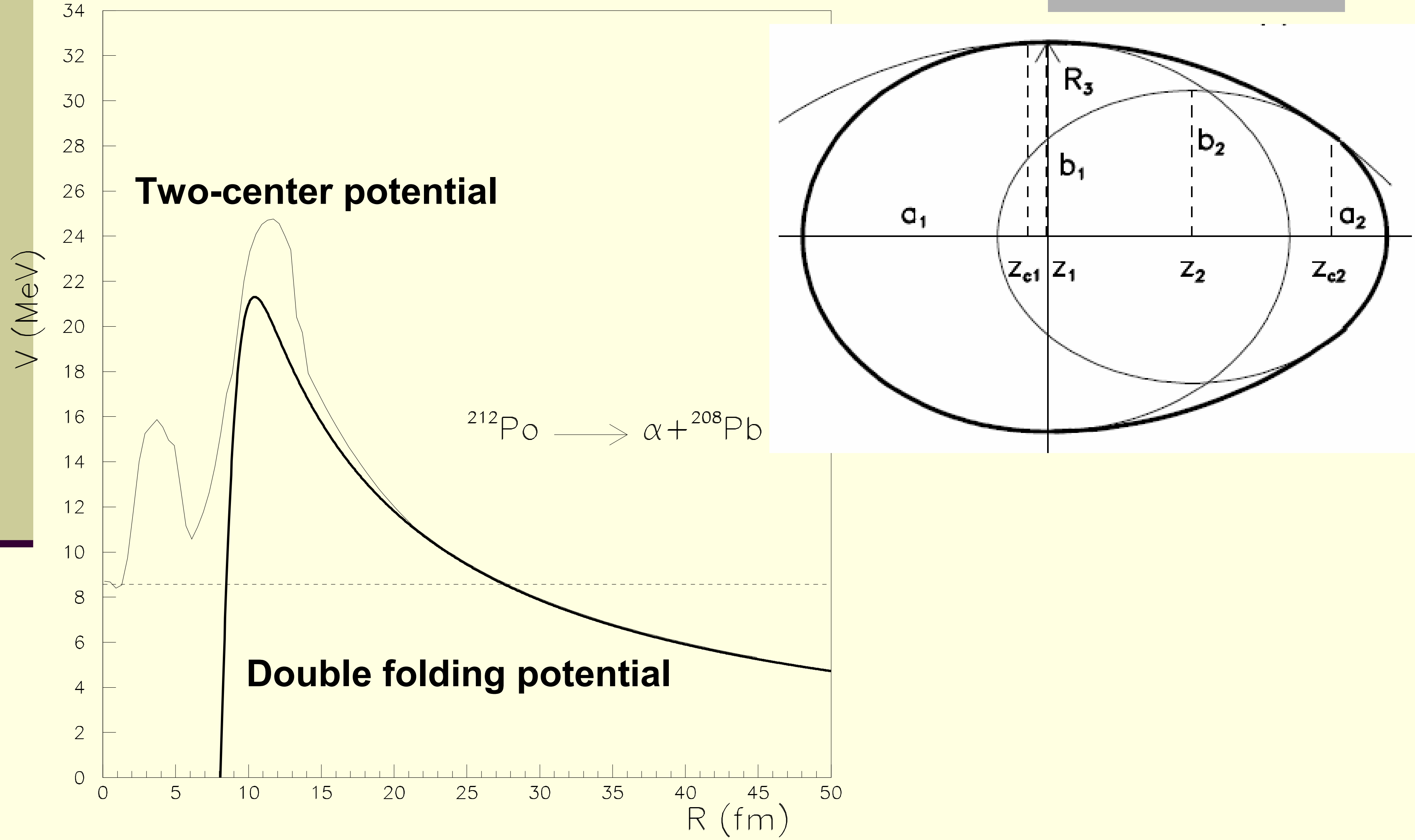
D.S. Delion

Universal decay rule for reduced widths

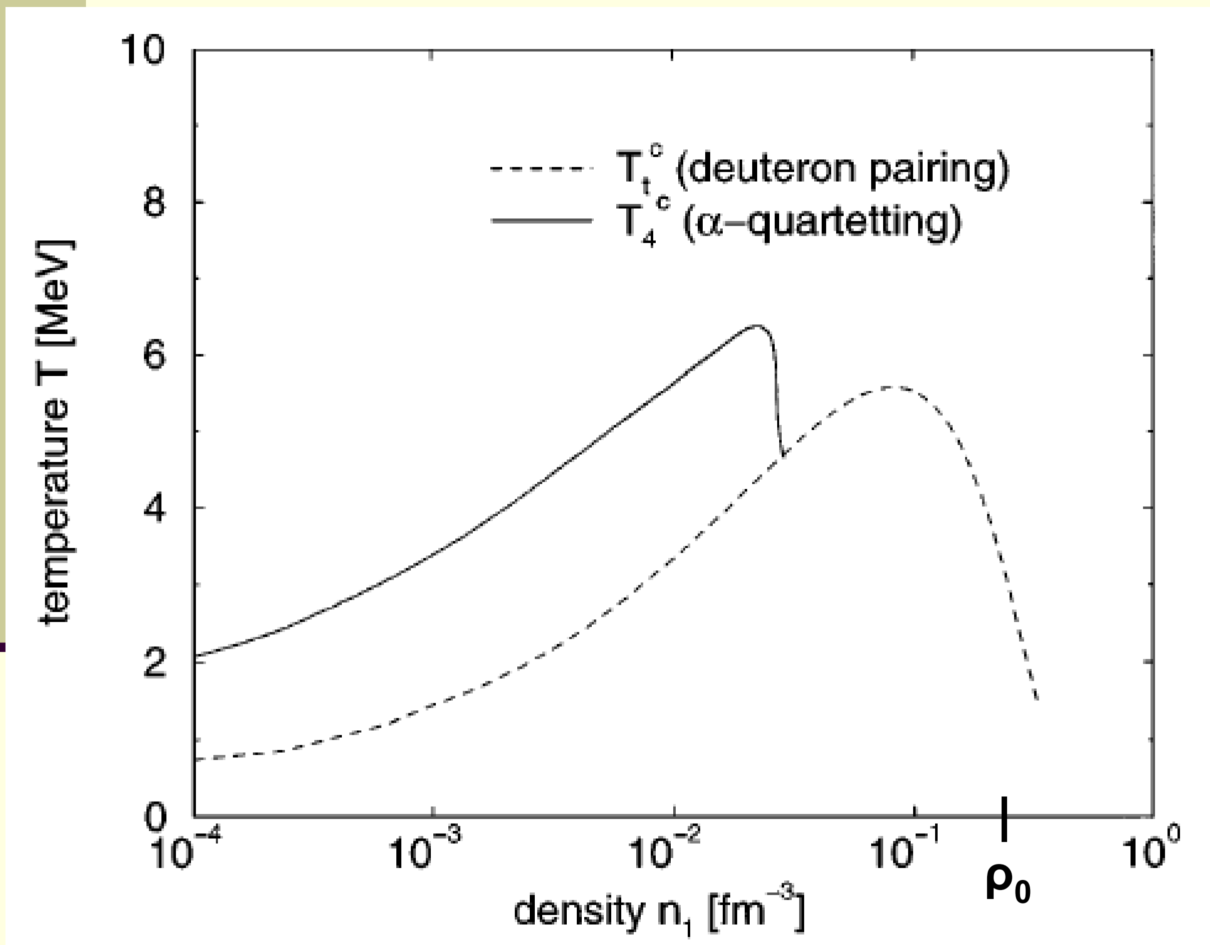
Physical Review **C80** (2009) 024310

Evidences for surface alpha-clustering

I. Two-center shell model predicts a pocket-like potential



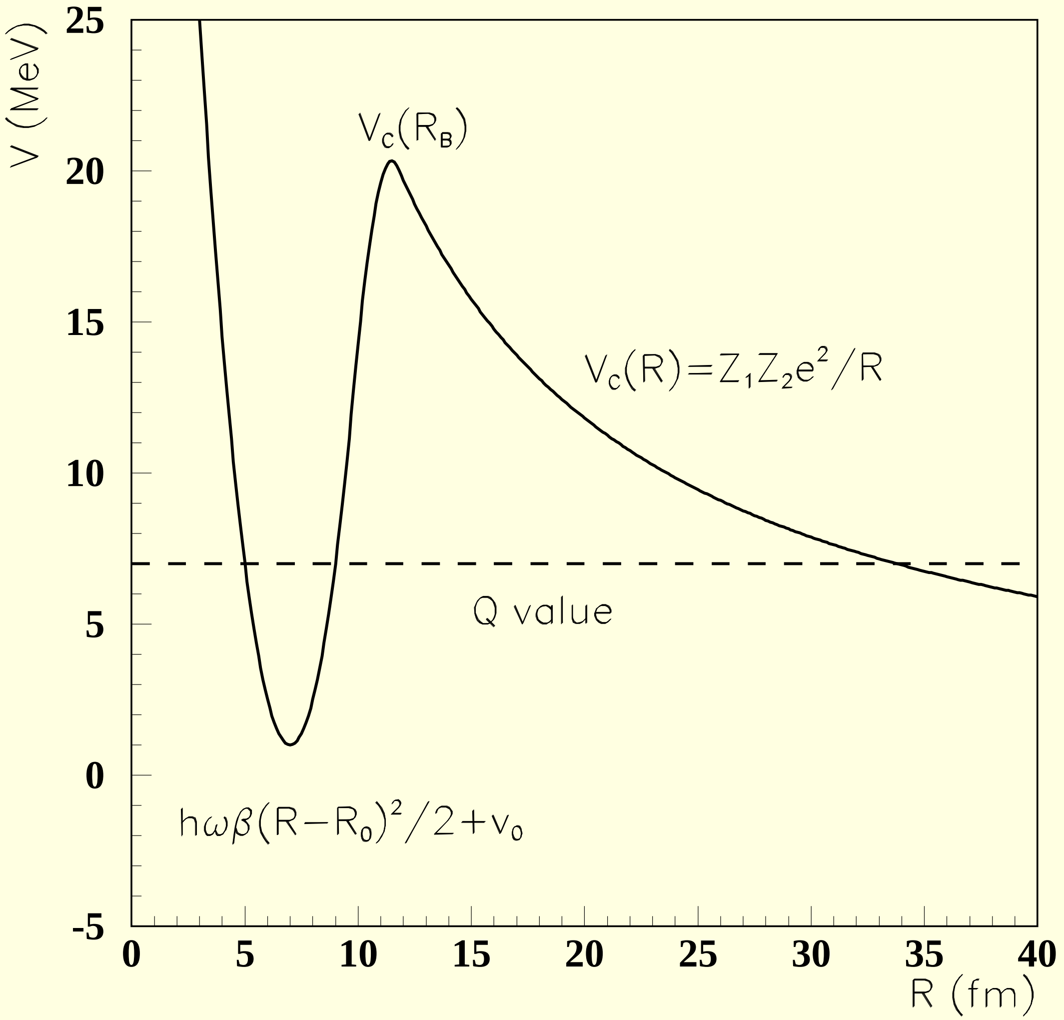
II. Phase diagram for deuteron and α-particle



G. Ropke, A. Schnell, P. Schuck, P. Nozieres
Four-particle condensate in strongly coupled fermion systems
Phys. Rev. Lett. 80, 3177 (1998).

Pairing survives at the equilibrium density ρ_0 and α-quarteting collapses at about 10% ρ_0 , i.e.
an α-particle can exist only beyond the nuclear surface

Conclusion: cluster-daughter interaction should be pocket-like on the nuclear surface: α-particle is hindered inside by the Pauli principle



Conditions for an α-particle moving in a shifted harmonic oscillator potential

1) The first eigenstate energy is the Q-value

$$Q = E = \frac{1}{2}\hbar\omega$$

2) Its wave function is given by

$$\Psi(R) = A_0 e^{-\beta(R-R_0)^2/2}$$

where the oscillator parameter is

$$\beta = \frac{m\omega}{\hbar}$$

Consequence:
Harmonic oscillator parameter depends linearly on the fragmentation potential

$$\beta(R_B - R_0)^2 = \frac{2}{\hbar\omega} V_{frag} + 1$$

where the fragmentation potential is defined as

$$V_{frag} = V_{Coul}(R_B) - Q$$

Reduced width depends linearly on the fragmentation potential

$$\log_{10}\gamma^2 = -\frac{\log_{10} e^2}{\hbar\omega} V_{frag} + \log_{10}\frac{\hbar^2 A_0^2}{2emR_B}$$

does not depend on the pocket radius and remains valid for any potential

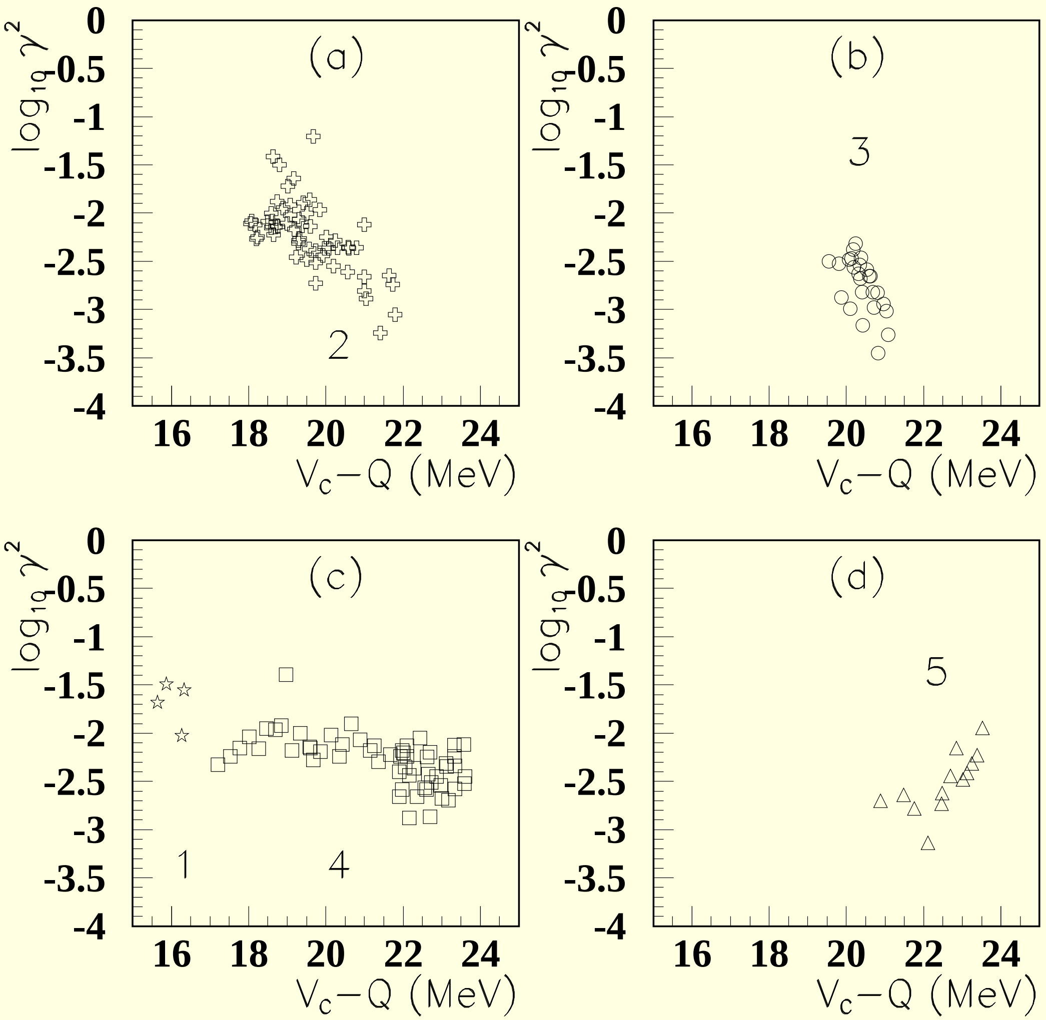
The fragmentation potential is given by:

$$V_{frag} = \frac{Z_D Z_C}{R_B} - Q$$

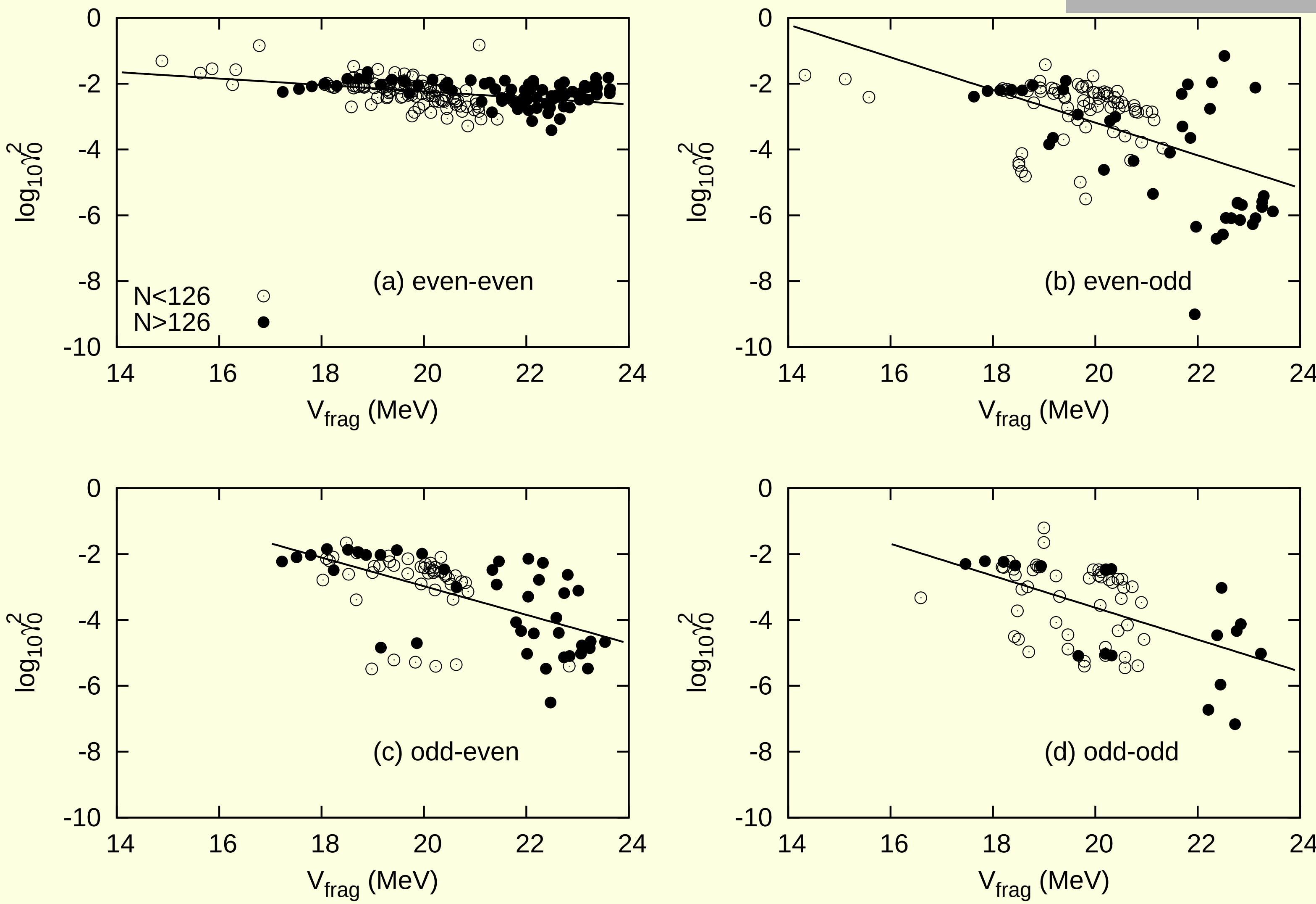
The slope should be negative

Reduced width for α-decay from even-even nuclei

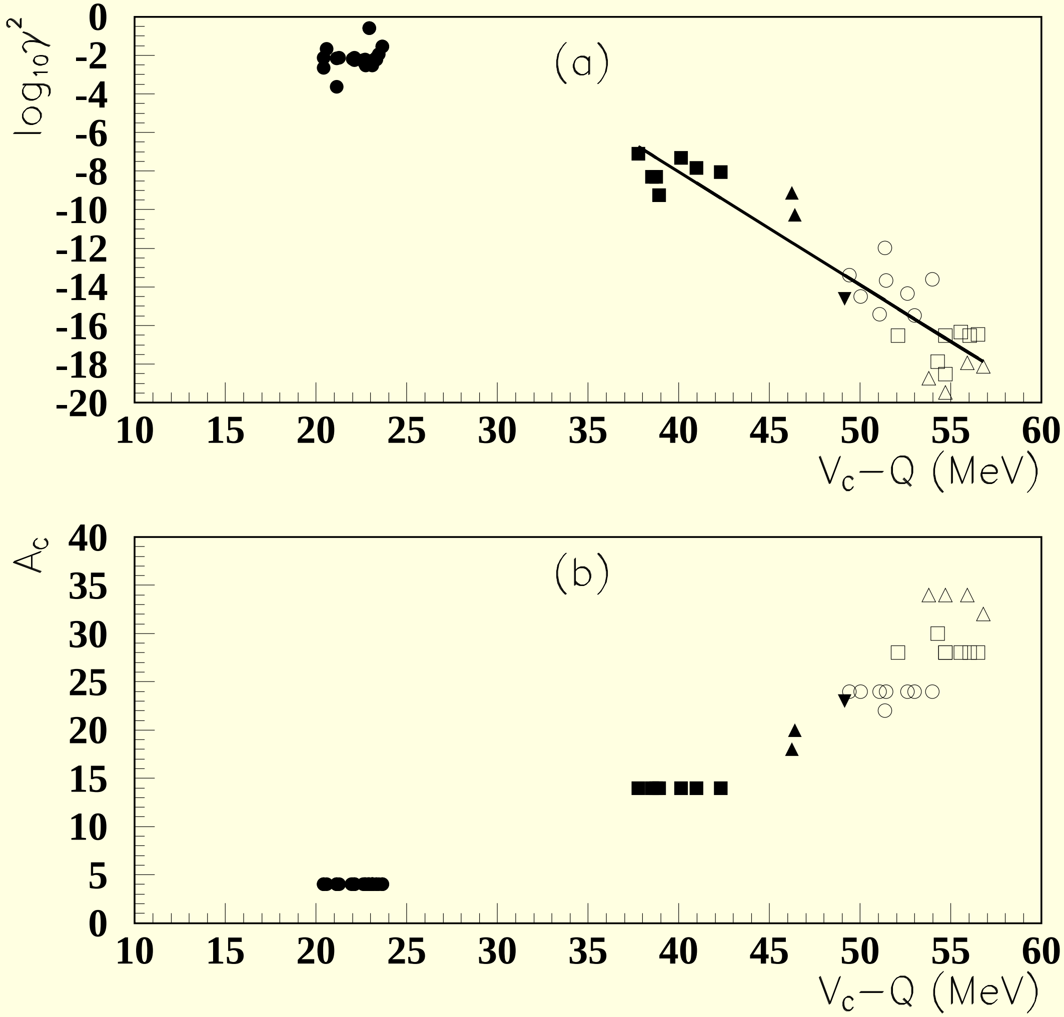
n	Z	N
1	Z<82	50<N<82
2		82<N<126
3	Z>82	82<N<126
4		126<N<152
5		N>152



The law for reduced width is valid for all transitions between ground states



The law for reduced width remains valid for cluster decays



and explains the Blendowske rule for spectroscopic factor

$$S = S_{\alpha}^{(A_C - 1)/3}$$

In terms of the fragmentation potential, because

$$S = c\gamma^2$$

II. Microscopic approach

How the emitted cluster is formed from protons and neutrons lying in different major shells ?

Microscopic estimate of the formation amplitude

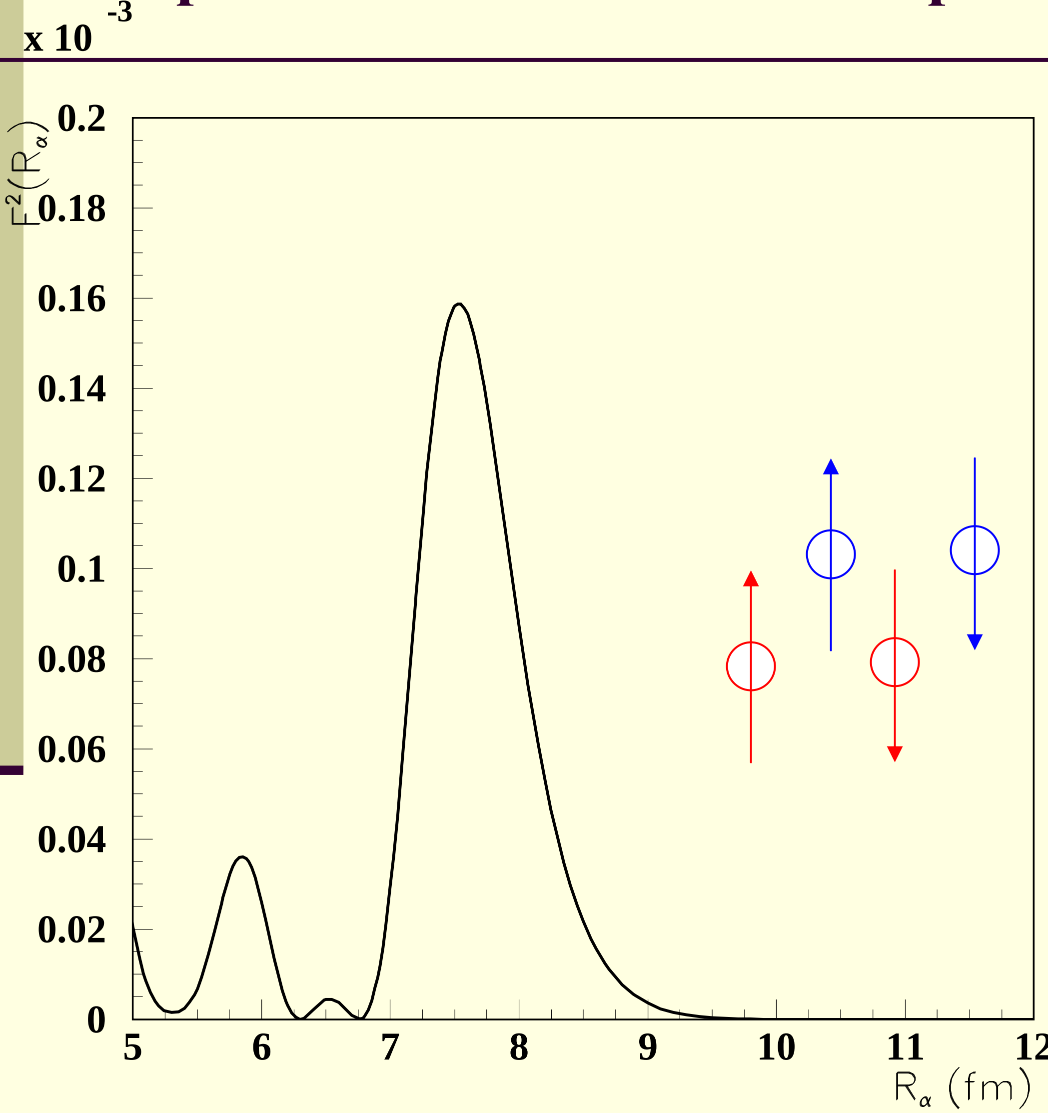
$$\Psi_P \rightarrow \Psi_D + \psi_\alpha$$

The first microscopic estimates of the α-particle formation amplitude were performed in:

H. J. Mang, Phys. Rev. **119**, 1069 (1960).

A. Sandulescu, Nucl. Phys. A **37**, 332 (1962).

Woods-Saxon mean field plus pairing approach predicts an α-formation amplitude peaked on surface

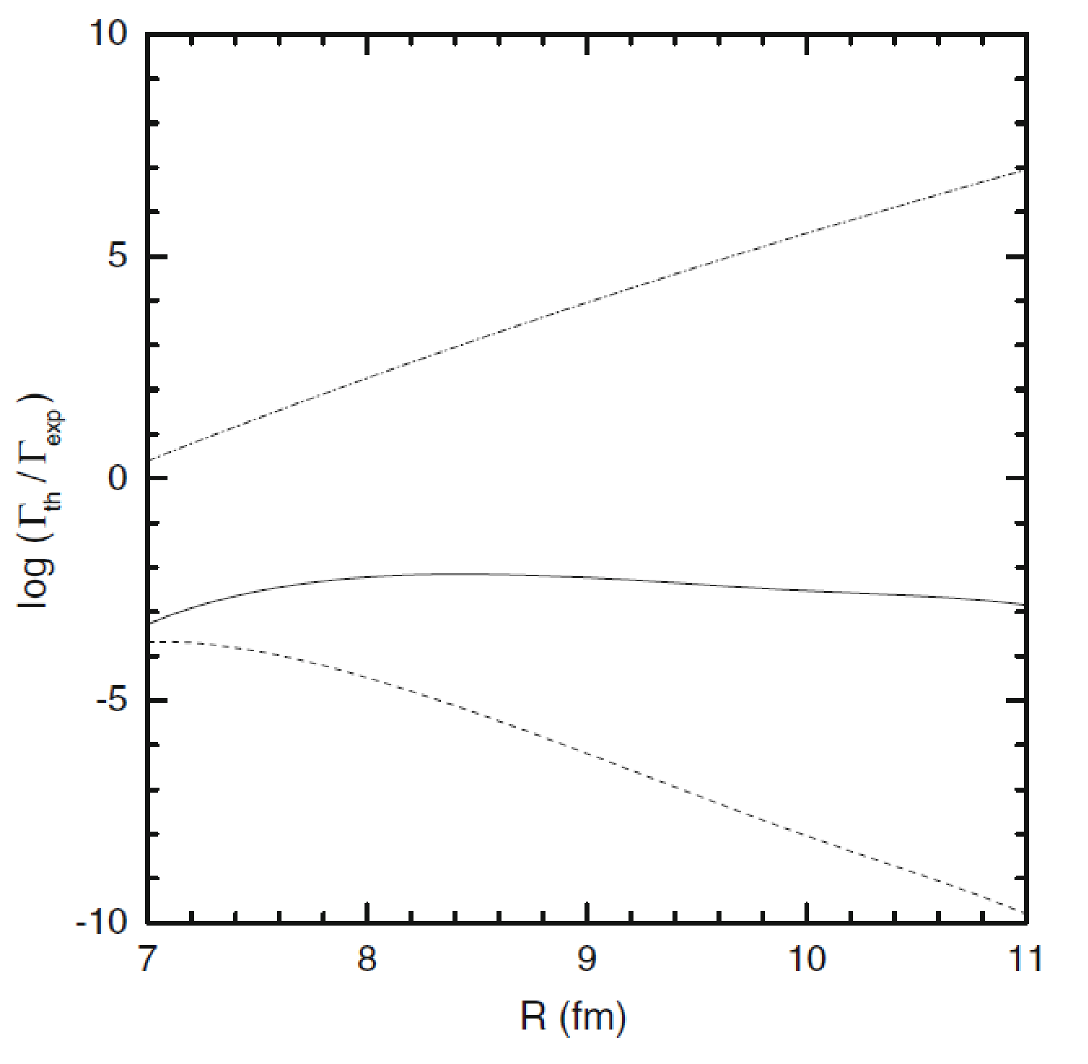


$$F(R_\alpha) = \langle \psi_\alpha \Psi_D | \Psi_P \rangle$$

$$\Psi_{D,P} = |BCS\rangle_{D,P}$$

$$\psi_\alpha = e^{-\beta_\alpha (r_\pi^2 + r_\nu^2 + r_\alpha^2)/2} \chi_{1/2}^{\pi} \chi_{1/2}^{\nu}$$

Decay width versus cm radius for N=12 major shells in the diagonalization basis underestimates the exp. value by two orders of magnitude



Penetrability

Decay width underestimates exp. value by two orders of magnitude

Reduced width ^{2}

How to increase the tail of the α-particle formation amplitude?

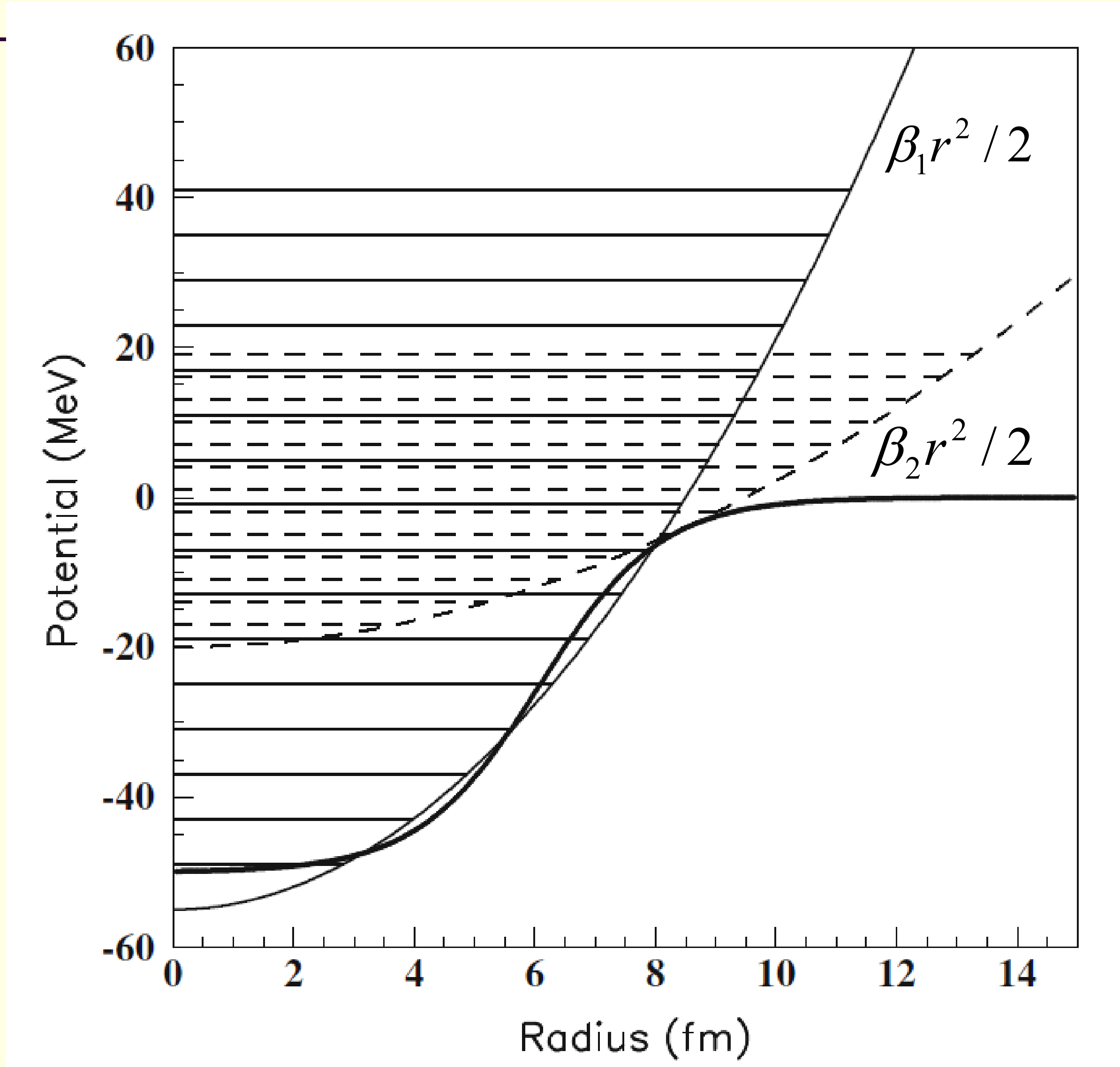
A. By keeping single particle (sp) mean field and changing the diagonalization sp basis.

D.S. Delion, A. Insolia, R.J. Liotta,
New single particle basis for microscopic description of decay processes,
Physical Review **C54**, 292 (1996).

B. By keeping the diagonalization sp basis and increasing p-n correlations.

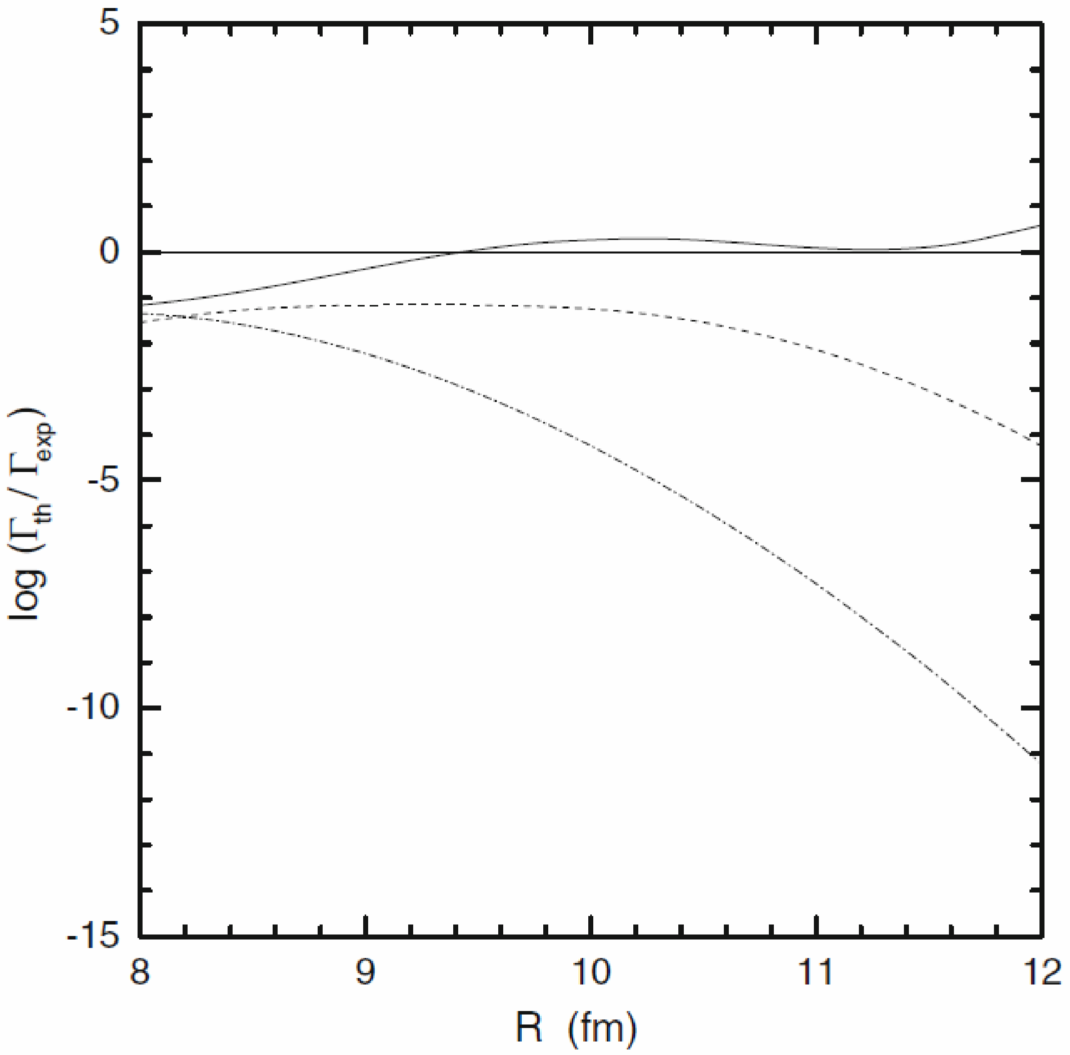
D.S. Delion and R.J. Liotta,
Shell-model representation to describe alpha emission
Physical Review **C87**, 041302(R) (2013).

A. Woods-Saxon mean field diagonalized within the two-harmonic oscillator basis



$$f = \frac{\beta_2}{\beta_1}$$

Last major shells with smaller ho parameter β_2 have the most important contribution



N=6 major shells, f=1
+3 major shells, f=0.2

N=9 major shells, f=1

N=6 major shells, f=1

Important observation:
matching condition between logarithmic derivatives of the Coulomb wave and formation amplitude

$$\frac{G_0'(R_B)}{G_0(R_B)} = \frac{\psi'(R_B)}{\psi(R_B)} = -\beta R_\alpha$$

where Coulomb wave is

$$G_0(R) = ctg\alpha \; e^{\chi(\alpha - sin\alpha \cos\alpha)}$$

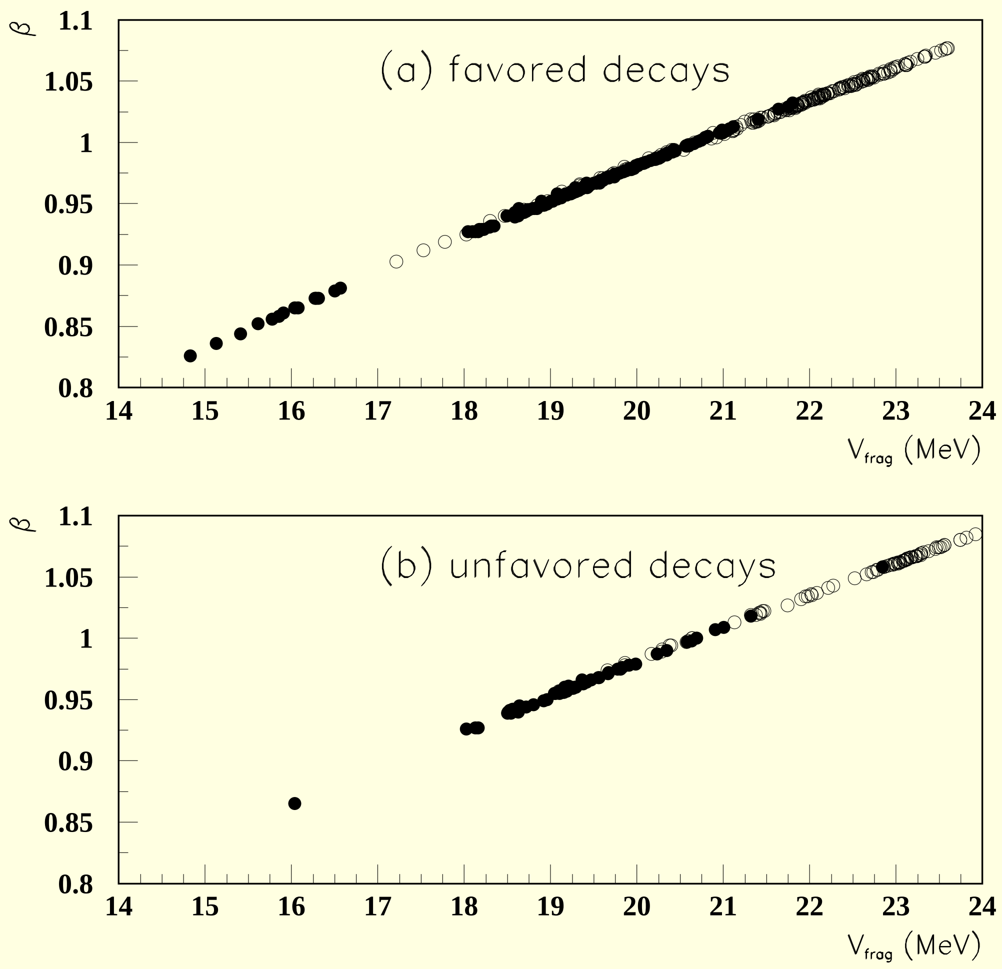
$$\cos^2\alpha = \frac{\rho}{\chi} = \frac{Q}{V(R)}$$

and formation amplitude is approximated by a Gaussian centered on nuclear surface

$$\psi(R) = AN_\beta \; e^{-\beta(R-R_0)^2/2}$$

$$N_\beta = \left(\frac{4\beta}{\pi}\right)^{1/4} ,$$

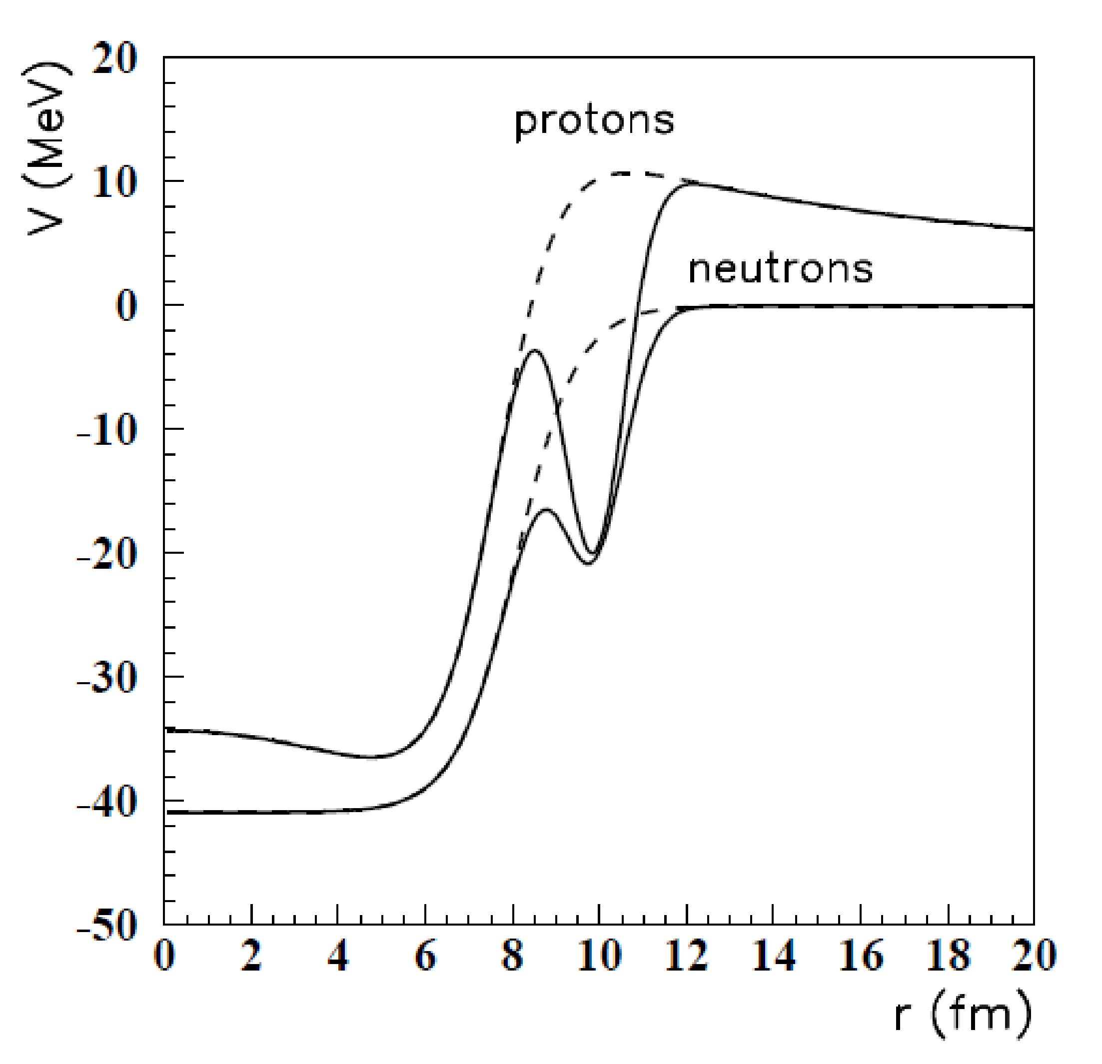
leads to a linear dependence between the α-particle harmonic oscillator parameter and fragmentation potential



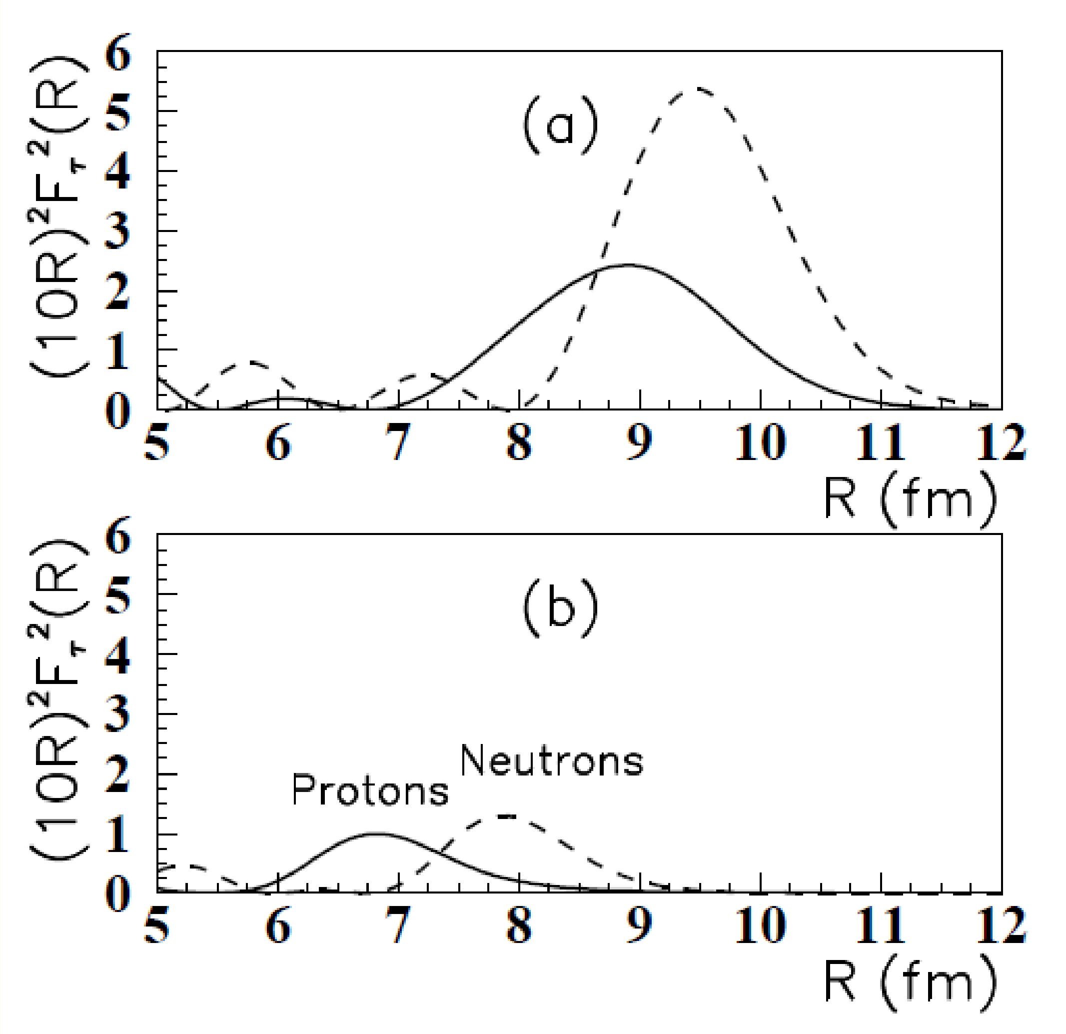
Thus, sp parameter $\beta_2=\beta/4$ should depend linearly upon the fragmentation potential

and therefore the standard sp mean field cannot describe the α-decay phenomenon

B. Woods-Saxon mean field plus a Gaussian surface component simulating p-n clustering correlations



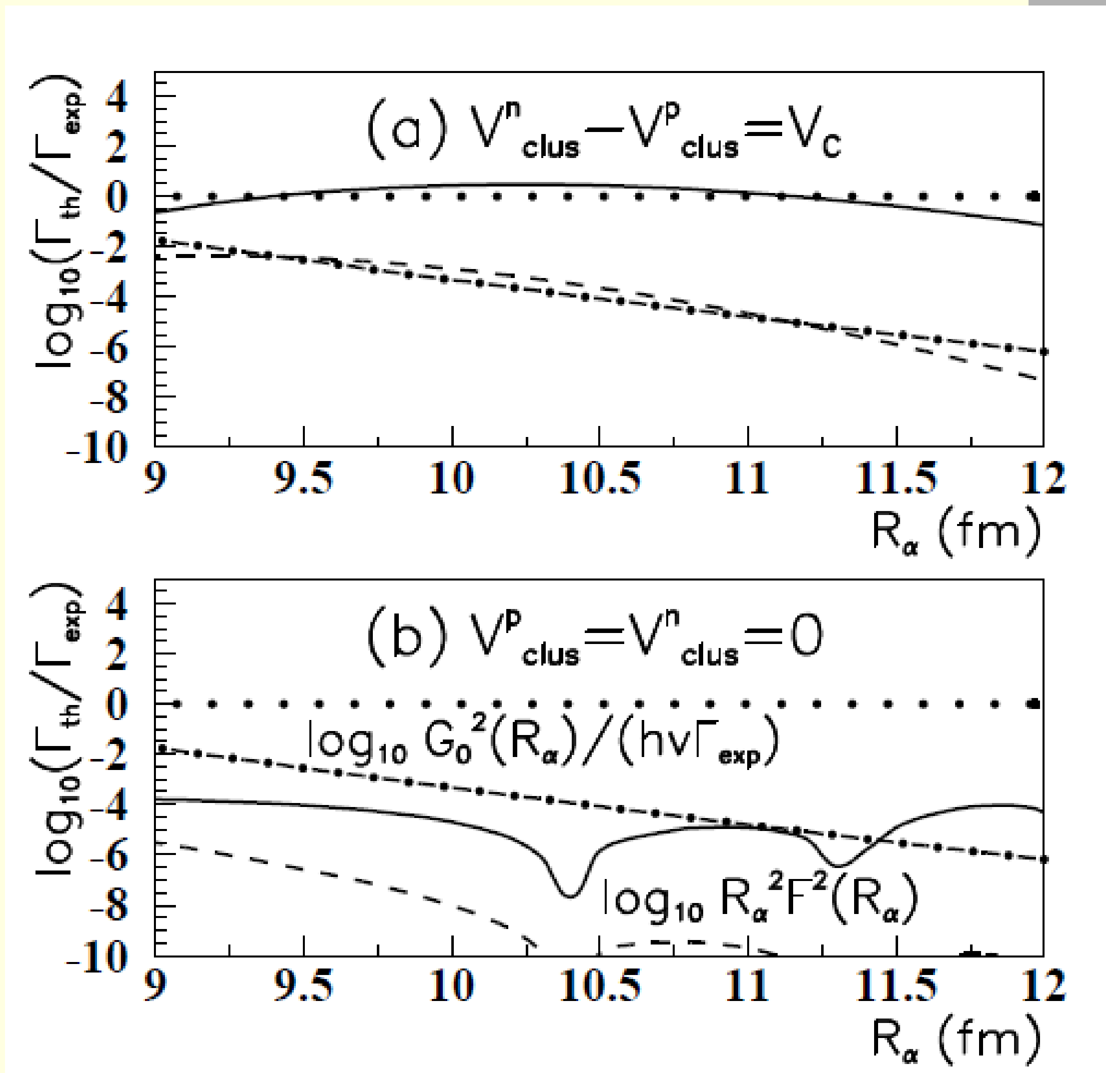
Proton and neutron formation probabilities with cluster component (a) and without cluster component (b)



Cluster component increases the p-n overlap by creating p & n orbitals with the same principal quantum number.

Thus, the effective p-n correlation increases.

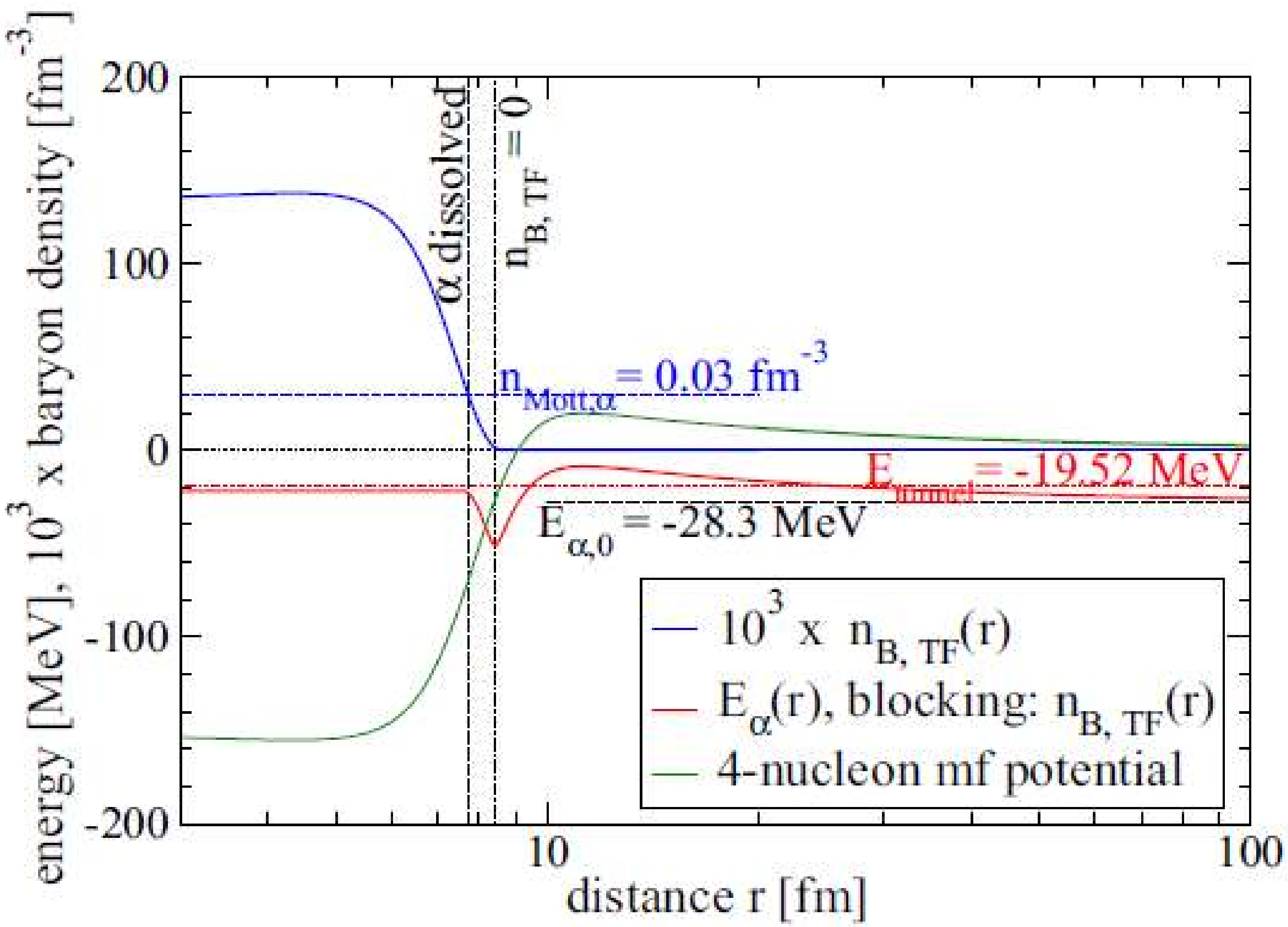
Decay width with cluster component (a) and without cluster component (b)



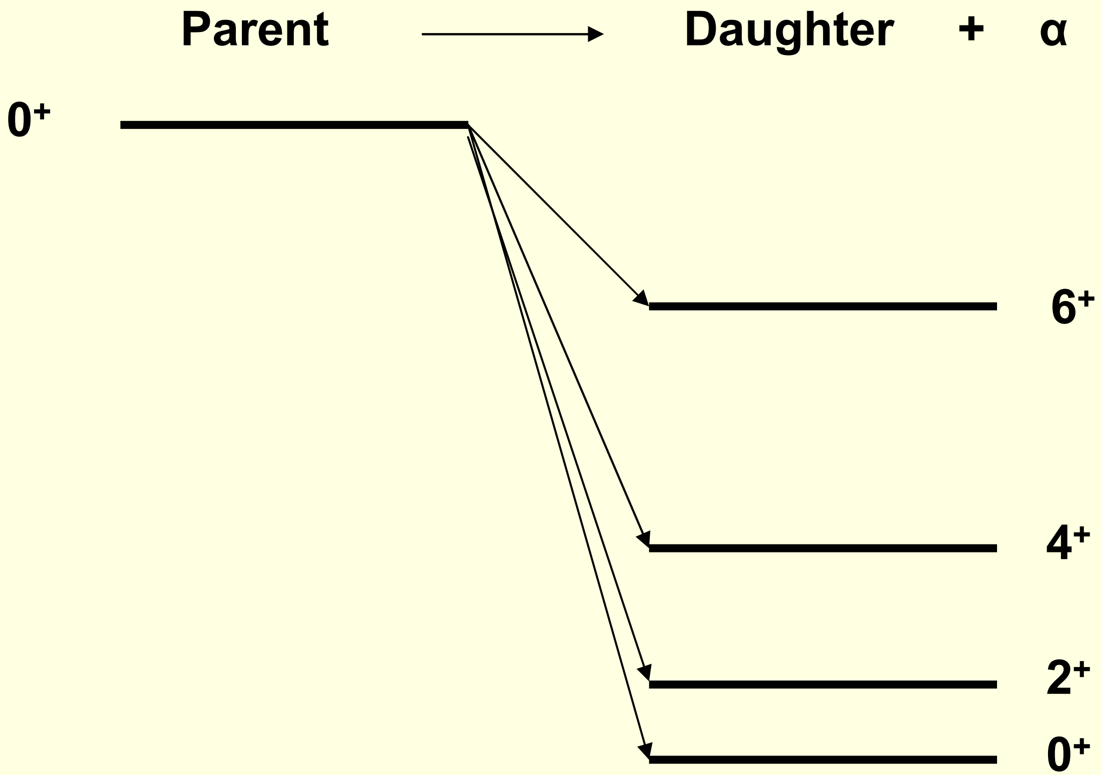
PHYSICAL REVIEW C **90**, 034304 (2014)

Nuclear clusters bound to doubly magic nuclei: The case of ^{212}Po

G. Röpke,[1,*] P. Schuck,[2,3,†] Y. Funaki,[4] H. Horiuchi,[5,6] Zhongzhou Ren,[7,8] A. Tohsaki,[5] Chang Xu,[7]
T. Yamada,[9] and Bo Zhou[5,7]



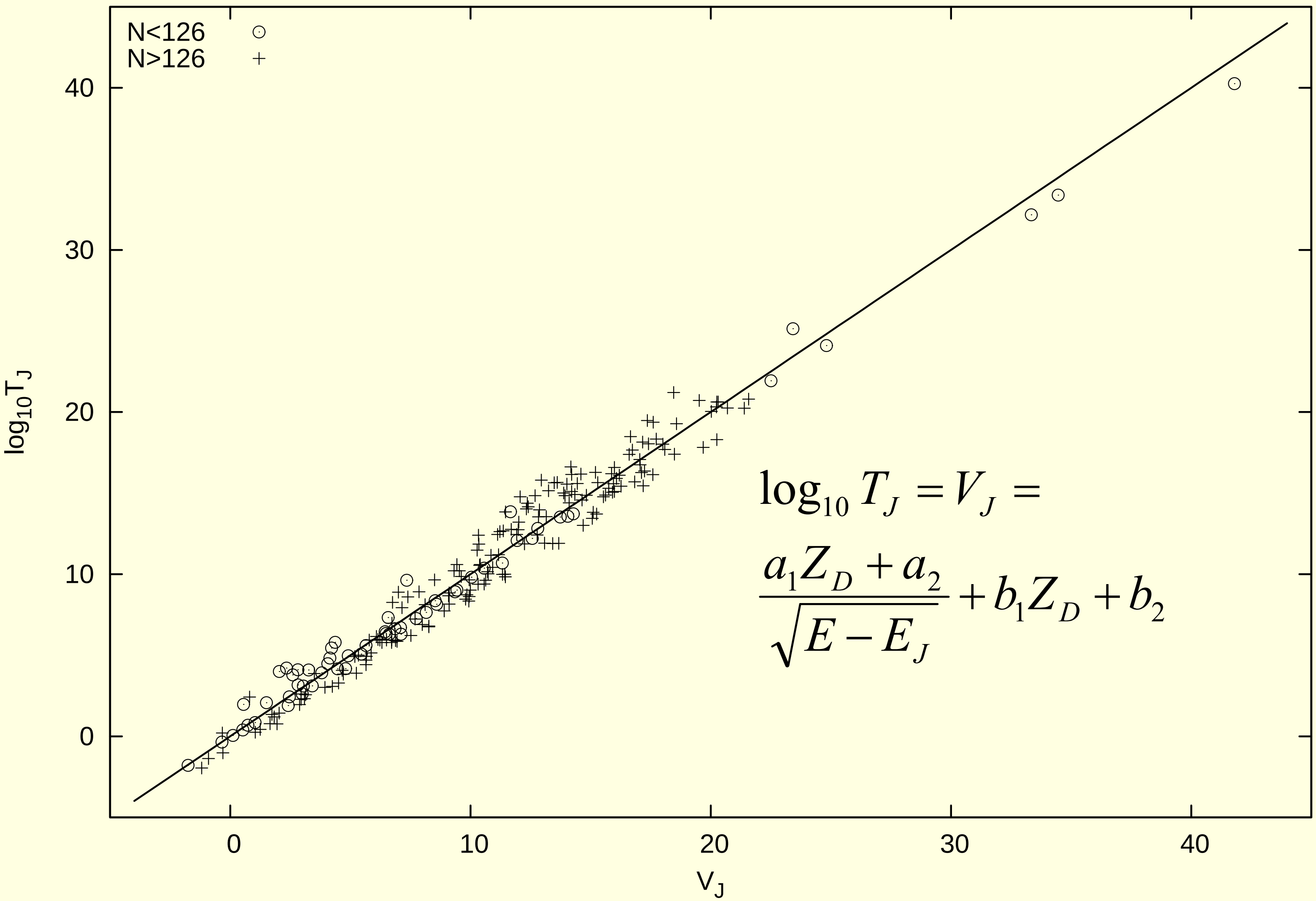
III. α-decay spectroscopy



Transitions to the ground band
in even-even nuclei

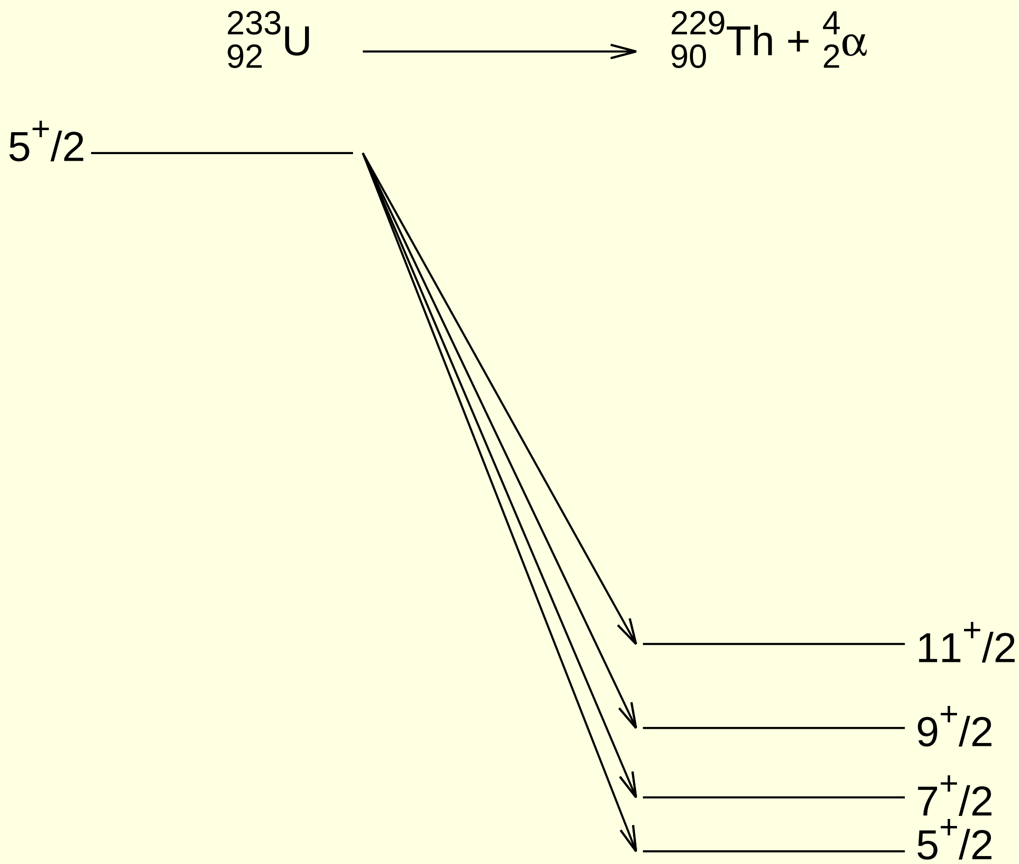
$$P \to D(J) + \alpha$$

Viola-Seaborg graph for α-decays to excited states in even-even nuclei

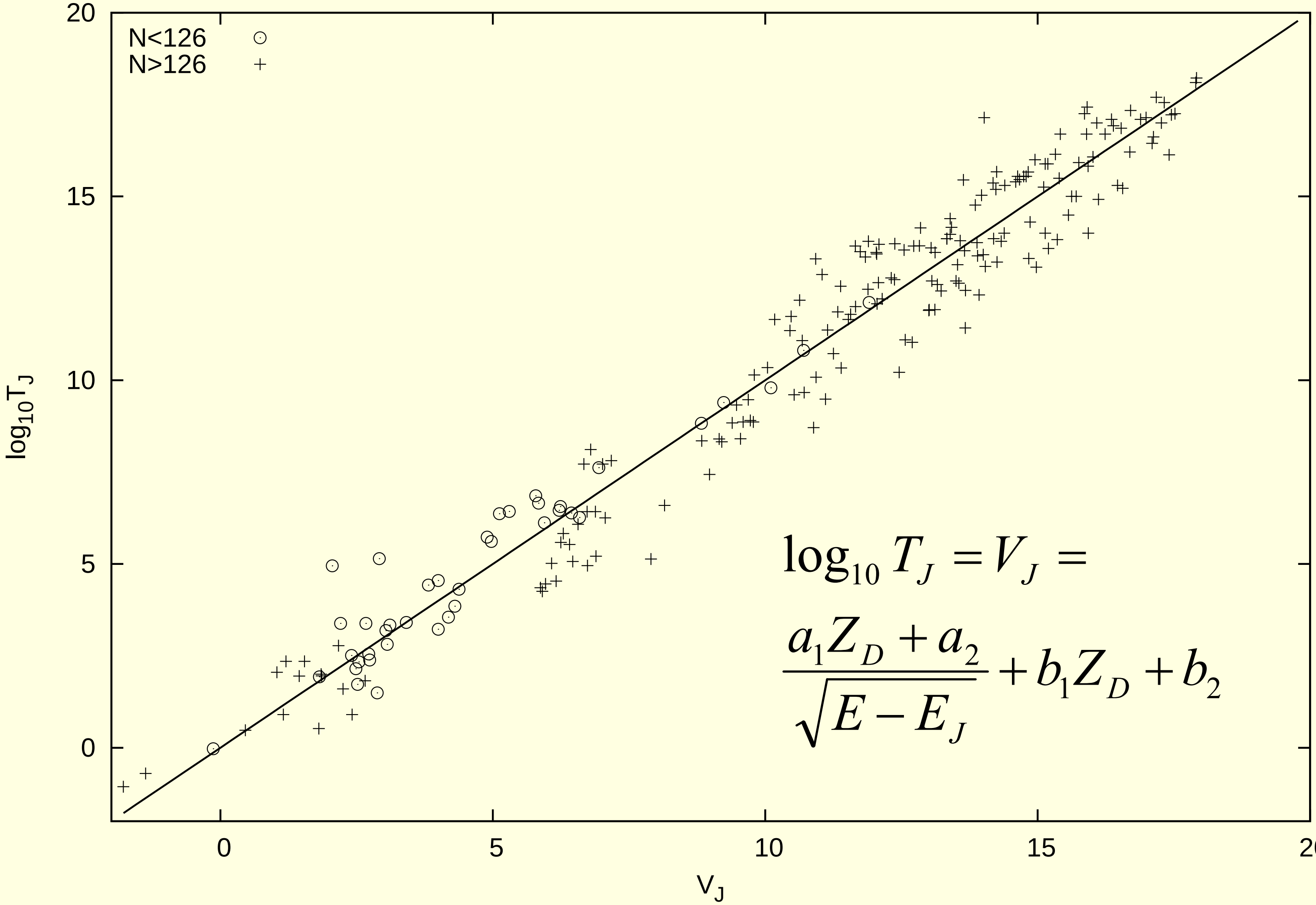


$$\log_{10} T_J = V_J = \frac{a_1 Z_D + a_2}{\sqrt{E - E_J}} + b_1 Z_D + b_2$$

Favored transitions in odd-mass nuclei: sp state does not change during transition

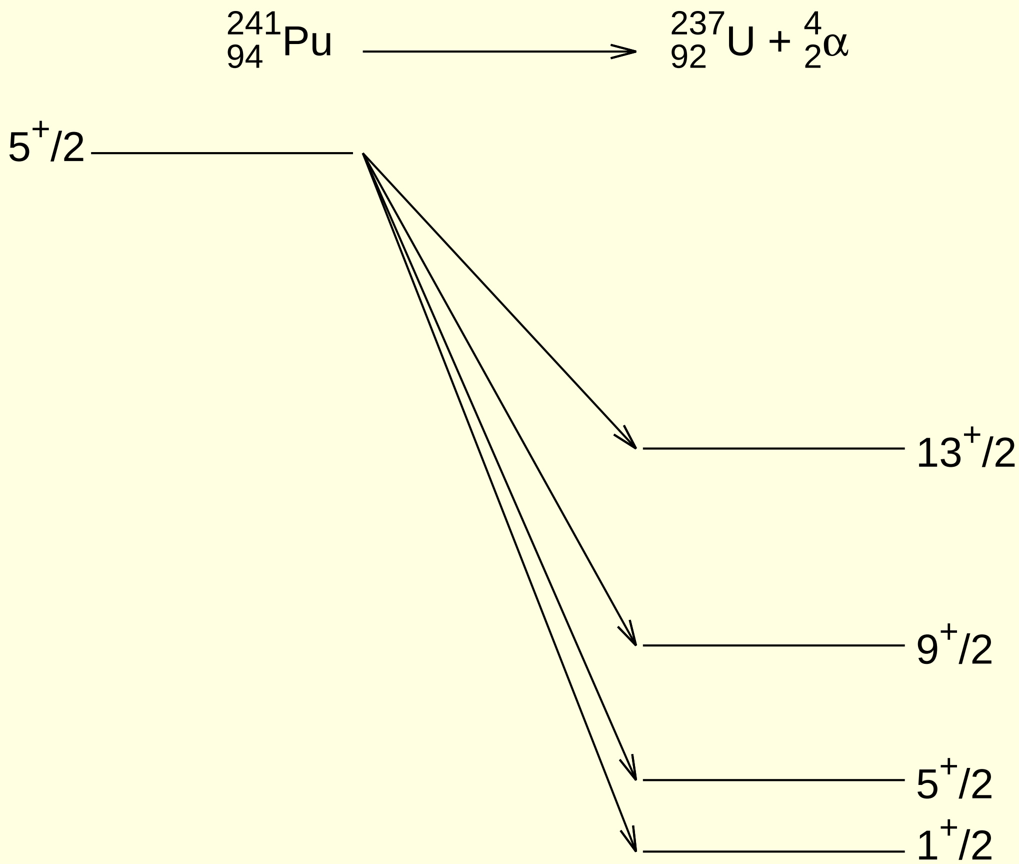


Viola-Seaborg graph for favored α-decays to excited states in odd-mass nuclei

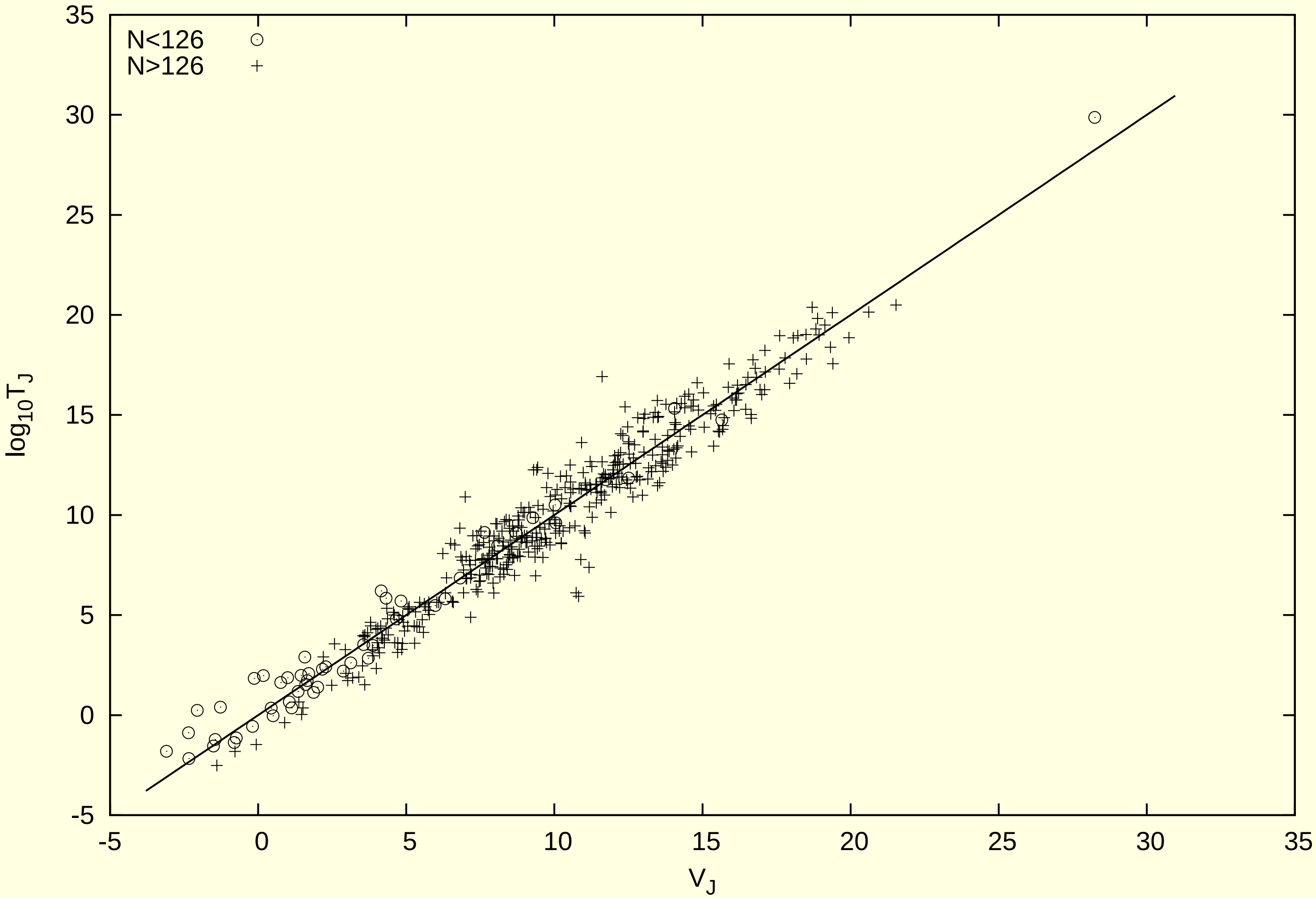


$$\log_{10} T_J = V_J = \frac{a_1 Z_D + a_2}{\sqrt{E - E_J}} + b_1 Z_D + b_2$$

Unfavored transitions in odd-mass nuclei: sp state changes during transition



Viola-Seaborg graph for unfavored α-decays to excited states in odd-mass nuclei



Observables

Hindrance factor

$$HF_J = \frac{\gamma_0^2}{\gamma_J^2} = \frac{\Gamma_0}{\Gamma_J}\frac{P_J}{P_0}$$

Intensity

$$I_J = \log_{10}\frac{\Gamma_0}{\Gamma_J} = \log_{10} HF_J + \log_{10}\frac{P_0}{P_J}$$

Universal law for hindrance factors

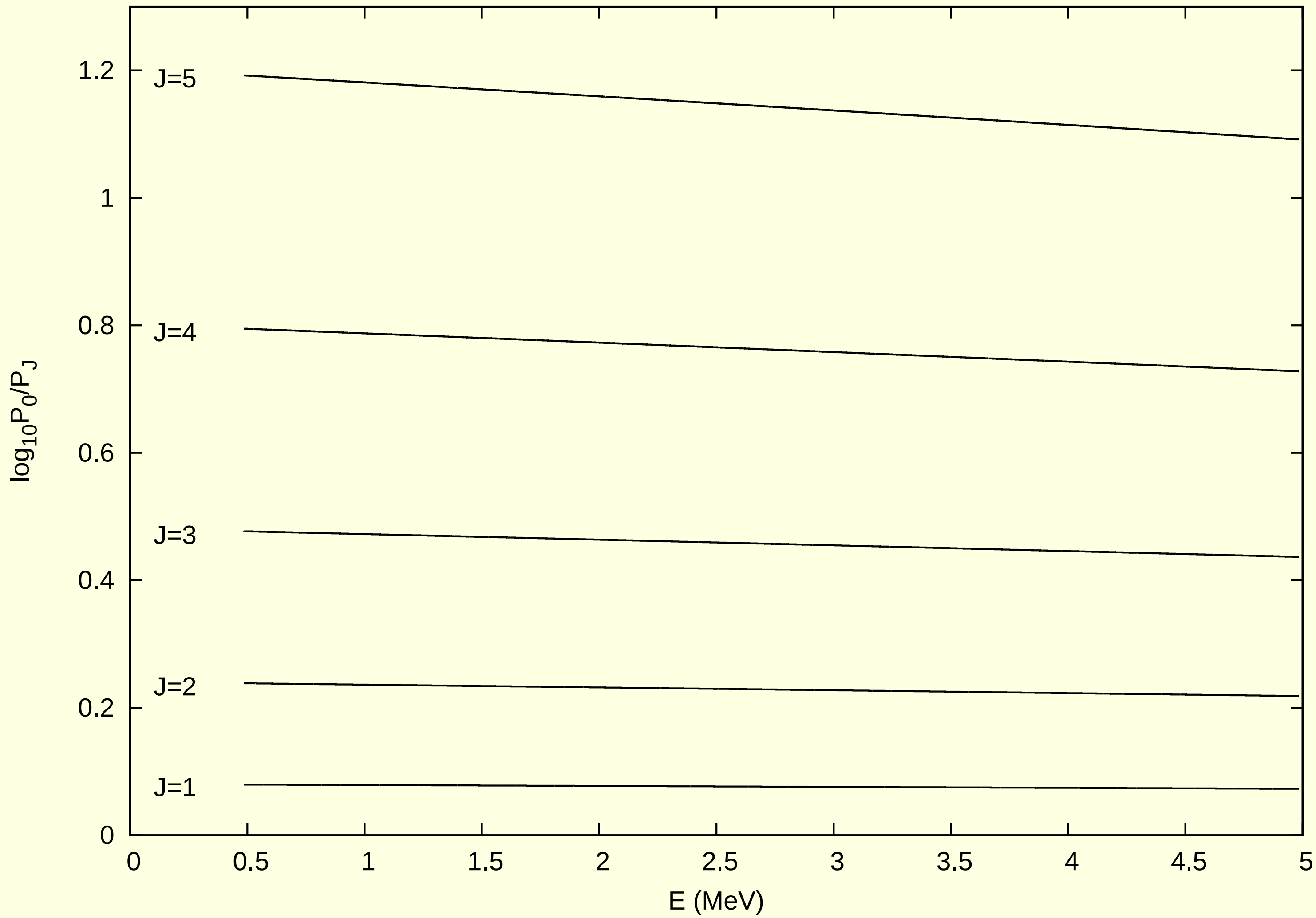
$$\log_{10} HF_J = \frac{\log_{10} e^2}{\hbar\omega} E_J + \log_{10} \frac{A_0^2}{A_J^2}$$

and intensities

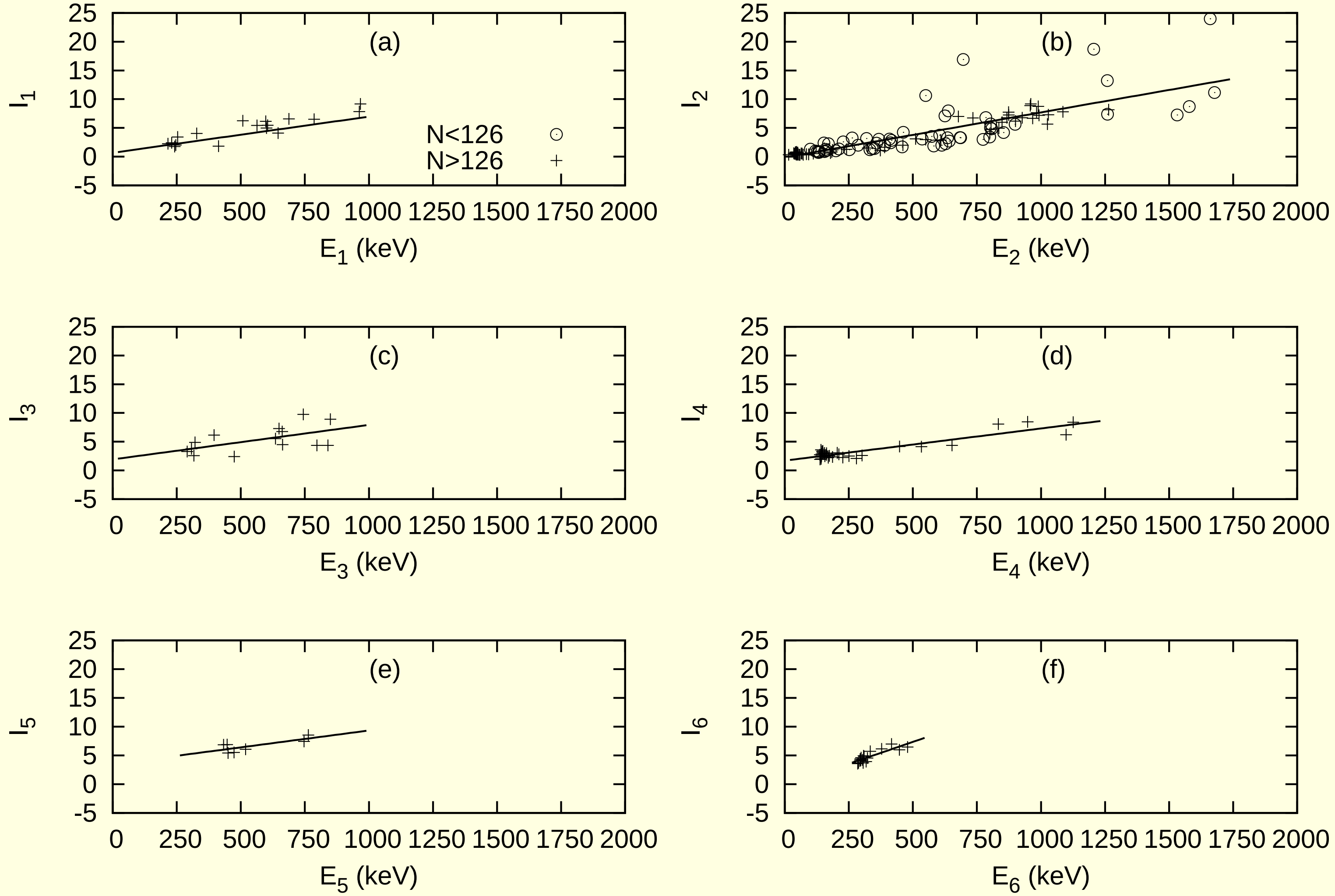
$$I_J = \frac{\log_{10} e^2}{\hbar\omega} E_J + \log_{10} \frac{A_0^2}{A_J^2} + \log_{10} \frac{P_0}{P_J}$$

The slope should be positive

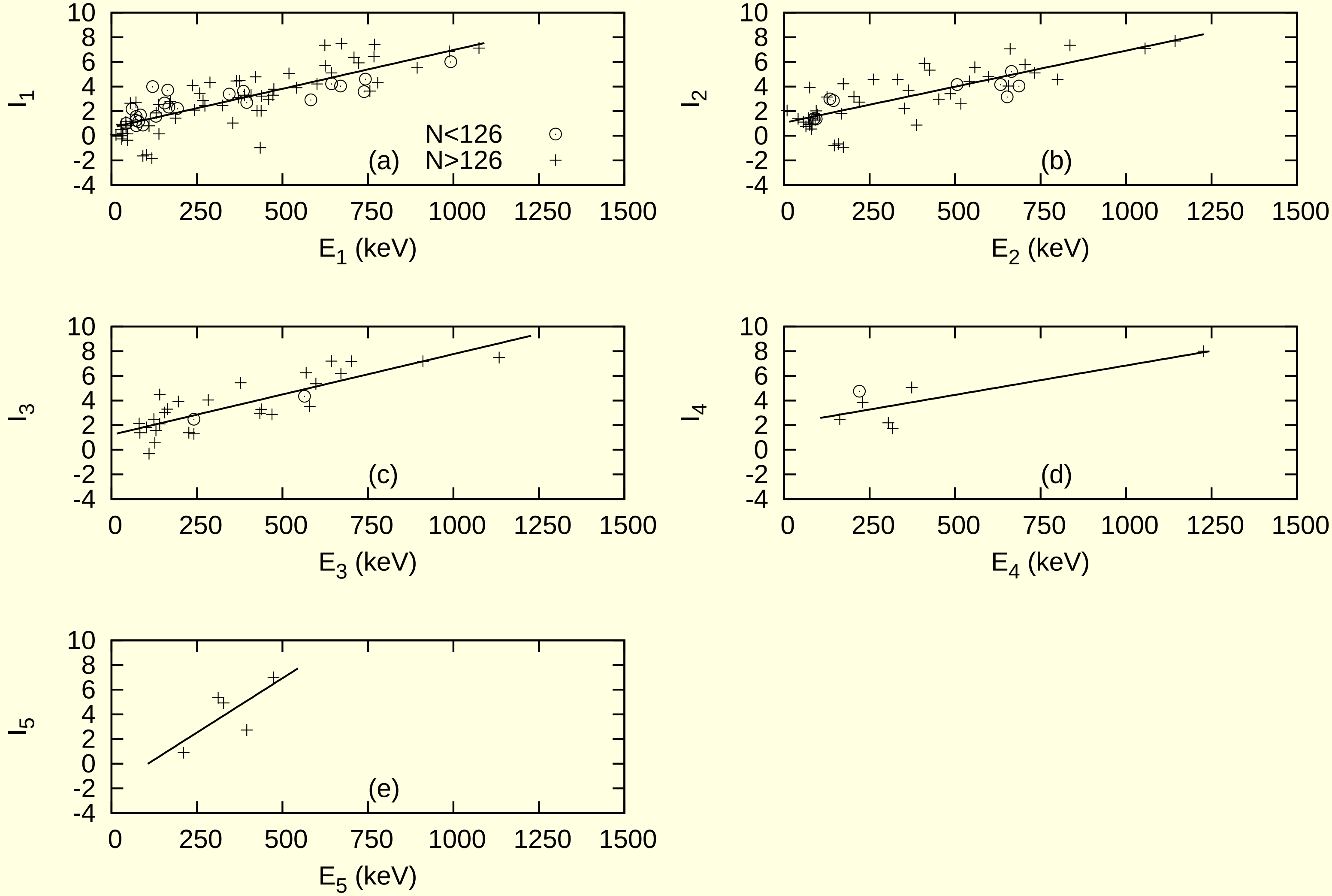
Ratio P_0/P_J weakly depends upon energy and therefore the universal rule for intensities is similar to the rule for HF's



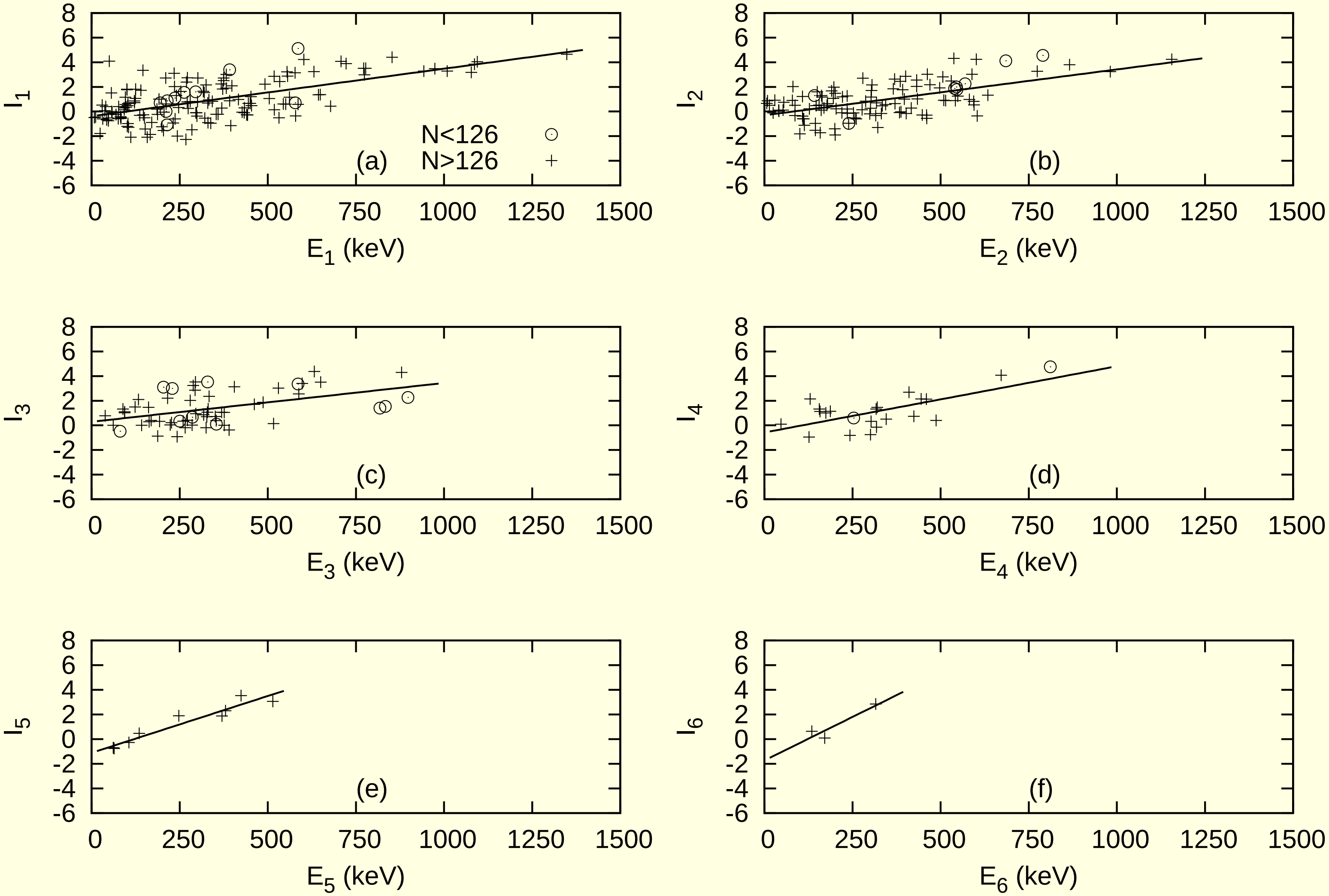
Universal law for intensities in even-even nuclei



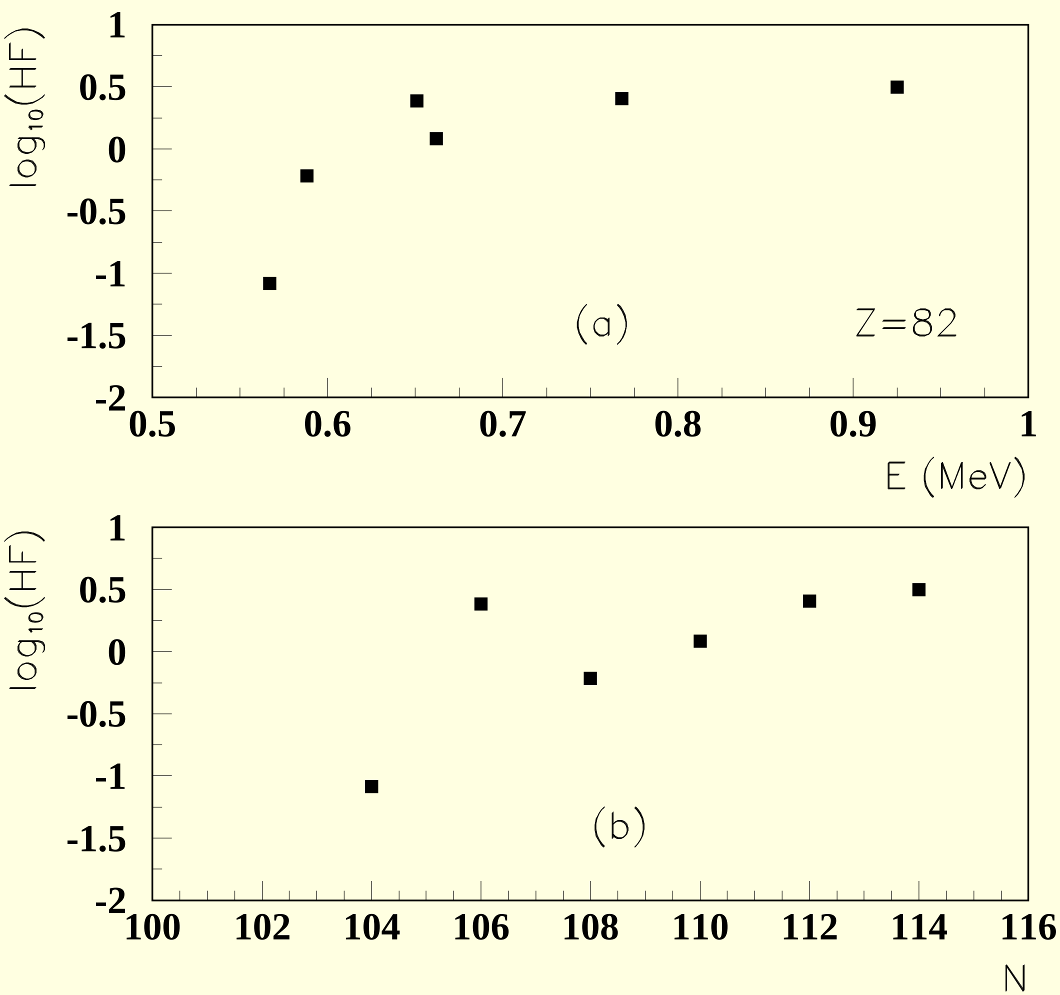
Universal law for intensities in odd-mass nuclei for favored transitions



Universal law for intensities in odd-mass nuclei for unfavored transitions



Universal law for hindrance factors to 0_2^+ levels in Pb isotopes



IV. Coupled channels description of α-transitions

PHYSICAL REVIEW C **87**, 044314 (2013)

Unified description of electromagnetic and α transitions in even-even nuclei

D. S. Delion[1,2,3] and A. Dumitrescu[1,4]

Schrodinger equation describing α-decay

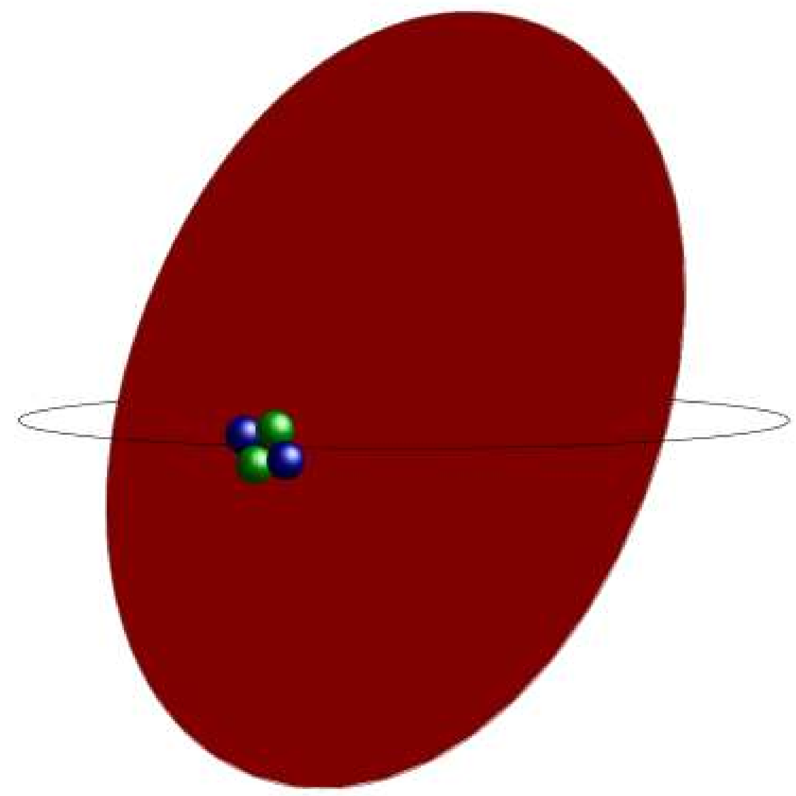
$$H\Psi(b_2,\mathrm{R}) = E\Psi(b_2,\mathrm{R})$$

has a Hamiltonian containing the sum of kinetic, daughter and α-daughter terms:

$$H = -\frac{\hbar^2}{2\mu}\nabla_R^2 + H_D(b_2) + V(b_2,\mathrm{R})$$

R **is the distance between α-particle and daughter nucleus**
b_2 **is the quadrupole coordinate describing core excitations**

α-transitions in even-even nuclei



α-daughter potential

$$V(b_2, \mathbf{R}) = V_0(R) + V_2(b_2, \mathbf{R})$$

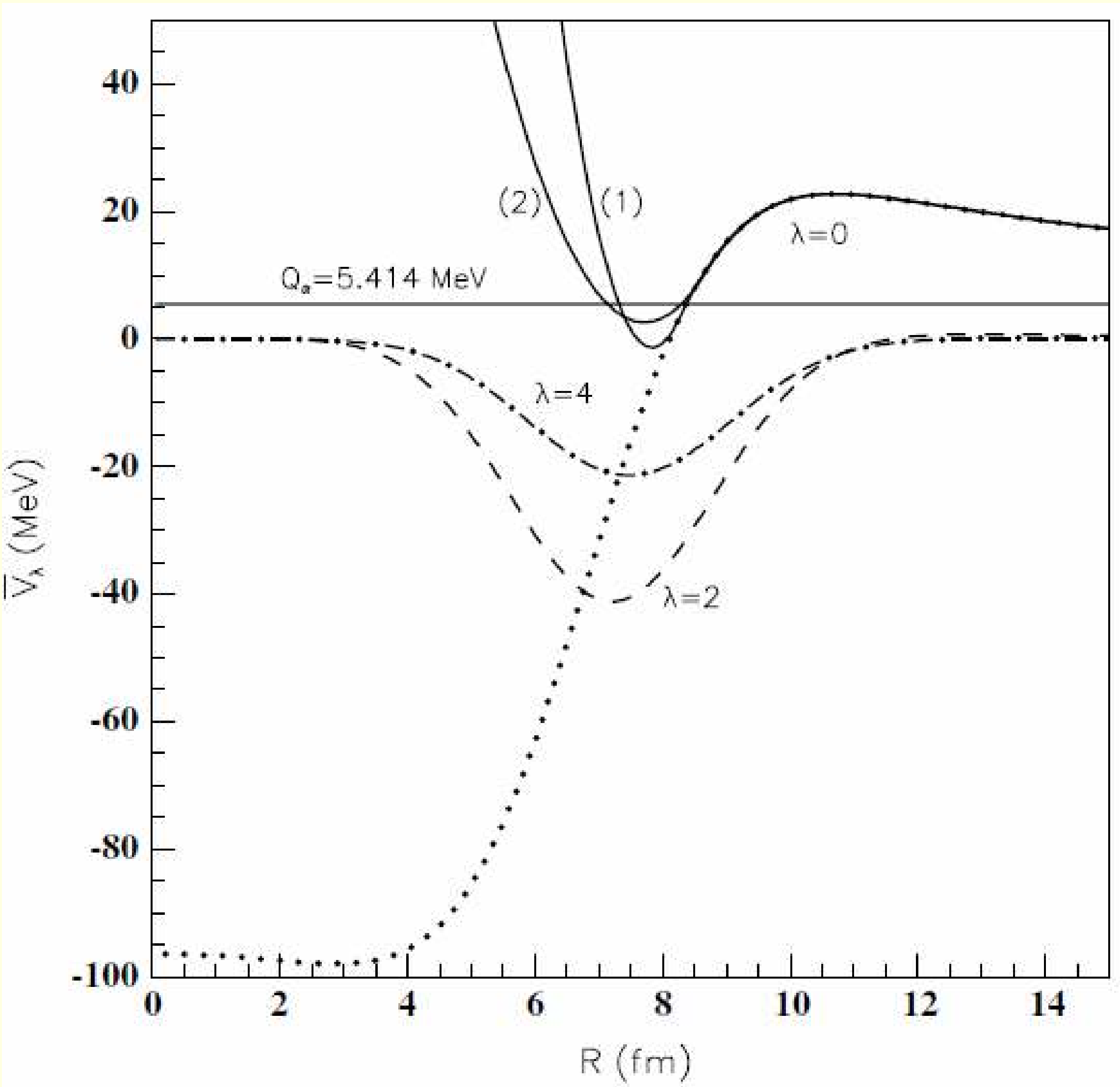
Spherical term is given by the double folding attraction plus quadratic repulsion (simulating Pauli principle) terms

$$\begin{aligned} V_0(R) &= v_a \overline{V}_0(R) \ , \quad R > R_m \\ &= c(R - R_{min})^2 - v_0 \ , \quad R \leq R_m \end{aligned}$$

Quadrupole term is given by QQ interaction between daughter nucleus and α-particle

$$\begin{aligned} V_2(b_2, \mathbf{R}) &= -C_0(R - R_{min}) \frac{dV_0(R)}{dR} \\ &\times \ \hat{2} \left[Q_2 \otimes Y_2(\Omega) \right]_0 \ . \end{aligned}$$

Double folding plus repulsion (simulating Pauli principle) potentials



Repulsive strength *c* is related to the depth of the potential v_0. The radial parameters R_m and R_{min} are fixed by matching repulsion to the double-folding potential.

By fixing the repulsive strength we use the depth of the potential to adjust the resonant energy to the Q-value.

Wave function for even-even nuclei has the total angular momentum = 0 which is conserved during transition

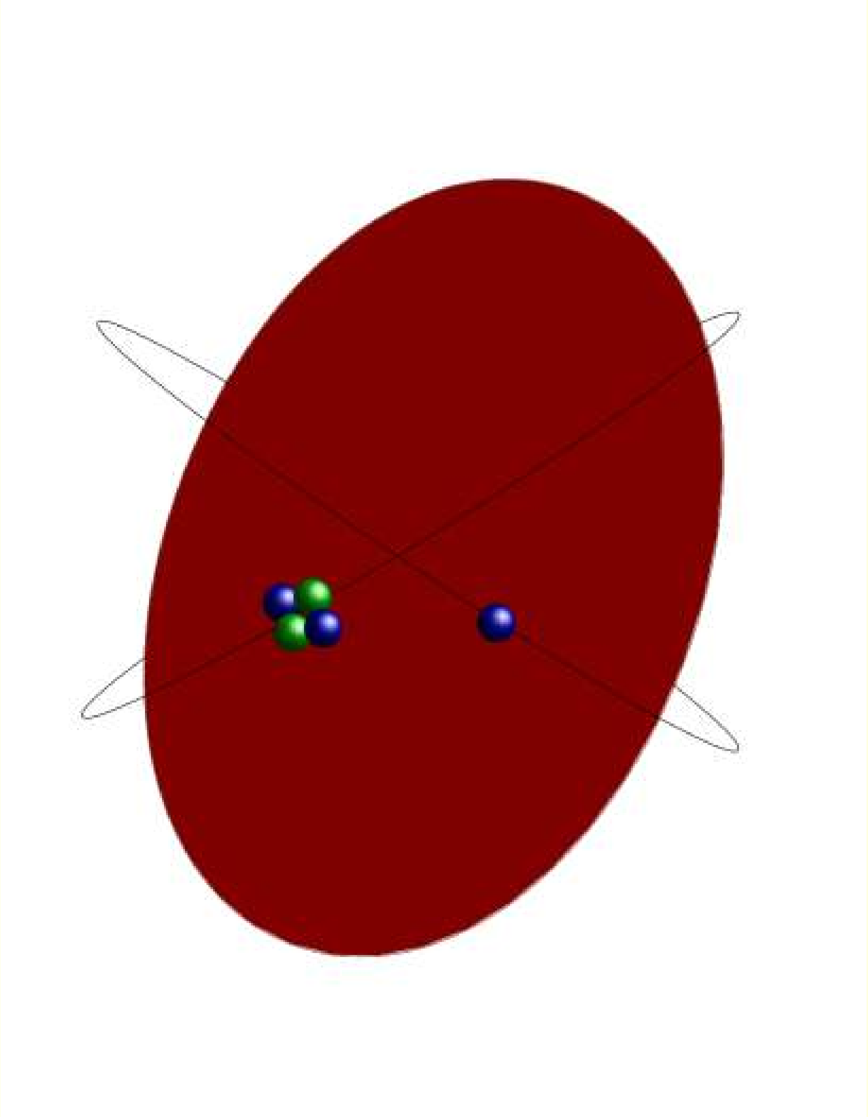
and it is given by superposition of terms with different angular momenta

$$\Psi(b_2, \mathrm{R}) = \sum_J \frac{f_J(R)}{R} \mathcal{Z}_J(b_2, \Omega)$$

$$\mathcal{Z}_J(b_2, \Omega) \equiv \left[\varphi_J^{(g)} \otimes Y_J(\Omega)\right]_0 .$$

where the first factor in the wave function of the daughter wave function ("ground band" for most cases) and the second one the α-particle angular wave function.
b_2 is the quadrupole coordinate describing core excitations

α-transitions in odd-mass nuclei



Wave function of an odd mass nucleus has the angular momentum *I* which is conserved during transition

$$\Psi_{IM}(b_2,\mathbf{r},\mathbf{R})=\sum_{Jl}\frac{f_I^{(Jl)}(r)}{r}Z_{IM}^{(Jl)}(b_2,\mathbf{r},\Omega)$$

$$Z_{IM}^{(Jl)}(b_2,\mathbf{r},\Omega)=\left[\Phi_J^{(j)}(b_2,\mathbf{r})\otimes Y_l(\Omega)\right]_{IM}$$

where the core (*J*) - sp particle (*j*) wave function is

$$\Phi_{J\mu}^{(j)}(b_2,\mathbf{r})=\sum_{J_1}X_J^{(J_1 j)}\left[\varphi_{J_1}^{(g)}\otimes\psi_j(\mathbf{r})\right]_{J\mu}$$

Favored transition: *j* is unchanged
Unfavored transition: *j* changes during transition
The diagonalisation basis is much larger than in the case of even-even nuclei!

The use of quadrupole boson coherent states to describe low-lying states in daughter nuclei

A coherent state describes an axially deformed even-even nucleus in the intrinsic system of coordinates

$$|\psi_g\rangle = e^{d\left(b^\dagger_{20} - b_{20}\right)}|0\rangle$$

where the deformation parameter is proportional to the standard quadrupole deformation

$$d = \kappa\beta_2$$

Coherent State Model (CSM)

Ground state band in even-even nuclei is obtained by projecting out the intrinsic coherent state

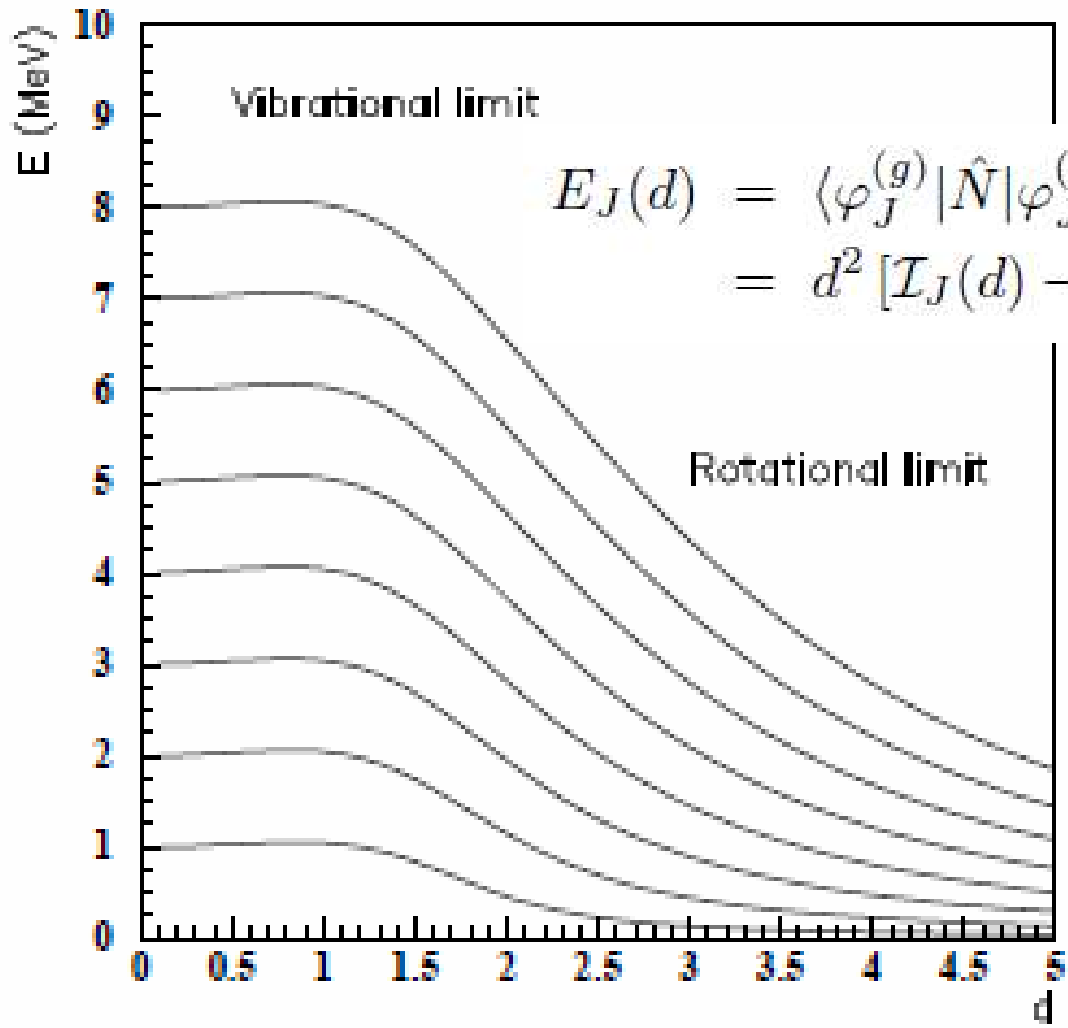
$$\varphi_J^{(g)} = \mathcal{N}_J^{(g)} P_{M0}^J \psi_g$$

where

$$P_{MK}^J = \sqrt{\frac{2J+1}{8\pi^2}} \int d\omega D_{MK}^J(\omega) \hat{R}(\omega)$$

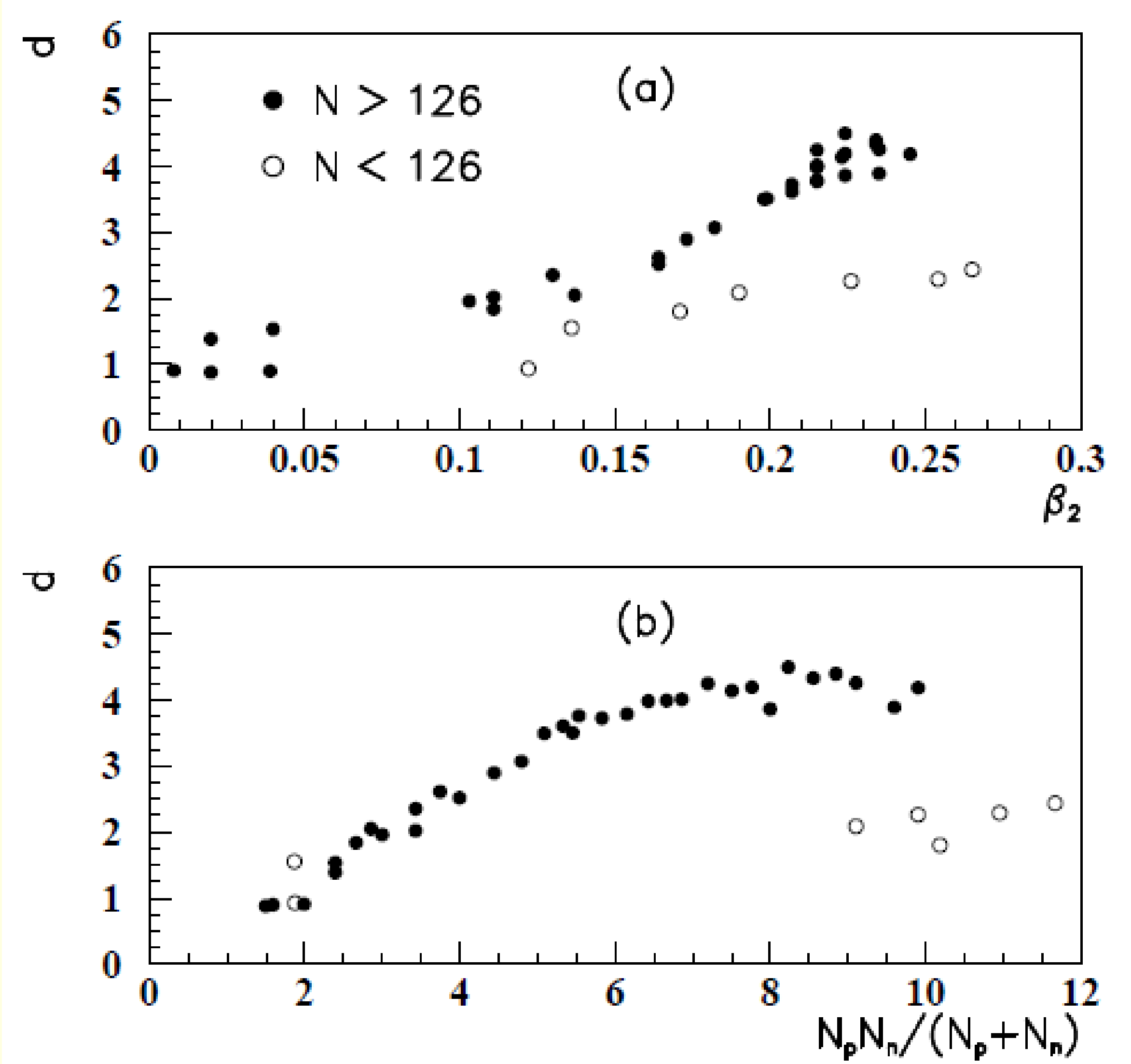
A.A. Raduta and R.M. Dreizler, Nucl. Phys. A258, 109 (1976)

Energy versus deformation parameter d has a vibrational shape for small d and rotational behavior for large d

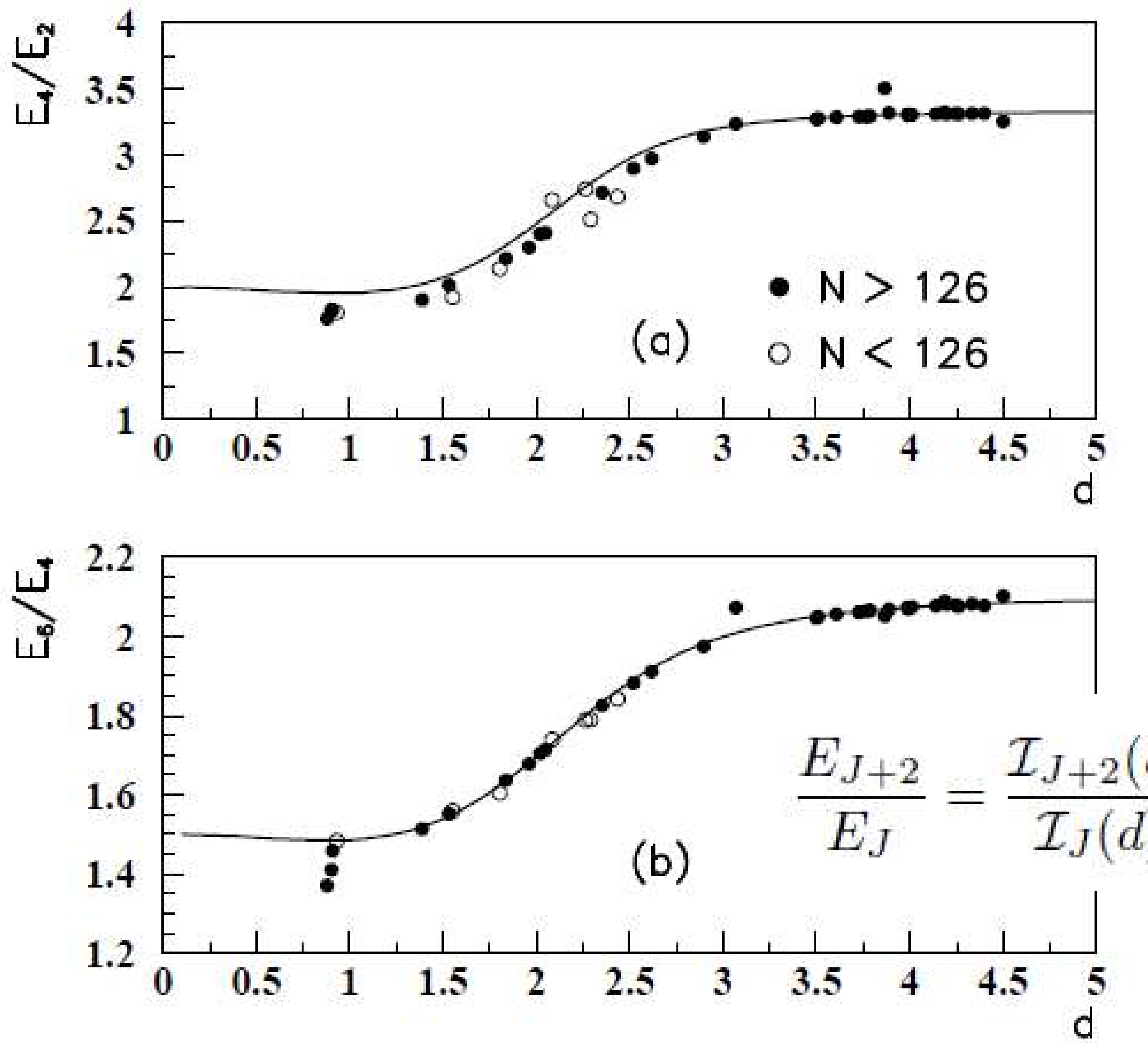


$$
\begin{aligned}
E_J(d) &= \langle \varphi_J^{(g)} | \hat{N} | \varphi_J^{(g)} \rangle - \langle \varphi_0^{(g)} | \hat{N} | \varphi_0^{(g)} \rangle \\
&= d^2 \left[\mathcal{I}_J(d) - \mathcal{I}_0(d) \right] \ ,
\end{aligned}
$$

Deformation parameter fitting exp. energies versus (a) standard quadrupole deformation (b) Casten parameter



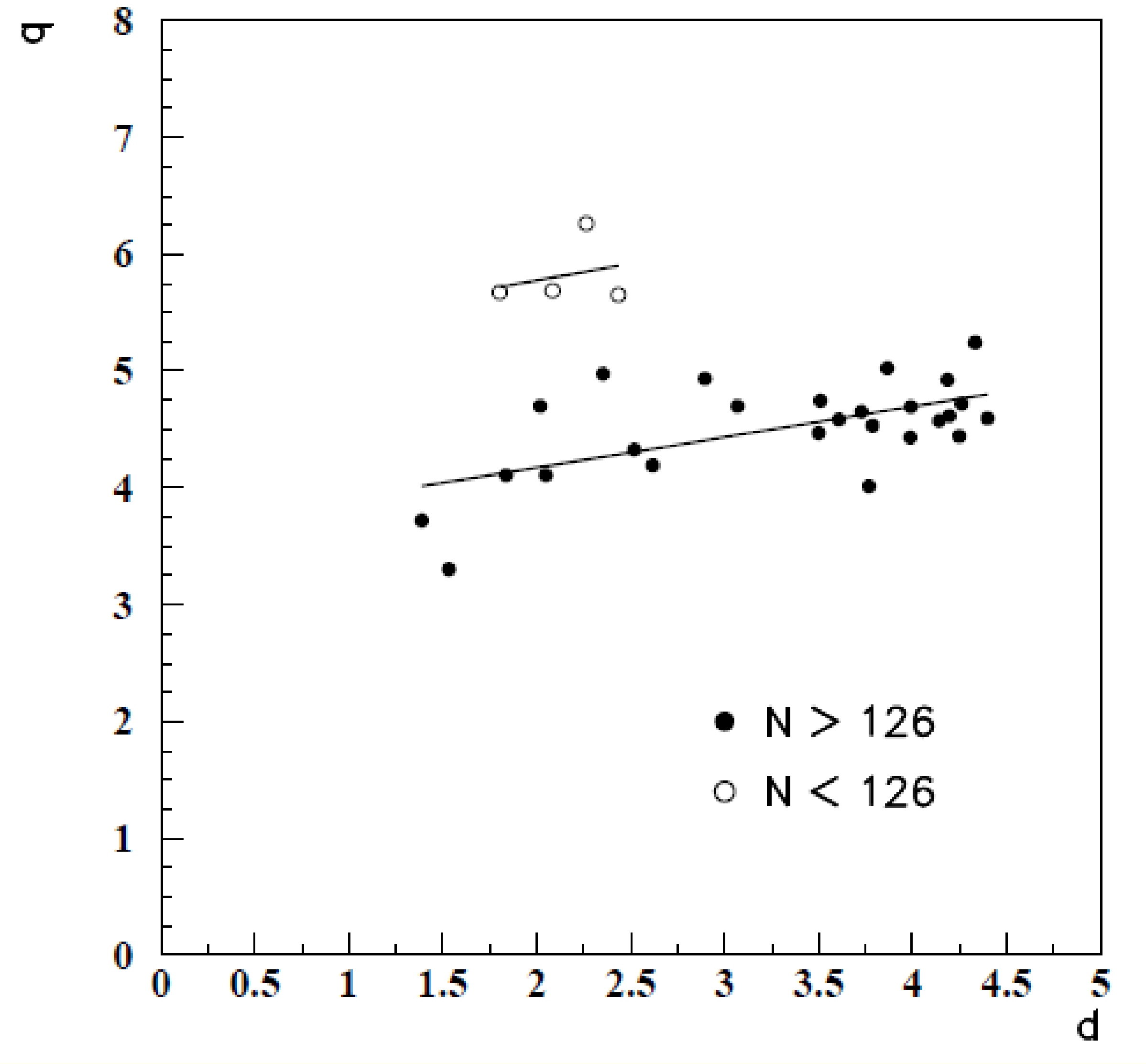
Energy ratios versus the CSM deformation parameter are universal functions



$$\frac{E_{J+2}}{E_J} = \frac{\mathcal{I}_{J+2}(d) - \mathcal{I}_0(d)}{\mathcal{I}_J(d) - \mathcal{I}_0(d)}$$

Effective charge versus the CSM deformation parameter

$$q(d) = q_0 \left(1 - \sqrt{\frac{2}{7}} a_q d\right)$$



Fundamental outgoing resonant states for a deformed nucleus

In the internal region asymptotics at small distances is regular:

$$\mathcal{R}_{JI}(R) \stackrel{R \to R_0}{\longrightarrow} \delta_{JI} \varepsilon_J$$

In the external region asymptotics at large distances is given by Gamow (Coulomb-Hankel) outgoing waves:

$$\mathcal{H}^{(+)}_{JI}(R) \equiv \mathcal{G}_{JI}(R) + i\mathcal{F}_{JI}(R)$$

$$\stackrel{R \to \infty}{\longrightarrow} \delta_{JI} H^{(+)}_J(\kappa_J R) \equiv \delta_{JI} \left[G_J(\kappa_J R) + iF_J(\kappa_J R) \right]$$

General solution is given by the superposition of fundamental outgoing resonances

Matching between internal and external solutions at some radius R_1

$$f_J(R_1) = \sum_I \mathcal{R}_{JI}(R_1)M_I = \sum_I \mathcal{H}_{JI}^{(+)}(R_1)N_I$$

and their derivatives

$$\frac{df_J(R_1)}{dR} = \sum_I \frac{d\mathcal{R}_{JI}(R_1)}{dR}M_I = \sum_I \frac{d\mathcal{H}_{JI}^{(+)}(R_1)}{dR}N_I$$

leads to:

The secular equation for outgoing resonant states

$$\left| \begin{array}{cc} \mathcal{R}(R_1) & \mathcal{H}^{(+)}(R_1) \\ \frac{d\mathcal{R}(R_1)}{dR} & \frac{d\mathcal{H}^{(+)}(R_1)}{dR} \end{array} \right| \approx \left| \begin{array}{cc} \mathcal{R}(R_1) & \mathcal{G}(R_1) \\ \frac{d\mathcal{R}(R_1)}{dR} & \frac{d\mathcal{G}(R_1)}{dR} \end{array} \right| = 0$$

because the regular waves are much smaller than the regular ones inside the barrier.

Resonant states are normalized in the internal region:

$$\sum_J \int_{R_0}^{R_2} |f_J(R)|^2 dR = 1$$

Channel decay widths

By using continuity equation one obtains total decay width as a sum of partial widths:

$$\Gamma = \sum_J \Gamma_J = \sum_J \hbar v_J \lim_{R\to\infty} |f_J(R)|^2$$
$$= \sum_J \hbar v_J |N_J|^2 ,$$

where channel velocity is given by:

$$v_J = \frac{\hbar \kappa_J}{\mu}$$

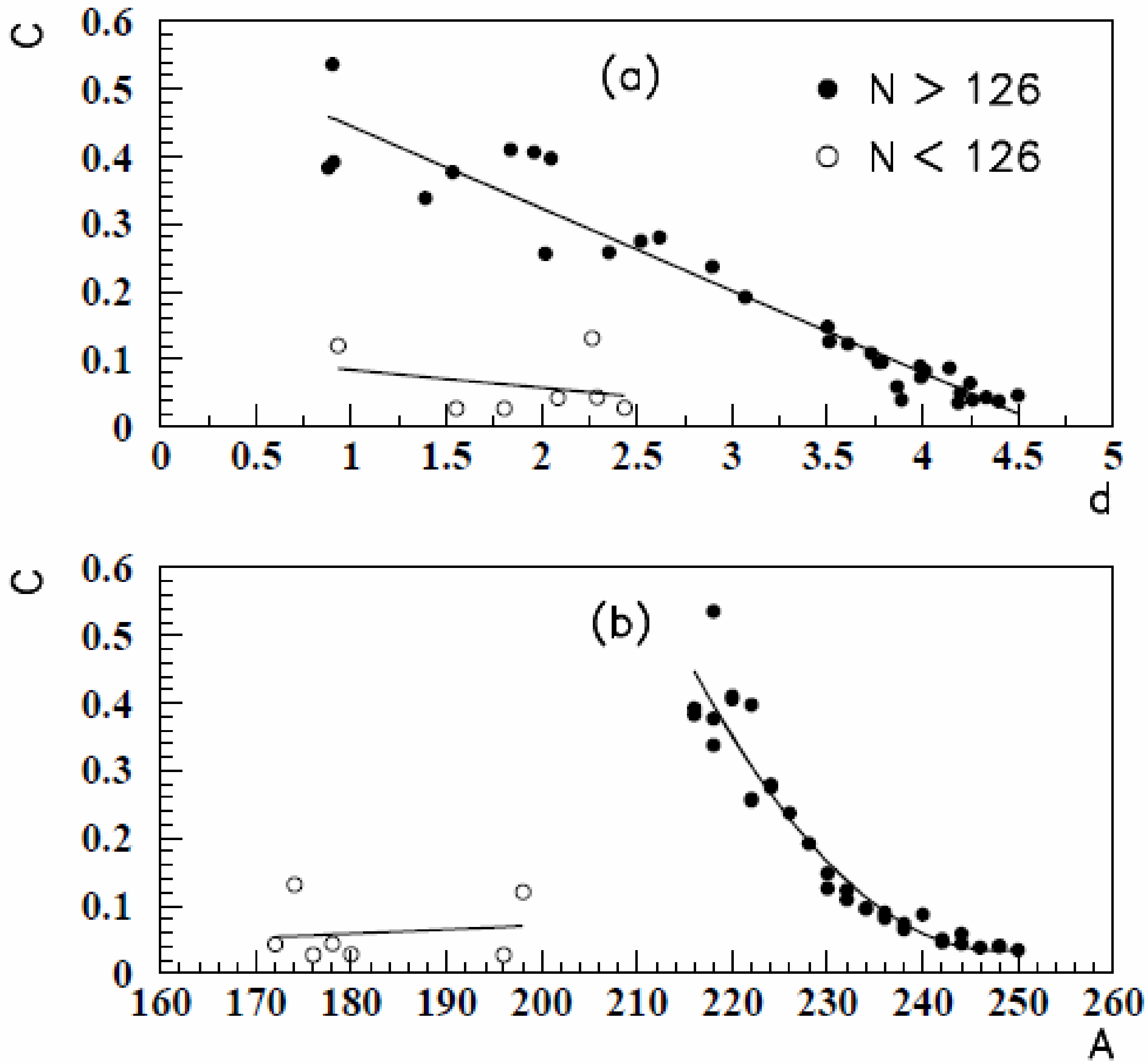
Channel intensities

$$I_J \equiv log_{10} \frac{\Gamma_0}{\Gamma_J}$$

define the strength of α-transitions to some excited state with spin J

The only free parameter is the α-daughter coupling strength which can be determined by the I_2 value for each transition

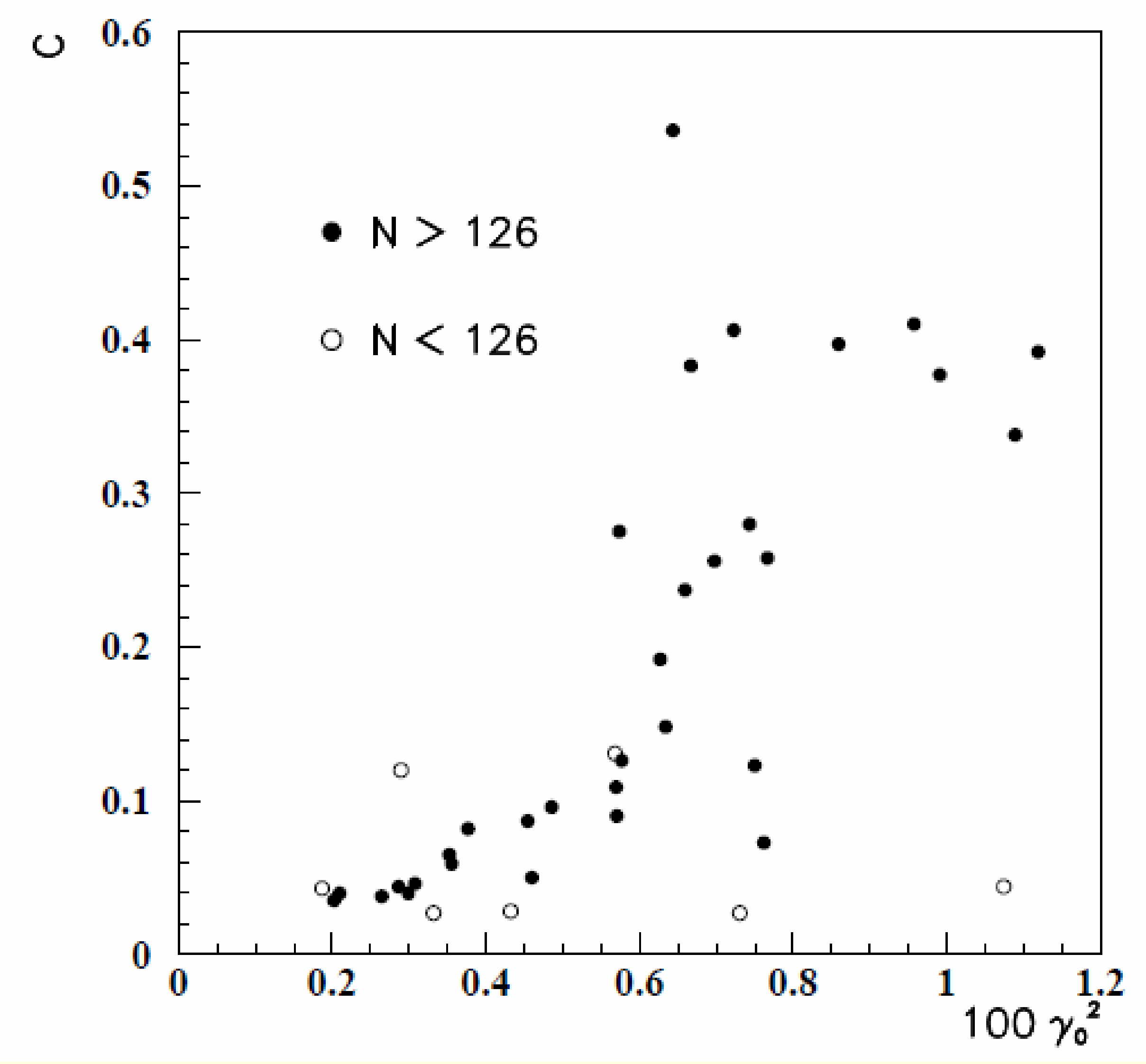
α-daughter coupling strength reproducing I_2 versus deformation parameter (a) and mass number (b)



confirms the CSM prediction:

$$C(d) = C_0\left(1 - \sqrt{\frac{2}{7}}a_\alpha d\right)$$

α-daughter coupling strength is proportional to the reduced width squared

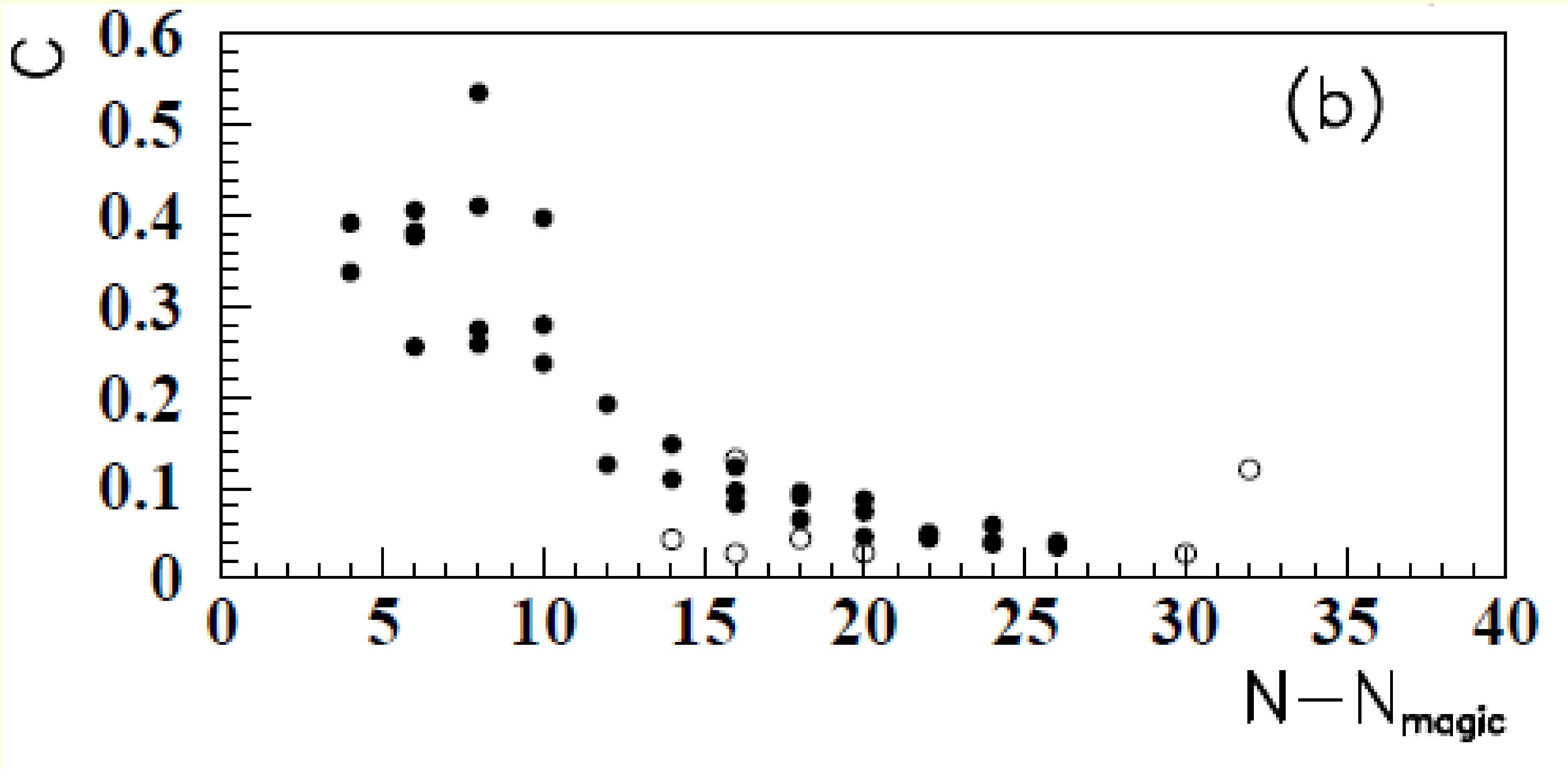


Coupling strength is proportional to the α-clustering probability (reduced width squared)

$$\gamma_0^2 = \frac{\Gamma_0}{2P_0}$$

α-daughter coupling strength versus the difference N-N_{magic}

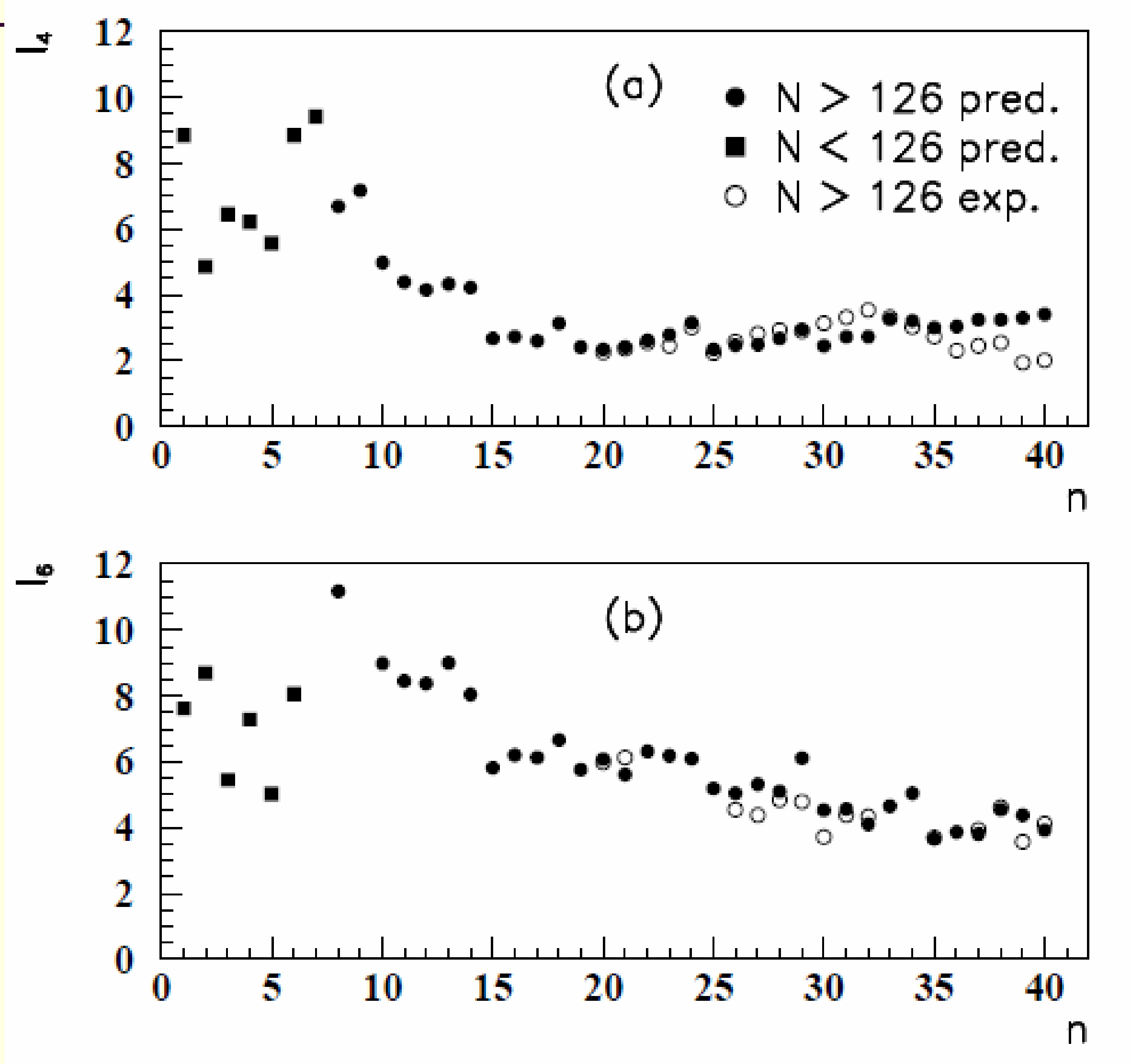
α-clustering is stronger above magic nuclei



The largest α-clustering is above the double magic ^{208}Pb

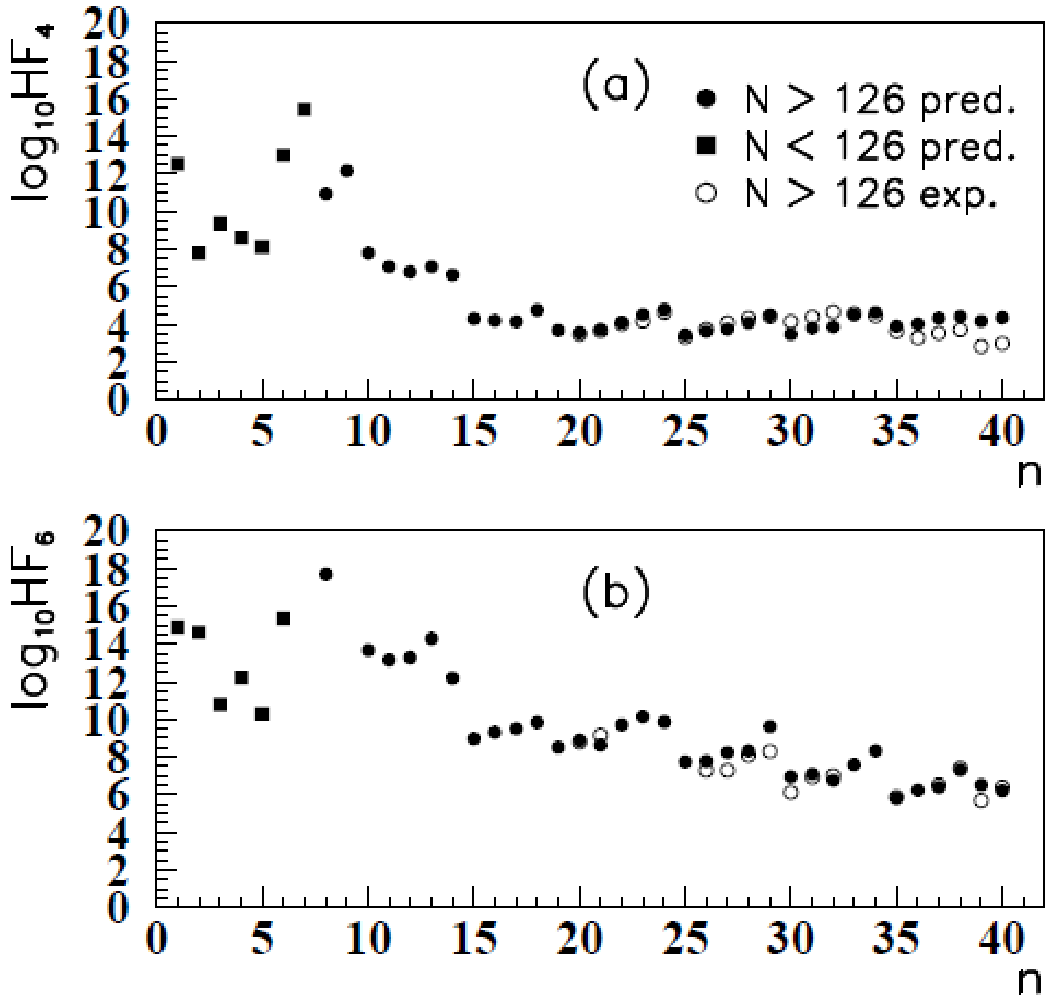
N_{magic} = 126, N>126
= 82, N<126

α-transition intensities versus decay label in the table of α-emitters



Hindrance factors

$$\log_{10} HF_J \equiv \log_{10} \frac{\gamma_0^2}{\gamma_J^2} = \log_{10} I_J - \log_{10} \frac{P_0}{P_J}$$



IV. Surface α-clustering in ^{212}Po

D.S. Delion, R.J. Liotta, P. Schuck, A. Astier, and M.-G. Porquet
Phys. Rev. C85, 064306 (2012)

Positive parity states 2^+, 4^+, 6^+, 8^+ are given by neutron broken pairs

$$|^{212}\mathrm{Po}(J^+)\rangle = |^{210}\mathrm{Pb}(J^+) \otimes {}^{210}\mathrm{Po(g.s.)}\rangle$$

Negative parity states 4^-, 6^-, 8^- are given by neutron broken pairs coupled to an octupole state

$$|^{212}\mathrm{Po}(I^-)\rangle = |[^{210}\mathrm{Pb}(J^+) \otimes {}^{210}\mathrm{Pb}(3^-)]_{I^-} \otimes {}^{210}\mathrm{Po(g.s.)}\rangle$$

Single particle basis contains two components. It is similar to the method B to compute α-decay widths

$$\psi_l(r) = \psi_l^{(SM)}(r) + \psi_l^{(clus)}(r)$$

where the cluster component is given by a Gaussian centered on the nuclear surface

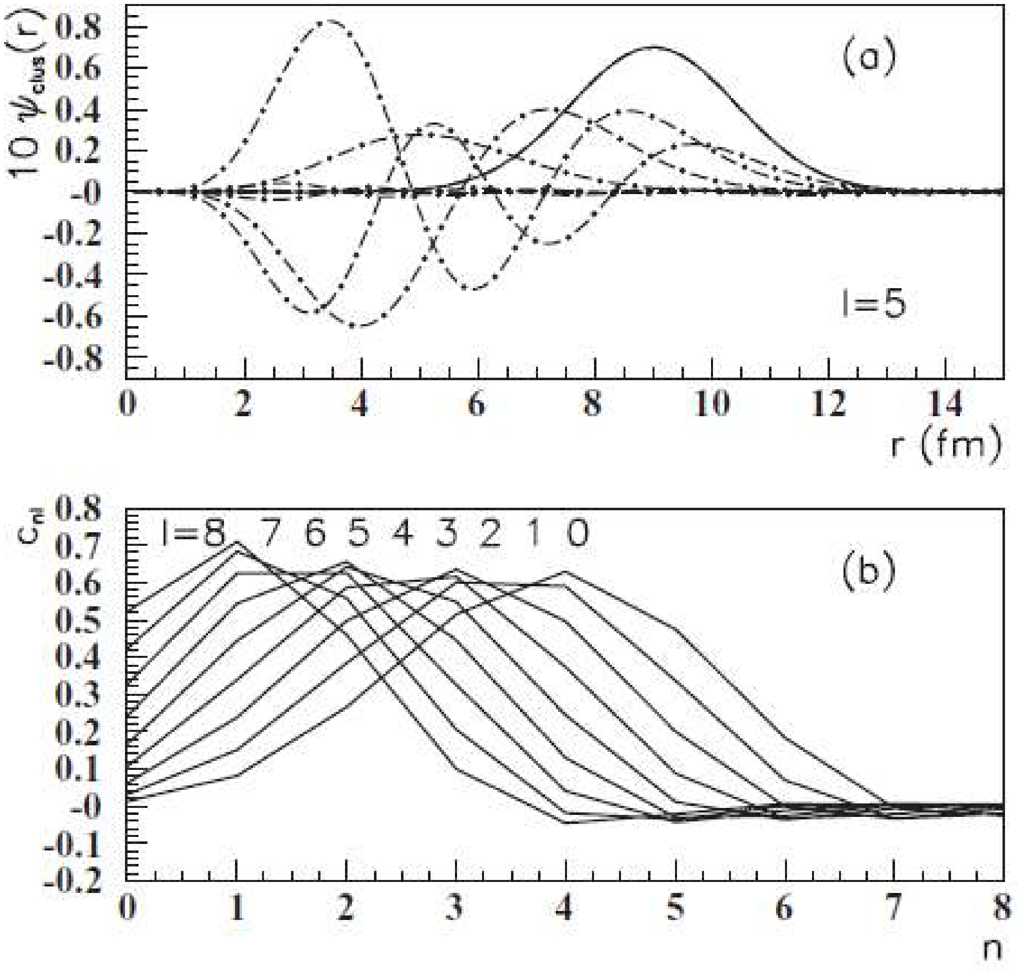
$$\psi_l^{(clus)}(r) = \mathcal{N}_l^{(clus)} e^{-\beta_c (r-r_0)^2/2}$$

containing components with larger principal quantum number N~8,9,10

$$\psi_l^{(clus)}(r) = \sum_N c_{nl} (-)^n \mathcal{R}_{nl}^{(\beta)}(r)$$

Transition operator is proportional to the principal quantum number

$$\langle \mathcal{R}_{nl}^{(\beta)} | r^{\lambda} | \mathcal{R}_{n'l'}^{(\beta)} \rangle \sim \left(\frac{N}{\beta} \right)^{\lambda/2}, \quad N = 2n + l$$



Surface clustering states have large values of the ang. momentum and N > 8

Bound states have low values of the ang. momentum and N < 8

Surface α-clustering term with the amplitude ≈ 0.3 explains large electromagnetic transitions in ^{212}Po

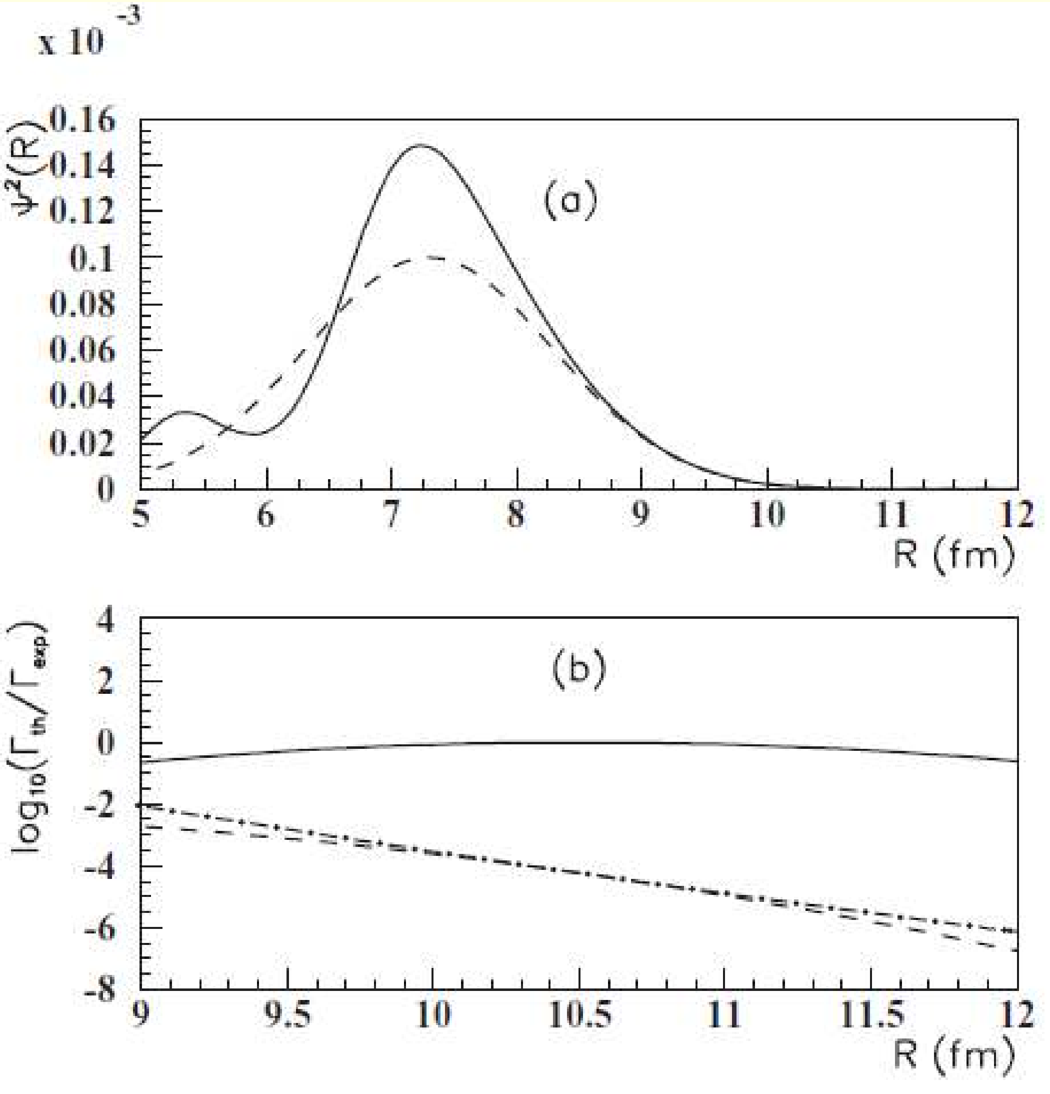
B(E2:J+2→J)-values

$J' \to J$	^{210}Po $B(E2)_{exp}$	$B(E2)_{th}$	^{210}Pb $B(E2)_{exp}$	$B(E2)_{th}$	^{212}Po $B(E2)_{exp}$	$B(E2)_{th}$
$2 \to 0$	0.56(12)	6.7	1.4(4)	3.9		9.2
$4 \to 2$	4.6(2)	12.9	3.2(7)	3.5		20.8
$6 \to 4$	3.0(1)	8.9	2.2(3)	2.4	13.5(36)	14.4
$8 \to 6$	1.18(3)	3.9	0.62(5)	1.0	4.60(9)	5.8

B(E1:I⁻→J⁺)-values

I^-	J^+	E_{MSM} (MeV)	$E(^{212}\text{Po}(I^-))$ (MeV)	$E_{exp}(^{212}\text{Po}(I^-))$ (MeV)	$B(E1)^{(1)}_{th}$ (10^4 W.u.)	$B(E1)^{(2)}_{th}$ (10^4 W.u.)	$B(E1)_{exp}$ (10^4 W.u.)
2^-	2^+	−0.407	1.236		5	1	
	4^+	−0.204	1.907		15	63	
4^-	4^+	−0.303	1.808	1.744	9	11	25
	6^+	−0.107	2.201	1.946	2	4	11
6^-	6^+	−0.213	1.886	1.787	37	122	66
	8^+	−0.490	2.197	2.016	3	8	19
8^-	6^+	−0.489	1.816	1.751	43	148	200
	8^+	−0.215	2.240	1.986	8	24	
10^-	8^+	−0.360	2.135	2.465	2	1	18

Surface α-clustering in ^{212}Po
explains decay width between ground states



Formation probability versus cm radius
(a): total
(b): cluster component

Log (width / exp.) versus cm radius

The same cluster amplitude ≈ 0.3 explains B(Eλ) values and absolute α-decay width

V. Probing shape coexistence by α-decay to excited 0_2^+ states

J. Wauters et al., Phys. Rev. Lett. **72**, 1329 (1994)

	E_x (keV)	E_α (MeV)	I_α (%)	HF	$T_{1/2\alpha}(\mathrm{th})/T_{1/2\alpha}(\mathrm{exp})$
$^{202}\mathrm{Rn} \xrightarrow{\alpha} {}^{198}\mathrm{Po}$	0	6.641	80–100	1	1.5–1.9
	816	5.841	$(1.4–1.8)\times10^{-3}$	19(6)	$(7.5\times9.4)\times10^{-2}$
$^{198}\mathrm{Po} \xrightarrow{\alpha} {}^{194}\mathrm{Pb}$	0	6.180	57	1	0.85
	931	5.273	7.6×10^{-4}	2.8(5)	0.27
$^{196}\mathrm{Po} \xrightarrow{\alpha} {}^{192}\mathrm{Pb}$	0	6.521	94	1	1.17
	769	5.769	2.1×10^{-2}	2.5(1)	0.39
$^{194}\mathrm{Po} \xrightarrow{\alpha} {}^{190}\mathrm{Pb}$	0	6.842	93	1	0.98
	658	6.194	0.22	1.1(1)	0.77
$^{188}\mathrm{Pb} \xrightarrow{\alpha} {}^{184}\mathrm{Hg}$	0	5.980	3–10	1	0.24–0.81
	375	5.614	$(2.9–9.5)\times10^{-2}$	21(3)	$(1.1–3.6)\times10^{-2}$
$^{186}\mathrm{Pb} \xrightarrow{\alpha} {}^{182}\mathrm{Hg}$	0	6.335	<100	1	<1.46
	328	6.014	<0.20	21(4)	$<6.6\times10^{-2}$
$^{184}\mathrm{Hg} \xrightarrow{\alpha} {}^{180}\mathrm{Pt}$	0	5.535	1.25	1	0.98
	478	5.067	2.0×10^{-3}	2.4(4)	0.39
$^{182}\mathrm{Hg} \xrightarrow{\alpha} {}^{178}\mathrm{Pt}$	0	5.865	8.6	1	0.58
	422	5.446	2.9×10^{-2}	3.5(6)	0.17
$^{180}\mathrm{Hg} \xrightarrow{\alpha} {}^{176}\mathrm{Pt}$	0	6.118	33	1	0.79
	443	5.689	2.6×10^{-2}	17(5)	4.3×10^{-2}

Pairing vibrations in Pb isotopes

VOLUME 74, NUMBER 20 PHYSICAL REVIEW LETTERS 15 MAY 1995

Microscopic Description of Alpha Decay to Intruder 0_2^+ States in Pb, Po, Hg, and Pt Isotopes

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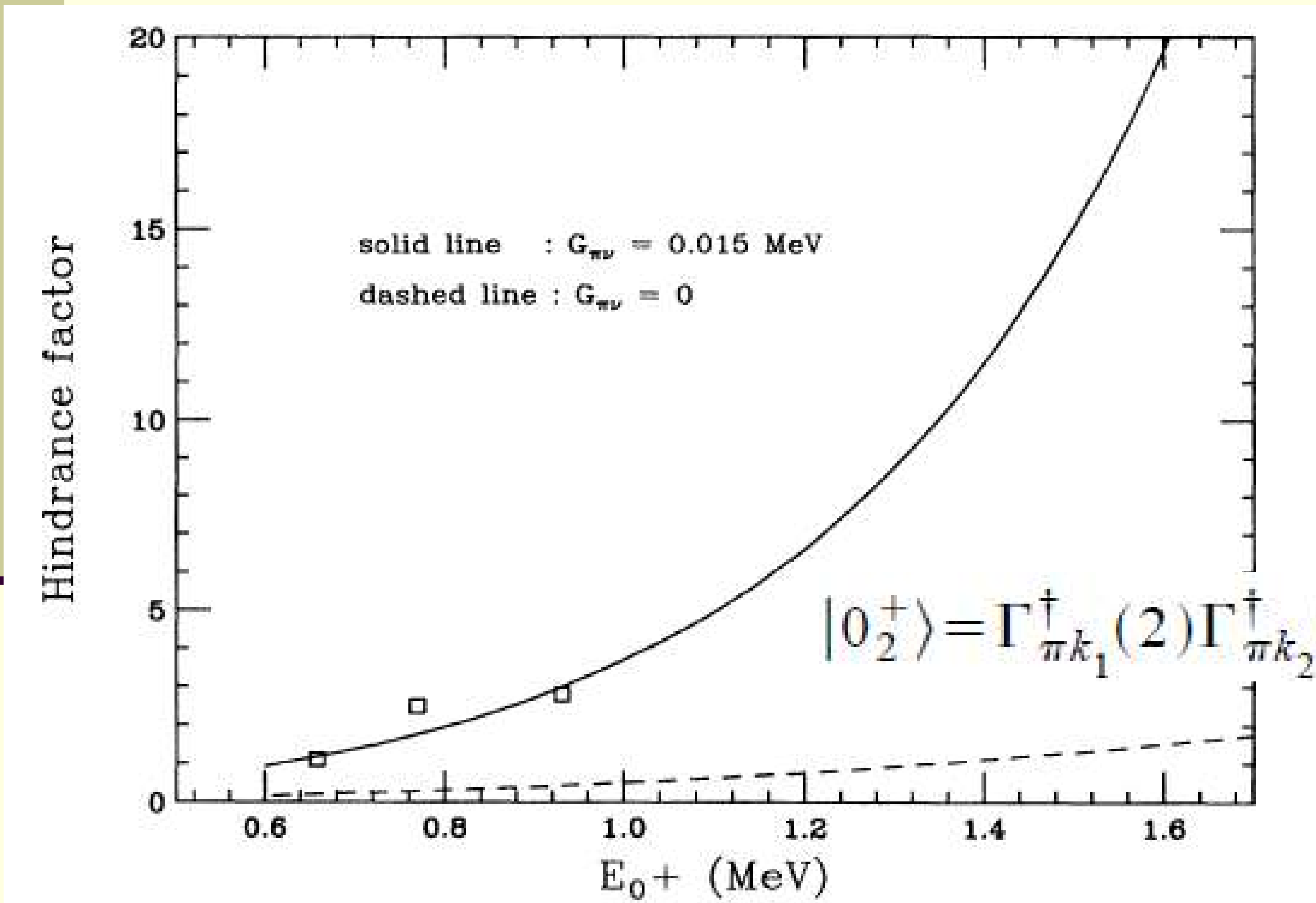
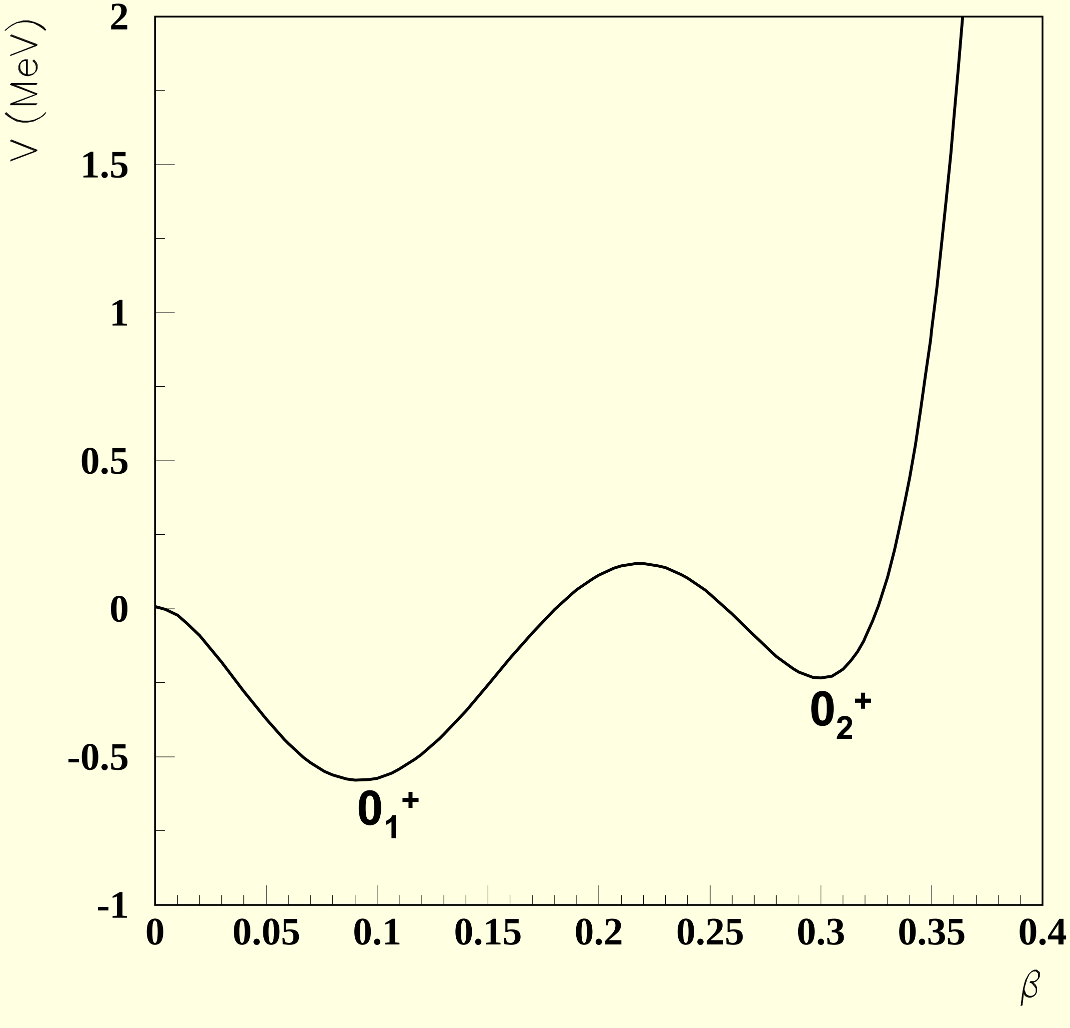
0_2^+ states are described as 2p-2h excitations across the magic shell Z=82
In addition we use a proton-neutron interaction

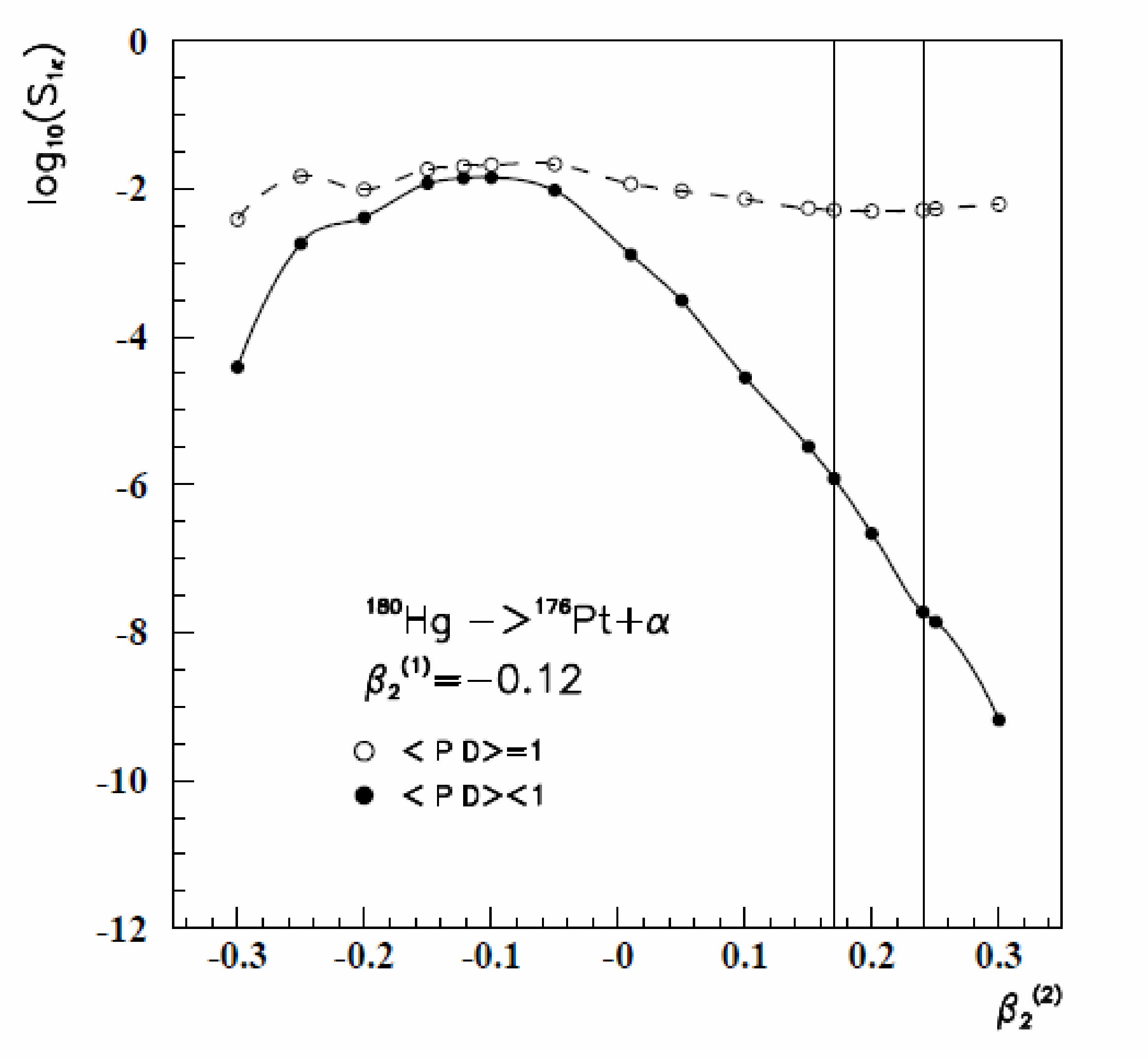


$$|0_2^+\rangle = \Gamma^\dagger_{\pi k_1}(2)\Gamma^\dagger_{\pi k_2}(-2)|RPA\rangle$$

α-decay to excited 0^+ states in superfluid nuclei



Spectroscopic factor versus the quadrupole deformation in daughter nucleus for a realistic BCS overlap (solid line) and BCS overlap=1 (dashed line)



Shape coexistence

Wave function is a superposition of BCS wave functions corresponding the two minima

$$|\varphi_1^A\rangle = X_A|\psi_1^A\rangle + Y_A|\psi_2^A\rangle$$

$$|\varphi_2^A\rangle = -Y_A|\psi_1^A\rangle + X_A|\psi_2^A\rangle \ , \quad A = P, D$$

Normalisation condition: $\left\{ \begin{matrix} X_A^2 \\ Y_A^2 \end{matrix} \right\} = \frac{1}{2} \pm \delta_A$

α transition operator

$$\langle \psi_{k'}^{D} | \hat{T} | \psi_{k}^{P} \rangle \equiv T_{k'k} = \sqrt{\langle S_{kk'} \rangle}$$

Theoretical HF between BCS states

$$HF_{th}(k) = \left| \frac{\langle \psi_{1}^{D} | \hat{T} | \psi_{k}^{P} \rangle}{\langle \psi_{2}^{D} | \hat{T} | \psi_{k}^{P} \rangle} \right|^{2} \equiv \left| \frac{T_{1k}}{T_{2k}} \right|^{2}$$

Experimental HF between superposed states

$$HF_{exp}(k) = \left| \frac{\langle \varphi_{1}^{D} | \hat{T} | \varphi_{k}^{P} \rangle}{\langle \varphi_{2}^{D} | \hat{T} | \varphi_{k}^{P} \rangle} \right|^{2}$$

Results

No.	Parent	0_k^+	E_k (keV)	β_k	Daughter	$0_{k'}^+$	$E_{k'}$ (keV)	$\beta_{k'}$	b_α (%)	$HF_{exp}(1)$	$HF_{th}(k)$	δ_P δ_D
1	^{180}Hg	0_1^+	0	-0.12	^{176}Pt	0_1^+	0	0.17	47.9	16.1	64.0	0.5
		0_2^+	-	-		0_2^+	443.0	0.24	0.038		-	0.03
2	^{182}Hg	0_1^+	0	-0.13	^{178}Pt	0_1^+	0	0.25	15.0	4.8	0.1	0.25
		0_2^+	328.0	0.27		0_2^+	421.0	0.18	0.036		1.1	0
3	^{184}Hg	0_1^+	0	-0.13	^{180}Pt	0_1^+	0	0.26	1.25	1.9	0.06	0.25
		0_2^+	375.1	0.25		0_2^+	478.1	0.18	0.002		1.7	0
4	^{202}Rn	0_1^+	0	0.09	^{198}Po	0_1^+	0	0.07	78.0	25.2	810.9	0.5
		0_2^+	-	-		0_2^+	816.0	-0.15	0.001		-	0.007

The 0^+ state are strongly mixed, giving a large shape coexistence

VII. Conclusions

1) A pocket-like α-daughter interaction leads to
an universal law for reduced widths versus the fragmentation potential
and hindrance factors versus the excitation energy.
2) Absolute decay widths can be described by using a mixed ho basis,
or a mean field with an additional pocket-like interaction.
3) CSM describes α-transitions to excited states in even-even nuclei
predicting a linear dependence of the α-daughter QQ strengths
on the deformation parameter d.
4) The α-daughter strength is proportional to reduced width squared
and has the largest value in the region above ^{208}Pb, where
the α-clustering explains large B(Eλ) values in ^{212}Po.
5) HF for transitions to excited 0^{+} states strongly depends upon the
difference between deformations of parent and daughter nuclei.
6) Exp. HF's predict a strong shape coexistence in Hg isotopes.

Collaboration with

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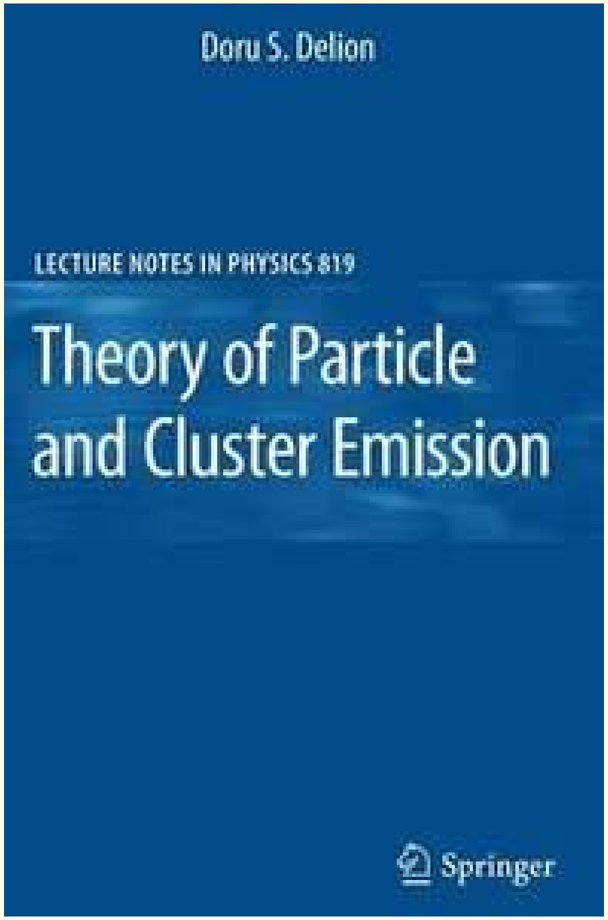
THANK YOU !



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