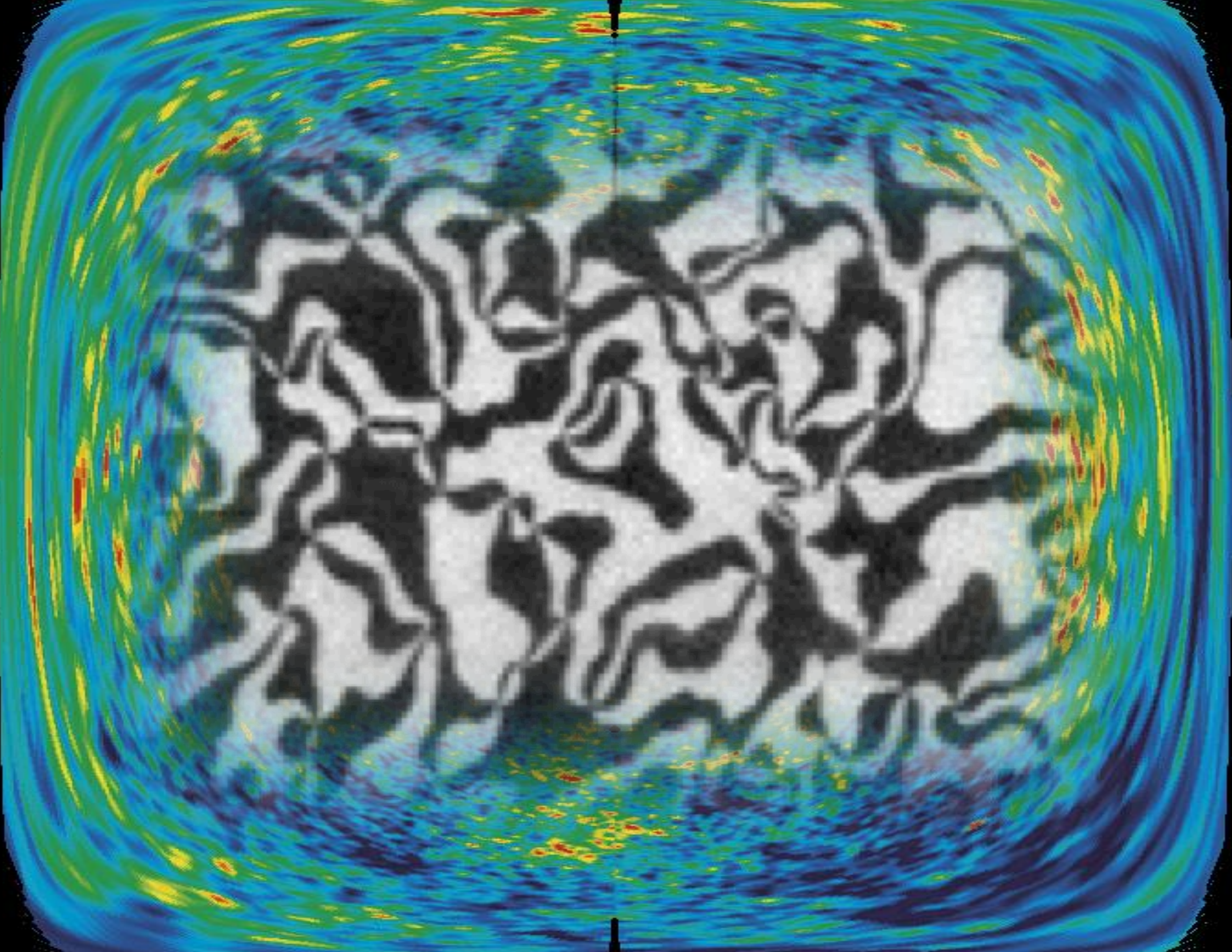
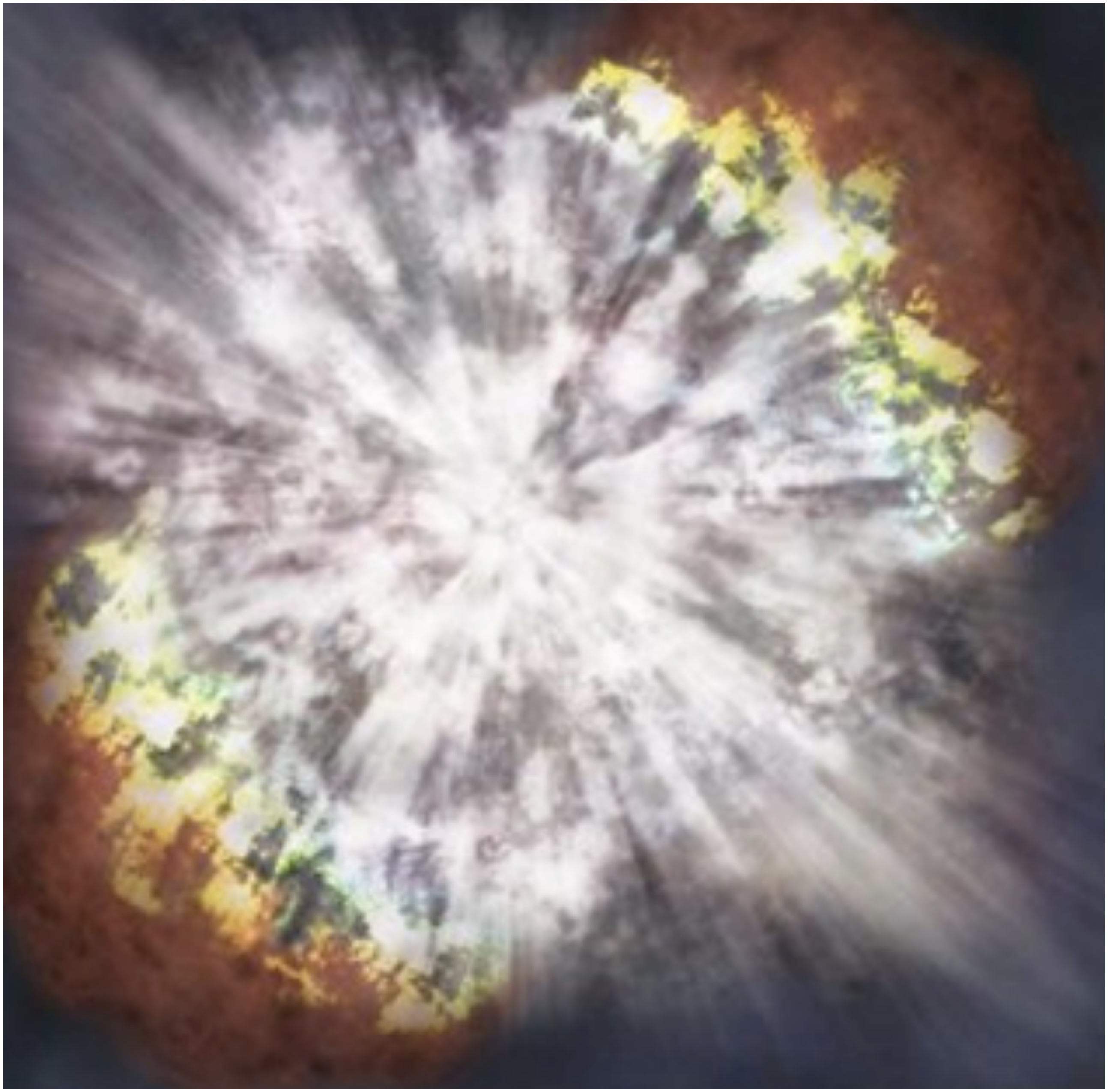
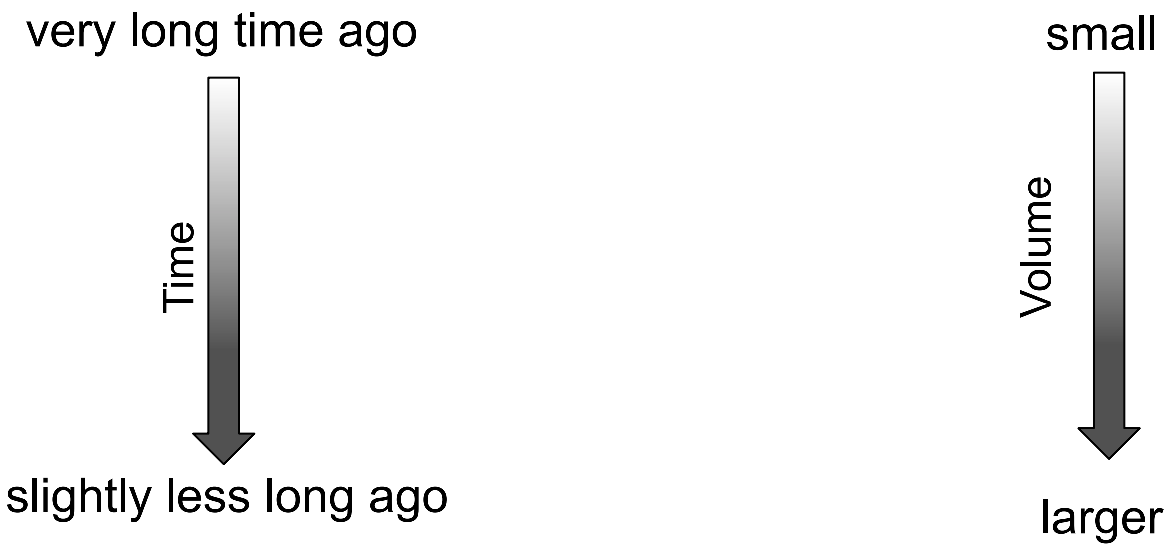
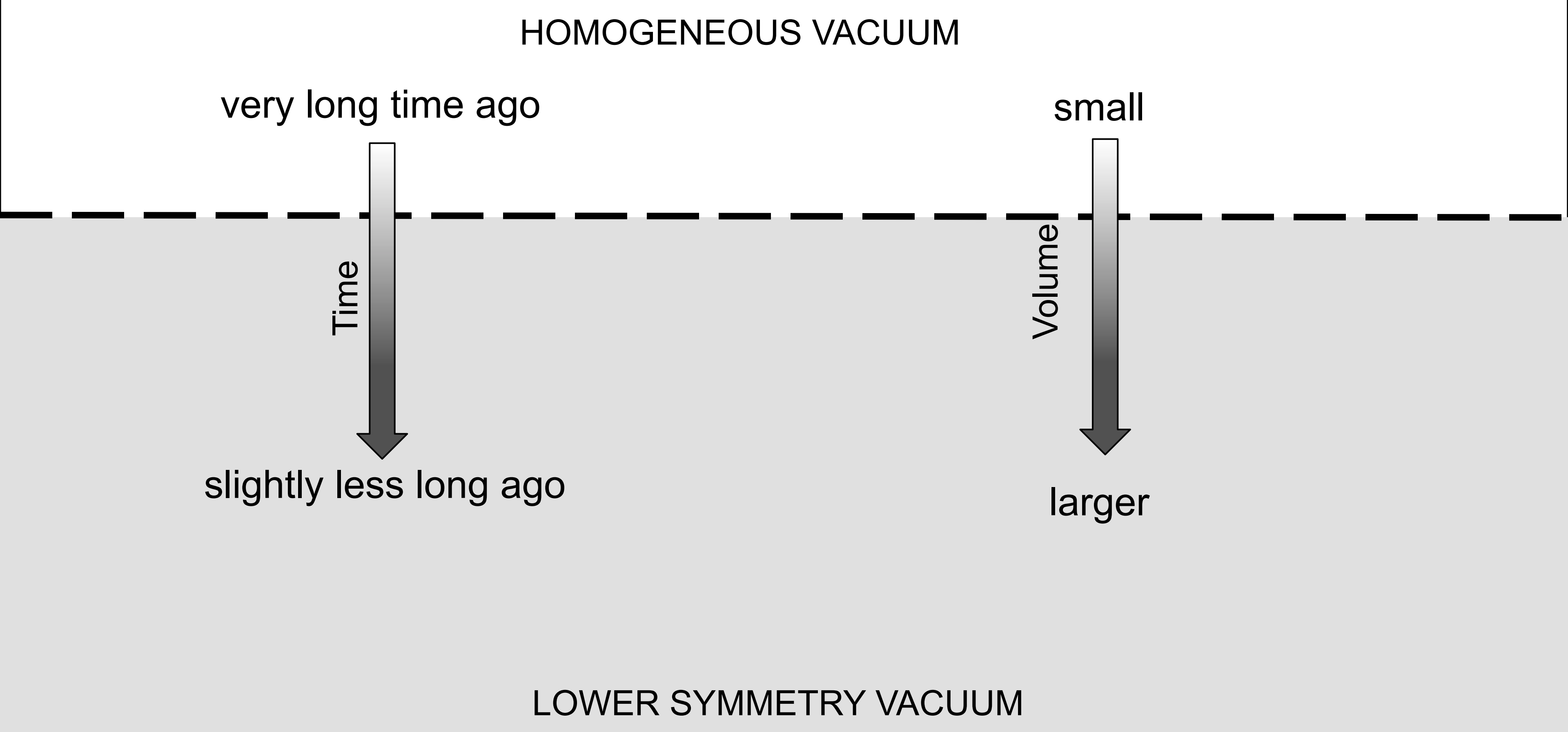




HOMOGENEOUS VACUUM

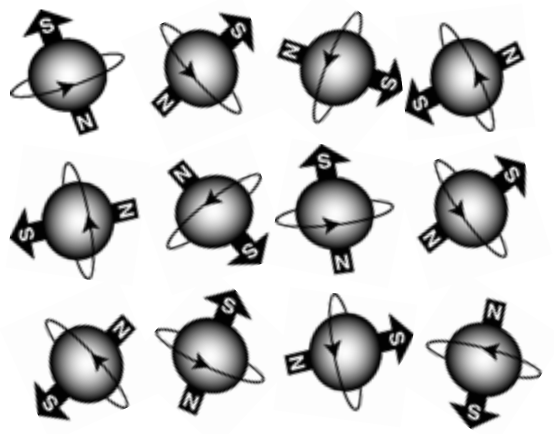


After 10^{-37} seconds: Grand Unification Transition



Symmetry lowering phase transition in a ferromagnet

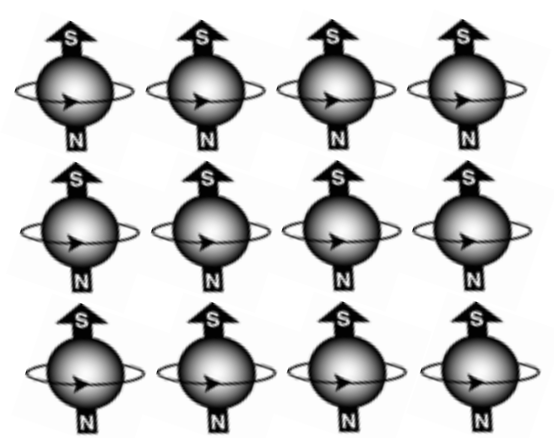
high temperature



high symmetry

Transition (Curie) temperature, Tc

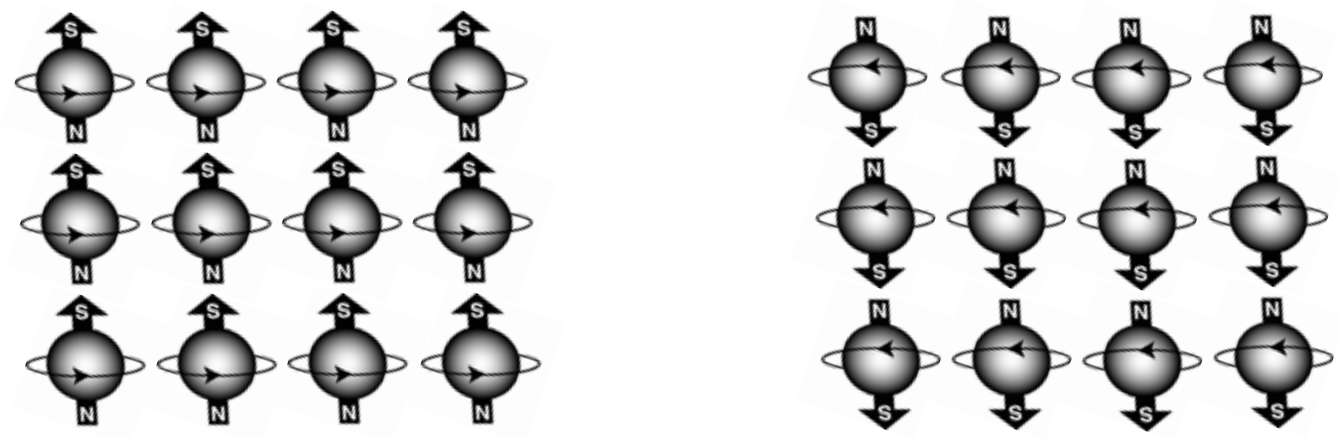
low temperature



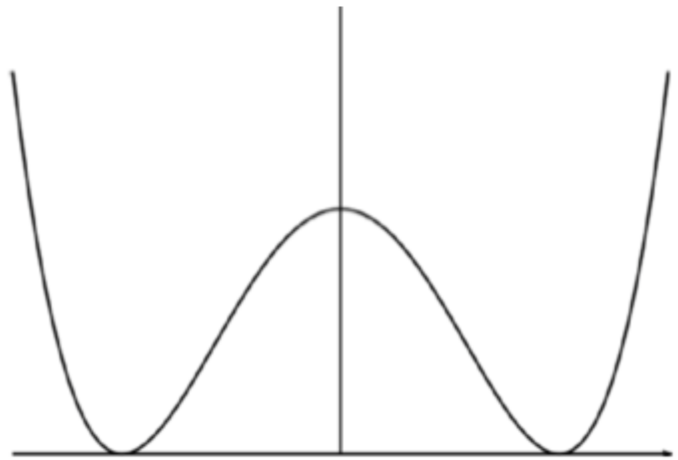
low symmetry

If the ferromagnet is uniaxial…

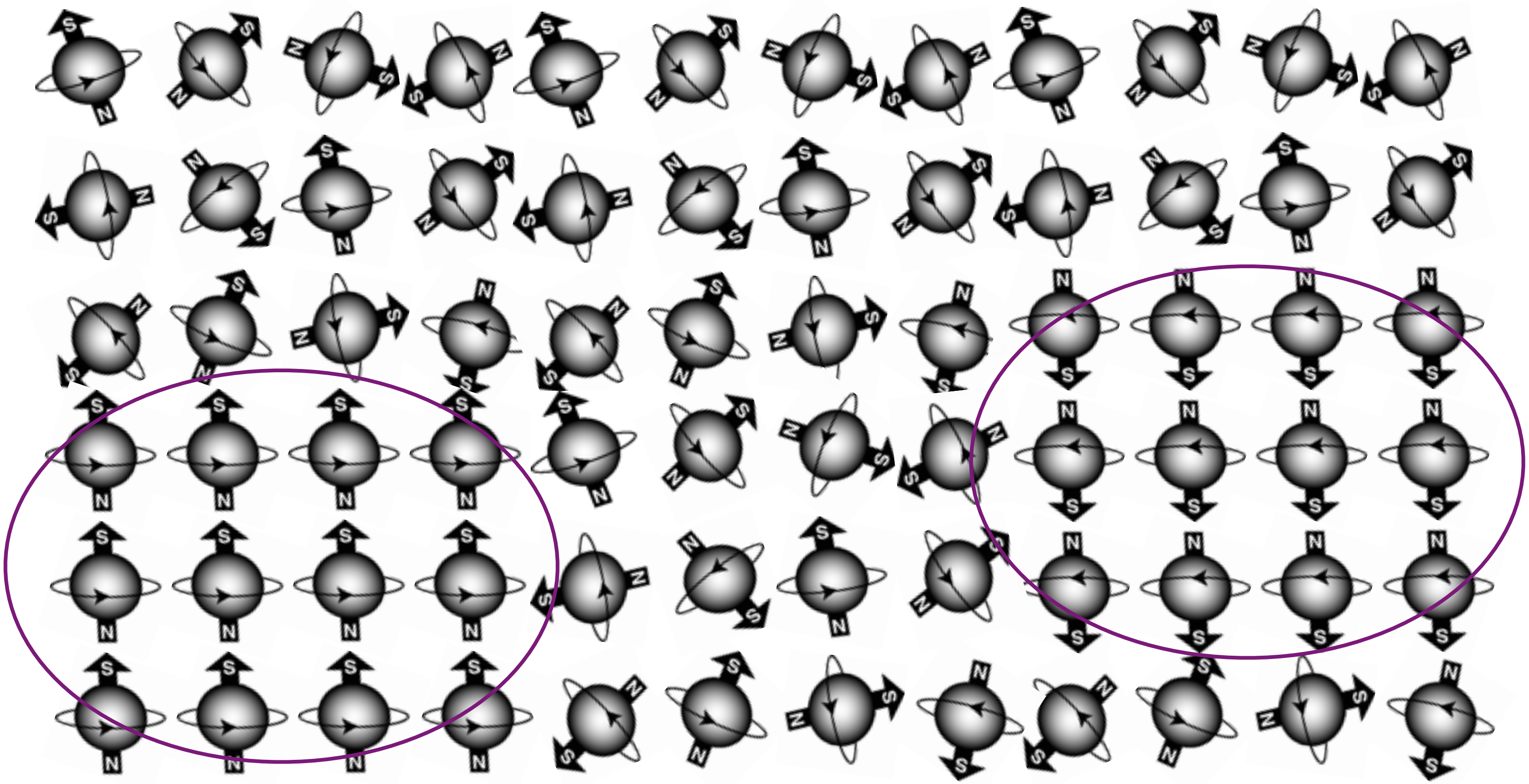
… then there are two equivalent low symmetry states



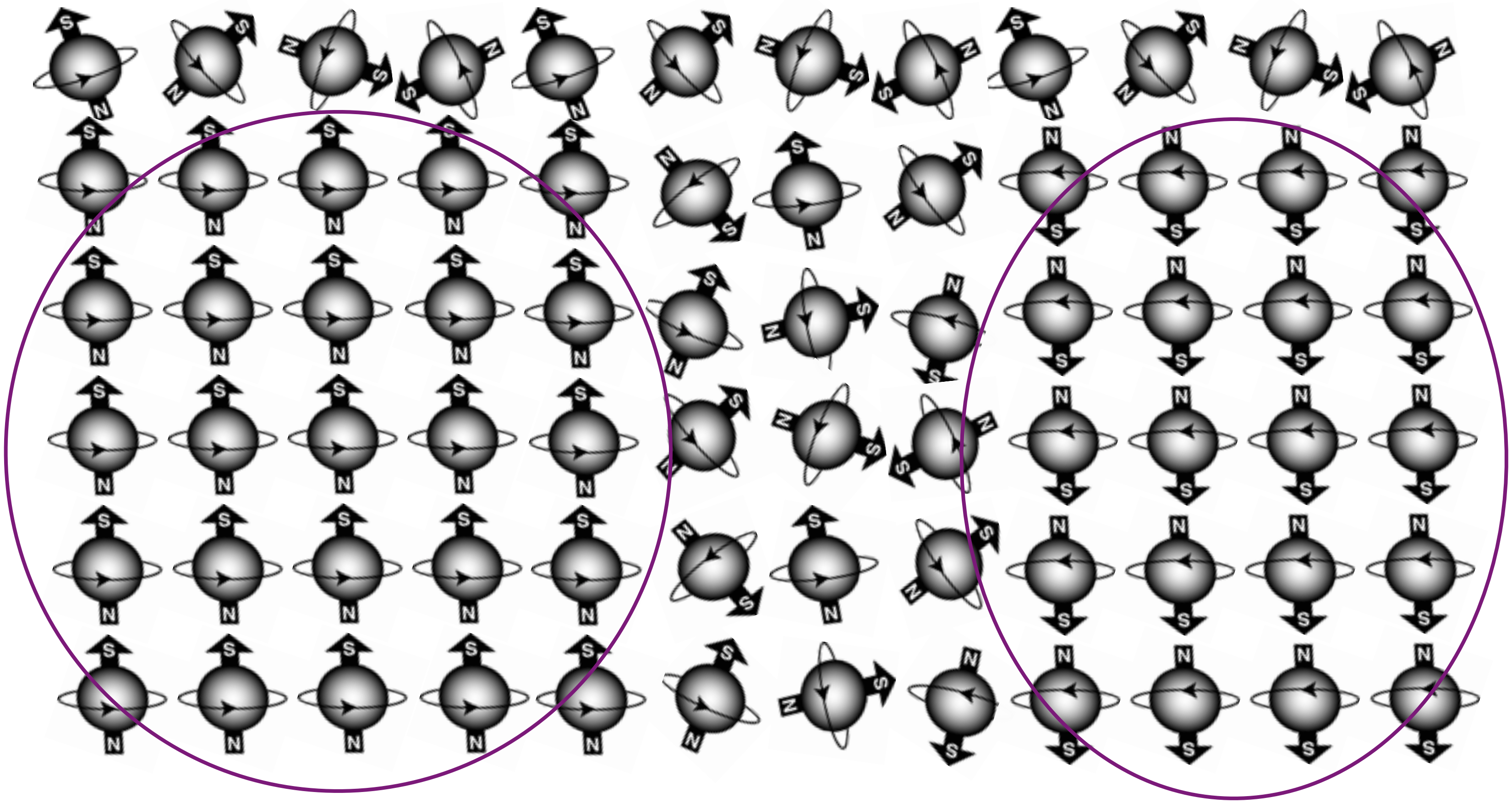
and the symmetry-lowering phase transition is described by a double well potential:



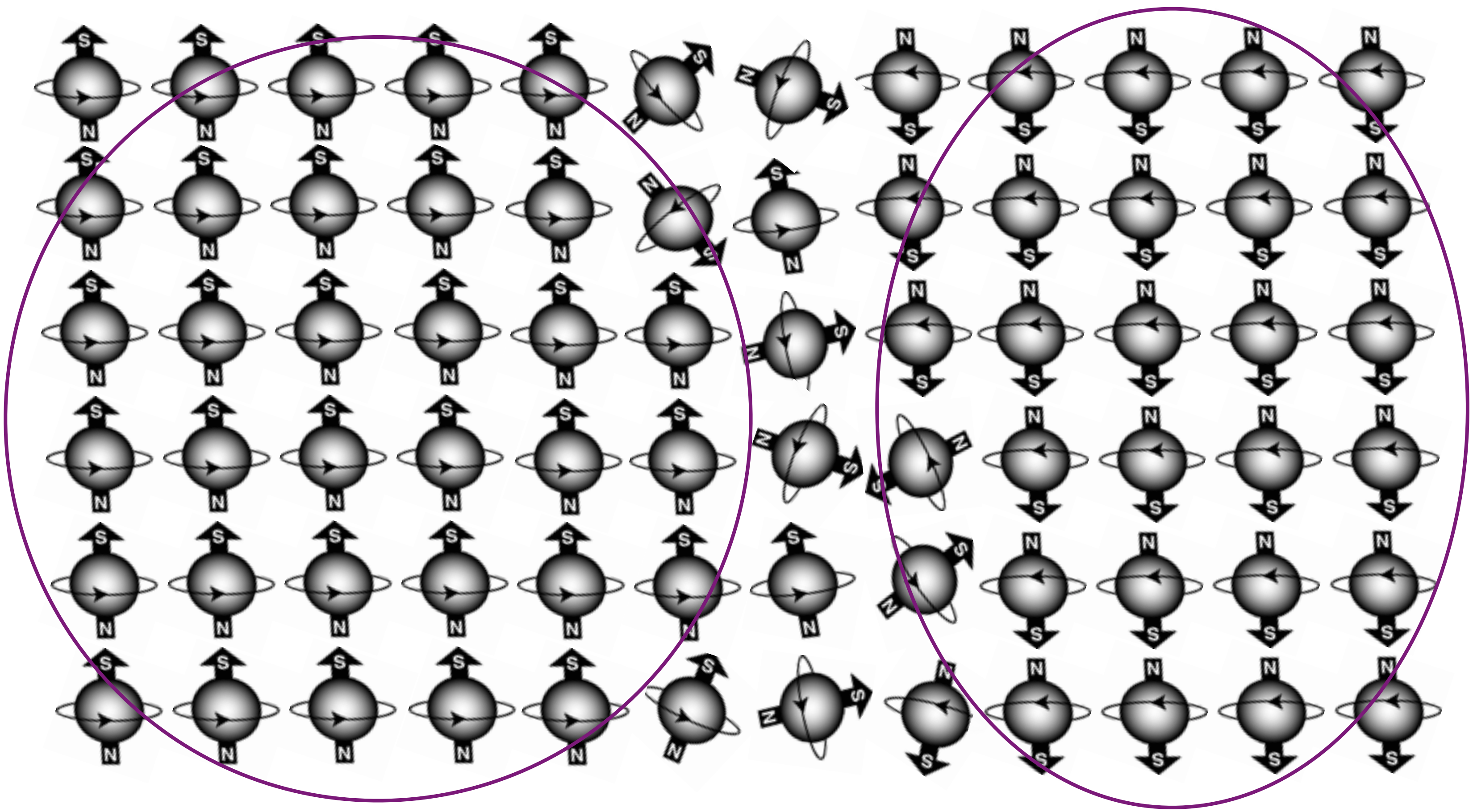
Defect formation at symmetry-lowering phase transitions



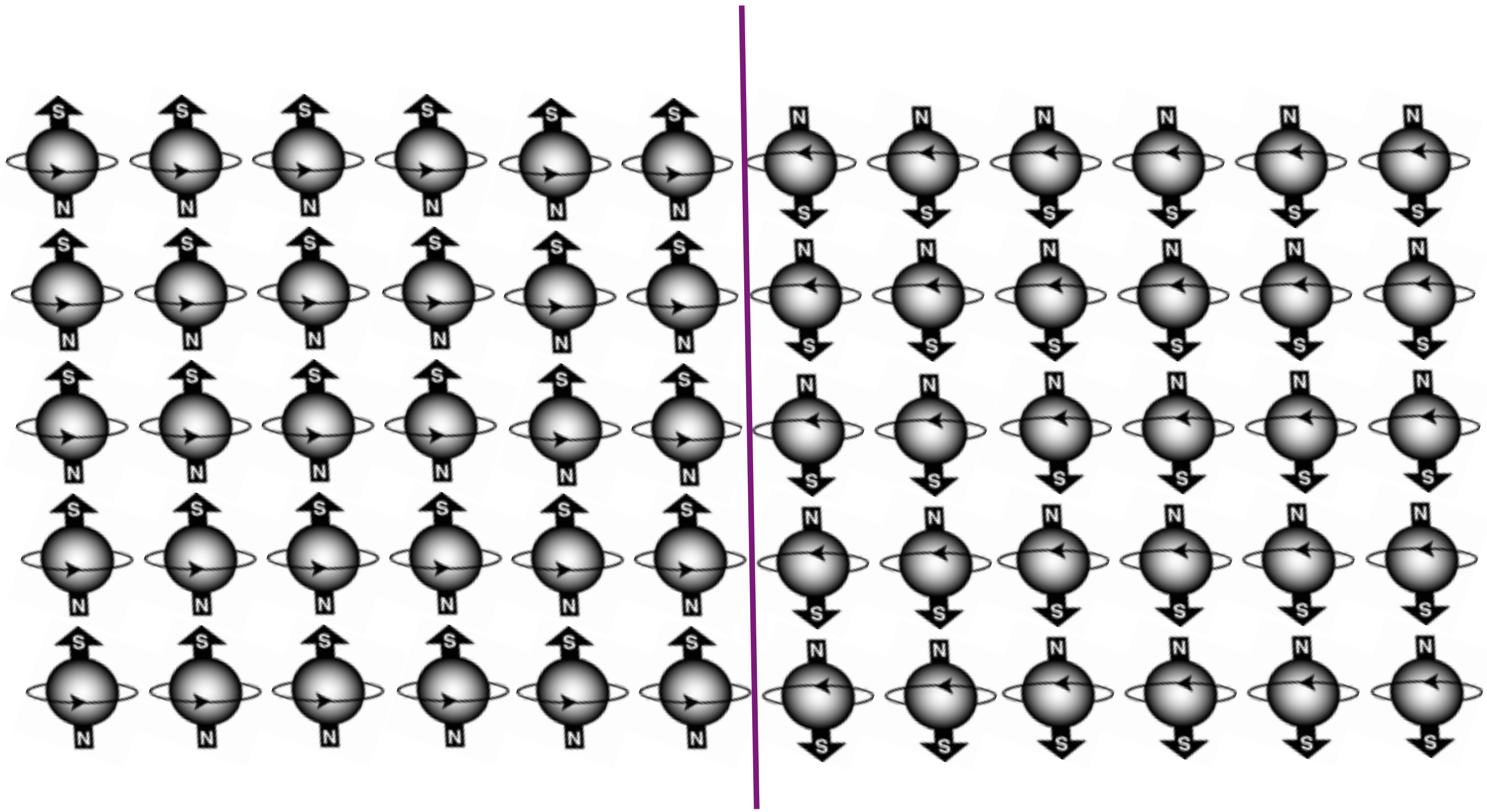
Defect formation at symmetry-lowering phase transitions



Defect formation at symmetry-lowering phase transitions

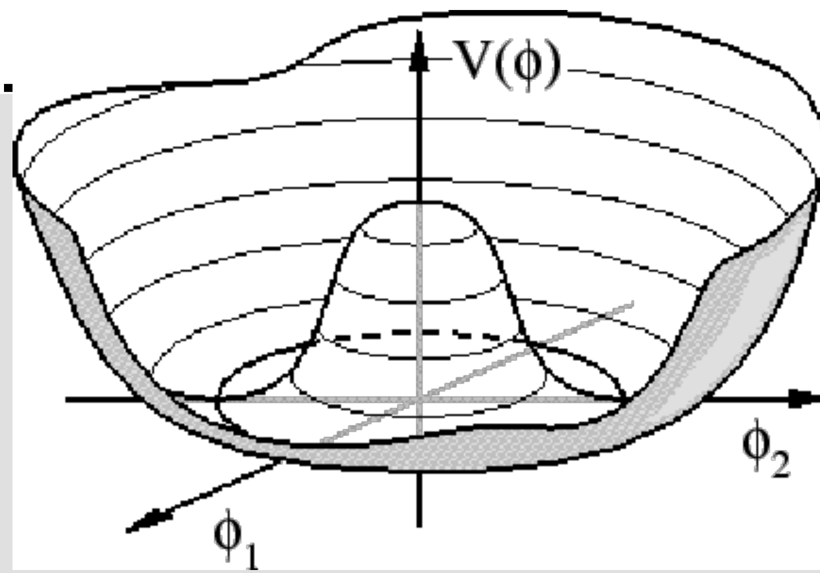


Defect formation at symmetry-lowering phase transitions



Symmetry lowering at the Grand Unification Transition

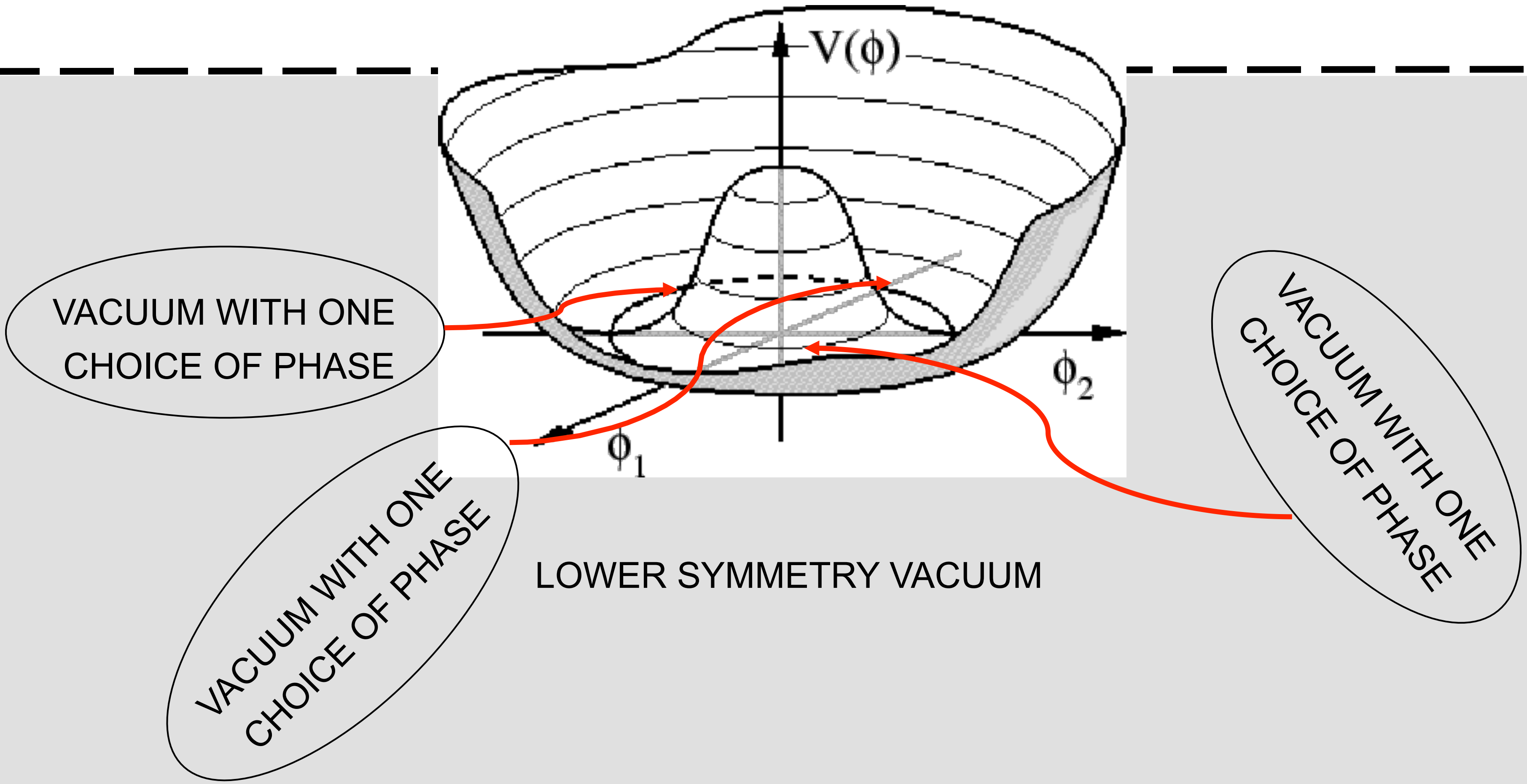
HOMOGENEOUS VACUUM



LOWER SYMMETRY VACUUM

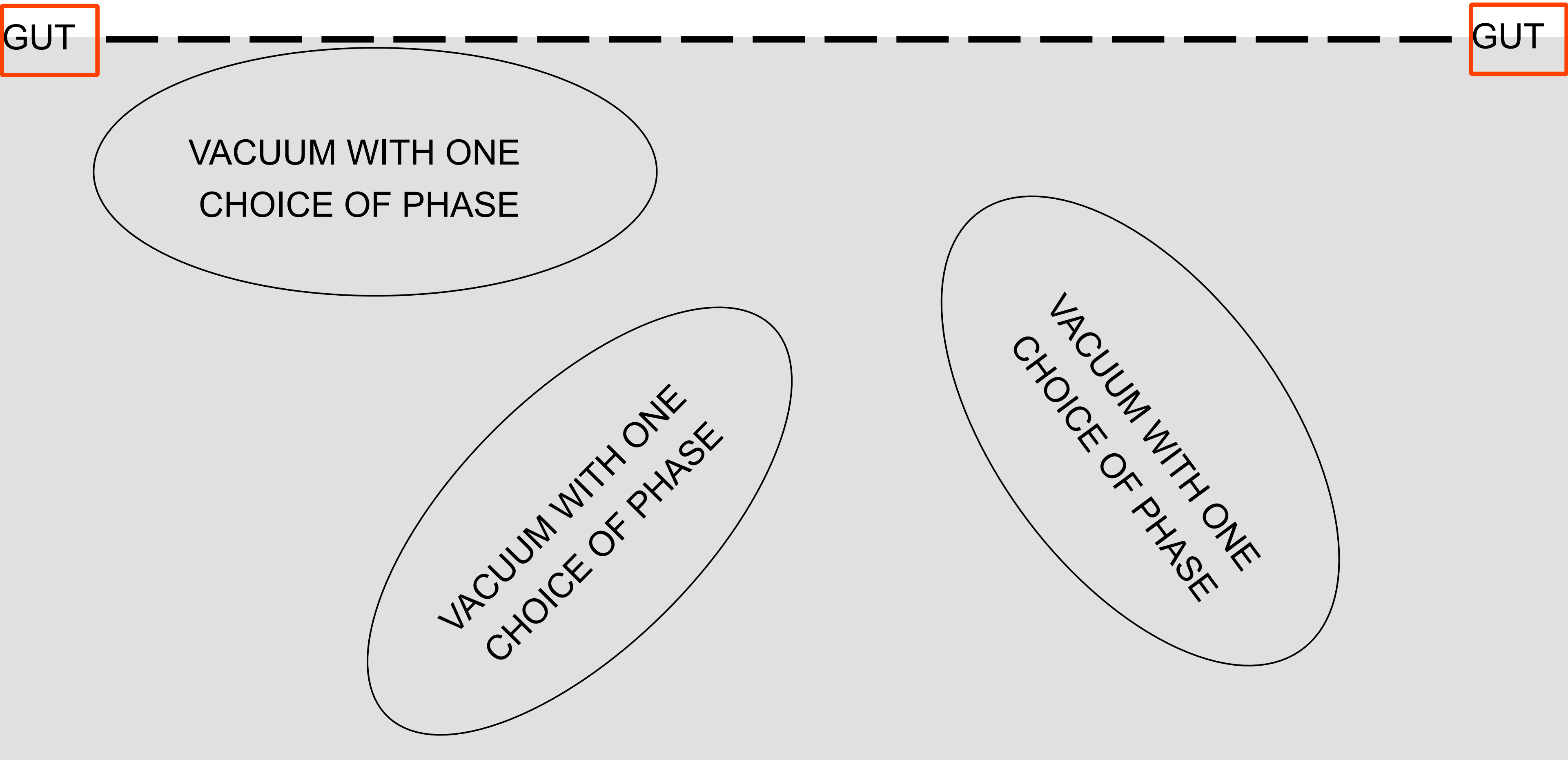
Symmetry lowering at the Grand Unification Transition

HOMOGENEOUS VACUUM

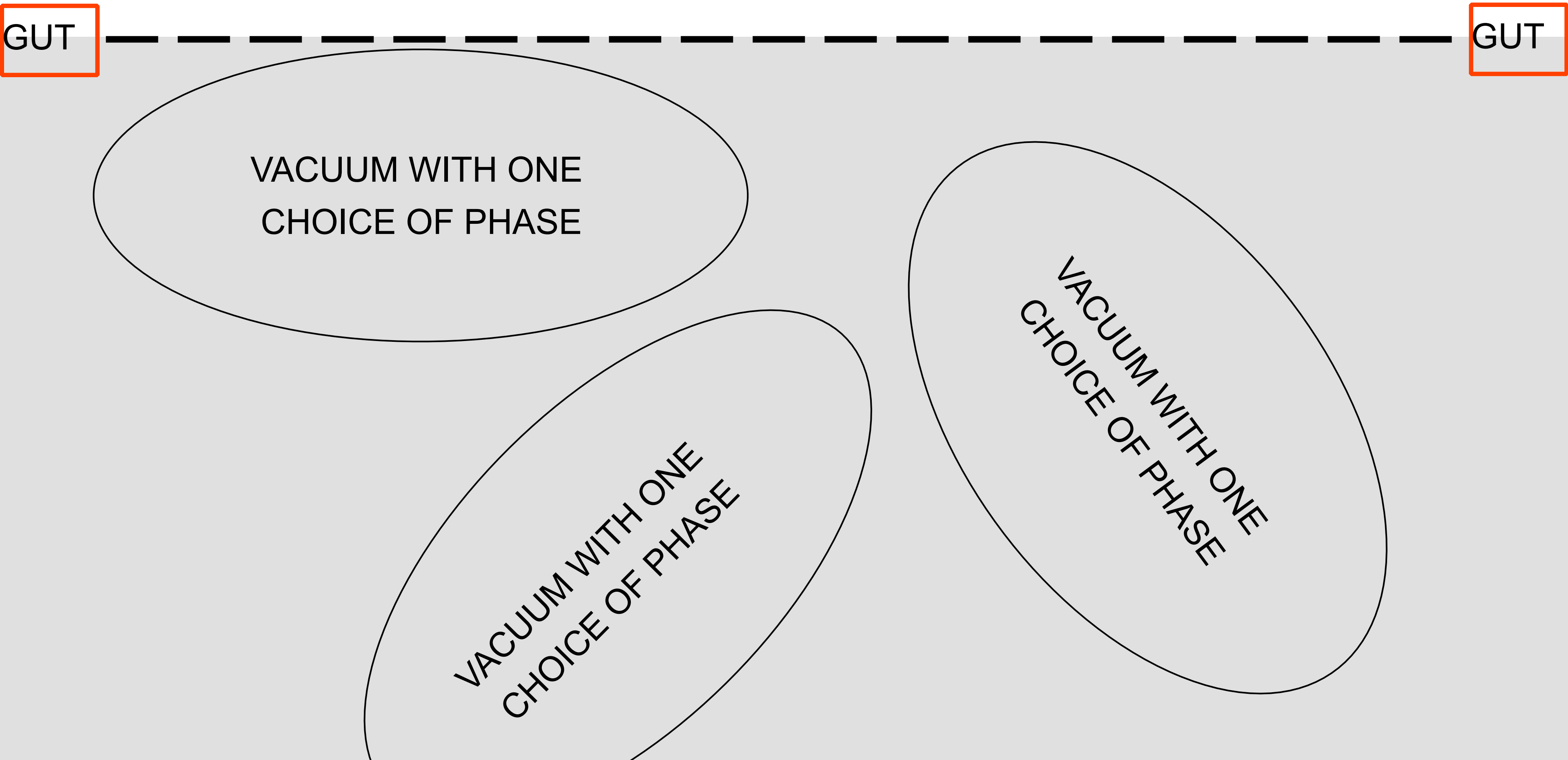


LOWER SYMMETRY VACUUM

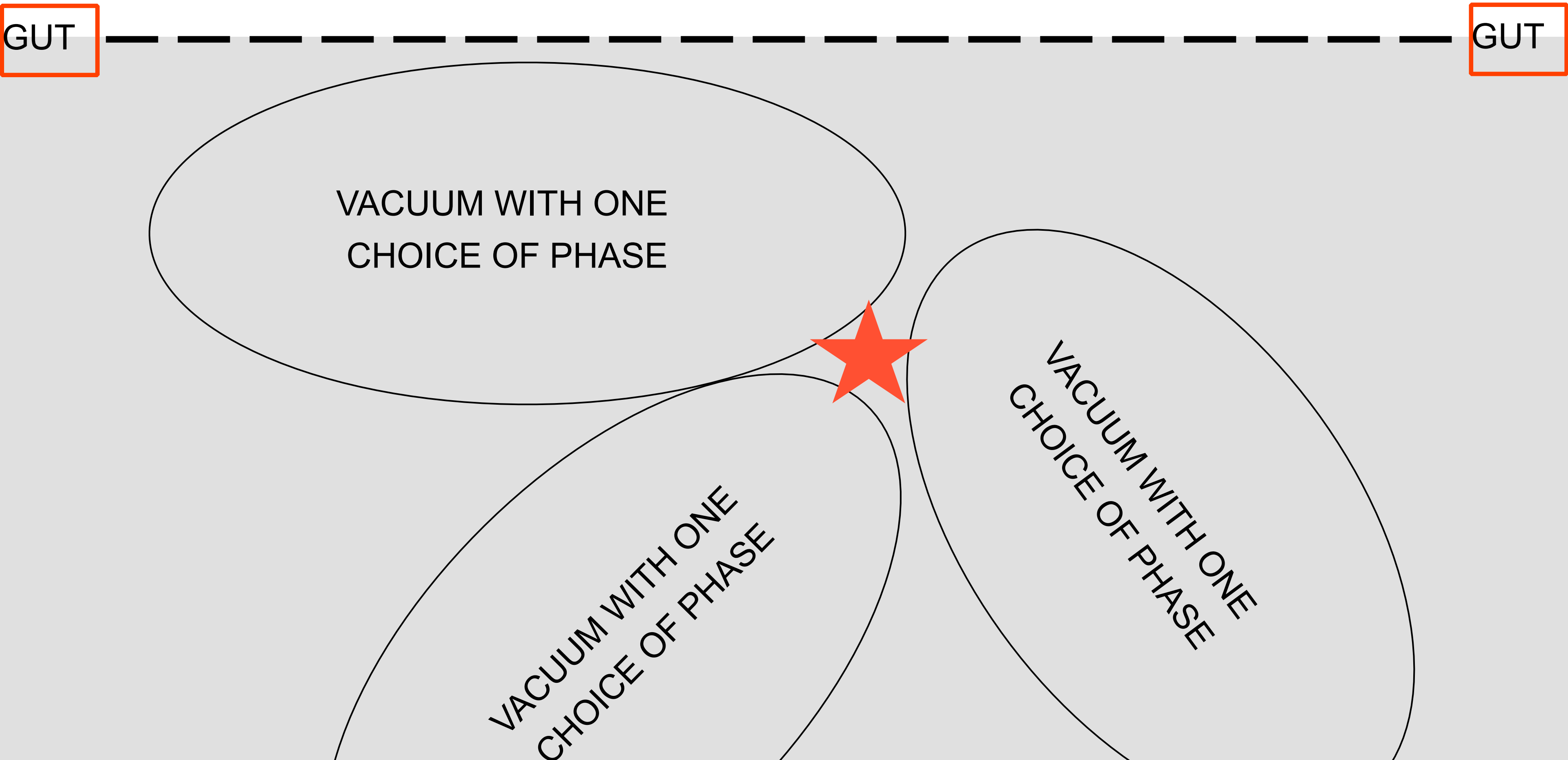
As the universe expands through the transition, the low symmetry regions grow…



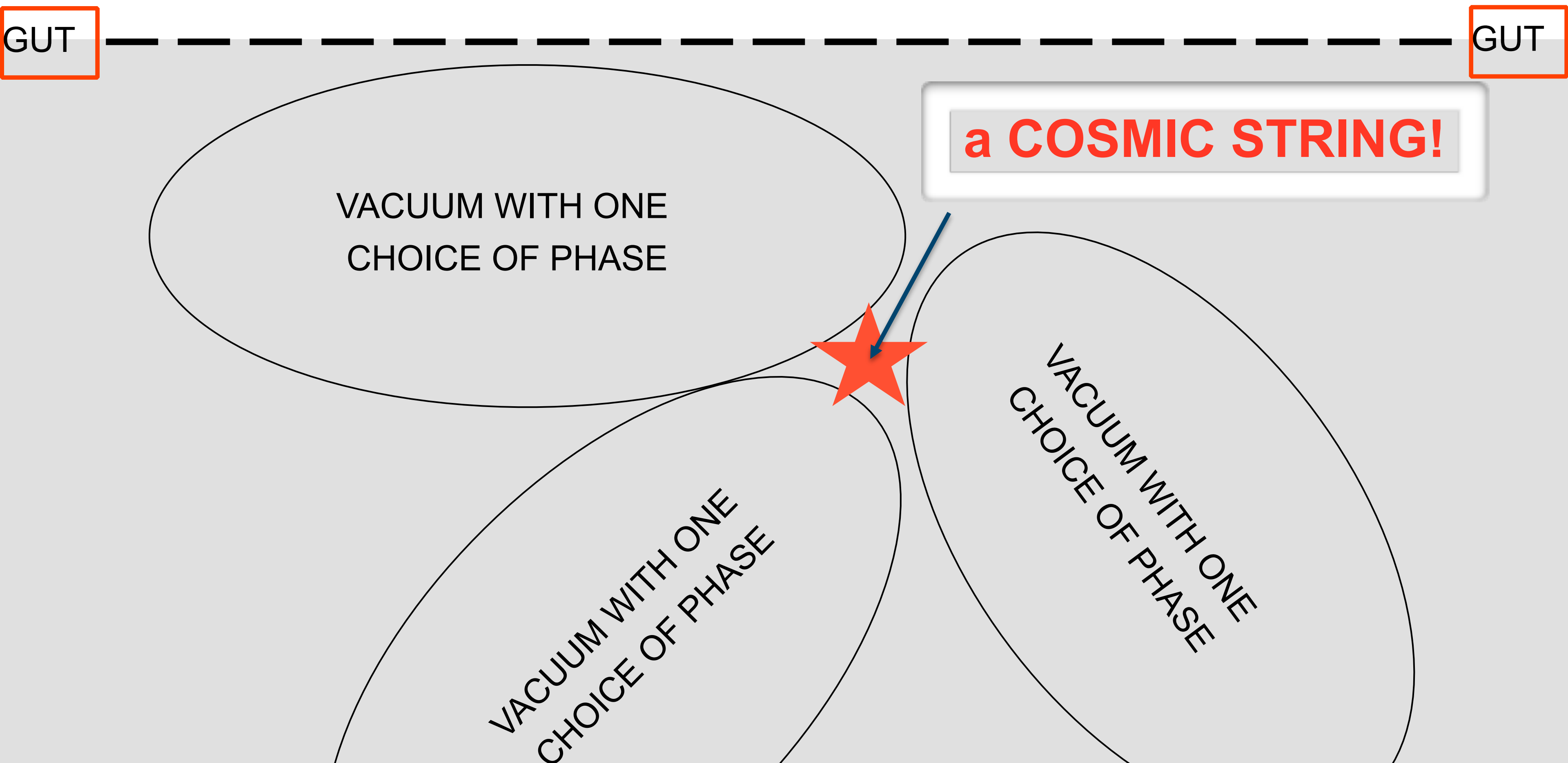
and grow…


GUT
GUT
VACUUM WITH ONE CHOICE OF PHASE
VACUUM WITH ONE CHOICE OF PHASE
VACUUM WITH ONE CHOICE OF PHASE

and eventually meet!



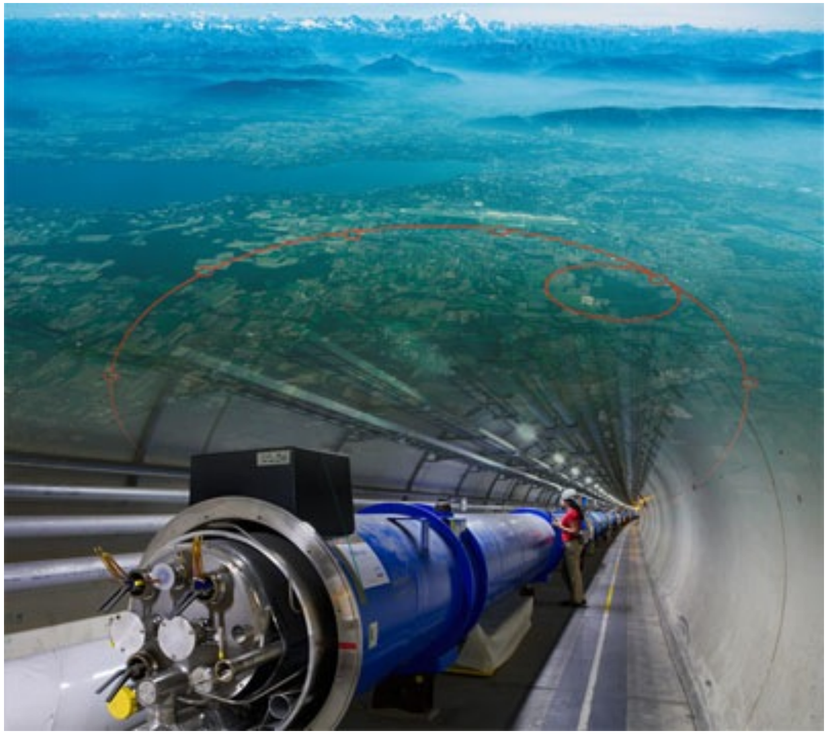
The phase mismatch in the vacuum is a *topologically protected defect* with energy and mass



Do cosmic strings exist? How can we study them?

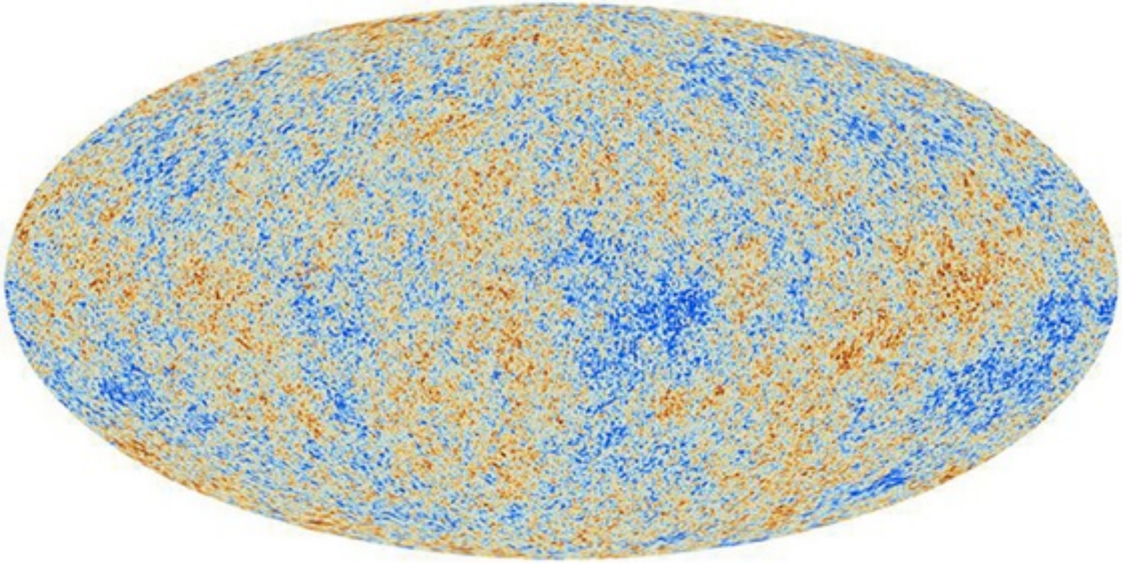
For direct study we need a probe with a similar energy, ~10^{15} GeV

Our highest energy probes, the largest hadronic colliders reach ~1 GeV

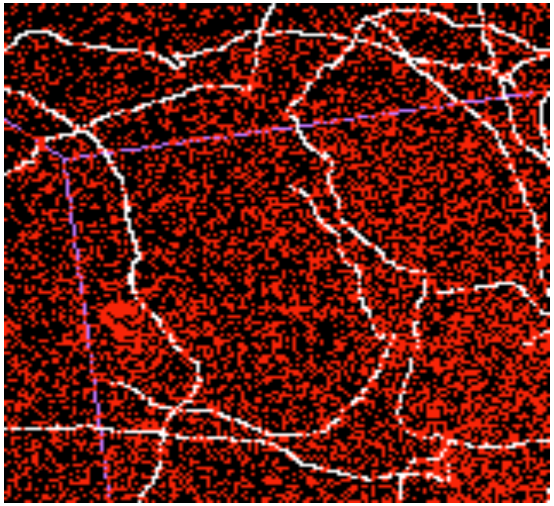


How is Cosmic String Formation at the Grand Unification Transition studied?

Analyzing the Cosmic Microwave Background



Computer Simulation



Instead we will study cosmic string formation in the lab.

What are the laboratory equivalent of cosmic strings?

Defects formed as a result of symmetry-lowering phase transitions

Plan for studying early universe processes in the lab.

First will identify a material with a symmetry-lowering phase transition described by the same mathematics as that proposed for the GUT

spontaneous symmetry breaking *described by a non-trivial homotopy group*

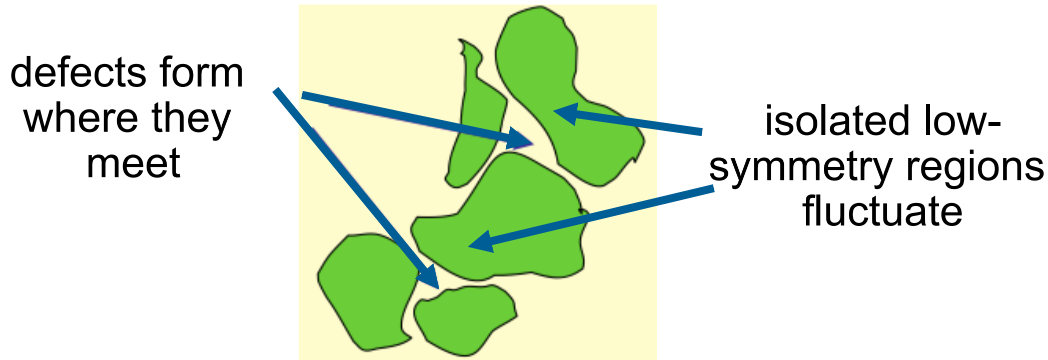
Results in formation of *topologically-protected defects* (Kibble)

Then we will do experiments on the material to test the behavior predicted for the GUT!

In particular the proposed *scaling laws* – the number of topologically-protected defects formed as a function of cooling rate

Kibble-Zurek scenario

Physics of a "GUT-like" transition (Zurek scaling):



Size of the resulting domains set by competition between:

Speed of information propagation

Rate of cooling through the transition

Cool slowly: Different regions can communicate their choice of phase

→ **Large regions of the same choice**

→ **Low density of defects**

Cool quickly: Not much time to communicate choice of phase

→ **Many smaller regions with different choice of phase**

→ **High density of defects**

Kibble-Zurek scaling law for defect formation

domain size (for linear quench)

defined as Tc / cooling rate

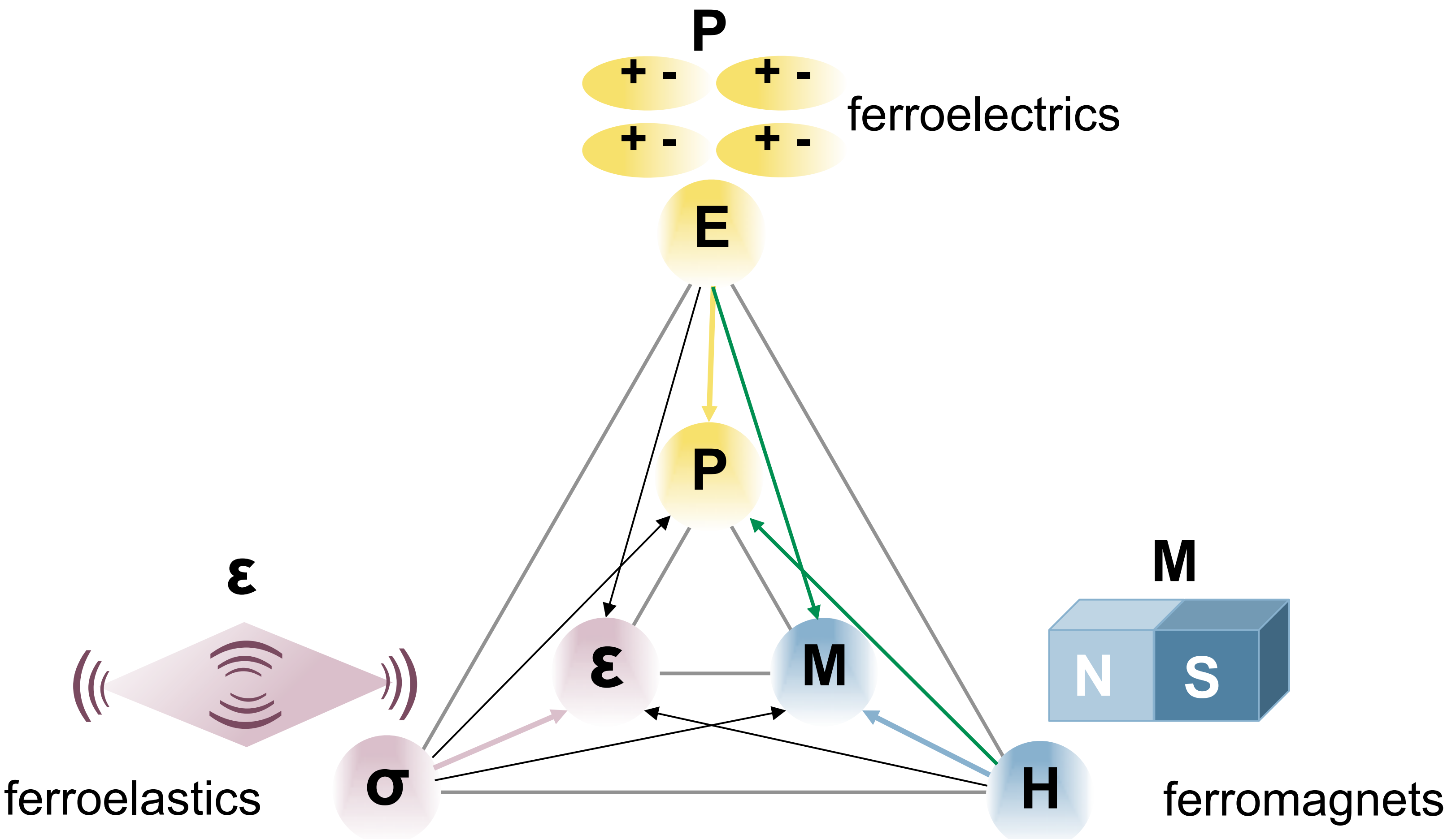
$$d = \xi_0 \left(\frac{\tau_q}{\tau_0} \right)^{\frac{\nu}{1+\mu}}$$

Critical exponents

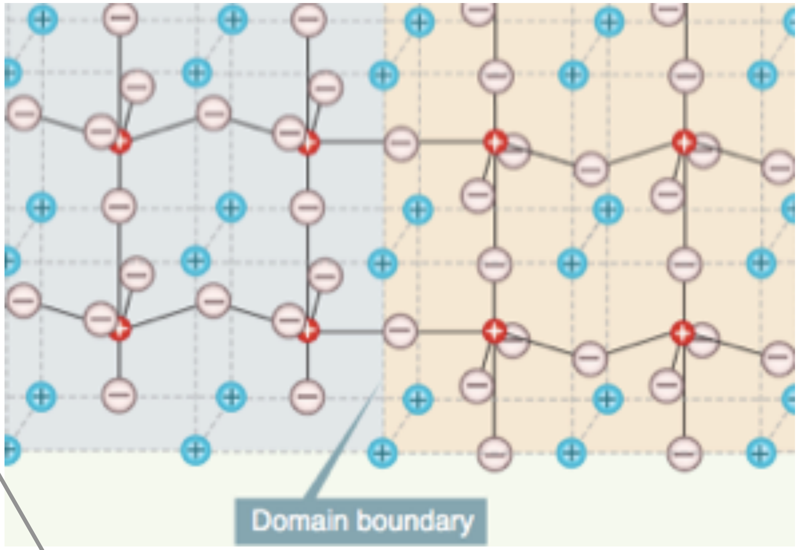
zero-temperature correlation length

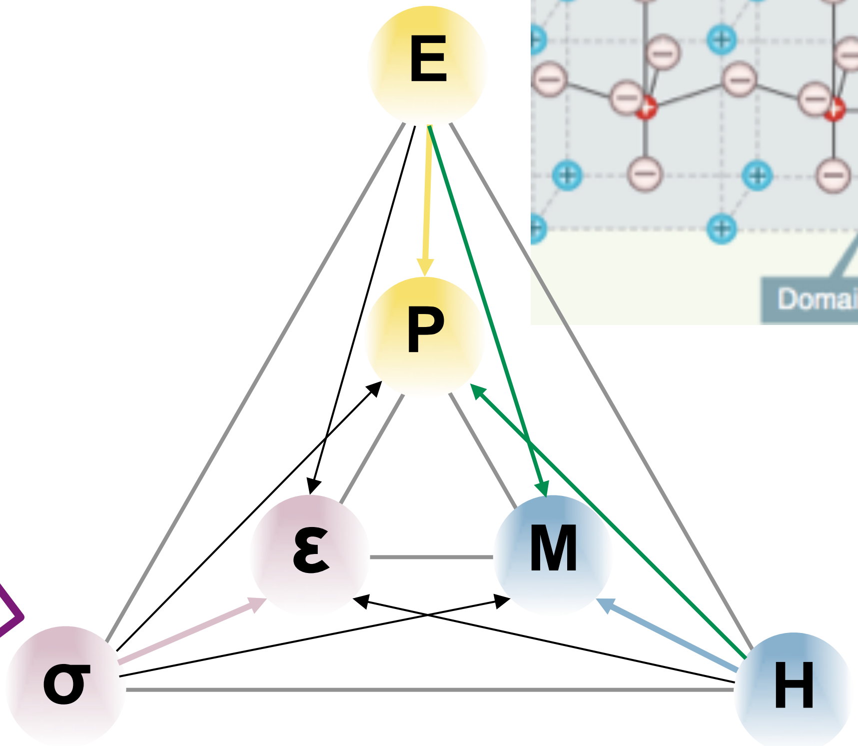
zero-temperature relaxation time = ξ_0 / speed of information transfer

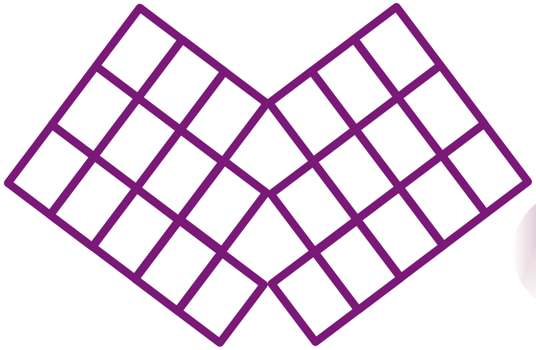
Multiferroics: Multiple ferroic orders…

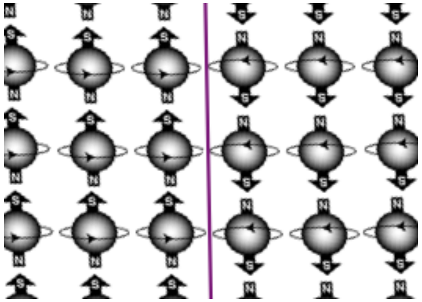


…and multiple defects from spontaneous symmetry-lowering transitions



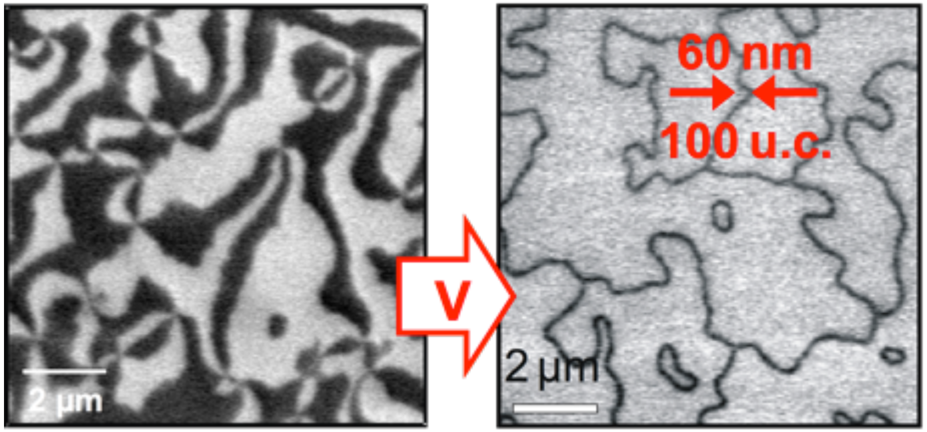






Our candidate multiferroic: $YMnO_3$

The ferroelectric domain intersections are somehow "protected":



Perhaps they are mathematically topologically protected too?

Outline

Use electronic structure calculations to understand the phase transition in $YMnO_3$

Check mathematically whether it meets the Kibble requirements

Use electronic structure calculations to calculate how many topologically protected defects should be formed as a function of cooling rate

Measure how many topologically protected defects are formed as a function of cooling rate

Does a system that is described by the same physics and symmetry as the GUT exhibit the predicted Kibble-Zurek behavior?!

The Materials Theory group at ETH

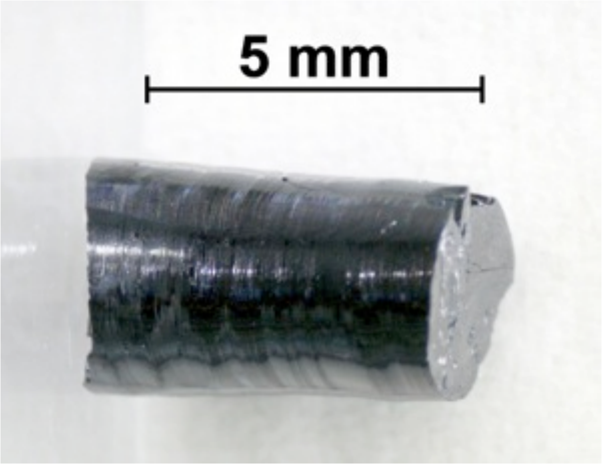
Yu Kumagai

Sinead Griffin

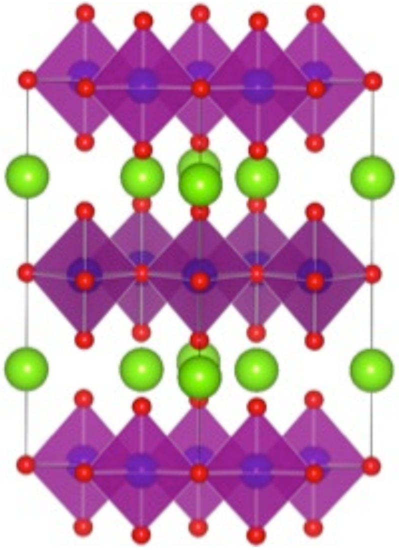
The Multifunctional Ferroic Materials group at ETH

Manfred Fiebig and Martin Lilienblum

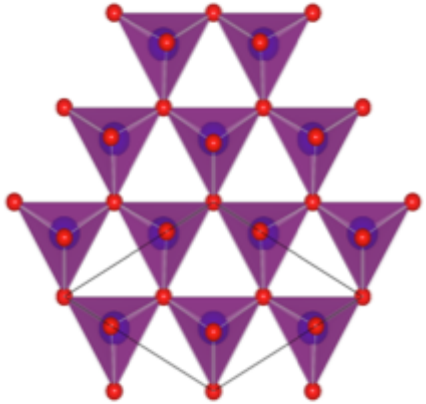
Our material: Multiferroic $YMnO_3$



Side view



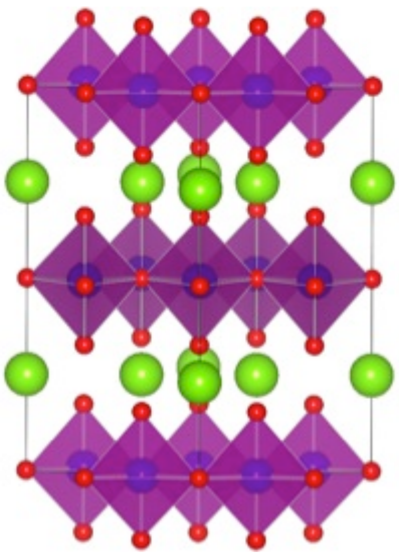
Top view

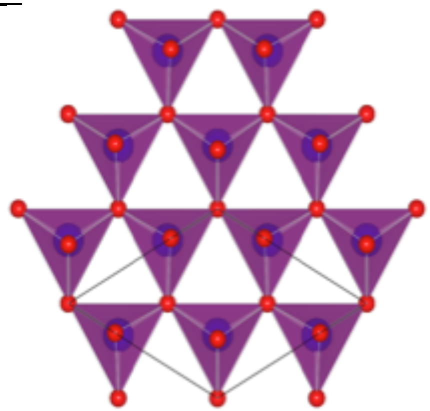


Structural phase transition in $YMnO_3$

High temperature

$P6_3/mmc$



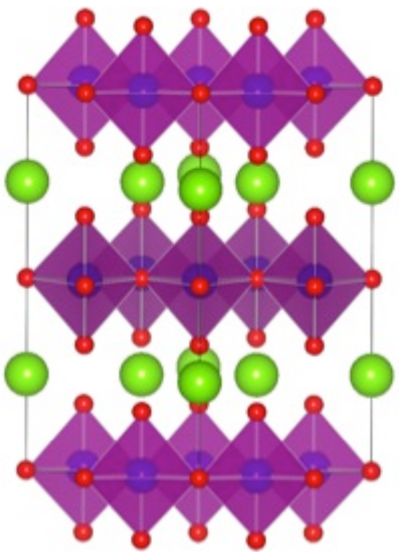


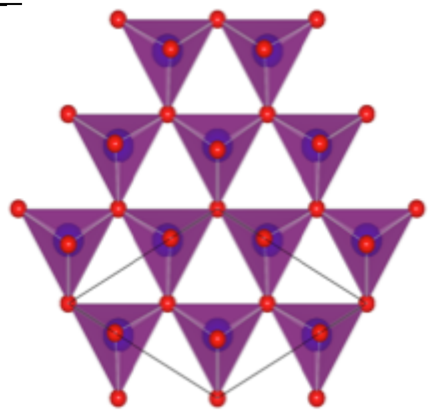
paraelectric

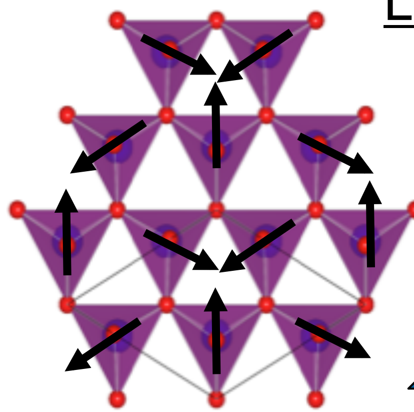
Structural phase transition in $YMnO_3$

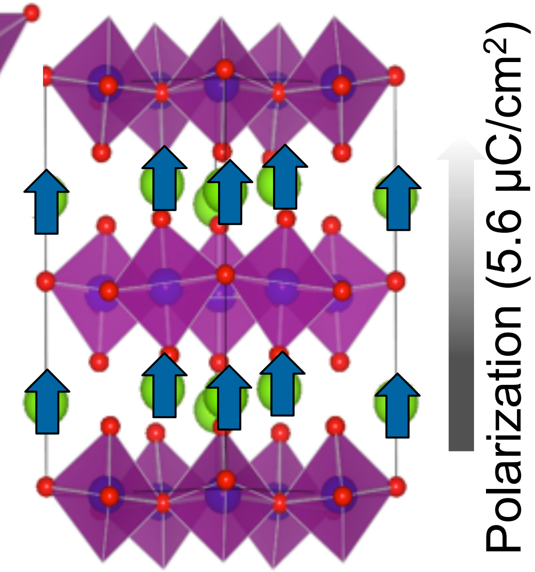
High temperature | ~1000K | Low temperature

$P6_3/mmc$ | $P6_3cm$







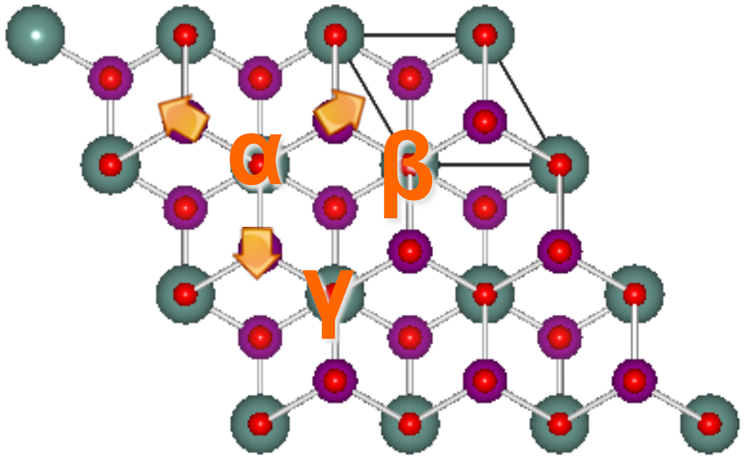


paraelectric

ferroelectric

Look at these distortions in more detail:

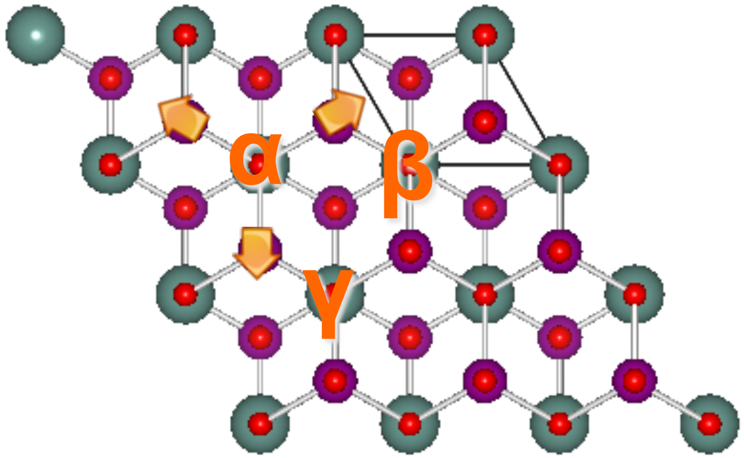
Trimerization / Tilting



Three possible origins

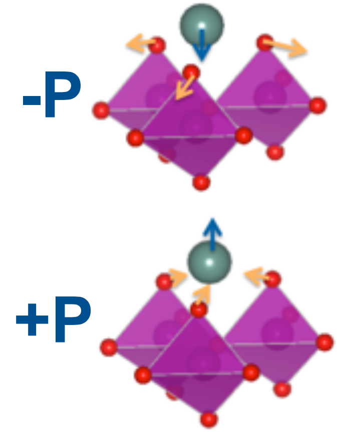
Look at these distortions in more detail:

Trimerization / Tilting



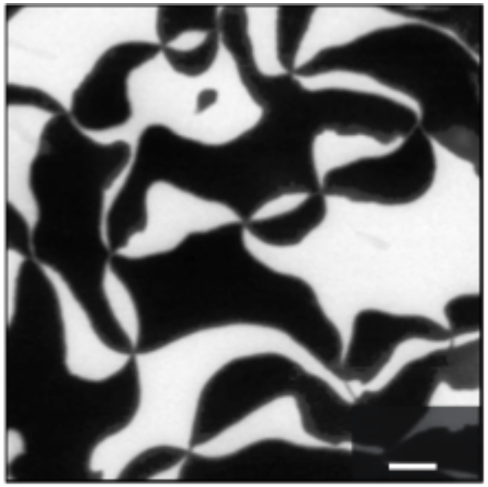
Three possible origins

Polarization



Two possible orientations

Results in six domains



Formalize with symmetry analysis and DFT

Landau free energy

$$f_{\mathrm{u}} = \frac{a}{2}Q^2 + \frac{b}{4}Q^4 + \frac{Q^6}{6}(c + c' \cos 6\Phi) - gQ^3 P_z \cos 3\Phi$$

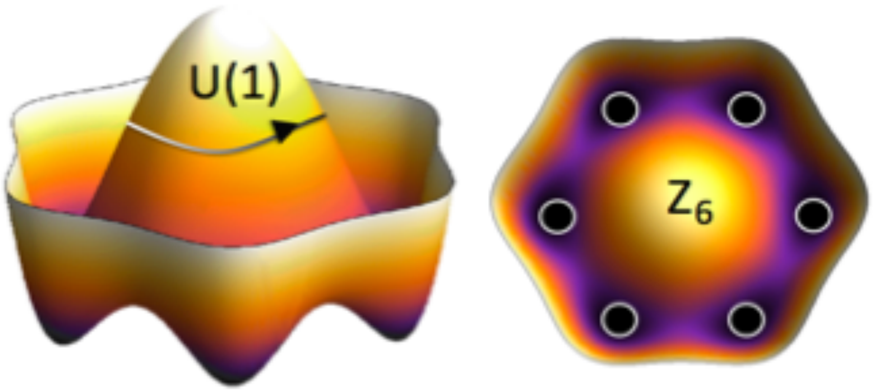
Q is amplitude of trimerization mode

Φ is phase of trimerization mode

P_z is polarization

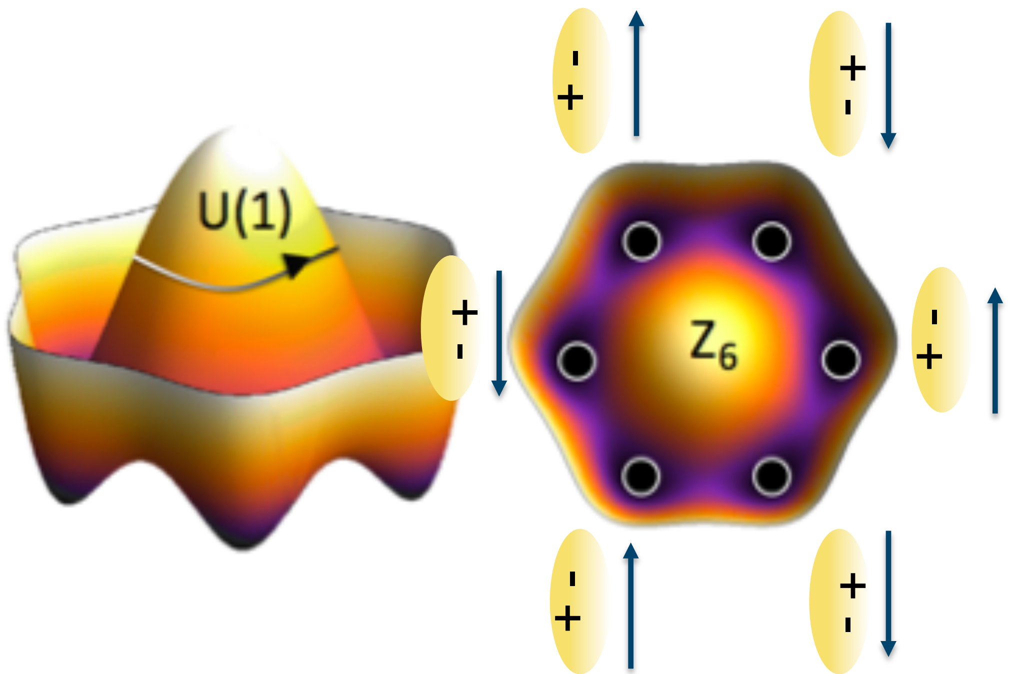
The ferroelectricity is *improper*

The phase transition is described by a Mexican-hat-like potential!



S. Artyukhin, K.T. Delaney, NAS and M. Mostovoy, *Landau theory of topological defects in multiferroic hexagonal manganites*, Nature Materials 13, 42 (2013)

And the details of our "Mexican Hat-like" potential make it easy for us to measure!

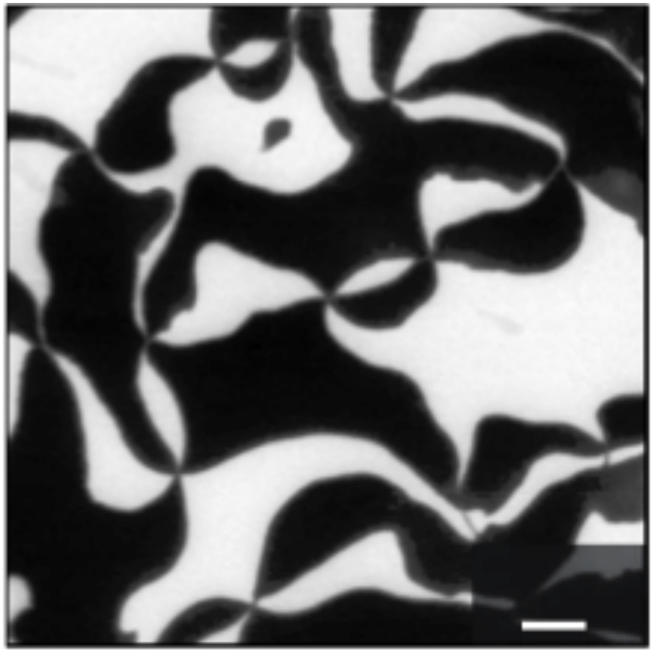


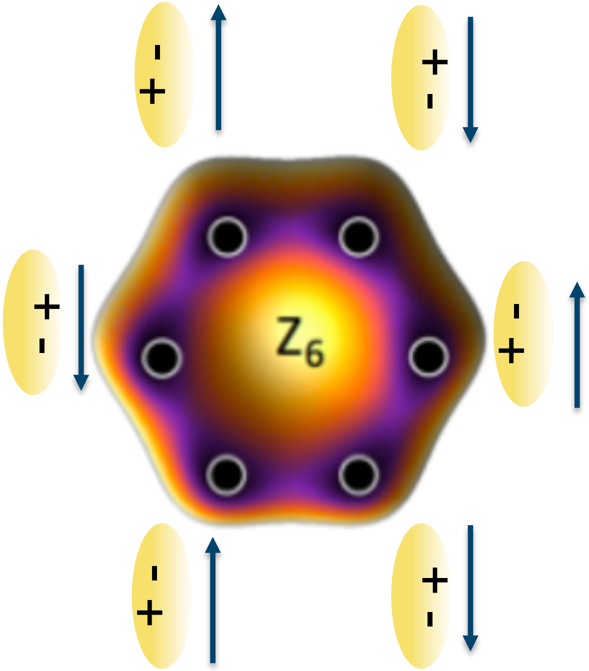
S. Artyukhin, K.T. Delaney, NAS and M. Mostovoy, *Landau theory of topological defects in multiferroic hexagonal manganites*, Nature Materials, 13, 42 (2014)

S. Griffin, M. Lilienblum, K. Delaney, Y. Kumagai, M. Fiebig and NAS, *Scaling behaviour and beyond equilibrium in the hexagonal manganites*, PRX **2**, 041022 (2012)

And the details of our "Mexican Hat-like" potential make it easy for us to measure!

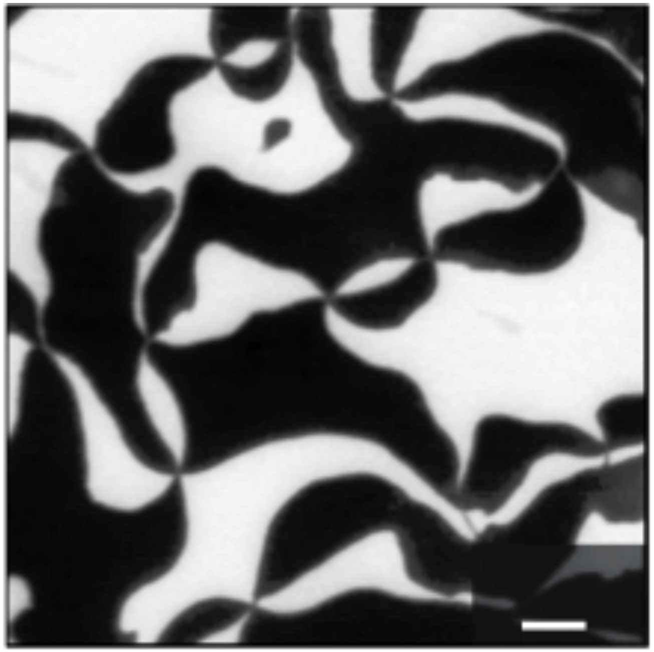
Piezoforce Microscopy Image of the Defects in $YMnO_3$



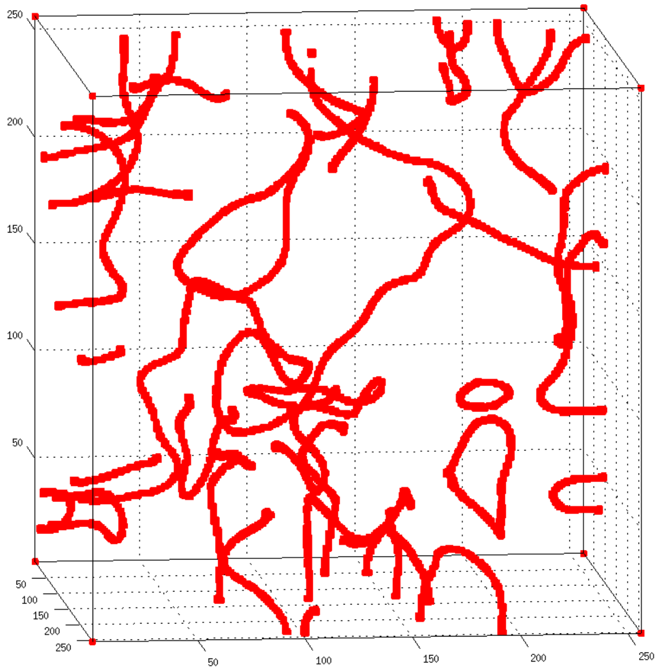


The meeting points of the ferroelectric domains are in fact one-dimensional "strings"

Piezoforce Microscopy Image of the Defects in $YMnO_3$



3-D Simulation



One last requirement:

The phase transition must be described by a non-trivial homotopy group (for the defects to be topologically protected)

When are two objects homotopic?

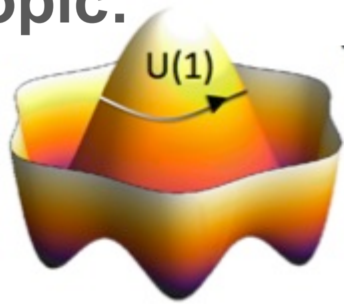
✔ Allowed: Stretch, twist, bend, pull, shrink, enlarge, squeeze

✗ Not allowed: Cutting, pasting, or volumes shrinking to zero

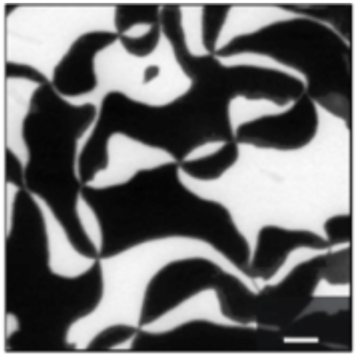
Homotopic

Not Homotopic

Fortunately there exists a "recipe" for working out whether a phase transition of continuous symmetry is homotopic:



1) Map the symmetry characteristic of the order parameter – U(1) – onto an n-dimensional sphere: one-dimensional circle S^1
2) Look up in a topology textbook whether the homotopy group π of S^1 is non-trivial or not
3) In fact $\pi_1(S^1)$ is non-trivial and produces 1-dimensional topologically protected singularities – these are the lines of intersection of the domains



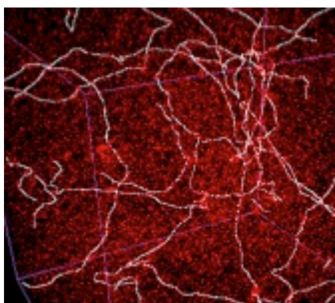
The domain intersections in $YMnO_3$ are formally mathematically topologically protected

The structural phase transition in multiferroic $YMnO_3$ fulfills the Kibble requirements!

spontaneous symmetry breaking described by a non-trivial homotopy group

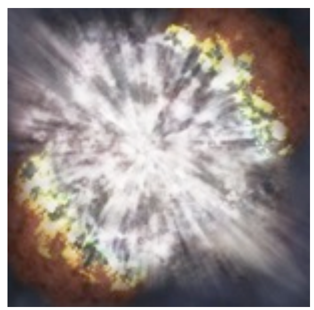
Results in formation of *topologically-protected defects*

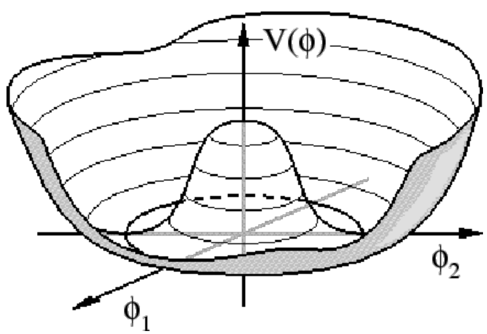
Early Universe



Low symmetry vacuum

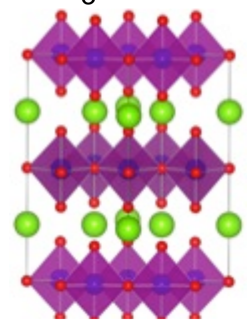
High symmetry vacuum

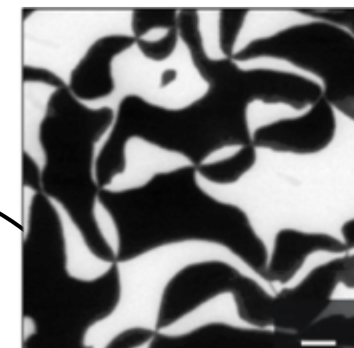


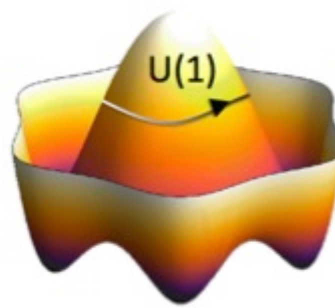


$YMnO_3$

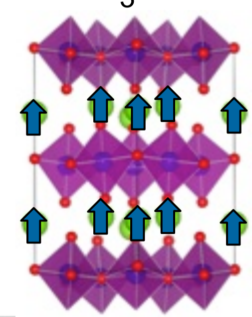
P6₃/mmc







P6₃cm

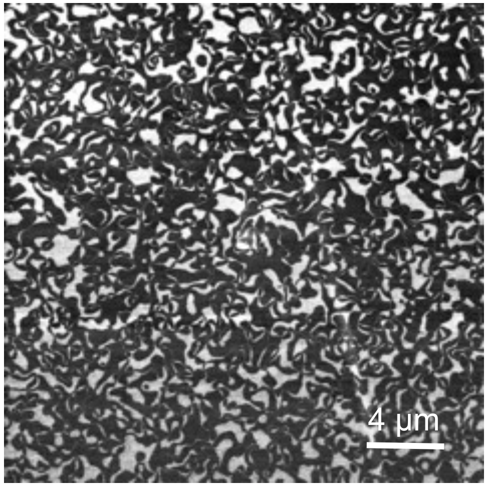


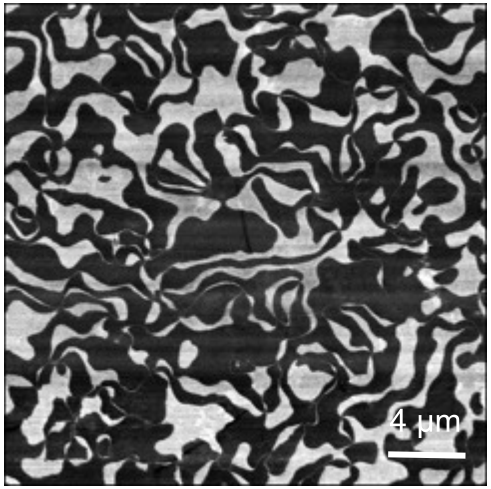
Does it behave as cosmologists predict?

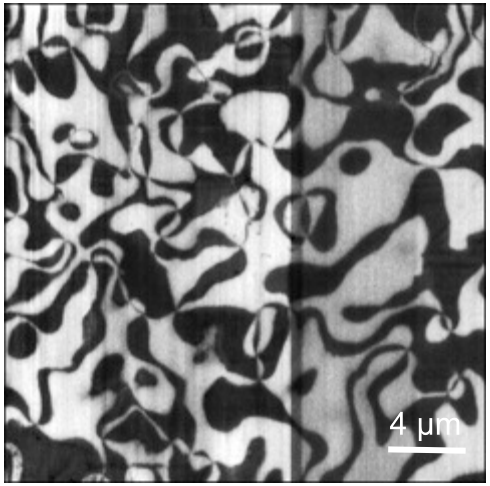
What experiment would we like to do on the early universe?

We'd like to expand it at different rates, crossing the GUT, and see how many cosmic strings form in each case

Instead we will cool $YMnO_3$ at different rates through the structural phase transition and count how many domain intersections form







Testing the Kibble-Zurek scaling law for defect formation

domain size,

defined as Tc (1400K) / cooling rate

$$d = \xi_0 \left(\frac{\tau_q}{\tau_0} \right)^{\frac{\nu}{1+\mu}}$$

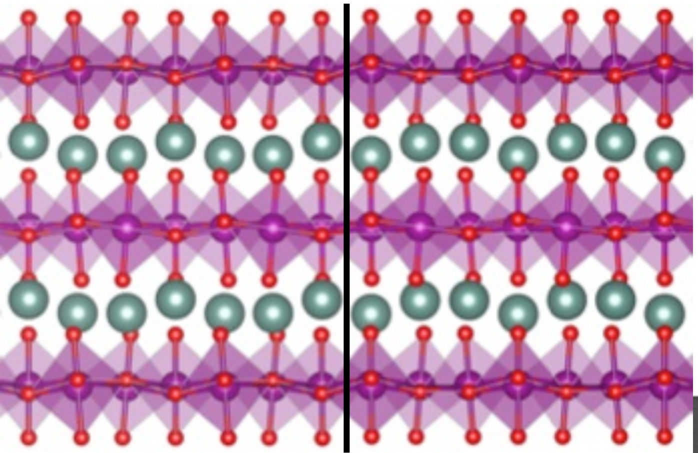
Critical exponents: $\nu = 0.6717$

$\mu = 1.3132$

from MC simulations for 3D XY model

M. Campostrini et al., Phys. Rev. B **74**, 144506 (2006)

zero-temperature correlation length ~ domain wall width in ferroelectrics (DFT)



zero-temperature relaxation time = ξ_0 / speed of sound

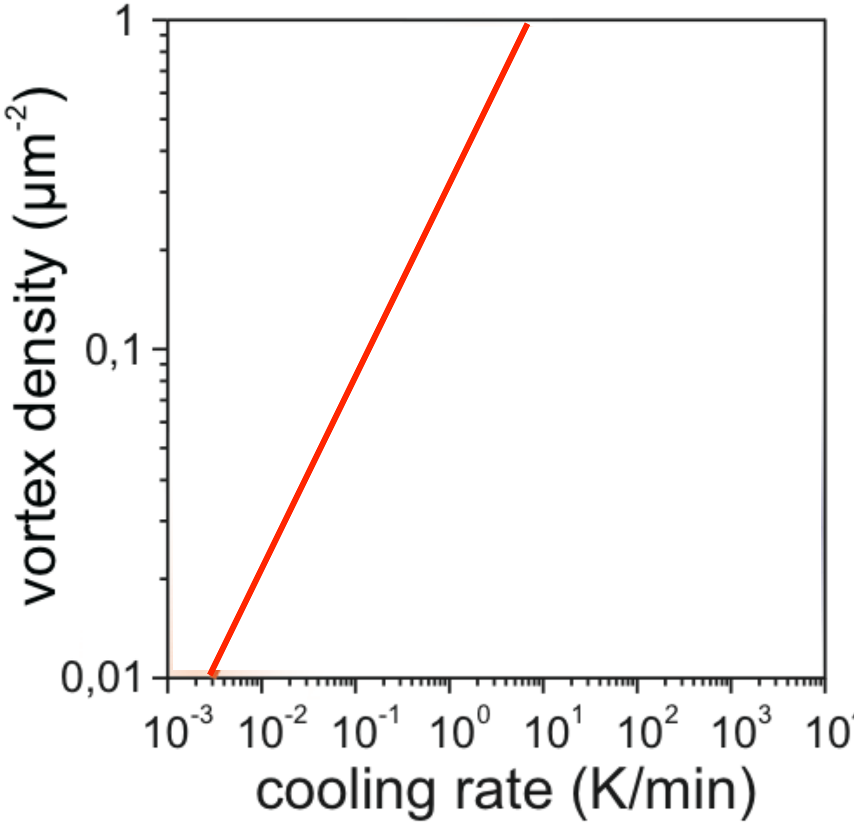
speed of sound = 640 m/s (DFT)

Yu Kumagai and NAS, *Structural domain walls in polar hexagonal manganites,* Nat. Comm. 4, 1540 (2013)

Comparison of KZ theory (DFT parameters) with experiment

Red line: our calculations

with ξ_0 = 0.06 A



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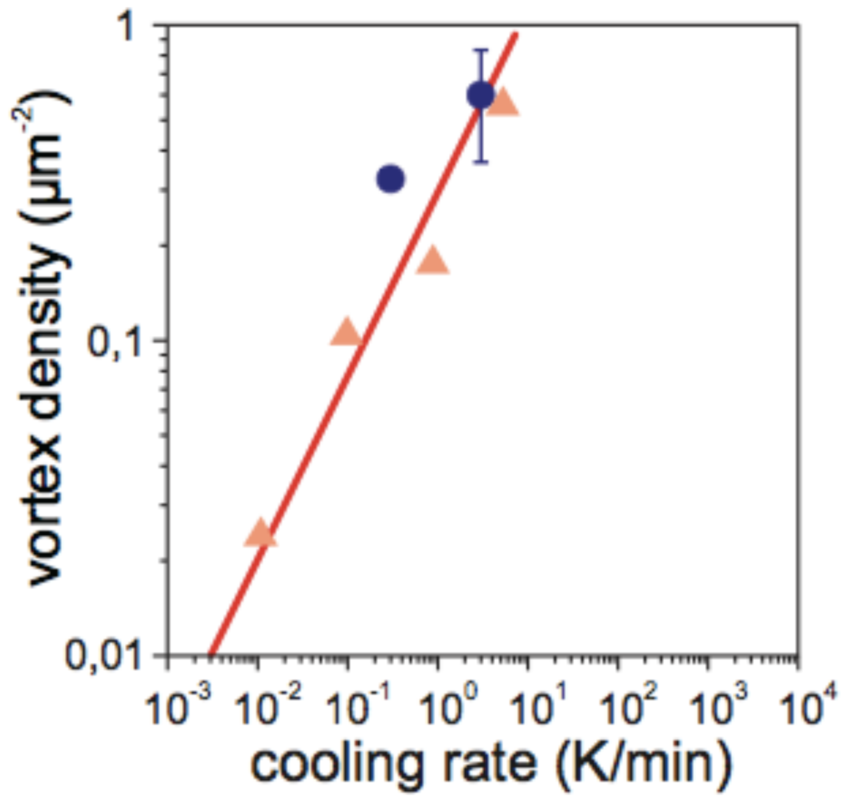
Red triangles: measured by Chae et al.

Blue circles: measured by Fiebig et al.

KIBBLE-ZUREK BEHAVIOR!



S. C.. Chae et al., *Direct observation of the proliferation of ferroelectric loop domains and vortex-antivortex pairs,* PRL **108**, 167603 (2012)

S. Griffin, M. Lilienblum, K. Delaney, Y. Kumagai, M. Fiebig and N. A. Spaldin, *Scaling behaviour and beyond equilibrium in the hexagonal manganites*, PRX **2**, 041022 (2012)

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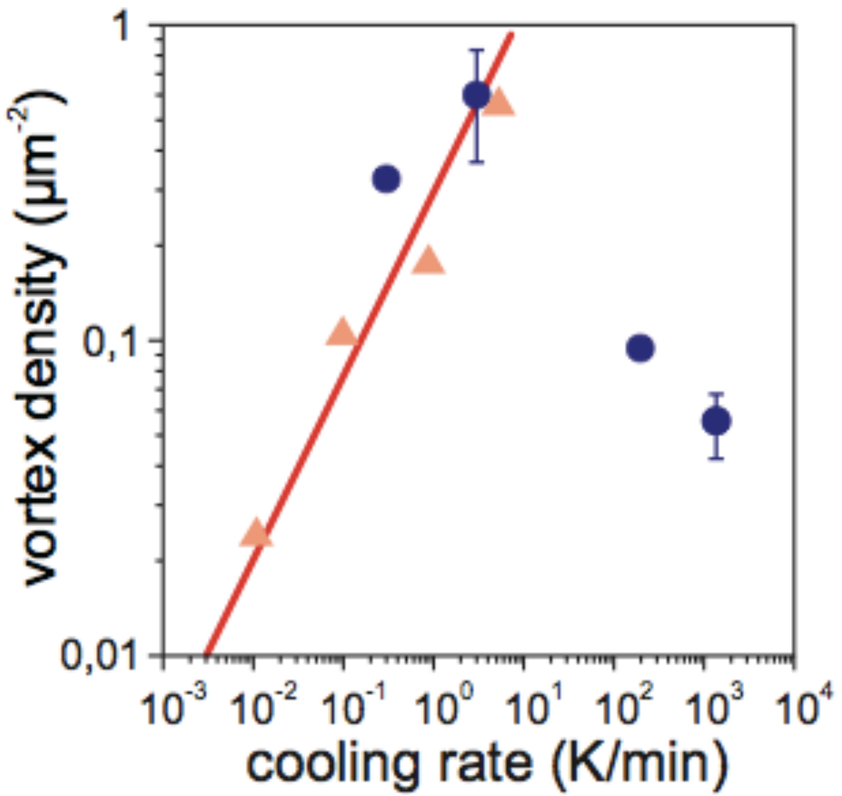
KIBBLE-ZUREK BEHAVIOR AT SLOW COOLING RATES!



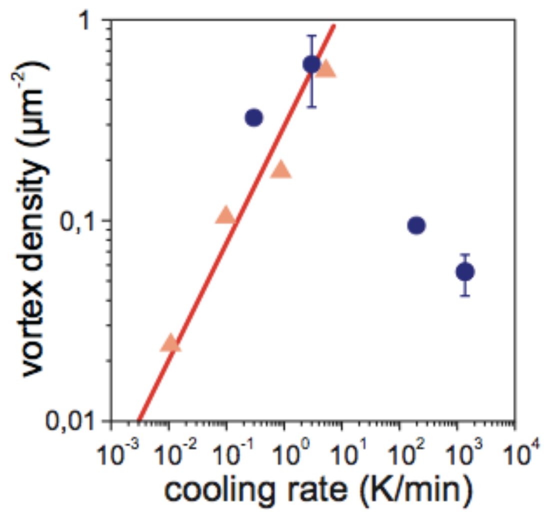
S. Griffin, M. Lilienblum, K. Delaney, Y. Kumagai, M. Fiebig and N. A. Spaldin, *Scaling behaviour and beyond equilibrium in the hexagonal manganites*, PRX **2**, 041022 (2012)

Open questions:

Are we now able to explore the “beyond-Kibble-Zurek” regime?

If so, what is the origin of the turnaround?

Or do we not have K-Z behavior at all?



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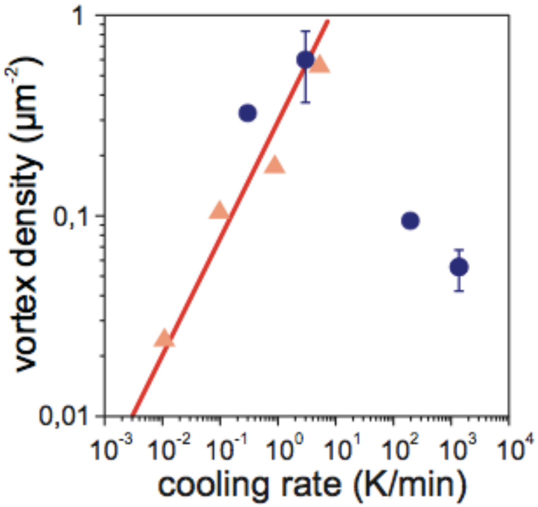

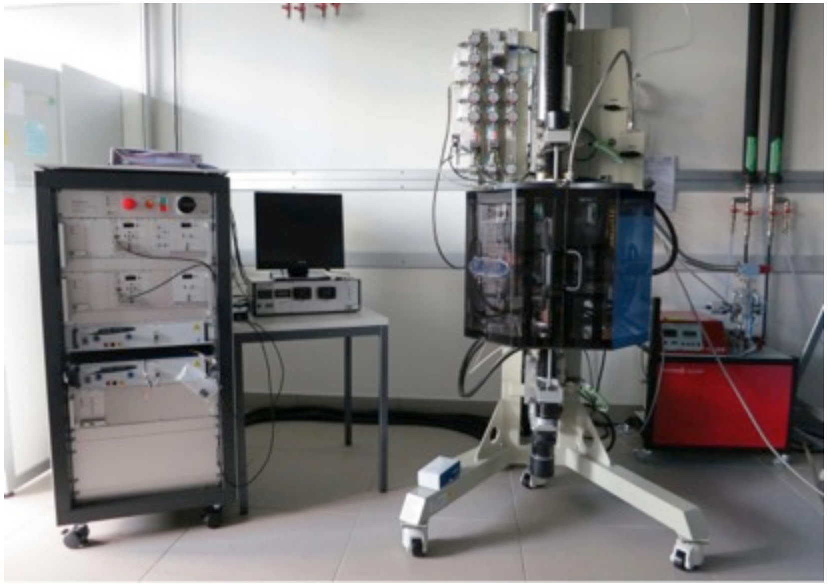

What next?

More samples / more measurements

A room-temperature system?

Early cosmic string dynamics / evolution?

Summary

$YMnO_3$ seems to provide the first example of Kibble-Zurek scaling in a condensed matter system

Cosmic strings formed the way cosmologists thought ;)

Use of real materials to explore questions in cosmology is a lot of fun

The Economist

Table-top astrophysics

How to build a multiverse

Small models of cosmic phenomena are shedding light on the real thing

Mar 16th 2013 | From the print edition

Whether all this ingenuity unravels any cosmic truth is uncertain. Cliff Burgess, a theorist at Perimeter Institute for Theoretical Physics in Ontario, has his doubts. But he thinks that such experiments are nevertheless worth pursuing. “Like tap-dancing snakes,” he says, “the point is not that they do it well, it is that they do it at all.”