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Intertwined orders in 2D strongly correlated fermion systems

"Computational Challenges in Nuclear and Many--Body Physics" Nordita program - October 2014



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LPC Caen & CRISMAT



Cuprates

Low energy effective model of electrons in CuO₂ planes

The half-filled Hubbard model

$$\hat{H} = -t \sum_{\langle \vec{r}, \vec{r}' \rangle, \sigma} \hat{c}^{+}_{\vec{r}\sigma} \hat{c}_{\vec{r}'\sigma} + U \sum_{\vec{r}} \hat{n}_{\vec{r}\uparrow} \hat{n}_{\vec{r}\downarrow}$$

Fermionic atoms loaded in optical lattices

Delocalized states

U >> t

Mott insulator One atom per site

M. Köhl & al, Phys. Rev. Lett. (2005)

R. Jördens & al, Nature (2008)

The half-filled Hubbard model

Antiferromagnetic order

$$\left\langle \hat{\vec{S}}_{\vec{0}} \cdot \hat{\vec{S}}_{\vec{r}} \right\rangle \propto (-1)^{x+y} = \cos(\pi x + \pi y)$$
$$S(\vec{q}) = \sum_{\vec{r}} \exp(i\vec{q}.\vec{r}) \left\langle \hat{\vec{S}}_{\vec{0}} \cdot \hat{\vec{S}}_{\vec{r}} \right\rangle \text{ peaked at } \vec{q} = (\pi,\pi)$$

R. A. Hart et al., arxiv/cond-mat (2014)

Conventional variational approaches to the Hubbard model

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The symmetry-projected Hartree-Fock/BCS wavefunctionNo assumed
charge, spin or
superconducting
ordersTotally unrestricted orthonormal
single-particle statesGeneral unitary
Bogliubov transformation
$$\prod_{i=1}^{N} \hat{c}_{\phi_i}^+ | \lambda \\ \text{with } \hat{c}_{\phi_i}^+ = \sum_{r \sigma} \phi_{i,r \sigma} \hat{c}_{r \sigma}^+$$
 $with \hat{\gamma}_{r \sigma} = \sum_{r \sigma} (U_{r \sigma', r \sigma}^* \hat{c}_{r \sigma'} + V_{r \sigma', r \sigma}^* \hat{c}_{r \sigma'}^+)$
$$| \Psi_0^{(\Gamma)} \rangle \rightarrow | \Phi_{HF-BCS}^{(\Gamma)} \rangle = \hat{P}^{(\Gamma)} (c_{HF} | \Phi_{HF} \rangle + c_{BCS} | \Phi_{BCS} \rangle)$$
Exact ground state
constrained by constr

characterized by several quantum numbers Γ reflecting symmetries of the Hamiltonian

$$\Gamma = \left(N, \vec{K}, S, S_z, \cdots\right)$$

$$\hat{P}^{(\Gamma)} \propto \sum_{g} \left(\chi^{(\Gamma)}_{g}
ight)^{*} \hat{U}_{g}$$

Quantum number projection by superposition of symmetry related wavefunctions

Correlations from restoration of deliberately broken symmetries by projection before variation

$$\delta E^{(\Gamma)} = \delta \left\langle \hat{H} \right\rangle_{\Phi^{(\Gamma)}} = 0$$

The symmetry-projected Hartree-Fock/BCS wavefunction

Generalized eigenvalue problem for the amplitudes

Wick's theorem for matrix elements between mean-field states

One-body transition density matrix

 $\mathcal{R}_{g}^{(a,b)} = \begin{pmatrix} \rho_{g}^{(a,b)} & \kappa_{g}^{(a,b)} \\ \tilde{\kappa}_{g}^{(a,b)} & \tilde{\rho}_{g}^{(a,b)} \end{pmatrix}$

Overlap between HF and/or BCS wave-functions (Determinant or pfaffian)

with
$$\begin{pmatrix} \left[\rho_{g}^{(a,b)} \right]_{\vec{r}\sigma,\vec{r}'\sigma'} & \left[\kappa_{g}^{(a,b)} \right]_{\vec{r}\sigma,\vec{r}'\sigma'} \\ \left[\tilde{\kappa}_{g}^{(a,b)} \right]_{\vec{r}\sigma,\vec{r}'\sigma'} & \left[\tilde{\rho}_{g}^{(a,b)} \right]_{\vec{r}\sigma,\vec{r}'\sigma'} \end{pmatrix} = \frac{1}{\mathcal{N}_{g}^{(a,b)}} \left\langle \Phi_{a} \right| \begin{pmatrix} \hat{c}_{\vec{r}'\sigma}^{+}\hat{c}_{\vec{r}\sigma} & \hat{c}_{\vec{r}'\sigma'}\hat{c}_{\vec{r}\sigma} \\ \hat{c}_{\vec{r}'\sigma'}^{+}\hat{c}_{\vec{r}\sigma}^{+} & \hat{c}_{\vec{r}'\sigma'}\hat{c}_{\vec{r}\sigma}^{+} \end{pmatrix} | \Phi_{b,g} \rangle$$

The symmetry-projected Hartree-Fock/BCS wavefunction

Optimal single-particle states and BCS quasiparticles

$$\mathcal{Q}^{(\Gamma,a,b)} = \begin{pmatrix} L^{(\Gamma,a,b)} & \Lambda^{(\Gamma,a,b)} \\ \tilde{\Lambda}^{(\Gamma,a,b)} & \tilde{L}^{(\Gamma,a,b)} \end{pmatrix} = \sum_{g} (\chi_{g}^{(\Gamma)})^{*} \mathcal{N}_{g}^{(a,b)} \Big[(1 - \mathcal{R}_{g}^{(a,b)}) \mathcal{D} \Big[\mathcal{R}_{g}^{(a,b)} \Big] \mathcal{R}_{g}^{(a,b)} + \mathcal{R}_{g}^{(a,b)} \Big(\mathfrak{E} \Big[\mathcal{R}_{g}^{(a,b)} \Big] - E^{(\Gamma)} \Big) \Big]$$
Hartree-Fock-Bogoliubov
Hamiltonian expressed in terms of the one-body transition density matrix
$$\begin{pmatrix} h_{g}^{(a,b)} & \tilde{h}_{g}^{(a,b)} \\ \tilde{\Lambda}_{g}^{(a,b)} & \tilde{h}_{g}^{(a,b)} \\ \tilde{\Lambda}_{g}^{(a,b)} & \tilde{h}_{g}^{(a,b)} \\ \tilde{\Lambda}_{g}^{(a,b)} & \tilde{h}_{g}^{(a,b)} \end{pmatrix} = \begin{pmatrix} \mathfrak{D} \mathfrak{E} \Big[\mathfrak{D} \big[\mathfrak{D} \big[\mathcal{R}_{g}^{(a,b)} \Big] - E^{(\Gamma)} \big] \Big]$$

$$\text{with} \begin{pmatrix} \left[h_{g}^{(a,b)} \right]_{r\sigma,r\sigma'} & \left[\Lambda_{g}^{(a,b)} \right]_{r\sigma,r\sigma'} \\ \left[\tilde{\Lambda}_{g}^{(a,b)} \right]_{r\sigma,r\sigma'} & \left[\tilde{\Lambda}_{g}^{(a,b)} \right]_{r\sigma,r\sigma'} \\ \tilde{\mathcal{D}} \big[\mathfrak{D} \big[\mathfrak{D} \big[\mathcal{D} \big[\mathcal{D}$$

A. Leprévost, O. Juillet, R. Frésard, to be published (2014).

Technical comments

Energy minimization by the conjugate gradient method through Thouless parameterization of HF and BCS wavefunctions

We use a 16x4 supercell with antiperiodic/periodic boundary conditions

 $\sim 10^4$ complex variables to be simultaneously determined

- Translational invariance and lattice symmetries (C_{2v} group) are restored + Particle number projection for the BCS state
- + Spin rotational invariance partially restored during the optimization :

 S_z and spin-parity $\varpi_s = (-1)^s$ projections

The variational N-body state corresponds to the superposition

of ~ 7.10^4 symmetry related wavefunctions

Full spin projection after basis optimization

Signatures of spin-density wave and spiral orderings

$$S(\vec{q}) = \sum_{\vec{r}} \exp(i\vec{q}\,\vec{r}) \left\langle \hat{\vec{S}}_{\vec{0}} \cdot \hat{\vec{S}}_{\vec{r}} \right\rangle \text{ peaked at } \vec{q} = \vec{q}_s$$
$$\mathscr{O}(\vec{r}) = \left\langle \hat{\vec{V}}_{\vec{0}} \cdot \hat{\vec{V}}_{\vec{r}} \right\rangle \xrightarrow[r \to \infty]{} Cte \Rightarrow \overline{\mathscr{O}(\vec{r})}_{r \to \infty} = \begin{cases} 0 \to \text{Spin density wave} \\ > 0 \to \text{Spiral} \end{cases}$$

Signatures of spin-density wave and spiral orderings

SDW with wavevector \vec{q}_s

 $S(\vec{q}) \propto \delta_{\vec{q},\vec{q}_s}$ and $\overline{\mathscr{O}(\vec{r})}_{r \to \infty} = 0$

Spiral with wavevector \vec{q}_s

$$S(\vec{q}) \propto \delta_{\vec{q},\vec{q}_s}$$
 and $\mathscr{N}(\vec{r})_{r\to\infty} > 0$

<u>Dependence of magnetic correlations on hole doping at U=8t</u>

Signatures of stripe ordering

Related peaks in the Fourier transform of the spin-spin and density-density correlation functions

0,6

0,5

0,4

0,3

0,2

0,1

 q_x

 \dot{q}_x

<u>Dependence of magnetic</u> <u>and charge correlations</u> <u>on hole doping</u>

Phase diagram from symmetry-projected HF/BCS wavefunctions

Comparison with exact results at half-filling

$\hat{P}^{(\Gamma)} \ket{\Phi_{_{HF}}}$ is the exact ground-state at any coupling for the 2x2 cluster

A. Leprévost, O. Juillet & R. Frésard, Ann. der Phys. (2014)

		Energy/t	$S(\pi,\pi)$	$C(\pi,\pi)$
$4 \times 4 - U = 4t - t' = 0$	Ex. Diag.	-13.6224	2.73	0.385
	HF - BCS (S = 0)	-13.618	2.743	0.386
$6 \times 6 - U = 4t - t' = 0$	QMC	-30.87(2)	4.365(3)	0.418(2)
	$HF - BCS \ (S = 0)$	-30.724	4.562	0.411

Comparison with exact results on doped small clusters

		Energy/t	$S(\pi,\pi)$	$C(\pi,\pi)$
$4 \times 4 - N = 14$	Ex. Diag.	-12.503	0.724	0.279
U = 8t - t' = -0.3t	$HF - BCS \ (S = 0)$	-12.439	0.727	0.288

Comparison with the Gutzwiller wavefunction

8x8 lattice - U=10t

T. Giamarchi & C. Lhuillier, Phys. Rev. B. (1991)

Comparison with quantum number projection on QMC and Gutzwiller wavefunctions

8x8 lattice - U=4t - N=50 electrons

Energy improvements on the symmetry projected HF/BCS scheme

$$\Psi_{0}^{(\Gamma)}\rangle \rightarrow \left|\Phi_{HF-BCS}^{(\Gamma)}\rangle\right| = \hat{P}^{(\Gamma)}\left(\sum_{i=1}^{\mathscr{H}_{HF}} c_{HF}^{(i)} \left|\Phi_{HF}^{(i)}\rangle\right| + \sum_{i=1}^{\mathscr{H}_{BCS}} c_{BCS}^{(i)} \left|\Phi_{BCS}^{(i)}\rangle\right|\right)$$

Full symmetry restoration during the optimization

$$\Gamma = \left(N, \vec{K}, S, S_z, \cdots\right)$$

Mixture of unrestricted and restricted HF/BCS wave functions

States with good quantum number S_z

$$\begin{cases} |\Phi_{HF}\rangle = |\Phi\rangle_{\uparrow} |\Phi\rangle_{\downarrow} \\ |\Phi_{BCS}\rangle \propto \exp\left(\sum_{\vec{r},\vec{r}'} A_{\vec{r},\vec{r}'} \hat{c}^{+}_{\vec{r}\uparrow} \hat{c}^{+}_{\vec{r}\downarrow}\right) |\rangle \end{cases}$$

Sequential optimization

16×4 lattice - U=12t - N=56 electrons

${\mathscr W}_{\scriptscriptstyle HF}$	\mathscr{W}_{BCS}	Simultaneous optimization	S=0 projection during the energy minimization	$\frac{\left\langle \hat{H} \right\rangle_{HF-BCS}^{(\Gamma)} - E_{HF}}{\left E_{HF} \right }$
1	1	YES	NO	~ -10.48 %
101	20	NO	YES	~ -17.13 %
40	0	NO	YES	~ -14.96 %

R. Rodríguez-Guzmán, C. Jiménez-Hoyos, G. Scuséria, arxiv/cond_mat (2014)

Energy improvements on the symmetry projected HF/BCS scheme

The symmetry projected HF/BCS scheme : new or not ?

	$\left \Psi_{0}^{(\Gamma)}\right\rangle \rightarrow \left \Phi_{HF}^{(\Gamma)}\right\rangle = \hat{P}^{(\Gamma)}\left(\sum_{i=1}^{\mathscr{H}_{HF}} c_{HF}^{(i)} \left \Phi_{HF}^{(i)}\right\rangle\right)$ (*), (*) : Full symmetry restoration	$\left \Psi_{0}^{(\Gamma)}\right\rangle \rightarrow \left \Phi_{BCS}^{(\Gamma)}\right\rangle = \hat{P}^{(\Gamma)}\left(\sum_{i=1}^{\mathscr{H}_{F}} c_{HF}^{(i)} \left \Phi_{BCS}^{(i)}\right\rangle\right)$ on with totally unrestricted wavefunctions
General features	➢ F. Fukutome, Prog. Theor. Phys. (1988)	 K.W. Schmid, F. Grümmer & A. Faessler, Phys. Rev. C (1984) H. Fukutome, Prog. Theor. Phys. (1991) J. A. Sheikh & P. Ring, Nucl. Phys. A (2000)
Nuclear shell model	➢ J. Bonnard & O. Juillet, Phys. Rev. Lett. (2013)	 K.W. Schmid, Prog. Part. Nucl. Phys. (2004) I. Maqbool <i>et al.</i>, J. Phys. G (2011)
Quantum chemistry	C. Jiménez-Hoyos, R. Rodríguez-Guzmán & G. E. Scuseria, J. Chem. Phys. (2012), arxiv/physics.chem-ph (2014)	G.E Scuseria et al., J. Chem. Phys. (2011)
Hubbard model	ID model▶ N. Tomita, Phys. Rev. B (2004)▶ K.W. Schmid et al., Phys. Rev. B (2005)▶ R. Rodríguez-Guzmán et al., Phys. Rev. B (2013)2D model▶ N. Tomita & S. Watanabe, Phys. Rev. Lett. (2009)▶ R. Rodríguez-Guzmán et al., Phys. Rev. B (2012)▶ O. Juillet & R. Frésard, Phys. Rev. B (2013)▶ H. Shi et al., Phys. Rev. B (2013 & 2014)▶ A. Leprévost, O. Juillet & R. Frésard, Ann. der Phys. (2014)▶ R. Rodríguez-Guzmán et al., arxiv/cond-mat (2014)▶ R. Rodríguez-Guzmán et al., arxiv/cond-mat (2014)	work

Spiral, SDW, stripes find their place in the phase diagram of the 2D Hubbard model.

They successively appear for decreasing doping at fixed U and for increasing U at fixed doping.

Coexistence with d-wave superconductivity has been evidenced for longranged pairing correlations at strong coupling (U~10t) and up to a hole doping ~0.2, EXCEPT when the holes are totally trapped in stripes.

These features are robust against extensions of the wavefunction.

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