Quantum Simulation of 2d U(1) critical systems: A Higgs particle and the possible help of string theory for the optical conductivity

Lode Pollet

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Ref: PRL 2012, PRL 2013, PRL 2014

in collaboration with:

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Youjin Deng, USTC Hefei, China

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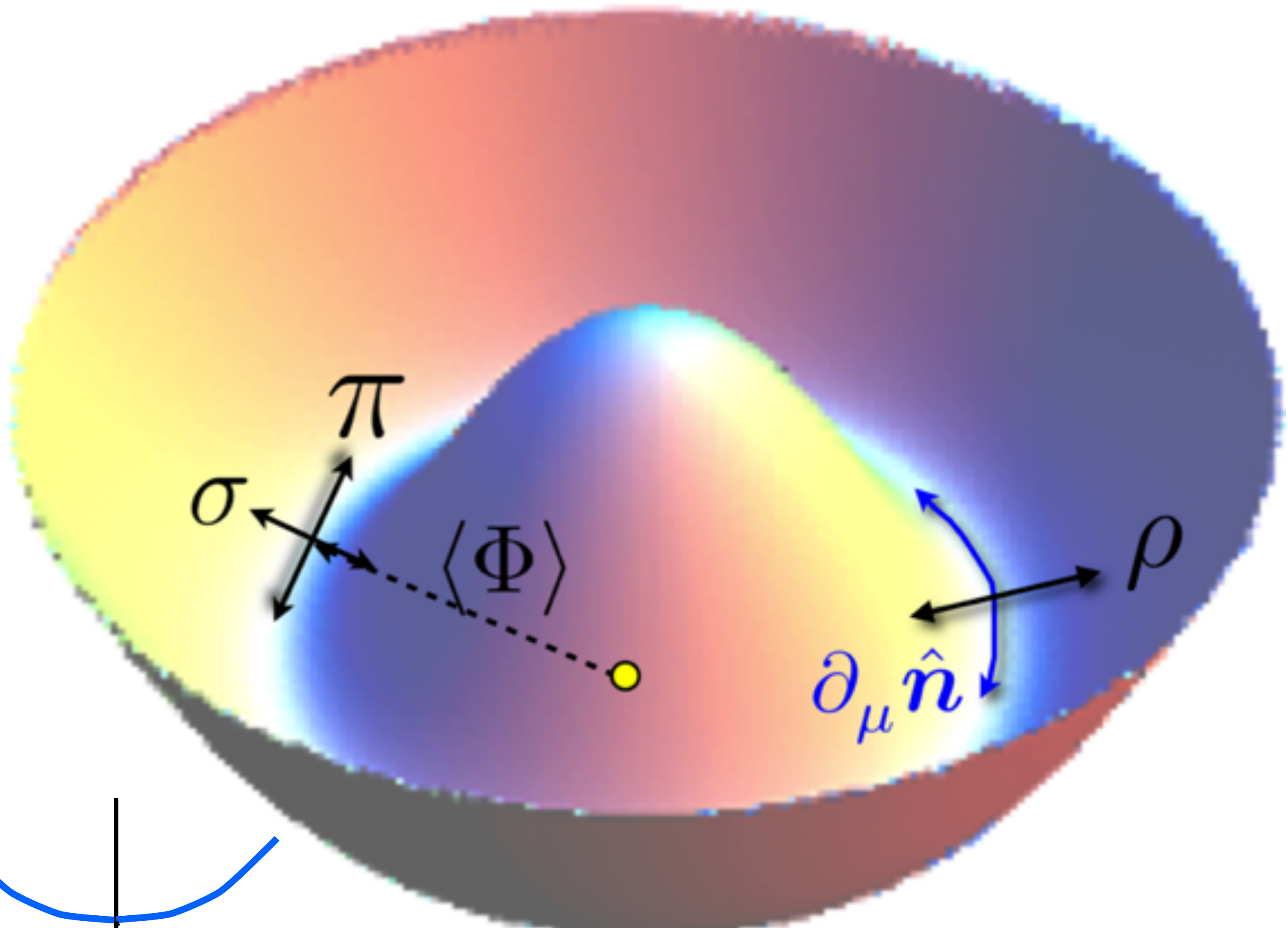
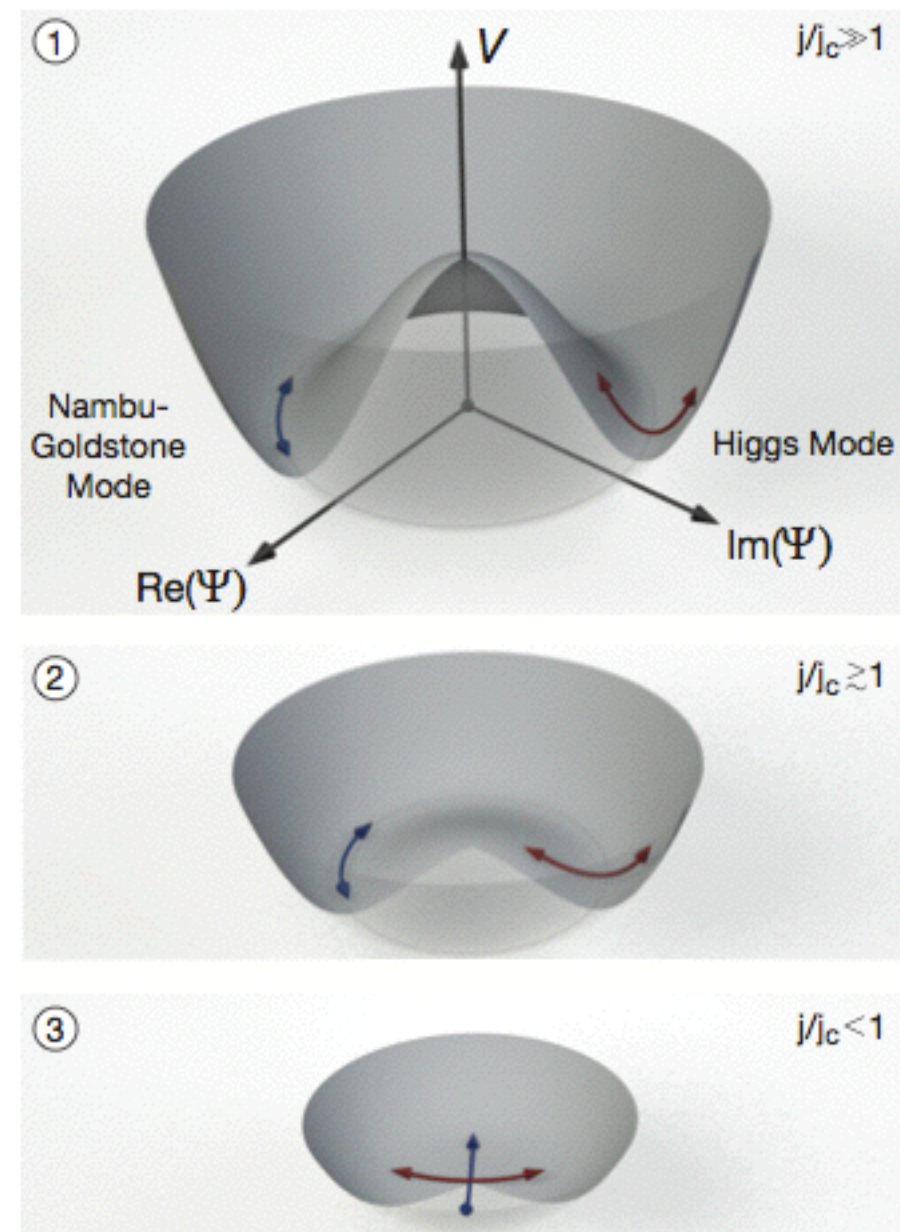
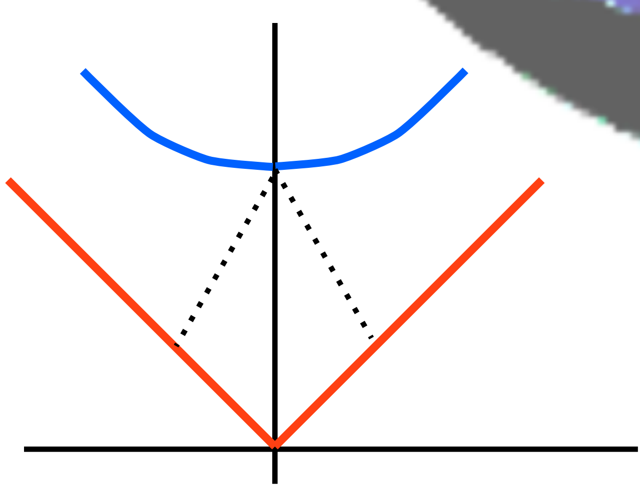
Mexican hat -- radial fluctuations

$$S = \frac{1}{2g}\int d^{d+1}r\left[\left|\partial_\mu \psi\right|^2 + \frac{1}{2}\left(\left|\psi\right|^2 - r\right)^2\right]$$ O(2) relativistic field theory







$$\Phi = (\Phi_0 + \sigma, \pi)$$

$$\Phi = \Phi_0(1 + \sqrt{N}\rho)\hat{n}$$

global gauge invariance

Consider a relativistic quantum field theory with mass m, and a complex scalar field

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - \frac{1}{2}\lambda(\phi^*\phi)^2$$

or, for negative mass,

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + m^2 \phi^* \phi - \frac{1}{2}\lambda(\phi^*\phi)^2$$

The Lagrangian has a global U(1) symmetry

$$\phi(x) \to \phi(x) e^{i\theta}$$

In terms of the Mexican hat potential,

$$V(\phi) = -\frac{1}{2}\lambda\nu\phi^*\phi + \frac{1}{2}\lambda(\phi^*\phi)^2 \qquad \nu = -\frac{-2m^2}{\lambda}$$

the minimum occurs for

$$|\phi|^2 = \frac{\nu^2}{2}$$

global gauge invariance

We pick one of the minima and expand around it,

$$\phi = \frac{1}{\sqrt{2}}(\nu + \varphi_1 + i\varphi_2)$$

The low-energy Lagrangian is then

$$\mathcal{L} = \frac{1}{2}\left[\boxed{(\partial_\mu \varphi_1)^2} + (\partial_\mu \varphi_2)^2\right] \boxed{- \frac{1}{2}\lambda \nu^2 \varphi_1^2} + \ldots$$

where we see a massless Goldstone mode and a massive Higgs mode.

local gauge invariance

Consider now the case of coupling to a gauge field and local gauge invariance,

$$\theta \to \theta(x)$$

$$A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \theta(x)$$

$$D_\mu \phi = \partial_\mu \phi + ieA_\mu \phi$$

$$\mathcal{L} = D_\mu \phi^* D^\mu \phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi)$$

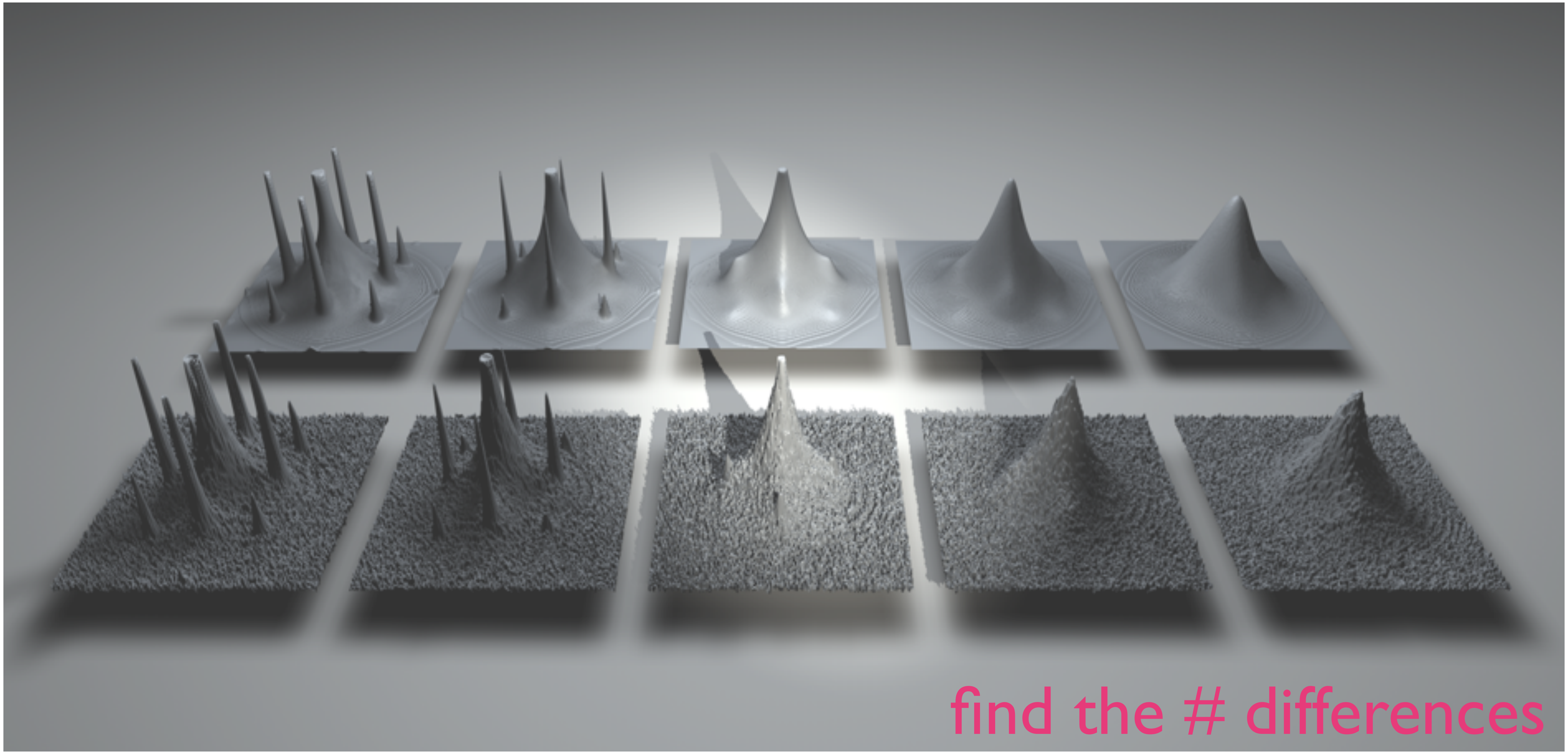
Breaking the symmetry now leads to

$$\mathcal{L} = \frac{1}{2}\boxed{\left[(\partial_\mu \varphi_1)^2\right.} + \underbrace{\left.(\partial_\mu \varphi_2 + e\nu A_\mu)^2\right]}_{\text{plasmon}} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \boxed{- \frac{1}{2}\lambda\nu^2\varphi_1^2} + \dots$$

exactly same terms as for global gauge invariance

Ultracold atom quantum simulators pass first key test

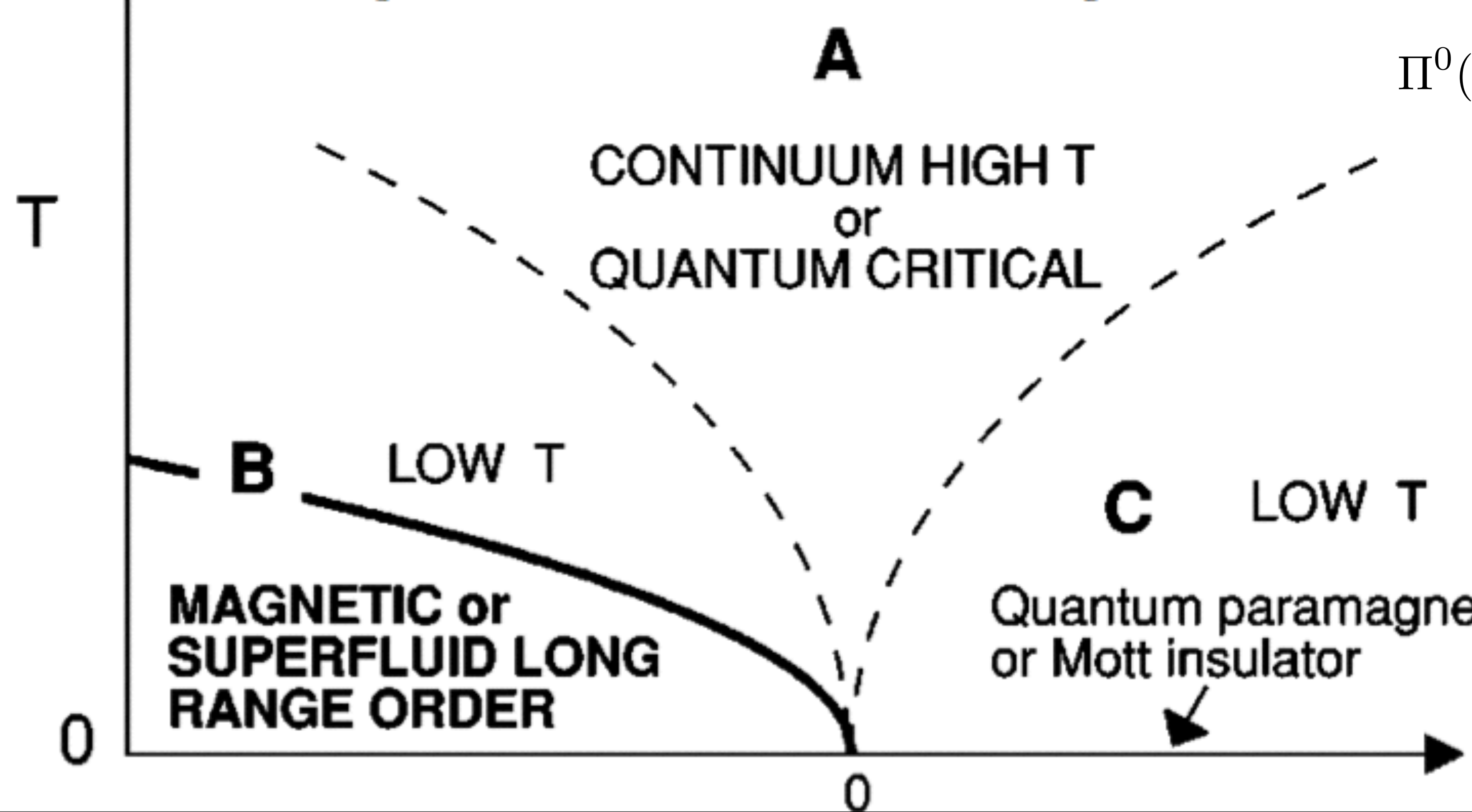
+ : control, clean, tunable
- : temperature, detection, light-matter effects



O(N) field theories

$$\mathcal{Z}_Q = \int \mathcal{D}\phi_\alpha(x,\tau) \exp\left(- \int d^d x \int_0^{1/T} d\tau \mathcal{L}_Q \right)$$

$$\mathcal{L}_Q = \frac{1}{2}\left[\frac{1}{c^2}(\partial_\tau \phi_\alpha)^2 + (\nabla_x \phi_\alpha)^2 + (r_c + r)\phi_\alpha^2 \right] + \frac{u}{4!}(\phi_\alpha^2)^2 .$$



d>3 : u is irrelevant (Gaussian free field theory)

$$\chi^0(q) = \frac{u}{q^2 + (r_c + r)}$$

mean-field pole at amplitude mass

$$\Pi^0(q) \sim \int \frac{1}{k^2(k+q)^2} \frac{d^{d+1}k}{(2\pi)^{d+1}}$$

d=2, n=1,2

hard to couple to Higgs mode

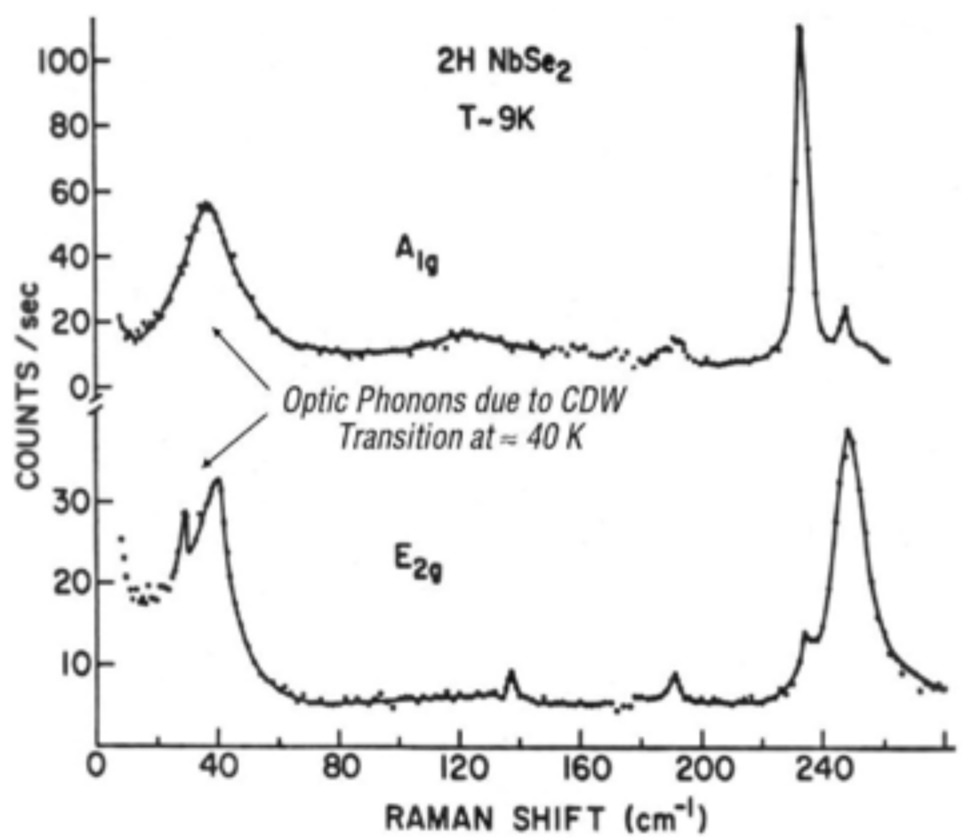
$$S = -r\Psi^*\Psi + \frac{U}{2}(\Psi^*\Psi)^2$$
$$+ (\tau_{GL}^{-1})\Psi^*\partial_t\Psi + iK_1\Psi^*\partial_t\Psi - K_2(\partial_t\Psi^*)(\partial_t\Psi) + \xi^{-2}(\nabla\Psi^*)(\nabla\Psi)$$

ph - symmetry needed (superconductors, not metals nor superfluids)

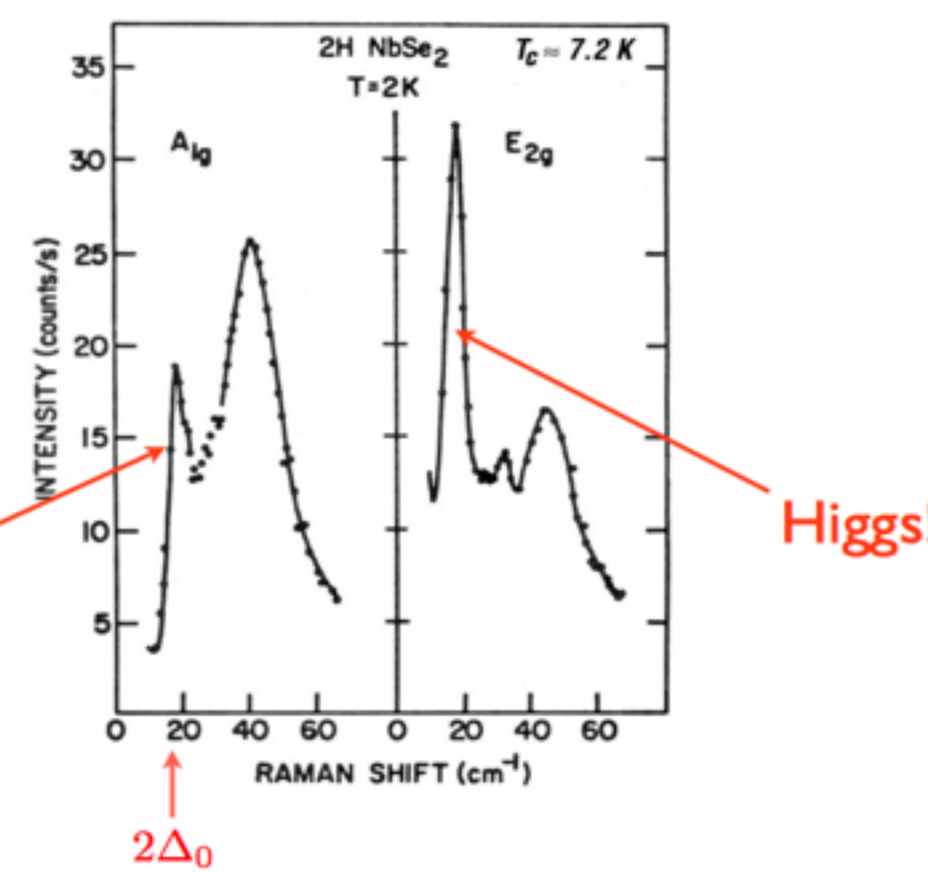
$$\Psi(\mathbf{r},t) - \Psi_0 \approx \delta\Psi(\mathbf{r},t) + i\Psi_0\phi(\mathbf{r},t) + \ldots \equiv \delta_a(\mathbf{r},t) + i\delta_{ph}(\mathbf{r},t) + \ldots$$

$$(2r + \xi^2q^2 - K_2\omega^2)\delta_a + (iK_1 + \tau_{GL}^{-1})\omega\delta_{ph} = 0;$$

Raman spectra of NbSe2



Higgs!



PRL 100, 205701 (2008) PHYSICAL REVIEW LETTERS week ending 23 MAY 2008

Quantum Magnets under Pressure: Controlling Elementary Excitations in $TlCuCl_3$

Ch. Rüegg,[1] B. Normand,[2,3] M. Matsumoto,[4] A. Furrer,[5] D. F. McMorrow,[1] K. W. Krämer,[6] H.-U. Güdel,[6] S. N. Gvasaliya,[5] H. Mutka,[7] and M. Boehm[7]

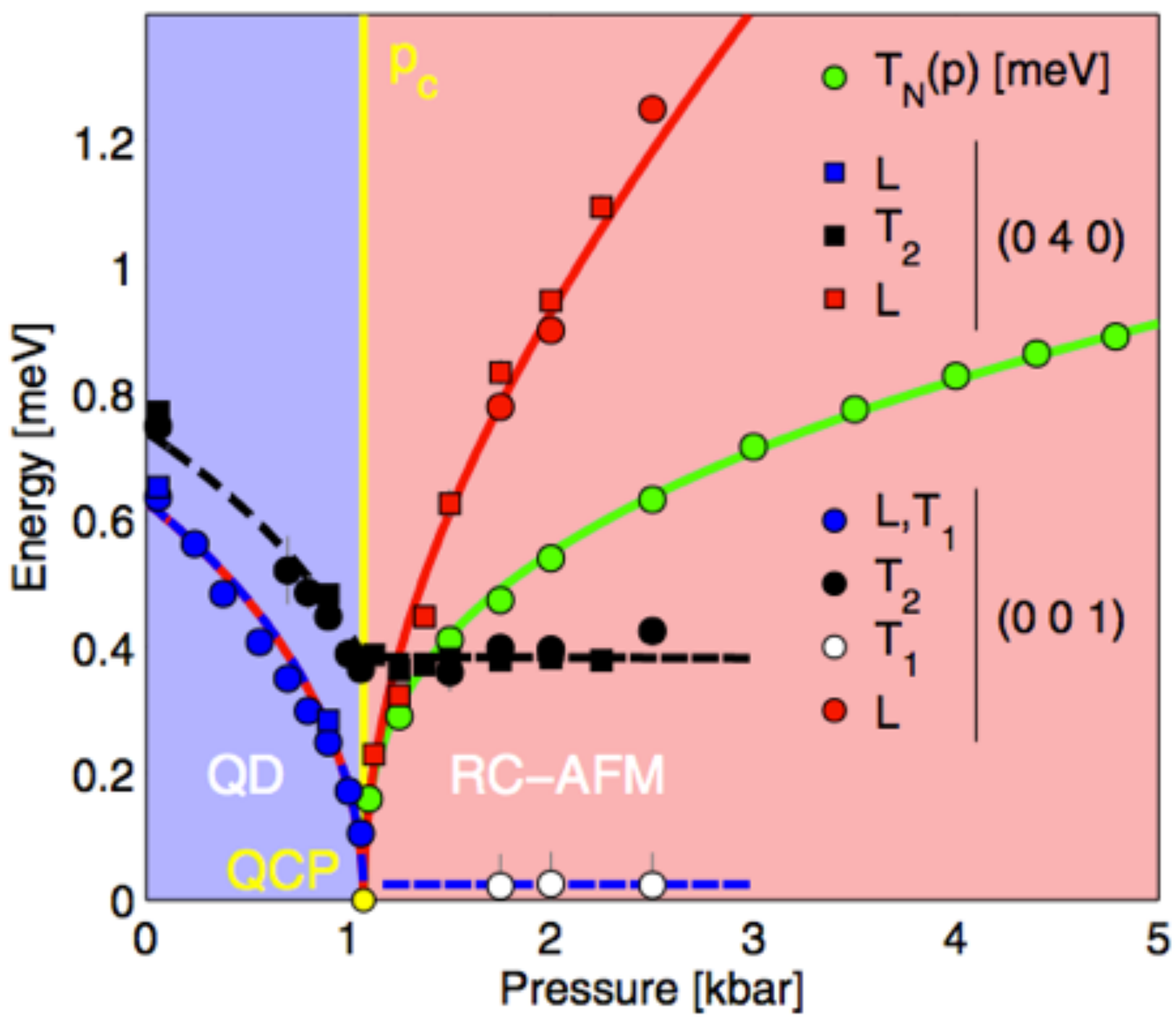


FIG. 3 (color online). Summary of INS results for the gaps of all three triplet excitations as functions of pressure at $T = 1.85$ K. Data for $T_N(p)$ from Ref. [5]. Modes L and T_1 are degenerate within experimental resolution at $p < p_c$. Red symbols show the longitudinal mode L at $p > p_c$. Solid and dashed lines are theoretical fits.

(3d quantum antiferromagnet)

two dimensions

PHYSICAL REVIEW B VOLUME 49, NUMBER 17 1 MAY 1994-I

Theory of two-dimensional quantum Heisenberg antiferromagnets with a nearly critical ground state

Andrey V. Chubukov
Departments of Physics and Applied Physics, P.O. Box 208120, Yale University, New Haven, Connecticut 06520-8120, and P.L. Kapitza Institute for Physical Problems, Moscow, Russia

Subir Sachdev and Jinwu Ye
Department of Physics and Applied Physics, P.O. Box 208120, Yale University, New Haven, Connecticut 06520-8120
(Received 21 April 1993; revised manuscript received 6 January 1994)

PHYSICAL REVIEW B VOLUME 59, NUMBER 21 1 JUNE 1999-I

Universal relaxational dynamics near two-dimensional quantum critical points

Subir Sachdev
Department of Physics, Yale University, P.O. Box 208120, New Haven, Connecticut 06520-8120
(Received 4 November 1998)

$$\chi_{\perp}(k,\omega)=\frac{N_0^2}{\rho_s(0)[k^2-(\omega/c)^2]},$$

$$\chi_{\parallel}(k,\omega)=\frac{N_0^2}{\rho_s(0)}\frac{1}{\sqrt{k^2-(\omega/c)^2}[\sqrt{k^2-(\omega/c)^2}+16\rho_s(0)/cn]}$$

longitudinal susceptibility has branch cut

no pole-like structure at a frequency of order $\rho_s(0)$

two dimensions

VOLUME 92, NUMBER 2 PHYSICAL REVIEW LETTERS week ending 16 JANUARY 2004

Anomalous Fluctuations in Phases with a Broken Continuous Symmetry

W. Zwerger
Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria
(Received 7 April 2003; published 16 January 2004)

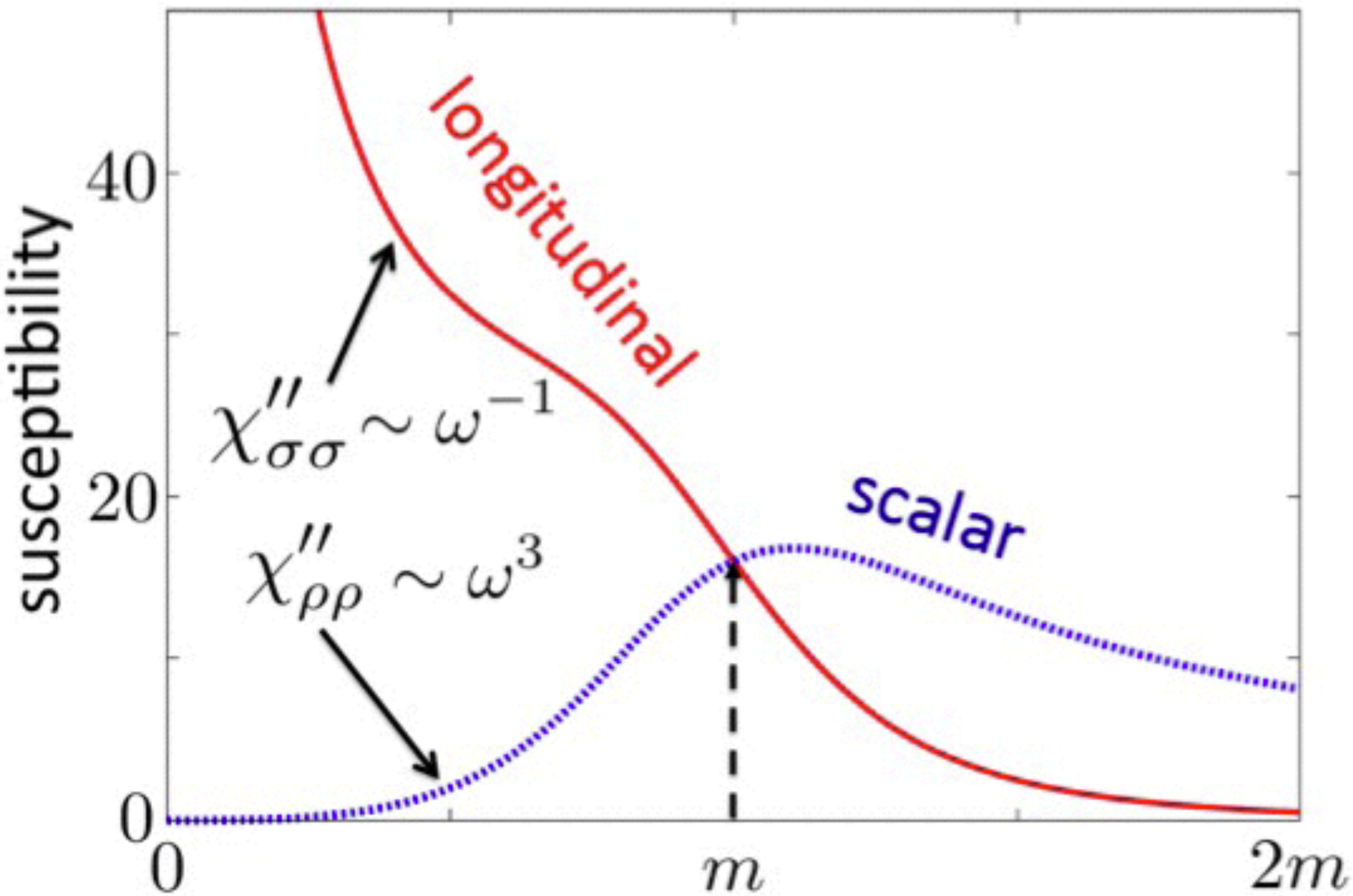
derived same formula's, and used them in the dynamic structure factor:

$$S(q,\omega) = 2m_s^2 \xi_J \frac{N-1}{N}\left[\frac{\pi}{2q}\delta(\omega - cq) + \frac{\xi_J}{16}\frac{\theta(\omega - cq)}{\sqrt{\omega^2 - c^2q^2}}\right]$$

``The longitudinal fluctuations of the Neel order thus lead to a critical continuum above the spin wave pole at w~ cq, which decays only algebraically. The continuum results from the decay of a normally massive amplitude mode with momentum p into a pair of spin waves with momenta q and p-q, which is possible for any w > cq, with a singular cross section because of the large phase space. The amplitude mode is thus completely overdamped in two dimensions."

Scalar and longitudinal susceptibility

Chubukov, Sachdev, Ye ’93
Podolsky, Auerbach, Arovas ’11
S. Huber, G. Blatter, E. Altman



open problems

1. Is there an amplitude (Higgs) mode near the U(1) QCP in 2d?

YES

amplitude modulation experiment for cold gases

Monte Carlo simulations, subject to analytic continuation confirm positively, current experiments also support this, but further experimental refinements would be welcome

2. How to obtain the optical conductivity near the U(1) QCP (ie CFT3) in 2d?

HARD

phase modulation experiment for cold gases

no experiments yet for cold gases

Universal scaling predictions

$$\chi''_{\rho\rho}(\omega) \propto \Delta^{3-2/\nu} F(\omega/\Delta)$$

$$\Delta \propto (U_C - U)^{\nu}, \quad \nu = 0.6717$$

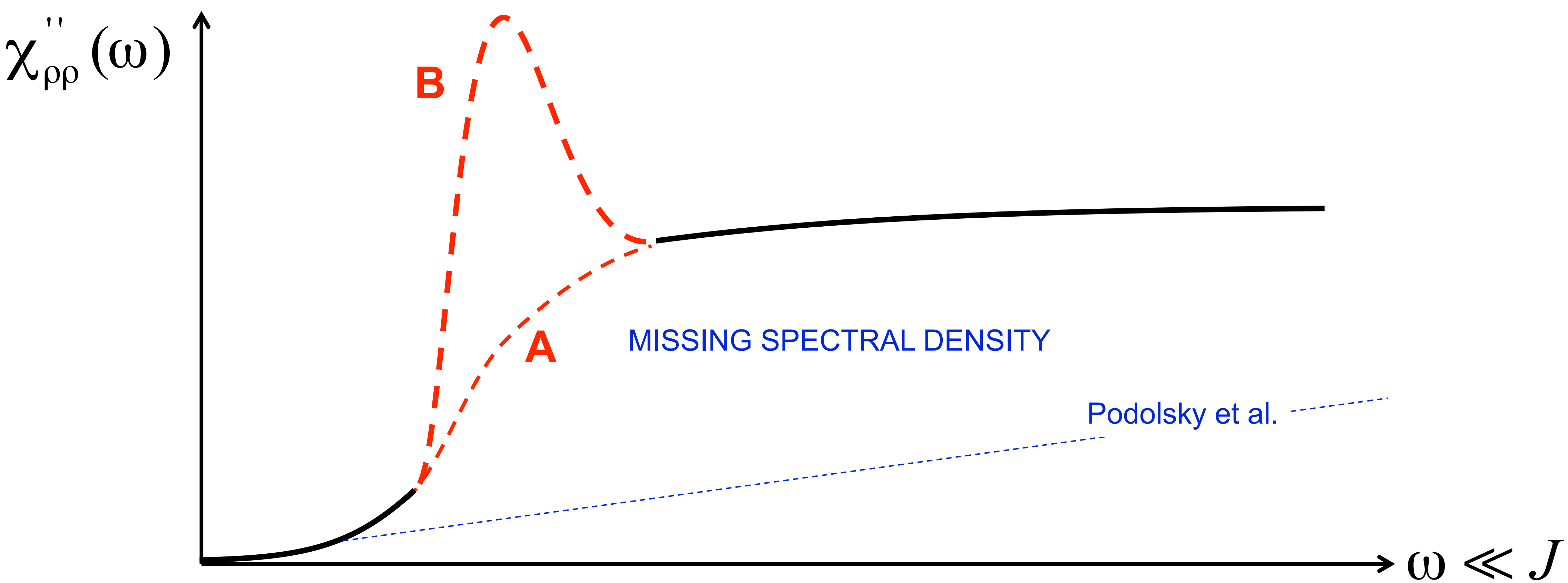
Chubukov, Sachdev, Ye '93

Sachdev '99

$\omega \ll \Delta$ $\qquad$ $\omega \gg \Delta$

$$\chi''_{\rho\rho}(\omega) \propto \omega^3/(1-U/U_C)^2$$

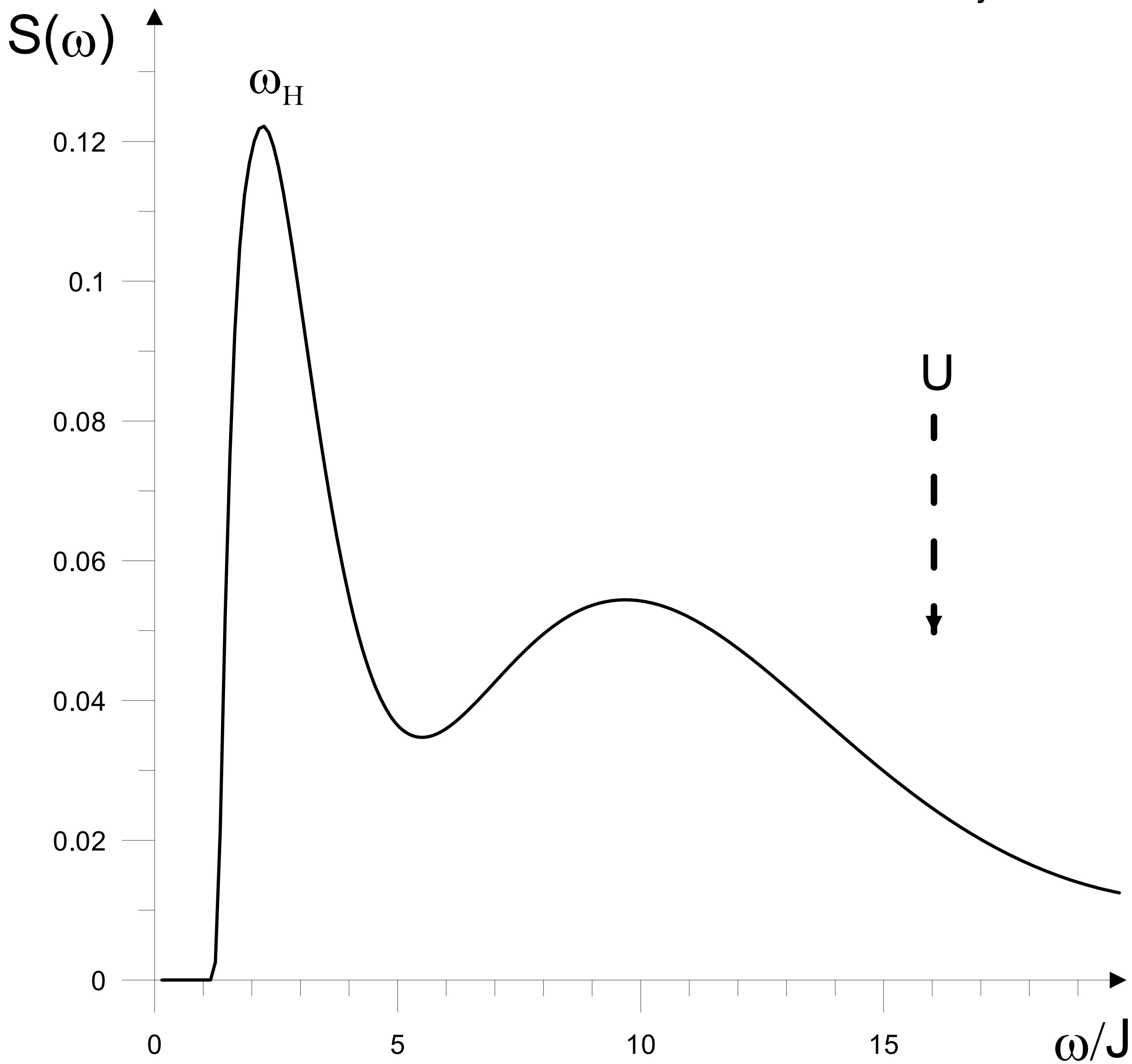
$$\chi''_{\rho\rho}(\omega) \propto \omega^{3-2/\nu} = \omega^{0.0225}$$

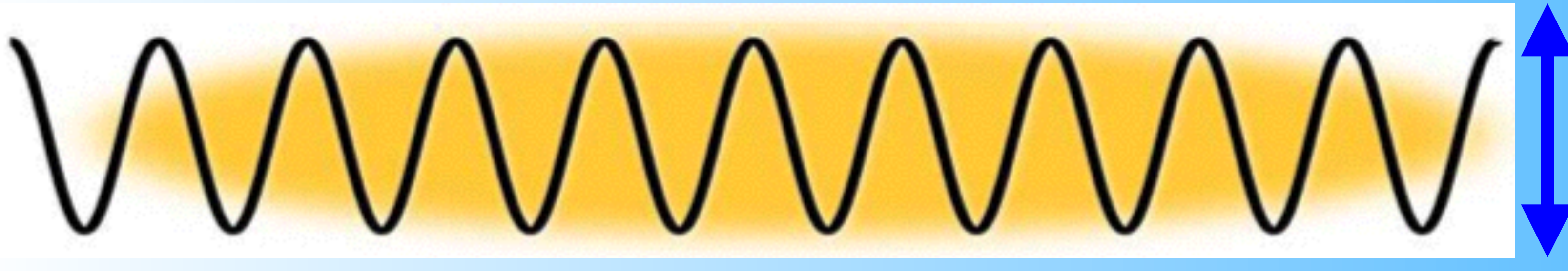


Our response

This involves analytic continuation

U=16





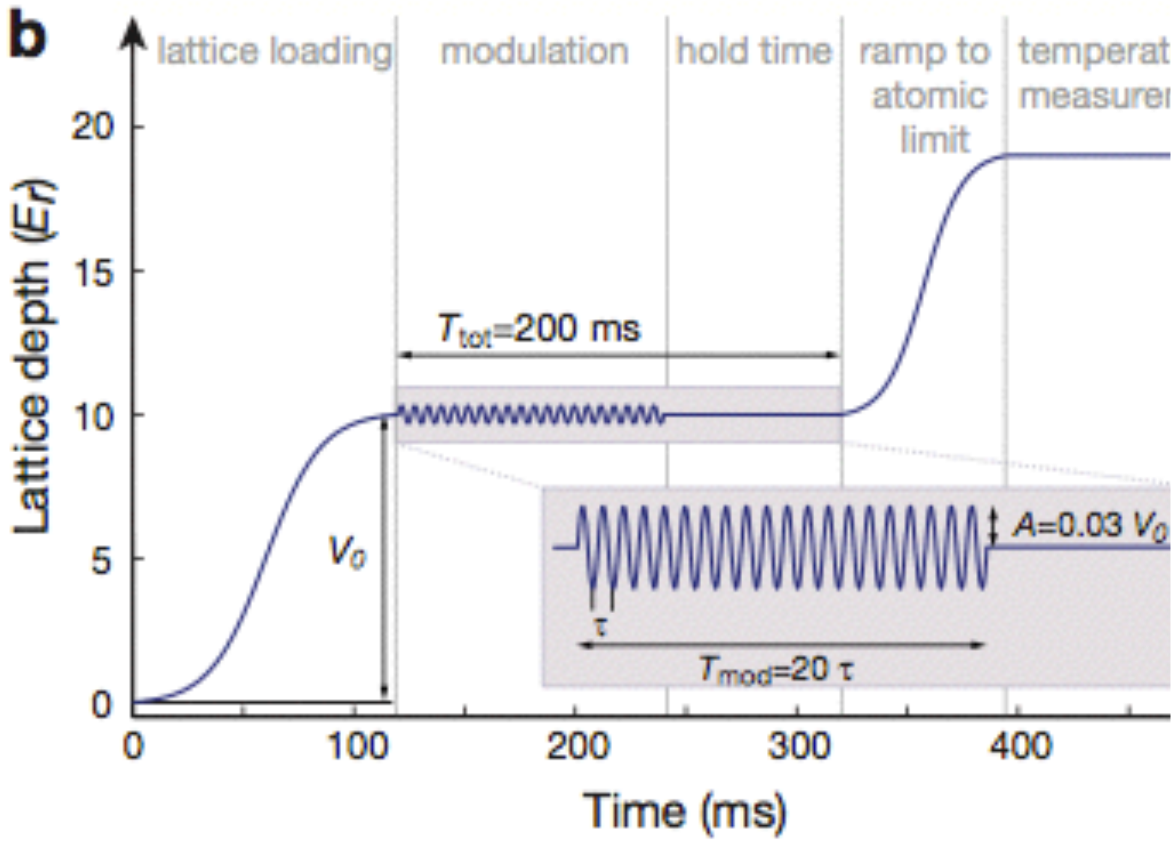
amplitude modulation

Technique pioneered in Zurich (Stoeferle et al);
see also Kollath et al, etc

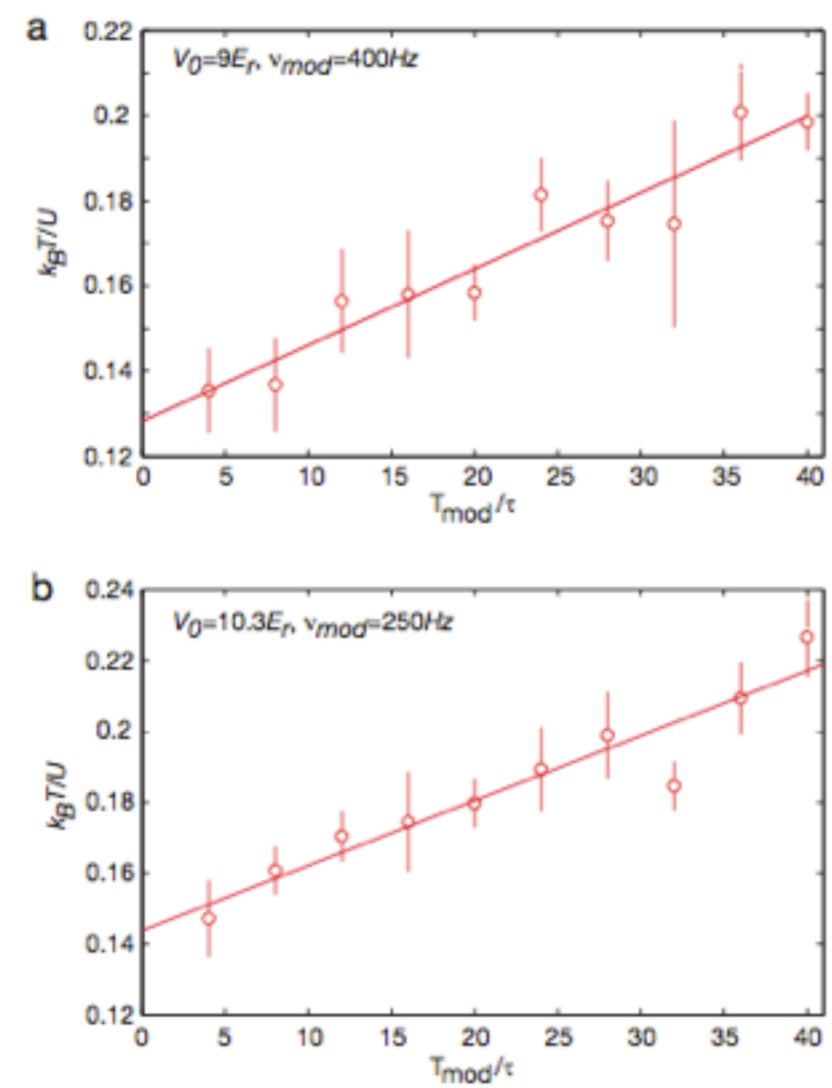
The 'Higgs' Amplitude Mode at the Two-Dimensional Superfluid-Mott Insulator Transition

Manuel Endres[1], Takeshi Fukuhara[1], David Pekker[2], Marc Cheneau[1], Peter Schauß[1], Christian Gross[1], Eugene Demler[3], Stefan Kuhr[1,4], and Immanuel Bloch[1,5]

Nature 2012



190(36) particles

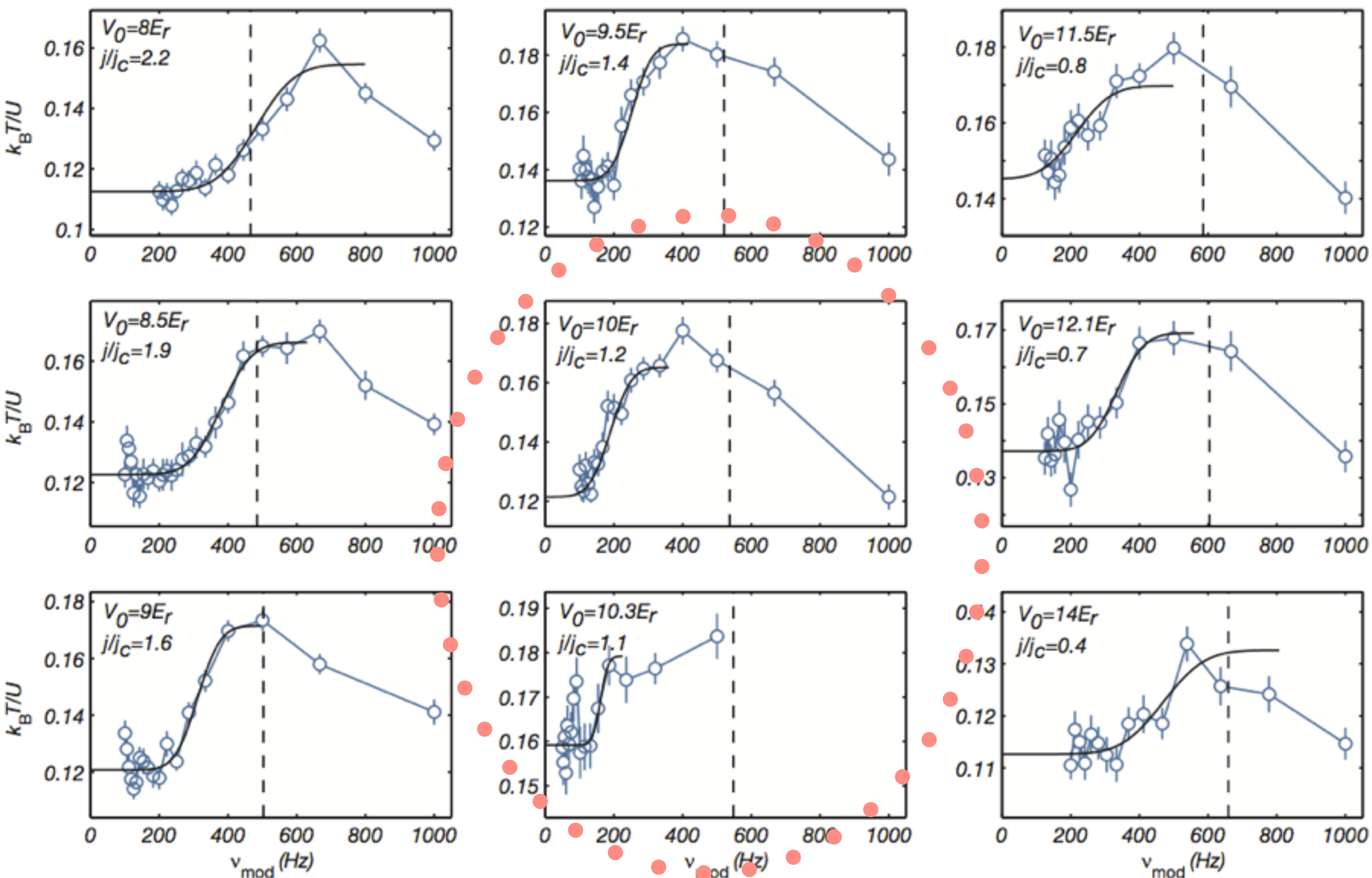


$$\chi_\omega = \langle K(\tau) K(0) \rangle_\omega + \langle K \rangle$$

Energy dissipation rate $\omega \, \mathrm{Im}\, \chi_\omega$

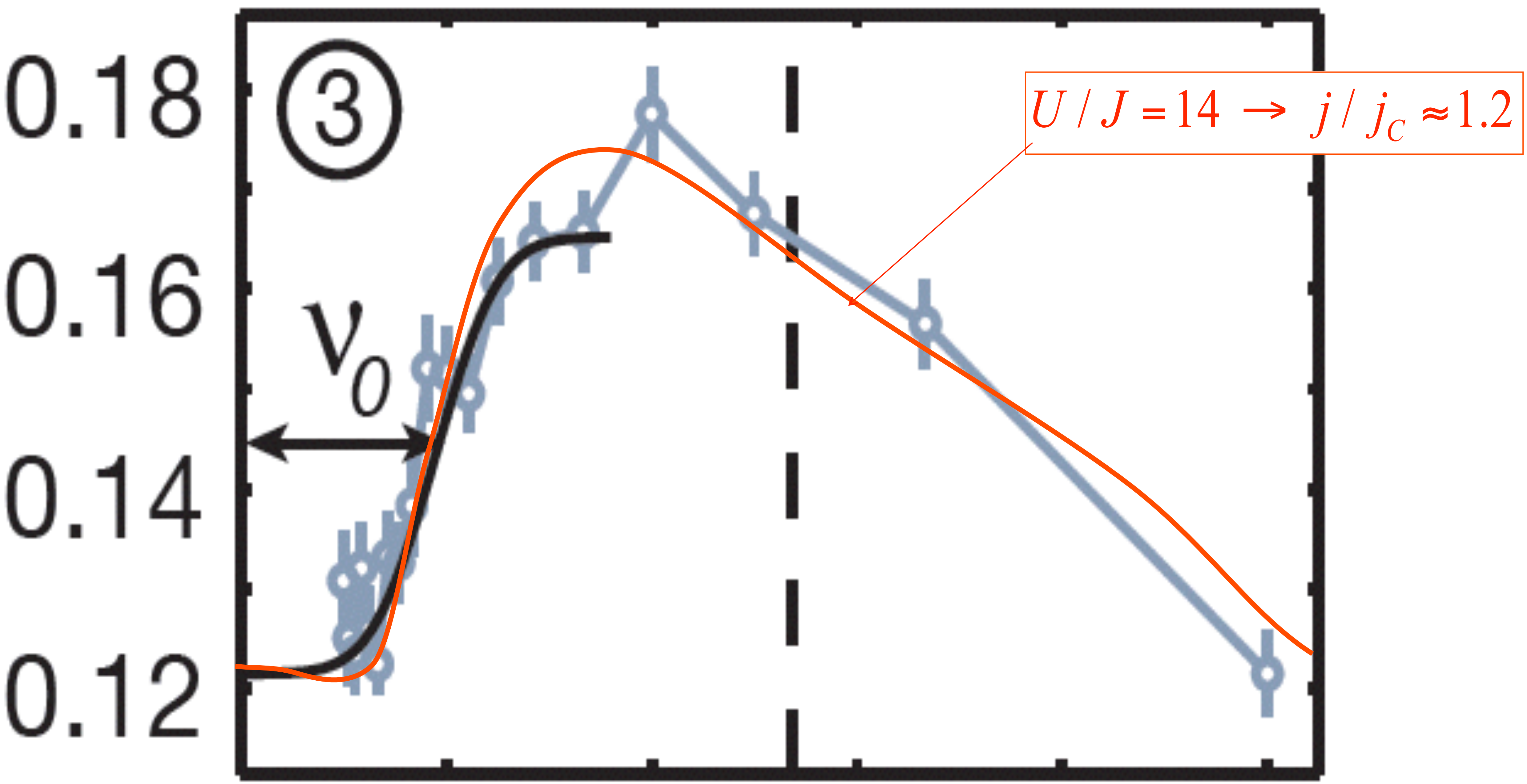
Total energy absorbed: $\Delta E = \omega \, \mathrm{Im}\, \chi_\omega \left(2\pi M / \omega \right) \propto \mathrm{Im}\, \chi_\omega$

The experimental results

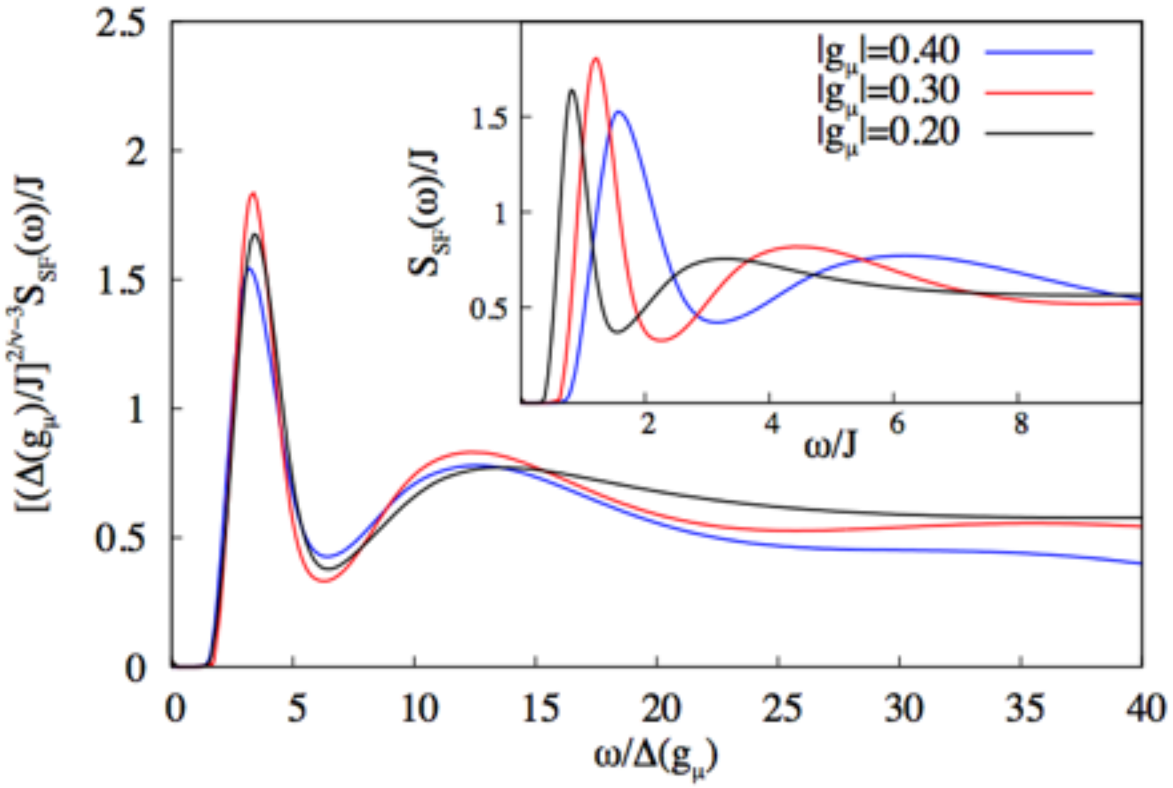


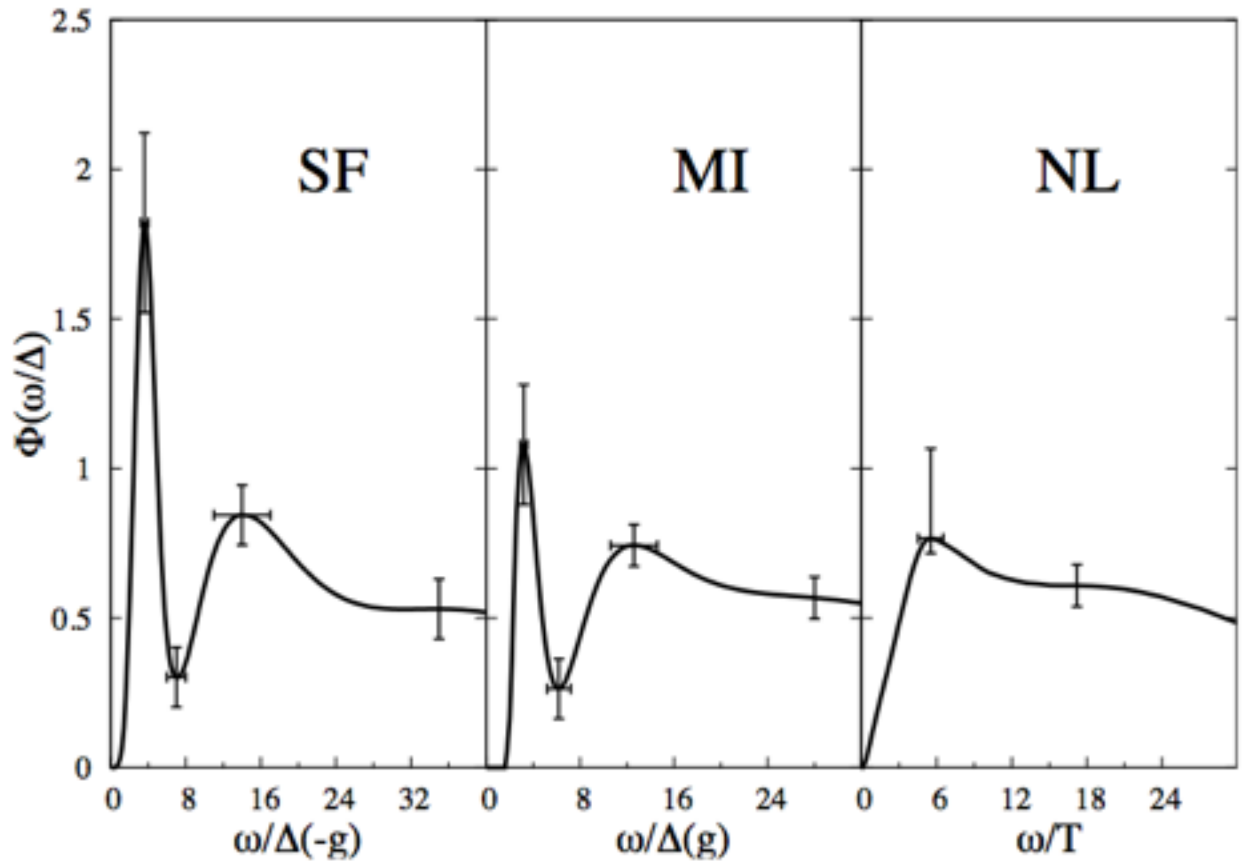
Attempt to compare signals (amplitude adjusted)

Take a realistic temperature and trapping parameters into account



universal scaling function





results by Podolsky et al

$$S_E = \frac{1}{g}\left[-\sum_{\langle i,j\rangle}\vec{\phi}_i\cdot\vec{\phi}_j - \mu\sum_i|\vec{\phi}_i|^2 + \sum_i(|\vec{\phi}_i|^2)^2\right]$$

S. Gazit et al, arXiv:1212.3759, PRL

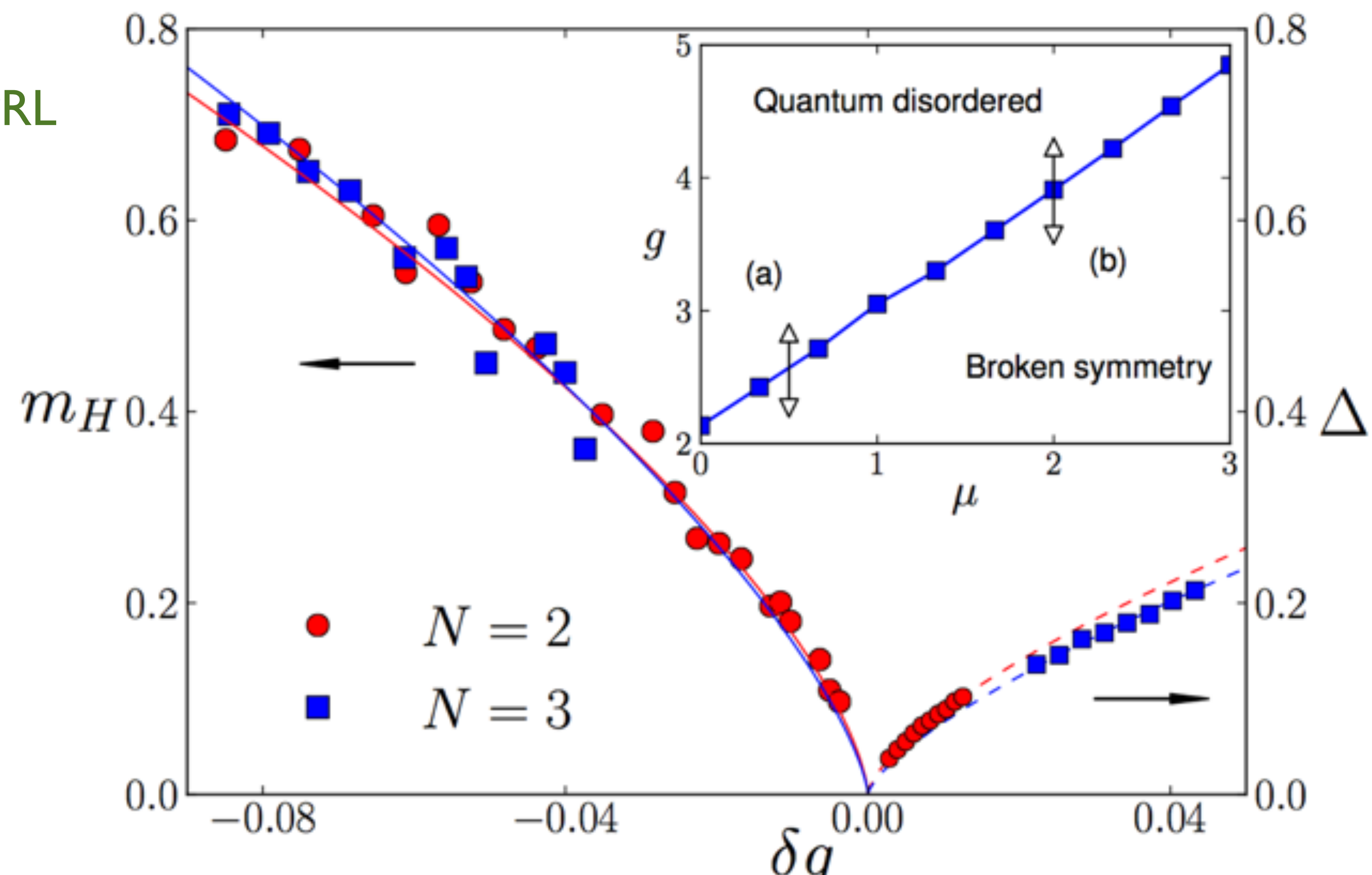
on SF side:

$\cancel{\omega_H = 1.9(1)\Delta}$

$\omega_H = 2.1(3)\Delta$

compare to ours:

$\omega_H = 3.2(8)\Delta$



open problems

1. Is there an amplitude (Higgs) mode near the U(1) QCP in 2d?

for a recent review: D. Pekker, C. Varma, arXiv:1406.2968

YES

amplitude modulation experiment for cold gases

Monte Carlo simulations, subject to analytic continuation confirm positively, current experiments also support this, but further experimental refinements would be welcome

2. How to obtain the optical conductivity near the U(1) QCP (ie CFT3) in 2d?

HARD

phase modulation experiment for cold gases

no experiments yet for cold gases

conductivity in CFT3

Universal behavior:

$$\sigma(\omega/T, T \to 0) \sim T^{(d-2)/z}\Sigma(\omega/T) \to \Sigma(\omega/T)$$

K. Damle and S. Sachdev, Phys. Rev. B **56**, 8714 (2003).

For $\omega/T \gg 1$ at fixed T, the system no longer feels the effect of the finite temperature

Then a crossover to a temperature-independent regime should occur,

$$\boxed{\sigma(\omega, T) \sim \sigma(\omega) \sim \Sigma(\infty) + A/\omega^2}$$

Old experiments and simulations remain ambiguous in their support of the scaling theory.

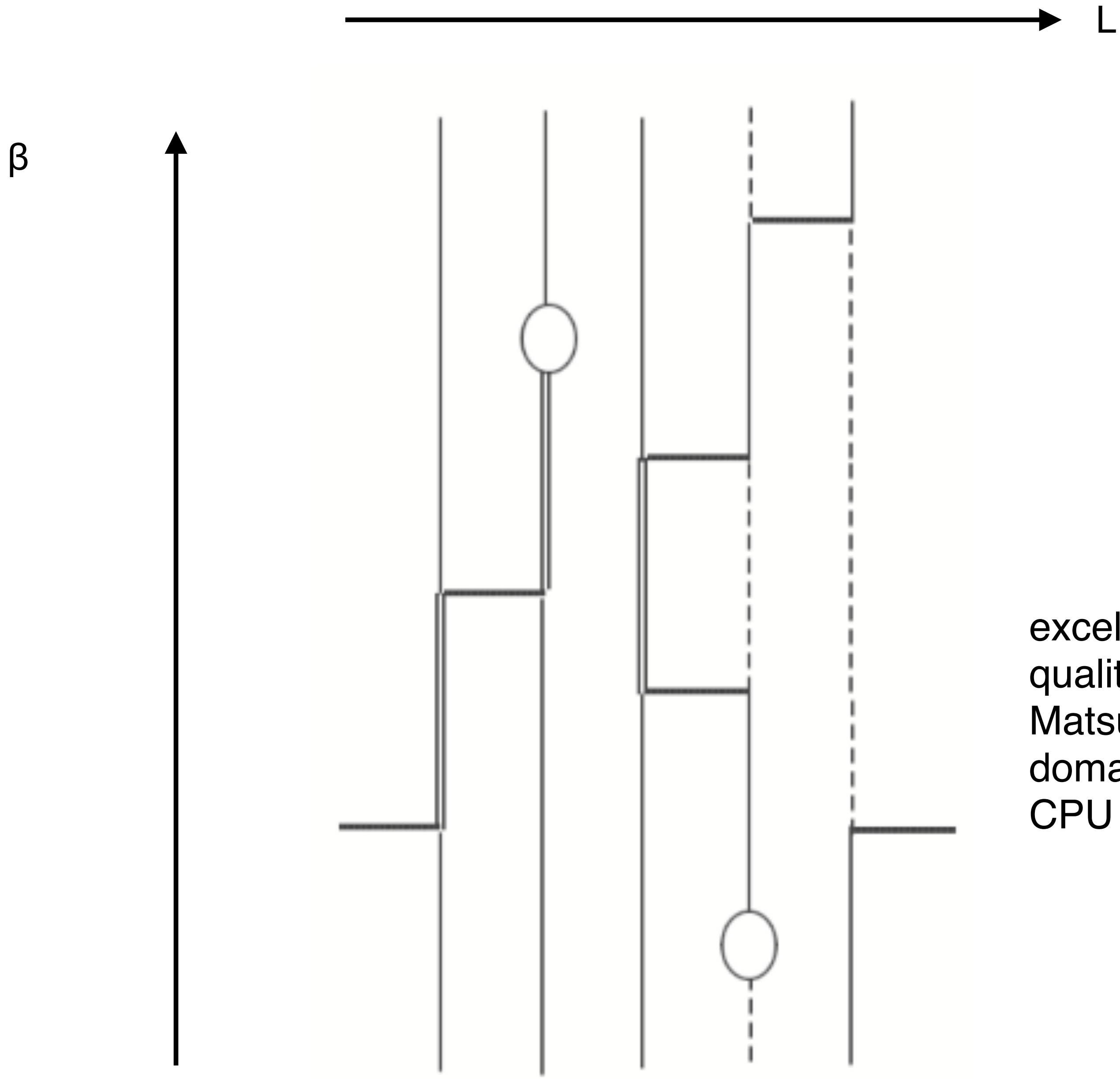
see Ref in J. Smakov, E. Sorensen, Phys. Rev. Lett. **95**, 180603 (2005)

Numerical simulations should however be able to see the scaling if

first $L \to \infty$

then $T \to 0$ at fixed ω/T are done.

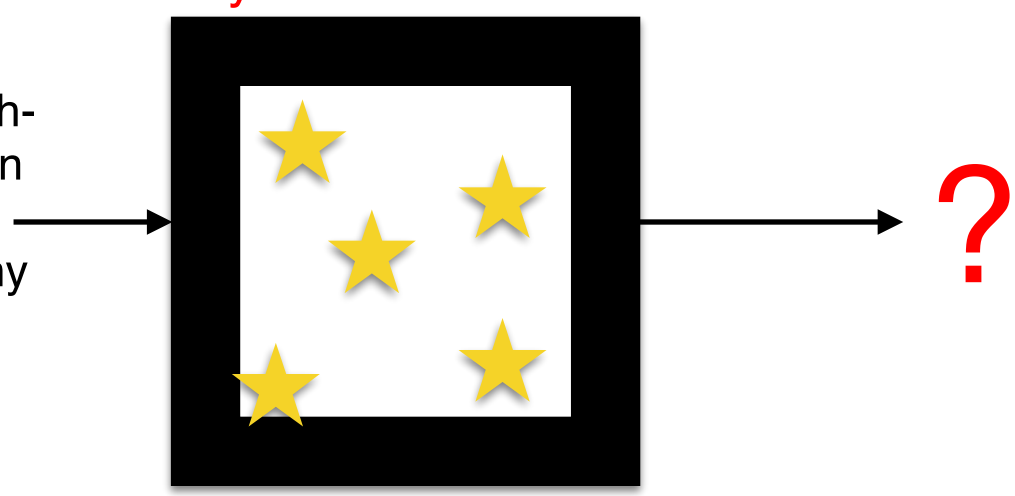
Path integral representation



sum over all kink-kink pairs
with appropriate factors
when there are no worms

Analytical continuation

excellent high-quality data in Matsubara domain; many CPU hours



$$K = -t\sum_{\langle i,j\rangle} b_i^\dagger b_j$$

note: for amplitude mode: $\chi_{KK}(i\omega_n) = \langle K(\tau)K(0)\rangle_{i\omega_n} + \langle K\rangle$

note: for conductivity:

$$\Lambda_{xx}(i\omega_n) = \frac{1}{\beta V}\langle\left(\sum_r j_x(r,i\omega_n)\right)\left(\sum_{r'} j_x(r',i\omega_n)\right)\rangle_{MC} - \langle k_x\rangle_{MC}$$

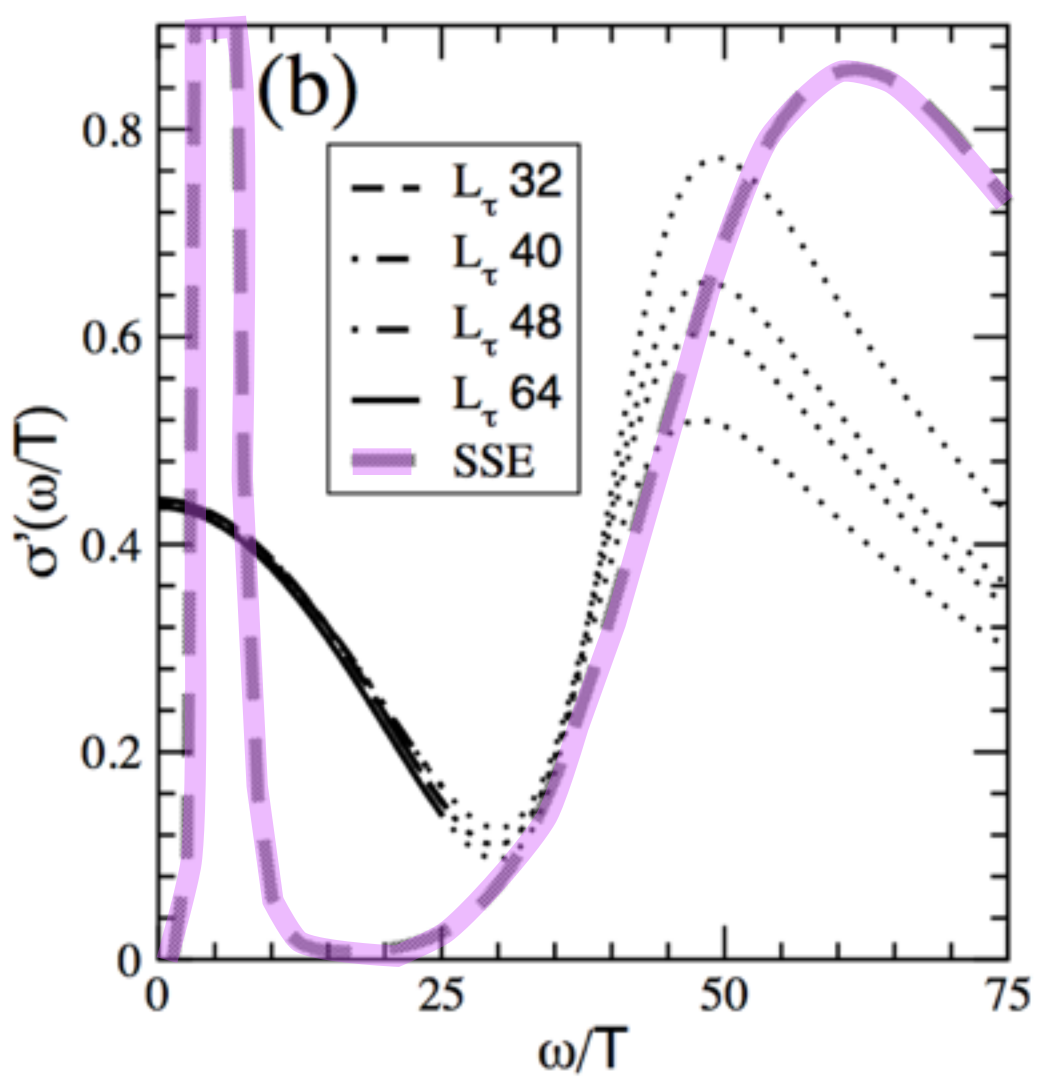
conductivity

one can also study a classical model with the same universal behavior (Adv: larger system sizes):

$$\mathcal{H}_V = \frac{1}{K}\sum_{(\mathbf{r},\tau)}\left[\frac{1}{2}\mathbf{J}^2_{(\mathbf{r},\tau)} - \mu J^\tau_{(\mathbf{r},\tau)}\right]$$

J. Smakov, E. Sorensen, Phys. Rev. Lett. **95**, 180603 (2005)

$$\sigma(i\omega_n) = 2\pi\sigma_Q \frac{\rho_s(i\omega_n)}{\omega_n}$$



2 extrapolations:

first $L \to \infty$

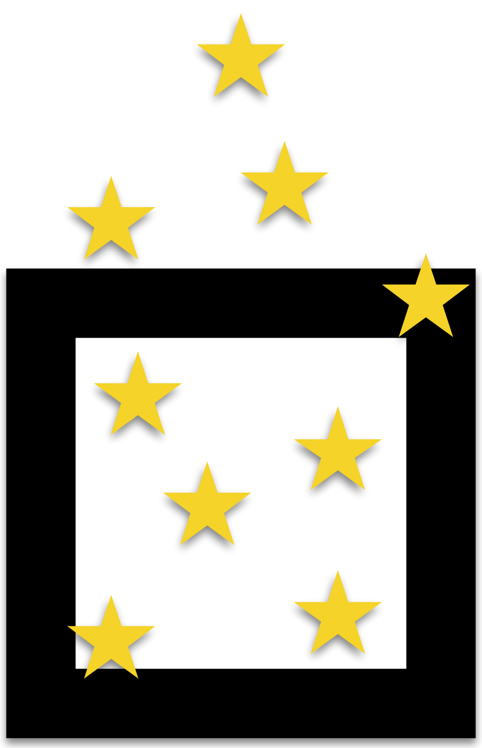
then $T \to 0$

other calculations: Fazio, Podolsky, …

bottom-to-top approach to AdS/CFT: can string theory be of help to condensed matter theory?

Back to reality? (J. Basheen)

AdS/CFT correspondence: transport properties in a strongly correlated system may be easier to compute in a gravity theory in a higher dimension



after holographic 'mapping' : 2 degrees of freedom $\gamma, \Sigma(\infty)$

$$|\gamma| \leq 1/12$$

J. M. Maldacena, Adv. Theor. Math. Phys. 2,231-252 (1998).
S. Sachdev, Annu. Rev. Condensed Matter Phys. 3,9 (2012).
A. Adams, L. D. Carr, T. Schaefer, P. Steinberg and J. E. Thomas, New J. Phys. 14, 115009 (2012).

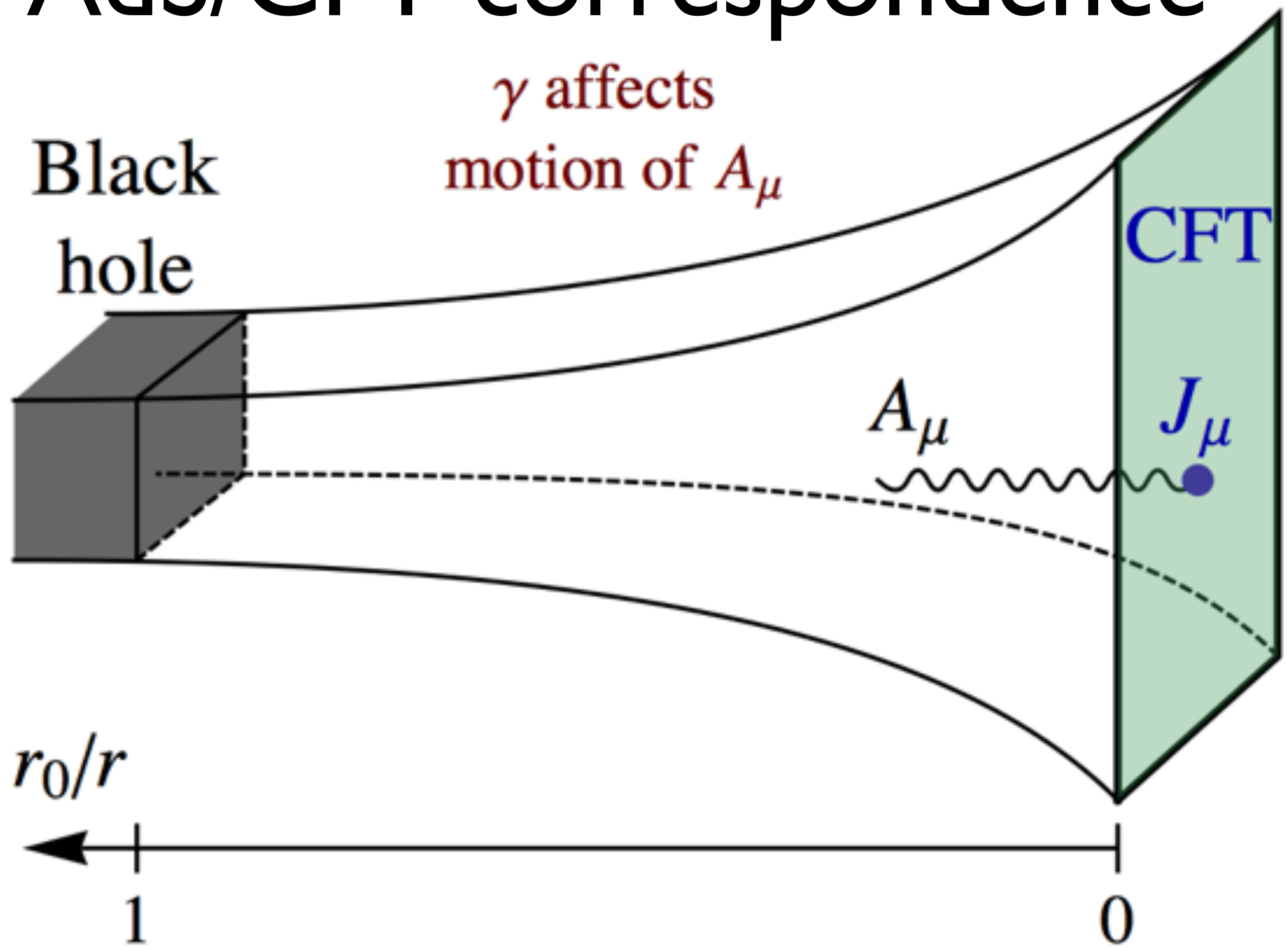
Quoted from Wikipedia:

So far some success has been achieved in using string theory methods to describe the transition of a superfluid to an insulator. A superfluid is a system of electrically neutral atoms that flows without any friction. Such systems are often produced in the laboratory using liquid helium, but recently experimentalists have developed new ways of producing artificial superfluids by pouring trillions of cold atoms into a lattice of criss-crossing lasers. These atoms initially behave as a superfluid, but as experimentalists increase the intensity of the lasers, they become less mobile and then suddenly transition to an insulating state. During the transition, the atoms behave in an unusual way. For example, the atoms slow to a halt at a rate that depends on the temperature and on Planck's constant, the fundamental parameter of quantum mechanics, which does not enter into the description of the other phases. This behavior has recently been understood by considering a dual description where properties of the fluid are described in terms of a higher dimensional black hole.[48]

S. Sachdev, Scientific American 2013
S. Sachdev, J. Basheen, J. McGreevy, lecture notes, AdS/CFT school Aug @LMU (ASC), 2013

AdS/CFT correspondence

W. Witczak-Krempa, E. Sørensen and S. Sachdev, arXiv:1309.2941 (2013)



note for experts:
- not clear that O(2) has a gravity dual
- neglects terms non-quadratic in F (why???)
- temperature is set by the planar black hole
- classical gravity requires the radius L of AdS4 to be much larger than effective Planck scale, usually done by l = g/N^2, with g going to 0 ('t Hooft) and N to infinity with l const, and finally letting l going to infinity — quite different from O(2). (PS N is not 2 here!!!)

metric:

$$ds^2 = \frac{r^2}{L^2}\left[-f(r)dt^2 + dx^2 + dy^2\right] + \frac{L^2 dr^2}{r^2 f(r)} \qquad f(r) = 1 - r_0^3/r^3$$

action:

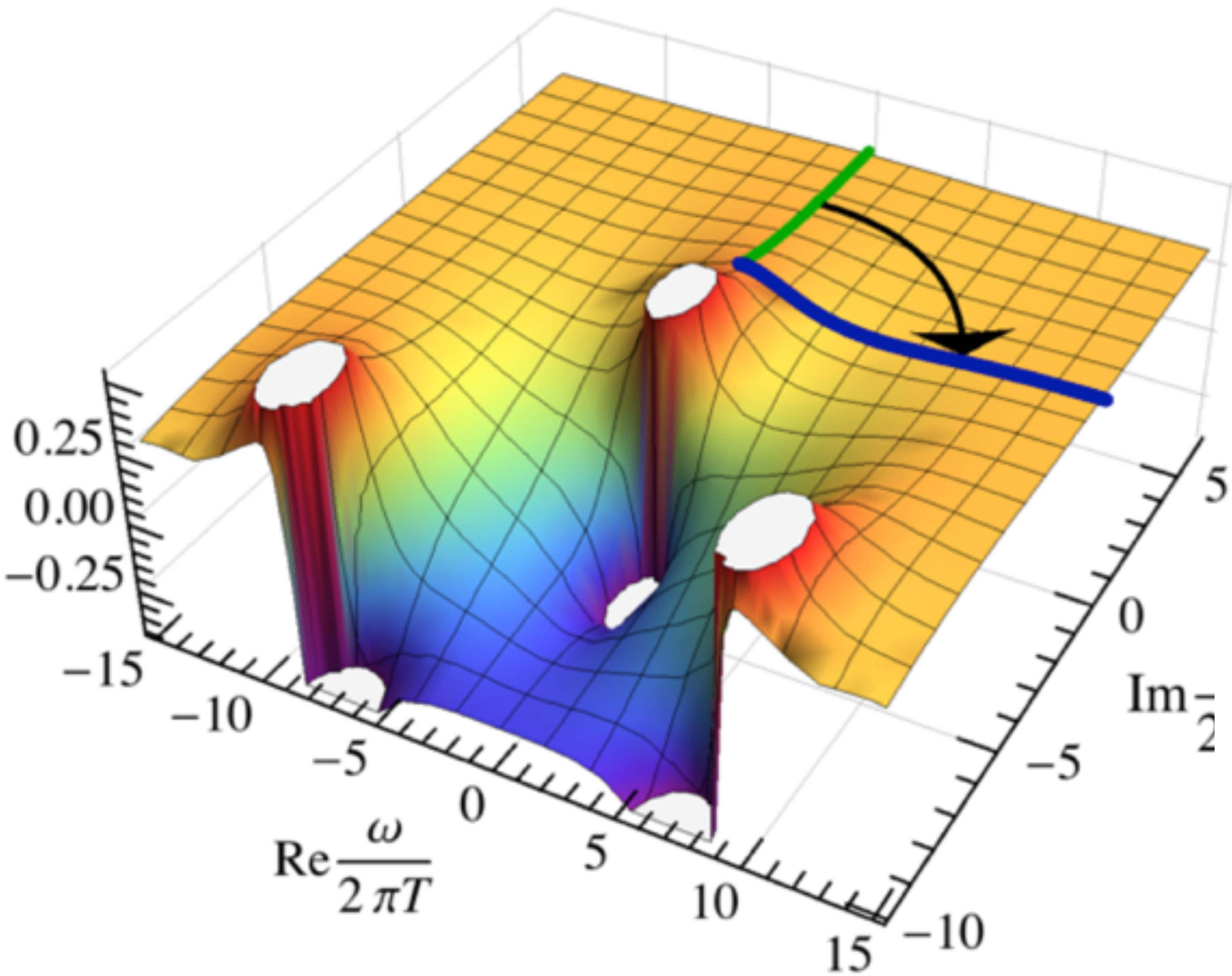
$$S_{\rm bulk} = \int d^4x\sqrt{-g}\left(-\frac{1}{4g_4^2}F_{ab}F^{ab} + \gamma\frac{L^2}{g_4^2}C_{abcd}F^{ab}F^{cd}\right)$$

with $F_{ab} = \partial_a A_b - \partial_b A_a$

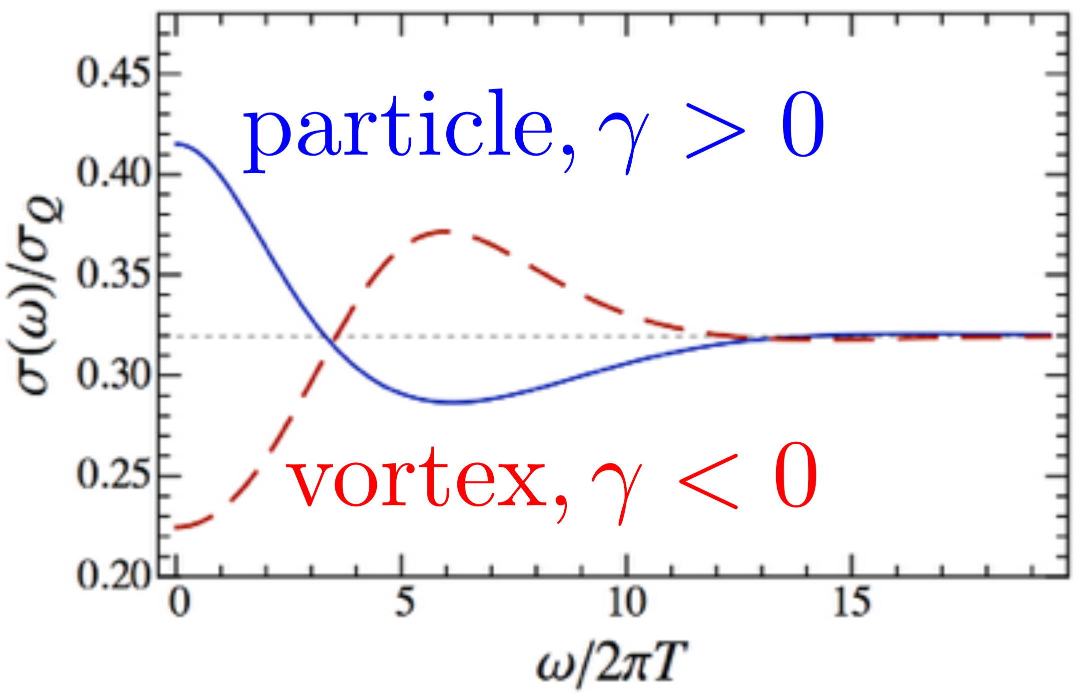
conductivity formula:

$$\sigma(\omega/T) = \frac{ir^2}{g_4^2 L^2\omega}\frac{\partial_r A_y(\omega,r)}{A_y(\omega,r)}\bigg|_{r=\infty}$$

A hand from string theory?

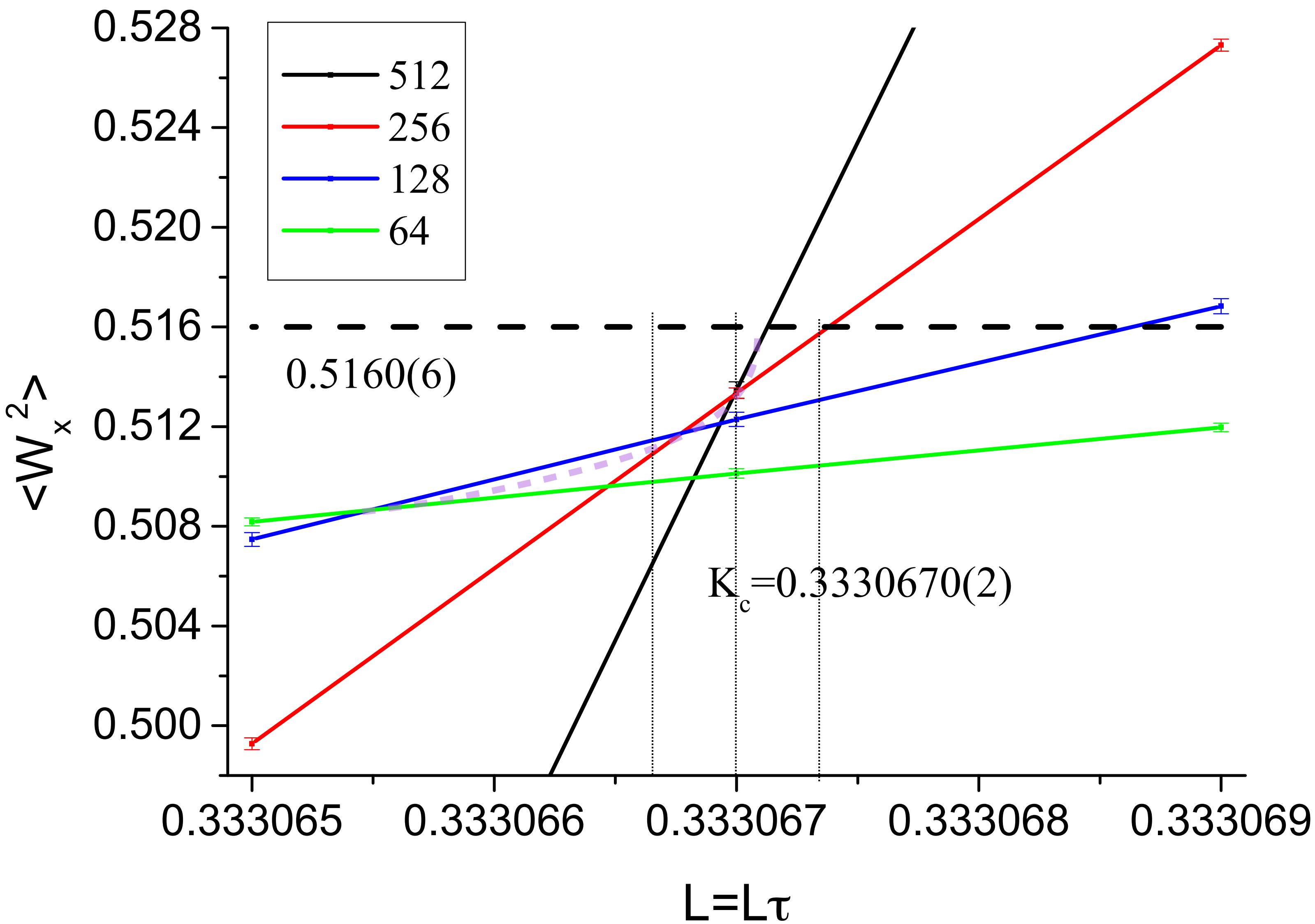


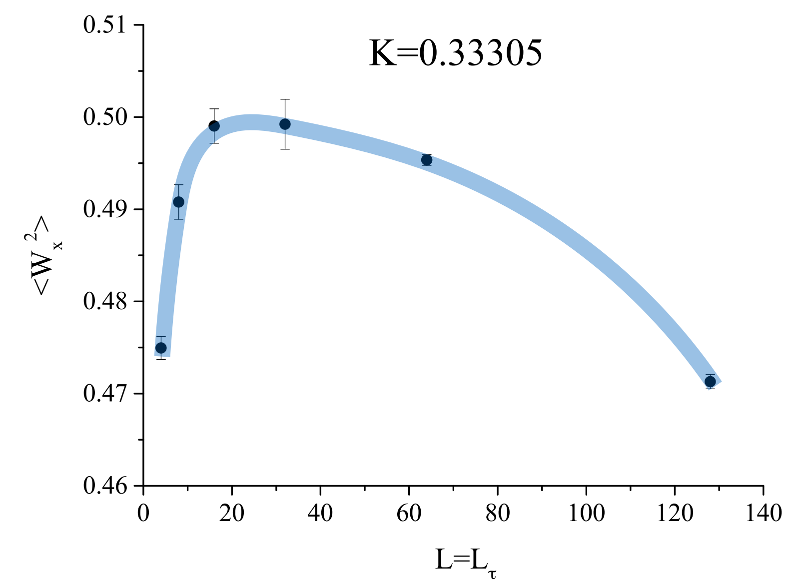
holographic continuation

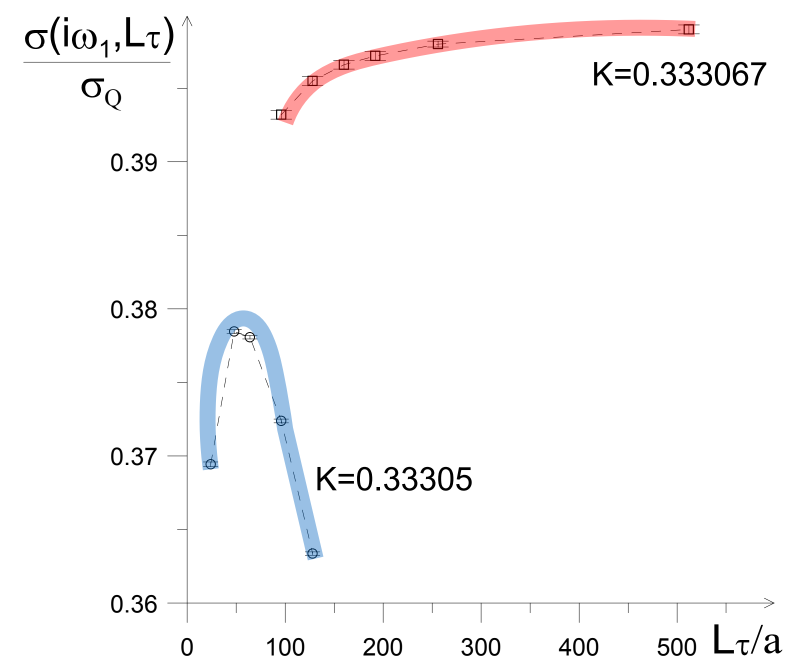


W. Witczak-Krempa, E. Sørensen and S. Sachdev, arXiv:1309.2941 (2013) Nat Phys (2014)

Surprise: a more precise determination of the critical point was needed (Villain model)





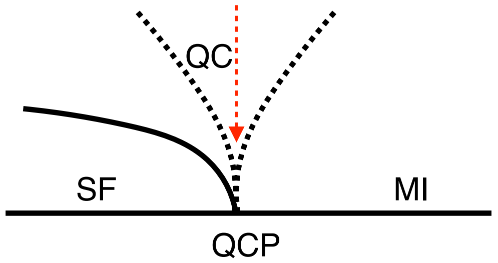


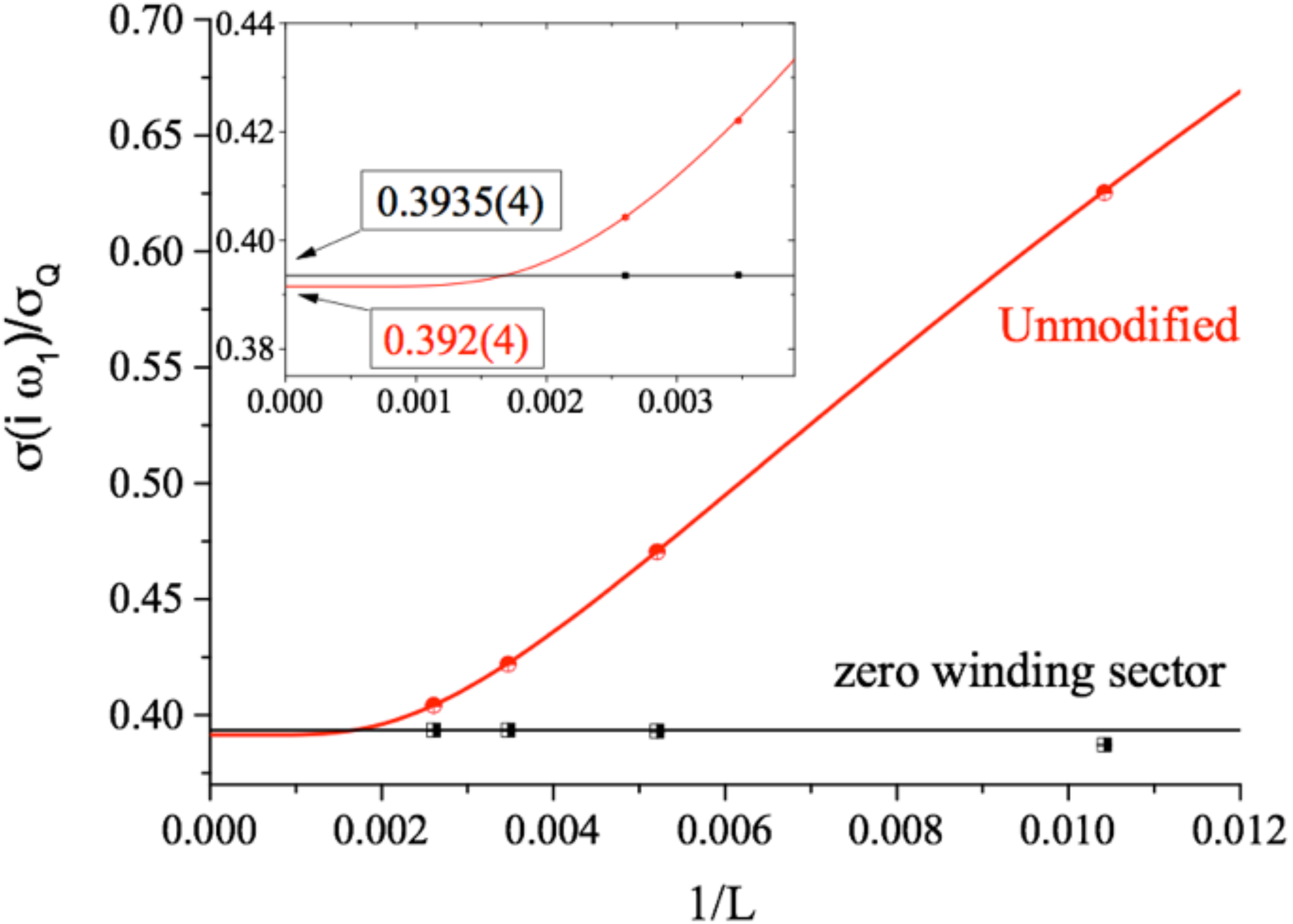
fit: $$a + b(K - K_c)L^{1/\nu} + cL^{-\omega}$$

$a = 0.5160(6)$

$\nu = 0.6717(3)$

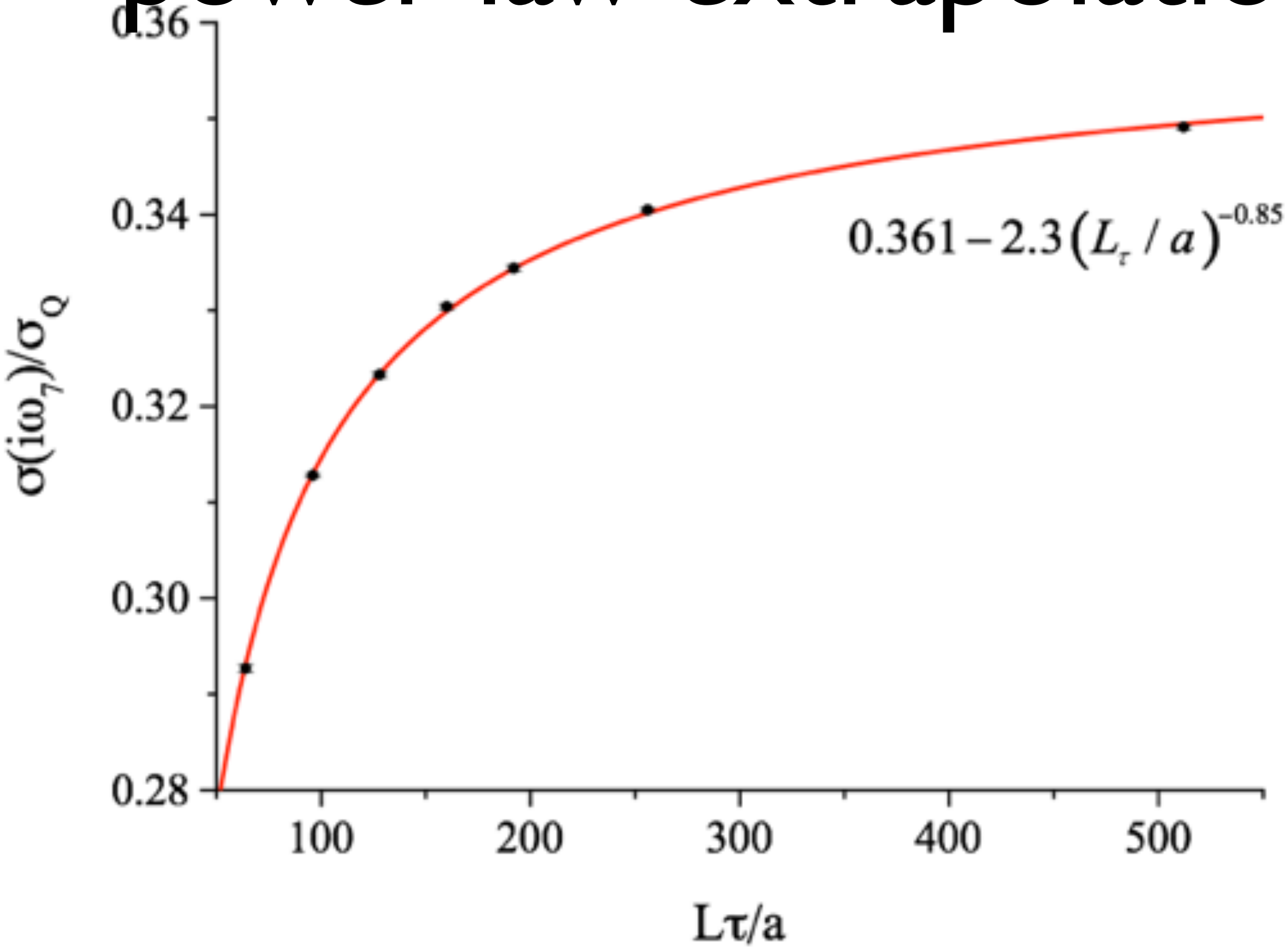
Trick: only collect data in zero winding sector





fixed temperature

power law extrapolation in temperature



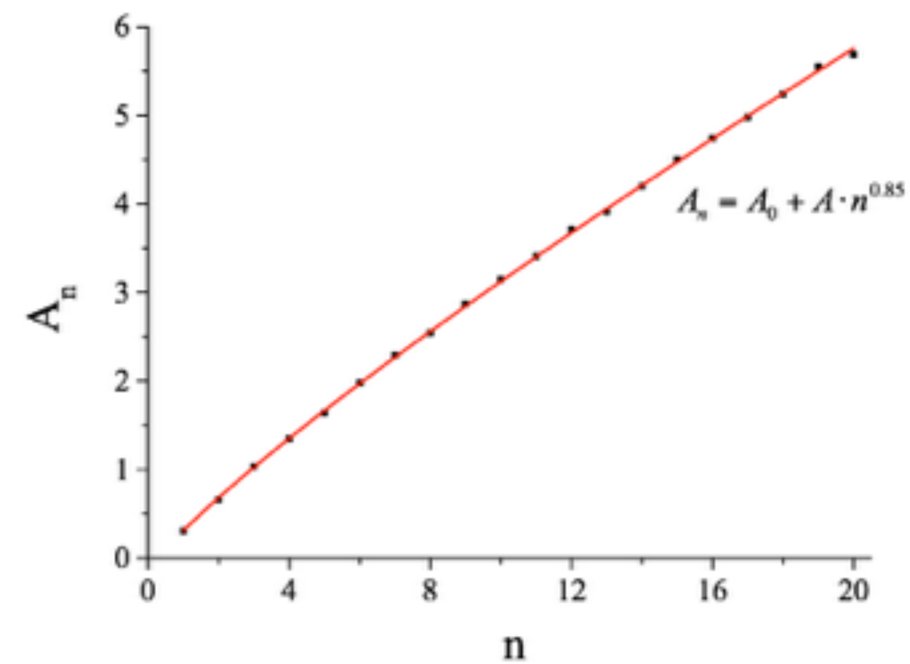


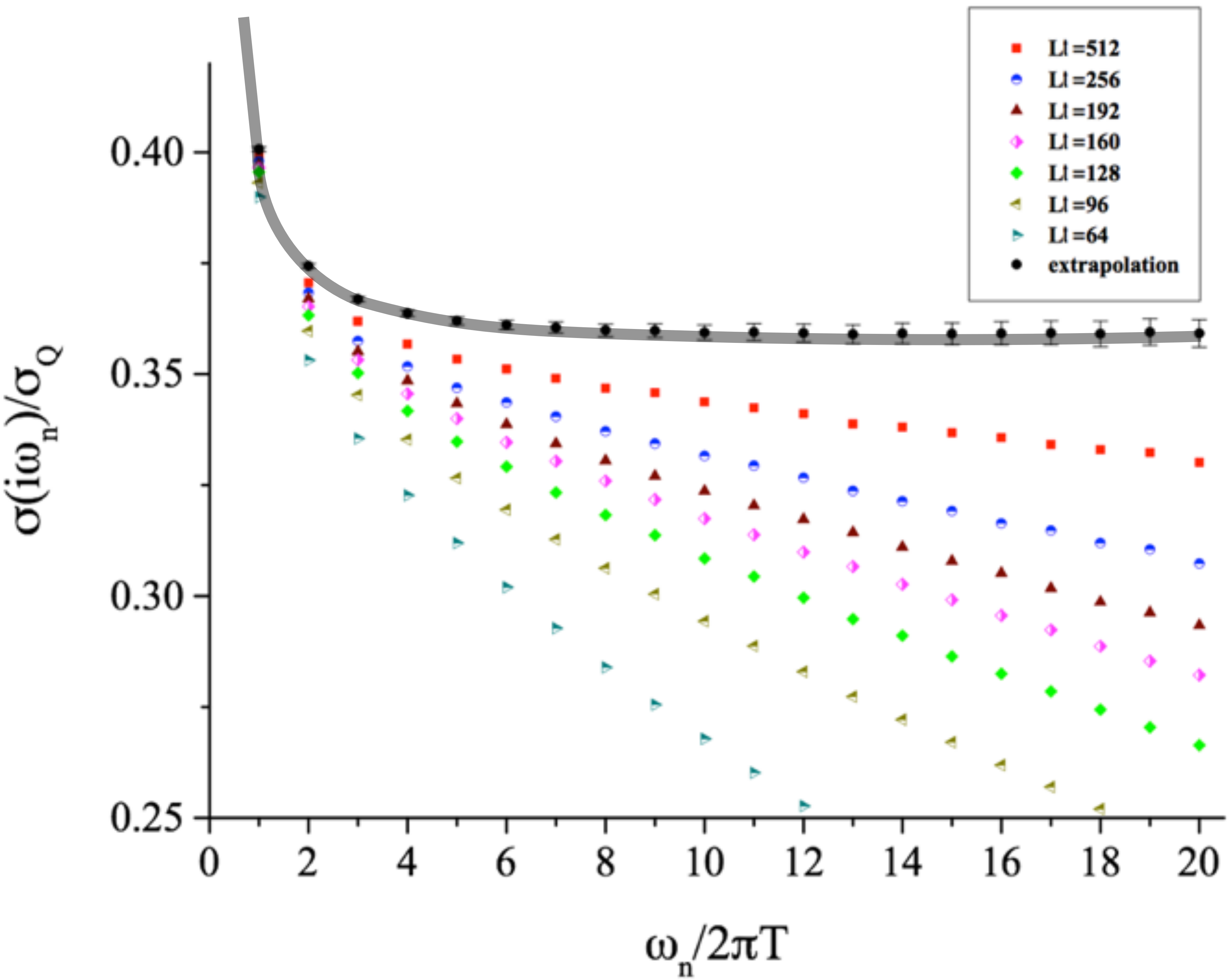
FIG. 9. A_n amplitudes deduced by fitting each Matsubara harmonic independently, see Fig. 8.

FIG. 8. Extrapolating the conductivity in Matsubara representation to the universal zero-temperature limit for fixed $\omega_n/2\pi T = 7$.

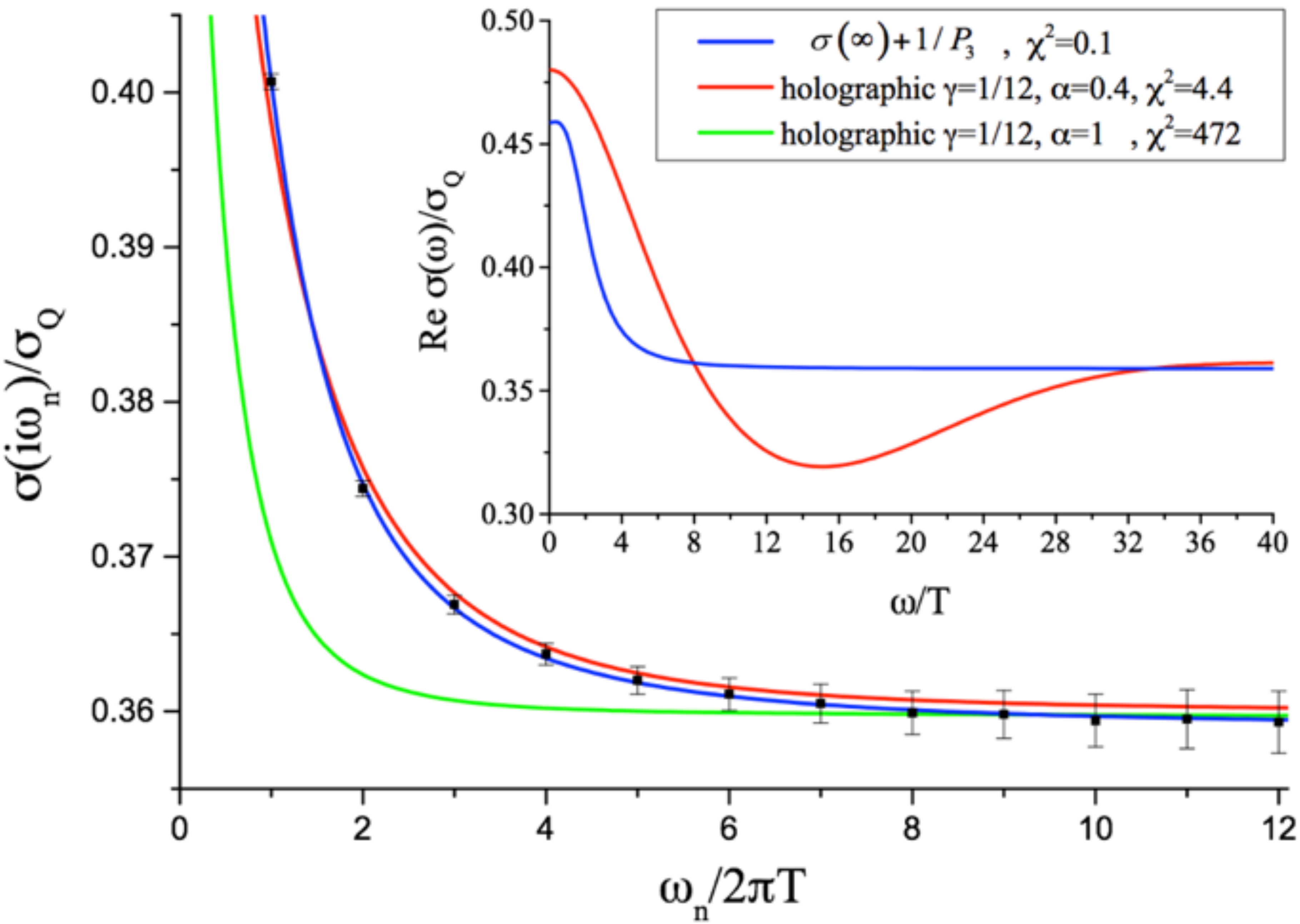
$$\sigma(i\omega_n/T, L_\tau) = \sigma(i\omega_n/T, \infty) + A_n/L_\tau^\omega$$

field-theoretically: $\omega = 0.80(2)$ we kept fixed: $\omega = 0.85$

final data in Matsubara space for different temperatures (Villain model)



analytical continuation



transport is particle-like

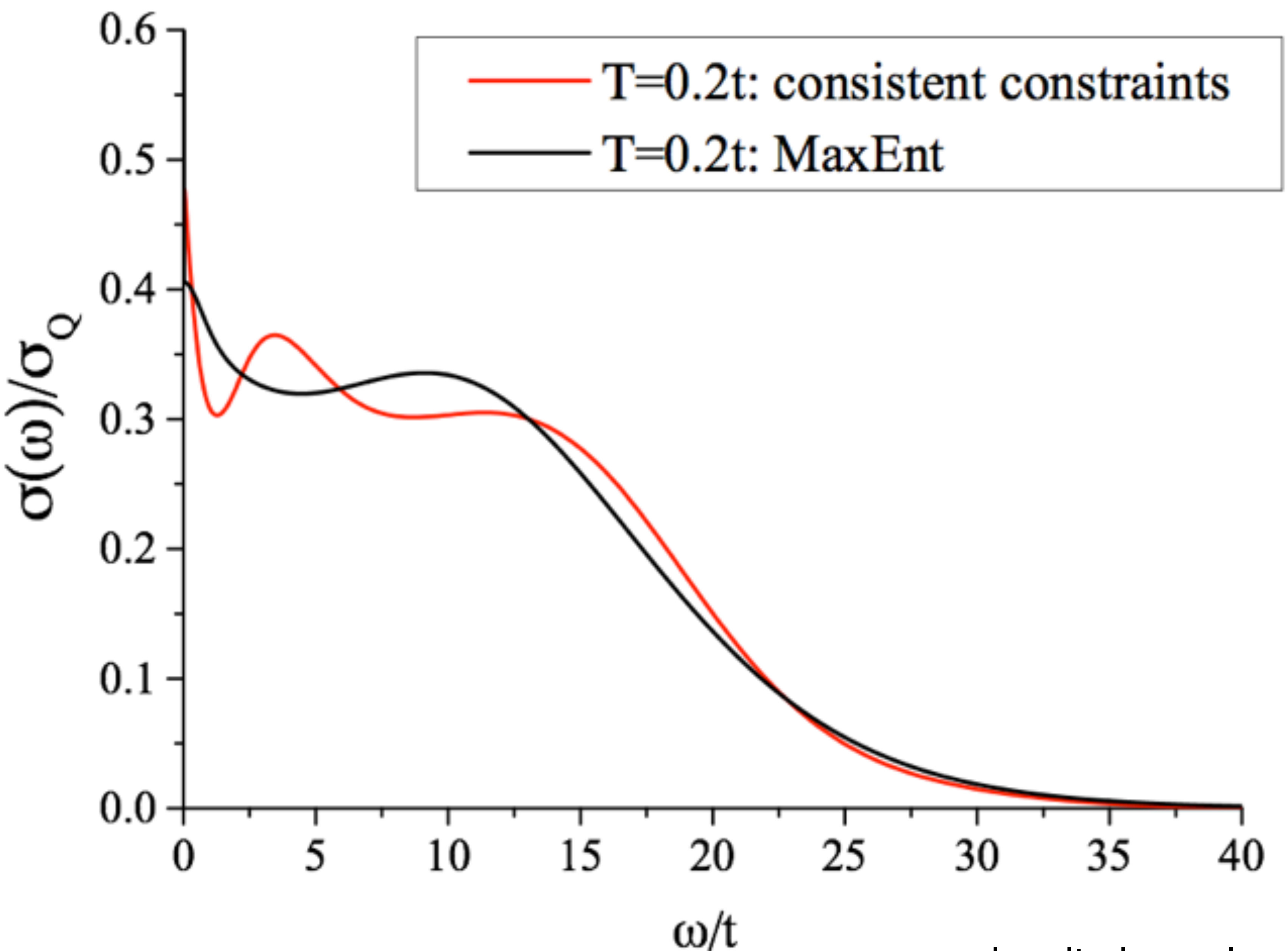
without rescaling of the temperature, the holographic fit does not work.

with rescaling of the temperature, the fit is reasonable

W. Witczak-Krempa, E. Sørensen and S. Sachdev, arXiv:1309.2941 (2013) Nat Phys 2014

„quantum renormalization for low gauge groups"

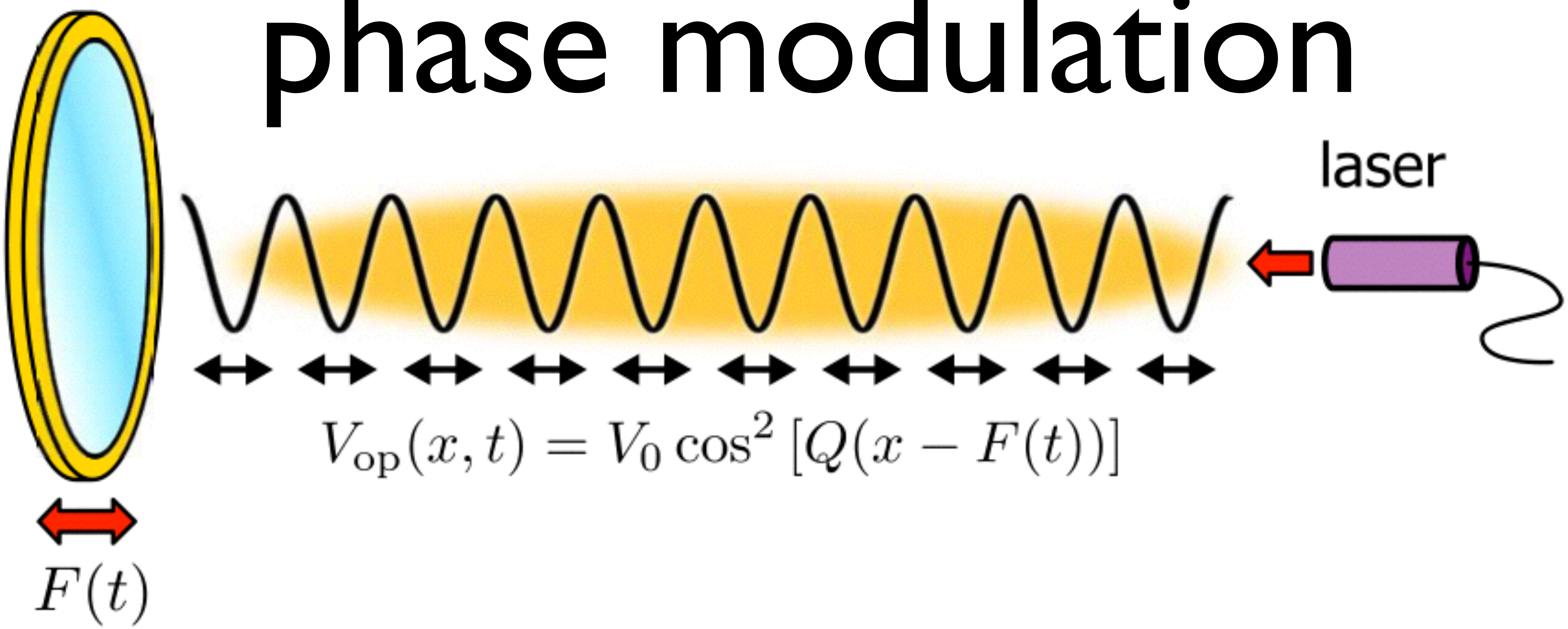
analytical continuation of the quantum model at finite temperature



without analytical theory, the analytical continuation seems unreliable. However, the plateau at high frequencies may be observed in a present-day cold atom experiment

despite long simulation ($$$) and high quality data in Matsubara domain

phase modulation



$$H(t) = H_0 - M\dot{\boldsymbol{F}}(t)\cdot\boldsymbol{J}$$

$$\boldsymbol{A}_{\rm ext} = \dot{\boldsymbol{F}}(t)$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{A}_{\rm ext} = 0$$

$$\boldsymbol{E}_{\rm ext}(t) = -\ddot{\boldsymbol{F}}(t)$$

$$\boldsymbol{F}(t) = \boldsymbol{f}\cos(\omega t)$$

A. Tokuno and T. Giamarchi, Phys. Rev. Lett. **106**, 205301 (2011)

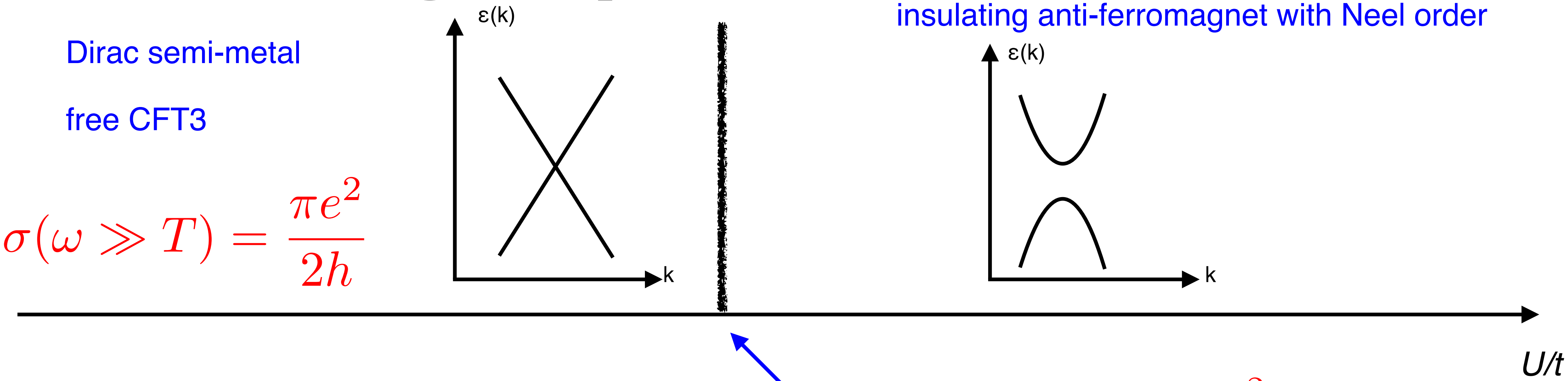
graphene

Dirac semi-metal

free CFT3

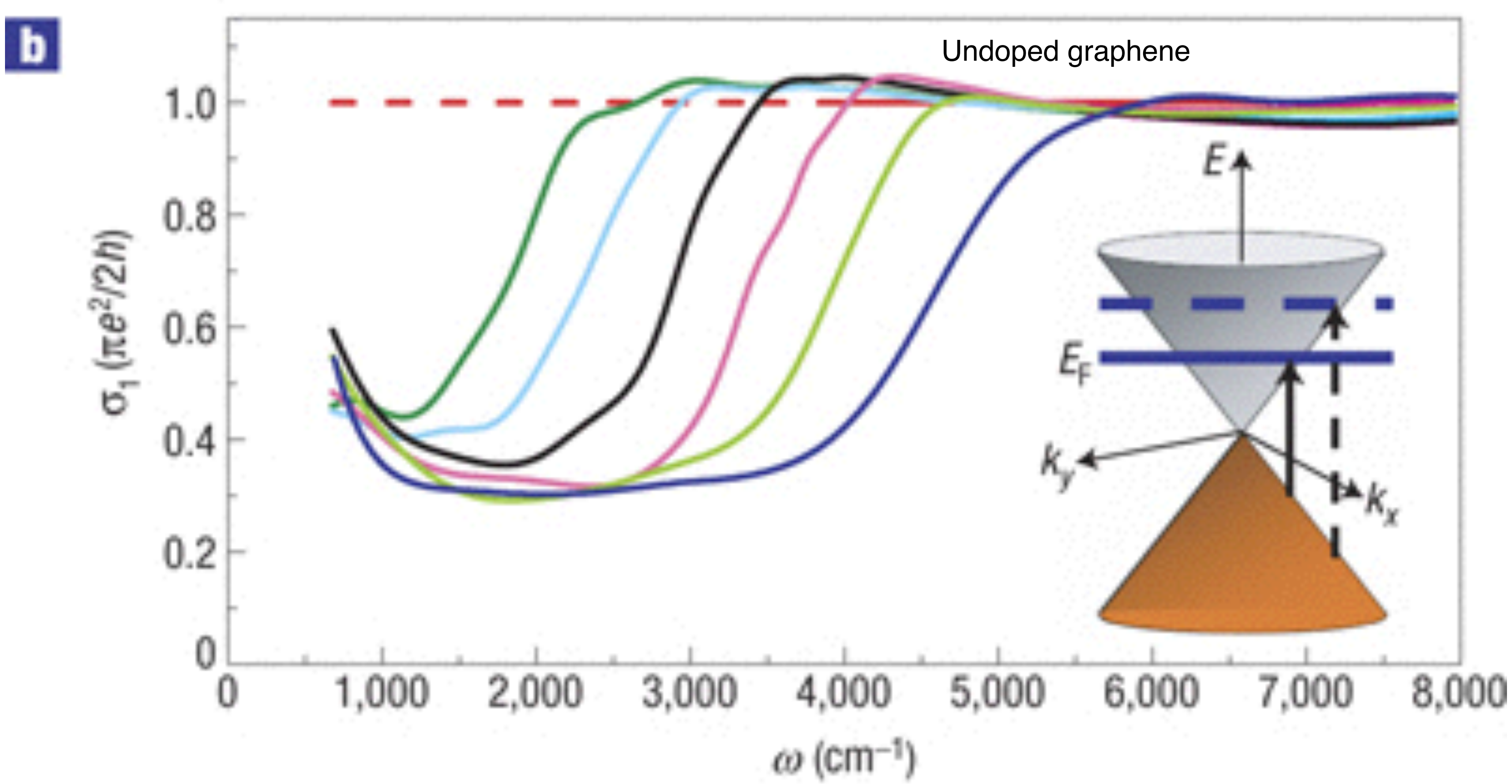
$$\sigma(\omega \gg T) = \frac{\pi e^2}{2h}$$

insulating anti-ferromagnet with Neel order



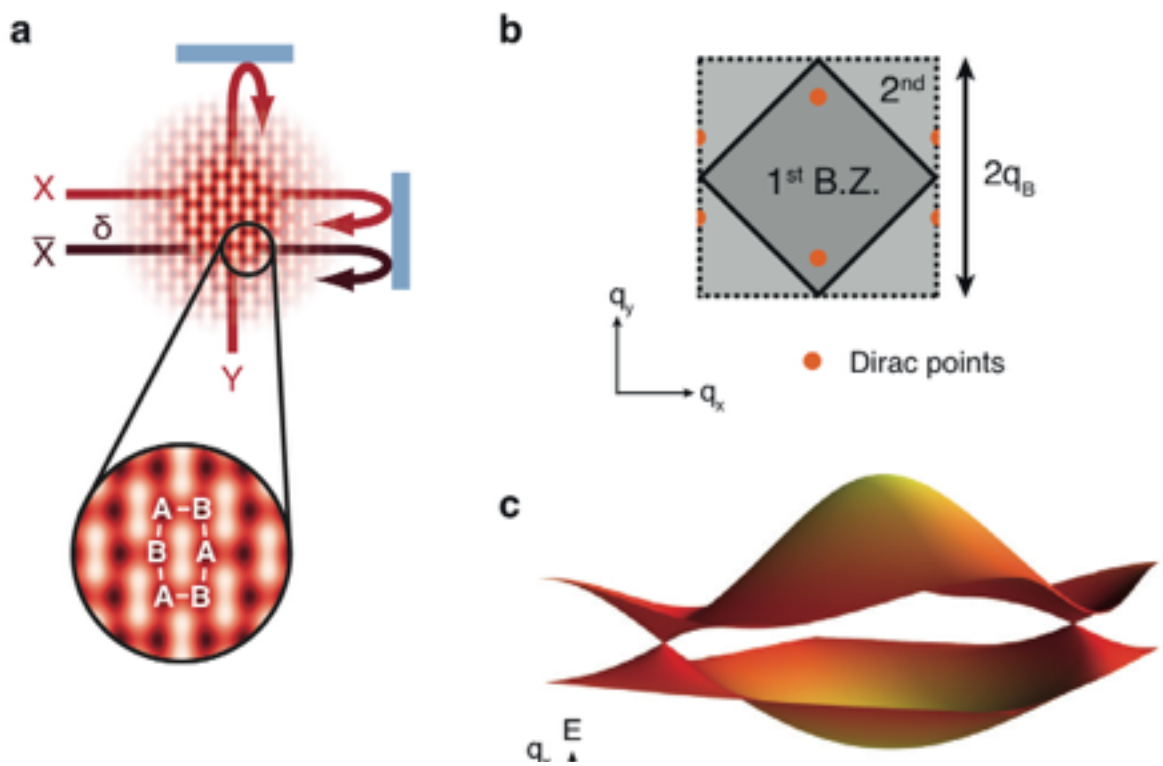
interacting CFT3
(Gross-Neveu criticality)

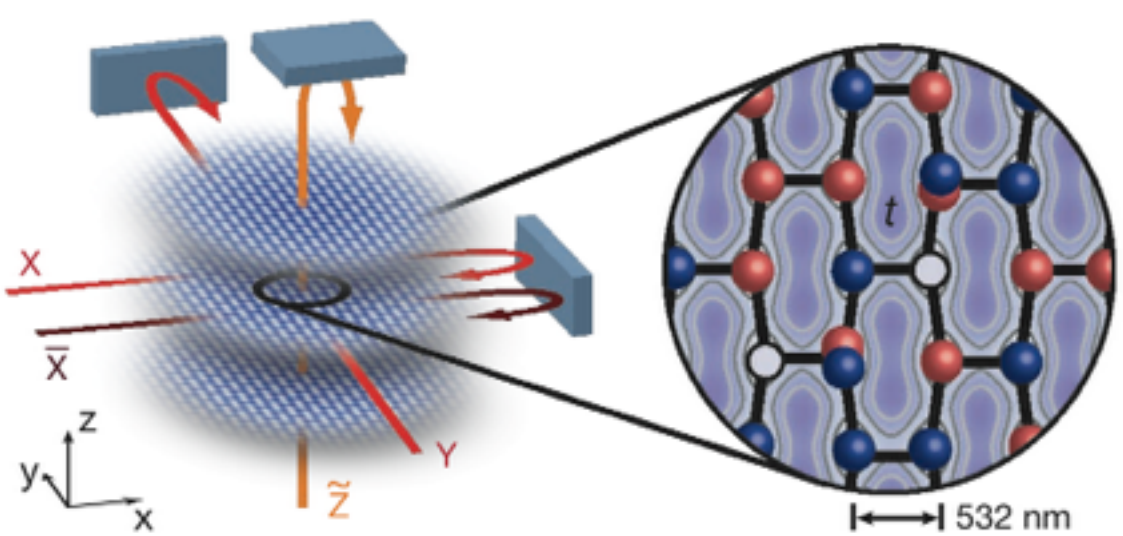
$$\sigma(\omega \gg T) = \frac{\mathcal{K} e^2}{2h}$$

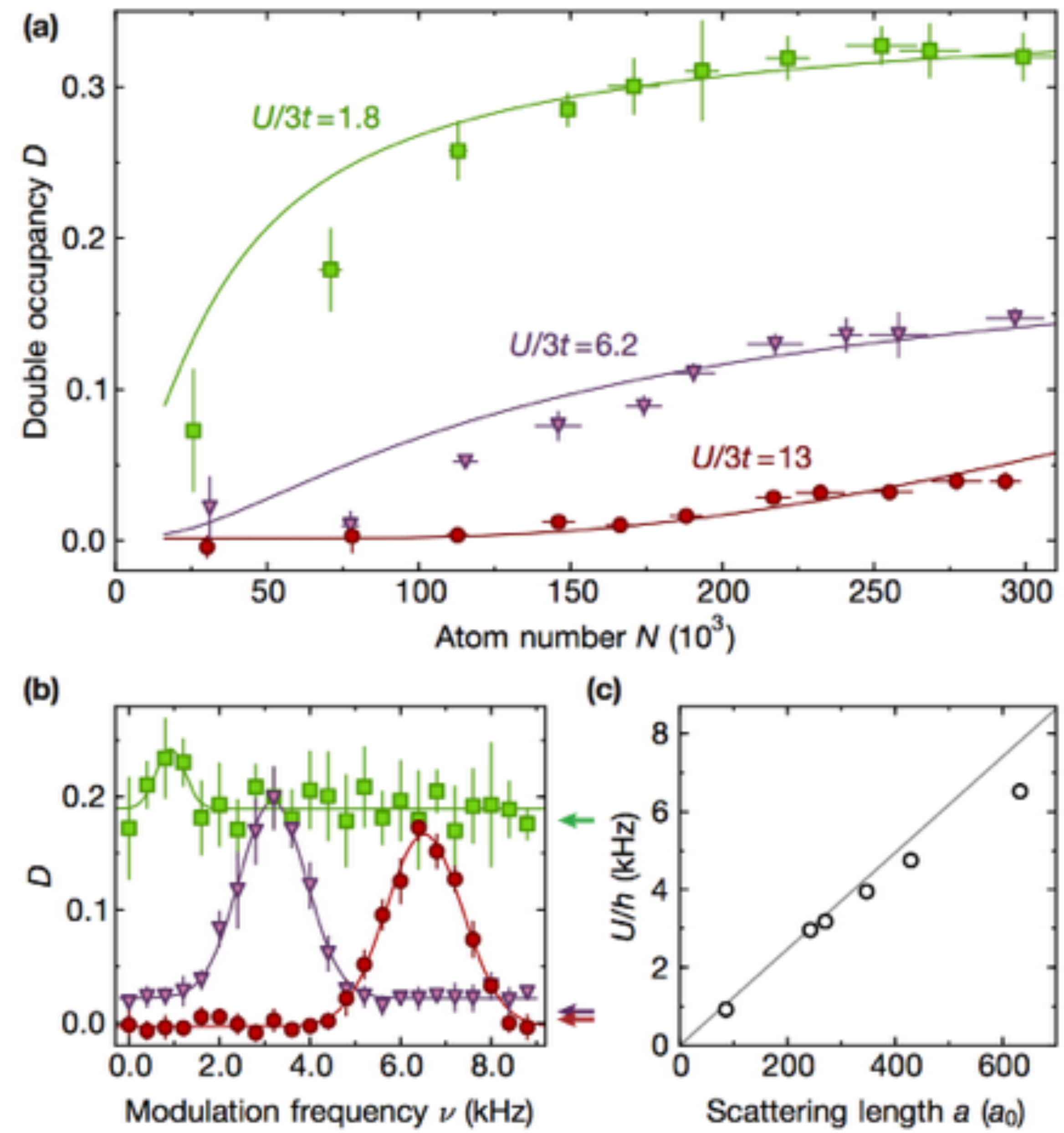


Z. Q. Li, E. A. Henriksen, Z. Jiang, Z. Hao, M. C. Martin, P. Kim, H. L. Stormer, and D. N. Basov, Nat. Phys. **4**, 532 (2008)

artificial graphene







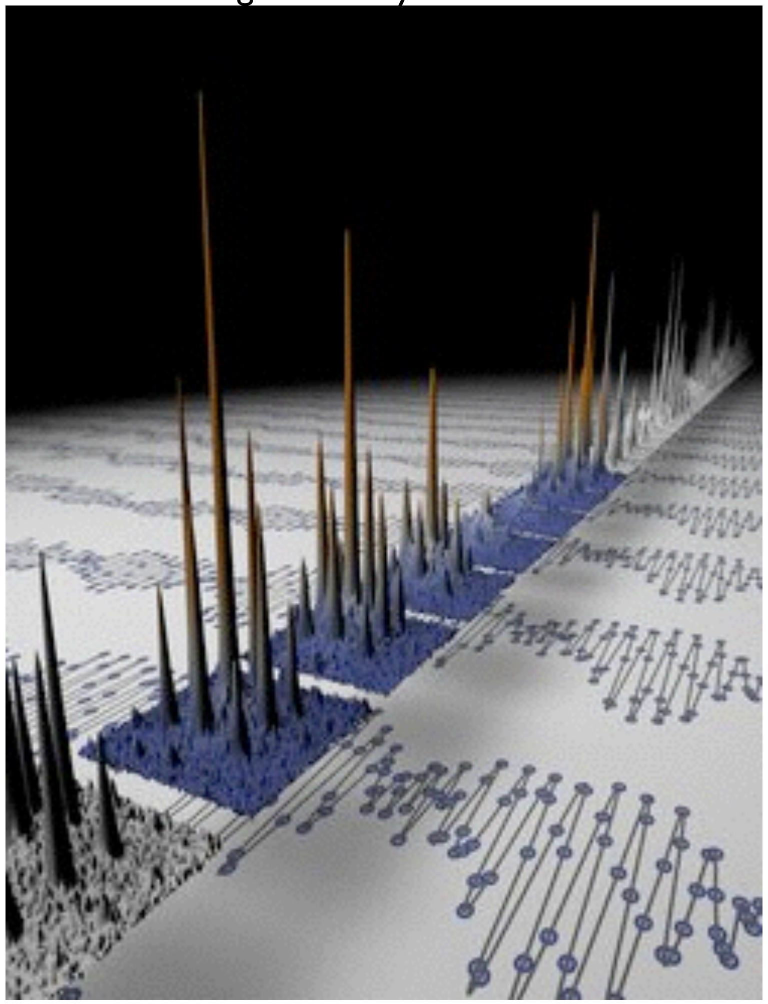
L. Tarruell, D. Greif, T. Uehlinger, G. Jotzu, and T. Esslinger, Nature **483**, 302 (2012)
T. Uehlinger, G. Jotzu, M. Messer, D. Greif, W. Hofstetter, U. Bissbort, and T. Esslinger, Phys. Rev. Lett. 111, 185307 (2013)

conclusion and future work

- identifiable Higgs mode in 2d; conductivity accurately computed
- further experiments are needed though challenging
- holographic duality may work after ‘rescaling’ temperature, but why???
- what about (artificial) graphene (Gross-Neveu criticality)?
- what about 1d?
- disorder? (cf M. Swanson, N. Trivedi et al. arXiv:1310.1073)

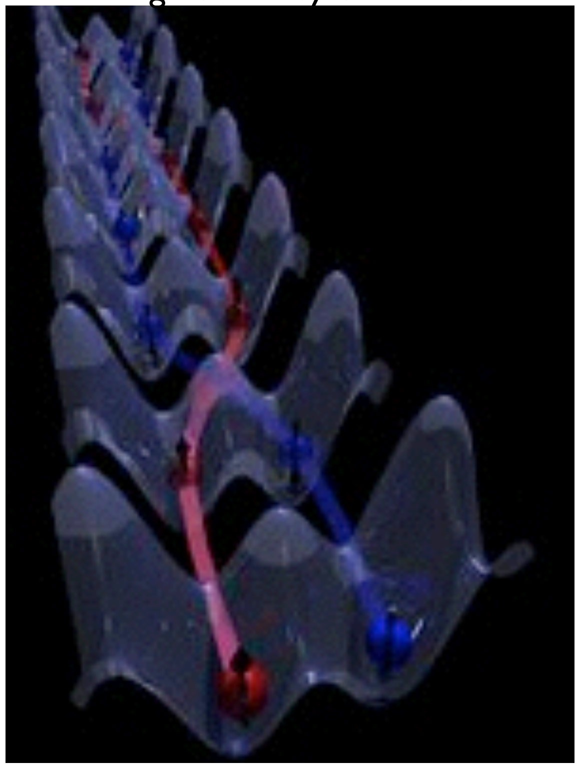
Special thanks:
Kun Chen, Longxiang Liu, Youjin Deng, Nikolay Prokof’ev
W. Witczak-Krempa. E. Sorensen, S. Sachdev, D. Pekker, M. Endres, I. Bloch
W. Zwerger

image courtesy of I. Bloch



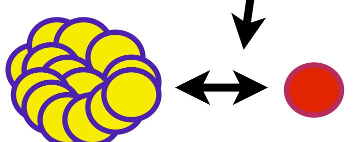
quantum Monte Carlo

image courtesy of I. Bloch

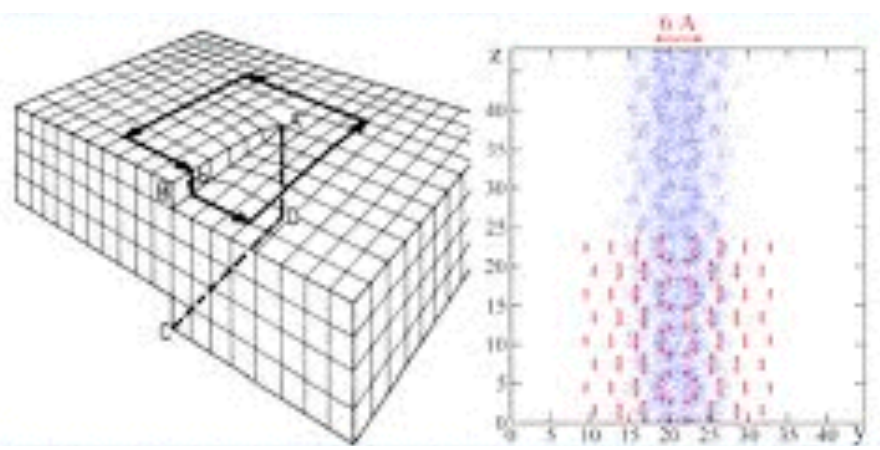


towards pseudo-gap phase in the Hubbard model

towards magnetism for spinor bosons

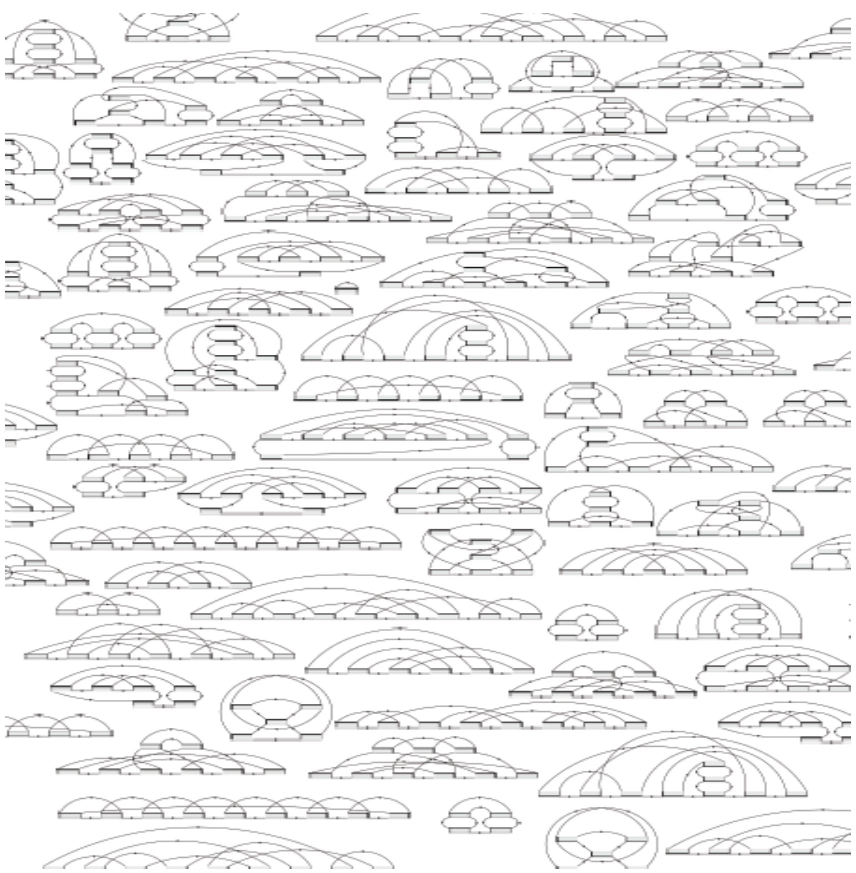


polarons and impurities



towards simulations of systems with long-range interactions

further developments in diagrammatic Monte Carlo



Anderson-Higgs mechanism

PHYSICAL REVIEW VOLUME 130, NUMBER 1 1 APRIL 1963

Plasmons, Gauge Invariance, and Mass

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received 8 November 1962)

VOLUME 13, NUMBER 16 PHYSICAL REVIEW LETTERS 19 OCTOBER 1964

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
(Received 31 August 1964)

"...The purpose of the present note is to report that, as a consequence of this coupling, the spin-one quanta of some of the gauge fields acquire mass; the longitudinal degrees of freedom of these particles (which would be absent if their mass were zero) go over into the Goldstone bosons when the coupling tends to zero. This phenomenon is just the relativistic analog of the plasmon phenomenon to which Anderson has drawn attention: that the scalar zero-mass excitations of a superconducting neutral Fermi gas become longitudinal plasmon modes of finite mass when the gas is charged."